

Factoring numbers

What does it mean to factor?

We want to *break down* a number into the *product* of *smaller numbers*.

$$10 = 5 \cdot 2$$

True or **false**: Every factorization of a number is unique

False!

$$100 = 10 \cdot 10 = 10 \cdot 5 \cdot 2 = 5 \cdot 2 \cdot 5 \cdot 2$$

$$100 = 50 \cdot 2 = \dots$$

Let's practice

With a partner, factor these numbers into the product of two numbers

$$10 = ? \cdot ?$$

$$25 = ? \cdot ?$$

$$30 = ? \cdot ?$$

$$42 = ? \cdot ?$$

Let's practice

With a partner, factor these numbers into the product of two numbers (un numero → il prodotto di due numeri)

$$10 = 5 \cdot 2$$

$$25 = 5 \cdot 5$$

$$30 = 10 \cdot 3$$

$$42 = 7 \cdot 6$$

Why factor?

It makes life easier!

$$\frac{1638}{63} = ?$$

$$(48)^2 = ?$$

Why factor?

It makes life easier!

$$\frac{1638}{63} = \frac{26 \cdot 9 \cdot 7}{\underbrace{9 \cdot 7}} = \frac{26 \cdot 7}{7} = 26$$

factors cancel out!

$$(48)^2 = (6 \cdot 8)^2 = 6^2 \cdot 8^2 = 36 \cdot 64 = 2304$$

A new, powerful idea

Is there a way to re-write this expression by factoring?

$$10 + 15 + 20$$

let's notice something...

$$5 \cdot 2 + 5 \cdot 3 + 5 \cdot 4$$

what now?

$$5 \cdot 2 + 5 \cdot 3 + 5 \cdot 4 = \boxed{5(2 + 3 + 4)}$$

wow! That looks a lot nicer!

What happened here?

- Notice that the 5 was a ***common factor*** to each number in our sum.
- This means we can ***factor it out*** and multiply the remaining sum by it!

$$\underbrace{5 \cdot 2 + 5 \cdot 3 + 5 \cdot 4}_{5 \text{ is a common factor}} = \underbrace{5(2 + 3 + 4)}_{\text{factor it out}}$$

True or **false**: You can factor a variable out of an expression

True!

A variable really just represents a number. If a variable is common to every term, we can factor it out!

$$5x + 12x + 4x = x(5 + 12 + 4)$$

This works for any number!

$$2x + 3x + 4x = x(2 + 3 + 4)$$

Notice that $x = 5$ becomes our example from before

Let's play a game: find the greatest common factor!

Team up with two other students and try to find the greatest common factor of each expression

$$12 + 28 : ?$$

$$7x + 14x : ?$$

$$5x^2 + 10x : ?$$

$$2 + 4x^2 + 32x : ?$$

$$36x + 12xy + 3x : ?$$

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$$12 + 28 : 4$$

$$7x + 14x : 7x$$

$$5x^2 + 10x : 5x$$

$$2 + 4x^2 + 32x : 2$$

$$36x + 12xy + 3x : 3x$$

Let's use these common factors to factor our expression!

Team up with two **different** students and factor each expression with our new common factors

$$12 + 28 : 4$$

$$7x + 14x : 7x$$

$$5x^2 + 10x : 5x$$

$$2 + 4x^2 + 32x : 2$$

$$36x + 12xy + 3x : 3x$$

$$12 + 28 = ?(? + ?)$$

$$7x + 14x = ?(? + ?)$$

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$$12 + 28 = 4(3 + 7)$$

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$$5x^2 + 10x = ?(? + ?)$$

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$$2 + 4x^2 + 32x = 2(1 + 2x^2 + 16x)$$

$$36x + 12xy + 3x = 3x(12 + 4y + 1)$$

True or **false**: Every expression has a greatest common factor > 1

False!

Some expressions just don't have anything in common 😞. Consider the following expressions

$$2 + 5 + 11$$

$$3x + yz - 6$$

$$14x^2 + 9x - 2$$

Factor the following expressions (if possible)

$$4x^3 + 6x^2y + 12x$$

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$$4x^3 + 6x^2y + 12x = 2x(2x^2 + 3xy + 6)$$

Factor the following expressions (if possible)

$$2x + 5y + 7z$$

Factor the following expressions (if possible)

$$2x + 5y + 7z$$

No common factor :(

Factor the following expressions (if possible)

$$25x^3y^5 - 15x^2y^3$$

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$$25x^3y^5 - 15x^2y^3$$
$$25x^3y^5 - 15x^2y^3 = 5x^2y^3(5xy^2 - 3)$$

Factor the following expressions (if possible)

$$12x^3y^5z^6 + 2y^5z^3 + 5x^2z^3 + 24xy$$

Factor the following expressions (if possible)

$$12x^3y^5z^6 + 2y^5z^3 + 5x^2z^3 + 24xy$$

No common factor :(

Factor the following expressions (if possible)

$$3x^2(y + z) + 6x(12 - z)$$

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$$3x^2(y + z) + 6x(12 - z) = 3x(x(y + z) + 2(12 - z))$$

Factor the following expressions (if possible)

$$25x(6 - x)^2 + 5z(6 - x)$$

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$$25x(6 - x)^2 + 5z(6 - x)$$

$$25x(6 - x) + 5z(6 - x) = 5(6 - x)(5x + z)$$

Factor the following expressions (if possible)

$$25x(6 - x)^2 + 2z(12 - x)$$

Factor the following expressions (if possible)

$$25x(6 - x)^2 + 5z(6 - x)$$

No common factor :(

Next time we will learn some more factoring techniques!