Factoring numbers

#### What does it mean to factor?

We want to **break down** a number into the **product** of **smaller numbers**.

$$10 = 5 \cdot 2$$

True or false: Every factorization of a number is unique

False!

$$100 = 10 \cdot 10 = 10 \cdot 5 \cdot 2 = 5 \cdot 2 \cdot 5 \cdot 2$$
  
 $100 = 50 \cdot 2 = \dots$ 

#### Let's practice

With a partner, factor these numbers into the product of two numbers

$$10 = ? \cdot ?$$

$$25 = ? \cdot ?$$

$$30 = ? \cdot ?$$

$$42 = ? \cdot ?$$

#### Let's practice

With a partner, factor these numbers into the product of two numbers (un numero  $\rightarrow$  il prodotto di due numeri)

$$10 = 5 \cdot 2$$

$$25 = 5 \cdot 5$$

$$30 = 10 \cdot 3$$

$$42 = 7 \cdot 6$$

### Why factor?

It makes life easier!

$$\frac{1638}{63} = ?$$
$$(48)^2 = ?$$

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It makes life easier!

$$\frac{1638}{63} = \underbrace{\frac{26 \cdot 9 \cdot 7}{9 \cdot 7}}_{\text{factors cancel out!}} = 26$$

$$(48)^2 = (6 \cdot 8)^2 = 6^2 \cdot 8^2 = 36 \cdot 64 = 2304$$

#### A new, powerful idea

Is there a way to re-write this expression by factoring?

$$10 + 15 + 20$$

let's notice something...

$$5 \cdot 2 + 5 \cdot 3 + 5 \cdot 4$$

what now?

$$5 \cdot 2 + 5 \cdot 3 + 5 \cdot 4 = \boxed{5(2+3+4)}$$

wow! That looks a lot nicer!

#### What happened here?

- Notice that the 5 was a **common factor** to each number in our sum.
- This means we can *factor it out* and multiply the remaining sum by it!

$$\underbrace{5 \cdot 2 + 5 \cdot 3 + 5 \cdot 4}_{5 \text{ is a common factor}} = \underbrace{5(2 + 3 + 4)}_{\text{factor it out}}$$

True or false: You can factor a variable out of an expression	



A variable really just represents a number. If a variable is common to every term, we can factor it out!

$$5x + 12x + 4x = x(5 + 12 + 4)$$

#### This works for any number!

$$2x + 3x + 4x = x(2 + 3 + 4)$$

Notice that x = 5 becomes our example from before

#### Let's play a game: find the greatest common factor!

Team up with two other students and try to find the greatest common factor of each expression

12 + 28 : ?

7x + 14x : ?

 $5x^2 + 10x : ?$ 

 $2 + 4x^2 + 32x$ : ?

36x + 12xy + 3x: ?

#### Let's play a game: find the greatest common factor!

Team up with two other students and try to find the greatest common factor of each expression

12 + 28 : 4

7x + 14x : 7x

 $5x^2 + 10x : 5x$ 

 $2 + 4x^2 + 32x : 2$ 

36x + 12xy + 3x : 3x

$$12 + 28 : 4$$

$$7x + 14x : 7x$$

$$5x^{2} + 10x : 5x$$

$$2 + 4x^{2} + 32x : 2$$

$$36x + 12xy + 3x : 3x$$

$$12 + 28 = ?(? + ?)$$

$$7x + 14x = ?(? + ?)$$

$$5x^{2} + 10x = ?(? + ?)$$

$$2 + 4x^{2} + 32x = ?(? + ? + ?)$$

$$12 + 28 : 4$$

$$7x + 14x : 7x$$

$$5x^{2} + 10x : 5x$$

$$2 + 4x^{2} + 32x : 2$$

$$36x + 12xy + 3x : 3x$$

$$12 + 28 = 4(3 + 7)$$

$$7x + 14x = ?(? + ?)$$

$$5x^{2} + 10x = ?(? + ?)$$

$$2 + 4x^{2} + 32x = ?(? + ? + ?)$$

$$12 + 28 : 4$$

$$7x + 14x : 7x$$

$$5x^{2} + 10x : 5x$$

$$2 + 4x^{2} + 32x : 2$$

$$36x + 12xy + 3x : 3x$$

$$12 + 28 = 4(3 + 7)$$

$$7x + 14x = 7x(1 + 2)$$

$$5x^{2} + 10x = ?(? + ?)$$

$$2 + 4x^{2} + 32x = ?(? + ? + ?)$$

$$12 + 28 : 4$$

$$7x + 14x : 7x$$

$$5x^{2} + 10x : 5x$$

$$2 + 4x^{2} + 32x : 2$$

$$36x + 12xy + 3x : 3x$$

$$12 + 28 = 4(3 + 7)$$

$$7x + 14x = 7x(1 + 2)$$

$$5x^{2} + 10x = 5x(x + 2)$$

$$2 + 4x^{2} + 32x = ?(? + ? + ?)$$

$$12 + 28 : 4$$

$$7x + 14x : 7x$$

$$5x^{2} + 10x : 5x$$

$$2 + 4x^{2} + 32x : 2$$

$$36x + 12xy + 3x : 3x$$

$$12 + 28 = 4(3 + 7)$$

$$7x + 14x = 7x(1 + 2)$$

$$5x^{2} + 10x = 5x(x + 2)$$

$$2 + 4x^{2} + 32x = 2(1 + 2x^{2} + 16x)$$

$$36x + 12xy + 3x = ?(? + ? + ?)$$

$$12 + 28 : 4$$

$$7x + 14x : 7x$$

$$5x^{2} + 10x : 5x$$

$$2 + 4x^{2} + 32x : 2$$

$$36x + 12xy + 3x : 3x$$

$$12 + 28 = 4(3 + 7)$$

$$7x + 14x = 7x(1 + 2)$$

$$5x^{2} + 10x = 5x(x + 2)$$

$$2 + 4x^{2} + 32x = 2(1 + 2x^{2} + 16x)$$

$$36x + 12xy + 3x = 3x(12 + 4y + 1)$$

True or false: Every expression has a greatest common factor > 1

#### False!

Some expressions just don't have anything in common ②. Consder the following expressions

$$2 + 5 + 11$$

$$3x + yz - 6$$

$$14x^2 + 9x - 2$$

## Factor the following expressions (if possible) $4x^3 + 6x^2y + 12x$

$$4x^3 + 6x^2y + 12x$$

$$4x^3 + 6x^2y + 12x = 2x(2x^2 + 3xy + 6)$$

## Factor the following expressions (if possible) 2x + 5y + 7z

No common factor :(

### Factor the following expressions (if possible) $25x^3y^5 - 15x^2y^3$

$$25x^3y^5 - 15x^2y^3$$

$$25x^3y^5 - 15x^2y^3 = 5x^2y^3(5xy^2 - 3)$$

# Factor the following expressions (if possible) $12x^3y^5z^6 + 2y^5z^3 + 5x^2z^3 + 24xy$

No common factor :(

### Factor the following expressions (if possible) $3x^2(y+z) + 6x(12-z)$

$$3x^2(y+z) + 6x(12-z)$$

$$3x^{2}(y+z) + 6x(12-z) = 3x(x(y+z) + 2(12-z))$$

## Factor the following expressions (if possible) $25x(6-x)^2 + 5z(6-x)$

$$25x(6-x)^2 + 5z(6-x)$$

$$25x(6-x) + 5z(6-x) = 5(6-x)(5x+z)$$

## Factor the following expressions (if possible) $25x(6-x)^2 + 2z(12-x)$

No common factor:(

