Factoring Part 3

Let's start by trying to simplify the following expression

$$\frac{50x + 20x^2 + 2x^3}{x^3 - 25x}$$

Factoring Part 3

We always begin by factoring out common factors

$$\frac{50x + 20x^2 + 2x^3}{x^3 - 25x} = \frac{2x\left(25 + 10x + x^2\right)}{x\left(x^2 - 25\right)}$$

$$= \frac{2x(x+5)^2}{x(x+5)(x-5)}$$

$$\rightarrow \frac{2(x+5)}{(x-5)}$$

Question: True or False:

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$$\frac{50x + 20x^2 + 2x^3}{x^3 - 25x} = \frac{2(x+5)}{(x-5)}$$

False! For example, if $x = 0$ we get

$$\frac{0}{0} \neq \frac{10}{-5}$$

What if it doesn't have a form we recognize?

Consider the following expression

$$x^2 + 5x + 6$$

And remember the special products we saw last time

$$\begin{aligned} x^2 + 2ax + a^2 &= (x+a)^2 \\ x^2 - 2ax + a^2 &= (x-a)^2 \\ x^2 - a^2 &= (x+a)(x-a) \end{aligned}$$

What's wrong?

Our expression does not have a value of a that lets us use a special product rule!

A New Idea

All the special products factored into the form

$$(x \pm a)(x \pm a)$$

But what if we can factor an expression into

$$(x \pm a)(x \pm b)$$

where a and b are different values??

A New Idea

Let's look at what happens when we have $(x + a)(x + b)$

$$\begin{aligned}(x + a)(x + b) &= x^2 + ax + bc + ab \\ &= x^2 + (a + b)x + ab\end{aligned}$$

😮😮😮

The *constant* term = ab and a and b add to be the coeffecient of the x term!

Factoring $x^2 + 5x + 6$

We need to find a, b that **multiply to 6 and add to 5**.

How can we do this?

Let's list all the factors of 6, and what those factors add to!

$$\begin{aligned} 6 &= 1 \cdot 6 & \quad 1 + 6 &= 6 \\ 6 &= 2 \cdot 3 & \quad 2 + 3 &= 5 \end{aligned}$$

Can you see the answer?

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

Let's practice

With a partner, try to factor the following expressions

$$x^2 + 6x + 8$$
$$x^2 + 5x - 6$$

Let's practice

With a partner, try to factor the following expressions

$$x^2 + 6x + 8 = (x+2)(x+4)$$

$$x^2 + 5x - 6 = (x-1)(x+6) \qquad \text{something new!}$$

Signs are important

The general form of the equation we are factoring is

$$ax^2 + bx + c$$

(For now, we will always let $a = 1$).

If c is positive, then we must consider all pairs with two positives and all pairs with two negatives. For example

$$c = 10 = (1)(10) = (-1)(-10) = (2)(5) = (-2)(-5)$$

If c is negative, we must consider all pairs with one negative and one positive

$$c = -10 = (-1)(10) = (1)(-10) = (-2)(5) = (2)(-5)$$

An Example

$$ax^2 + bx + c$$

For example, if we have

$$x^2 + 7x - 30$$

then we list all the facotrs of -30 as

$$\begin{aligned}
-30 &= 1 \cdot -30 & 1 - 30 &= -29 \\
-30 &= -1 \cdot 30 & -1 + 30 &= 29 \\
-30 &= 2 \cdot -15 & 2 - 15 &= -13 \\
-30 &= -2 \cdot 15 & -2 + 15 &= 13 \\
-30 &= 3 \cdot -10 & 3 - 10 &= -7
\end{aligned}$$

$$\boxed{-30 = -3 \cdot 10 \qquad -3 + 10 = 7}$$

Thus,

$$\boxed{x^2 + 7x - 30 = (x - 3)(x + 7)}$$

Some more practice!

In ***groups of three*** try to factor these expressions

$$
\begin{aligned}
x^2 - 3x - 4 \\
x^2 - 12x + 35 \\
x^2 + 10x + 16 \\
x^2 + 4x - 32
\end{aligned}
$$

Some more practice!

In ***groups of three*** try to factor these expressions

$$
\begin{aligned}
x^2 - 3x - 4 &= (x-4)(x+1) \\
x^2 - 12x + 35 &= (x-5)(x-7) \\
x^2 + 10x + 16 &= (x+8)(x+2) \\
x^2 + 4x - 32 &= (x+8)(x-4)
\end{aligned}
$$

Some trivia

True or False: we can always factor an expression $x^2 + bx + c$ into the product of $(x \pm i)(x \pm j)$, for integers b, c, i, j.

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True or False: we can always factor an expression $x^2 + bx + c$ into the product of $(x \pm i)(x \pm j)$, for integers b, c, i, j.

False. What if the factors of ab do not add up to c? For example,

$$x^2 + 10x + 7$$

does not factor nicely.

Some trivia

True or False: The special products we saw yesterday, $(x \pm a)(x \pm a)$ are just a specific case of what we learned today.

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True or False: The special products we saw yesterday, $(x \pm a)(x \pm a)$ are just a specific case of what we learned today.

True. If you get really good at factoring, there's no need to memorize the special products. But, for large numbers, recognizing special products is ***much*** faster than today's method.

A hard example

Let's work through simplifying the following expression using all the techniques we've learned so far.

$$\frac{3x^2y - 12y}{3x^3y + 24x^2y - 60xy}$$

Let's first try to factor the top. What can we factor out?

$3y$! So now we get

$$\frac{3y(x^2 - 4)}{3x^3y + 24x^2y - 60xy}$$

can we do anything else?

A hard example

Special form (difference of two perfect squares)

$$\frac{3y(x-2)(x+2)}{3x^3y+24x^2y-60xy}$$

Let's look at the bottom now. What can we factor out?

$3xy$! So now we get

$$\frac{3y(x-2)(x+2)}{3xy(x^2+8x-20)}$$

What now?

A hard example

We can factor the bottom more.

$$\frac{3y(x-2)(x+2)}{3xy(x-2)(x+10)}$$

Are we done?

Cancel things out! ☺

$$\frac{(x+2)}{x(x+10)}$$

Now we are done.

Wrapping Up

If we have an expression that looks like

$$x^2 + bx + c$$

We can factor this into

$$(x \pm i)(x \pm j)$$

where

$$c = (i)(j) \qquad b = i + j$$

We should also remember the *special products* we saw last time and factoring by looking for the *common factor* we saw two lessons ago.