Factoring Part 4 (Last Part!)

So far, we've seen a lot of problems with 2 or 3 terms. Let's try a problem with 4 terms. Let's try to factor

$$x^3 + 7x^2 + 2x + 14$$

Uh oh... there's no common factor!

What can we do? 

If we look closely, we can notice something!

$$\underbrace{x^3 + 7x^2}_{\text{have a common factor}} + \overbrace{2x + 14}^{\text{have a common factor}}$$

Factor by Grouping

Let us **group** terms together and factor them each independently!

$$\underbrace{\left(x^3 + 7x^2\right)}_{\text{group 1}} + \underbrace{(2x + 14)}_{\text{group 2}} = \underbrace{x^2(x + 7)}_{\text{group 1}} + \underbrace{2(x + 7)}_{\text{group 2}}$$

😮 😮 😮 😮 Something cool just happened! 😮 😮 😮 😮 😮

$$\underbrace{\underbrace{x^2(x + 7)}_{\text{group 1}} + \underbrace{2(x + 7)}_{\text{group 2}}}_{\text{these terms have a common factor!}}$$

Factor by Grouping

$$\underbrace{\underbrace{x^2(x+7)}_{\text{group 1}} + \underbrace{2(x+7)}_{\text{group 2}}}_{\text{these terms have a common factor!}}$$

We can factor out $(x+7)$

$$x^2(x+7) + 2(x+7) = \boxed{(x+7)(x^2+2)}$$

True or False: factoring by grouping always lets us factor 4 terms into the product of 2 numbers.

False!! In our last step, we were only able to factor because group 1 and group 2 *shared* a common factor of $(x+7)$! This is **not always the case.**

Factor by Grouping Steps

1. Group together terms that, when factored, will share a common factor
2. Factor each group *independently*
3. Factor out the common factor of each group

In *groups of 3*, try to factor the following expressions

$$x^4 + 5x + x^3 + 5$$

$$x^5 + 10x^2 + x^4 + 10x$$

$$x^3 + 5x^2 + 2x + 10$$

Hint: re-order terms!

$$x^5 + 2x^4 + 2x^3 + 4x^2 + 5x + 10$$

Factor by Grouping Example 1

We have a common factor of $x^3 + 5$ in both groups

$$x^4 + 5x + x^3 + 5$$
$$x(x^3 + 5) + (x^3 + 5)$$
$$(x^3 + 5)(x + 1)$$

Factor by Grouping Example 2

We have a common factor of $x^3 + 10$ in both groups

$$x^5 + 10x^2 + x^4 + 10x$$
$$x^2(x^3 + 10) + x(x^3 + 10)$$
$$(x^3 + 10)(x^2 + x)$$

Factor by Grouping Example 3

If we re-arrange terms, we see that we have a common factor of $x^2 + 2$

$$x^3 + 5x^2 + 2x + 10$$
$$x^3 + 2x + 5x^2 + 10$$
$$x(x^2 + 2) + 5(x^2 + 2)$$
$$(x^2 + 2)(x + 5)$$

Factor by Grouping Example 4

We have 3 groups with a common factor of $(x+2)$!

$$x^5 + 2x^4 + 2x^3 + 4x^2 + 5x + 10$$
$$x^5 + 2x^4 \quad + \quad 2x^3 + 4x^2 \quad + \quad 5x + 10$$
$$x^4(x+2) + 2x^2(x+2) + 5(x+2)$$
$$(x+2)(x^4 + 2x^2 + 5)$$

Fact check

True or False: when factoring a group, we can use any method of factorization we want

True! We treat each group as an independent expression. We can use any method we learned to factor them.

Fact check

True or False: Given an expression, the first thing we should try to do is factor by grouping.

False! ***Always always always*** see if you can factor out a common factor first. 99.9% of the time it will help!

Some Harder Examples

Remember all the ways we've learned how to factor in the first three lessons.

$$3x^2y + 6x^2 + 5y + 10$$

$$x^2 - 4 + x^2 + 7x + 10$$

$$4x^2 + 12x + 9 + 2x + 3$$

$$12x^2 + 15x + 36x + 45$$

Harder Problems Example 1

$$3x^2y + 6x^2 + 5y + 10$$

$$3x^2(y + 2) + 5(y + 2)$$

$$(y + 2)(3x^2 + 5)$$

Harder Problems Example 2

$$x^2 - 4 + x^2 + 7x + 10$$

$$(x + 2)(x - 2) + (x + 2)(x + 5)$$

$$(x + 2)\,[(x - 2) + (x + 5)]$$

$$(x + 2)(2x + 3)$$

Harder Problems Example 3

$$4x^2 + 12x + 9 + 2x + 3$$

$$(2x + 3)^2 + (2x + 3)$$

$$(2x + 3)\,[(2x + 3) + 1]$$

$$(2x + 3)(2x + 4)$$

Harder Problems Example 4

$$12x^2 + 15x + 36x + 45$$

$$3(4x^2 + 5x + 12x + 15)$$

$$3\,[x(4x + 5) + 3(4x + 5)]$$

$$3\,[(4x + 5)(x + 3)]$$

$$3(4x + 5)(x + 3)$$

Review: What we've learned

- Factoring involves taking a number (or expression) and writing it as the product of two or more numbers (or expressions!)
- A factorization of a number (or expression) is not neccessarily unique
- Factoring out a common factor is a great first step For example, $5x^2 + x = x(5 + 1)$

Review: What we've learned

- Keep an eye out for special products
  - $x^2 + 2ax + a^2 = (x + a)^2$
  - $x^2 - 2ax + a^2 = (x - a)^2$
  - $x^2 - a * 2 = (x + a)(x - a)$
- If we don't have a special product, try to factor into the form $(x \pm a)(x \pm b)$
  - Given an expression $x^2 + cx + d$, we need to find a and b such that $a \cdot b = d$ and $a + b = c$
- If we have four or more terms, try factoring by grouping
  - Look for a way to group terms such that after all groups are factored, each term shares a common factor

A finito!

That's everything! Thank you everyone for an amazing time these past three weeks 😄