

# Factoring Part 2: Il Prodotto Speciale

More advanced techniques!

## Before we start...

Pair up with somebody and try to simplify the expression (*hint: look for common factors!*)

$$\frac{36x^2 + 24xy + 18x^3}{6xyz + 48x^2}$$

We have a common factor of  $6x$  on both the top and bottom!

$$\frac{36x^2 + 24xy + 18x^3}{6xyz + 48x^2} = \frac{6x(6x + 4y + 3x^2)}{6x(yz + 8x)} = \boxed{\frac{6x + 4y + 3x^2}{yz + 8x}}$$

# What *is* factoring?

- Last week:

*breaking down a number a number into the product of smaller numbers.*

- But what is a *number*?
  1. The number 5 is a number
  2. The expression  $5x$  represents a number, for some  $x$
  3. But.... what about the expression  $(x^2 + 6x + 9)$ 
    - There's no common factor we can take out 😞

# The Special Product

Let us consider the following expression

$$(x + a)(x + a) = (x + a)^2$$

here is what happens when we multiply everything out

$$(x + a)(x + a) = x^2 + ax + ax + a^2 = x^2 + 2ax + a^2$$

Look what happens when we let  $a = 3$ !

$$(x + 3)(x + 3) = x^2 + 6x + 9$$



# What to look for

Let's study the equation

$$x^2 + 2ax + a^2$$

There are four things to look for

1. All terms are positive (they have a +)
2. The first term has  $x^2$
3. The second term is  $2x$  times some number  $a$
4. The third term is the ***square*** of that number  $a$

If we see this form, we know we can factor into  $(x + a)^2$  ! 😊

## An example

The formula is

$$x^2 + 2ax + a^2 = (x + a)^2$$

Let us try to factor

$$\begin{aligned} & x^2 + 6x + 9 \\ & \underbrace{x^2 + 2(3)x + 3(3)}_{a=3} \\ & (x + 3)(x + 3) \end{aligned}$$

**Let's practice!**

$$x^2 + 2ax + a^2 = (x + a)^2$$

With a partner, try to factor these expressions

$$x^2 + 4x + 4 = (x + ?)^2$$

$$x^2 + 12x + 36 = (x + ?)^2$$

$$x^2 + 24x + 144 = (x + ?)$$

$$2x^2 + 16x + 32 = ?$$



**Let's practice!**

$$x^2 + 2ax + a^2 = (x + a)^2$$

With a partner, try to factor these expressions

$$x^2 + 4x + 4 = (x + 2)^2$$

$$x^2 + 12x + 36 = (x + ?)^2$$

$$x^2 + 24x + 144 = (x + ?)$$

$$2x^2 + 16x + 32 = ?$$

**Let's practice!**

$$x^2 + 2ax + a^2 = (x + a)^2$$

With a partner, try to factor these expressions

$$x^2 + 4x + 4 = (x + 2)^2$$

$$x^2 + 12x + 36 = (x + 6)^2$$

$$x^2 + 24x + 144 = (x + ?)$$

$$2x^2 + 16x + 32 = ?$$

**Let's practice!**

$$x^2 + 2ax + a^2 = (x + a)^2$$

With a partner, try to factor these expressions

$$x^2 + 4x + 4 = (x + 2)^2$$

$$x^2 + 12x + 36 = (x + 6)^2$$

$$x^2 + 24x + 144 = (x + 12)^2$$

$$2x^2 + 16x + 32 = ?$$

**Let's practice!**

$$x^2 + 2ax + a^2 = (x + a)^2$$

With a partner, try to factor these expressions

$$x^2 + 4x + 4 = (x + 2)^2$$

$$x^2 + 12x + 36 = (x + 6)^2$$

$$x^2 + 24x + 144 = (x + 12)^2$$

$$2x^2 + 16x + 32 = 2(x^2 + 8x + 16) = 2(x + 4)^2$$

## A harder example

Recall the formula is

$$x^2 + 2ax + a^2 = (x + a)^2$$

Let us take a look at

$$4x^2 + 12x + 9$$

True or False: this is also the case where  $a = 3$

## A harder example

Recall the formula is

$$x^2 + 2ax + a^2 = (x + a)^2$$

Let us take a look at

$$4x^2 + 12x + 9$$

True or False: this is also the case where  $a = 3$

True! We can always look at the last term

## A harder example

Recall the formula is

$$x^2 + 2ax + a^2 = (x + a)^2$$

Let us take a look at

$$4x^2 + 12x + 9$$

We can write this as

$$(2x)^2 + 2(3)(2x) + 3^2$$

## A harder example

Recall the formula is

$$x^2 + 2ax + a^2 = (x + a)^2$$

Let us take a look at

$$4x^2 + 12x + 9$$

We can write this as

$$(2x)^2 + 2(3)(2x) + 3^2 = (2x + 3)^2$$



## Let's Practice

With a partner, factor the following expression (*Hint: what is  $a$ ?*)

$$9x^2 + 12x + 4$$

## Let's Practice

Recall the formula is

$$x^2 + 2ax + a^2 = (x + a)^2$$

Let us re-write our expression after seeing  $a = 2$

$$\begin{aligned} &9x^2 + 12x + 4 \\ &(3x)^2 + 2(2)(3x) + 2^2 \\ &(3x + 2)^2 \end{aligned}$$

## Some more special products

Work together with the *same* partner to try and factor the following two equations

$$x^2 + 18x + 81$$

$$x^2 - 18x + 81$$

Hint: how can we change our answer from above?

## Some more special products

Work together with the *same* partner to try and factor the following two equations

$$x^2 + 18x + 81 = (x + 9)^2$$

$$x^2 - 18x + 81$$

## Some more special products

Work together with the *same* partner to try and factor the following two equations

$$x^2 + 18x + 81 = (x + 9)^2$$

$$x^2 - 18x + 81 = (x - 9)^2$$

Let's review this second example

## A different special form

What if we expand

$$\begin{aligned}(x - a)(x - a) \\ x^2 - ax - ax + a^2 \\ x^2 - 2ax + a^2\end{aligned}$$

This is almost the same as before! We get

$$x^2 - 2ax + a^2 = (x - a)^2$$

This is why  $x^2 - 18x + 81 = (x - 9)^2$

## A very special form

Let's look at what happens if signs are opposite now!

$$\begin{array}{r} (x + a)(x - a) \\ x^2 - ax + ax - a^2 \\ \hline x^2 - a^2 \end{array}$$

We're left with only  $x^2 - a^2$  ☹️☹️☹️!

This means that we can factor

$$x^2 - a^2 = (x + a)(x - a)$$

$x^2 - a^2$  is known as the ***difference of two perfect squares***

## An example

We can factor

$$x^2 - 9 = (x + 3)(x - 3)$$



## Let's Practice

Recall the formula is

$$x^2 - a^2 = (x + a)(x - a)$$

With a partner, factor the following equations

$$x^2 - 4$$

$$x^2 - 81$$

$$2x^2 - 32$$

$$4x^2 - 9$$

$$x^4y^8 - 16$$

$$x^2y^4 - 4z^6$$

## Let's Practice

Recall the formula is

$$x^2 - a^2 = (x + a)(x - a)$$

With a partner, factor the following equations

$$x^2 - 4 = (x + 2)(x - 2)$$

$$x^2 - 81$$

$$2x^2 - 32$$

$$4x^2 - 9$$

$$x^4y^8 - 16$$

$$x^2y^4 - 4z^6$$

## Let's Practice

Recall the formula is

$$x^2 - a^2 = (x + a)(x - a)$$

With a partner, factor the following equations

$$x^2 - 4 = (x + 2)(x - 2)$$

$$x^2 - 81 = (x + 9)(x - 9)$$

$$2x^2 - 32$$

$$4x^2 - 9$$

$$x^4y^8 - 16$$

$$x^2y^4 - 4z^6$$

## Let's Practice

Recall the formula is

$$x^2 - a^2 = (x + a)(x - a)$$

With a partner, factor the following equations

$$x^2 - 4 = (x + 2)(x - 2)$$

$$x^2 - 81 = (x + 9)(x - 9)$$

$$2x^2 - 32 = 2(x^2 - 16) = 2(x + 4)(x - 4)$$

$$4x^2 - 9$$

$$x^4y^8 - 16$$

$$x^2y^4 - 4z^6$$

## Let's Practice

Recall the formula is

$$x^2 - a^2 = (x + a)(x - a)$$

With a partner, factor the following equations

$$x^2 - 4 = (x + 2)(x - 2)$$

$$x^2 - 81 = (x + 9)(x - 9)$$

$$2x^2 - 32 = 2(x^2 - 16) = 2(x + 4)(x - 4)$$

$$4x^2 - 9 = (2x)^2 - 3^2 = (2x + 3)(2x - 3)$$

$$x^4y^8 - 16$$

$$x^2y^4 - 4z^6$$

## Let's Practice

Recall the formula is

$$x^2 - a^2 = (x + a)(x - a)$$

With a partner, factor the following equations

$$x^2 - 4 = (x + 2)(x - 2)$$

$$x^2 - 81 = (x + 9)(x - 9)$$

$$2x^2 - 32 = 2(x^2 - 16) = 2(x + 4)(x - 4)$$

$$4x^2 - 9 = (2x)^2 - 3^2 = (2x + 3)(2x - 3)$$

$$x^4y^8 - 16 = ((x^2y^4)^2 - 4^2) = (x^2y^4 + 4)(x^2y^4 - 4)$$

$$x^2y^4 - 4z^6$$

## Let's Practice

Recall the formula is

$$x^2 - a^2 = (x + a)(x - a)$$

With a partner, factor the following equations

$$x^2 - 4 = (x + 2)(x - 2)$$

$$x^2 - 81 = (x + 9)(x - 9)$$

$$2x^2 - 32 = 2(x^2 - 16) = 2(x + 4)(x - 4)$$

$$4x^2 - 9 = (2x)^2 - 3^2 = (2x + 3)(2x - 3)$$

$$x^4y^8 - 16 = ((x^2y^4)^2 - 4^2) = (x^2y^4 + 4)(x^2y^4 - 4)$$

$$x^2y^4 - 4z^6 = ((xy^2)^2 - (2z^3)^2) = (xy^2 + 2z^3)(xy^2 - 2z^3)$$

## Wrapping Up

We've seen some really useful equations for factoring today!

$$x^2 + 2ax + a^2 = (x + a)^2$$

$$x^2 - 2ax + a^2 = (x - a)^2$$

$$x^2 - a^2 = (x + a)(x - a)$$

Don't forget what we saw last time: ***factoring out a common factor***.