Second Degree Equations Part 4

Cool Applications of Second Degree Equations!

But First: Example Word Problems

We want to build a fence enclosing a $12\mathrm{m}^2$ rectangle. If the width of the rectangle is 4 more than its length, how much fence do we need?

Let

$$x = \text{length of rectangle}$$

We know that the area is length·width. So

$$\begin{aligned} x(x+4) &= 12 \\ x^2 + 4x &= 12 \\ x^2 + 4x - 12 &= 0 \end{aligned}$$

Now we can solve two ways. We can use the quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{64}}{2}$$

$$x = 2, -6$$

Or we can factor directly

$$x^2 + 4x - 12 = 0$$

$$(x + 6)(x - 2) = 0 \rightarrow x = -6, 2$$

We take $x = 2$ (since lengths must be positive) and find the permiter is

$$\boxed{P = 2 + 2 + 6 + 6 = 16}$$

A second word problem

In seven years, Mario will be five more than the square of his current age. How old is Mario?

Let

$$x = \text{the current age of Mario}$$

We are solving the following equation for x

$$x^2 + 5 = x + 7$$
$$x^2 - x - 2 = 0$$
$$(x - 2)(x + 1) = 0 \rightarrow x = 2, -1$$

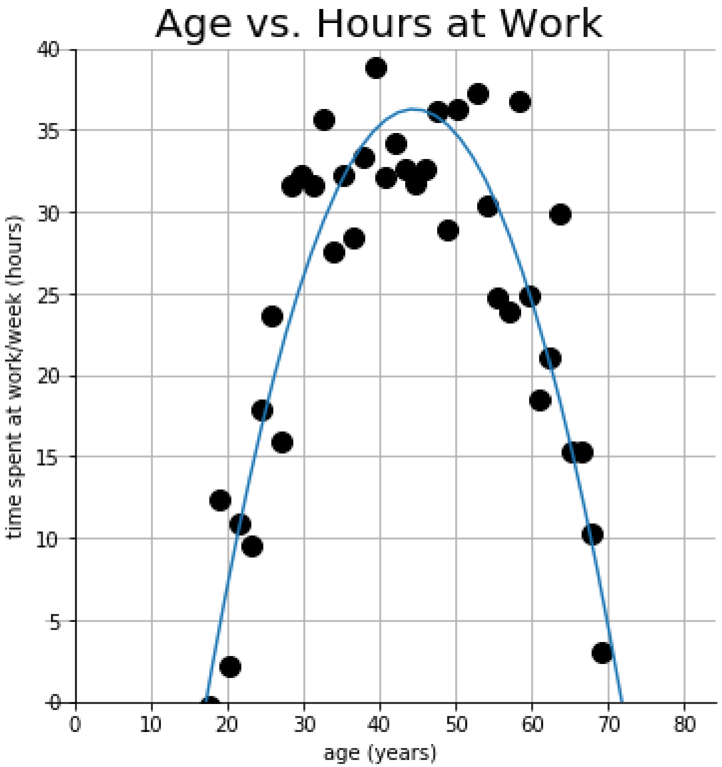
Mario is 2 years old!

Some Cool Applications of Second Degree Equations

Linear Regression

Let's say we recorded the following data.

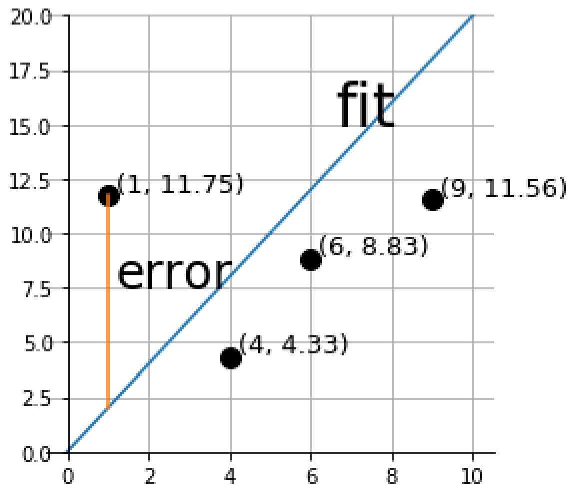
It looks like we can possibly model this with a parabola!



Let's learn how the computer figures out this fit!

Error

We need to figure out a way to measure how "bad" a fit is. Let's say we have a guess to our answer and want to calculate how bad one point is.



If our point is located at $(x_i, y_i))$ then we can calculate the error as

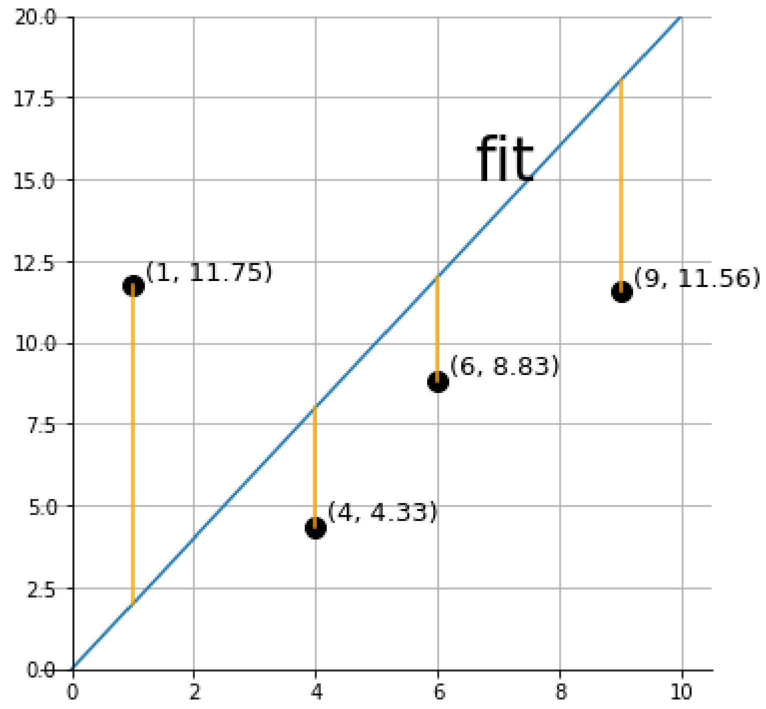
$$\text{error} = (y_i - fit(x_i))^2$$

Why is this squared?

Error

We can calculate how bad a fit is by summing up the error of each point!

$$\text{badness} = \sum_{\text{all points}} (y_i - fit(x_i))^2$$



Our goal is to ***find a fit that minimizes badness***.

Machine Learning

Given our original data set, we want our computer to figure out the values for a b and c for a parabola that minimizes badness.

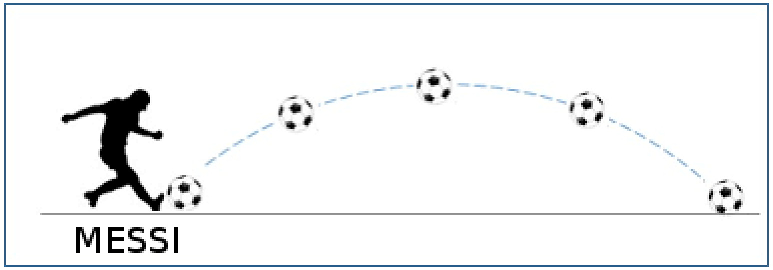
$$fit = ax^2 + bx + c$$

The process of figuring out these values is callled *linear regression*, which is a topic of *machine learning*.

Learning the exact way machines do this is a complicated topic, but just know there's many many different methods to do this.

Projectile Motion

When things are thrown into the air, they follow a parabolic arc!



$$y = -\frac{1}{2}gt^2 + v_0 t + y_0$$

where

$$\begin{aligned} g &= \text{acceleration due to gravity (9.8)} \\ v_0 &= \text{initial velocity in y-direction} \\ y_0 &= \text{initial y position} \end{aligned}$$

Example Projectile Motion

A ball is thrown upwards at 3 meters/second at a height of 2 meters above the ground. How long does it take to hit the ground?

We have $v_0 = 3, y_0 = 2$, and we are solving for when $y = 0$

$$y = -\frac{1}{2}gt^2 + v_0t + y_0$$
$$0 = -\frac{1}{2}(9.8)t^2 + 3t + 2$$

Now we can use the quadratic equation to find t!

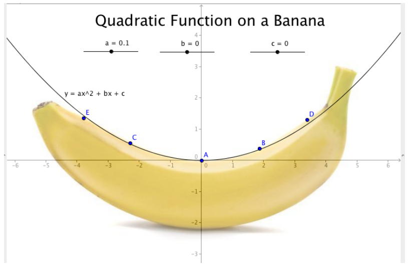
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$t = \frac{-3 \pm \sqrt{3^2 - 4(-4.9)(2)}}{2(-4.9)}$$
$$t = 1.01, -0.4$$

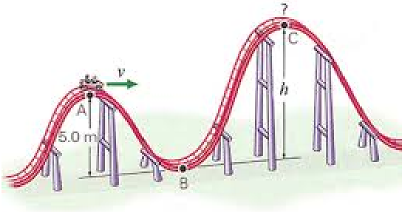
Our answer is

$$\boxed{t = 1.01 \text{ seconds}}$$

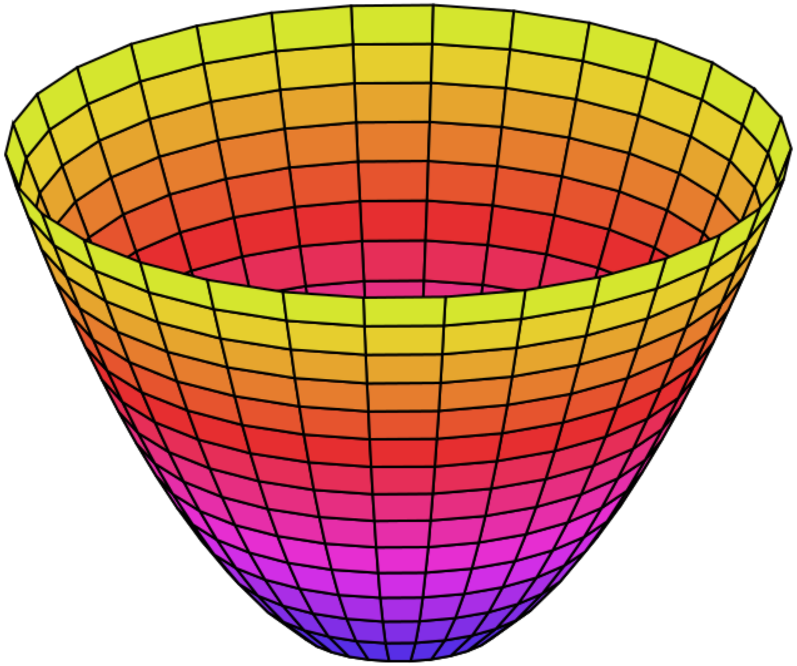
Parabolas in Real Life

Parabolas are everywhere!





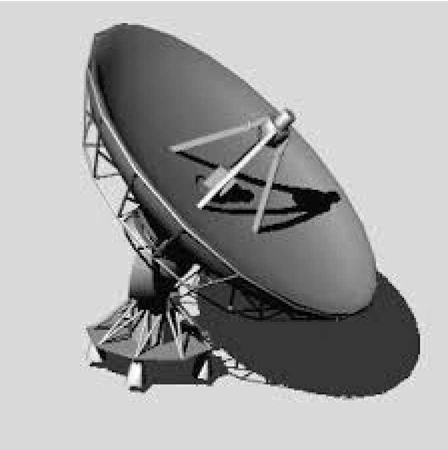
Paraboloid



$$z = x^2 + y^2$$

Imagine we rotated a parabola 360° around the y axis.

Paraboloids in Real Life



Wrapping Up

- We learned a lot about how to solve second degree equations in the past 3 lessons
- Mathematics is more than just learning how to solve an equation, though
- Every day, people are using the math that you learn to solve real world problems and create real things
- Parabolas occur in the world far more often then you'd think! Keep an eye out for them!

It's been very fun teaching you all. Thank you for a great three weeks!