# Factoring Part 2: Il Prodotto Speciale

More advanced techniques!

#### Before we start...

Pair up with somebody and try to simplify the expression (hint: look for common factors!)

$$\frac{36x^2 + 24xy + 18x^3}{6xyz + 48x^2}$$

We have a common factor of 6x on both the top and bottom!

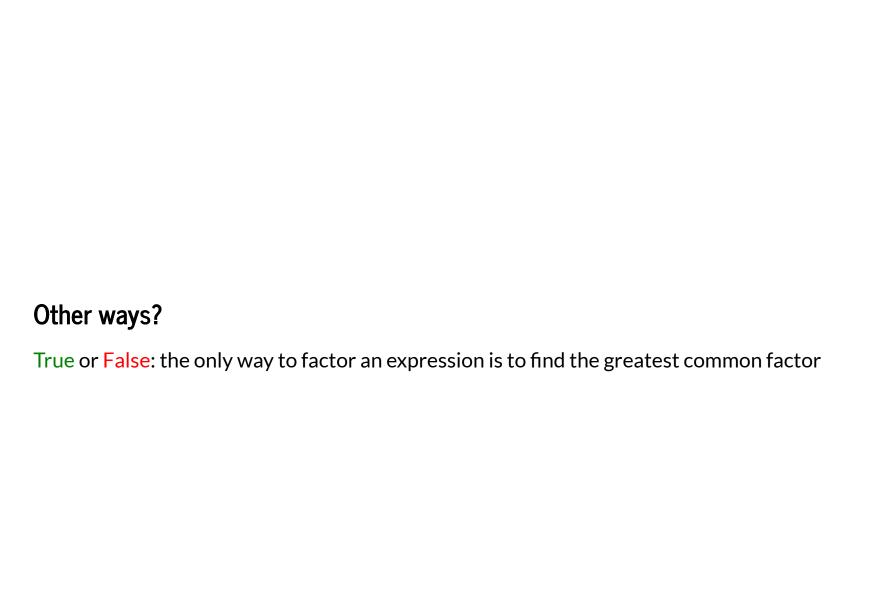
$$\frac{36x^2 + 24xy + 18x^3}{6xyz + 48x^2} = \frac{6x(6x + 4y + 3x^2)}{6x(yz + 8x)} = \boxed{\frac{6x + 4y + 3x^2}{yz + 8x}}$$

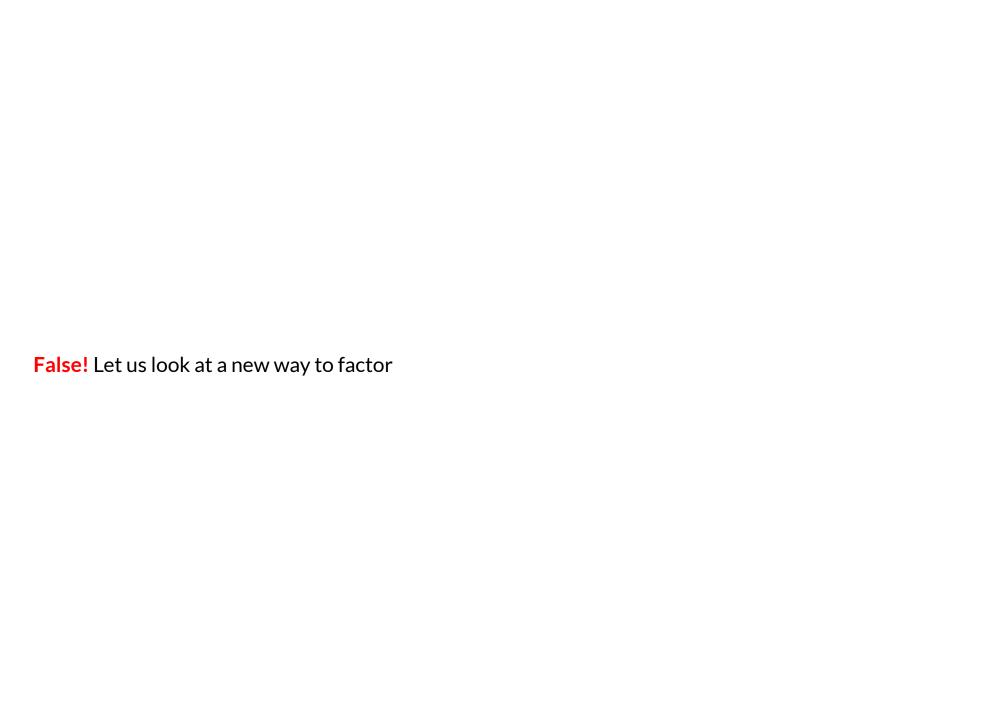
# What *is* factoring?

• Last week:

**breaking down** a number a number into the **product** of **smaller numbers**.

- But what is a number?
- 1. The number 5 is a number
- 2. The expression 5x represents a number, for some x
- 3. But... what about the expression  $(x^2 + 6x + 9)$ 
  - There's no common factor we can take out ②





# The Special Product

Let us consider the following expression

$$(x+a)(x+a) = (x+a)^2$$

here is what happens when we multiply everything out

$$(x + a)(x + a) = x^2 + ax + ax + a^2 = x^2 + 2ax + a^2$$

Look what happens when we let a = 3!

$$(x+3)(x+3) = x^2 + 6x + 9$$







#### What to look for

Let's study the equation

$$x^2 + 2ax + a^2$$

There are four things to look for

- 1. All terms are positive (they have a +)
- 2. The first term has  $x^2$
- 3. The second term is 2x times some number a
- 4. The third term is the **square** of that number a

If we see this form, we know we can factor into  $(x + a)^2$ !

# An example

The formula is

Let us try to factor

$$x^{2} + 2ax + a^{2} = (x + a)^{2}$$

$$x^{2} + 6x + 9$$

$$\underbrace{x^{2} + 2(3)x + 3(3)}_{a=3}$$

$$(x + 3)(x + 3)$$

# Let's practice!

$$x^2 + 2ax + a^2 = (x + a)^2$$

With a partner, try to factor these expressions

$$x^{2} + 4x + 4 = (x + ?)^{2}$$
$$x^{2} + 12x + 36 = (x + ?)^{2}$$

# Let's practice!

$$x^2 + 2ax + a^2 = (x + a)^2$$

With a partner, try to factor these expressions

$$x^{2} + 4x + 4 = (x + 2)^{2}$$
$$x^{2} + 12x + 36 = (x + ?)^{2}$$

# Let's practice!

$$x^2 + 2ax + a^2 = (x + a)^2$$

With a partner, try to factor these expressions

$$x^{2} + 4x + 4 = (x + 2)^{2}$$
$$x^{2} + 12x + 36 = (x + 6)^{2}$$

Recall the formula is

$$x^2 + 2ax + a^2 = (x + a)^2$$

Let us take a look at

$$4x^2 + 12x + 9$$

True or False: this is also the case where a=3

Recall the formula is

$$x^2 + 2ax + a^2 = (x + a)^2$$

Let us take a look at

$$4x^2 + 12x + 9$$

True or False: this is also the case where a=3

True! We can always look at the last term

Recall the formula is

$$x^2 + 2ax + a^2 = (x + a)^2$$

Let us take a look at

$$4x^2 + 12x + 9$$

We can write this as

$$(2x)^2 + 2(3)(2x) + 3^2$$

Recall the formula is

$$x^2 + 2ax + a^2 = (x + a)^2$$

Let us take a look at

$$4x^2 + 12x + 9$$

We can write this as

$$(2x)^2 + 2(3)(2x) + 3^2 = (2x + 3)^2$$

With a partner, factor the following expression (*Hint: what is a?*)

$$9x^2 + 12x + 4$$

Recall the formula is

$$x^2 + 2ax + a^2 = (x + a)^2$$

Let us re-write our expression after seeing a=2

$$9x^{2} + 12x + 4$$

$$(3x)^{2} + 2(2)(3x) + 2^{2}$$

$$(3x + 2)^{2}$$

# Some more special products

Work together with the *same* partner to try and factor the following two equations

$$x^2 + 18x + 81$$

 $x^2 - 18x + 81$  Hint: how can we change our answer from above?

# Some more special products

Work together with the *same* partner to try and factor the following two equations

$$x^{2} + 18x + 81 = (x + 9)^{2}$$
$$x^{2} - 18x + 81$$

# Some more special products

Work together with the *same* partner to try and factor the following two equations

$$x^{2} + 18x + 81 = (x + 9)^{2}$$
$$x^{2} - 18x + 81 = (x - 9)^{2}$$

Let's review this second example

# A different special form

What if we expand

$$(x-a)(x-a)$$

$$x^{2}-ax-ax+a^{2}$$

$$x^{2}-2ax+a^{2}$$

This is almost the same as before! We get

$$x^2 - 2ax + a^2 = (x - a)^2$$

This is why  $x^2 - 18x + 81 = (x - 9)^2$ 

# A very special form

Let's look at what happens if signs are opposite now!

$$(x+a)(x-a)$$

$$x^{2} - ax + ax - a^{2}$$

$$x^{2} - a^{2}$$

We're left with only  $x^2 - a^2 \otimes \odot \odot!$ 

This means that we can factor

$$x^2 - a^2 = (x + a)(x - a)$$

 $x^2 - a^2$  is known as the **difference of two perfect squares** 

# An example

We can factor

$$x^2 - 9 = (x+3)(x-3)$$

Recall the formula is

$$x^2 - a^2 = (x + a)(x - 1)$$

$$x^{2} - 4$$

$$x^{2} - 81$$

$$2x^{2} - 32$$

$$4x^{2} - 9$$

Recall the formula is

$$x^2 - a^2 = (x + a)(x - 1)$$

$$x^{2} - 4 = (x + 2)(x - 2)$$

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$$x^{2} - 81 = (x + 9)(x - 9)$$

$$2x^{2} - 32 = 2(x^{2} - 16) = 2(x + 4)(x - 4)$$

$$4x^{2} - 9 = (2x)^{2} - 3^{2} = (2x + 3)(2x - 3)$$

### Wrapping Up

We've seen some really useful equations for factoring today!

$$x^{2} + 2ax + a^{2} = (x + a)^{2}$$

$$x^{2} - 2ax + a^{2} = (x - a)^{2}$$

$$x^{2} - a^{2} = (x + a)(x - a)$$

Don't forget what we saw last time: factoring out a common factor.