Second Degree Equations Part 3

Graphing a Parabola Continued

Given a parabola

$$y = ax^2 + bx + c$$

we can find three things.

First, we can find x_+ and x_- such that $ax^2 + bx + c = 0$ with

$$x_+, x_- = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Then, we can find the vertex of the parabola with

$$(x_{vertex}, y_{vertex}) = \left(\frac{-b}{2a}, \frac{-\left(b^2 - 4ac\right)}{4a} \right)$$

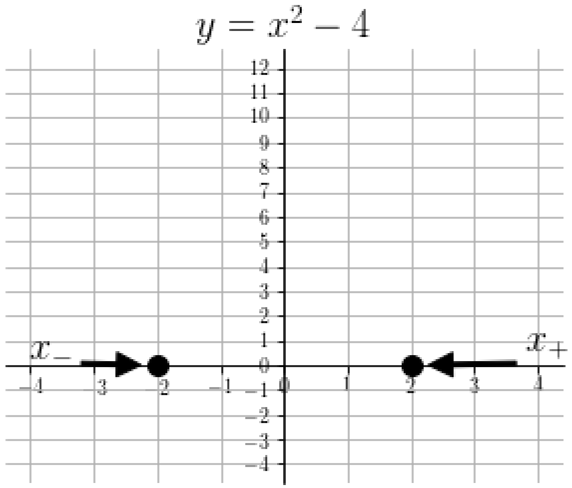
Graphing a Parabola Continued

The most straigt-forward way to graph a parabola is to plot a couple of points on it, and then connect them.

Let's try to graph $y = x^2 - 4$

First we can calculate and graph

$$x_+, x_- = 2, -2$$

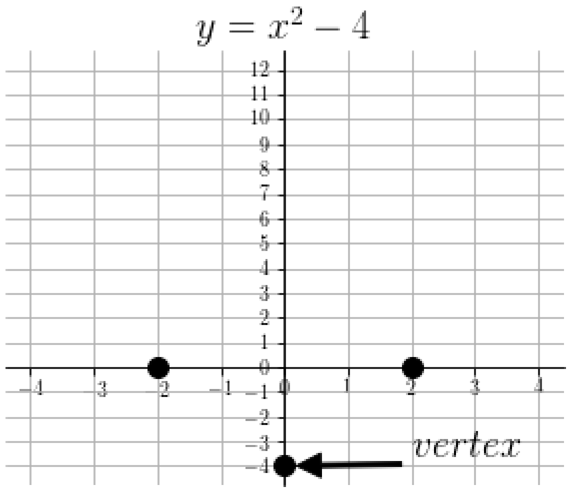


Graphing a Parabola Continued

$$y = x^2 - 4$$

Now let's calclate the vertex

$$\begin{aligned}(x_{vertex}, y_{vertex}) &= \left(\frac{-b}{2a}, \frac{-\left(b^2 - 4ac\right)}{4a}\right) \\ &= \left(\frac{0}{2}, \frac{-(0 - 4(1)(-4))}{4}\right) \\ &= \left(0, \frac{-(16)}{4}\right) = (0, -4)\end{aligned}$$



We need to calculate some additional points to the *left* and *right* of the vertex!

Graphing a Parabola Continued

$$y = x^2 - 4$$

Let's make a chart to record our points. We'll start with the vertex at the center.

x	y
--	--
-2	0
--	--
0	-4
--	--
2	0
--	--

Graphing a Parabola Continued

$$y = x^2 - 4$$

Now we can compute $x = -1$

$$(-1)^2 - 4 = -3$$

x	y
--	--
-2	0
-1	-3
0	-4
--	--
2	0
--	--

Graphing a Parabola Continued

$$y = x^2 - 4$$

Now let's compute $x = 1$

$$(1)^2 - 4 = -3$$

x	y
--	--
-2	0
-1	-3
0	-4
1	-3
2	0
--	--

Graphing a Parabola Continued

$$y = x^2 - 4$$

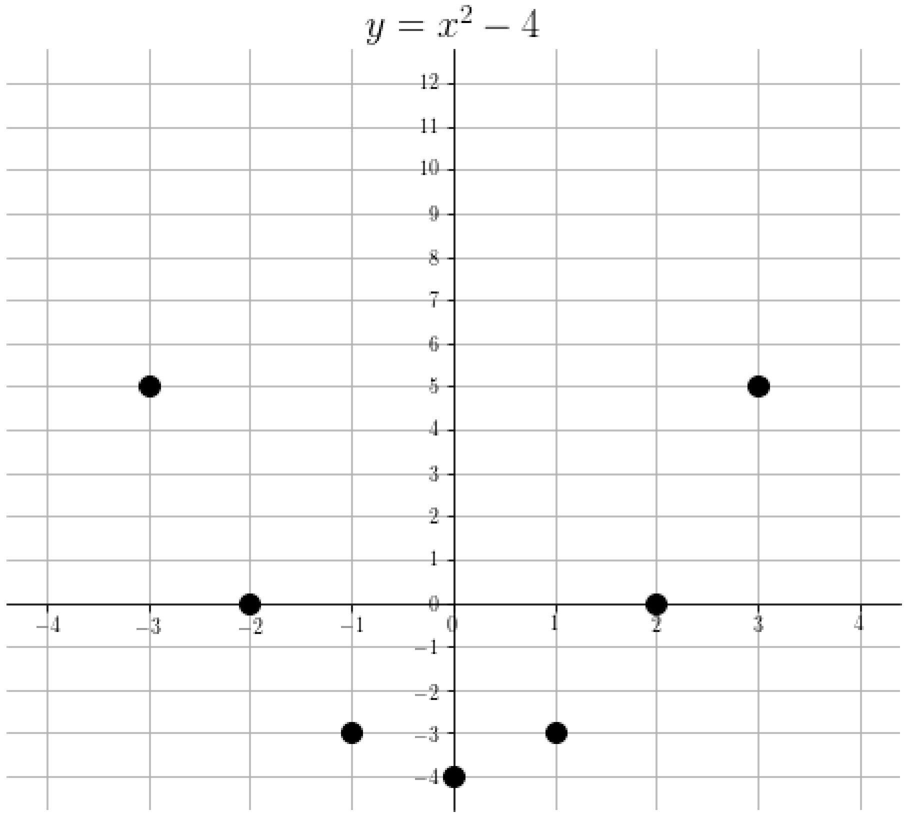
Lastly let's do $x = -3$, and $x = 3$

$$(-3)^2 - 4 = 5 \qquad (3)^2 - 4 = 5$$

x	y
-3	5
-2	0
-1	-3
0	-4
1	-3
2	0
3	5

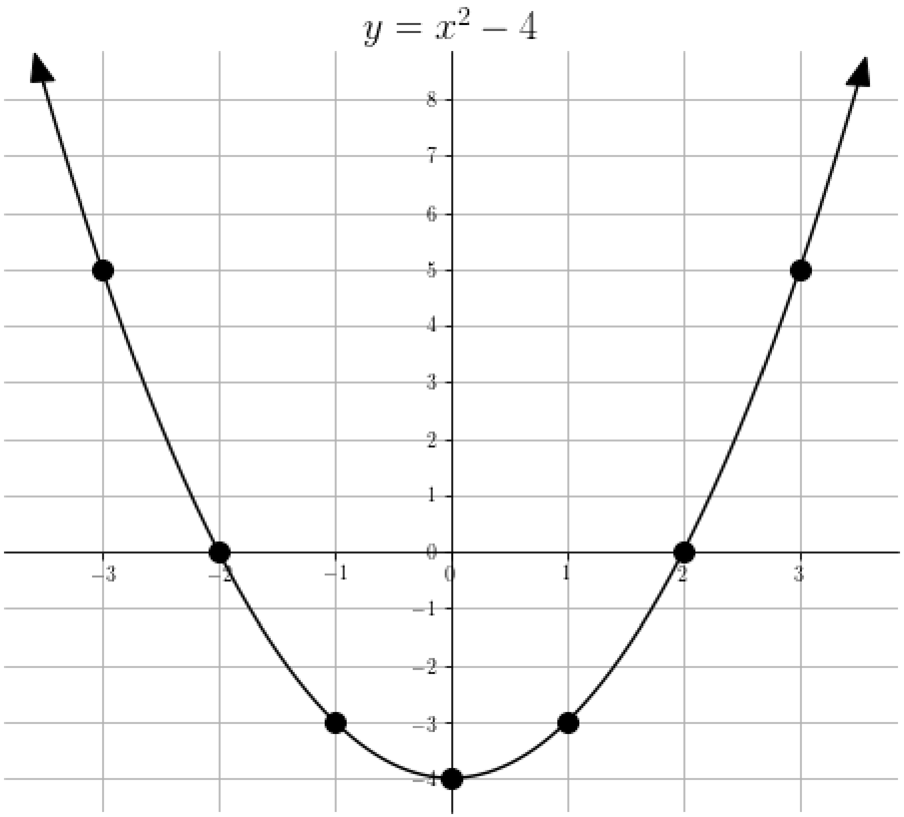
Plotting the points

With this chart, let's plot the points we found!



All that's left to do now is draw in the line!

Plotting the points



True or False: we only have to compute values on one side of the vertex

True! The parabola is symmetric about x_{vertex}, so the value for $x_{vertex} - k$ is the same value for $x_{vertex} + k$.

Trivia Question

True or False: we should **always** graph x_+ and x_-

False! If x_+ and x_- are not rational numers, we shouldn't really try to plot them.

For example, if

$$x_+, x_- = \frac{3 \pm \sqrt{12}}{6}$$

we shouldn't bother plotting the points.

Let’s practice!

In *groups of 3* graph the following parabolas using the method we've just seen

$$x^2 + 2x - 3$$
$$x^2 - 4x + 2$$
$$-2x^2 - 4x + 2$$

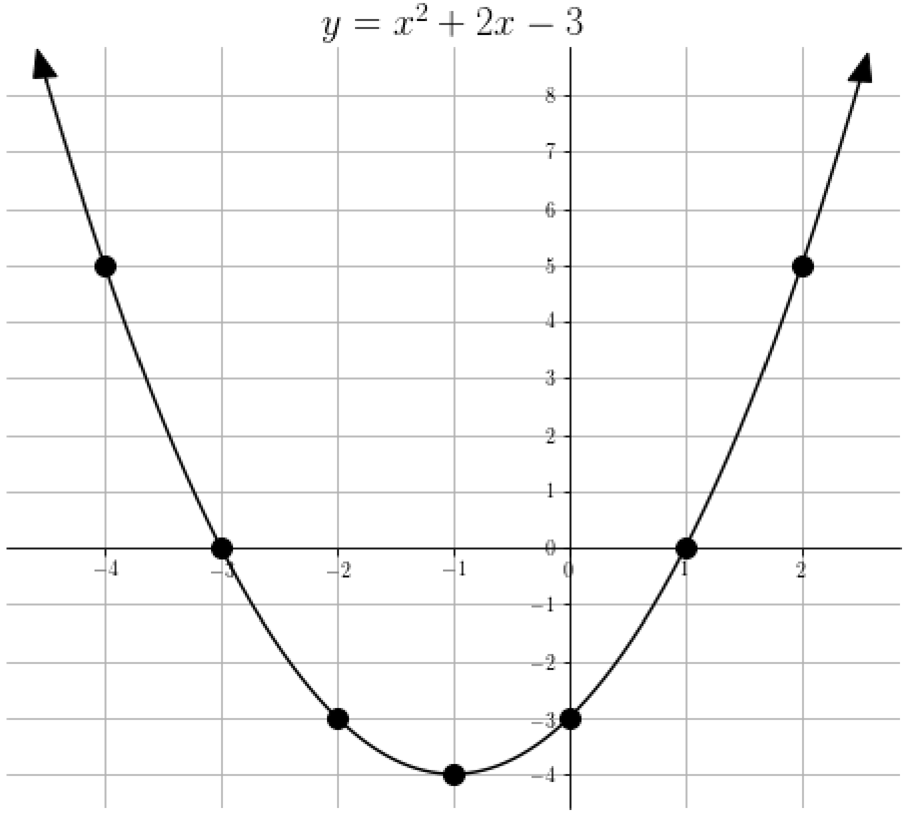
Problem 1

$$y = x^2 + 2x - 3$$

$$x_+, x_- = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 + 12}}{2} = \frac{-2 \pm 4}{2} = 1, -3$$

$$(x_{vertex}, y_{vertex}) = \left(\frac{-b}{2a}, \frac{-\left(b^2 - 4ac\right)}{4a} \right) = \left(\frac{-2}{2}, \frac{-(4 + 12)}{4} \right) = (-1, -4)$$

x	-4	-3	-2	-1	0	1	2
y	5	0	-3	-4	-3	0	5


$y = x^2 + 2x - 3$
8
7
6
5
4
3
2
1
0
−1
−2
−3
−4
−4
−3
−2
−1
0
1
2

Problem 2

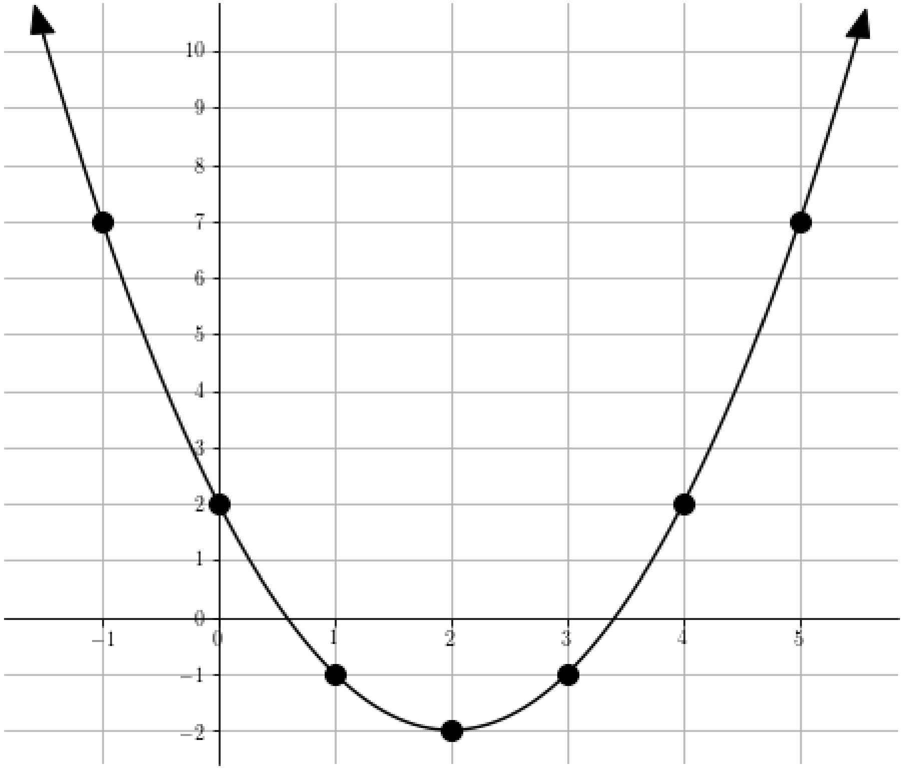
$$y = x^2 - 4x + 2$$

$$x_+, x_- = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 - 8}}{2} = \frac{4 \pm \sqrt{8}}{2} = \text{not nice}$$

$$(x_{vertex}, y_{vertex}) = \left(\frac{-b}{2a}, \frac{-\left(b^2 - 4ac\right)}{4a} \right) = \left(\frac{4}{2}, \frac{-(16-8)}{4} \right) = (2, -2)$$

x	-1	0	1	2	3	4	5
y	7	2	-1	-2	-1	2	7

$y = x^2 - 4x + 2$



Problem 3

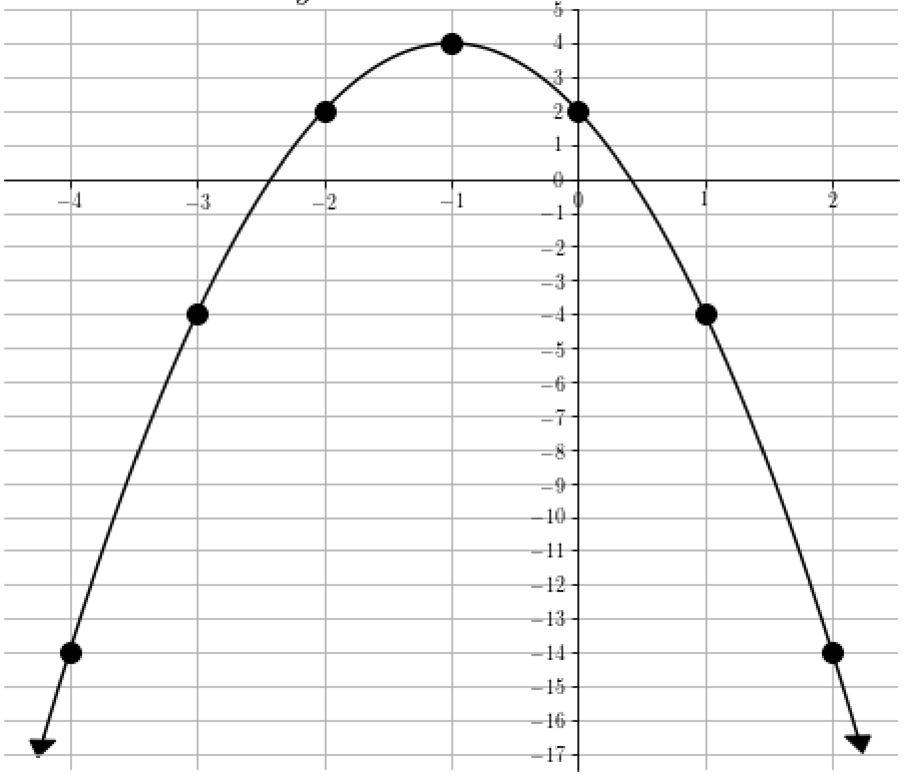
$$y = -2x^2 - 4x + 2$$

$$x_+, x_- = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 + 16}}{-4} = \frac{4 \pm \sqrt{32}}{-4} = \text{not nice}$$

$$(x_{vertex}, y_{vertex}) = \left(\frac{-b}{2a}, \frac{-\left(b^2 - 4ac\right)}{4a} \right) = \left(\frac{4}{-4}, \frac{-(16+16)}{-8} \right) = (-1, 4)$$

x	-4	-3	-2	-1	0	1	2
y	-14	-4	2	4	2	-4	-14

$y = -2x^2 - 4x + 2$



Vertex Form of a Parabola

We have been working with parabolas with equations that look like

$$y = ax^2 + bx + c$$

But there is a way to re-arrange our terms to get our expression into ***vertex form***.

Given a parabola, we can find

$$(x_{vertex}, y_{vertex}) = (h, k)$$

We can then write our parabola as

$$y = a(x - h)^2 + k$$

This makes the symmetry of the parabola easy to see!

Vertex Form of a Parabola

True or False: The *vertex form* of a quadratic expression is easier to work with.

True! We don't have to calculate the vertex, only read it. Calculating points is also easier!

Vertex Form of a Parabola - Practice

$$(x_{vertex}, y_{vertex}) = (h, k)$$
$$y = a(x - h)^2 + k$$

What is the vertex for each of the following parabolas?

$$(x - 2)^2 + 4$$
$$5(x + 5)^2 - 10$$
$$-2(x - 12)^2 + 6$$

Vertex Form of a Parabola - Practice

What is the vertex for each of the following parabolas?

$$\begin{aligned} (x-2)^2+4 &\rightarrow (2,4) \\ 5(x+5)^2-10 & \\ -2(x-12)^2+6 & \end{aligned}$$

Vertex Form of a Parabola - Practice

What is the vertex for each of the following parabolas?

$$\begin{aligned} (x-2)^2+4 &\rightarrow (2,4) \\ 5(x+5)^2-10 &\rightarrow (-5,-10) \\ -2(x-12)^2+6 & \end{aligned}$$

Vertex Form of a Parabola - Practice

What is the vertex for each of the following parabolas?

$$\begin{aligned}(x-2)^2+4 &\rightarrow (2,4)\\ 5(x+5)^2-10 &\rightarrow (-5,-10)\\ -2(x-12)^2+6 &\rightarrow (12,6)\end{aligned}$$

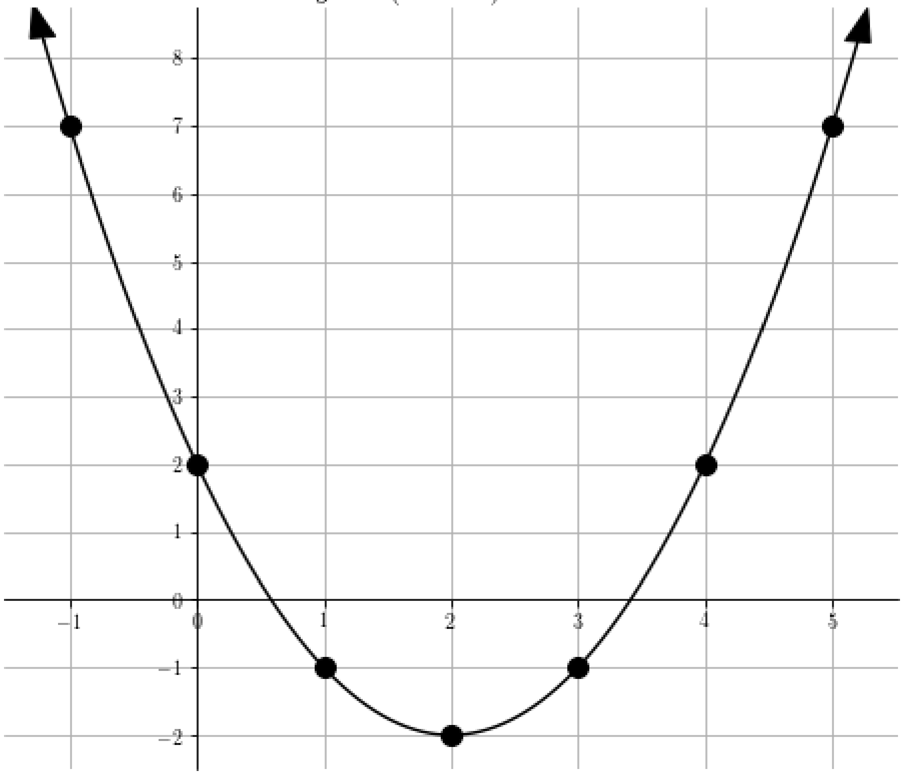
Practice!

Spend the last few minutes graphing the equation

$$y = (x - 2)^2 - 2$$

x	-1	0	1	2	3	4	5
y	7	2	-1	-2	-1	2	7

$$y = (x - 2)^2 - 2$$



Wrapping up

Today we learned how to graph parabolas by calculating points.

Next time we will learn how to use our knowledge to solve real world problems!