Factoring numbers

What does it mean to factor?

We want to ***break down*** a number into the ***product*** of ***smaller numbers***.

$$10 = 5 \cdot 2$$

True or **false**: **Every factorization of a number is unique**

False!

$$100 = 10 \cdot 10 = 10 \cdot 5 \cdot 2 = 5 \cdot 2 \cdot 5 \cdot 2$$
$$100 = 50 \cdot 2 = \ldots$$

Let's practice

With a partner, factor these numbers into the product of two numbers

$$10 = \ ? \cdot \ ?$$
$$25 = \ ? \cdot \ ?$$
$$30 = \ ? \cdot \ ?$$
$$42 = \ ? \cdot \ ?$$

Let's practice

With a partner, factor these numbers into the product of two numbers (un numero → il prodotto di due numeri)

$$10 = 5 \cdot 2$$
$$25 = 5 \cdot 5$$
$$30 = 10 \cdot 3$$
$$42 = 7 \cdot 6$$

Why factor?

It makes life easier!

$$\frac{1638}{63} = \ ?$$

$$(48)^2 = \ ?$$

Why factor?

It makes life easier!

$$\frac{1638}{63} = \underbrace{\frac{26 \cdot 9 \cdot 7}{9 \cdot 7} = \frac{26 \cdot 7}{7}}_{\text{factors cancel out!}} = 26$$

$$(48)^2 = (6 \cdot 8)^2 = 6^2 \cdot 8^2 = 36 \cdot 64 = 2304$$

A new, powerful idea

Is there a way to re-write this expression by factoring?

$$10 + 15 + 20$$

let's notice something...

$$5 \cdot 2 + 5 \cdot 3 + 5 \cdot 4$$

what now?

$$5 \cdot 2 + 5 \cdot 3 + 5 \cdot 4 = \boxed{5(2 + 3 + 4)}$$

wow! That looks a lot nicer!

What happened here?

- Notice that the 5 was a ***common factor*** to each number in our sum.
- This means we can ***factor it out*** and multiply the remaining sum by it!

$$\underbrace{5 \cdot 2 + 5 \cdot 3 + 5 \cdot 4}_{\text{5 is a common factor}} = \underbrace{5(2 + 3 + 4)}_{\text{factor it out}}$$

True or false: You can factor a variable out of an expression

True!

A variable really just represents a number. If a variable is common to every term, we can factor it out!

$$5x + 12x + 4x = x(5 + 12 + 4)$$

This works for any number!

$$2x + 3x + 4x = x(2 + 3 + 4)$$

Notice that $x = 5$ becomes our example from before

Let's play a game: find the greatest common factor!

Team up with two other students and try to find the greatest common factor of each expression

$$12 + 28 : \ ?$$
$$7x + 14x : \ ?$$
$$5x^2 + 10x : \ ?$$
$$2 + 4x^2 + 32x : \ ?$$
$$36x + 12xy + 3x : \ ?$$

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$$12 + 28 : 4$$
$$7x + 14x : 7x$$
$$5x^2 + 10x : 5x$$
$$2 + 4x^2 + 32x : 2$$
$$36x + 12xy + 3x : 3x$$

Let's use these common factors to factor our expression!

Team up with two **different** students and factor each expression with our new common factors

$$12 + 28 : 4$$
$$7x + 14x : 7x$$
$$5x^2 + 10x : 5x$$
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$$36x + 12xy + 3x : 3x$$

$$12 + 28 = \ ?(\ ? + \ ?)$$
$$7x + 14x = \ ?(\ ? + \ ?)$$
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$$12 + 28 = 4(3 + 7)$$
$$7x + 14x = \ ?(\ ? + \ ?)$$
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$$12 + 28 : 4 \qquad 12 + 28 = 4(3 + 7)$$
$$7x + 14x : 7x \qquad 7x + 14x = 7x(1 + 2)$$
$$5x^2 + 10x : 5x \qquad 5x^2 + 10x = \ ?(\ ? + \ ?)$$
$$2 + 4x^2 + 32x : 2 \qquad 2 + 4x^2 + 32x = \ ?(\ ? + \ ? + \ ?)$$
$$36x + 12xy + 3x : 3x \qquad 36x + 12xy + 3x = \ ?(\ ? + \ ? + \ ?)$$

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$$7x + 14x : 7x \qquad 7x + 14x = 7x(1 + 2)$$
$$5x^2 + 10x : 5x \qquad 5x^2 + 10x = 5x(x + 2)$$
$$2 + 4x^2 + 32x : 2 \qquad 2 + 4x^2 + 32x = \ ?(\ ? + \ ? + \ ?)$$
$$36x + 12xy + 3x : 3x \qquad 36x + 12xy + 3x = \ ?(\ ? + \ ? + \ ?)$$

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$$2 + 4x^2 + 32x = 2(1 + 2x^2 + 16x)$$
$$36x + 12xy + 3x = 3x(12 + 4y + 1)$$

True or false: Every expression has a greatest common factor > 1

False!

Some expressions just don't have anything in common ☹. Consder the following expressions

$$2 + 5 + 11$$
$$3x + yz - 6$$
$$14x^2 + 9x - 2$$

Factor the following expressions (if possible)

$$4x^3 + 6x^2y + 12x$$

$$4x^3 + 6x^2y + 12x = 2x(2x^2 + 3xy + 6)$$

Factor the following expressions (if possible)

$$2x + 5y + 7z$$

No common factor :(

Factor the following expressions (if possible)

$$25x^3y^5 - 15x^2y^3$$

$$25x^3y^5 - 15x^2y^3 = 5x^2y^3(5xy^2 - 3)$$

Factor the following expressions (if possible)

$$12x^3y^5z^6 + 2y^5z^3 + 5x^2z^3 + 24xy$$

No common factor :(

Factor the following expressions (if possible)

$$3x^2(y+z)+6x(12-z)$$

$$3x^2(y+z)+6x(12-z)=3x\left(x(y+z)+2(12-z)\right)$$

Factor the following expressions (if possible)

$$25x(6-x)^2 + 5z(6-x)$$

$$25x(6-x) + 5z(6-x) = 5(6-x)\,(5x+z)$$

Factor the following expressions (if possible)

$$25x(6-x)^2 + 2z(12-x)$$

No common factor :(

Next time we will learn some more factoring techniques!