

## 1 Contest

## 2 Mathematics

## 3 Data structures

## 4 Numerical

## 5 Number theory

## 6 Combinatorial

## 7 Graph

## 8 Geometry

## 9 Strings

## 10 Various

# Contest (1)

### template.cpp

25 lines

```
#define range(v, i, j) v.begin() + i, v.begin() + j + 1
#define rep(i, a, b) for(int i = a; i < (b); ++i)
#define sz(x) (long long)(x).size()
#define R(i, a, b) for(long long i = (a); i >= (b); --(i))
#define extract(m, x) {auto it=m.find(x); if(it != m.end()) m.erase(it);}
#define isSet(x, i) ((x>>i)&1)
#define setbit(x, i) (x | (1LL<<i))
#define resetbit(x, i) (x & (~(1LL << i)))
#define toggleBit(x, i) ((x) ^ (1LL << (i)))
#define clz(x) __builtin_clzll(x)
#define ctz(x) __builtin_ctzll(x)
#define csb(x) __builtin_popcountll(x)
#define msb(x) ((x) ? (63 - __builtin_clzll((1ll)(x)) ) : -1)
#define lsb(x) ((x) ? (__builtin_ctzll((1ll)(x)) ) : -1)

#ifndef LOCAL
#include "debug.h"
#else
#define deb(...) (void)0
#endif
int main() {
    ios_base::sync_with_stdio(false); cin.tie(NULL);
    prep(); int tcase = 1;
    int t; cin >> t; for(; tcase <= t; ++tcase)
        solve(tcase); }
```

### setup

7 lines

```
mkdir -p ~/bin
code ~/bin/d
chmod +x ~/bin/d
vim ~/.bashrc
add line : export PATH="$HOME/bin:$PATH" [save and exit :wq]
source ~/.bashrc
for i in {A..J}; do cp -r template_folder $i; done;
```

1 d.sh  
11 lines

```
#!/bin/bash
g++ -std=c++23 -DLOCAL -Wshadow -Wall -g -fsanitize=address \
-fsanitize=undefined -D_GLIBCXX_DEBUG -fmax-errors=2 code.cpp \
-o a.out || { echo "Compilation failed!"; exit 1; }
echo
echo Input:
cat $1
echo
echo Output:
./a.out < $1
echo
```

10 f.sh  
1 lines

```
g++ -std=c++17 -O2 -Wno-unused-result code.cpp -o a.out
```

11 run.sh  
11 lines

```
#!/bin/bash
for inp in ${in[0-9]*}; do
    if [ -f $inp ]; then
        echo
        echo "Input: $inp"
        cat $inp
        echo Output:
        ./a.out < $inp
        echo
    fi
done
```

debug.h  
21 lines

```
#define deb(...) debug(__VA_ARGS__, __LINE__, __VA_ARGS__)
template<class T>
auto pr(T x) -> decltype(cout<<x, void()); {cout<<x;}
void pr(string s) {cout << '\'' << s << '\'';}
template<class T, size_t N>
void pr(array<T, N> a) {cout << "{"; for(size_t i=0; i<N; i++)
    {pr(a[i]); if(i+1 < N) cout << ", "; } cout << "}";}
template<class T, class U>
void pr(pair<T,U> x) {cout << "{"; pr(x.first);
    cout << ", "; pr(x.second); cout << "}";}
template<class T>
auto pr(T v) -> decltype(v.begin(), v.end(), void()){
    cout << "["; for(auto it = v.begin(); it != v.end(); )
        {pr(*it); if(++it != v.end()) cout << ", "; } cout << "]";}
#define eb emplace_back
template<class... A>
void debug(const char* s, int l, A... a){
    cout << l << "| "; vector<string> names; int i = 0;
    for(stringstream ss(s); getline(ss, names.eb(), ',' ); ){}
        (cout << " " << names[i] << ":" , pr(a), i++, ...);
    cout << "\n"; }
```

stress.sh  
16 lines

```
set -e
g++ gen.cpp -o gen
g++ brute.cpp -o bru
for((i = 1; ; ++i)); do
    ./gen $1 > tcase
    ./a.out < tcase > myans
    ./bru < tcase > corans
    diff -Z myans corans > /dev/null || break
    echo "Passed test: " $i
done
echo "WA on test:"
cat tcase
```

echo "Your answer:"  
cat myans  
echo "Correct answer:"  
cat corans

gen.cpp  
4 lines

```
mt19937_64 rng(chrono::steady_clock::now().time_since_epoch().count());
inline ll rnd(ll l, ll r) {
    return uniform_int_distribution<ll>(l, r)(rng);
}
```

brute.cpp  
1 lines

```
// Write brute force for stress testing
```

f.bat  
8 lines

```
@echo off /// set SRC=code.cpp // set EXE=a
g++ -std=c++17 -O2 -Wno-unused-result %SRC% -o %EXE%
if errorlevel 1 (
    echo Compilation failed! // exit /b
)
echo. // echo Input: // type %1 // echo.
echo. // echo Output: // %EXE% < %1 // echo.
/// alt : gnu++14 or ++11 or ++1z
```

d.bat  
1 lines

```
g++ -std=c++17 -DLOCAL -Wshadow -Wall -g -D_GLIBCXX_DEBUG -fmax-
-errors=2 %SRC% -o %EXE%
```

run.bat  
7 lines

```
@echo off
for %%f in (in1 in2 in3 in4 in5 in6 in7 in8 in9) do (
    if exist %%f (
        echo. // echo Input: %%f // type %%f
        echo. // echo Output: //a.exe < %%f // echo.
    )
)
```

stress.bat  
15 lines

```
@echo off
setlocal enabledelayedexpansion
g++ gen.cpp -o gen.exe /// g++ brute.cpp -o brute.exe
set i=1
:loop
gen.exe !i! > stress /// a.exe < stress > myAnswer
brute.exe < stress > correctAnswer
fc /w myAnswer correctAnswer > nul
if errorlevel 1 goto fail
echo Passed test: !i! // set /a i+=1 // goto loop
:fail
echo WA on the following test:
type stress /// echo Your answer is:
type myAnswer /// echo Correct answer is:
type correctAnswer
```

mint.h  
36 lines

```
const int mod=1000000007;
struct mint{
    ll x; mint() { x=0; }
    mint(ll _x) { x = _x%mod; if(x<0) x+=mod; }
    mint operator+=(mint b){ x= (x + b.x)%mod; return *this; }
    mint operator-=(mint b){ x= (x-b.x +mod)%mod; return *this; }
```

```

mint operator*(mint b){ x= (x * b.x)%mod; return *this; }
mint operator/(mint b){ return *this *= b.inv(); }

friend mint operator+(mint a,mint b){ return a+=b; }
friend mint operator-(mint a,mint b){ return a-=b; }
friend mint operator*(mint a,mint b){ return a*=b; }
friend mint operator/(mint a,mint b){ return a/=b; }
mint operator-() { return mint()-*this; }
mint inv() { return power(mod-2); }

mint power(ll n) {
    mint a=*this, res=1;
    while(n) { if(n&1)res*=a; a*=a; n>>=1; }
    return res;
}
mint power(mint a,ll n){ return a.power(n); }

const int N=2000005; mint fact[N],inv_fact[N];
void prep_factorials(){
fact[0] = 1;
for(int i=1;i<N;++i) fact[i] = fact[i-1] * i;
inv_fact[N-1] = fact[N-1].inv();
for(int i=N-2;i>=0;--i) inv_fact[i]=inv_fact[i+1]*(i+1);
}

mint nCr(int n,int r){
if(r<0 || r>n) return mint();
return fact[n] * inv_fact[r] * inv_fact[n-r];
}

```

## Mathematics (2)

### 2.1 Equations

$$ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The extremum is given by  $x = -b/2a$ .

$$\begin{aligned} ax + by = e &\Rightarrow x = \frac{ed - bf}{ad - bc} \\ cx + dy = f &\Rightarrow y = \frac{af - ec}{ad - bc} \end{aligned}$$

In general, given an equation  $Ax = b$ , the solution to a variable  $x_i$  is given by

$$x_i = \frac{\det A'_i}{\det A}$$

where  $A'_i$  is  $A$  with the  $i$ 'th column replaced by  $b$ .

### 2.2 Recurrences

If  $a_n = c_1 a_{n-1} + \dots + c_k a_{n-k}$ , and  $r_1, \dots, r_k$  are distinct roots of  $x^k - c_1 x^{k-1} - \dots - c_k$ , there are  $d_1, \dots, d_k$  s.t.

$$a_n = d_1 r_1^n + \dots + d_k r_k^n.$$

Non-distinct roots  $r$  become polynomial factors, e.g.  
 $a_n = (d_1 n + d_2) r^n$ .

### 2.3 Trigonometry

$$\begin{aligned} \sin(v+w) &= \sin v \cos w + \cos v \sin w \\ \cos(v+w) &= \cos v \cos w - \sin v \sin w \end{aligned}$$

$$\begin{aligned} \tan(v+w) &= \frac{\tan v + \tan w}{1 - \tan v \tan w} \\ \sin v + \sin w &= 2 \sin \frac{v+w}{2} \cos \frac{v-w}{2} \\ \cos v + \cos w &= 2 \cos \frac{v+w}{2} \cos \frac{v-w}{2} \end{aligned}$$

$$(V+W) \tan(v-w)/2 = (V-W) \tan(v+w)/2$$

where  $V, W$  are lengths of sides opposite angles  $v, w$ .

$$\begin{aligned} a \cos x + b \sin x &= r \cos(x - \phi) \\ a \sin x + b \cos x &= r \sin(x + \phi) \end{aligned}$$

where  $r = \sqrt{a^2 + b^2}, \phi = \text{atan2}(b, a)$ .

### 2.4 Geometry

#### 2.4.1 Triangles

Side lengths:  $a, b, c$

Semiperimeter:  $p = \frac{a+b+c}{2}$

Area:  $A = \sqrt{p(p-a)(p-b)(p-c)}$

Circumradius:  $R = \frac{abc}{4A}$

Inradius:  $r = \frac{A}{p}$

Length of median (divides triangle into two equal-area triangles):

$$m_a = \frac{1}{2}\sqrt{2b^2 + 2c^2 - a^2}$$

Length of bisector (divides angles in two):

$$s_a = \sqrt{bc \left[ 1 - \left( \frac{a}{b+c} \right)^2 \right]}$$

Law of sines:  $\frac{\sin \alpha}{a} = \frac{\sin \beta}{b} = \frac{\sin \gamma}{c} = \frac{1}{2R}$

Law of cosines:  $a^2 = b^2 + c^2 - 2bc \cos \alpha$

Law of tangents:  $\frac{a+b}{a-b} = \frac{\tan \frac{\alpha+\beta}{2}}{\tan \frac{\alpha-\beta}{2}}$

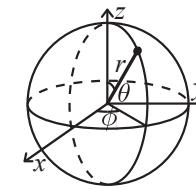
#### 2.4.2 Quadrilaterals

With side lengths  $a, b, c, d$ , diagonals  $e, f$ , diagonals angle  $\theta$ , area  $A$  and magic flux  $F = b^2 + d^2 - a^2 - c^2$ :

$$4A = 2ef \cdot \sin \theta = F \tan \theta = \sqrt{4e^2 f^2 - F^2}$$

For cyclic quadrilaterals the sum of opposite angles is  $180^\circ$ ,  $ef = ac + bd$ , and  $A = \sqrt{(p-a)(p-b)(p-c)(p-d)}$ .

### 2.4.3 Spherical coordinates



$$\begin{aligned} x &= r \sin \theta \cos \phi & r &= \sqrt{x^2 + y^2 + z^2} \\ y &= r \sin \theta \sin \phi & \theta &= \arccos(z/\sqrt{x^2 + y^2 + z^2}) \\ z &= r \cos \theta & \phi &= \text{atan2}(y, x) \end{aligned}$$

### 2.5 Derivatives/Integrals

$$\begin{aligned} \frac{d}{dx} \arcsin x &= \frac{1}{\sqrt{1-x^2}} & \frac{d}{dx} \arccos x &= -\frac{1}{\sqrt{1-x^2}} \\ \frac{d}{dx} \tan x &= 1 + \tan^2 x & \frac{d}{dx} \arctan x &= \frac{1}{1+x^2} \\ \int \tan ax \, dx &= -\frac{\ln |\cos ax|}{a} & \int x \sin ax \, dx &= \frac{\sin ax - ax \cos ax}{a^2} \\ \int e^{-x^2} \, dx &= \frac{\sqrt{\pi}}{2} \operatorname{erf}(x) & \int x e^{ax} \, dx &= \frac{e^{ax}}{a^2} (ax - 1) \end{aligned}$$

Integration by parts:

$$\int_a^b f(x)g(x) \, dx = [F(x)g(x)]_a^b - \int_a^b F(x)g'(x) \, dx$$

### 2.6 Sums

$$c^a + c^{a+1} + \dots + c^b = \frac{c^{b+1} - c^a}{c-1}, c \neq 1$$

$$\begin{aligned} 1 + 2 + 3 + \dots + n &= \frac{n(n+1)}{2} \\ 1^2 + 2^2 + 3^2 + \dots + n^2 &= \frac{n(2n+1)(n+1)}{6} \\ 1^3 + 2^3 + 3^3 + \dots + n^3 &= \frac{n^2(n+1)^2}{4} \\ 1^4 + 2^4 + 3^4 + \dots + n^4 &= \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30} \end{aligned}$$

### 2.7 Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1)$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots, (-\infty < x < \infty)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots, (-\infty < x < \infty)$$

## 2.8 Probability theory

Let  $X$  be a discrete random variable with probability  $p_X(x)$  of assuming the value  $x$ . It will then have an expected value (mean)  $\mu = \mathbb{E}(X) = \sum_x x p_X(x)$  and variance  $\sigma^2 = V(X) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2 = \sum_x (x - \mathbb{E}(X))^2 p_X(x)$  where  $\sigma$  is the standard deviation. If  $X$  is instead continuous it will have a probability density function  $f_X(x)$  and the sums above will instead be integrals with  $p_X(x)$  replaced by  $f_X(x)$ .

Expectation is linear:

$$\mathbb{E}(aX + bY) = a\mathbb{E}(X) + b\mathbb{E}(Y)$$

For independent  $X$  and  $Y$ ,

$$V(aX + bY) = a^2 V(X) + b^2 V(Y).$$

### 2.8.1 Discrete distributions

#### Binomial distribution

The number of successes in  $n$  independent yes/no experiments, each which yields success with probability  $p$  is  $\text{Bin}(n, p)$ ,  $n = 1, 2, \dots$ ,  $0 \leq p \leq 1$ .

$$p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$\mu = np, \sigma^2 = np(1-p)$$

$\text{Bin}(n, p)$  is approximately  $\text{Po}(np)$  for small  $p$ .

#### First success distribution

The number of trials needed to get the first success in independent yes/no experiments, each which yields success with probability  $p$  is  $\text{Fs}(p)$ ,  $0 \leq p \leq 1$ .

$$p(k) = p(1-p)^{k-1}, k = 1, 2, \dots$$

$$\mu = \frac{1}{p}, \sigma^2 = \frac{1-p}{p^2}$$

#### Poisson distribution

The number of events occurring in a fixed period of time  $t$  if these events occur with a known average rate  $\kappa$  and independently of the time since the last event is  $\text{Po}(\lambda)$ ,  $\lambda = t\kappa$ .

$$p(k) = e^{-\lambda} \frac{\lambda^k}{k!}, k = 0, 1, 2, \dots$$

$$\mu = \lambda, \sigma^2 = \lambda$$

## OrderStatisticTree OrderedMultiset HashMap

### 2.8.2 Continuous distributions

#### Uniform distribution

If the probability density function is constant between  $a$  and  $b$  and 0 elsewhere it is  $U(a, b)$ ,  $a < b$ .

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{otherwise} \end{cases}$$

$$\mu = \frac{a+b}{2}, \sigma^2 = \frac{(b-a)^2}{12}$$

#### Exponential distribution

The time between events in a Poisson process is  $\text{Exp}(\lambda)$ ,  $\lambda > 0$ .

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\mu = \frac{1}{\lambda}, \sigma^2 = \frac{1}{\lambda^2}$$

#### Normal distribution

Most real random values with mean  $\mu$  and variance  $\sigma^2$  are well described by  $\mathcal{N}(\mu, \sigma^2)$ ,  $\sigma > 0$ .

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

If  $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$  and  $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$  then

$$aX_1 + bX_2 + c \sim \mathcal{N}(\mu_1 + \mu_2 + c, a^2\sigma_1^2 + b^2\sigma_2^2)$$

## 2.9 Markov chains

A *Markov chain* is a discrete random process with the property that the next state depends only on the current state. Let  $X_1, X_2, \dots$  be a sequence of random variables generated by the Markov process. Then there is a transition matrix  $\mathbf{P} = (p_{ij})$ , with  $p_{ij} = \Pr(X_n = i | X_{n-1} = j)$ , and  $\mathbf{p}^{(n)} = \mathbf{P}^n \mathbf{p}^{(0)}$  is the probability distribution for  $X_n$  (i.e.,  $p_i^{(n)} = \Pr(X_n = i)$ ), where  $\mathbf{p}^{(0)}$  is the initial distribution.

$\pi$  is a stationary distribution if  $\pi = \pi\mathbf{P}$ . If the Markov chain is *irreducible* (it is possible to get to any state from any state), then  $\pi_i = \frac{1}{\mathbb{E}(T_i)}$  where  $\mathbb{E}(T_i)$  is the expected time between two visits in state  $i$ .  $\pi_j/\pi_i$  is the expected number of visits in state  $j$  between two visits in state  $i$ .

For a connected, undirected and non-bipartite graph, where the transition probability is uniform among all neighbors,  $\pi_i$  is proportional to node  $i$ 's degree.

A Markov chain is *ergodic* if the asymptotic distribution is independent of the initial distribution. A finite Markov chain is ergodic iff it is irreducible and *aperiodic* (i.e., the gcd of cycle lengths is 1).  $\lim_{k \rightarrow \infty} \mathbf{P}^k = \mathbf{1}\pi$ .

A Markov chain is an *A-chain* if the states can be partitioned into two sets **A** and **G**, such that all states in **A** are absorbing ( $p_{ii} = 1$ ), and all states in **G** leads to an absorbing state in **A**. The probability for absorption in state  $i \in \mathbf{A}$ , when the initial state is  $j$ , is  $a_{ij} = p_{ij} + \sum_{k \in \mathbf{G}} a_{ik} p_{kj}$ . The expected time until absorption, when the initial state is  $i$ , is  $t_i = 1 + \sum_{k \in \mathbf{G}} p_{ik} t_k$ .

## Data structures (3)

### OrderStatisticTree.h

**Description:** A set (not multiset!) with support for finding the  $n$ 'th element, and finding the index of an element. To get a map, change `null_type`.

**Time:**  $\mathcal{O}(\log N)$

`<ext/pbds/assoc.container.hpp>, <ext/pbds/tree_policy.hpp>` 5eb830, 5 lines

```
#include <bits/extc++.h>
#define ordered_set tree<int, null_type, less<int>, rb_tree_tag,
                     tree_order_statistics_node_update>
using namespace __gnu_pbds;
// *st.find_by_order(i) : ith largest element
// st.order_of_key(k) : number of items strictly smaller than k
```

### OrderedMultiset.h

**Description:** idk

**Time:** idk

f2915a, 18 lines

```
template <typename T>
struct ordered_multiset {
    using Tpair = pair<T, int>;
    tree<Tpair, null_type, less<Tpair>, rb_tree_tag,
         tree_order_statistics_node_update> t;
    int _idx = 0;
    void insert(const T &x) { t.insert({x, _idx++}); }
    void erase(const T &x) {
        auto it = t.lower_bound({x, 0});
        if (it != t.end() && it->first == x) t.erase(it);
    }
    int order_of_key(const T &x) const {
        return t.order_of_key({x, 0});
    }
    T find_by_order(int k) const {
        auto it = t.find_by_order(k);
        if (it == t.end()) return 1000000000;
        return it->first;
    }
    int size() const { return t.size(); }
    bool empty() const { return t.empty(); }
};
```

### HashMap.h

**Description:** Hash map with mostly the same API as `unordered_map`, but ~3x faster. Uses 1.5x memory. Initial capacity must be a power of 2 (if provided).

d77092, 7 lines

```
#include <bits/extc++.h>
// To use most bits rather than just the lowest ones:
struct chash { // large odd number for C
    const uint64_t C = 11(4e18 * acos(0)) | 71;
    ll operator()(ll x) const { return __builtin_bswap64(x*C); }
};
```

`__gnu_pbds::gp_hash_table<ll, int, chash> h({}, {}, {}, {}, {1<<16});`

## SegmentTree.h

**Description:** Zero-indexed max-tree. Bounds are inclusive to the left and exclusive to the right. Can be changed by modifying T, f and unit.

**Time:**  $\mathcal{O}(\log N)$

e0c6b9, 31 lines

```
struct Seg { // left child even, right odd
    ll unit = -inf; // identity element
    int n, m=1; vector<ll> t;

    Seg(int _n) : n(_n) { while(m < n) m *= 2; t.resize(4 * m, unit); }
    ll f(ll a, ll b) { return max(a, b); } // change operation

    void update(int p, ll value) {
        for (t[p += m] = value; p > 1; p >>= 1) {
            t[p > 1] = f(t[p], t[p ^ 1]); // p>1 is parent
        }
    }

    ll query(int l, int r) { // [l, r)
        ll resL = unit, resR = unit;
        for (l += m, r -= m; l < r; l >>= 1, r >>= 1) {
            if (l & 1) resL = f(resL, t[l++]);
            if (r & 1) resR = f(t[--r], resR);
        }
        return f(resL, resR);
    }

    // find first position having element >= val in a range
    ll myQ(int val) { return myQ(val, 1, 0, m, 0, n); } // whole range
    ll myQ(int val, int pos, int l, int r, int ql, int qr) {
        if (r <= ql || qr <= l) return -1; // no overlap
        if (t[pos] < val) return -1; // all values < val
        if (l + 1 == r) return l; // leaf node
        int mid = (l + r) / 2;
        int left = myQ(val, 2 * pos, l, mid, ql, qr);
        if (left != -1) return left;
        return myQ(val, 2 * pos + 1, mid, r, ql, qr);
    }
}
```

## LazySegmentTree.h

**Description:** can use set and add together. ranges are all [], EVEN IN CONSTRUCTOR TOO Use with a bump allocator for better performance, and SmallIPtr or implicit indices to save memory.

**Usage:** Lseg seg(a, 1, n+1); / seg(1, n+1) / seg(0, n)

**Time:**  $\mathcal{O}(\log N)$ .

.../various/BumpAllocator.h" 0f5cd4, 63 lines

```
struct Node { ll mn, mx, sum; // ignore variables you dont need
    Node() : mn(inf), mx(-inf), sum(0) {}
    Node(ll x) : mn(x), mx(x), sum(x) {}
    Node(ll x, ll y, ll z) : mn(x), mx(y), sum(z) {}

};

struct Lseg {
    Lseg *l = 0, *r = 0; int lo, hi; ll mset = inf, madd = 0;
    Node val;
    Node unite(Node a, Node b) {
        return Node(min(a.mn, b.mn), max(a.mx, b.mx), a.sum + b.sum);
    }

    Lseg(int lo, int hi) : l(lo), hi(hi) {} // Huge interval of neutrals
    Lseg(vll& v, int _lo, int _hi) : l(_lo), hi(_hi) {
        if (_lo + 1 < _hi) {
            int mid = _lo + (_hi - _lo) / 2;
            l = new Lseg(v, _lo, mid); r = new Lseg(v, mid, _hi);
            val = unite(l->val, r->val);
        } else val = Node(v[_lo]);
    }

    Node query(int L, int R) {
        if (R <= lo || hi <= L) return Node();
        if (L <= lo && hi <= R) return val;
        push();
        return unite(l->query(L, R), r->query(L, R));
    }
}
```

```

    void set(int L, int R, ll x) { // skip if not needed
        if (R <= lo || hi <= L) return;
        if (L <= lo && hi <= R) {
            mset = val.mn = val.mx = x, madd = 0;
            val.sum = x * (hi - lo);
        } else {
            push(), l->set(L, R, x), r->set(L, R, x);
            val = unite(l->val, r->val);
        }
    }

    void add(int L, int R, ll x) { // skip if not needed
        if (R <= lo || hi <= L) return;
        if (L <= lo && hi <= R) {
            if (mset != inf) mset += x;
            else madd += x;
            val.mn += x, val.mx += x, val.sum += x * (hi - lo);
        } else {
            push(), l->add(L, R, x), r->add(L, R, x);
            val = unite(l->val, r->val);
        }
    }

    void push() {
        if (!l) {
            int mid = lo + (hi - lo) / 2;
            l = new Lseg(lo, mid); r = new Lseg(mid, hi);
        }
        if (mset != inf)
            l->set(lo, hi, mset), r->set(lo, hi, mset), mset = inf;
        else if (madd)
            l->add(lo, hi, madd), r->add(lo, hi, madd), madd = 0;
    }

    int myQ(int val, int L, int R) { // first idx >= val
        if (R <= lo || hi <= L) return -1;
        if (val > this->val.mx) return -1;
        if (lo + 1 == hi) return lo;
        push();
        int res = l->myQ(val, L, R); if (res != -1) return res;
        return r->myQ(val, L, R);
    }

    ~Lseg() { delete l; delete r; } // must if you want to debug
}
```

## PersistentSeg.h

**Description:** ...

**Usage:** vector<Node\*> pref(n+1);

```
pref[0] = build(0, unqvals); // [0, unqvals)
trees[k] = change(trees[k-1], 0, unqvals, pos, x);
cout << query(trees[k], 0, unqvals, l, r+1) << nl;
```

**Time:** ...

a4e183, 33 lines

```
struct Node {
    Node *left = nullptr, *right = nullptr; ll sum = 0;
};

// Builds a segment tree on [l,r). use same [l,r) in qry, upd
// Returns a pointer to the constructed tree.
Node *build(int l, int r) {
    Node *p = new Node();
    if (r - l == 1) return p;
    int m = (l + r) / 2;
    p->left = build(l, m);
    p->right = build(m, r);
    return p;
}

// Changes value in position k to x in tree p.
// Returns a pointer to the new tree.
Node *change(Node *p, int l, int r, int k, int x) {
    Node *n = new Node(*p);
    if (r - l == 1) { n->sum = x; }
    else {
        int m = (l + r) / 2;
        if (k < m) { n->left = change(p->left, l, m, k, x); }
        else { n->right = change(p->right, m, r, k, x); }
    }
    n->sum = n->left->sum + n->right->sum;
    return n;
}

// Returns the sum of values in range [ql, qr].
ll query(Node *p, int l, int r, int ql, int qr) {
    if (qr <= l || r <= ql) return 0;
    if (ql <= l && r <= qr) return p->sum;
    int m = (l + r) / 2;
    return query(p->left, l, m, ql, qr) +
        query(p->right, m, r, ql, qr);
}
```

## SegTree2D.h

**Description:** ...

**Usage:** one based grid, and all inclusive

**Time:** log square n

3e4e8d, 30 lines

```
const int TREE_SIZE = 1 << 10;
int trees[TREE_SIZE * 2][TREE_SIZE * 2];
// Adds d to the position x in the segment tree 'tree'.
void add(int *tree, int x, int d) {
    x += TREE_SIZE;
    while (x > 0) { tree[x] += d; x /= 2; }
}

// Adds d to the position (i, j).
void add(int y, int x, int d) {
    y += TREE_SIZE;
    while (y > 0) { add(trees[y], x, d); y /= 2; }
}

// Returns the sum of value in range [x1, x2] in 'tree'.
int query(int *tree, int x1, int x2) {
    x1 += TREE_SIZE; x2 += TREE_SIZE; int sum = 0;
    while (x1 <= x2) {
        if (x1 % 2 == 1) sum += tree[x1++];
        if (x2 % 2 == 0) sum += tree[x2--];
        x1 /= 2; x2 /= 2;
    }
    return sum;
}

// Returns the sum of value in the rectangle [y1, y2]x[x1, x2]
int query(int y1, int y2, int x1, int x2) {
    y1 += TREE_SIZE; y2 += TREE_SIZE; int sum = 0;
    while (y1 <= y2) {
        if (y1 % 2 == 1) sum += query(trees[y1++], x1, x2);
        if (y2 % 2 == 0) sum += query(trees[y2--], x1, x2);
        y1 /= 2; y2 /= 2;
    }
    return sum;
}
```

## UnionFindRollback.h

**Description:** Disjoint-set data structure with undo. If undo is not needed, skip st, time() and rollback().

**Usage:** int t = uf.time(); ...; uf.rollback(t);

**Time:**  $\mathcal{O}(\log N)$

de4ad0, 21 lines

```
struct RollbackUF {
    vi e; vector<pii> st;
    RollbackUF(int n) : e(n, -1) {}
    int size(int x) { return -e[find(x)]; }
    int find(int x) { return e[x] < 0 ? x : find(e[x]); }
    int time() { return sz(st); }
    void rollback(int t) {
        for (int i = time(); i --> t;)
            e[st[i].first] = st[i].second;
        st.resize(t);
    }

    bool join(int a, int b) {
        a = find(a), b = find(b);
        if (a == b) return false;
    }
}
```

```

if (e[a] > e[b]) swap(a, b);
st.push_back({a, e[a]});
st.push_back({b, e[b]});
e[a] += e[b]; e[b] = a;
return true;
}

```

**SubMatrix.h**

**Description:** Calculate submatrix sums quickly, given upper-left and lower-right corners (half-open).

**Usage:** SubMatrix<int> m(matrix);  
m.sum(0, 0, 2, 2); // top left 4 elements  
**Time:**  $\mathcal{O}(N^2 + Q)$

c59ada, 13 lines

```

template<class T>
struct SubMatrix {
    vector<vector<T>> p;
    SubMatrix(vector<vector<T>> v) {
        int R = sz(v), C = sz(v[0]);
        p.assign(R+1, vector<T>(C+1));
        rep(r,0,R) rep(c,0,C)
            p[r+1][c+1] = v[r][c] + p[r][c+1] + p[r+1][c] - p[r][c];
    }
    T sum(int u, int l, int d, int r) {
        return p[d][r] - p[d][l] - p[u][r] + p[u][l];
    }
};

```

**Matrix.h**

**Description:** Basic operations on square matrices.

**Usage:** Matrix<int, 3> A;  
A.d = {{{1,2,3}}, {{4,5,6}}, {{7,8,9}}};  
array<int, 3> vec = {1,2,3};  
vec = (A^N) \* vec;

4da5a2, 26 lines

```

template<class T, int N> struct Matrix {
    typedef Matrix M;
    array<array<T, N>, N> d{};
    M operator*(const M& m) const {
        M a;
        rep(i,0,N) rep(j,0,N)
            rep(k,0,N) a.d[i][k] += d[i][j] * m.d[j][k];
        return a;
    }
    array<T, N> operator*(const array<T, N>& vec) const {
        array<T, N> ret{};
        rep(i,0,N) rep(j,0,N) ret[i] += d[i][j] * vec[j];
        return ret;
    }
    M operator^(ll p) const {
        assert(p >= 0);
        M a, b(*this);
        rep(i,0,N) a.d[i][i] = 1;
        while (p) {
            if (p&1) a = a*b;
            b = b*b;
            p >>= 1;
        }
        return a;
    }
};

```

**LineContainer.h**

**Description:** Container where you can add lines of the form  $kx+m$ , and query maximum values at points  $x$ . Useful for dynamic programming (“convex hull trick”).

**Time:**  $\mathcal{O}(\log N)$

8ec1c7, 30 lines

```

struct Line {
    mutable ll k, m, p;
    bool operator<(const Line& o) const { return k < o.k; }
    bool operator<(ll x) const { return p < x; }
};

struct LineContainer : multiset<Line, less<>> {
    // (for doubles, use inf = 1./0., div(a,b) = a/b)
    static const ll inf = LLONG_MAX;
    ll div(ll a, ll b) { // floored division
        return a / b - ((a ^ b) < 0 && a % b); }
    bool isect(iterator x, iterator y) {
        if (y == end()) return x->p = inf, 0;
        if (x->k == y->k) x->p = x->m > y->m ? inf : -inf;
        else x->p = div(y->m - x->m, x->k - y->k);
        return x->p >= y->p;
    }
    void add(ll k, ll m) {
        auto z = insert({k, m, 0}), y = z++, x = y;
        while (isect(y, z)) z = erase(z);
        if (x != begin() && isect(--x, y)) isect(x, y = erase(y));
        while ((y = x) != begin() && (--x)->p >= y->p)
            isect(x, erase(y));
    }
    ll query(ll x) {
        assert(!empty());
        auto l = *lower_bound(x);
        return l.k * x + l.m;
    }
};

```

**Treap.h**

**Description:** A short self-balancing tree. It acts as a sequential container with log-time splits/joins, and is easy to augment with additional data.

**Time:**  $\mathcal{O}(\log N)$

1754b4, 53 lines

```

struct Node {
    Node *l = 0, *r = 0;
    int val, y, c = 1;
    Node(int val) : val(val), y(rand()) {}
    void recalc();
};

int cnt(Node* n) { return n ? n->c : 0; }
void Node::recalc() { c = cnt(l) + cnt(r) + 1; }

template<class F> void each(Node* n, F f) {
    if (n) { each(n->l, f); f(n->val); each(n->r, f); }
}

pair<Node*, Node*> split(Node* n, int k) {
    if (!n) return {};
    if (cnt(n->l) >= k) { // "n->val >= k" for lower_bound(k)
        auto [L,R] = split(n->l, k);
        n->l = R;
        n->recalc();
        return {L, n};
    } else {
        auto [L,R] = split(n->r, k - cnt(n->l) - 1); // and just "k"
        n->r = L;
        n->recalc();
        return {n, R};
    }
}

Node* merge(Node* l, Node* r) {
    if (!l) return r;
    if (!r) return l;
    if (l->y > r->y) {

```

```

        l->r = merge(l->r, r);
        return l->recalc(), l;
    } else {
        r->l = merge(l, r->l);
        return r->recalc(), r;
    }
}

Node* ins(Node* t, Node* n, int pos) {
    auto [l,r] = split(t, pos);
    return merge(merge(l, n), r);
}

// Example application: move the range [l, r) to index k
void move(Node*& t, int l, int r, int k) {
    Node* a, *b, *c;
    tie(a,b) = split(t, l); tie(b,c) = split(b, r - 1);
    if (k <= l) t = merge(ins(a, b, k), c);
    else t = merge(a, ins(c, b, k - r));
}

```

**RMQ.h**

**Description:** Range Minimum Queries on an array. Returns  $\min(V[a], V[a+1], \dots, V[b-1])$  in constant time.

**Usage:** RMQ rmq(values);

rmq.query(inclusive, exclusive);

**Time:**  $\mathcal{O}(|V| \log |V| + Q)$

510c32, 16 lines

```

template<class T>
struct RMQ {
    vector<vector<T>> jmp;
    RMQ(const vector<T>& V) : jmp(1, V) {
        for (int pw = 1, k = 1; pw * 2 <= sz(V); pw *= 2, ++k) {
            jmp.emplace_back(sz(V) - pw * 2 + 1);
            rep(j,0,sz(jmp[k]))
                jmp[k][j] = min(jmp[k - 1][j], jmp[k - 1][j + pw]);
        }
    }
    T query(int a, int b) {
        assert(a < b); // or return inf if a == b
        int dep = 31 - __builtin_clz(b - a);
        return min(jmp[dep][a], jmp[dep][b - (1 << dep)]);
    }
};

```

**MoQueries.h**

**Description:** Answer interval or tree path queries by finding an approximate TSP through the queries, and moving from one query to the next by adding/removing points at the ends. If values are on tree edges, change step to add/remove the edge  $(a, c)$  and remove the initial add call (but keep in).

**Time:**  $\mathcal{O}(N\sqrt{Q})$

a12ef4, 49 lines

```

void add(int ind, int end) { ... } // add a[ind] (end = 0 or 1)
void del(int ind, int end) { ... } // remove a[ind]
int calc() { ... } // compute current answer

vi mo(vector<pii> Q) {
    int L = 0, R = 0, blk = 350; // ~N/sqrt(Q)
    vi s(sz(Q)), res = s;
#define K(x) pii(x.first/blk, x.second ^ -(x.first/blk & 1))
    iota(all(s), 0);
    sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
    for (int qi : s) {
        pii q = Q[qi];
        while (L > q.first) add(-L, 0);
        while (R < q.second) add(R++, 1);
        while (L < q.first) del(L++, 0);
        while (R > q.second) del(--R, 1);
        res[qi] = calc();
    }
}

```

```

    }
    return res;
}

vi moTree(vector<array<int, 2>> Q, vector<vi>& ed, int root=0) {
    int N = sz(ed), pos[2] = {}, blk = 350; // ~N/sqrt(Q)
    vi s(sz(Q)), res = s, I(N), L(N), R(N), in(N), par(N);
    add(0, 0), in[0] = 1;
    auto dfs = [&](int x, int p, int dep, auto& f) -> void {
        par[x] = p;
        L[x] = N;
        if (dep) I[x] = N++;
        for (int y : ed[x]) if (y != p) f(y, x, !dep, f);
        if (!dep) I[x] = N++;
        R[x] = N;
    };
    dfs(root, -1, 0, dfs);
#define K(x) pii(I[x[0]] / blk, I[x[1]] ^ -(I[x[0]] / blk & 1))
    iota(all(s), 0);
    sort(all(s), [&](int s, int t){ return K(Q[s]) < K(Q[t]); });
    for (int qi : s) rep(end,0,2) {
        int &a = pos[end], b = Q[qi][end], i = 0;
#define step(c) { if (in[c]) { del(a, end); in[a] = 0; } \
        else { add(c, end); in[c] = 1; } a = c; \
        while (! (L[b] <= L[a] && R[a] <= R[b])) \
            I[i++] = b, b = par[b]; \
        while (a != b) step(par[a]); \
        while (i--) step(I[i]); \
        if (end) res[qi] = calc(); \
    }
    return res;
}

```

## Numerical (4)

### 4.1 Polynomials and recurrences

Polynomial.h

c9b7b0, 17 lines

```

struct Poly {
    vector<double> a;
    double operator()(double x) const {
        double val = 0;
        for (int i = sz(a); i--;) (val *= x) += a[i];
        return val;
    }
    void diff() {
        rep(i,1,sz(a)) a[i-1] = i*a[i];
        a.pop_back();
    }
    void divroot(double x0) {
        double b = a.back(), c; a.back() = 0;
        for(int i=sz(a)-1; i--;) c = a[i], a[i] = a[i+1]*x0+b, b=c;
        a.pop_back();
    }
};

```

PolyRoots.h

**Description:** Finds the real roots to a polynomial.**Usage:** polyRoots({{2,-3,1}},-1e9,1e9) // solve  $x^2-3x+2 = 0$ **Time:**  $\mathcal{O}(n^2 \log(1/\epsilon))$ 

"Polynomial.h"

b00bfe, 23 lines

```

vector<double> polyRoots(Poly p, double xmin, double xmax) {
    if (sz(p.a) == 2) { return {-p.a[0]/p.a[1]}; }
    vector<double> ret;
    Poly der = p;
    der.diff();
    auto dr = polyRoots(der, xmin, xmax);

```

```

        dr.push_back(xmin-1);
        dr.push_back(xmax+1);
        sort(all(dr));
        rep(i,0,sz(dr)-1) {
            double l = dr[i], h = dr[i+1];
            bool sign = p(l) > 0;
            if (sign ^ (p(h) > 0)) {
                rep(it,0,60) { // while (h - l > 1e-8)
                    double m = (l + h) / 2, f = p(m);
                    if ((f <= 0) ^ sign) l = m;
                    else h = m;
                }
                ret.push_back((l + h) / 2);
            }
        }
        return ret;
    }

```

PolyInterpolate.h

**Description:** Given  $n$  points  $(x[i], y[i])$ , computes an  $n-1$ -degree polynomial  $p$  that passes through them:  $p(x) = a[0] * x^0 + \dots + a[n-1] * x^{n-1}$ . For numerical precision, pick  $x[k] = c * \cos(k/(n-1) * \pi)$ ,  $k = 0 \dots n-1$ .**Time:**  $\mathcal{O}(n^2)$ 

```

typedef vector<double> vd;
vd interpolate(vd x, vd y, int n) {
    vd res(n), temp(n);
    rep(k,0,n-1) rep(i,k+1,n)
        y[i] = (y[i] - y[k]) / (x[i] - x[k]);
    double last = 0; temp[0] = 1;
    rep(k,0,n) rep(i,0,n) {
        res[i] += y[k] * temp[i];
        swap(last, temp[i]);
        temp[i] -= last * x[k];
    }
    return res;
}

```

BerlekampMassey.h

**Description:** Recovers any  $n$ -order linear recurrence relation from the first  $2n$  terms of the recurrence. Useful for guessing linear recurrences after brute-forcing the first terms. Should work on any field, but numerical stability for floats is not guaranteed. Output will have size  $\leq n$ .**Usage:** berlekampMassey({0, 1, 1, 3, 5, 11}) // {1, 2}**Time:**  $\mathcal{O}(N^2)$ 

"../number-theory/ModPow.h"

96548b, 20 lines

```

vector<ll> berlekampMassey(vector<ll> s) {
    int n = sz(s), L = 0, m = 0;
    vector<ll> C(n), B(n), T;
    C[0] = B[0] = 1;

    ll b = 1;
    rep(i,0,n) { ++m;
        ll d = s[i] % mod;
        rep(j,1,L+1) d = (d + C[j] * s[i - j]) % mod;
        if (!d) continue;
        T = C; ll coef = d * modpow(b, mod-2) % mod;
        rep(j,m,n) C[j] = (C[j] - coef * B[j - m]) % mod;
        if (2 * L > i) continue;
        L = i + 1 - L; B = T; b = d; m = 0;
    }

    C.resize(L + 1); C.erase(C.begin());
    for (ll & x : C) x = (mod - x) % mod;
    return C;
}

```

LinearRecurrence.h

**Description:** Generates the  $k$ 'th term of an  $n$ -order linear recurrence  $S[i] = \sum_j S[i - j - 1]tr[j]$ , given  $S[0 \dots \geq n-1]$  and  $tr[0 \dots n-1]$ . Faster than matrix multiplication. Useful together with Berlekamp-Massey.**Usage:** linearRec({0, 1}, {1, 1}, k) //  $k$ 'th Fibonacci number

f4e444, 26 lines

```

typedef vector<ll> Poly;
ll linearRec(Poly S, Poly tr, ll k) {
    int n = sz(tr);

    auto combine = [&](Poly a, Poly b) {
        Poly res(n * 2 + 1);
        rep(i,0,n+1) rep(j,0,n+1)
            res[i + j] = (res[i + j] + a[i] * b[j]) % mod;
        for (int i = 2 * n; i > n; --i) rep(j,0,n)
            res[i - 1 - j] = (res[i - 1 - j] + res[i] * tr[j]) % mod;
        res.resize(n + 1);
        return res;
    };

    Poly pol(n + 1), e(pol);
    pol[0] = e[1] = 1;

    for (++k; k; k /= 2) {
        if (k % 2) pol = combine(pol, e);
        e = combine(e, e);
    }

    ll res = 0;
    rep(i,0,n) res = (res + pol[i + 1] * S[i]) % mod;
    return res;
}

```

### 4.2 Optimization

GoldenSectionSearch.h

**Description:** Finds the argument minimizing the function  $f$  in the interval  $[a, b]$  assuming  $f$  is unimodal on the interval, i.e. has only one local minimum and no local maximum. The maximum error in the result is  $eps$ . Works equally well for maximization with a small change in the code. See TernarySearch.h in the Various chapter for a discrete version.**Usage:** double func(double x) { return 4+x+.3\*x\*x; }

double xmin = gss(-1000,1000,func);

**Time:**  $\mathcal{O}(\log((b-a)/\epsilon))$ 

31d45b, 14 lines

```

double gss(double a, double b, double (*f)(double)) {
    double r = (sqrt(5)-1)/2, eps = 1e-7;
    double x1 = b - r*(b-a), x2 = a + r*(b-a);
    double f1 = f(x1), f2 = f(x2);
    while (b-a > eps) {
        if (f1 < f2) { //change to > to find maximum
            b = x2; x2 = x1; f2 = f1;
            x1 = b - r*(b-a); f1 = f(x1);
        } else {
            a = x1; x1 = x2; f1 = f2;
            x2 = a + r*(b-a); f2 = f(x2);
        }
    }
    return a;
}

```

HillClimbing.h

**Description:** Poor man's optimization for unimodal functions

Seeefaf, 14 lines

```

typedef array<double, 2> P;

template<class F> pair<double, P> hillClimb(P start, F f) {
    pair<double, P> cur(f(start), start);
    for (double jmp = 1e9; jmp > 1e-20; jmp /= 2) {
        rep(j,0,100) rep(dx,-1,2) rep(dy,-1,2) {

```

```

P p = cur.second;
p[0] += dx*jmp;
p[1] += dy*jmp;
cur = min(cur, make_pair(f(p), p));
}
return cur;
}

```

## Integrate.h

**Description:** Simple integration of a function over an interval using Simpson's rule. The error should be proportional to  $h^4$ , although in practice you will want to verify that the result is stable to desired precision when epsilon changes.

4756fc, 7 lines

```

template<class F>
double quad(double a, double b, F f, const int n = 1000) {
    double h = (b - a) / 2 / n, v = f(a) + f(b);
    rep(i, 1, n*2)
        v += f(a + i*h) * (i&1 ? 4 : 2);
    return v * h / 3;
}

```

## IntegrateAdaptive.h

**Description:** Fast integration using an adaptive Simpson's rule.

**Usage:** double sphereVolume = quad(-1, 1, [] (double x) {
 return quad(-1, 1, [&] (double y) {
 return quad(-1, 1, [&] (double z) {
 return x\*x + y\*y + z\*z < 1; })});});

92dd79, 15 lines

```

typedef double d;
#define S(a,b) (f(a) + 4*f((a+b) / 2) + f(b)) * (b-a) / 6

template <class F>
d rec(F& f, d a, d b, d eps, d S) {
    d c = (a + b) / 2;
    d S1 = S(a, c), S2 = S(c, b), T = S1 + S2;
    if (abs(T - S) <= 15 * eps || b - a < 1e-10)
        return T + (T - S) / 15;
    return rec(f, a, c, eps / 2, S1) + rec(f, c, b, eps / 2, S2);
}

template<class F>
d quad(d a, d b, F f, d eps = 1e-8) {
    return rec(f, a, b, eps, S(a, b));
}

```

## Simplex.h

**Description:** Solves a general linear maximization problem: maximize  $c^T x$  subject to  $Ax \leq b$ ,  $x \geq 0$ . Returns -inf if there is no solution, inf if there are arbitrarily good solutions, or the maximum value of  $c^T x$  otherwise. The input vector is set to an optimal  $x$  (or in the unbounded case, an arbitrary solution fulfilling the constraints). Numerical stability is not guaranteed. For better performance, define variables such that  $x = 0$  is viable.

**Usage:** vvd A = {{1,-1}, {-1,1}, {-1,-2}};

vd b = {1,1,-4}, c = {-1,-1}, x;

T val = LPSolver(A, b, c).solve(x);

**Time:**  $\mathcal{O}(NM * \#pivots)$ , where a pivot may be e.g. an edge relaxation.

$\mathcal{O}(2^n)$  in the general case.

aa8530, 68 lines

```

typedef double T; // long double, Rational, double + modP>...
typedef vector<T> vd;
typedef vector<vd> vvd;
const T eps = 1e-8, inf = 1/.0;
#define MP make_pair
#define ltj(X) if(s == -1 || MP(X[j], N[j]) < MP(X[s], N[s])) s=j

struct LPSolver {
    int m, n;
}

```

```

vi N, B;
vvd D;

LPSolver(const vvd& A, const vd& b, const vd& c) :
    m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n+2)) {
    rep(i, 0, m) rep(j, 0, n) D[i][j] = A[i][j];
    rep(i, 0, m) { B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i]; }
    rep(j, 0, n) { N[j] = j; D[m][j] = -c[j]; }
    N[n] = -1; D[m+1][n] = 1;
}

```

```

void pivot(int r, int s) {
    T *a = D[r].data(), inv = 1 / a[s];
    rep(i, 0, m+2) if (i != r && abs(D[i][s]) > eps) {
        T *b = D[i].data(), inv2 = b[s] * inv;
        rep(j, 0, n+2) b[j] -= a[j] * inv2;
        b[s] = a[s] * inv2;
    }
    rep(j, 0, n+2) if (j != s) D[r][j] *= inv;
    rep(i, 0, m+2) if (i != r) D[i][s] *= -inv;
    D[r][s] = inv;
    swap(B[r], N[s]);
}

```

```

bool simplex(int phase) {
    int x = m + phase - 1;
    for (;;) {
        int s = -1;
        rep(j, 0, n+1) if (N[j] != -phase) ltj(D[x]);
        if (D[x][s] >= -eps) return true;
        int r = -1;
        rep(i, 0, m) {
            if (D[i][s] <= eps) continue;
            if (r == -1 || MP(D[i][n+1] / D[i][s], B[i]) < MP(D[r][n+1] / D[r][s], B[r])) r = i;
        }
        if (r == -1) return false;
        pivot(r, s);
    }
}

```

```

T solve(vd &x) {
    int r = 0;
    rep(i, 1, m) if (D[i][n+1] < D[r][n+1]) r = i;
    if (D[r][n+1] < -eps) {
        pivot(r, n);
        if (!simplex(2) || D[m+1][n+1] < -eps) return -inf;
        rep(i, 0, m) if (B[i] == -1) {
            int s = 0;
            rep(j, 1, n+1) ltj(D[i]);
            pivot(i, s);
        }
    }
    bool ok = simplex(1); x = vd(n);
    rep(i, 0, m) if (B[i] < n) x[B[i]] = D[i][n+1];
    return ok ? D[m][n+1] : inf;
}

```

## 4.3 Matrices

## Determinant.h

**Description:** Calculates determinant of a matrix. Destroys the matrix.

**Time:**  $\mathcal{O}(N^3)$

bd5cec, 15 lines

```

double det(vector<vector<double>>& a) {
    int n = sz(a); double res = 1;
    rep(i, 0, n) {
        int b = i;
        rep(j, i+1, n) if (fabs(a[j][i]) > fabs(a[b][i])) b = j;

```

```

        if (i != b) swap(a[i], a[b]), res *= -1;
        res *= a[i][i];
        if (res == 0) return 0;
        rep(j, i+1, n) {
            double v = a[j][i] / a[i][i];
            if (v != 0) rep(k, i+1, n) a[j][k] -= v * a[i][k];
        }
    }
    return res;
}

```

## IntDeterminant.h

**Description:** Calculates determinant using modular arithmetics. Modulos can also be removed to get a pure-integer version.

**Time:**  $\mathcal{O}(N^3)$

3313dc, 18 lines

```

const ll mod = 12345;
ll det(vector<vector<ll>>& a) {
    int n = sz(a); ll ans = 1;
    rep(i, 0, n) {
        rep(j, i+1, n) {
            while (a[j][i] != 0) { // gcd step
                ll t = a[i][i] / a[j][i];
                if (t) rep(k, i, n)
                    a[i][k] = (a[i][k] - a[j][k] * t) % mod;
                swap(a[i], a[j]);
                ans *= -1;
            }
            ans = ans * a[i][i] % mod;
            if (!ans) return 0;
        }
        return (ans + mod) % mod;
    }
}

```

## SolveLinear.h

**Description:** Solves  $A * x = b$ . If there are multiple solutions, an arbitrary one is returned. Returns rank, or -1 if no solutions. Data in  $A$  and  $b$  is lost.

**Time:**  $\mathcal{O}(n^2m)$

44c9ab, 38 lines

```

typedef vector<double> vd;
const double eps = 1e-12;

int solveLinear(vvd &A, vd &b, vd &x) {
    int n = sz(A), m = sz(x), rank = 0, br, bc;
    if (n) assert(sz(A[0]) == m);
    vi col(m); iota(all(col), 0);

    rep(i, 0, n) {
        double v, bv = 0;
        rep(r, i, n) rep(c, i, m)
            if ((v = fabs(A[r][c])) > bv)
                br = r, bc = c, bv = v;
        if (bv <= eps) {
            rep(j, i, n) if (fabs(b[j]) > eps) return -1;
            break;
        }
        swap(A[i], A[br]);
        swap(b[i], b[br]);
        swap(col[i], col[bc]);
        rep(j, 0, n) swap(A[j][i], A[j][bc]);
        bv = 1/A[i][i];
        rep(j, i+1, n) {
            double fac = A[j][i] * bv;
            b[j] -= fac * b[i];
            rep(k, i+1, m) A[j][k] -= fac * A[i][k];
        }
        rank++;
    }
}

```

```
x.assign(m, 0);
for (int i = rank; i--;) {
    b[i] /= A[i][i];
    x[col[i]] = b[i];
    rep(j, 0, i) b[j] -= A[j][i] * b[i];
}
return rank; // (multiple solutions if rank < m)
```

## SolveLinear2.h

**Description:** To get all uniquely determined values of  $x$  back from SolveLinear, make the following changes:

"SolveLinear.h" 08e495, 7 lines

```
rep(j, 0, n) if (j != i) // instead of rep(j, i+1, n)
// ... then at the end:
x.assign(m, undefined);
rep(i, 0, rank) {
    rep(j, rank, m) if (fabs(A[i][j]) > eps) goto fail;
    x[col[i]] = b[i] / A[i][i];
}
fail; }
```

## SolveLinearBinary.h

**Description:** Solves  $Ax = b$  over  $\mathbb{F}_2$ . If there are multiple solutions, one is returned arbitrarily. Returns rank, or -1 if no solutions. Destroys  $A$  and  $b$ .

**Time:**  $\mathcal{O}(n^2m)$  fa2d7a, 34 lines

**typedef** bitset<1000> bs;

```
int solveLinear(vector<bs>& A, vi& b, bs& x, int m) {
    int n = sz(A), rank = 0, br;
    assert(m <= sz(x));
    vi col(m); iota(all(col), 0);
    rep(i, 0, n) {
        for (br=i; br<n; ++br) if (A[br].any()) break;
        if (br == n) {
            rep(j, i, n) if (b[j]) return -1;
            break;
        }
        int bc = (int)A[br]._Find_next(i-1);
        swap(A[i], A[br]);
        swap(b[i], b[br]);
        swap(col[i], col[bc]);
        rep(j, 0, n) if (A[j][i] != A[j][bc]) {
            A[j].flip(i); A[j].flip(bc);
        }
        rep(j, i+1, n) if (A[j][i]) {
            b[j] ^= b[i];
            A[j] ^= A[i];
        }
        rank++;
    }

    x = bs();
    for (int i = rank; i--;) {
        if (!b[i]) continue;
        x[col[i]] = 1;
        rep(j, 0, i) b[j] ^= A[j][i];
    }
    return rank; // (multiple solutions if rank < m)
}
```

## MatrixInverse.h

**Description:** Invert matrix  $A$ . Returns rank; result is stored in  $A$  unless singular ( $\text{rank} < n$ ). Can easily be extended to prime moduli; for prime powers, repeatedly set  $A^{-1} = A^{-1}(2I - AA^{-1}) \pmod{p^k}$  where  $A^{-1}$  starts as the inverse of  $A$  mod  $p$ , and  $k$  is doubled in each step.

**Time:**  $\mathcal{O}(n^3)$  ebffff6, 35 lines

```
int matInv(vector<vector<double>>& A) {
    int n = sz(A); vi col(n);
    vector<vector<double>> tmp(n, vector<double>(n));
    rep(i, 0, n) tmp[i][i] = 1, col[i] = i;

    rep(i, 0, n) {
        int r = i, c = i;
        rep(j, i, n) rep(k, i, n)
            if (fabs(A[j][k]) > fabs(A[r][c]))
                r = j, c = k;
        if (fabs(A[r][c]) < 1e-12) return i;
        A[i].swap(A[r]); tmp[i].swap(tmp[r]);
        rep(j, 0, n)
            swap(A[j][i], A[j][c]), swap(tmp[j][i], tmp[j][c]);
        swap(col[i], col[c]);
        double v = A[i][i];
        rep(j, i+1, n) {
            double f = A[j][i] / v;
            A[j][i] = 0;
            rep(k, i+1, n) A[j][k] -= f*A[i][k];
            rep(k, 0, n) tmp[j][k] -= f*tmp[i][k];
        }
        rep(j, i+1, n) A[i][j] /= v;
        rep(j, 0, n) tmp[i][j] /= v;
        A[i][i] = 1;
    }

    for (int i = n-1; i > 0; --i) rep(j, 0, i) {
        double v = A[j][i];
        rep(k, 0, n) tmp[j][k] -= v*tmp[i][k];
    }

    rep(i, 0, n) rep(j, 0, n) A[col[i]][col[j]] = tmp[i][j];
    return n;
}
```

## Tridiagonal.h

**Description:**  $x = \text{tridiagonal}(d, p, q, b)$  solves the equation system

$$\begin{pmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{n-1} \end{pmatrix} = \begin{pmatrix} d_0 & p_0 & 0 & 0 & \cdots & 0 \\ q_0 & d_1 & p_1 & 0 & \cdots & 0 \\ 0 & q_1 & d_2 & p_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & q_{n-3} & d_{n-2} & p_{n-2} \\ 0 & 0 & \cdots & 0 & q_{n-2} & d_{n-1} \end{pmatrix} \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \end{pmatrix}.$$

This is useful for solving problems on the type

$$a_i = b_i a_{i-1} + c_i a_{i+1} + d_i, \quad 1 \leq i \leq n,$$

where  $a_0, a_{n+1}, b_i, c_i$  and  $d_i$  are known.  $a$  can then be obtained from

$$\{a_i\} = \text{tridiagonal}(\{1, -1, -1, \dots, -1, 1\}, \{0, c_1, c_2, \dots, c_n\}, \{b_1, b_2, \dots, b_n, 0\}, \{a_0, d_1, d_2, \dots, d_n, a_{n+1}\}).$$

Fails if the solution is not unique.

If  $|d_i| > |p_i| + |q_{i-1}|$  for all  $i$ , or  $|d_i| > |p_{i-1}| + |q_i|$ , or the matrix is positive definite, the algorithm is numerically stable and neither  $\text{tr}$  nor the check for  $\text{diag}[i] == 0$  is needed.

**Time:**  $\mathcal{O}(N)$  8f9fa8, 26 lines

```
typedef double T;
vector<T> tridiagonal(vector<T> diag, const vector<T>& super,
                       const vector<T>& sub, vector<T> b) {
    int n = sz(b); vi tr(n);
    rep(i, 0, n-1) {
        if (abs(diag[i]) < 1e-9 * abs(super[i])) { // diag[i] == 0
            b[i+1] -= b[i] * diag[i+1] / super[i];
            if (i+2 < n) b[i+2] -= b[i] * sub[i+1] / super[i];
            diag[i+1] = sub[i]; tr[i+1] = 1;
        }
    }
}
```

```
} else {
    diag[i+1] -= super[i]*sub[i]/diag[i];
    b[i+1] -= b[i]*sub[i]/diag[i];
}
}
for (int i = n; i--;) {
    if (tr[i]) {
        swap(b[i], b[i-1]);
        diag[i-1] = diag[i];
        b[i] /= super[i-1];
    } else {
        b[i] /= diag[i];
        if (i) b[i-1] -= b[i]*super[i-1];
    }
}
return b;
}
```

## 4.4 Fourier transforms

## FastFourierTransform.h

**Description:**  $\text{fft}(a)$  computes  $\hat{f}(k) = \sum_x a[x] \exp(2\pi i \cdot kx/N)$  for all  $k$ .  $N$  must be a power of 2. Useful for convolution:  $\text{conv}(a, b) = c$ , where  $c[x] = \sum a[i]b[x-i]$ . For convolution of complex numbers or more than two vectors: FFT, multiply pointwise, divide by  $n$ , reverse(start+1, end), FFT back. Rounding is safe if  $(\sum a_i^2 + \sum b_i^2) \log_2 N < 9 \cdot 10^{14}$  (in practice  $10^{16}$ ; higher for random inputs). Otherwise, use NTT/FFTMod.

**Time:**  $\mathcal{O}(N \log N)$  with  $N = |A| + |B|$  ( $\sim 1s$  for  $N = 2^{22}$ ) 00ced6, 35 lines

```
typedef complex<double> C;
typedef vector<double> vd;
void fft(vector<C>& a) {
    int n = sz(a), L = 31 - __builtin_clz(n);
    static vector<complex<long double>> R(2, 1);
    static vector<C> rt(2, 1); // (^ 10% faster if double)
    for (static int k = 2; k < n; k *= 2)
        R.resize(n); rt.resize(n);
        auto x = polar(1.0L, acos(-1.0L) / k);
        rep(i, k, 2*k) rt[i] = R[i] = i&1 ? R[i/2] * x : R[i/2];
    }
    vi rev(n);
    rep(i, 0, n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
    rep(i, 0, n) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2)
        for (int i = 0; i < n; i += 2 * k) rep(j, 0, k) {
            C z = rt[j+k] * a[i+j+k]; // (25% faster if hand-rolled)
            a[i + j + k] = a[i + j] - z;
            a[i + j] += z;
        }
}
vd conv(const vd& a, const vd& b) {
    if (a.empty() || b.empty()) return {};
    vd res(sz(a) + sz(b) - 1);
    int L = 32 - __builtin_clz(sz(res)), n = 1 << L;
    vector<C> in(n), out(n);
    copy(all(a), begin(in));
    rep(i, 0, sz(b)) in[i].imag(b[i]);
    fft(in);
    for (C& x : in) x *= x;
    rep(i, 0, n) out[i] = in[-i & (n - 1)] - conj(in[i]);
    fft(out);
    rep(i, 0, sz(res)) res[i] = imag(out[i]) / (4 * n);
    return res;
}
```

## FastFourierTransformMod.h

**Description:** Higher precision FFT, can be used for convolutions modulo arbitrary integers as long as  $N \log_2 N \cdot \text{mod} < 8.6 \cdot 10^{14}$  (in practice  $10^{16}$  or higher). Inputs must be in  $[0, \text{mod}]$ .

**Time:**  $\mathcal{O}(N \log N)$ , where  $N = |A| + |B|$  (twice as slow as NTT or FFT)  
**FastFourierTransform.h**  

```
b82773, 22 lines
```

```
typedef vector<ll> vl;
template<int M> vl convMod(const vl &a, const vl &b) {
    if (a.empty() || b.empty()) return {};
    vl res(sz(a) + sz(b) - 1);
    int B=32-_builtin_clz(sz(res)), n=1<<B, cut=int(sqrt(M));
    vector<C> L(n), R(n), outs(n), outl(n);
    rep(i,0,sz(a)) L[i] = C((int)a[i] / cut, (int)a[i] % cut);
    rep(i,0,sz(b)) R[i] = C((int)b[i] / cut, (int)b[i] % cut);
    fft(L), fft(R);
    rep(i,0,n) {
        int j = -i & (n - 1);
        outl[j] = (L[i] + conj(L[j])) * R[i] / (2.0 * n);
        outs[j] = (L[i] - conj(L[j])) * R[i] / (2.0 * n) / li;
    }
    fft(outl), fft(outs);
    rep(i,0,sz(res)) {
        ll av = 11(real(outl[i])+.5), cv = 11(imag(outs[i])+.5);
        ll bv = 11(imag(outl[i])+.5) + 11(real(outs[i])+.5);
        res[i] = ((av % M * cut + bv) % M * cut + cv) % M;
    }
    return res;
}
```

## NumberTheoreticTransform.h

**Description:** ntt(a) computes  $\hat{f}(k) = \sum_x a[x]g^{xk}$  for all  $k$ , where  $g = \text{root}^{(mod-1)/N}$ .  $N$  must be a power of 2. Useful for convolution modulo specific nice primes of the form  $2^a b + 1$ , where the convolution result has size at most  $2^a$ . For arbitrary modulo, see FFTMod. conv(a, b) = c, where  $c[x] = \sum a[i]b[x-i]$ . For manual convolution: NTT the inputs, multiply pointwise, divide by n, reverse(start+1, end), NTT back. Inputs must be in [0, mod].

**Time:**  $\mathcal{O}(N \log N)$

**ModPow.h**  

```
.../number-theory/ModPow.h" ced03d, 35 lines
```

```
const ll mod = (119 << 23) + 1, root = 62; // = 998244353
// For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 << 21
// and 483 << 21 (same root). The last two are > 10^9.
typedef vector<ll> vl;
void ntt(vl &a) {
    int n = sz(a), L = 31 - _builtin_clz(n);
    static vl rt(2, 1);
    for (static int k = 2, s = 2; k < n; k *= 2, s++) {
        rt.resize(n);
        ll z[] = {1, modpow(root, mod >> s)};
        rep(i,k,2*k) rt[i] = rt[i / 2] * z[i & 1] % mod;
    }
    vi rev(n);
    rep(i,0,n) rev[i] = (rev[i / 2] | (i & 1) << L) / 2;
    rep(i,0,n) if (i < rev[i]) swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k *= 2)
        for (int i = 0; i < n; i += 2 * k) rep(j,0,k) {
            ll z = rt[j + k] * a[i + j + k] % mod, &ai = a[i + j];
            a[i + j + k] = ai - z + (z > ai ? mod : 0);
            ai += (ai + z >= mod ? z - mod : z);
        }
}
vl conv(const vl &a, const vl &b) {
    if (a.empty() || b.empty()) return {};
    int s = sz(a) + sz(b) - 1, B = 32 - _builtin_clz(s),
        n = 1 << B;
    int inv = modpow(n, mod - 2);
    vl L(a), R(b), out(n);
    L.resize(n), R.resize(n);
    ntt(L), ntt(R);
    rep(i,0,n)
        out[-i & (n - 1)] = (ll)L[i] * R[i] % mod * inv % mod;
    ntt(out);
}
```

```
return {out.begin(), out.begin() + s};
}
```

## FastSubsetTransform.h

**Description:** Transform to a basis with fast convolutions of the form  $c[z] = \sum_{z=x \oplus y} a[x] \cdot b[y]$ , where  $\oplus$  is one of AND, OR, XOR. The size of  $a$  must be a power of two.

**Time:**  $\mathcal{O}(N \log N)$

**464cf3, 16 lines**

```
void FST(vi& a, bool inv) {
    for (int n = sz(a), step = 1; step < n; step *= 2) {
        for (int i = 0; i < n; i += 2 * step) rep(j,i,i+step) {
            int &u = a[j], &v = a[j + step]; tie(u, v) =
                inv ? pii(v - u, u) : pii(v, u + v); // AND
                inv ? pii(v, u - v) : pii(u + v, u); // OR
                pii(u + v, u - v); // XOR
        }
        if (inv) for (int& x : a) x /= sz(a); // XOR only
    }
    vi conv(vi a, vi b) {
        FST(a, 0); FST(b, 0);
        rep(i,0,sz(a)) a[i] *= b[i];
        FST(a, 1); return a;
    }
}
```

## Number theory (5)

## 5.1 Modular arithmetic

## ModLog.h

**Description:** Returns the smallest  $x > 0$  s.t.  $a^x \equiv b \pmod{m}$ , or  $-1$  if no such  $x$  exists. modLog(a, l, m) can be used to calculate the order of  $a$ .

**Time:**  $\mathcal{O}(\sqrt{m})$

**c040b8, 11 lines**

```
11 modLog(ll a, ll b, ll m) {
    11 n = (11) sqrt(m) + 1, e = 1, f = 1, j = 1;
    unordered_map<ll, ll> A;
    while (j <= n && (e = f = e * a % m) != b % m)
        A[e % m] = j++;
    if (e == b % m) return j;
    if (__gcd(m, e) == __gcd(m, b))
        rep(i,2,n+2) if (A.count(e = e * f % m))
            return n * i - A[e];
    return -1;
}
```

## ModSqrt.h

**Description:** Tonelli-Shanks algorithm for modular square roots. Finds  $x$  s.t.  $x^2 \equiv a \pmod{p}$  ( $-x$  gives the other solution).

**Time:**  $\mathcal{O}(\log^2 p)$  worst case,  $\mathcal{O}(\log p)$  for most  $p$

**ModPow.h**  

```
.../ModPow.h" 19a793, 24 lines
```

```
11 sqrt(ll a, ll p) {
    a %= p; if (a < 0) a += p;
    if (a == 0) return 0;
    assert(modpow(a, (p-1)/2, p) == 1); // else no solution
    if (p % 4 == 3) return modpow(a, (p+1)/4, p);
    // a^(n+3)/8 or 2^(n+3)/8 * 2^(n-1)/4 works if p % 8 == 5
    11 s = p - 1, n = 2;
    int r = 0, m;
    while (s % 2 == 0)
        ++r, s /= 2;
    while (modpow(n, (p - 1) / 2, p) != p - 1) ++n;
    11 x = modpow(a, (s + 1) / 2, p);
    11 b = modpow(a, s, p), g = modpow(n, s, p);
    for (; r = m) {
        11 t = b;
```

```
for (m = 0; m < r && t != 1; ++m)
    t = t * t % p;
if (m == r) return x;
11 gs = modpow(g, 1LL << (r - m - 1), p);
g = gs * gs % p;
x = x * gs % p;
b = b * g % p;
}
```

## 5.2 Primality

## FastEratosthenes.h

**Description:** Prime sieve for generating all primes smaller than LIM.

**Time:**  $LIM=1e9 \approx 1.5s$

**6b2912, 20 lines**

```
const int LIM = 1e6;
bitset<LIM> isPrime;
vi eratosthenes() {
    const int S = (int)round(sqrt(LIM)), R = LIM / 2;
    vi pr = {2}, sieve(S+1); pr.reserve(int(LIM/log(LIM)*1.1));
    vector<pii> cp;
    for (int i = 3; i <= S; i += 2) if (!sieve[i]) {
        cp.push_back({i, i * i / 2});
        for (int j = i * i; j <= S; j += 2 * i) sieve[j] = 1;
    }
    for (int L = 1; L <= R; L += S) {
        array<bool, S> block{};
        for (auto &p, idx : cp)
            for (int i=idx; i < S+L; idx = (i+=p)) block[i-L] = 1;
        rep(i,0,min(S, R - L))
            if (!block[i]) pr.push_back((L + i) * 2 + 1);
    }
    for (int i : pr) isPrime[i] = 1;
    return pr;
}
```

## MillerRabin.h

**Description:** Deterministic Miller-Rabin primality test. Guaranteed to work for numbers up to  $7 \cdot 10^{18}$ ; for larger numbers, use Python and extend A randomly.

**Time:** 7 times the complexity of  $a^b \pmod{c}$ .

**"ModMulLL.h"**  

```
.../ModMulLL.h" 60dcfd1, 12 lines
```

```
bool isPrime(ull n) {
    if (n < 2 || n % 6 != 1) return (n | 1) == 3;
    ull A[] = {2, 325, 9375, 28178, 450775, 9780504, 1795265022},
        s = __builtin_ctzll(n-1), d = n >> s;
    for (ull a : A) { // ^ count trailing zeroes
        ull p = modpow(a, n, d), i = s;
        while (p != 1 && p != n - 1 && a % n && i--)
            p = modmul(p, p, n);
        if (p != n-1 && i != s) return 0;
    }
    return 1;
}
```

## Factor.h

**Description:** Pollard-rho randomized factorization algorithm. Returns prime factors of a number, in arbitrary order (e.g. 2299 -> {11, 19, 11}).

**Time:**  $\mathcal{O}(n^{1/4})$ , less for numbers with small factors.

**"ModMulLL.h", "MillerRabin.h"**  

```
.../ModMulLL.h", "MillerRabin.h" d8d98d, 18 lines
```

```
ull pollard(ull n) {
    ull x = 0, y = 0, t = 30, prd = 2, i = 1, q;
    auto f = [&x](ull x) { return modmul(x, x, n) + i; };
    while (t++ % 40 || __gcd(prd, n) == 1) {
        if (x == y) x = ++i, y = f(x);
        if ((q = modmul(prd, max(x,y) - min(x,y), n)) prd = q;
            x = f(x), y = f(f(y));
    }
```

```

    }
    return __gcd(prd, n);
}

vector<ull> factor(ull n) {
    if (n == 1) return {};
    if (isPrime(n)) return {n};
    ull x = pollard(n);
    auto l = factor(x), r = factor(n / x);
    l.insert(l.end(), all(r));
    return l;
}

```

## PrimeFact.h

Description: idk  
Time: idk

bcd355, 16 lines

```

const int N = 1e7;
vector<int> spf(N);
void prep() { // modified sieve
    L(i, 2, N-1) {
        if(spf[i] == 0) {
            spf[i] = i;
            for(ll j = i * i; j <= N-1; j += i) {
                if(spf[j] == 0) spf[j] = i; }}
```

// prime factorize [if you need 1e9, use loop upto sqrt]

```

vector<pll> prime_factorize(ll x) {
    vector<pll> ans;
    while(x!=1) {
        ll cur = spf[x]; ll cnt = 0;
        while(x%cur==0) cnt++, x/=cur;
        ans.push_back({cur, cnt}); }
    return ans; }
```

## 5.3 Divisibility

### euclid.h

Description: Finds two integers  $x$  and  $y$ , such that  $ax + by = \gcd(a, b)$ . If you just need  $\gcd$ , use the built in `__gcd` instead. If  $a$  and  $b$  are coprime, then  $x$  is the inverse of  $a$  (mod  $b$ ).

33ba8f, 5 lines

```

ll euclid(ll a, ll b, ll &x, ll &y) {
    if (!b) return x = 1, y = 0, a;
    ll d = euclid(b, a % b, y, x);
    return y -= a/b * x, d;
}
```

### CRT.h

Description: Chinese Remainder Theorem.

`crt(a, m, b, n)` computes  $x$  such that  $x \equiv a \pmod{m}$ ,  $x \equiv b \pmod{n}$ . If  $|a| < m$  and  $|b| < n$ ,  $x$  will obey  $0 \leq x < \text{lcm}(m, n)$ . Assumes  $mn < 2^{62}$ .

Time:  $\log(n)$

04d93a, 7 lines

```

ll crt(ll a, ll m, ll b, ll n) {
    if (n > m) swap(a, b), swap(m, n);
    ll x, y, g = euclid(m, n, x, y);
    assert((a - b) % g == 0); // else no solution
    x = (b - a) % n * x % n / g * m + a;
    return x < 0 ? x + m*n/g : x;
}
```

### 5.3.1 Bézout's identity

For  $a \neq 0, b \neq 0$ , then  $d = \gcd(a, b)$  is the smallest positive integer for which there are integer solutions to

$$ax + by = d$$

## PrimeFact euclid CRT phiFunction IntPerm

If  $(x, y)$  is one solution, then all solutions are given by

$$\left( x + \frac{kb}{\gcd(a, b)}, y - \frac{ka}{\gcd(a, b)} \right), \quad k \in \mathbb{Z}$$

### phiFunction.h

Description: Euler's  $\phi$  function is defined as  $\phi(n) := \#$  of positive integers  $\leq n$  that are coprime with  $n$ .  $\phi(1) = 1$ ,  $p$  prime  $\Rightarrow \phi(p^k) = (p-1)p^{k-1}$ ,  $m, n$  coprime  $\Rightarrow \phi(mn) = \phi(m)\phi(n)$ . If  $n = p_1^{k_1}p_2^{k_2}\dots p_r^{k_r}$  then  $\phi(n) = (p_1-1)p_1^{k_1-1}\dots(p_r-1)p_r^{k_r-1}$ .  $\phi(n) = n \cdot \prod_{p|n} (1 - 1/p)$ .

$\sum_{d|n} \phi(d) = n$ ,  $\sum_{1 \leq k \leq n, \gcd(k, n)=1} k = n\phi(n)/2$ ,  $n > 1$

Euler's thm:  $a, n$  coprime  $\Rightarrow a^{\phi(n)} \equiv 1 \pmod{n}$ .

Fermat's little thm:  $p$  prime  $\Rightarrow a^{p-1} \equiv 1 \pmod{p} \forall a$ .

$$g(n) = \sum_{d|n} f(d) \Leftrightarrow f(n) = \sum_{d|n} \mu(d/n)g(d)$$

$$g(n) = \sum_{1 \leq m \leq n} f(\lfloor \frac{n}{m} \rfloor) \Leftrightarrow f(n) = \sum_{1 \leq m \leq n} \mu(m)g(\lfloor \frac{n}{m} \rfloor)$$

## Combinatorial (6)

### 6.1 Permutations

#### 6.1.1 Factorial

$n$	1	2	3	4	5	6	7	8	9	10
$n!$	1	2	6	24	120	720	5040	40320	362880	3628800
$n$		11	12	13	14	15	16	17		
$n!$	4.0e7	4.8e8	6.2e9	8.7e10	1.3e12	2.1e13	3.6e14			
$n$	20	25	30	40	50	100	150	171		
$n!$	2e18	2e25	3e32	8e47	3e64	9e157	6e262	>DBL_MAX		

### IntPerm.h

Description: Permutation  $\rightarrow$  integer conversion. (Not order preserving.) Integer  $\rightarrow$  permutation can use a lookup table.

Time:  $\mathcal{O}(n)$

044568, 6 lines

```

int permToInt(vi& v) {
    int use = 0, i = 0, r = 0;
    for(int x:v) r = r * ++i + __builtin_popcount(use & -(1<<x)),
        use |= 1 << x; // (note: minus, not ~!)
    return r;
}
```

#### 6.1.2 Cycles

Let  $gs(n)$  be the number of  $n$ -permutations whose cycle lengths all belong to the set  $S$ . Then

$$\sum_{n=0}^{\infty} gs(n) \frac{x^n}{n!} = \exp \left( \sum_{n \in S} \frac{x^n}{n} \right)$$

#### 6.1.3 Derangements

Permutations of a set such that none of the elements appear in their original position.

$$D(n) = (n-1)(D(n-1) + D(n-2)) = nD(n-1) + (-1)^n = \left[ \frac{n!}{e} \right]$$

#### 6.1.4 Burnside's lemma

Given a group  $G$  of symmetries and a set  $X$ , the number of elements of  $X$  up to symmetry equals

$$\frac{1}{|G|} \sum_{g \in G} |X^g|,$$

where  $X^g$  are the elements fixed by  $g$  ( $g \cdot x = x$ ).

If  $f(n)$  counts “configurations” (of some sort) of length  $n$ , we can ignore rotational symmetry using  $G = \mathbb{Z}_n$  to get

$$g(n) = \frac{1}{n} \sum_{k=0}^{n-1} f(\gcd(n, k)) = \frac{1}{n} \sum_{k|n} f(k)\phi(n/k).$$

Other useful formulas/forms:

$$\sum_{d|n} \mu(d) = [n = 1] \text{ (very useful)}$$

## 6.2 Partitions and subsets

### 6.2.1 Partition function

Number of ways of writing  $n$  as a sum of positive integers, disregarding the order of the summands.

$$p(0) = 1, p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$$

$$p(n) \sim 0.145/n \cdot \exp(2.56\sqrt{n})$$

$n$	0	1	2	3	4	5	6	7	8	9	20	50	100
$p(n)$	1	1	2	3	5	7	11	15	22	30	627	$\sim 2e5$	$\sim 2e8$

### 6.2.2 Lucas' Theorem

Let  $n, m$  be non-negative integers and  $p$  a prime. Write  $n = n_k p^k + \dots + n_1 p + n_0$  and  $m = m_k p^k + \dots + m_1 p + m_0$ . Then  $\binom{n}{m} \equiv \prod_{i=0}^k \binom{n_i}{m_i} \pmod{p}$ .

### 6.2.3 Binomials

multinomial.h

Description: Computes  $\binom{k_1 + \dots + k_n}{k_1, k_2, \dots, k_n} = \frac{(\sum k_i)!}{k_1! k_2! \dots k_n!}$ .

```
ll multinomial(vi& v) {
    ll c = 1, m = v.empty() ? 1 : v[0];
    rep(i, 1, sz(v)) rep(j, 0, v[i]) c = c * ++m / (j+1);
    return c;
}
```

## 6.3 General purpose numbers

### 6.3.1 Bernoulli numbers

EGF of Bernoulli numbers is  $B(t) = \frac{t}{e^t - 1}$  (FFT-table).

$$B[0, \dots] = [1, -\frac{1}{2}, \frac{1}{6}, 0, -\frac{1}{30}, 0, \frac{1}{42}, \dots]$$

Sums of powers:

$$\sum_{i=1}^n i^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k \cdot (n+1)^{m+1-k}$$

Euler-Maclaurin formula for infinite sums:

$$\begin{aligned} \sum_{i=m}^{\infty} f(i) &= \int_m^{\infty} f(x) dx - \sum_{k=1}^{\infty} \frac{B_k}{k!} f^{(k-1)}(m) \\ &\approx \int_m^{\infty} f(x) dx + \frac{f(m)}{2} - \frac{f'(m)}{12} + \frac{f'''(m)}{720} + O(f^{(5)}(m)) \end{aligned}$$

### 6.3.2 Stirling numbers of the first kind

Number of permutations on  $n$  items with  $k$  cycles.

$$c(n, k) = c(n-1, k-1) + (n-1)c(n-1, k), c(0, 0) = 1$$

$$\sum_{k=0}^n c(n, k)x^k = x(x+1)\dots(x+n-1)$$

$$c(8, k) = 8, 0, 5040, 13068, 13132, 6769, 1960, 322, 28, 1$$

$$c(n, 2) = 0, 0, 1, 3, 11, 50, 274, 1764, 13068, 109584, \dots$$

## multinomial Dijkstra BellmanFord FloydWarshall

### 6.3.3 Eulerian numbers

Number of permutations  $\pi \in S_n$  in which exactly  $k$  elements are greater than the previous element.  $k$   $j$ :s s.t.  $\pi(j) > \pi(j+1)$ ,  $k+1$   $j$ :s s.t.  $\pi(j) \geq j$ ,  $k$   $j$ :s s.t.  $\pi(j) > j$ .

$$E(n, k) = (n-k)E(n-1, k-1) + (k+1)E(n-1, k)$$

$$E(n, 0) = E(n, n-1) = 1$$

$$E(n, k) = \sum_{j=0}^k (-1)^j \binom{n+1}{j} (k+1-j)^n$$

### 6.3.4 Stirling numbers of the second kind

Partitions of  $n$  distinct elements into exactly  $k$  groups.

$$S(n, k) = S(n-1, k-1) + kS(n-1, k)$$

$$S(n, 1) = S(n, n) = 1$$

$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$

### 6.3.5 Bell numbers

Total number of partitions of  $n$  distinct elements.  $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$  For  $p$  prime,

$$B(p^m + n) \equiv mB(n) + B(n+1) \pmod{p}$$

### 6.3.6 Labeled unrooted trees

# on  $n$  vertices:  $n^{n-2}$

# on  $k$  existing trees of size  $n_i$ :  $n_1 n_2 \dots n_k n^{k-2}$

# with degrees  $d_i$ :  $(n-2)! / ((d_1-1)! \dots (d_n-1)!)$

### 6.3.7 Catalan numbers

$$C_n = \frac{1}{n+1} \binom{2n}{n} = \binom{2n}{n} - \binom{2n}{n+1} = \frac{(2n)!}{(n+1)!n!}$$

$$C_0 = 1, C_{n+1} = \frac{2(2n+1)}{n+2} C_n, C_{n+1} = \sum C_i C_{n-i}$$

$$C_n = 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, \dots$$

- sub-diagonal monotone paths in an  $n \times n$  grid.
- strings with  $n$  pairs of parenthesis, correctly nested.
- binary trees with  $n+1$  leaves (0 or 2 children).
- ordered trees with  $n+1$  vertices.
- ways a convex polygon with  $n+2$  sides can be cut into triangles by connecting vertices with straight lines.
- permutations of  $[n]$  with no 3-term increasing subseq.

## Graph (7)

### 7.1 Fundamentals

### Dijkstra.h

Description: idk

Time: idk

---

```
bbab71, 15 lines
vector<ll> dijkstra(ll start, vector<vector<array<ll, 2>>> &gr,
                     int n) {
    vector<ll> d(n); For(i, 1, n) d[i] = inf; d[start] = 0;
    bool vis[n+1] = {0};
    priority_queue<array<ll, 2>, vector<array<ll, 2>>, greater<
array<ll, 2>> pq;
    pq.push({0LL, start});
    while(!pq.empty()) {
        auto [dist, node] = pq.top(); pq.pop();
        if(vis[node]) continue;
        vis[node] = 1;
        for(auto [nextNode, weight] : gr[node]) {
            if(weight + dist < d[nextNode]){
                d[nextNode] = weight + dist;
                pq.push({d[nextNode], nextNode}); }}
```

### BellmanFord.h

Description: Calculates shortest paths from  $s$  in a graph that might have negative edge weights. Unreachable nodes get dist = inf; nodes reachable through negative-weight cycles get dist = -inf. Assumes  $V^2 \max|w_i| < \sim 2^{63}$ .

Time:  $\mathcal{O}(VE)$

---

```
830a8f, 23 lines
const ll inf = LLONG_MAX;
struct Ed { int a, b, w, s() { return a < b ? a : -a; }};
struct Node { ll dist = inf; int prev = -1; };

void bellmanFord(vector<Node>& nodes, vector<Ed>& eds, int s) {
    nodes[s].dist = 0;
    sort(all(eds), [] (Ed a, Ed b) { return a.s() < b.s(); });

    int lim = sz(nodes) / 2 + 2; // /3+100 with shuffled vertices
    rep(i, 0, lim) for (Ed ed : eds) {
        Node cur = nodes[ed.a], &dest = nodes[ed.b];
        if (abs(cur.dist) == inf) continue;
        ll d = cur.dist + ed.w;
        if (d < dest.dist) {
            dest.prev = ed.a;
            dest.dist = (i < lim-1 ? d : -inf);
        }
    }
    rep(i, 0, lim) for (Ed e : eds) {
        if (nodes[e.a].dist == -inf)
            nodes[e.b].dist = -inf;
    }
}
```

### FloydWarshall.h

Description: Calculates all-pairs shortest path in a directed graph that might have negative edge weights. Input is an distance matrix  $m$ , where  $m[i][j] = \text{inf}$  if  $i$  and  $j$  are not adjacent. As output,  $m[i][j]$  is set to the shortest distance between  $i$  and  $j$ ,  $\text{inf}$  if no path, or  $-\text{inf}$  if the path goes through a negative-weight cycle.

Time:  $\mathcal{O}(N^3)$

---

```
531245, 12 lines
const ll inf = 1LL << 62;
void floydWarshall(vector<vector<ll>>& m) {
    int n = sz(m);
    rep(i, 0, n) m[i][i] = min(m[i][i], 0LL);
    rep(k, 0, n) rep(i, 0, n) rep(j, 0, n)
        if (m[i][k] != inf && m[k][j] != inf) {
            auto newDist = max(m[i][k] + m[k][j], -inf);
            m[i][j] = min(m[i][j], newDist);
        }
    rep(k, 0, n) if (m[k][k] < 0) rep(i, 0, n) rep(j, 0, n)
```

```

    if (m[i][k] != inf && m[k][j] != inf) m[i][j] = -inf;
}

```

## TopoSort.h

**Description:** Topological sorting. Given is an oriented graph. Output is an ordering of vertices, such that there are edges only from left to right. If there are cycles, the returned list will have size smaller than  $n$  – nodes reachable from cycles will not be returned.

**Time:**  $\mathcal{O}(|V| + |E|)$

d678d8, 8 lines

```

vi topoSort(const vector<vi>& gr) {
    vi indeg(sz(gr)), q;
    for (auto& li : gr) for (int x : li) indeg[x]++;
    rep(i, 0, sz(gr)) if (indeg[i] == 0) q.push_back(i);
    rep(j, 0, sz(q)) for (int x : gr[q[j]]) {
        if (--indeg[x] == 0) q.push_back(x);
    }
    return q;
}

```

## 7.2 Network flow

## PushRelabel.h

**Description:** Push-relabel using the highest label selection rule and the gap heuristic. Quite fast in practice. To obtain the actual flow, look at positive values only.

**Time:**  $\mathcal{O}(V^2\sqrt{E})$

0ae1d4, 48 lines

```

struct PushRelabel {
    struct Edge {
        int dest, back;
        ll f, c;
    };
    vector<vector<Edge>> g;
    vector<ll> ec;
    vector<Edge*> cur;
    vector<vi> hs; vi H;
    PushRelabel(int n) : g(n), ec(n), cur(n), hs(2*n), H(n) {}

    void addEdge(int s, int t, ll cap, ll rcap=0) {
        if (s == t) return;
        g[s].push_back({t, sz(g[t]), 0, cap});
        g[t].push_back({s, sz(g[s])-1, 0, rcap});
    }

    void addFlow(Edge& e, ll f) {
        Edge &back = g[e.dest][e.back];
        if (!ec[e.dest] && f) hs[H[e.dest]].push_back(e.dest);
        e.f += f; e.c -= f; ec[e.dest] += f;
        back.f -= f; back.c += f; ec[back.dest] -= f;
    }

    ll calc(int s, int t) {
        int v = sz(g); H[s] = v; ec[t] = 1;
        vi co(2*v); co[0] = v-1;
        rep(i, 0, v) cur[i] = g[i].data();
        for (Edge& e : g[s]) addFlow(e, e.c);

        for (int hi = 0;;) {
            while (hs[hi].empty()) if (!hi--) return -ec[s];
            int u = hs[hi].back(); hs[hi].pop_back();
            while (ec[u] > 0) // discharge u
                if (cur[u] == g[u].data() + sz(g[u])) {
                    H[u] = 1e9;
                    for (Edge& e : g[u]) if (e.c && H[u] > H[e.dest]+1)
                        H[u] = H[e.dest]+1, cur[u] = &e;
                    if (++co[H[u]], !--co[hi] && hi < v)
                        rep(i, 0, v) if (hi < H[i] && H[i] < v)
                            --co[H[i]], H[i] = v + 1;
                    hi = H[u];
                } else if (cur[u]->c && H[u] == H[cur[u]->dest]+1)

```

## TopoSort PushRelabel MinCostMaxFlow EdmondsKarp

```

        addFlow(*cur[u], min(ec[u], cur[u]->c));
    }
    bool leftOfMinCut(int a) { return H[a] >= sz(g); }
}

```

## MinCostMaxFlow.h

**Description:** Min-cost max-flow. If costs can be negative, call setpi before maxflow, but note that negative cost cycles are not supported. To obtain the actual flow, look at positive values only.

**Time:**  $\mathcal{O}(FE \log(V))$  where  $F$  is max flow.  $\mathcal{O}(VE)$  for setpi. 58385b, 79 lines

```

#include <bits/extc++.h>

const ll INF = numeric_limits<ll>::max() / 4;

struct MCMF {
    struct edge {
        int from, to, rev;
        ll cap, cost, flow;
    };
    int N;
    vector<vector<edge>> ed;
    vi seen;
    vector<ll> dist, pi;
    vector<edge*> par;
    MCMF(int N) : N(N), ed(N), seen(N), dist(N), pi(N), par(N) {}

    void addEdge(int from, int to, ll cap, ll cost) {
        if (from == to) return;
        ed[from].push_back(edge{from, to, sz(ed[to]), cap, cost, 0 });
        ed[to].push_back(edge{to, from, sz(ed[from])-1, 0, -cost, 0 });
    }

    void path(int s) {
        fill(all(seen), 0);
        fill(all(dist), INF);
        dist[s] = 0; ll di;
        __gnu_pbds::priority_queue<pair<ll, int>> q;
        vector<decltype(q)::point_iterator> its(N);
        q.push({0, s});

        while (!q.empty()) {
            s = q.top().second; q.pop();
            seen[s] = 1; di = dist[s] + pi[s];
            for (edge& e : ed[s]) if (!seen[e.to]) {
                ll val = di - pi[e.to] + e.cost;
                if (e.cap - e.flow > 0 && val < dist[e.to]) {
                    dist[e.to] = val;
                    par[e.to] = &e;
                    if (its[e.to] == q.end())
                        its[e.to] = q.push({-dist[e.to], e.to });
                    else
                        q.modify(its[e.to], {-dist[e.to], e.to });
                }
            }
            rep(i, 0, N) pi[i] = min(pi[i] + dist[i], INF);
        }

        pair<ll, ll> maxflow(int s, int t) {
            ll totflow = 0, totcost = 0;
            while (path(s), seen[t]) {
                ll fl = INF;
                for (edge* x = par[t]; x; x = par[x->from])
                    fl = min(fl, x->cap - x->flow);

```

```

                totflow += fl;
                for (edge* x = par[t]; x; x = par[x->from]) {
                    x->flow += fl;
                    ed[x->to][x->rev].flow -= fl;
                }
            }
            rep(i, 0, N) for (edge& e : ed[i]) totcost += e.cost * e.flow;
            return {totflow, totcost/2};
        }

        // If some costs can be negative, call this before maxflow:
        void setpi(int s) { // (otherwise, leave this out)
            fill(all(pi), INF); pi[s] = 0;
            int it = N, ch = 1; ll v;
            while (ch-- && it--) {
                rep(i, 0, N) if (pi[i] != INF)
                    for (edges e : ed[i]) if (e.cap)
                        if ((v = pi[i] + e.cost) < pi[e.to])
                            pi[e.to] = v, ch = 1;
                assert(it >= 0); // negative cost cycle
            }
        }
    };

```

## EdmondsKarp.h

**Description:** Flow algorithm with guaranteed complexity  $O(VE^2)$ . To get edge flow values, compare capacities before and after, and take the positive values only.

482fe0, 36 lines

```

template<class T> T edmondsKarp(vector<unordered_map<int, T>>& graph, int source, int sink) {
    assert(source != sink);
    T flow = 0;
    vi par(sz(graph)), q = par;

    for (;;) {
        fill(all(par), -1);
        par[source] = 0;
        int ptr = 1;
        q[0] = source;

        rep(i, 0, ptr) {
            int x = q[i];
            for (auto e : graph[x]) {
                if (par[e.first] == -1 && e.second > 0) {
                    par[e.first] = x;
                    q[ptr++] = e.first;
                    if (e.first == sink) goto out;
                }
            }
        }
        return flow;
    out:
        T inc = numeric_limits<T>::max();
        for (int y = sink; y != source; y = par[y]) {
            int p = par[y];
            if ((graph[p][y] -= inc) <= 0) graph[p].erase(y);
            graph[y][p] += inc;
        }
    }
}

```

## MinCut.h

**Description:** After running max-flow, the left side of a min-cut from  $s$  to  $t$  is given by all vertices reachable from  $s$ , only traversing edges with positive residual capacity.

## GlobalMinCut.h

**Description:** Find a global minimum cut in an undirected graph, as represented by an adjacency matrix.

**Time:**  $\mathcal{O}(V^3)$

8b0e19, 21 lines

```
pair<int, vi> globalMinCut(vector<vi> mat) {
    pair<int, vi> best = {INT_MAX, {}};
    int n = sz(mat);
    vector<vi> co(n);
    rep(i, 0, n) co[i] = {i};
    rep(ph, 1, n) {
        vi w = mat[0];
        size_t s = 0, t = 0;
        rep(it, 0, n-ph) { //  $O(V^2)$   $\rightarrow O(E \log V)$  with prio. queue
            w[t] = INT_MIN;
            s = t, t = max_element(all(w)) - w.begin();
            rep(i, 0, n) w[i] += mat[t][i];
        }
        best = min(best, {w[t] - mat[t][t], co[t]} );
        co[s].insert(co[s].end(), all(co[t]));
        rep(i, 0, n) mat[s][i] += mat[t][i];
        rep(i, 0, n) mat[i][s] = mat[s][i];
        mat[0][t] = INT_MIN;
    }
    return best;
}
```

## GomoryHu.h

**Description:** Given a list of edges representing an undirected flow graph, returns edges of the Gomory-Hu tree. The max flow between any pair of vertices is given by minimum edge weight along the Gomory-Hu tree path.

**Time:**  $\mathcal{O}(V)$  Flow Computations

PushRelabel.h<sup>\*</sup>

0418b3, 13 lines

```
typedef array<ll, 3> Edge;
vector<Edge> gomoryHu(int N, vector<Edge> ed) {
    vector<Edge> tree;
    vi par(N);
    rep(i, 1, N) {
        PushRelabel D(N); // Dinic also works
        for (Edge t : ed) D.addEdge(t[0], t[1], t[2], t[2]);
        tree.push_back({i, par[i], D.calc(i, par[i])});
        rep(j, i+1, N)
            if (par[j] == par[i] && D.leftOfMinCut(j)) par[j] = i;
    }
    return tree;
}
```

## 7.3 Matching

## hopcroftKarp.h

**Description:** Fast bipartite matching algorithm. Graph  $g$  should be a list of neighbors of the left partition, and  $btoa$  should be a vector full of -1's of the same size as the right partition. Returns the size of the matching,  $btoa[i]$  will be the match for vertex  $i$  on the right side, or -1 if it's not matched.

**Usage:**  $vi btoa(m, -1);$  hopcroftKarp( $g$ ,  $btoa$ );

**Time:**  $\mathcal{O}(\sqrt{VE})$

f612e4, 42 lines

```
bool dfs(int a, int L, vector<vi> &g, vi &btoa, vi &A, vi &B) {
    if (A[a] != L) return 0;
    A[a] = -1;
    for (int b : g[a]) if (B[b] == L + 1) {
        B[b] = 0;
        if (btoa[b] == -1 || dfs(btoa[b], L + 1, g, btoa, A, B))
            return true;
    }
}
return sz(btoa) - (int)count(all(btoa), -1);
}
```

```
    return btoa[b] = a, 1;
}
return 0;
}

int hopcroftKarp(vector<vi> &g, vi &btoa) {
    int res = 0;
    vi A(g.size()), B(btoa.size()), cur, next;
    for (;;) {
        fill(all(A), 0);
        fill(all(B), 0);
        cur.clear();
        for (int a : btoa) if (a != -1) A[a] = -1;
        rep(a, 0, sz(g)) if (A[a] == 0) cur.push_back(a);
        for (int lay = 1; lay++) {
            bool islast = 0;
            next.clear();
            for (int a : cur) for (int b : g[a]) {
                if (btoa[b] == -1) {
                    B[b] = lay;
                    islast = 1;
                }
                else if (btoa[b] != a && !B[b]) {
                    B[b] = lay;
                    next.push_back(btoa[b]);
                }
            }
            if (islast) break;
            if (next.empty()) return res;
            for (int a : next) A[a] = lay;
            cur.swap(next);
        }
        rep(a, 0, sz(g))
            res += dfs(a, 0, g, btoa, A, B);
    }
}
```

## DFSMatching.h

**Description:** Simple bipartite matching algorithm. Graph  $g$  should be a list of neighbors of the left partition, and  $btoa$  should be a vector full of -1's of the same size as the right partition. Returns the size of the matching.  $btoa[i]$  will be the match for vertex  $i$  on the right side, or -1 if it's not matched.

**Usage:**  $vi btoa(m, -1);$  dfsMatching( $g$ ,  $btoa$ );

**Time:**  $\mathcal{O}(VE)$

522b98, 22 lines

```
bool find(int j, vector<vi> &g, vi &btoa, vi &vis) {
    if (btoa[j] == -1) return 1;
    vis[j] = 1; int di = btoa[j];
    for (int e : g[di])
        if (!vis[e] && find(e, g, btoa, vis)) {
            btoa[e] = di;
            return 1;
        }
    return 0;
}
int dfsMatching(vector<vi> &g, vi &btoa) {
    vi vis;
    rep(i, 0, sz(g)) {
        vis.assign(sz(btoa), 0);
        for (int j : g[i])
            if (find(j, g, btoa, vis)) {
                btoa[j] = i;
                break;
            }
    }
    return sz(btoa) - (int)count(all(btoa), -1);
}
```

## MinimumVertexCover.h

**Description:** Finds a minimum vertex cover in a bipartite graph. The size is the same as the size of a maximum matching, and the complement is a maximum independent set.

"DFSMatching.h"

```
vi cover(vector<vi> &g, int n, int m) {
    vi match(m, -1);
    int res = dfsMatching(g, match);
    vector<bool> lfound(n, false);
    for (int it : match) if (it != -1) lfound[it] = false;
    vi q, cover;
    rep(i, 0, n) if (!lfound[i]) q.push_back(i);
    while (!q.empty()) {
        int i = q.back(); q.pop_back();
        lfound[i] = 1;
        for (int e : g[i]) if (!lfound[e] && match[e] != -1) {
            seen[e] = true;
            q.push_back(match[e]);
        }
    }
    rep(i, 0, n) if (!lfound[i]) cover.push_back(i);
    rep(i, 0, m) if (seen[i]) cover.push_back(n+i);
    assert(sz(cover) == res);
    return cover;
}
```

da4196, 20 lines

## WeightedMatching.h

**Description:** Given a weighted bipartite graph, matches every node on the left with a node on the right such that no nodes are in two matchings and the sum of the edge weights is minimal. Takes cost[N][M], where cost[i][j] = cost for  $L[i]$  to be matched with  $R[j]$  and returns (min cost, match), where  $L[i]$  is matched with  $R[\text{match}[i]]$ . Negate costs for max cost. Requires  $N \leq M$ .

**Time:**  $\mathcal{O}(N^2 M)$

1e0fe9, 31 lines

```
pair<int, vi> hungarian(const vector<vi> &a) {
    if (a.empty()) return {0, {}};
    int n = sz(a) + 1, m = sz(a[0]) + 1;
    vi u(n), v(m), p(m), ans(n - 1);
    rep(i, 1, n) {
        p[0] = i;
        int j0 = 0; // add "dummy" worker 0
        vi dist(m, INT_MAX), pre(m, -1);
        vector<bool> done(m + 1);
        do { // dijkstra
            done[j0] = true;
            int i0 = p[j0], j1, delta = INT_MAX;
            rep(j, 1, m) if (!done[j]) {
                auto cur = a[i0 - 1][j - 1] - u[i0] - v[j];
                if (cur < dist[j]) dist[j] = cur, pre[j] = j0;
                if (dist[j] < delta) delta = dist[j], j1 = j;
            }
        } while (j1);
        rep(j, 0, m) {
            if (done[j]) u[p[j]] += delta, v[j] -= delta;
            else dist[j] -= delta;
        }
        j0 = j1;
    } while (p[j0]);
    while (j0) { // update alternating path
        int j1 = pre[j0];
        p[j0] = p[j1], j0 = j1;
        if (dist[j1] < delta) delta = dist[j1], j1 = j;
    }
    rep(j, 1, m) if (p[j]) ans[p[j] - 1] = j - 1;
    return {-v[0], ans}; // min cost
}
```

## GeneralMatching.h

**Description:** Matching for general graphs. Fails with probability  $N/\text{mod}$ .

**Time:**  $\mathcal{O}(N^3)$

```
...numerical/MatrixInverse-mod.h" cb1912, 40 lines
vector<pii> generalMatching(int N, vector<pii>& ed) {
    vector<vector<ll>> mat(N, vector<ll>(N)), A;
    for (pii pa : ed) {
        int a = pa.first, b = pa.second, r = rand() % mod;
        mat[a][b] = r, mat[b][a] = (mod - r) % mod;
    }

    int r = matInv(A = mat), M = 2*N - r, fi, fj;
    assert(r % 2 == 0);

    if (M != N) do {
        mat.resize(M, vector<ll>(M));
        rep(i, 0, N) {
            mat[i].resize(M);
            rep(j, N, M) {
                int r = rand() % mod;
                mat[i][j] = r, mat[j][i] = (mod - r) % mod;
            }
        }
    } while (matInv(A = mat) != M);

    vi has(M, 1); vector<pii> ret;
    rep(it, 0, M/2) {
        rep(i, 0, M) if (has[i])
            rep(j, i+1, M) if (A[i][j] && mat[i][j]) {
                fi = i; fj = j; goto done;
            } assert(0); done:
        if (fj < N) ret.emplace_back(fi, fj);
        has[fi] = has[fj] = 0;
    }
    rep(sw, 0, 2) {
        ll a = modpow(A[fi][fj], mod-2);
        rep(i, 0, M) if (has[i] && A[i][fj]) {
            ll b = A[i][fj] * a % mod;
            rep(j, 0, M) A[i][j] = (A[i][j] - A[fi][j] * b) % mod;
        }
        swap(fi, fj);
    }
    return ret;
}
```

## 7.4 DFS algorithms

## SCC.h

**Description:** Finds strongly connected components in a directed graph. If vertices  $u, v$  belong to the same component, we can reach  $u$  from  $v$  and vice versa.

**Usage:** `scc(graph, [&](vi& v) { ... })` visits all components in reverse topological order. `comp[i]` holds the component index of a node (a component only has edges to components with lower index). `ncomps` will contain the number of components.

**Time:**  $\mathcal{O}(E + V)$

```
76b5c9, 24 lines
vi val, comp, z, cont;
int Time, ncomps;
template<class G, class F> int dfs(int j, G& g, F& f) {
    int low = val[j] = ++Time, x; z.push_back(j);
    for (auto e : g[j]) if (comp[e] < 0)
        low = min(low, val[e] ?: dfs(e, g, f));

    if (low == val[j]) {
        do {
            x = z.back(); z.pop_back();
            comp[x] = ncomps;
            cont.push_back(x);
        } while (x != j);
    }
}
```

## GeneralMatching SCC BiconnectedComponents 2sat EulerWalk

```
f(cont); cont.clear();
ncomps++;
}
return val[j] = low;
}

template<class G, class F> void scc(G& g, F f) {
    int n = sz(g);
    val.assign(n, 0); comp.assign(n, -1);
    Time = ncomps = 0;
    rep(i, 0, n) if (comp[i] < 0) dfs(i, g, f);
}

BiconnectedComponents.h
```

**Description:** Finds all biconnected components in an undirected graph, and runs a callback for the edges in each. In a biconnected component there are at least two internally disjoint paths between any two nodes (a cycle exists through them). Note that a node can be in several components. An edge which is not in a component is a bridge, i.e., not part of any cycle.

**Usage:** `int eid = 0; ed.resize(N); for each edge (a,b) { ed[a].emplace_back(b, eid); ed[b].emplace_back(a, eid++); } bicomps([&](const vi& edgelist) {...});`

**Time:**  $\mathcal{O}(E + V)$

c6b7c7, 32 lines

```
vi num, st;
vector<vector<pii>> ed;
int Time;
template<class F>
int dfs(int at, int par, F& f) {
    int me = num[at] = ++Time, top = me;
    for (auto [y, e] : ed[at]) if (e != par) {
        if (num[y]) {
            top = min(top, num[y]);
            if (num[y] < me)
                st.push_back(e);
        } else {
            int si = sz(st);
            int up = dfs(y, e, f);
            top = min(top, up);
            if (up == me) {
                st.push_back(e);
                f(vi(st.begin() + si, st.end()));
                st.resize(si);
            }
            else if (up < me) st.push_back(e);
            else /* e is a bridge */
        }
    }
    return top;
}
```

```
template<class F>
void bicomps(F f) {
    num.assign(sz(ed), 0);
    rep(i, 0, sz(ed)) if (!num[i]) dfs(i, -1, f);
}
```

## 2sat.h

**Description:** Calculates a valid assignment to boolean variables  $a, b, c, \dots$  to a 2-SAT problem, so that an expression of the type  $(a \mid\mid b) \&\& (\neg a \mid\mid c) \&\& (\neg d \mid\mid b) \&\& \dots$  becomes true, or reports that it is unsatisfiable. Negated variables are represented by bit-inversions ( $\sim x$ ).

**Usage:** `TwoSat ts(number of boolean variables); ts.either(0, ~3); // Var 0 is true or var 3 is false ts.setValue(2); // Var 2 is true ts.atMostOne({0, ~1, 2}); // <= 1 of vars 0, ~1 and 2 are true ts.solve(); // Returns true iff it is solvable ts.values[0..N-1] holds the assigned values to the vars`

**Time:**  $\mathcal{O}(N + E)$ , where  $N$  is the number of boolean variables, and  $E$  is the number of clauses.

484741, 56 lines

// variables are zero indexed

```
struct twosat{
    int n;
    vector<vector<int>> g, gt;
    vector<bool> vis, res;
    vector<int> comp;
    stack<int> ts;
    twosat(int vars=0) {
        n=vars<<1; g.resize(n); gt.resize(n); }
```

```
//if you want to force variable a to be true : addOR (a,1,a,1)
//if you want to force variable a to be false : addOR (a,0,a,0);
```

// adds an implication

```
void _add(int a, bool sign_a, int b, bool sign_b) {
    a+=a+(sign_a^1);
    b+=b+(sign_b^1);
    g[a].push_back(b); gt[b].push_back(a); }
```

```
//(x-a or (not x-b))> sign_a=1,sign_b=0
void addOR(int a, bool sign_a, int b, bool sign_b) {
    a+=a+(sign_a^1);
    b+=b+(sign_b^1);
    g[a^1].push_back(b); // !a => b
    g[b^1].push_back(a); // !b => a
    gt[b].push_back(a^1);
    gt[a].push_back(b^1); }
```

//(!x-a xor !x-b)> sign\_a=0, sign\_b=0

```
void addXOR(int a, bool sign_a, int b, bool sign_b) {
    addOR(a, sign_a, b, sign_b);
    addOR(a, !sign_a, b, !sign_b); }
```

```
void dfs1(int u) {
    vis[u]=true;
    for(int v:g[u]) if(!vis[v]) dfs1(v);
    ts.push(u); }
```

```
void dfs2(int u,int c) {
    comp[u]=c;
    for(int v:gt[u]) if(comp[v]==-1) dfs2(v,c); }
```

```
bool run(){ // returns true if possible, else false
    vis.resize(n, false);
    for(int i=0; i<n; ++i) if(!vis[i]) dfs1(i);
    int scc=0;
    comp.resize(n, -1);
    while(!ts.empty()) {
        int u=ts.top(); ts.pop();
        if(comp[u]==-1) dfs2(u, scc++);
        res.resize(n/2);
        for(int i=0; i<n; i+=2) {
            if(comp[i]==comp[i+1]) return false;
            res[i/2]=(comp[i]>comp[i+1]);
        }
    }
}
```

## EulerWalk.h

**Description:** Eulerian undirected/directed path/cycle algorithm. Input should be a vector of (dest, global edge index), where for undirected graphs, forward/backward edges have the same index. Returns a list of nodes in the Eulerian path/cycle with src at both start and end, or empty list if no cycle/path exists. To get edge indices back, add .second to s and ret.

**Time:**  $\mathcal{O}(V + E)$

780b64, 15 lines

```
vi eulerWalk(vector<vector<pii>& gr, int nedges, int src=0) {
    int n = sz(gr);
    vi D(n), its(n), eu(nedges), ret, s = {src};
    D[src]++;
    while (!s.empty()) {
        int x = s.back(), y, e, &it = its[x], end = sz(gr[x]);
        if (it == end) { ret.push_back(x); s.pop_back(); continue; }
        tie(y, e) = gr[x][it++];
        if (!eu[e]) {
            D[x]--;
            D[y]++;
            eu[e] = 1;
            s.push_back(y);
        }
    }
    for (int x : D) if (x < 0 || sz(ret) != nedges+1) return {};
    return {ret.rbegin(), ret.rend()};
}
```

## Bridges.h

Description: idk  
Time: idk

98ffe3, 36 lines

```
struct BridgesAndCuts{
    int n, timer=0;
    vector<vector<int>> g;
    vector<int> tin, low;
    vector<char> vis, is_cut;
    vector<pair<int, int>> bridges;
    BridgesAndCuts(int n=0) : n(n), g(n), tin(n, -1),
    low(n, -1), vis(n, 0), is_cut(n, 0) {}

    void addEdge(int u, int v) { g[u].push_back(v); g[v].push_back(u) }

    void run(){
        timer=0;
        fill(tin.begin(), tin.end(), -1);
        fill(low.begin(), low.end(), -1);
        fill(vis.begin(), vis.end(), 0);
        fill(is_cut.begin(), is_cut.end(), 0);
        bridges.clear();
        for(int v=0; v<n; ++v) if(!vis[v]) dfs(v, -1);
    }

    private:
    void dfs(int v, int p){
        vis[v]=1;
        tin[v]=low[v]=timer++;
        int children=0;
        for(int to:g[v]){
            if(to==p) continue;
            if(vis[to]) low[v]=min(low[v], tin[to]);
            else{
                ++children; dfs(to, v);
                low[v]=min(low[v], low[to]);
                if(low[to]>in[v]) bridges.emplace_back(min(v, to), max(v, to));
                if(p!=-1&&low[to]>=tin[v]) is_cut[v]=1;
            }
            if(p==-1&&children>1) is_cut[v]=1;
        }
    }
}
```

## 7.5 Coloring

### EdgeColoring.h

Description: Given a simple, undirected graph with max degree  $D$ , computes a  $(D + 1)$ -coloring of the edges such that no neighboring edges share a color. ( $D$ -coloring is NP-hard, but can be done for bipartite graphs by repeated matchings of max-degree nodes.)

Time:  $\mathcal{O}(NM)$

e210e2, 31 lines

```
vi edgeColoring(int N, vector<pii> eds) {
```

```
vi cc(N + 1), ret(sz(eds)), fan(N), free(N), loc;
for (pii e : eds) ++cc[e.first], ++cc[e.second];
int u, v, ncols = *max_element(all(cc)) + 1;
vector<vi> adj(N, vi(ncols, -1));
for (pii e : eds) {
    tie(u, v) = e;
    fan[0] = v;
    loc.assign(ncols, 0);
    int at = u, end = u, d, c = free[u], ind = 0, i = 0;
    while (d == free[v], !loc[d] && (v = adj[u][d]) != -1)
        loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
    cc[loc[d]] = c;
    for (int cd = d; at != -1; cd ^= c ^ d, at = adj[at][cd])
        swap(adj[at][cd], adj[end = at][cd ^ c ^ d]);
    while (adj[fan[i]][d] != -1) {
        int left = fan[i], right = fan[++i], e = cc[i];
        adj[u][e] = left;
        adj[left][e] = u;
        adj[right][e] = -1;
        free[right] = e;
    }
    adj[u][d] = fan[i];
    adj[fan[i]][d] = u;
    for (int y : {fan[0], u, end})
        for (int z = free[y] = 0; adj[y][z] != -1; z++)
            adj[y][z] = -1;
}
rep(i, 0, sz(eds))
    for (tie(u, v) = eds[i]; adj[u][ret[i]] != v;) ++ret[i];
return ret;
}
```

## 7.6 Heuristics

### MaximalCliques.h

Description: Runs a callback for all maximal cliques in a graph (given as a symmetric bitset matrix; self-edges not allowed). Callback is given a bitset representing the maximal clique.

Time:  $\mathcal{O}(3^{n/3})$ , much faster for sparse graphs

b0d5b1, 12 lines

```
typedef bitset<128> B;
template<class F>
void cliques(vector<B>& eds, F f, B P = ~B(), B X={}, B R={}) {
    if (!P.any()) { if (!X.any()) f(R); return; }
    auto q = (P | X).find_first();
    auto cands = P & ~eds[q];
    rep(i, 0, sz(eds)) if (cands[i]) {
        R[i] = 1;
        cliques(eds, f, P & eds[i], X & eds[i], R);
        R[i] = P[i] = 0; X[i] = 1;
    }
}
```

### MaximumClique.h

Description: Quickly finds a maximum clique of a graph (given as symmetric bitset matrix; self-edges not allowed). Can be used to find a maximum independent set by finding a clique of the complement graph.

Time: Runs in about 1s for  $n=155$  and worst case random graphs ( $p=.90$ ). Runs faster for sparse graphs.

f7c0bc, 49 lines

```
typedef vector<bitset<200>> vb;
struct Maxclique {
    double limit=0.025, pk=0;
    struct Vertex { int i, d=0; };
    typedef vector<Vertex> vv;
    vb e;
    vv V;
    vector<vi> C;
    vi qmax, q, S, old;
    void init(vv& r) {
```

```
        for (auto& v : r) v.d = 0;
        for (auto& v : r) for (auto j : r) v.d += e[v.i][j.i];
        sort(all(r), [](auto a, auto b) { return a.d > b.d; });
        int mxD = r[0].d;
        rep(i, 0, sz(r)) r[i].d = min(i, mxD) + 1;
    }
    void expand(vv& R, int lev = 1) {
        S[lev] += S[lev - 1] - old[lev];
        old[lev] = S[lev - 1];
        while (sz(R)) {
            if (sz(q) + R.back().d <= sz(qmax)) return;
            q.push_back(R.back().i);
            vv T;
            for (auto v:R) if (e[R.back().i][v.i]) T.push_back({v.i});
            if (sz(T)) {
                if (S[lev]++ / ++pk < limit) init(T);
                int j = 0, mxk = 1, mnk = max(sz(qmax) - sz(q) + 1, 1);
                C[1].clear(), C[2].clear();
                for (auto v : T) {
                    int k = 1;
                    auto f = [&](int i) { return e[v.i][i]; };
                    while (any_of(all(C[k]), f)) k++;
                    if (k > mxk) mxk = k, C[mxk + 1].clear();
                    if (k < mnk) T[j++].i = v.i;
                    C[k].push_back(v.i);
                }
                if (j > 0) T[j - 1].d = 0;
                rep(k, mnk, mxk + 1) for (int i : C[k])
                    T[j].i = i, T[j].d = k;
                expand(T, lev + 1);
            } else if (sz(q) > sz(qmax)) qmax = q;
            q.pop_back(), R.pop_back();
        }
    }
    vi maxClique() { init(V), expand(V); return qmax; }
    Maxclique(vb conn) : e(conn), C(sz(e)+1), S(sz(C)), old(S) {
        rep(i, 0, sz(e)) V.push_back({i});
    }
};
```

### MaximumIndependentSet.h

Description: To obtain a maximum independent set of a graph, find a max clique of the complement. If the graph is bipartite, see MinimumVertex-Cover.

## 7.7 Trees

### BinaryLifting.h

Description: Calculate power of two jumps in a tree, to support fast upward jumps and LCAs. Assumes the root node points to itself.

Time: construction  $\mathcal{O}(N \log N)$ , queries  $\mathcal{O}(\log N)$

bfce85, 25 lines

```
vector<vi> treeJump(vi& P){
    int on = 1, d = 1;
    while (on < sz(P)) on *= 2, d++;
    vector<vi> jmp(d, P);
    rep(i, 1, d) rep(j, 0, sz(P))
        jmp[i][j] = jmp[i-1][jmp[i-1][j]];
    return jmp;
}
```

```
int jmp(vector<vi>& tbl, int nod, int steps){
    rep(i, 0, sz(tbl))
        if (steps & (1 << i)) nod = tbl[i][nod];
    return nod;
}
```

```
int lca(vector<vi>& tbl, vi& depth, int a, int b) {
    if (depth[a] < depth[b]) swap(a, b);
```

```
a = jmp(tbl, a, depth[a] - depth[b]);
if (a == b) return a;
for (int i = sz(tbl); i--;) {
    int c = tbl[i][a], d = tbl[i][b];
    if (c != d) a = c, b = d;
}
return tbl[0][a];
```

**LCA.h**

**Description:** Data structure for computing lowest common ancestors in a tree (with 0 as root). C should be an adjacency list of the tree, either directed or undirected.

**Time:**  $\mathcal{O}(N \log N + Q)$

```
.../data-structures/RMQ.h" 0f62fb, 21 lines
struct LCA {
    int T = 0;
    vi time, path, ret;
    RMQ<int> rmq;

    LCA(vector<vi>& C) : time(sz(C)), rmq((dfs(C, 0, -1), ret)) {}
    void dfs(vector<vi>& C, int v, int par) {
        time[v] = T++;
        for (int y : C[v]) if (y != par) {
            path.push_back(v), ret.push_back(time[v]);
            dfs(C, y, v);
        }
    }

    int lca(int a, int b) {
        if (a == b) return a;
        tie(a, b) = minmax(time[a], time[b]);
        return path[rmq.query(a, b)];
    }
    //dist(a,b){return depth[a] + depth[b] - 2*depth[lca(a,b)];}
};
```

**CompressTree.h**

**Description:** Given a rooted tree and a subset S of nodes, compute the minimal subtree that contains all the nodes by adding all (at most  $|S| - 1$ ) pairwise LCA's and compressing edges. Returns a list of (par, orig\_index) representing a tree rooted at 0. The root points to itself.

**Time:**  $\mathcal{O}(|S| \log |S|)$

```
"LCA.h" 9775a0, 21 lines
typedef vector<pair<int, int>> vpi;
vpi compressTree(LCA& lca, const vi& subset) {
    static vi rev; rev.resize(sz(lca.time));
    vi li = subset, &T = lca.time;
    auto cmp = [&](int a, int b) { return T[a] < T[b]; };
    sort(all(li), cmp);
    int m = sz(li)-1;
    rep(i, 0, m) {
        int a = li[i], b = li[i+1];
        li.push_back(lca.lca(a, b));
    }
    sort(all(li), cmp);
    li.erase(unique(all(li)), li.end());
    rep(i, 0, sz(li)) rev[li[i]] = i;
    vpi ret = {pii(0, li[0])};
    rep(i, 0, sz(li)-1) {
        int a = li[i], b = li[i+1];
        ret.emplace_back(rev[lca.lca(a, b)], b);
    }
    return ret;
}
```

**HLD.h**

**Description:** Decomposes a tree into vertex disjoint heavy paths and light edges such that the path from any leaf to the root contains at most  $\log(n)$  light edges. Code does additive modifications and max queries, but can support commutative segtree modifications/queries on paths and subtrees. Takes as input the full adjacency list. VALS\_EDGES being true means that values are stored in the edges, as opposed to the nodes. All values initialized to the segtree default. Root must be 0.

**Time:**  $\mathcal{O}((\log N)^2)$

.../data-structures/LazySegmentTree.h" cb0b6c, 49 lines

```
template <bool VALS_EDGES> struct HLD {
    int N, tim = 0;
    vector<vi> adj;
    vi par, siz, rt, pos;
    Lseg *tree;
    HLD(vector<vi> adj_) {
        : N(sz(adj_)), adj(adj_), par(N, -1), siz(N, 1),
          rt(N), pos(N), tree(new Lseg(0, N)){ dfsSz(0); dfsHld(0); }
    }
    void dfsSz(int v) {
        for (int& u : adj[v]) {
            adj[u].erase(find(all(adj[u]), v));
            par[u] = v;
            dfsSz(u);
            siz[v] += siz[u];
            if (siz[u] > siz[adj[v][0]]) swap(u, adj[v][0]);
        }
    }
    void dfsHld(int v) {
        pos[v] = tim++;
        for (int u : adj[v]) {
            rt[u] = (u == adj[v][0] ? rt[v] : u);
            dfsHld(u);
        }
    }
    template <class B> void process(int u, int v, B op) {
        for (;;) v = par[rt[v]];
        if (pos[u] > pos[v]) swap(u, v);
        if (rt[u] == rt[v]) break;
        op(pos[rt[v]], pos[v] + 1);
        op(pos[u] + VALS_EDGES, pos[v] + 1);
    }
    void modifyPath(int u, int v, int val) {
        process(u, v, [&](int l, int r) { tree->add(l, r, val); });
    }
    void modifySubtree(int v, ll val) {
        tree->add(pos[v] + VALS_EDGES, pos[v] + siz[v], val);
    }
    int queryPath(int u, int v) { // Modify depending on problem
        int res = -1e9;
        process(u, v, [&](int l, int r) {
            res = max(res, (tree->query(l, r)).mx );
        });
        return res;
    }
    int querySubtree(int v) { // modifySubtree is similar
        return (tree->query(pos[v] + VALS_EDGES, pos[v] + siz[v])).mx;
    }
};
```

**LinkCutTree.h**

**Description:** Represents a forest of unrooted trees. You can add and remove edges (as long as the result is still a forest), and check whether two nodes are in the same tree.

**Time:** All operations take amortized  $\mathcal{O}(\log N)$ .

.../data-structures/LinkCutTree.h ofb462, 90 lines

```
struct Node { // Splay tree. Root's pp contains tree's parent.
    Node *p = 0, *pp = 0, *c[2];
```

```
bool flip = 0;
Node() { c[0] = c[1] = 0; fix(); }
void fix() {
    if (c[0]) c[0]->p = this;
    if (c[1]) c[1]->p = this;
    // (+ update sum of subtree elements etc. if wanted)
}
void pushFlip() {
    if (!flip) return;
    flip = 0; swap(c[0], c[1]);
    if (c[0]) c[0]->flip ^= 1;
    if (c[1]) c[1]->flip ^= 1;
}
int up() { return p ? p->c[1] == this : -1; }
void rot(int i, int b) {
    int h = i ^ b;
    Node *x = c[i], *y = b == 2 ? x : x->c[h], *z = b ? y : x;
    if ((y->p = p) p->c[up()] = y;
    c[i] = z->c[i ^ 1];
    if (b < 2) {
        x->c[h] = y->c[h ^ 1];
        y->c[h ^ 1] = x;
    }
    z->c[i ^ 1] = this;
    fix(); x->fix(); y->fix();
    if (p) p->fix();
    swap(pp, y->pp);
}
void splay() {
    for (pushFlip(); p; ) {
        if (p->p) p->p->pushFlip();
        p->pushFlip(); pushFlip();
        int c1 = up(), c2 = p->up();
        if (c2 == -1) p->rot(c1, 2);
        else p->p->rot(c2, c1 != c2);
    }
}
Node* first() {
    pushFlip();
    return c[0] ? c[0]->first() : (splay(), this);
}
struct LinkCut {
    vector<Node> node;
    LinkCut(int N) : node(N) {}

    void link(int u, int v) { // add an edge (u, v)
        assert(!connected(u, v));
        makeRoot(&node[u]);
        node[u].pp = &node[v];
    }
    void cut(int u, int v) { // remove an edge (u, v)
        Node *x = &node[u], *top = &node[v];
        makeRoot(top); x->splay();
        assert(top == (x->pp ?: x->c[0]));
        if (x->pp) x->pp = 0;
        else {
            x->c[0] = top->p = 0;
            x->fix();
        }
    }
    bool connected(int u, int v) { // are u, v in the same tree?
        Node* nu = access(&node[u])->first();
        return nu == access(&node[v])->first();
    }
    void makeRoot(Node* u) {
        access(u);
        u->splay();
    }
```

```

if(u->c[0]) {
    u->c[0]->p = 0;
    u->c[0]->flip ^= 1;
    u->c[0]->pp = u;
    u->c[0] = 0;
    u->fix();
}

Node* access(Node* u) {
    u->splay();
    while (Node* pp = u->pp) {
        pp->splay(); u->pp = 0;
        if (pp->c[1]) {
            pp->c[1]->p = 0; pp->c[1]->pp = pp; }
        pp->c[1] = u; pp->fix(); u = pp;
    }
    return u;
}

DirectedMST.h
Description: Finds a minimum spanning tree/arborescence of a directed graph, given a root node. If no MST exists, returns -1.
Time:  $\mathcal{O}(E \log V)$ 
.../data-structures/UnionFindRollback.h" 39e620, 59 lines
struct Edge { int a, b; ll w; };
struct Node {
    Edge key;
    Node *l, *r;
    ll delta;
    void prop() {
        key.w += delta;
        if (l) l->delta += delta;
        if (r) r->delta += delta;
        delta = 0;
    }
    Edge top() { prop(); return key; }
};

Node *merge(Node *a, Node *b) {
    if (!a || !b) return a ?: b;
    a->prop(), b->prop();
    if (a->key.w > b->key.w) swap(a, b);
    swap(a->l, (a->r = merge(b, a->r)));
    return a;
}

void pop(Node*& a) { a->prop(); a = merge(a->l, a->r); }

pair<ll, vi> dmst(int n, int r, vector<Edge>& g) {
    RollbackUF uf(n);
    vector<Node*> heap(n);
    for (Edge e : g) heap[e.b] = merge(heap[e.b], new Node{e});
    ll res = 0;
    vi seen(n, -1), path(n), par(n);
    seen[r] = r;
    vector<Edge> Q(n), in(n, {-1, -1}), comp;
    deque<tuple<int, int, vector<Edge>> cycs;
    rep(s, 0, n) {
        int u = s, qi = 0, w;
        while (seen[u] < 0) {
            if (!heap[u]) return {-1, {}};
            Edge e = heap[u]->top();
            heap[u]->delta -= e.w, pop(heap[u]);
            Q[qi] = e, path[qi++] = u, seen[u] = s;
            res += e.w, u = uf.find(e.a);
            if (seen[u] == s) {
                Node* cyc = 0;
                int end = qi, time = uf.time();
                do cyc = merge(cyc, heap[w = path[--qi]]);
                while (uf.join(u, w));
    }
}

```

```

        u = uf.find(u), heap[u] = cyc, seen[u] = -1;
        cycs.push_front({u, time, {&Q[qi], &Q[end]}});
    }
    rep(i, 0, qi) in[uf.find(Q[i].b)] = Q[i];
}

for (auto& [u,t,comp] : cycs) { // restore sol (optional)
    uf.rollback(t);
    Edge inEdge = in[u];
    for (auto& e : comp) in[uf.find(e.b)] = e;
    in[uf.find(inEdge.b)] = inEdge;
}
rep(i, 0, n) par[i] = in[i].a;
return {res, par;};
}

TreeFlatten.h
Description: idk
Time: idk
b731c2, 14 lines
vll in(N), out(N); ll timer = -1;
void euler_tour(ll node, ll par) {
    in[node] = ++timer;
    for(ll ch : gr[node]) {
        if(ch == par) continue;
        euler_tour(ch, node);
    }
    out[node] = timer;
}
// usage :
euler_tour(1, 0);
vll flat_tree(n);
L(i, 1, n) flat_tree[in[i]] = a[i];
sumSeg seg(flat_tree);

CentDecom.h
Description: idk
Time: idk
b47437, 35 lines
ll n,m,x,y,z,q,k,u,v,w;
vector<int> gr[N];
int sz[N];
int tot,done[N],cenpar[N];

void calc_sz(int node,int p){
    tot++; sz[node]=1;
    for(auto ch:gr[node]){
        if(ch==p||done[ch]) continue;
        calc_sz(ch,node); sz[node]+=sz[ch]; } }

int find_cen(int node,int p){ // find centroid
    for(auto ch:gr[node]){
        if(ch==p||done[ch]) continue;
        else if(sz[ch]>tot/2) return find_cen(ch,node); }
    return node; }

void decompose(int node,int p){ // find centroid of subtree
    tot=0; calc_sz(node,p);
    int cen=find_cen(node,p);
    cenpar[cen]=p; done[cen]=1;
    for(auto ch:gr[cen]){
        if(ch==p||done[ch]) continue;
        decompose(ch,cen); } }

vll cen_tree[N]; // centroid tree

int form_cen_tree(){ // form graph, return root(centroid)
    decompose(1,0); int root;
    L(i,1,n) {

```

```

        if(cenpar[i]==0) root=i;
        if(cenpar[i]!=0){
            cen_tree[i].push_back(cenpar[i]);
            cen_tree[cenpar[i]].push_back(i); }
    }
return root;
}

7.8 Math
7.8.1 Number of Spanning Trees
Create an  $N \times N$  matrix mat, and for each edge  $a \rightarrow b \in G$ , do  $\text{mat}[a][b]--$ ,  $\text{mat}[b][b]++$  (and  $\text{mat}[b][a]--$ ,  $\text{mat}[a][a]++$  if  $G$  is undirected). Remove the  $i$ th row and column and take the determinant; this yields the number of directed spanning trees rooted at  $i$  (if  $G$  is undirected, remove any row/column).

```

## 7.8.2 Erdős–Gallai theorem

A simple graph with node degrees  $d_1 \geq \dots \geq d_n$  exists iff  $d_1 + \dots + d_n$  is even and for every  $k = 1 \dots n$ ,

$$\sum_{i=1}^k d_i \leq k(k-1) + \sum_{i=k+1}^n \min(d_i, k).$$

# Geometry (8)

## 8.1 Geometric primitives

### Point.h

**Description:** Class to handle points in the plane. T can be e.g. double or long long. (Avoid int.)

47ec0a, 28 lines

```

template <class T> int sgn(T x) { return (x > 0) - (x < 0); }
template<class T>
struct Point {
    typedef Point P;
    T x, y;
    explicit Point(T x=0, T y=0) : x(x), y(y) {}
    bool operator<(P p) const { return tie(x,y) < tie(p.x,p.y); }
    bool operator==(P p) const { return tie(x,y)==tie(p.x,p.y); }
    P operator+(P p) const { return P(x+p.x, y+p.y); }
    P operator-(P p) const { return P(x-p.x, y-p.y); }
    P operator*(T d) const { return P(x*d, y*d); }
    P operator/(T d) const { return P(x/d, y/d); }
    T dot(P p) const { return x*p.x + y*p.y; }
    T cross(P p) const { return x*p.y - y*p.x; }
    T cross(P a, P b) const { return (a-*this).cross(b-*this); }
    T dist2() const { return x*x + y*y; }
    double dist() const { return sqrt((double)dist2()); }
    // angle to x-axis in interval [-pi, pi]
    double angle() const { return atan2(y, x); }
    P unit() const { return *this/dist(); } // makes dist()=1
    P perp() const { return P(-y, x); } // rotates +90 degrees
    P normal() const { return perp().unit(); }
    // returns point rotated 'a' radians ccw around the origin
    P rotate(double a) const {
        return P(x*cos(a)-y*sin(a), x*sin(a)+y*cos(a)); }
    friend ostringstream& operator<<(ostream& os, P p) {
        return os << "(" << p.x << ", " << p.y << ")"; }
};

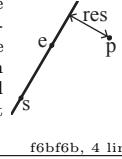
lineDistance.h

```

**Description:**

Returns the signed distance between point p and the line containing points a and b. Positive value on left side and negative on right as seen from a towards b. a==b gives nan. P is supposed to be Point<T> or Point3D<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long. Using Point3D will always give a non-negative distance. For Point3D, call .dist on the result of the cross product.

"Point.h"



f6bf6b, 4 lines

```
template<class P>
double lineDist(const P& a, const P& b, const P& p) {
    return (double)(b-a).cross(p-a)/(b-a).dist();}
```

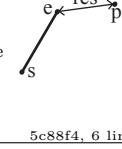
## SegmentDistance.h

**Description:**

Returns the shortest distance between point p and the line segment from point s to e.

**Usage:** Point<double> a, b(2,2), p(1,1);  
bool onSegment = segDist(a,b,p) < 1e-10;

"Point.h"



5c88f4, 6 lines

```
typedef Point<double> P;
double segDist(P& s, P& e, P& p) {
    if (s==e) return (p-s).dist();
    auto d = (e-s).dist2(), t = min(d,max(.0,(p-s).dot(e-s)));
    return ((p-s)*d-(e-s)*t).dist()/d;
}
```

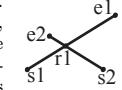
## SegmentIntersection.h

**Description:**

If a unique intersection point between the line segments going from s1 to e1 and from s2 to e2 exists then it is returned. If no intersection point exists an empty vector is returned. If infinitely many exist a vector with 2 elements is returned, containing the endpoints of the common line segment. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or long long.

**Usage:** vector<P> inter = segInter(s1,e1,s2,e2);  
if (sz(inter)==1)  
cout << "segments intersect at " << inter[0] << endl;

"Point.h", "OnSegment.h"



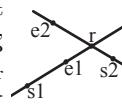
9d57f2, 13 lines

```
template<class P> vector<P> segInter(P a, P b, P c, P d) {
    auto oa = c.cross(d, a), ob = c.cross(d, b),
        oc = a.cross(b, c), od = a.cross(b, d);
    // Checks if intersection is single non-endpoint point.
    if (sgn(oa) * sgn(ob) < 0 && sgn(oc) * sgn(od) < 0)
        return {(a * ob - b * oa) / (ob - oa)};
    set<P> s;
    if (onSegment(c, d, a)) s.insert(a);
    if (onSegment(c, d, b)) s.insert(b);
    if (onSegment(a, b, c)) s.insert(c);
    if (onSegment(a, b, d)) s.insert(d);
    return {all(s)};
```

## lineIntersection.h

**Description:**

If a unique intersection point of the lines going through s1,e1 and s2,e2 exists {1, point} is returned. If no intersection point exists {0, (0,0)} is returned and if infinitely many exists {-1, (0,0)} is returned. The wrong position will be returned if P is Point<ll> and the intersection point does not have integer coordinates. Products of three coordinates are used in intermediate steps so watch out for overflow if using int or ll.



f06062, 35 lines

**Usage:** auto res = lineInter(s1,e1,s2,e2);  
if (res.first == 1)  
cout << "intersection point at " << res.second << endl;

"Point.h"

a01f81, 8 lines

```
template<class P>
pair<int, P> lineInter(P s1, P e1, P s2, P e2) {
    auto d = (e1 - s1).cross(e2 - s2);
    if (d == 0) // if parallel
        return {-(s1.cross(e1, s2) == 0), P(0, 0)};
    auto p = s2.cross(e1, e2), q = s2.cross(e2, s1);
    return {1, (s1 * p + e1 * q) / d};}
```

## sideOf.h

**Description:** Returns where p is as seen from s towards e. 1/0/-1  $\Leftrightarrow$  left/on line/right. If the optional argument eps is given 0 is returned if p is within distance eps from the line. P is supposed to be Point<T> where T is e.g. double or long long. It uses products in intermediate steps so watch out for overflow if using int or long long.

**Usage:** bool left = sideOf(p1,p2,q)==1;  
"Point.h"

3af81c, 9 lines

```
template<class P>
int sideOf(P s, P e, P p) { return sgn(s.cross(e, p)); }

template<class P>
int sideOf(const P& s, const P& e, const P& p, double eps) {
    auto a = (e-s).cross(p-s);
    double l = (e-s).dist()*eps;
    return (a > l) - (a < -l);}
```

## OnSegment.h

**Description:** Returns true iff p lies on the line segment from s to e. Use (segDist(s,e,p)<=epsilon) instead when using Point<double>.

"Point.h"

```
c597e8, 3 lines
template<class P> bool onSegment(P s, P e, P p) {
    return p.cross(s, e) == 0 && (s - p).dot(e - p) <= 0;
```

## linearTransformation.h

**Description:**

Apply the linear transformation (translation, rotation and scaling) which takes line p0-p1 to line q0-q1 to point r.

"Point.h"

```
03a306, 6 lines
typedef Point<double> P;
P linearTransformation(const P& p0, const P& p1,
                      const P& q0, const P& q1, const P& r) {
    P dp = p1-p0, dq = q1-q0, num(dp.cross(dq), dp.dot(dq));
    return q0 + P((r-p0).cross(num), (r-p0).dot(num))/dp.dist2();}
```

## Angle.h

**Description:** A class for ordering angles (as represented by int points and a number of rotations around the origin). Useful for rotational sweeping. Sometimes also represents points or vectors.

**Usage:** vector<Angle> v = {w[0], w[0].t360() ...}; // sorted  
int j = 0; rep(i,0,n) { while (v[j] < v[i].t180()) ++j; }  
// sweeps j such that (j-i) represents the number of positively oriented triangles with vertices at 0 and i

0f0602, 35 lines

```
struct Angle {
    int x, y;
    int t;
    Angle(int x, int y, int t=0) : x(x), y(y), t(t) {}
    Angle operator-(Angle b) const { return {x-b.x, y-b.y, t}; }
    int half() const {
        assert(x || y);}
```

```
return y < 0 || (y == 0 && x < 0);

Angle t90() const { return {-y, x, t + (half() && x >= 0)}; }
Angle t180() const { return {-x, -y, t + half()}; }
Angle t360() const { return {x, y, t + 1}; }

};

bool operator<(Angle a, Angle b) {
    // add a.dist2() and b.dist2() to also compare distances
    return make_tuple(a.t, a.half(), a.y * (ll)b.x) <
        make_tuple(b.t, b.half(), a.x * (ll)b.y);
}

// Given two points, this calculates the smallest angle between
// them, i.e., the angle that covers the defined line segment.
pair<Angle, Angle> segmentAngles(Angle a, Angle b) {
    if (b < a) swap(a, b);
    return (b < a.t180()) ?
        make_pair(a, b) : make_pair(b, a.t360()));

}

Angle operator+(Angle a, Angle b) { // point a + vector b
    Angle r(a.x + b.x, a.y + b.y, a.t);
    if (a.t180() < r) r.t--;
    return r.t180() < a ? r.t360() : r;
}

Angle angleDiff(Angle a, Angle b) { // angle b - angle a
    int tu = b.t - a.t; a.t = b.t;
    return {a.x*b.x + a.y*b.y, a.x*b.y - a.y*b.x, tu - (b < a)};
```

## 8.2 Circles

## CircleIntersection.h

**Description:** Computes the pair of points at which two circles intersect. Returns false in case of no intersection.

"Point.h"

84d6d3, 11 lines

```
typedef Point<double> P;
bool circleInter(P a, P b, double r1, double r2, pair<P, P>* out) {
    if (a == b) { assert(r1 != r2); return false; }
    P vec = b - a;
    double d2 = vec.dist2(), sum = r1+r2, dif = r1-r2,
           p = (d2 + r1*r1 - r2*r2)/(d2*2), h2 = r1*r1 - p*p*d2;
    if (sum*sum < d2 || dif*dif > d2) return false;
    P mid = a + vec*p, per = vec.perp() * sqrt(fmax(0, h2) / d2);
    *out = {mid + per, mid - per};
    return true;
}
```

## CircleTangents.h

**Description:** Finds the external tangents of two circles, or internal if r2 is negated. Can return 0, 1, or 2 tangents – 0 if one circle contains the other (or overlaps it, in the internal case, or if the circles are the same); 1 if the circles are tangent to each other (in which case .first = .second and the tangent line is perpendicular to the line between the centers). .first and .second give the tangency points at circle 1 and 2 respectively. To find the tangents of a circle with a point set r2 to 0.

"Point.h"

b0153d, 13 lines

```
template<class P>
vector<pair<P, P>> tangents(P c1, double r1, P c2, double r2) {
    P d = c2 - c1;
    double dr = r1 - r2, d2 = d.dist2(), h2 = d2 - dr * dr;
    if (d2 == 0 || h2 < 0) return {};
    vector<pair<P, P>> out;
    for (double sign : {-1, 1}) {
        P v = (d * dr + d.perp() * sqrt(h2) * sign) / d2;
        out.push_back({c1 + v * r1, c2 + v * r2});
    }
    if (h2 == 0) out.pop_back();
    return out;
}
```

**CirclePolygonIntersection.h**

**Description:** Returns the area of the intersection of a circle with a ccw polygon.

**Time:**  $\mathcal{O}(n)$

"../../../../content/geometry/Point.h"

19add1, 19 lines

```
typedef Point<double> P;
#define arg(p, q) atan2(p.cross(q), p.dot(q))
double circlePoly(P c, double r, vector<P> ps) {
    auto tri = [&](P p, P q) {
        auto r2 = r * r / 2;
        P d = q - p;
        auto a = d.dot(p)/d.dist2(), b = (p.dist2()-r*r)/d.dist2();
        auto det = a * a - b;
        if (det <= 0) return arg(p, q) * r2;
        auto s = max(0., -a-sqrt(det)), t = min(1., -a+sqrt(det));
        if (t < 0 || 1 <= s) return arg(p, q) * r2;
        P u = p + d * s, v = q + d * (t-1);
        return arg(p,u) * r2 + u.cross(v)/2 + arg(v,q) * r2;
    };
    auto sum = 0.0;
    rep(i, 0, sz(ps)) {
        sum += tri(ps[i] - c, ps[(i + 1) % sz(ps)] - c);
    }
    return sum;
}
```

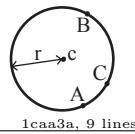
**circumcircle.h**

**Description:**

The circumcircle of a triangle is the circle intersecting all three vertices. ccRadius returns the radius of the circle going through points A, B and C and ccCenter returns the center of the same circle.

"Point.h"

```
typedef Point<double> P;
double ccRadius(const P& A, const P& B, const P& C) {
    return (B-A).dist()*(C-B).dist()*(A-C).dist() /
        abs((B-A).cross(C-A))/2;
}
P ccCenter(const P& A, const P& B, const P& C) {
    P b = C-A, c = B-A;
    return A + (b*c.dist2()-c*b.dist2()).perp()/b.cross(c)/2;
}
```



1caa3a, 9 lines

**MinimumEnclosingCircle.h**

**Description:** Computes the minimum circle that encloses a set of points.

**Time:** expected  $\mathcal{O}(n)$

"circumcircle.h"

09dd0a, 17 lines

```
pair<P, double> mec(vector<P> ps) {
    shuffle(all(ps), mt19937(time(0)));
    P o = ps[0];
    double r = 0, EPS = 1 + 1e-8;
    rep(i, 0, sz(ps)) if ((o - ps[i]).dist() > r * EPS) {
        o = ps[i], r = 0;
        rep(j, 0, i) if ((o - ps[j]).dist() > r * EPS) {
            o = (ps[i] + ps[j]) / 2;
            r = (o - ps[i]).dist();
            rep(k, 0, j) if ((o - ps[k]).dist() > r * EPS) {
                o = ccCenter(ps[i], ps[j], ps[k]);
                r = (o - ps[i]).dist();
            }
        }
    }
    return {o, r};
}
```

**8.3 Polygons****InsidePolygon.h**

**Description:** Returns true if p lies within the polygon. If strict is true, it returns false for points on the boundary. The algorithm uses products in intermediate steps so watch out for overflow.

**Usage:** `vector<P> v = {P{4,4}, P{1,2}, P{2,1}};`  
`bool in = inPolygon(v, P{3, 3}, false);`

**Time:**  $\mathcal{O}(n)$

"Point.h", "OnSegment.h", "SegmentDistance.h"

2bf504, 11 lines

**template<class P>**

```
bool inPolygon(vector<P> &p, P a, bool strict = true) {
    int cnt = 0, n = sz(p);
    rep(i, 0, n) {
        P q = p[(i + 1) % n];
        if (onSegment(p[i], q, a)) return !strict;
        //or: if (segDist(p[i], q, a) <= eps) return !strict;
        cnt += ((a.y < p[i].y) - (a.y < q.y)) * a.cross(p[i], q) > 0;
    }
    return cnt;
}
```

**PolygonArea.h**

**Description:** Returns twice the signed area of a polygon. Clockwise enumeration gives negative area. Watch out for overflow if using int as T!

"Point.h"

f12300, 6 lines

**template<class T>**

```
T polygonArea2(vector<Point<T>> &v) {
    T a = v.back().cross(v[0]);
    rep(i, 0, sz(v)-1) a += v[i].cross(v[i+1]);
    return a;
}
```

**PolygonCenter.h**

**Description:** Returns the center of mass for a polygon.

**Time:**  $\mathcal{O}(n)$

"Point.h"

9706dc, 9 lines

**typedef Point<double> P;**

```
P polygonCenter(const vector<P>& v) {
    P res(0, 0); double A = 0;
    for (int i = 0, j = sz(v) - 1; i < sz(v); j = i++) {
        res = res + (v[i] + v[j]) * v[j].cross(v[i]);
        A += v[j].cross(v[i]);
    }
    return res / A / 3;
}
```

**PolygonCut.h**

**Description:**

Returns a vector with the vertices of a polygon with everything to the left of the line going from s to e cut away.

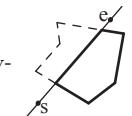
**Usage:** `vector<P> p = ...;`  
`p = polygonCut(p, P(0,0), P(1,0));`

"Point.h"

d07181, 13 lines

**typedef Point<double> P;**

```
vector<P> polygonCut(const vector<P>& poly, P s, P e) {
    vector<P> res;
    rep(i, 0, sz(poly)) {
        P cur = poly[i], prev = i ? poly[i-1] : poly.back();
        auto a = s.cross(e, cur), b = s.cross(e, prev);
        if ((a < 0) != (b < 0))
            res.push_back(cur + (prev - cur) * (a / (a - b)));
        if (a < 0)
            res.push_back(cur);
    }
    return res;
}
```

**ConvexHull.h****Description:**

Returns a vector of the points of the convex hull in counter-clockwise order. Points on the edge of the hull between two other points are not considered part of the hull.

**Time:**  $\mathcal{O}(n \log n)$

"Point.h"



310954, 13 lines

**typedef Point<ll> P;**

```
vector<P> convexHull(vector<P> pts) {
    if (sz(pts) <= 1) return pts;
    sort(all(pts));
    vector<P> h(sz(pts)+1);
    int s = 0, t = 0;
    for (int it = 2; it--> s == --t, reverse(all(pts))) {
        for (P p : pts) {
            while (t >= s + 2 && h[t-2].cross(h[t-1], p) <= 0) t--;
            h[t++] = p;
        }
    }
    return {h.begin(), h.begin() + t - (t == 2 && h[0] == h[1])};
}
```

**HullDiameter.h**

**Description:** Returns the two points with max distance on a convex hull (ccw, no duplicate/collinear points).

**Time:**  $\mathcal{O}(n)$

"Point.h"

c571b8, 12 lines

**typedef Point<ll> P;**

```
array<P, 2> hullDiameter(vector<P> S) {
    int n = sz(S), j = n < 2 ? 0 : 1;
    pair<ll, array<P, 2>> res({0, {S[0], S[0]}});
    rep(i, 0, j)
        for (;;) j = (j + 1) % n {
            res = max(res, {(S[i] - S[j]).dist2(), {S[i], S[j]}});
            if ((S[j + 1] % n - S[j]).cross(S[i + 1] - S[i]) >= 0)
                break;
        }
    return res.second;
}
```

**PointInsideHull.h**

**Description:** Determine whether a point t lies inside a convex hull (CCW order, with no collinear points). Returns true if point lies within the hull. If strict is true, points on the boundary aren't included.

**Time:**  $\mathcal{O}(\log N)$

"Point.h", "sideOf.h", "OnSegment.h"

71446b, 14 lines

**typedef Point<ll> P;**

```
bool inHull(const vector<P>& l, P p, bool strict = true) {
    int a = 1, b = sz(l) - 1, r = !strict;
    if (sz(l) < 3) return r && onSegment(l[0], l.back(), p);
    if (sideOf(l[0], l[a], l[b]) > 0) swap(a, b);
    if (sideOf(l[0], l[a], p) >= r || sideOf(l[0], l[b], p) <= -r)
        return false;
    while (abs(a - b) > 1) {
        int c = (a + b) / 2;
        (sideOf(l[0], l[c], p) > 0 ? b : a) = c;
    }
    return sgn(l[a].cross(l[b], p)) < r;
}
```

**LineHullIntersection.h**

**Description:** Line-convex polygon intersection. The polygon must be ccw and have no collinear points. `lineHull(line, poly)` returns a pair describing the intersection of a line with the polygon:  $\bullet(-1, -1)$  if no collision,  $\bullet(i, -1)$  if touching the corner  $i$ ,  $\bullet(i, i)$  if along side  $(i, i+1)$ ,  $\bullet(i, j)$  if crossing sides  $(i, i+1)$  and  $(j, j+1)$ . In the last case, if a corner  $i$  is crossed, this is treated as happening on side  $(i, i+1)$ . The points are returned in the same order as the line hits the polygon. `extrVertex` returns the point of a hull with the max projection onto a line.

**Time:**  $\mathcal{O}(\log n)$

```
"Point.h"          7cf45b, 39 lines
#define cmp(i, j) sgn(dir.perp().cross(poly[(i)%n]-poly[(j)%n]))
#define extr(i) cmp(i + 1, i) >= 0 && cmp(i, i - 1 + n) < 0
template <class P> int extrVertex(vector<P>& poly, P dir) {
    int n = sz(poly), lo = 0, hi = n;
    if (extr(0)) return 0;
    while (lo + 1 < hi) {
        int m = (lo + hi) / 2;
        if (extr(m)) return m;
        int ls = cmp(lo + 1, lo), ms = cmp(m + 1, m);
        (ls < ms || (ls == ms && ls == cmp(lo, m))) ? hi : lo) = m;
    }
    return lo;
}

#define cmpL(i) sgn(a.cross(poly[i], b))
template <class P>
array<int, 2> lineHull(P a, P b, vector<P>& poly) {
    int endA = extrVertex(poly, (a - b).perp());
    int endB = extrVertex(poly, (b - a).perp());
    if (cmpL(endA) < 0 || cmpL(endB) > 0)
        return {-1, -1};
    array<int, 2> res;
    rep(i, 0, 2) {
        int lo = endB, hi = endA, n = sz(poly);
        while ((lo + 1) % n != hi) {
            int m = ((lo + hi + (lo < hi ? 0 : n)) / 2) % n;
            (cmpL(m) == cmpL(endB) ? lo : hi) = m;
        }
        res[i] = (lo + !cmpL(hi)) % n;
        swap(endA, endB);
    }
    if (res[0] == res[1]) return {res[0], -1};
    if (!cmpL(res[0]) && !cmpL(res[1]))
        switch ((res[0] - res[1] + sz(poly) + 1) % sz(poly)) {
            case 0: return {res[0], res[0]};
            case 2: return {res[1], res[1]};
        }
    return res;
}
```

## 8.4 Misc. Point Set Problems

ClosestPair.h

**Description:** Finds the closest pair of points.

**Time:**  $\mathcal{O}(n \log n)$

```
"Point.h"          ac41a6, 17 lines
typedef Point<ll> P;
pair<P, P> closest(vector<P> v) {
    assert(sz(v) > 1);
    set<P> S;
    sort(all(v), [](P a, P b) { return a.y < b.y; });
    pair<ll, pair<P, P>> ret{LLONG_MAX, {P(), P()}};
    int j = 0;
    for (P p : v) {
        P d{l + (ll)sqrt(ret.first), 0};
        while (v[j].y <= p.y - d.x) S.erase(v[j++]);
        auto lo = S.lower_bound(p - d), hi = S.upper_bound(p + d);
        for (; lo != hi; ++lo)
            ret = min(ret, {(*lo - p).dist2(), {*lo, p}});
    }
}
```

## ClosestPair kdTree FastDelaunay

```
S.insert(p);
}
return ret.second;
}

kdTree.h
Description: KD-tree (2d, can be extended to 3d)
"Point.h"          bac5b0, 63 lines
typedef long long T;
typedef Point<T> P;
const T INF = numeric_limits<T>::max();

bool on_x(const P& a, const P& b) { return a.x < b.x; }
bool on_y(const P& a, const P& b) { return a.y < b.y; }

struct Node {
    P pt; // if this is a leaf, the single point in it
    T x0 = INF, x1 = -INF, y0 = INF, y1 = -INF; // bounds
    Node *first = 0, *second = 0;

    T distance(const P& p) { // min squared distance to a point
        T x = (p.x < x0 ? x0 : p.x > x1 ? x1 : p.x);
        T y = (p.y < y0 ? y0 : p.y > y1 ? y1 : p.y);
        return (P(x,y) - p).dist2();
    }

    Node(vector<P>&& vp) : pt(vp[0]) {
        for (P p : vp) {
            x0 = min(x0, p.x); x1 = max(x1, p.x);
            y0 = min(y0, p.y); y1 = max(y1, p.y);
        }
        if (vp.size() > 1) {
            // split on x if width >= height (not ideal...)
            sort(all(vp), x1 - x0 >= y1 - y0 ? on_x : on_y);
            // divide by taking half the array for each child (not
            // best performance with many duplicates in the middle)
            int half = sz(vp)/2;
            first = new Node({vp.begin(), vp.begin() + half});
            second = new Node({vp.begin() + half, vp.end()});
        }
    }
};

struct KDTree {
    Node* root;
    KDTree(const vector<P>& vp) : root(new Node({all(vp)})) {}

    pair<T, P> search(Node *node, const P& p) {
        if (!node->first) {
            // uncomment if we should not find the point itself:
            // if (p == node->pt) return {INF, P()};
            return make_pair((p - node->pt).dist2(), node->pt);
        }

        Node *f = node->first, *s = node->second;
        T bfist = f->distance(p), bsec = s->distance(p);
        if (bfist > bsec) swap(bsec, bfist), swap(f, s);

        // search closest side first, other side if needed
        auto best = search(f, p);
        if (bsec < best.first)
            best = min(best, search(s, p));
        return best;
    }

    // find nearest point to a point, and its squared distance
    // (requires an arbitrary operator< for Point)
    pair<T, P> nearest(const P& p) {
        return search(root, p);
    }
}
```

```
}
```

```
};
```

## FastDelaunay.h

**Description:** Fast Delaunay triangulation. Each circumcircle contains none of the input points. There must be no duplicate points. If all points are on a line, no triangles will be returned. Should work for doubles as well, though there may be precision issues in 'circ'. Returns triangles in order  $\{t[0][0], t[0][1], t[0][2], t[1][0], \dots\}$ , all counter-clockwise.

**Time:**  $\mathcal{O}(n \log n)$

```
"Point.h"          eedfd5, 88 lines
typedef Point<ll> P;
typedef struct Quad* Q;
typedef __int128_t lll; // (can be ll if coords are < 2e4)
P arb(LLONG_MAX, LLONG_MAX); // not equal to any other point

struct Quad {
    Q rot, o; P p = arb; bool mark;
    P F() { return r()>p; }
    Q& r() { return rot->rot; }
    Q prev() { return rot->o->rot; }
    Q next() { return r()>prev(); }
} *H;

bool circ(P p, P a, P b, P c) { // is p in the circumcircle?
    lll p2 = p.dist2(), A = a.dist2() - p2,
    B = b.dist2() - p2, C = c.dist2() - p2;
    return p.cross(a,b)*C + p.cross(b,c)*A + p.cross(c,a)*B > 0;
}

Q makeEdge(P orig, P dest) {
    Q r = H ? H : new Quad{new Quad{new Quad{new Quad{0}}}};
    H = r->o; r->r()>r() = r;
    rep(i, 0, 4) r = r->rot, r->p = arb, r->o = i & 1 ? r : r->r();
    r->p = orig; r->F() = dest;
    return r;
}

void splice(Q a, Q b) {
    swap(a->o->rot->o, b->o->rot->o); swap(a->o, b->o);
}

Q connect(Q a, Q b) {
    Q q = makeEdge(a->F(), b->p);
    splice(q, a->next());
    splice(q->r(), b);
    return q;
}

pair<Q, Q> rec(const vector<P>& s) {
    if (sz(s) <= 3) {
        Q a = makeEdge(s[0], s[1]), b = makeEdge(s[1], s.back());
        if (sz(s) == 2) return {a, a->r()};
        splice(a->r(), b);
        auto side = s[0].cross(s[1], s[2]);
        Q c = side ? connect(b, a) : 0;
        return {side < 0 ? c->r() : a, side < 0 ? c : b->r()};
    }
}

#define H(e) e->F(), e->p
#define valid(e) (e->F().cross(H(base)) > 0)
Q A, B, ra, rb;
int half = sz(s) / 2;
tie(ra, A) = rec({all(s) - half});
tie(B, rb) = rec({sz(s) - half + all(s)});
while ((B->p.cross(H(A)) < 0 && (A = A->next()) ||
        (A->p.cross(H(B)) > 0 && (B = B->r()>o)));
Q base = connect(B->r(), A);
if (A->p == ra->p) ra = base->r();
if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; if (valid(e)) \
```

```

while (circ(e->dir->F(), H(base), e->F())) { \
    Q t = e->dir; \
    splice(e, e->prev()); \
    splice(e->r(), e->r()->prev()); \
    e->o = H; H = e; e = t; \
}
for (;;) {
    DEL(LC, base->r(), o); DEL(RC, base, prev());
    if (!valid(LC) && !valid(RC)) break;
    if (!valid(LC) || (valid(RC) && circ(H(RC), H(LC))))
        base = connect(RC, base->r());
    else
        base = connect(base->r(), LC->r());
}
return { ra, rb };
}

vector<P> triangulate(vector<P> pts) {
    sort(all(pts)); assert(unique(all(pts)) == pts.end());
    if (sz(pts) < 2) return {};
    Q e = rec(pts).first;
    vector<Q> q = {e};
    int qi = 0;
    while (e->o->F().cross(e->F(), e->p) < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; pts.push_back(c->p); \
    q.push_back(c->r()); c = c->next(); } while (c != e); }
    ADD; pts.clear();
    while (qi < sz(q)) if (!(e = q[qi++])->mark) ADD;
    return pts;
}

```

## 8.5 3D

### PolyhedronVolume.h

**Description:** Magic formula for the volume of a polyhedron. Faces should point outwards.

3058c3, 6 lines

```

template<class V, class L>
double signedPolyVolume(const V& p, const L& trilist) {
    double v = 0;
    for (auto i : trilist) v += p[i.a].cross(p[i.b]).dot(p[i.c]);
    return v / 6;
}

```

### Point3D.h

**Description:** Class to handle points in 3D space. T can be e.g. double or long long.

8058ae, 32 lines

```

template<class T> struct Point3D {
    typedef Point3D P;
    typedef const P& R;
    T x, y, z;
    explicit Point3D(T x=0, T y=0, T z=0) : x(x), y(y), z(z) {}
    bool operator<(R p) const {
        return tie(x, y, z) < tie(p.x, p.y, p.z); }
    bool operator==(R p) const {
        return tie(x, y, z) == tie(p.x, p.y, p.z); }
    P operator+(R p) const { return P(x+p.x, y+p.y, z+p.z); }
    P operator-(R p) const { return P(x-p.x, y-p.y, z-p.z); }
    P operator*(T d) const { return P(x*d, y*d, z*d); }
    P operator/(T d) const { return P(x/d, y/d, z/d); }
    T dot(R p) const { return x*p.x + y*p.y + z*p.z; }
    P cross(R p) const {
        return P(y*p.z - z*p.y, z*p.x - x*p.z, x*p.y - y*p.x);
    }
    T dist2() const { return x*x + y*y + z*z; }
    double dist() const { return sqrt(double)dist2(); }
    //Azimuthal angle (longitude) to x-axis in interval [-pi, pi]
    double phi() const { return atan2(y, x); }
    //Zenith angle (latitude) to the z-axis in interval [0, pi]
}

```

```

double theta() const { return atan2(sqrt(x*x+y*y), z); }
P unit() const { return *this/(T)dist(); } //makes dist()=1
//returns unit vector normal to *this and p
P normal(P p) const { return cross(p).unit(); }
//returns point rotated 'angle' radians ccw around axis
P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle); P u = axis.unit();
    return u.dot(u)*(1-c) + (*this)*c - cross(u)*s;
}
}

```

### 3dHull.h

**Description:** Computes all faces of the 3-dimension hull of a point set. \*No four points must be coplanar\*, or else random results will be returned. All faces will point outwards.

Time:  $\mathcal{O}(n^2)$ 

"Point3D.h" 5b45fc, 49 lines

```

typedef Point3D<double> P3;
struct PR {
    void ins(int x) { (a == -1 ? a : b) = x; }
    void rem(int x) { (a == x ? a : b) = -1; }
    int cnt() { return (a != -1) + (b != -1); }
    int a, b;
};

struct F { P3 q; int a, b, c; };

vector<F> hull3d(const vector<P3>& A) {
    assert(sz(A) >= 4);
    vector<vector<PR>> E(sz(A)), vector<PR>(sz(A), {-1, -1});
#define E(x,y) E[f.x][f.y]
    vector<F> FS;
    auto mf = [&](int i, int j, int k, int l) {
        P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
        if (q.dot(A[l]) > q.dot(A[i])) {
            q = q * -1;
            F f{q, i, j, k};
            E(a,b).ins(k); E(a,c).ins(j); E(b,c).ins(i);
            FS.push_back(f);
        }
        rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
            mf(i, j, k, 6 - i - j - k);
    };
    rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
        mf(i, j, k, 6 - i - j - k);

    rep(i,4,sz(A)) {
        rep(j,0,sz(FS)) {
            F f = FS[j];
            if (f.q.dot(A[i]) > f.q.dot(A[f.a])) {
                E(a,b).rem(f.c);
                E(a,c).rem(f.b);
                E(b,c).rem(f.a);
                swap(FS[j--], FS.back());
                FS.pop_back();
            }
        }
        int nw = sz(FS);
        rep(j,0,nw) {
            F f = FS[j];
#define C(a, b, c) if (E(a,b).cnt() != 2) mf(f.a, f.b, i, f.c);
            C(a, b, c); C(a, c, b); C(b, c, a);
        }
        for (F& it : FS) if ((A[it.b] - A[it.a]).cross(
            A[it.c] - A[it.a]).dot(it.q) <= 0) swap(it.c, it.b);
    }
    return FS;
}

```

### sphericalDistance.h

**Description:** Returns the shortest distance on the sphere with radius radius between the points with azimuthal angles (longitude) f1 ( $\phi_1$ ) and f2 ( $\phi_2$ ) from x axis and zenith angles (latitude) t1 ( $\theta_1$ ) and t2 ( $\theta_2$ ) from z axis (0 = north pole). All angles measured in radians. The algorithm starts by converting the spherical coordinates to cartesian coordinates so if that is what you have you can use only the two last rows. dx\*radius is then the difference between the two points in the x direction and d\*radius is the total distance between the points.

611f07, 8 lines

```

double sphericalDistance(double f1, double t1,
    double f2, double t2, double radius) {
    double dx = sin(t2)*cos(f2) - sin(t1)*cos(f1);
    double dy = sin(t2)*sin(f2) - sin(t1)*sin(f1);
    double dz = cos(t2) - cos(t1);
    double d = sqrt(dx*dx + dy*dy + dz*dz);
    return radius*2*asin(d/2);
}

```

## Strings (9)

### KMP.h

Time:  $\mathcal{O}(n)$ 

3ac13f, 21 lines

```

// for each position (0 based ***) of s, what is the best match
// of a suffix at that position with a prefix of s
vector<int> prefixFunction(const string& s){
    int n=s.size(); vector<int> pi(n); pi[0]=0;
    for(int i=1;i<n;++i){
        int j=pi[i-1];
        while(j>0 and s[j]!=s[i]) j=pi[j-1];
        if(s[j]==s[i]) j++; pi[i]=j;
    }
    return pi;
}

// if the automaton is built on string s, then :
// aut[j][ch] = if i'm scanning/constructing some string and rn
// the best match between prefix_of_s and
// suffix_of_MY_string
// is of LENGTH j, and the next scanned/put character is ch,
// then what would be the best matched LENGTH
vector<vector<int>> prefixAutomaton(const string& s){
    int m=s.size(); vector<int> pi=prefixFunction(s);
    const int alph=26; vector<vector<int>> aut(m+1,vector<int>(alph));
    for(int j=0;j<=m;j++) {
        for(int c=0;c<alph;c++) {
            if(j<m&&s[j]==char('a'+c)) aut[j][c]=j+1;
            else if(j==0) aut[j][c]=0;
            else aut[j][c]=aut[pi[j-1]][c];
        }
    }
    return aut;
}

```

### Zfunc.h

**Description:** z[i] computes the length of the longest common prefix of s[i:] and s, except z[0] = 0. (abacaba -> 0010301)

Time:  $\mathcal{O}(n)$ 

ee09e2, 12 lines

```

vi Z(const string& S) {
    vi z(sz(S));
    int l = -1, r = -1;
    rep(i,1,sz(S)) {
        z[i] = i >= r ? 0 : min(r - i, z[i - 1]);
        while (i + z[i] < sz(S) && S[i + z[i]] == S[z[i]])
            z[i]++;
        if (i + z[i] > r)
            l = i, r = i + z[i];
    }
    return z;
}

```

## Manacher.h

**Description:** For each position in a string, computes  $p[0][i]$  = radius of longest even palindrome where central pair is  $(i-1, i)$ ,  $p[1][i]$  = radius of longest odd (center EXCLUDED).

**Time:**  $\mathcal{O}(N)$

e7ad79, 13 lines

```
array<vi, 2> manacher(const string& s) {
    int n = sz(s);
    array<vi, 2> p = {vi(n+1), vi(n)};
    rep(z, 0, 2) for (int i=0, l=0, r=0; i < n; i++) {
        int t = r-i+1;
        if (i < r) p[z][i] = min(t, p[z][l+t]);
        int L = i-p[z][i], R = i+p[z][i]-1;
        while (L>=1 && R+1<n && s[L-1] == s[R+1])
            p[z][i]++;
        L--;
        R++;
        if (R>r) l=L, r=R;
    }
    return p;
}
```

## Trie.h

**Description:** idk

**Time:** idk

6db610, 53 lines

```
struct Trie{
    static const int ALPHABET=26;
    static const char BASE='a';
    struct Node{
        int next[ALPHABET];
        int ends_here_cnt, count; // passes this node
        Node(){}
        fill(next, next+ALPHABET, -1);
        ends_here_cnt=0; count=0; } };
    vector<Node> nodes;
    Trie(): nodes.emplace_back(); } // root node

void insert(const string& s){
    int cur=0;
    for(char ch:s){
        int c=ch-BASE;
        if(nodes[cur].next[c]==-1){
            nodes[cur].next[c]=nodes.size(); nodes.emplace_back();
            cur=nodes[cur].next[c]; nodes[cur].count++;
            nodes[cur].ends_here_cnt++;
        }

        bool erase(const string& s){
            if(!search(s)) return false;
            int cur=0;
            for(char ch:s){
                int c=ch-BASE;
                cur=nodes[cur].next[c]; nodes[cur].count--;
                nodes[cur].ends_here_cnt--;
                return true;
            }

            bool search(const string& s) const{
                int cur=0;
                for(char ch:s){
                    int c=ch-BASE;
                    if(nodes[cur].next[c]==-1||nodes[nodes[cur].next[c]].count
                        ==0) return false;
                    cur=nodes[cur].next[c];
                }
                return nodes[cur].ends_here_cnt>0;
            }

            bool starts_with(const string& prefix) const{
                int cur=0;
                for(char ch:prefix){
                    int c=ch-BASE;
                    if(nodes[cur].next[c]==-1||nodes[nodes[cur].next[c]].count
                        ==0) return false;
                    cur=nodes[cur].next[c];
                }
                return true;
            }
        }
    }
}
```

```
int count_prefix(const string& prefix) const{
    int cur=0;
    for(char ch:prefix){
        int c=ch-BASE;
        if(nodes[cur].next[c]==-1||nodes[nodes[cur].next[c]].count
            ==0) return 0;
        cur=nodes[cur].next[c];
    }
    return nodes[cur].count;
}
```

## Hashing.h

**Description:** Self-explanatory methods for string hashing.

2d2a67, 44 lines

```
// Arithmetic mod  $2^{64}-1$ . 2x slower than mod  $2^{64}$  and more
// code, but works on evil test data (e.g. Thue-Morse, where
// ABBA... and BAAB... of length  $2^{10}$  hash the same mod  $2^{64}$ ).
// "typedef ull H;" instead if you think test data is random,
// or work mod  $10^{9+7}$  if the Birthday paradox is not a problem.
typedef uint64_t ull;
struct H {
    ull x; H(ull x=0) : x(x) {}
    H operator+(H o) { return x + o.x + (x + o.x < x); }
    H operator-(H o) { return *this + ~o.x; }
    H operator*(H o) { auto m = (_uint128_t)x * o.x;
        return H((ull)m) + (ull)(m > 64); }
    ull get() const { return x + !~x; }
    bool operator==(H o) const { return get() == o.get(); }
    bool operator<(H o) const { return get() < o.get(); }
};
static const H C = (ll)1e11+3; // (order ~ 3e9; random also ok)

struct HashInterval {
    vector<H> ha, pw;
    HashInterval(string& str) : ha(sz(str)+1), pw(ha) {
        pw[0] = 1;
        rep(i, 0, sz(str))
            ha[i+1] = ha[i] * C + str[i],
            pw[i+1] = pw[i] * C;
    }
    H hashInterval(int a, int b) { // hash [a, b)
        return ha[b] - ha[a] * pw[b-a];
    }
};

vector<H> getHashes(string& str, int length) {
    if (sz(str) < length) return {};
    H h = 0, pw = 1;
    rep(i, 0, length)
        h = h * C + str[i], pw = pw * C;
    vector<H> ret = {h};
    rep(i, length, sz(str)) {
        ret.push_back(h = h * C + str[i] - pw * str[i-length]);
    }
    return ret;
}

H hashString(string& s){H h{}; for(char c:s) h=h*C+c;return h;}
```

## SuffixArray.h

**Description:** Builds suffix array for a string.  $sa[i]$  is the starting index of the suffix which is  $i$ 'th in the sorted suffix array. The returned vector is of size  $n+1$ , and  $sa[0] = n$ . The  $lcp$  array contains longest common prefixes for neighbouring strings in the suffix array:  $lcp[i] = lcp(sa[i], sa[i-1])$ ,  $lcp[0] = 0$ . The input string must not contain any nul chars.

**Time:**  $\mathcal{O}(N \log n)$

635552, 22 lines

```
struct SuffixArray {
    vi sa, lcp;
```

```
SuffixArray(string s, int lim=256) { // or vector<int>
    s.push_back(0); int n = sz(s), k = 0, a, b;
    vi x(all(s)), y(n), ws(max(n, lim));
    sa = lcp = y, iota(all(sa), 0);
    for (int j = 0, p = 0; p < n; j = max(1, j * 2), lim = p) {
        p = j, iota(all(y), n - j);
        rep(i, 0, n) if (sa[i] >= j) y[p++] = sa[i] - j;
        fill(all(ws), 0);
        rep(i, 0, n) ws[x[i]]++;
        rep(i, 1, lim) ws[i] += ws[i - 1];
        for (int i = n; i-->0;) sa[--ws[x[y[i]]]] = y[i];
        swap(x, y), p = 1, x[sa[0]] = 0;
        rep(i, 1, n) a = sa[i-1], b = sa[i], x[b] =
            (y[a] == y[b] && y[a+j] == y[b+j]) ? p - 1 : p++;
    }
    for (int i = 0, j; i < n - 1; lcp[x[i++]] = k) {
        for (k && k--, j = sa[x[i] - 1];
            s[i + k] == s[j + k]; k++);
    }
}
```

## AhoCorasick.h

**Description:** Aho-Corasick automaton, used for multiple pattern matching. Initialize with AhoCorasick ac(patterns); the automaton start node will be at index 0. find(word) returns for each position the index of the longest word that ends there, or -1 if none. findAll(-, word) finds all words (up to  $N\sqrt{N}$  many if no duplicate patterns) that start at each position (shortest first). Duplicate patterns are allowed; empty patterns are not. To find the longest words that start at each position, reverse all input. For large alphabets, split each symbol into chunks, with sentinel bits for symbol boundaries.

**Time:** construction takes  $\mathcal{O}(26N)$ , where  $N = \text{sum of length of patterns}$ .  $\text{find}(x)$  is  $\mathcal{O}(N)$ , where  $N = \text{length of } x$ .  $\text{findAll}$  is  $\mathcal{O}(NM)$ .

```
29052a, 43 lines
```

```
struct Aho{
    static const int K=26; // alphabet size
    struct Vertex{
        int next[K], link, term_link; // suffix link, terminal link
        bool output; // true if this node is end of some pattern
        vector<int> ids; // pattern ids ending here
        int len;
        Vertex(){}
        fill(begin(next), end(next), -1);
        link=-1; term_link=-1; output=false; len=0; } };
    vector<Vertex> t={Vertex()} // root = state 0

void add_string(const string& s,int id){
    int v=0;
    for(char ch:s{
        int c=c-ch-'a';
        if(t[v].next[c]==-1) t[v].next[c]=t.size(); t.emplace_back();
        v=t[v].next[c];
    }
    v=t[v].next[c]; t[v].output=true; t[v].ids.push_back(id); }

void build_automaton(){ // build the automaton T.T
    queue<int> q;
    t[0].link=0; t[0].term_link=0;
    for(int c=0;c<K;c++){
        int u=t[0].next[c];
        if(u!=-1) { t[u].link=0; q.push(u); t[u].len=1; }
        else t[0].next[c]=0; } // missing edge from root loops to
        root
    while(!q.empty()){
        int v=q.front(); q.pop();
        for(int c=0;c<K;c++){
            int u=t[v].next[c];
            if(u!= -1){ // compute suffix link
                t[u].link=t[t[v].link].next[c];
                t[u].len=t[v].len+1; q.push(u); }
```

```

else t[v].next[c]=t[t[v].link].next[c]; }
// compute terminal link
if(t[t[v].link].output) t[v].term_link=t[v].link;
else t[v].term_link=t[t[v].link].term_link; } }

void form_tree(){
gr.resize(t.size());
L(state,l,t.size()-1){ gr[t[state].term_link].push_back(state);
}; } }
};

Eertree.h
Description: idk
Time: idk
f2cef9, 48 lines

```

```

// len = length of palindrome represented by this node
// link = points to longest proper palindromic suffix
// st, en = some starting and ending index of this palindrome
// oc = number of occurrences of this palindrome
// cnt = number of palindromic suffixes of this palindrome
// [used in problems like: how many palindromes end at each
pos]
struct PalindromicTree {
    struct node { int nxt[26], len, st, en, link; ll cnt, oc; };
    string s; vector<node> t; int sz, last;
    PalindromicTree() {}
    PalindromicTree(string _s) {
        s = _s; int n = s.size();
        t.clear(); t.resize(n + 9);
        sz = 2, last = 2;
        t[1].len = -1, t[1].link = 1;
        t[2].len = 0, t[2].link = 1;
    }
    int extend(int pos){//returns 1 if it creates a new palindrome
        int cur = last, curlen = 0;
        int ch = s[pos] - 'a';
        while (1) {
            curlen = t[cur].len;
            if(pos-1-curlen >= 0 && s[pos-1-curlen] == s[pos]) break;
            cur = t[cur].link;
        }
        if (t[cur].nxt[ch]) {
            last = t[cur].nxt[ch];
            t[last].oc++; return 0;
        }
        sz++; last = sz;
        t[sz].oc = 1;
        t[sz].len = t[cur].len + 2;
        t[cur].nxt[ch] = sz;
        t[sz].en = pos;
        t[sz].st = pos - t[sz].len + 1;
        if (t[sz].len == 1) {
            t[sz].link = 2; t[sz].cnt = 1; return 1;
        }
        while (1) {
            cur = t[cur].link; curlen = t[cur].len;
            if (pos - 1 - curlen >= 0 && s[pos-1-curlen] == s[pos]) {
                t[sz].link = t[cur].nxt[ch]; break;
            }
        }
        t[sz].cnt = 1 + t[t[sz].link].cnt; return 1;
    }
    void calc_occurrences() {
        for (int i = sz; i >= 3; i--) t[t[i].link].oc += t[i].oc;
    }
};

SuffixAutomaton.h
Description: idk
Time: idk
b94687, 51 lines

```

```

struct SAM{
    struct State{
        int link,maxlen;
        map<char,int> next;
        State(){ link=-1; maxlen=0; } };
    vector<State> st;
    vector<vector<int>> gr;
    vector<ll> koybar;
    vector<bool> was_terminal;
    int last,sz;
    ll total; // number of unique substrings
};

SAM(int n){
    st.resize(2*n); gr.resize(2*n); koybar.resize(2*n);
    was_terminal.resize(2*n);
    st[0]=State(); sz=1; last=0; total=0; }

void extend(char c){
    int cur=sz++;
    st[cur].maxlen=st[last].maxlen+1;
    int p=last;
    while(p!=1&&st[p].next.count(c)){
        st[p].next[c]=cur; p=st[p].link; }
    if(p==1) st[cur].link=0; }
    else{
        int q=st[p].next[c];
        if(st[p].maxlen+1==st[q].maxlen){ st[cur].link=q; }
        else{
            int clone=sz++;
            st[clone]=st[q];
            st[clone].maxlen=st[p].maxlen+1;
            while(p!=1&&st[p].next[c]==q){
                st[p].next[c]=clone; p=st[p].link; }
            st[q].link=st[cur].link=clone; }
        last=cur; was_terminal[last]=true;
        total+=st[cur].maxlen-st[st[cur].link].maxlen; }

void build_tree(){ // USE (sam.sz-1) when you need to visit
    all states
    L(i,1,sz-1) gr[st[i].link].push_back(i); }

void guno(){ // count repetitions in the string for each node
    int cur=last;
    L(i,1,sz-1) koybar[i]=was_terminal[i];
    function<void(int)> dfs=[&](int node){
        for(int ch:gr[node]){ dfs(ch); koybar[node]+=koybar[ch]; } }
    ;
    dfs(0); }

ll koyta(int i){ return st[i].maxlen-st[st[i].link].maxlen; }

// after adding the entire string, if you need terminal states
:
// start from last, go back by suffix links. all visited
states are terminal states
};

MinRotation.h

```

Description: Finds the lexicographically smallest rotation of a string.  
Usage: rotate(v.begin(), v.begin() + minRotation(v), v.end());  
Time:  $\mathcal{O}(N)$

```

d07a42, 8 lines
int minRotation(string s) {
    int a=0, N=sz(s); s += s;
    rep(b,0,N) rep(k,0,N) {
        if (a+k == b || s[a+k] < s[b+k]) { b += max(0, k-1); break; }
        if (s[a+k] > s[b+k]) { a = b; break; }
    }
    return a;
}

```

```

}

```

## SuffixTree.h

**Description:** Ukkonen's algorithm for online suffix tree construction. Each node contains indices  $[l, r)$  into the string, and a list of child nodes. Suffixes are given by traversals of this tree, joining  $[l, r)$  substrings. The root is 0 (has  $l = -1, r = 0$ ), non-existent children are -1. To get a complete tree, append a dummy symbol – otherwise it may contain an incomplete path (still useful for substring matching, though).

**Time:**  $\mathcal{O}(26N)$

aae0b8, 50 lines

```

struct SuffixTree {
    enum { N = 200010, ALPHA = 26 }; // N ~ 2*maxlen+10
    int toi(char c) { return c - 'a'; }
    string a; // v = cur node, q = cur position
    int t[N][ALPHA], l[N], r[N], p[N], s[N], v=0, q=0, m=2;
};

```

```

void ukkadd(int i, int c) { suff:
    if (r[v]<=q) {
        if (t[v][c]==-1) { t[v][c]=m; l[m]=i;
            p[m+]=v; v=s[v]; q=r[v]; goto suff; }
        v=t[v][c]; q=l[v];
    }
    if (q== -1 || c==toi(a[q])) q++; else {
        l[m+1]=i; p[m+1]=m; l[m]=l[v]; r[m]=q;
        p[m]=p[v]; t[m][c]=m+1; t[m][toi(a[q])]=v;
        l[v]=q; p[v]=m; t[p[m]][toi(a[l[m]])]=m;
        v=s[p[m]]; q=l[m];
        while (q<r[m]) { v=t[v][toi(a[q])]; q+=r[v]-1[v]; }
        if (q==r[m]) s[m]=v; else s[m]=m+2;
        q=r[v]-(q-r[m]); m+=2; goto suff;
    }
}

```

```

SuffixTree(string a) : a(a) {
    fill(r,r+N,sz(a));
    memset(s, 0, sizeof s);
    memset(t, -1, sizeof t);
    fill(t[1],t[1]+ALPHA,0);
    s[0] = 1; l[0] = l[1] = -1; r[0] = r[1] = p[0] = p[1] = 0;
    rep(i,0,sz(a)) ukkadd(i, toi(a[i]));
}

// example: find longest common substring (uses ALPHA = 28)
pii best;
int lcs(int node, int il, int i2, int olen) {
    if (l[node] <= il && il < r[node]) return 1;
    if (l[node] <= i2 && i2 < r[node]) return 2;
    int mask = 0, len = node ? olen + (r[node] - l[node]) : 0;
    rep(c,0,ALPHA) if (t[node][c] != -1)
        mask |= lcs(t[node][c], il, i2, len);
    if (mask == 3)
        best = max(best, {len, r[node] - len});
    return mask;
}
static pii LCS(string s, string t) {
    SuffixTree st(s + (char)('z' + 1) + t + (char)('z' + 2));
    st.lcs(0, sz(s), sz(s) + 1 + sz(t), 0);
    return st.best;
}
};

BitTrie.h

```

Description: idk  
Time: idk

5391dd, 40 lines

```

struct BitTrie{
    struct Node{
        int child[2],cnt; // trie[0]=total numbers inserted in trie
    };
};

```

```

Node() { child[0]=child[1]=-1; cnt=0; } };
vector<Node> trie;
int BITS;

BitTrie(int maxBits=30){ // 30 for numbers up to 1e9
    BITS=maxBits; trie.push_back(Node()); } // root

void insert(int x){
    int node=0;
    trie[node].cnt++;
    for(int b=BITS;b>=0;b--) {
        int bit=(x>>b)&1;
        if(trie[node].child[bit]==-1) {
            trie[node].child[bit]=trie.size(); trie.push_back(Node());
        }
        node=trie[node].child[bit]; trie[node].cnt++; } }

void erase(int x){
    int node=0;
    trie[node].cnt--;
    for(int b=BITS;b>=0;b--) {
        int bit=(x>>b)&1;
        int nxt=trie[node].child[bit];
        node=nxt; trie[node].cnt--; } }

ll max_xor(ll x){ // find y from this trie that maximizes x^y
if(trie[0].cnt==0) return 0; // empty trie
ll cur=0,y=0;
R(bit,BITS,0){
    int nextbit=getBit(x,bit); // at root, nextbit=30
    if(trie[cur].child[!nextbit]!=-1&&trie[trie[cur].child[!nextbit]].cnt){
        cur=trie[cur].child[!nextbit];
        if(!nextbit) y=setbit(y,bit);
    }else{ // has to exist, since tree is not empty
        cur=trie[cur].child[nextbit];
        if(nextbit) y=setbit(y,bit); } }
return x^y; }
};


```

## Various (10)

### 10.1 Misc. algorithms

#### TernarySearch.h

**Description:** Find the smallest  $i$  in  $[a, b]$  that maximizes  $f(i)$ , assuming that  $f(a) < \dots < f(i) \geq \dots \geq f(b)$ . To reverse which of the sides allows non-strict inequalities, change the  $<$  marked with (A) to  $\leq$ , and reverse the loop at (B). To minimize  $f$ , change it to  $>$ , also at (B).

**Usage:** `int ind = ternSearch(0,n-1,[&](int i){return a[i];});`

**Time:**  $\mathcal{O}(\log(b-a))$

9155b4, 11 lines

```

template<class F>
int ternSearch(int a, int b, F f) {
    assert(a <= b);
    while (b - a >= 5) {
        int mid = (a + b) / 2;
        if (f(mid) < f(mid+1)) a = mid; // (A)
        else b = mid+1;
    }
    rep(i,a+1,b+1) if (f(a) < f(i)) a = i; // (B)
    return a;
}

```

#### LIS.h

**Description:** Compute indices for the longest increasing subsequence.

**Time:**  $\mathcal{O}(N \log N)$

2932a0, 17 lines

```

template<class I> vi lis(const vector<I>& S) {
    if (S.empty()) return {};
    vi prev(sz(S));
    typedef pair<I, int> p;
    vector<p> res;
    rep(i,0,sz(S)) {
        // change 0->i for longest non-decreasing subsequence
        auto it = lower_bound(all(res), p{S[i], 0});
        if (it == res.end()) res.emplace_back(), it = res.end()-1;
        *it = {S[i], i};
        prev[i] = it == res.begin() ? 0 : (it-1)->second;
    }
    int L = sz(res), cur = res.back().second;
    vi ans(L);
    while (L--) ans[L] = cur, cur = prev[cur];
    return ans;
}

```

#### Diophantine.h

**Description:** idk

**Time:** idk

ac879d, 23 lines

```

// Extended Euclidean Algorithm
ll ext_gcd(ll a,ll b,ll& x,ll& y) {
    if(b==0) { x=1,y=0; return a; }
    ll x1,y1,g=ext_gcd(b,a%b,x1,y1);
    x=y1; y=x1-(a/b)*y1; return g; }
// Solve ax + by = c
// general solution : x + k*b/g, y - k*a/g
bool solve_diophantine(ll a,ll b,ll c,ll& x,ll& y,ll& g) {
    g=ext_gcd(abs(a),abs(b),x,y);
    if(c%g!=0) return false;
    x*=c/g; y*=c/g;
    if(a<0)x=-x; if(b<0)y=-y; return true; }

bool solve_dioph_non_neg(ll a,ll b,ll c,ll& x,ll& y,ll& g) {
    if(!solve_diophantine(a,b,c,x,y,g)) return false;
    ll dx=(b/g),dy=(a/g);
    if(x<0){
        ll k=(-x+abs(dx)-1)/abs(dx); // ceil
        if(dx<0)k=-k; x+=k*dx; y-=k*dy; }
    if(y<0){
        ll k=(-y+abs(dy)-1)/abs(dy);
        if(dy<0)k=-k; y+=k*dy; x-=k*dx; }
    return x>=0 and y>=0; }

```

### 10.2 Dynamic programming

#### SOSDP.h

**Description:** idk

**Time:** idk

f78b69, 30 lines

```

// for problems like sum/count involving submasks/supermask,
// we use sos dp to avoid overcounting.
// x | y = x : y is submask of x
// x & y = x : y is supermask of x
// x & y = 0 : y is submask of ~x

const int N=1<<20;
vll cnt(N);
vll dp1(N);
vll dp2(N);

void fwd1(vll& dp){ // dp[x] = cnt of submask of x
    L(bit,0,19){
        L(mask,0,N-1){
            if(getBit(mask,bit)) dp[mask]+=dp[resetbit(mask,bit)]; } } }

void bak1(vll& dp){ // return from submask count to mask count
    R(bit,19,0){
        L(mask,0,N-1){
            if(getBit(mask,bit)) dp[resetbit(mask,bit)]+=dp[mask]; } } }


```

```

R(mask,N-1,0){
    if(getBit(mask,bit)) dp[mask]-=dp[resetbit(mask,bit)]; } } }

void fwd2(vll& dp){ // dp[x] = cnt of supermask of x
    L(bit,0,19){
        R(mask,N-1,0){
            if(getBit(mask,bit)) dp[resetbit(mask,bit)]+=dp[mask]; } } }

void bak2(vll& dp){ // return from supermask count to mask count
    R(bit,19,0){
        L(mask,0,N-1){
            if(getBit(mask,bit)) dp[resetbit(mask,bit)]-=dp[mask]; } } }

DigitDP.h
Description: idk
Time: idk
ea364a, 26 lines
```

```

11 n,m,k,q;
11 a[N];
11 dp[] [11][2][2]; // put max len of string + 1

bool is_valid(){ return 1; } // for even sum, return sum % 2 == 0;

11 cnt(string s,11 pos,11 prev,bool started,bool tight){
    if(pos==Size(s)) return is_valid(); // number of such numbers = 1
    if(dp[pos][prev][started][tight]!=-1) return dp[pos][prev][started][tight];
    11 ans=0;
    11 digMax=tight?s[pos]-'0':9;
    for(11 dig=0;dig<=digMax;dig++){
        // ***
        if(started and dig==prev) continue; // no repetition
        bool startedNext=started?1:dig!=0; // started || (dig != 0)
        bool tightNext=tight?(dig==digMax):0; // tight && (dig == digMax)
        ans+=cnt(s,pos+1,dig,startedNext,tightNext);
    }
    return dp[pos][prev][started][tight]=ans;
}

// usage:
string s1 = to_string(n1 - 1);
memset(dp, -1, sizeof(dp));
11 ans1; if(n1-1 < 0) ans1 = 0;
// 10 is a dummy value for prev.
else ans1 = cnt(s1, 0, 10, 0, 1);
// similarly, find ans2, and output : ans2-ans1
```

#### DivideAndConquerDP.h

**Description:** Given  $a[i] = \min_{lo(i) \leq k < hi(i)} (f(i, k))$  where the (minimal) optimal  $k$  increases with  $i$ , computes  $a[i]$  for  $i = L..R - 1$ .

**Time:**  $\mathcal{O}((N + (hi - lo)) \log N)$

```

d38d2b, 18 lines

struct DP { // Modify at will:
    int lo(int ind) { return 0; }
    int hi(int ind) { return ind; }
    11 f(int ind, int k) { return dp[ind][k]; }
    void store(int ind, int k, 11 v) { res[ind] = pii(k, v); }

    void rec(int L, int R, int LO, int HI) {
        if (L >= R) return;
        int mid = (L + R) >> 1;
        pair<11, int> best(LLONG_MAX, LO);
        rep(k, max(LO, lo(mid)), min(HI, hi(mid)))
            best = min(best, make_pair(f(mid, k), k));
        store(mid, best.second, best.first);
        rec(L, mid, LO, best.second+1);
        rec(mid+1, R, best.second, HI);
    }
}
```

```
}
void solve(int L, int R) { rec(L, R, INT_MIN, INT_MAX); }
};
```

**BumpAllocator.h**

**Description:** When you need to dynamically allocate many objects and don't care about freeing them. "new X" otherwise has an overhead of something like 0.05us + 16 bytes per allocation.

745db2, 8 lines

```
// Either globally or in a single class:
static char buf[450 << 20];
void* operator new(size_t s) {
    static size_t i = sizeof buf;
    assert(s < i);
    return (void*)&buf[i -= s];
}
void operator delete(void*) {}
```

**submasks.h**

**Description:** ...

**Time:** ...

b0dc54, 2 lines

```
for (int mask = 0; mask < (1<<n); mask++)
    for(int i = mask; i; i = (i-1) & mask)
```

**int128.h**

**Description:** idk

**Time:** idk

6125c0, 14 lines

```
using i128=__int128_t;
i128 read(){
    string s; cin>>s; bool neg=false; size_t i=0;
    if(s[0]=='-') { neg=true; i=1; } i128 mag=0;
    for(;i<s.size();i++) { mag=mag*10+(unsigned)(s[i]-'0'); }
    return neg?mag:-mag; }
void print(i128 x){
    if(x==0){cout<<0<<n;return;}
    bool neg=x<0; if(neg)x=-x;
    string s;while(x){ int d=(int)(x%10);
        s.push_back(char('0'+d)); x/=10; }
    if(neg)s.push_back('-'); reverse(s.begin(),s.end());
    cout<<s<<n; }
bool cmp(i128 x,i128 y){ return x>y; } // for sorting
```

**FloorCeilDiv.h**

**Description:** ...

**Time:** ...

d77b32, 10 lines

```
ll floor_div(ll a, ll b) {
    if (b < 0) a = -a, b = -b;
    if (a >= 0) return a / b;
    return (a - b + 1) / b;
}
ll ceil_div(ll a, ll b) {
    if (b < 0) a = -a, b = -b;
    if (a >= 0) return (a + b - 1) / b;
    return a / b;
}
```

**LinearSieve.h**

**Description:** ...

**Time:** ...

84f2ca, 34 lines

```
const int N = 1000000;
vector<int> primes;
int spf[N + 1]; int phi[N + 1]; int mu[N + 1];
bool is_composite[N + 1];
// int func[MAXN], cnt[MAXN]; // Extension
```

```
void linear_sieve(int n) {
    std::fill(is_composite, is_composite + n, false);
    phi[1] = 1; mu[1] = 1;
    for (int i = 2; i <= n; i++) {
        if (!is_composite[i]) {
            primes.push_back(i);
            spf[i] = i; phi[i] = i - 1; mu[i] = -1;
        }
        for (int p : primes) {
            if (i * 1LL * p > n) break;
            is_composite[i * p] = true;
            spf[i * p] = p;

            if (i % p == 0) {
                phi[i * p] = phi[i] * p;
                mu[i * p] = 0;
                // func[i*prime[j]] = func[i]/cnt[i] * (cnt[i]+1);
                // cnt[i * prime[j]] = cnt[i] + 1;
                break;
            } else {
                phi[i * p] = phi[i] * (p - 1);
                mu[i * p] = -mu[i];
                // func[i * prime[j]] = func[i] * func[prime[j]];
                // cnt[i * prime[j]] = 1;
            }
        }
    }
}
```

**XorBasis.h**

**Description:** ...

**Time:** ...

5be624, 59 lines

```
struct XorBasis {
    static const int B = 60; // for 64-bit numbers
    long long b[B]; int sz;
    XorBasis() {
        memset(b, 0, sizeof b); sz = 0;
    }
    // Insert number into basis
    void insert(long long x) {
        for (int i = B-1; i >= 0; i--) {
            if (!(x >> i & 1)) continue;
            if (b[i]) x ^= b[i]; // eliminate bit
            else { // new basis vector
                b[i] = x; sz++; return;
            }
        }
        // Check if some subset has xor = x
        bool can(long long x) {
            for(int i = B-1; i>=0; i--) x = min(x, x ^ b[i]);
            return x == 0;
        }
        // Maximum possible xor with optional start value
        long long max_xor(long long x = 0) {
            for(int i = B-1; i>=0; i--) x = max(x, x ^ b[i]);
            return x;
        }
        // k-th smallest subset xor (1-indexed, 1st = 0)
        long long kth(long long k) {
            if (k < 1 || k > (1LL << sz)) return -1;
            long long x = 0, cnt = 1LL << sz;
            for (int i = B-1; i >= 0; i--) if (b[i]) {
                if (k > cnt/2) {
                    if (!(x >> i & 1)) x ^= b[i]; k -= cnt/2;
                } else {
                    if (x >> i & 1) x ^= b[i];
                }
                cnt /= 2;
            }
        }
}
```

```

        return x;
    }
    // Count how many subset xors are < x
    long long count_lt(long long x) {
        if (x < 0) return 0;
        long long ans = 0, cnt = 1LL << sz;
        long long mask = 0;
        for (int i = B-1; i >= 0; i--) {
            if (b[i]) {
                if (x >> i & 1) {
                    ans += cnt/2;
                    if (!(mask >> i & 1)) mask ^= b[i];
                } else if (mask >> i & 1) {
                    mask ^= b[i];
                }
                cnt /= 2;
            }
        }
        else if ((x >> i & 1) != (mask >> i & 1)) {
            return (x >> i & 1) ? ans + cnt : ans;
        }
    }
    return ans;
}}
```

**SubsetSumSqrt.h**

**Description:** ...

**Time:** ...

f9ccce, 28 lines

```
main:
// Sum of elements <= N implies that every element is <= N
vector<int> freq(N + 1, 0);
for (int i = 0; i < N; i++) {
    int x; cin >> x; freq[x]++;
}
vector<pair<int, int>> compressed;
for (int i = 1; i <= N; i++) {
    if (freq[i] > 0) compressed.emplace_back(i, freq[i]);
}
vector<int> dp(N + 1, 0); dp[0] = 1;
for (const auto &[w, k] : compressed) {
    vector<int> ndp = dp;
    for (int p = 0; p < w; p++) {
        int sum = 0;
        for(int mult = p, cnt = 0; mult <= N; mult += w, cnt++){
            if (cnt > k) {
                sum -= dp[mult - w * cnt]; cnt--;
            }
        }
        if (sum > 0) ndp[mult] = 1; sum += dp[mult];
    }
    swap(dp, ndp);
}
cout << "Possible subset sums are:\n";
for (int i = 0; i <= N; i++) {
    if (dp[i] > 0) cout << i << " ";
}
```