Limits Day 3

Warm-Up Questions!

1. What is

$$\lim_{x \to 10} \frac{-1}{(x-10)^4}$$

$$\lim_{x \to 10} \frac{-1}{(x-10)^4} = -\infty$$

Warm-Up Questions!

1. What is

$$\lim_{x \to \infty} \log(x)$$

$$\lim_{x \to \infty} \log(x) = \infty$$

Warm-Up Questions!

1. What is

$$\lim_{x \to \infty} \frac{-(x^3 + 2)}{x^2}$$

$$\lim_{x \to \infty} \frac{-(x^3 + 2)}{x^2} = -\infty$$

Converging Functions

So far we've seen graphs that *diverge* as x grows large. Let's look now at some graphs that *converge* as x grows large.

Consider the function

$$f(x) = \frac{1}{x}$$

What happens as $x \to \infty$?

Converging Functions

$$f(x) = \frac{1}{x}$$

As $x \to \infty$, the denominator gets larger and larger and $f(x) \to 0$. So,

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

This is different from what we've seen so far!

Our function is ***converging*** to a value of 0 as x grows infinitely large!

Another example

Let's look at another example. Consider the function

$$f(x) = \frac{3x^2 + 2x - 5}{x^5}$$

What happens as $x \to \infty$?

Like last time, we can just consider the highest order terms. Since x^5 grows faster than x^2, the denominator is growing faster. Thus

$$\lim_{x\to\infty} \frac{3x^2 + 2x - 5}{x^5} = \lim_{x\to\infty} \frac{3x^2}{x^5} = 0$$

Concept Question

True or False: If a function converges to a value, that value will always be zero.

False! Let's look at the function

$$f(x) = \frac{2x^3 + 25x - 19352}{x^3 - 5}$$

When taking the limit, we just consider the highest order terms

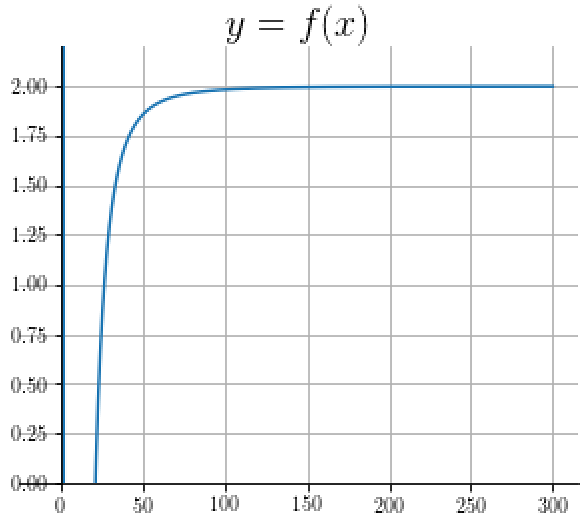
$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{2x^3}{x^3}$$

Now we can ignore any constants

$$\lim_{x \to \infty} \frac{2x^3}{x^3} = 2 \lim_{x \to \infty} \frac{x^3}{x^3}$$

That limit is just one! Thus

$$\lim_{x \to \infty} \frac{2x^3 + 25x - 19352}{x^3 - 5} = 2$$


$y = f(x)$
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
0
50
100
150
200
250
300

The Overall Rule

When taking

$$\lim_{x \to \infty} \frac{p(x)}{q(x)}$$

- If $p(x)$ grows faster than $q(x)$, then the limit diverges to $\pm\infty$
    - Check by either plugging in a value, or looking at the sign of the highest order terms!

- If $q(x)$ grows faster than $p(x)$, then the limit converges to 0

- If $p(x)$ and $q(x)$ grow at the same rate, then the limit converges to the ratio of the coeffecients of the highest order terms
    - For example, if $p(x) = 7x^5 + 12x^2 - \log(x)$ and $q(x) = 9x^5 - 1321x + 25$, then the limit converges to $\frac{7}{9}$

Let's Practice

In ***groups of 3*** evaluate the following limits

$$\lim_{x\to\infty} \frac{5x^3 - 12x + 21}{7x^3 - 21x + 9}$$

$$\lim_{x\to\infty} \frac{\log(x) + x}{\log(x)}$$

$$\lim_{x\to\infty} \frac{2^x}{x^{20}\log(x)}$$

$$\lim_{x\to\infty} \frac{16\log(x)}{20\log(x)}$$

$$\lim_{x\to\infty} \frac{2\log(x)}{4\log(x)^2}$$

Let's Practice

In ***groups of 3*** evaluate the following limits

$$\lim_{x\to\infty} \frac{5x^3 - 12x + 21}{7x^3 - 21x + 9} = \frac{5}{7}$$

$$\lim_{x\to\infty} \frac{\log(x) + x}{\log(x)}$$

$$\lim_{x\to\infty} \frac{2^x}{x^{20}\log(x)}$$

$$\lim_{x\to\infty} \frac{16\log(x)}{20\log(x)}$$

$$\lim_{x\to\infty} \frac{2\log(x)}{4\log(x)^2}$$

Let's Practice

In ***groups of 3*** evaluate the following limits

$$\lim_{x \to \infty} \frac{5x^3 - 12x + 21}{7x^3 - 21x + 9} = \frac{5}{7}$$

$$\lim_{x \to \infty} \frac{\log(x) + x}{\log(x)} = \infty$$

$$\lim_{x \to \infty} \frac{2^x}{x^{20} \log(x)}$$

$$\lim_{x \to \infty} \frac{16 \log(x)}{20 \log(x)}$$

$$\lim_{x \to \infty} \frac{2 \log(x)}{4 \log(x)^2}$$

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$$\lim_{x\to\infty} \frac{16\log(x)}{20\log(x)} = \frac{4}{5}$$

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$$\lim_{x\to\infty} \frac{\log(x) + x}{\log(x)} = \infty$$

$$\lim_{x\to\infty} \frac{2^x}{x^{20}\log(x)} = \infty$$

$$\lim_{x\to\infty} \frac{16\log(x)}{20\log(x)} = \frac{4}{5}$$

$$\lim_{x\to\infty} \frac{2\log(x)}{4\log(x)^2} = 0$$

Let's Practice

In ***groups of 3*** evaluate the following limits

$$\lim_{x\to\infty} \frac{1}{(x+4)}$$

$$\lim_{x\to\infty} \frac{3e^x + \log(x)}{2e^x + 4x^3}$$

$$\lim_{x\to\infty} x\log(x)$$

$$\lim_{x\to\infty} 3$$

$$\lim_{x\to\infty} \frac{x - \log(x)}{x}$$

Let's Practice

In ***groups of 3*** evaluate the following limits

$$\lim_{x\to\infty} \frac{1}{(x+4)} = 0$$

$$\lim_{x\to\infty} \frac{3e^x + \log(x)}{2e^x + 4x^3}$$

$$\lim_{x\to\infty} x\log(x)$$

$$\lim_{x\to\infty} 3$$

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Let's Practice

In ***groups of 3*** evaluate the following limits

$$\lim_{x\to\infty} \frac{1}{(x+4)} = 0$$

$$\lim_{x\to\infty} \frac{3e^x + \log(x)}{2e^x + 4x^3} = \frac{3}{2}$$

$$\lim_{x\to\infty} x\log(x)$$

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Let's Practice

In ***groups of 3*** evaluate the following limits

$$\lim_{x\to\infty} \frac{1}{(x+4)} = 0$$

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$$\lim_{x\to\infty} x\log(x) = \infty$$

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In ***groups of 3*** evaluate the following limits

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$$\lim_{x\to\infty} x\log(x) = \infty$$

$$\lim_{x\to\infty} 3 = 3$$

$$\lim_{x\to\infty} \frac{x - \log(x)}{x}$$

Let's Practice

In ***groups of 3*** evaluate the following limits

$$\lim_{x\to\infty} \frac{1}{(x+4)} = 0$$

$$\lim_{x\to\infty} \frac{3e^x + \log(x)}{2e^x + 4x^3} = \frac{3}{2}$$

$$\lim_{x\to\infty} x\log(x) = \infty$$

$$\lim_{x\to\infty} 3 = 3$$

$$\lim_{x\to\infty} \frac{x - \log(x)}{x} = 1$$

Review

- We have seen how to evaluate a limit at a given point, $\lim_{x \to a} f(x)$
  - First try to directly plug in a and calculate $f(a)$
  - If $f(a)$ is a value, that is the limit.
  - If $f(a)$ is indeterminant, rewrite the limit and re-evaluate
  - If $f(a)$ is not a value but is not indeterminant, it is an asymptote and is either $\pm\infty$ or undefined (in most cases)
- We have also seen how to evaluate a limit at ∞, $\lim_{x \to \infty} f(x)$
  - Consider the highest order/fastest growing terms (exponential $>>$ polynomial $>>$ logarithmic)
  - If numerator grows faster, limit diverges
  - If denominator grows faster, limit converges to 0
  - If both grow at same rate, limit converges to ratio of coeffecients