Factoring Part 4 (Last Part!)

So far, we've seen a lot of problems with 2 or 3 terms. Let's try a problem with 4 terms. Let's try to factor

$$x^3 + 7x^2 + 2x + 14$$

Uh oh... there's no common factor!



What can we do? @ @ @

If we look closely, we can notice something!

have a common factor
$$\underbrace{x^3 + 7x^2}_{\text{have a common factor}} + \underbrace{2x + 14}_{\text{have a common factor}}$$

# Factor by Grouping

Let us **group** terms together and factor them each independently!

$$\underbrace{(x^3 + 7x^2)}_{\text{group 1}} + \underbrace{(2x + 14)}_{\text{group 2}} = \underbrace{x^2(x + 7)}_{\text{group 1}} + \underbrace{2(x + 7)}_{\text{group 2}}$$

$$\underbrace{x^2(x+7)}_{\text{group 1}} + \underbrace{2(x+7)}_{\text{group 2}}$$

these terms have a common factor!

# Factor by Grouping

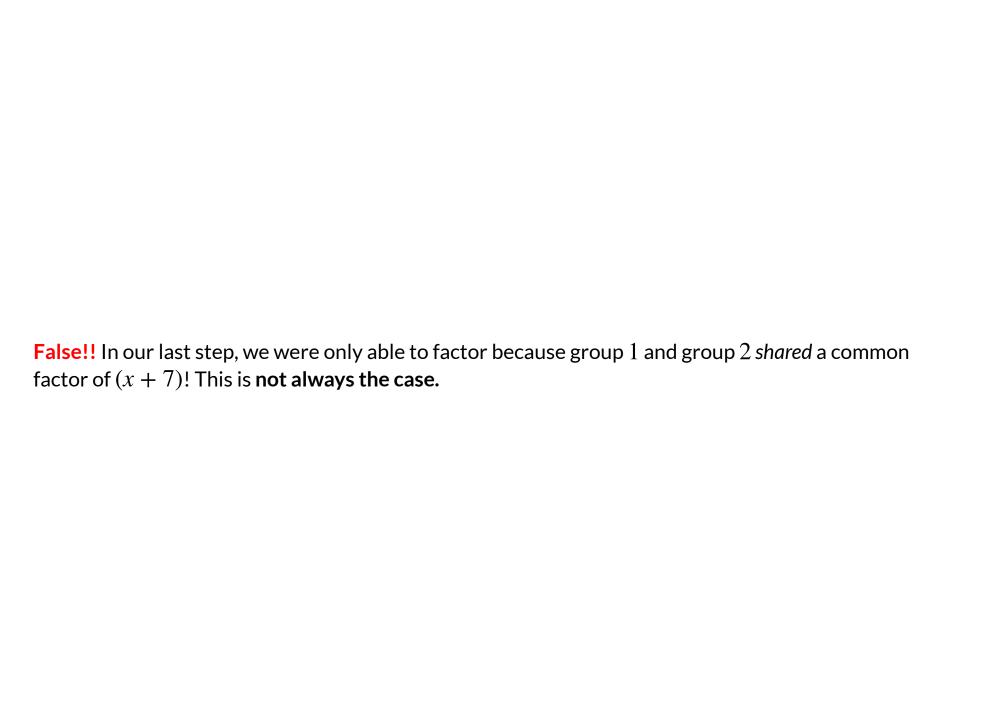
$$\underbrace{x^2(x+7)}_{\text{group 1}} + \underbrace{2(x+7)}_{\text{group 2}}$$

these terms have a common factor!

We can factor out (x + 7)

$$x^{2}(x+7) + 2(x+7) = (x+7)(x^{2}+2)$$

True or False: factoring by grouping always lets us factor 4 terms into the product of 2 numbers.



### **Factor by Grouping Steps**

- 1. Group together terms that, when factored, will share a common factor
- 2. Factor each group independently
- 3. Factor out the common factor of each group

In groups of 3, try to factor the following expressions

$$x^{4} + 5x + x^{3} + 5$$
  
 $x^{5} + 10x^{2} + x^{4} + 10x$   
 $x^{3} + 5x^{2} + 2x + 10$  *Hint: re-order terms!*  
 $x^{5} + 2x^{4} + 2x^{3} + 4x^{2} + 5x + 10$ 

We have a common factor of  $x^3 + 5$  in both groups

$$x^{4} + 5x + x^{3} + 5$$
$$x(x^{3} + 5) + (x^{3} + 5)$$
$$(x^{3} + 5)(x + 1)$$

We have a common factor of  $x^3 + 10$  in both groups

$$x^{5} + 10x^{2} + x^{4} + 10x$$

$$x^{2}(x^{3} + 10) + x(x^{3} + 10)$$

$$(x^{3} + 10)(x^{2} + x)$$

If we re-arrange terms, we see that we have a common factor of  $x^2 + 2$ 

$$x^{3} + 5x^{2} + 2x + 10$$

$$x^{3} + 2x + 5x^{2} + 10$$

$$x(x^{2} + 2) + 5(x^{2} + 2)$$

$$(x^{2} + 2)(x + 5)$$

We have 3 groups with a common factor of (x + 2)!

$$x^{5} + 2x^{4} + 2x^{3} + 4x^{2} + 5x + 10$$

$$x^{5} + 2x^{4} + 2x^{3} + 4x^{2} + 5x + 10$$

$$x^{4}(x+2) + 2x^{2}(x+2) + 5(x+2)$$

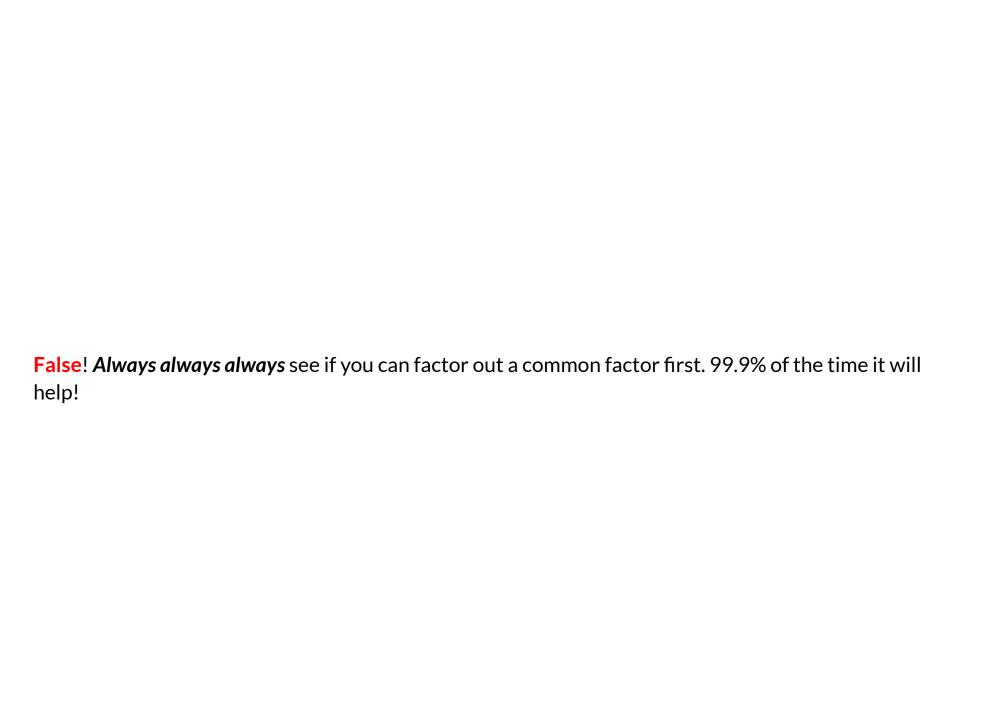
$$(x+2)(x^{4} + 2x^{2} + 5)$$

#### Fact check

True or False: when factoring a group, we can use any method of factorization we want

<b>True!</b> We treat each group as an independent expression. We can use any method we learned to factor them.

# Fact check True or False: Given an expression, the first thing we should try to do is factor by grouping.



### Some Harder Examples

Remember all the ways we've learned how to factor in the first three lessons.

$$3x^{2}y + 6x^{2} + 5y + 10$$

$$x^{2} - 4 + x^{2} + 7x + 10$$

$$4x^{2} + 12x + 9 + 2x + 3$$

$$12x^{2} + 15x + 36x + 45$$

$$3x^{2}y + 6x^{2} + 5y + 10$$
$$3x^{2}(y + 2) + 5(y + 2)$$
$$(y + 2)(3x^{2} + 5)$$

$$x^{2} - 4 + x^{2} + 7x + 10$$

$$(x + 2)(x - 2) + (x + 2)(x + 5)$$

$$(x + 2) [(x - 2) + (x + 5)]$$

$$(x + 2)(2x + 3)$$

$$4x^{2} + 12x + 9 + 2x + 3$$

$$(2x + 3)^{2} + (2x + 3)$$

$$(2x + 3) [(2x + 3) + 1]$$

$$(2x + 3)(2x + 4)$$

$$12x^{2} + 15x + 36x + 45$$

$$3(4x^{2} + 5x + 12x + 15)$$

$$3[x(4x + 5) + 3(4x + 5)]$$

$$3[(4x + 5)(x + 3)]$$

$$3(4x + 5)(x + 3)$$

#### Review: What we've learned

- Factoring involves taking a number (or expression) and writing it as the product of two or more numbers (or expressions!)
- A factorization of a number (or expression) is not neccessarily unique
- Factoring out a common factor is a great first step For example,  $5x^2 + x = x(5+1)$

#### Review: What we've learned

- Keep an eye out for special products
  - $x^2 + 2ax + a^2 = (x + a)^2$
  - $x^2 2ax + a^2 = (x a)^2$
  - $x^2 a * 2 = (x + a)(x a)$
- If we don't have a special product, try to factor into the form  $(x \pm a)(x \pm b)$ 
  - Given an expression  $x^2 + cx + d$ , we need to find a and b such that  $a \cdot b = d$  and a + b = c
- If we have four or more terms, try factoring by grouping
  - Look for a way to group terms such that after all groups are factored, each term shares a common factor

# A finito!

That's everything! Thank you everyone for an amazing time these past three weeks 🕲