SoSe 2020 Kiel University
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# Solution homework exercise 3.1:

*Statement:*

$$f : \mathbb{R}^2 \to \mathbb{R}, f(x, y) = (e^x + y)y - x^3$$

*has neither minima nor maxima.*

*Proof:* Assume that $f$ has a minimum or maximum $(\hat{x}, \hat{y})^T \in \mathbb{R}^2$. Due to the necessary first order optimality condition,

$$\nabla f \begin{pmatrix} \hat{x} \\ \hat{y} \end{pmatrix} = 0.$$

For all $(x, y)^T \in \mathbb{R}^2$,

$$\nabla f \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} e^x y - 3x^2 \\ e^x + 2y \end{pmatrix}.$$

Thus

$$e^{\hat{x}}\hat{y} - 3\hat{x}^2 = 0$$

and

$$e^{\hat{x}} + 2\hat{y} = 0.$$

Hence, it follows

$$\hat{y} = -\tfrac{1}{2}e^{\hat{x}} < 0$$

from the last equation. But then

$$e^{\hat{x}}\hat{y} < 0$$

and thus

$$e^{\hat{x}}\hat{y} - 3\hat{x}^2 < 0.$$

This is a contradiction to $e^{\hat{x}}\hat{y} - 3\hat{x}^2 = 0$. Hence $f$ has no minimum or maximum. $\square$