Second Degree Equations Part 1

Luke Bordonaro

Topics

- Understanding the degree of an equation
- How to use the *quadratic formula*
- How to factor any second degree expression with the *quadratic formula*
- How to graph a parabola
- How to solve a 2nd degree word problem by
    - Graphing
    - Factoring directly
    - Using the quadratic equation

Some Review on Polynomials

- Recall that a polynomial is a collection of terms. For example, $x^4 + 2x^2 + 5$ is a polynomial.
- The ***degree*** of a polynomial is the highest exponent it has.
    - $x^4 + 2x^2 + 5$ is a ***fourth***, 4^{th}, degree polynomial, because the highest exponent is 4
- A *second degree equation* is an equation where the maximum degree on either side is 2
    - $x^2 + 2x + 3 = 0$ is an example of a second degree equation

Practice

For each of the following equations, determine if it is a second degree equation

$$x^3 + 2x^2 + 5x- = 0$$
$$x^2 - 10x + 2 = 0$$
$$x - 105 = 0$$
$$x^2 = 25$$
$$x^2 + 2x = 0$$
$$0x^3 + 2x^2 + 8x - 5 = 2$$

Practice

For each of the following equations, determine if it is a second degree equation

$$x^3 + 2x^2 + 5x- = 0 \qquad \text{NO}$$
$$x^2 - 10x + 2 = 0$$
$$x - 105 = 0$$
$$x^2 = 25$$
$$x^2 + 2x = 0$$
$$0x^3 + 2x^2 + 8x - 5 = 2$$

Practice

For each of the following equations, determine if it is a second degree equation

$$x^3 + 2x^2 + 5x- = 0 \qquad \text{NO}$$

$$x^2 - 10x + 2 = 0 \qquad \text{YES}$$

$$x - 105 = 0$$

$$x^2 = 25$$

$$x^2 + 2x = 0$$

$$0x^3 + 2x^2 + 8x - 5 = 2$$

Practice

For each of the following equations, determine if it is a second degree equation

$$x^3 + 2x^2 + 5x- = 0 \qquad \text{NO}$$
$$x^2 - 10x + 2 = 0 \qquad \text{YES}$$
$$x - 105 = 0 \qquad \text{NO}$$
$$x^2 = 25$$
$$x^2 + 2x = 0$$
$$0x^3 + 2x^2 + 8x - 5 = 2$$

Practice

For each of the following equations, determine if it is a second degree equation

$$
\begin{array}{cc}
x^3 + 2x^2 + 5x- = 0 & \text{NO} \\
x^2 - 10x + 2 = 0 & \text{YES} \\
x - 105 = 0 & \text{NO} \\
x^2 = 25 & \text{YES} \\
x^2 + 2x = 0 & \\
0x^3 + 2x^2 + 8x - 5 = 2 &
\end{array}
$$

Practice

For each of the following equations, determine if it is a second degree equation

$x^3 + 2x^2 + 5x- = 0$	NO
$x^2 - 10x + 2 = 0$	YES
$x - 105 = 0$	NO
$x^2 = 25$	YES
$x^2 + 2x = 0$	YES
$0x^3 + 2x^2 + 8x - 5 = 2$	

Practice

For each of the following equations, determine if it is a second degree equation

$$\begin{array}{cc} x^3 + 2x^2 + 5x- = 0 & \text{NO} \\ x^2 - 10x + 2 = 0 & \text{YES} \\ x - 105 = 0 & \text{NO} \\ x^2 = 25 & \text{YES} \\ x^2 + 2x = 0 & \text{YES} \\ 0x^3 + 2x^2 + 8x - 5 = 2 & \text{YES} \end{array}$$

Solving Second Degree (Quadratic) Equations

We are going to be interested in solving the equation

$$ax^2 + bx + c = 0$$

Why $= 0$?

Suppose we have a problem involving any quadratic equation. For example, the forms

$$\begin{aligned} x^2 &= bx + c \\ x^2 + bc &= c \end{aligned}$$

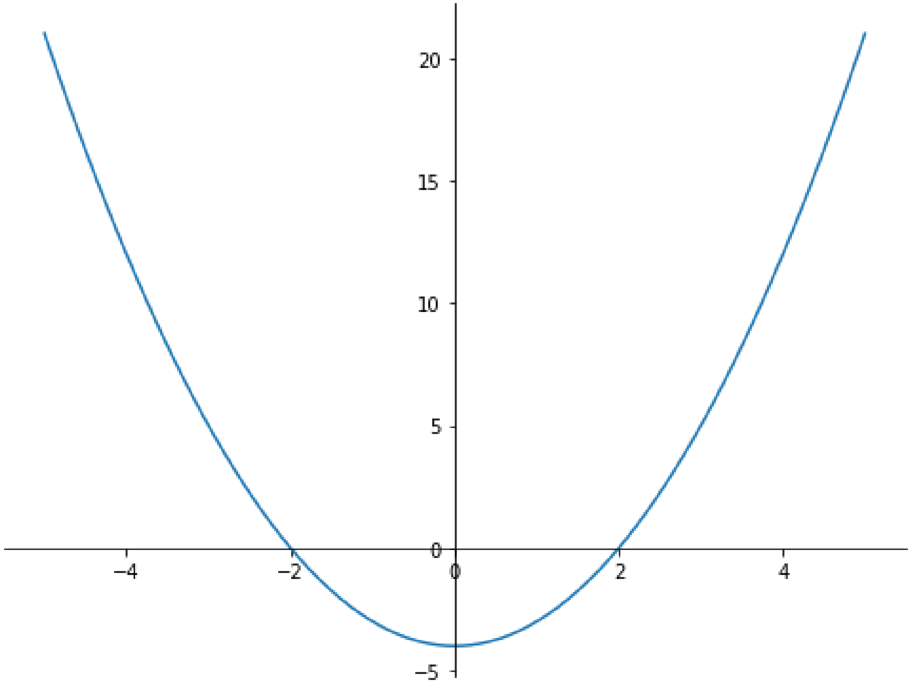
where we are trying to solve for x. We can always re-arrange terms to set it equal to 0

$$\begin{aligned} x^2 &= bx + c \;\rightarrow\; x^2 - bx - c = 0 \\ x^2 + bc &= c \;\rightarrow\; x^2 + bx - c = 0 \end{aligned}$$

Solving these equations gives the answers x that solve the original equations!

Graphically solve the equation

Let's say we are trying to solve $x^2 - 4 = 0$. Here is what the graph of $x^2 - 4$ looks like (we will learn how to graph later)



The shape of this graph is called a ***parabola***. **All second degree equations have the same shape.** What do we notice about the solution to $x^2 - 4 = 0$?

Introducing: the *Quadratic Formula* (the Quadratic Equation)

Given an equation of the form

$$ax^2 + bx + c = 0$$

we can calculate the values of x that solve the equation with

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Notice the $\pm$ here. This gives us two values, as we expect!

These values of x are called the ***roots of the parabola***.

Quadratic Formula Example

Let's try it out on our previous example $x^2 - 4 = 0$. Here we have $a = 1, b = 0, c = -4$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{0 \pm \sqrt{0 - 4 \cdot 1 \cdot -4}}{2 \cdot 1}$$
$$x = \frac{\pm\sqrt{16}}{2}$$
$$x = \frac{\pm 4}{2}$$
$$x = \pm 2$$

We get $x = 2, x = -2$, like we expected!

Let's practice!

In groups of three, solve each of the following equations for x using the quadratic formula

$$
\begin{aligned}
x^2 - 1 &= 0 \\
x^2 - 3x - 10 &= 0 \\
x^2 + 4x - 5 &= 0 \\
x^2 + 1 &= 1 \\
-x^2 + x + 4 &= -2
\end{aligned}
$$

Let's practice example 1

$$x^2 - 1 = 0$$

$a = 1, b = 0, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{0 \pm \sqrt{0 - 4 \cdot 1 \cdot -1}}{2 \cdot 1}$$
$$x = \frac{\pm\sqrt{4}}{2}$$
$$x = \frac{\pm 2}{2}$$
$$x = 1, -1$$

Let's practice example 2

$$x^2 - 3x - 10 = 0$$

$a = 1, b = -3, c = -10$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{3 \pm \sqrt{9 - 4 \cdot 1 \cdot -10}}{2 \cdot 1}$$
$$x = \frac{3 \pm \sqrt{9 + 40}}{2}$$
$$x = \frac{3 \pm 7}{2}$$
$$x = 5, -2$$

Let's practice example 3

$$x^2 + 4x - 5 = 0$$

$a = 1, b = 4, c = -5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-4 \pm \sqrt{16 - 4 \cdot 1 \cdot -5}}{2 \cdot 1}$$
$$x = \frac{-4 \pm \sqrt{16 + 20}}{2}$$
$$x = \frac{-4 \pm 6}{2}$$
$$x = 1, -5$$

Let's practice example 4

$$
\begin{aligned}
x^2 + 1 &= 0 \\
x^2 &= 0
\end{aligned}
$$

$a = 1, b = 0, c = 0$

$$
\begin{aligned}
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
x &= \frac{0 \pm \sqrt{0 - 4 \cdot 1 \cdot 0}}{2 \cdot 1} \\
x &= \frac{0 \pm \sqrt{0 + 0}}{2} \\
x &= \frac{0 \pm \sqrt{0}}{2} \\
x &= 0, 0
\end{aligned}
$$

Let's practice example 5

$$-x^2 + x + 4 = -2 \ \rightarrow \ -x^2 + x + 6 = 0$$

$a = -1, b = 1, c = 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-1 \pm \sqrt{1 - 4 \cdot -1 \cdot 6}}{2 \cdot -1}$$
$$x = \frac{-1 \pm \sqrt{1 + 24}}{-2}$$
$$x = \frac{-1 \pm 5}{-2}$$
$$x = -2, 3$$

The Discriminant

$$b^2 - 4ac$$

is a *very* important number in our calculation (why?). It is called ***the discriminant***.

- If $b^2 - 4ac > 0$ then we have two *real roots*
- If $b^2 - 4ac = 0$ then we have *repeated roots*
- If $b^2 - 4ac < 0$ then roots our roots are complex (not real)

True or False: Roots are always rational numbers

False. If the discriminant is not a perfect square (i.e. $4, 9, 16...$) then $\sqrt{b^2 - 4ac}$ will be *irrational*. This means are roots themselves are also irrational.

Let's practice!

In groups of three, determine if the roots of each parabola are real, complex, or repeated. If they are real, determine if they are rational or irrational.

$$x^2 - 5x + 4 = 0$$
$$x^2 - 3x + 1 = 0$$
$$x^2 + 4x + 4 = 0$$

Let’s practice!

In groups of three, determine if the roots of each parabola are real, complex, or repeated. If they are real, determine if they are rational or irrational.

$$x^2 - 5x + 4 = 0 \qquad b^2 - 4ac = 25 - 16 = 9 \rightarrow \text{ real, rational}$$
$$x^2 - 3x + 1 = 0$$
$$x^2 + 4x + 4 = 0$$

Let's practice!

In groups of three, determine if the roots of each parabola are real, complex, or repeated. If they are real, determine if they are rational or irrational.

$$x^2 - 5x + 4 = 0 \qquad b^2 - 4ac = 25 - 16 = 9 \rightarrow \text{ real, rational}$$
$$x^2 - 3x + 1 = 0 \qquad b^2 - 4ac = 9 - 4 = 5 \rightarrow \text{ real, irrational}$$
$$x^2 + 4x + 4 = 0$$

Let's practice!

In groups of three, determine if the roots of each parabola are real, complex, or repeated. If they are real, determine if they are rational or irrational.

$$x^2 - 5x + 4 = 0 \qquad b^2 - 4ac = 25 - 16 = 9 \rightarrow \text{ real, rational}$$
$$x^2 - 3x + 1 = 0 \qquad b^2 - 4ac = 9 - 4 = 5 \rightarrow \text{ real, irrational}$$
$$x^2 + 4x + 4 = 0 \qquad b^2 - 4ac = 16 - 16 = 0 \rightarrow \text{ repeated}$$

Tomorrow

Tomorrow we will see how we can use the quadratic formula to factor ***any*** quadratic expression.