Report : Control Theory and applications laboratory

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Note : refer to https://github.com/Muten-Roshi-Sama/TCLab_PID_Controller_FeedForward

1 Introduction

This control laboratory is about applying the control theory Of the theoretical course "Control theory and applications" in practice on the Temperature Control Lab kit also known as TCI,ab.

Refer to http://apmonitor.com/pdc/index.php/Main/ArduinoTemperatureControI.

TCLab is a pocket-sized lab With software in Python, MATLAB, and Simulink for the purpose Of reinforcing control theory for students. Refer to Figure 1.

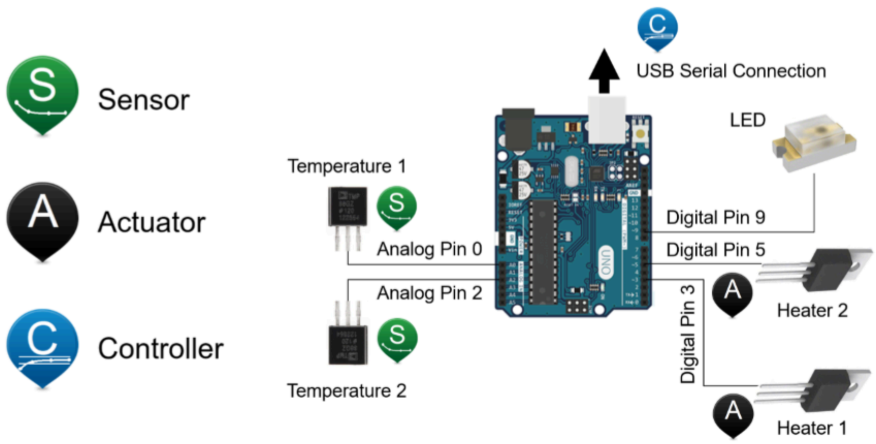


Figure 1: TCLab is a simple multivariable system comprised of **2 heaters** and **2 temperature sensors**.

1.1 Objectives

In this control laboratory, we will consider the single variable system in between the manipulated variable MV, the heating power of heater 1 [%], and the process variable PV, the temperature of sensor 1 [C°]. We will use DV, the heating power Of heater 2 [%] as a disturbance variable. through a coupling effect, heater 2 has an effect on the temperature sensor 1 through conduction', convection and radiation:

- MV : heating power of heater 1 [%].
- PV : temperature of sensor 1 [C°].
- DV : heating power Of heater 2 [%].

1.2 Identifying System's Parameters

Note : ALWAYS restart the kernel before running cells. This will clear the cache and ensure correct value prints and graphing.

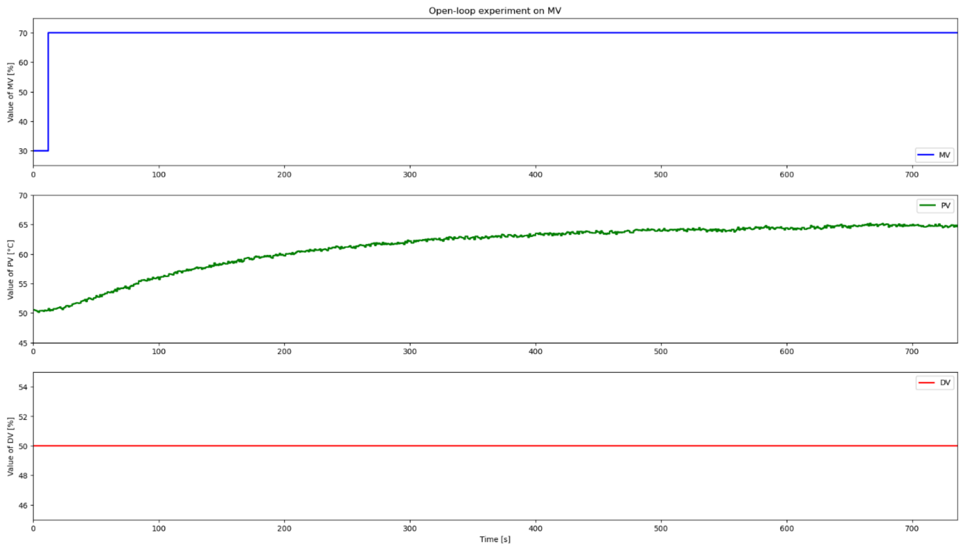


Figure 2: Open-loop experiment on MV.

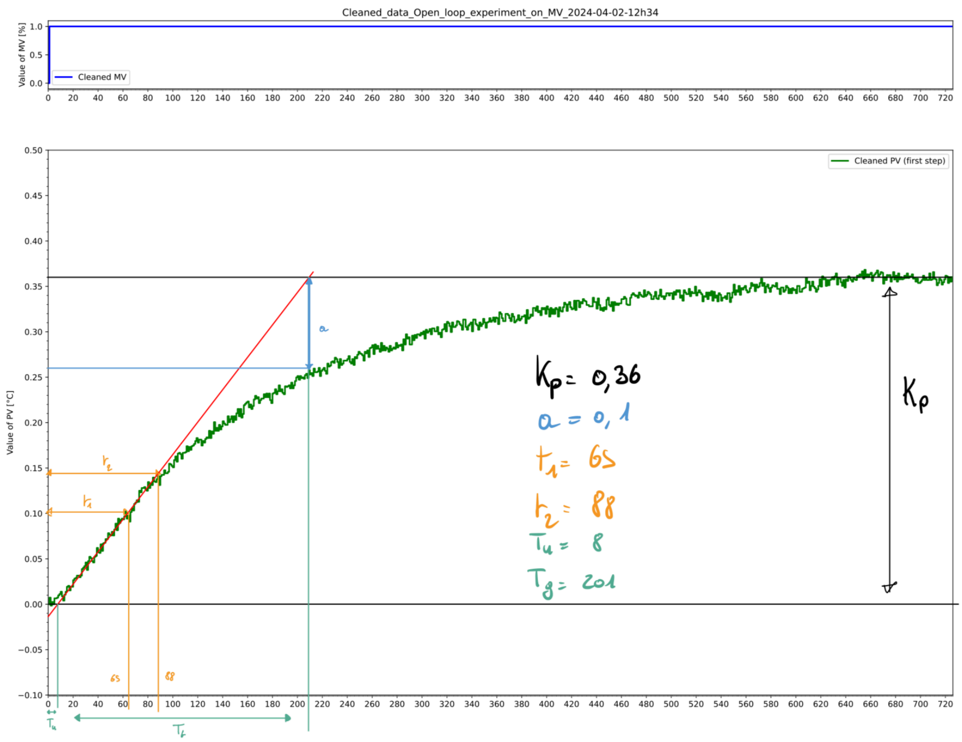


Figure 3: Tangent at inflexion method.

2 MV step response

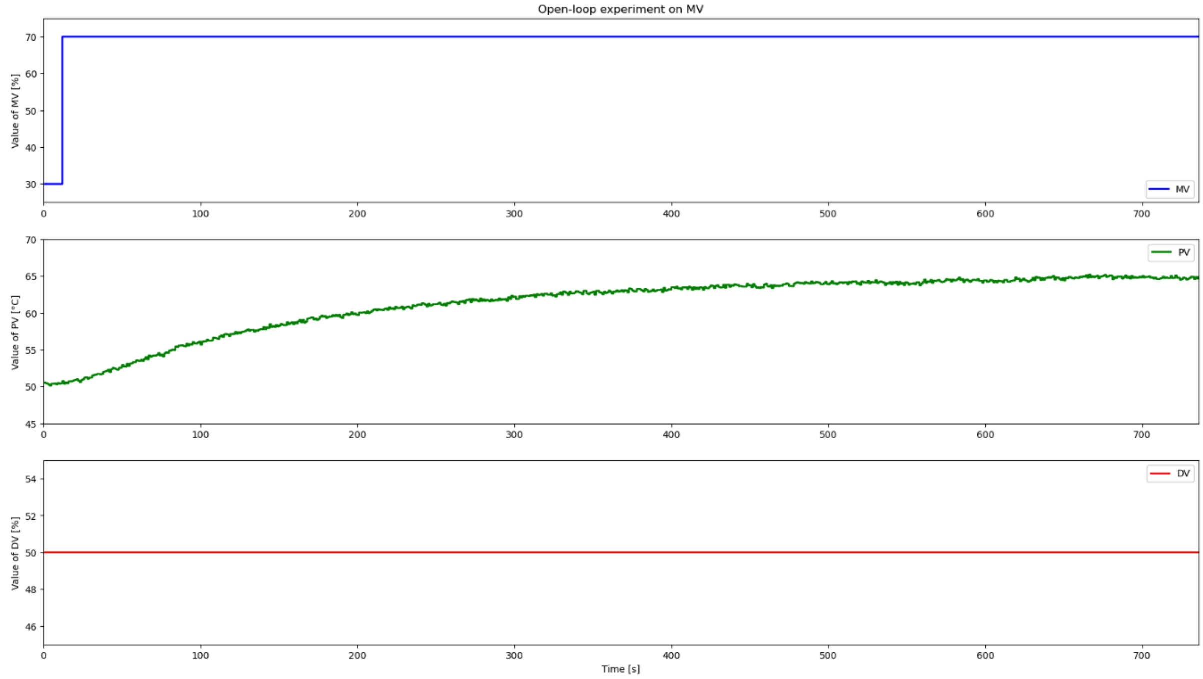


Figure 4: Effects of a MV perturbation on the system.

from TCLAB_OLP_Experiment_manual_button.ipynb

2.1 Graphical Approximations

We will now use several approximation methods relative to our experimental values, discuss and compare them.

Tangent at inflexion method.

- K_p = 0.36
- T_1 = 65
- T_2 = 88
- α = 0.1
- α_V = 0.123
- T_g = 201
- T_u = 8

The selection of the variable α_V[1] for Van Der Grinten's model was a result of our careful consideration. Given that a negative value for $T_{1\mathrm{Vdg}}$ in the Grinten model would lack physical significance, we adjusted our initial value from α=0.1 to α=0.123. This decision was driven by the expression for the first time constant, $T_{1\mathrm{Vdg}}$, in Van Der Grinten's formula (see lab assignment). Solving for "α" and ensuring $T_{1\mathrm{Vdg}} > 0$, we derived $\alpha > \frac{1}{3*e} =$ 0.1223… Consequently, this adjustment yielded a very small, yet positive value for $T_{1\mathrm{Vdg}}$.

[1]any value of α inferior to 0.123 would result in a negative $T_{1\mathrm{Vdg}}$. Following Van Der Grinten's equations from our lab assignment.

2.1.1 Broida's Models :

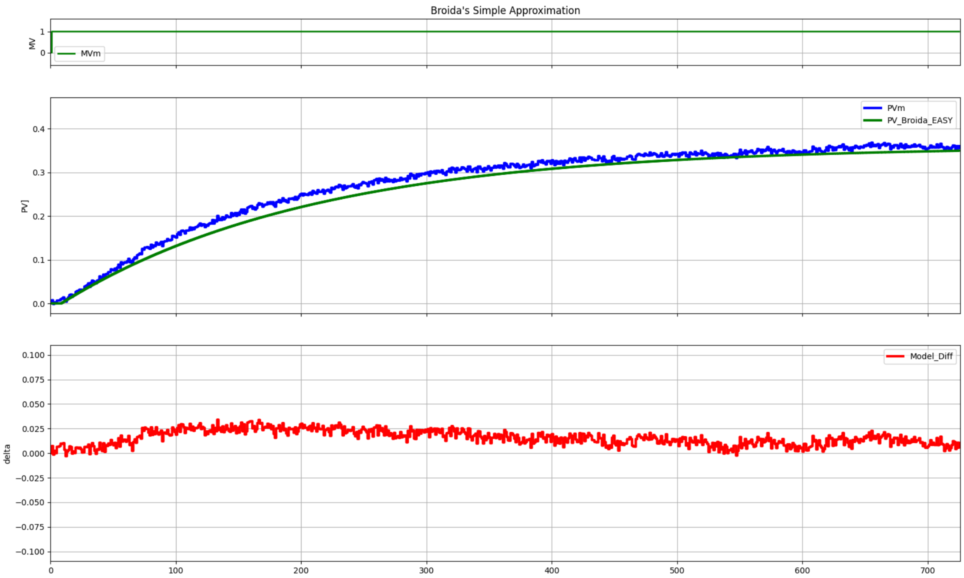


Figure 5: Broida's **1st** Model

from Simulation_OLP.ipynb

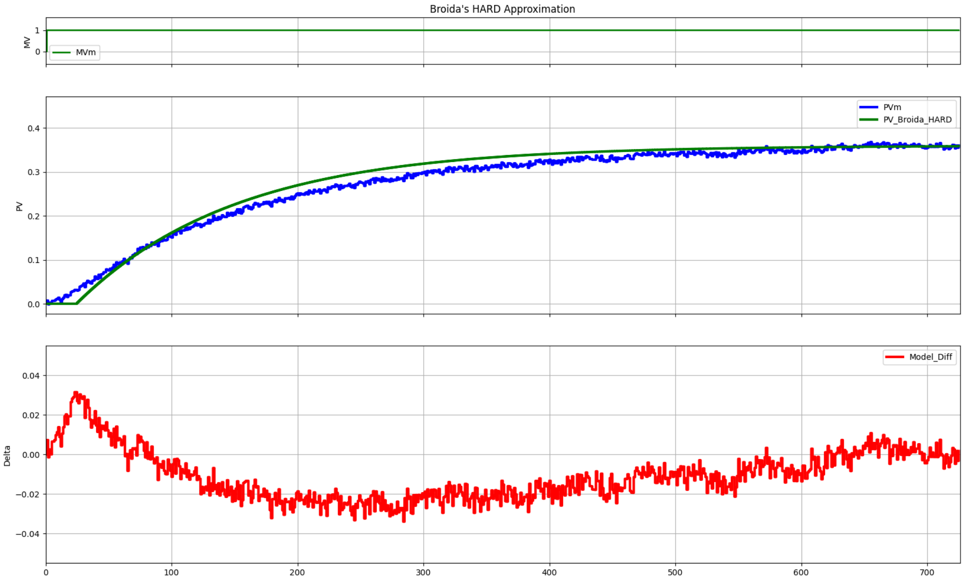


Figure 6: Broida's **2d** Model

from Simulation_OLP.ipynb

2.1.2 Van der Grinten's Model :

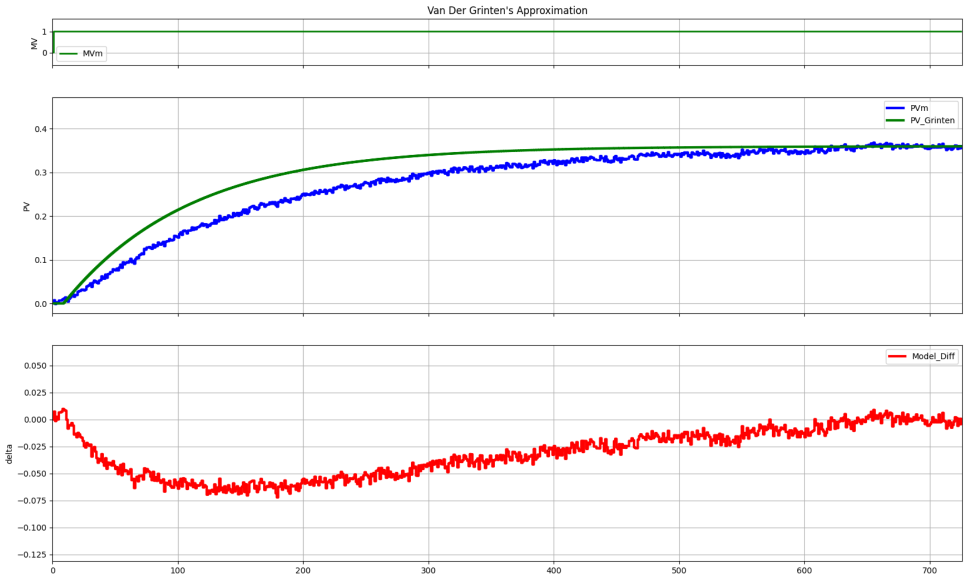


Figure 7: Van Der Grinten's model with $\alpha_V = 0.123$.

from Simulation_OLP.ipynb

2.1.3 Strejc's Model :

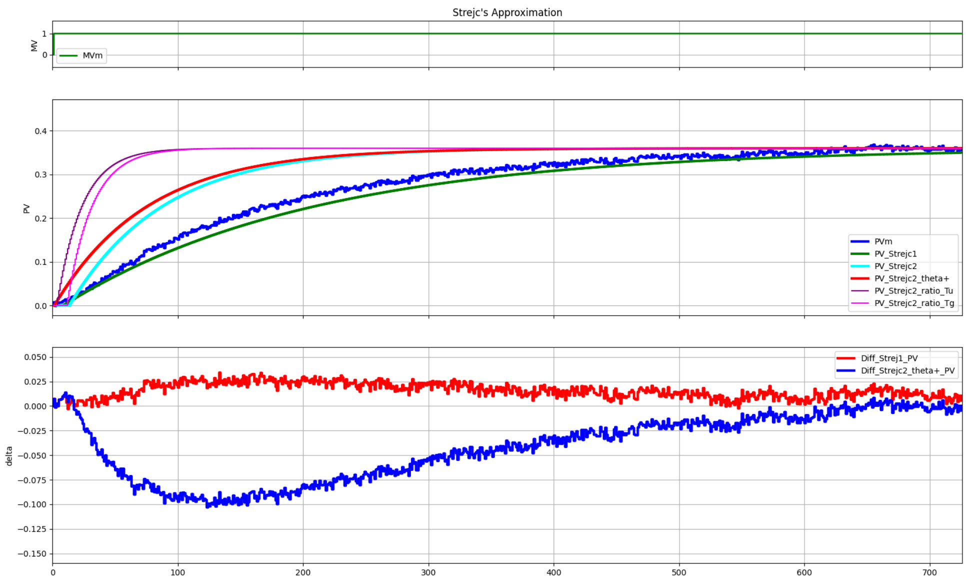


Figure 8: Strejc's first and second order approximations

from Simulation_OLP.ipynb

Our analysis of the system's step response using Strejc's model revealed valuable insights. The calculated ratio of dead time (T_u) to time constant (T_g) was approximately 0.09 (refer to the note below), suggesting a first-order model based on Strejc's table. However, since this ratio was close to the threshold for a second-order model (0.10), we attempted to fit a second-order Strejc model to the experimental data using the initial T_u and T_g experimental values.

The second-order model initially produced a constant zero response (or a python error from the Delay function), which could be attributed to the negative theta value (-12) obtained from the 2d order approximation. To explore this further, we simulated the model with an adjusted positive theta value (+12) named 'PV_Strejc2_theta+'. Although this adjustment yielded a response curve (in red), it significantly overestimated the actual system experimental behavior. This observation confirmed that a second-order Strejc model was not suitable for approximating the experimental data in this case.

To validate our calculations and explore the influence of T_u and T_g, we additionally simulated the second-order Strejc model with adjusted parameter values that resulted in a $\frac{T_u}{T_g}$ ratio of 0.15. This involved fixing either T_u or T_g while adjusting the other to achieve the desired ratio :

- Ratio_Tu (T_u fixed) : T_u = 8, T_g = $\frac{T_u}{\text{ratio}}$ = $\frac{8}{0.15}$ = 53.333
- Ratio_Tg (T_g fixed) : T_g = 201, T_u = T_g ∗ ratio = 201 ∗ 0.15 = 30.15

Influence of T_u and T_g :

- $\theta = T_u$, changing T_g (purple) result in a lesser delay (theta=2.66) compared to only changing T_u (magenta, theta=10.05). Which isn't a surprise since theta influences the time delay.
- $T = T_g$ is the time constant of the first-order dynamic response of our system. It indicates how fast the system responds to changes in input. A larger T would mean a slower rise or decay, which might not be apparent in our specific experiment.

—–Note—–– Our initial T_u/T_g ratio was 0.09, close to the 0.10 threshold for a second-order model. We then explored the second-order model to understand its behavior. However, our updated experimental values resulted in a ratio of 0.04, further strengthening the case for a first-order model. While the graphs in this report utilize the updated values, we believed interesting to keep testing the limits of Strejc's Model even using our new values, which proved to be not that distant from the curves obtained with a 0.09 ratio.

2.1.4 Model key differences :

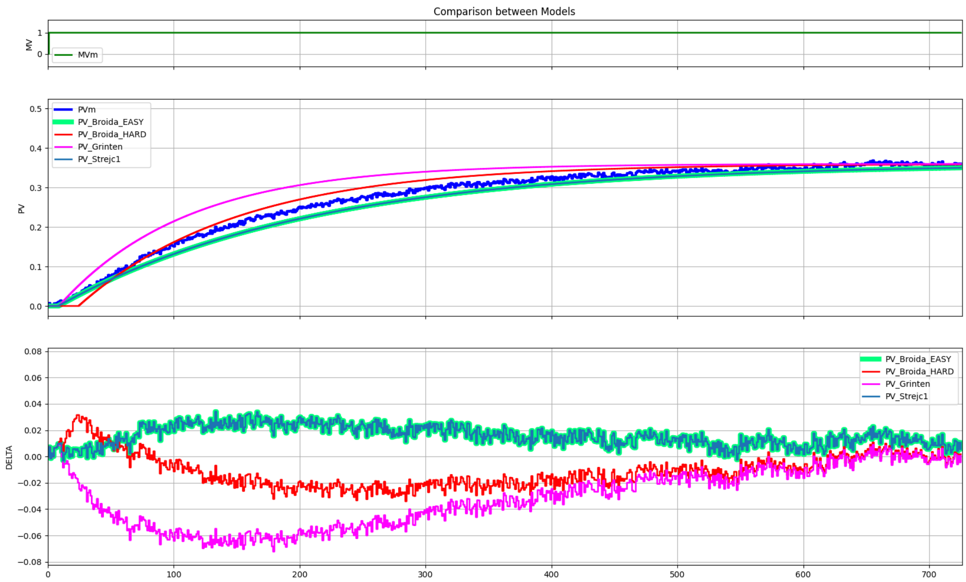


Figure 9: Error comparison between models.

from Simulation_OLP.ipynb

We observe that Strejc's and Broida's 1st model are one and the same, which makes sense since Strejc's 1st order model is equivalent to Broida's first model.

Since the 1st order Strejc model (same as Broida's FOPDT) fits our experimental curve best, we'll use its parameters as a reference. This means the delay in our system is 8 seconds ($\theta_p = T_u = 8.$).

(See Chapter 3 below. Our next Tangent approximation on DV will yield : $\theta_d = T_u = 31.$)

Note : Grinten's model might be the least suited approximation with the worst relative error since it is more suited for an "S-shape" step response.[2]

[2]Refer to lab assignment.

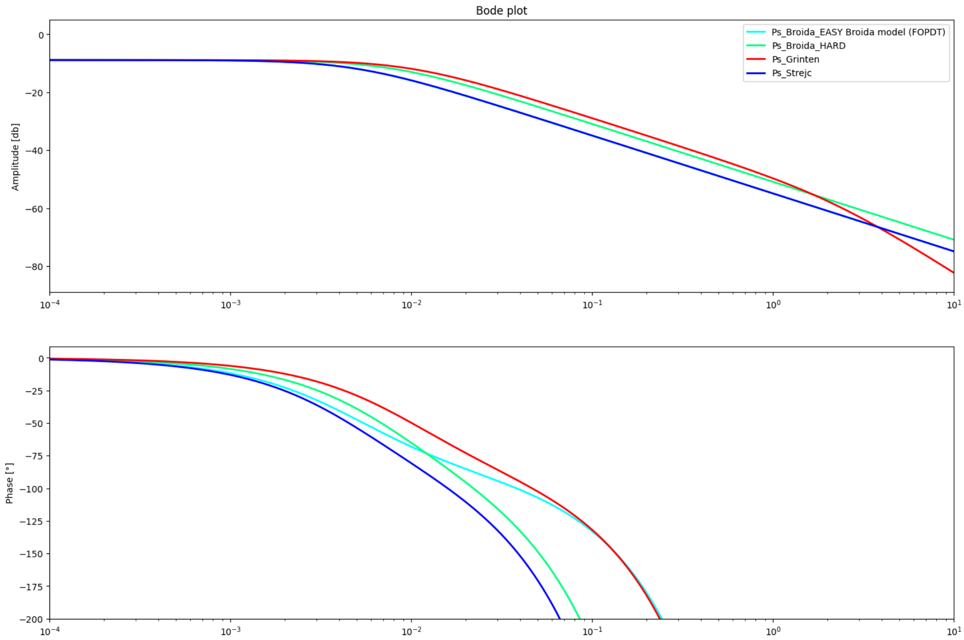


Figure 10: Error comparison between models.

from Simulation_OLP.ipynb

Strejc's and Broida's 2d model have the least amount of gain attenuation, especially in high frequencies. All model seem to have a similar phase behaviour.

3 DV step response

- Operating point : DV = 50.
- The process is not linear.

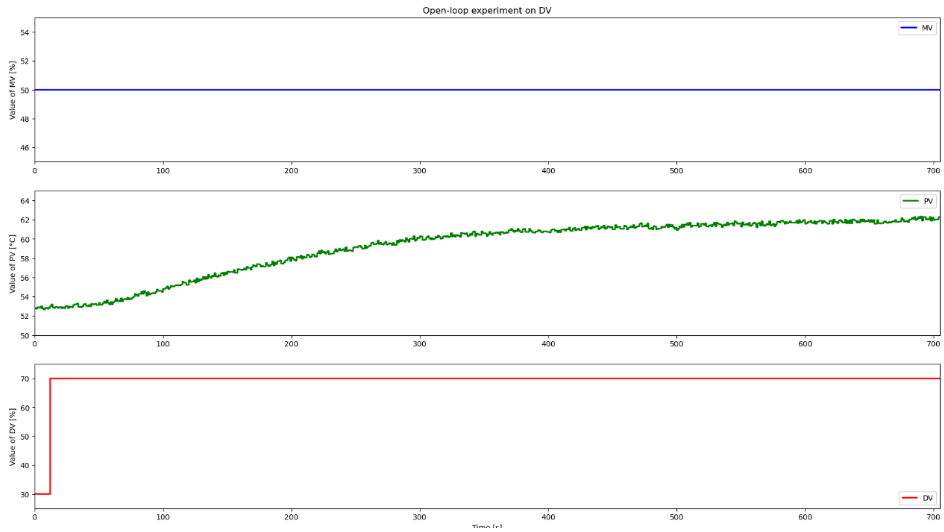


Figure 11: Effects of a DV perturbation on the system.

from TCLAB_OLP_Experiment_manual_button.ipynb

The tangent at inflexion method (Figure 12) for a DV disturbance on our system resulted in the following parameters :

- $K_p = 0.23$
- $a = 0.045$
- $T_1 = 106$
- $T_2 = 138$
- $T_u = 31$
- $T_g = 270$

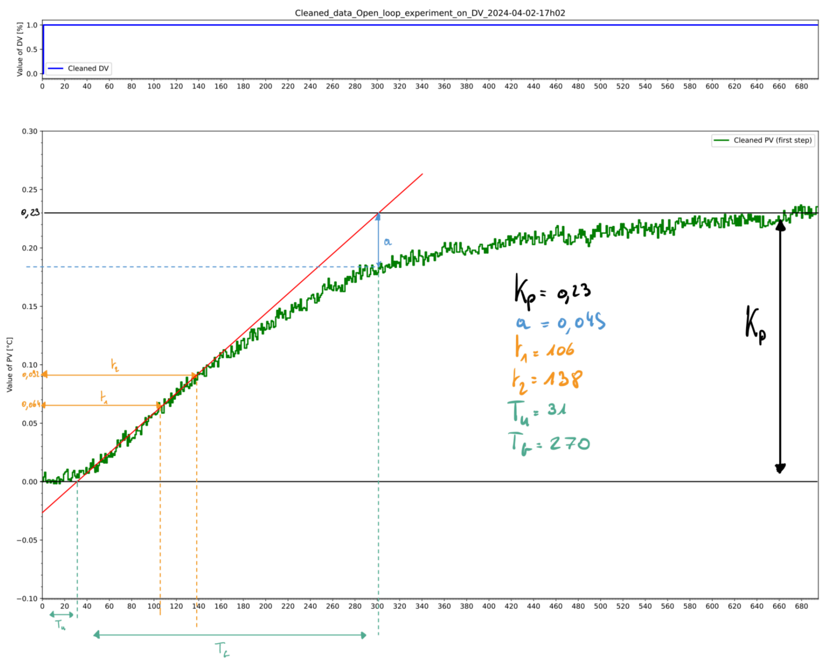


Figure 12: Tangent at inflexion method for a DV disturbance

4 Margins

As we have chosen the Strejc model, we will now need to calculate its gain and phase margins. This will give us an idea of its robustness to changes in gain or phase within the closed loop.

To do this, we have developed a function called "margin" that calculates $L(s) = P(s)C(s)$. Here is the diagram for the Strejc model (Figure 13).

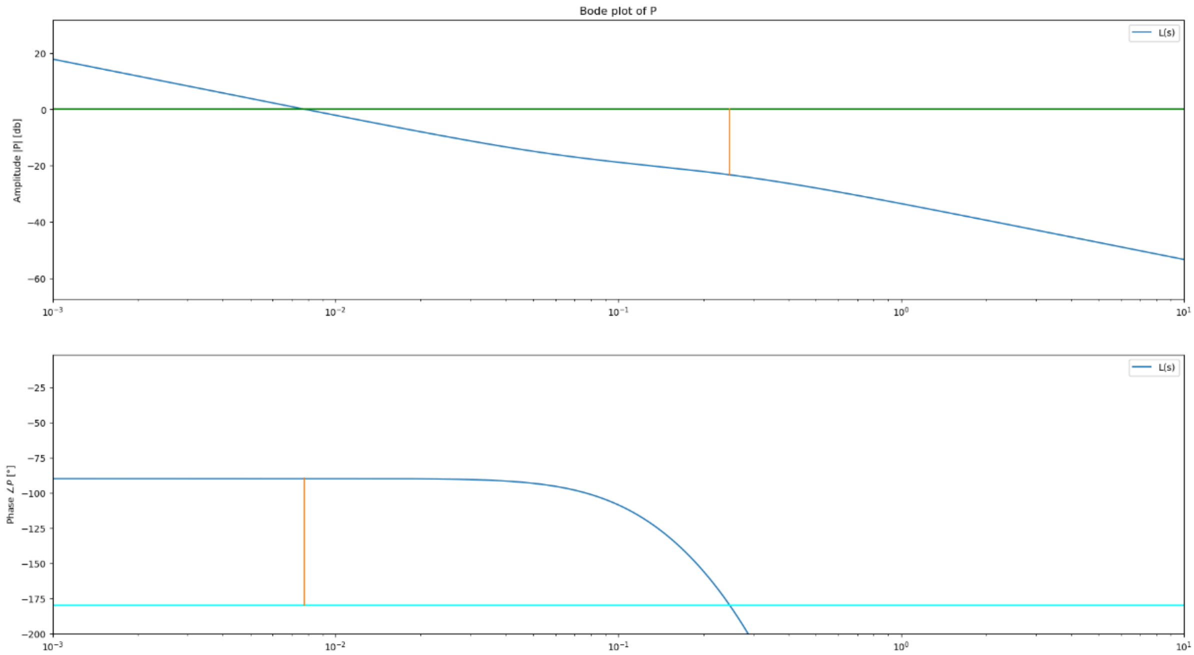


Figure 13: Phase and gain margins for the Strejc model.

To find the phase margin, we position ourselves at the frequency corresponding to a gain of 1 (at 0 dB). The phase margin is the distance in orange on the lower graph between L(s) in blue and −180°. The gain margin is found at the frequency where we have a phase shift of −180°. It's the distance compared to 0 dB.

We can compare the gain and phase values of the different models:

Model	Gain margin[dB]	Phase margin[°]
Broida easy	23	90
Broida hard	14	91.5
Van der grinten	17	106
Strejc	23	90

- Although all the models are similar, the Van der Grinten model appears to be the most robust in phase.
- Broida' easy (Strejc's) are the better models in terms of gain robustness.

5 Lead Lag responses

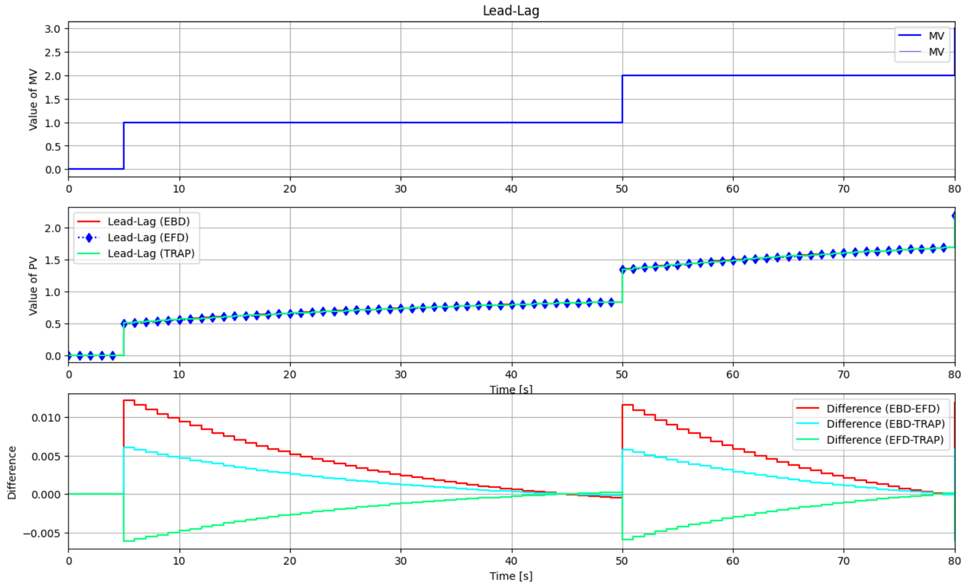


Figure 14: Lead-Lag responses comparison.

We observe a direct systematic error between EBD and EFD, which is normal since :

- Convex functions : (f''(x) > 0) : EBD overestimates, EFD underestimates.
- Concave functions (f''(x) < 0) : EBD underestimates, EFD overestimates.

Using T_Lag > T_Lead, we obtain a convex function, giving us a positive declining error (EBD - EFD > 0). Since the Trapezoidal is the in-between method, we indeed observe an equidistant error between the EBD (cyan) and EFD Models (springgreen).

The Trapezoidal approximation method seem to approximate our experimental curve best.

6 PID : Closed-loop response to a setpoint change SP

In this experiment, we implemented a PID controller to regulate a system's response to setpoint changes and disturbances. The PID controller was combined with a feedforward controller to anticipate disturbances and provide a corrective action beforehand.

To evaluate the performance of the PID controller, we applied a unit step input to the controller's error signal (E = SP - PV). We then observed the responses of the three components of the PID controller (proportional, integral, and derivative) as well as the manipulated variable (MV).

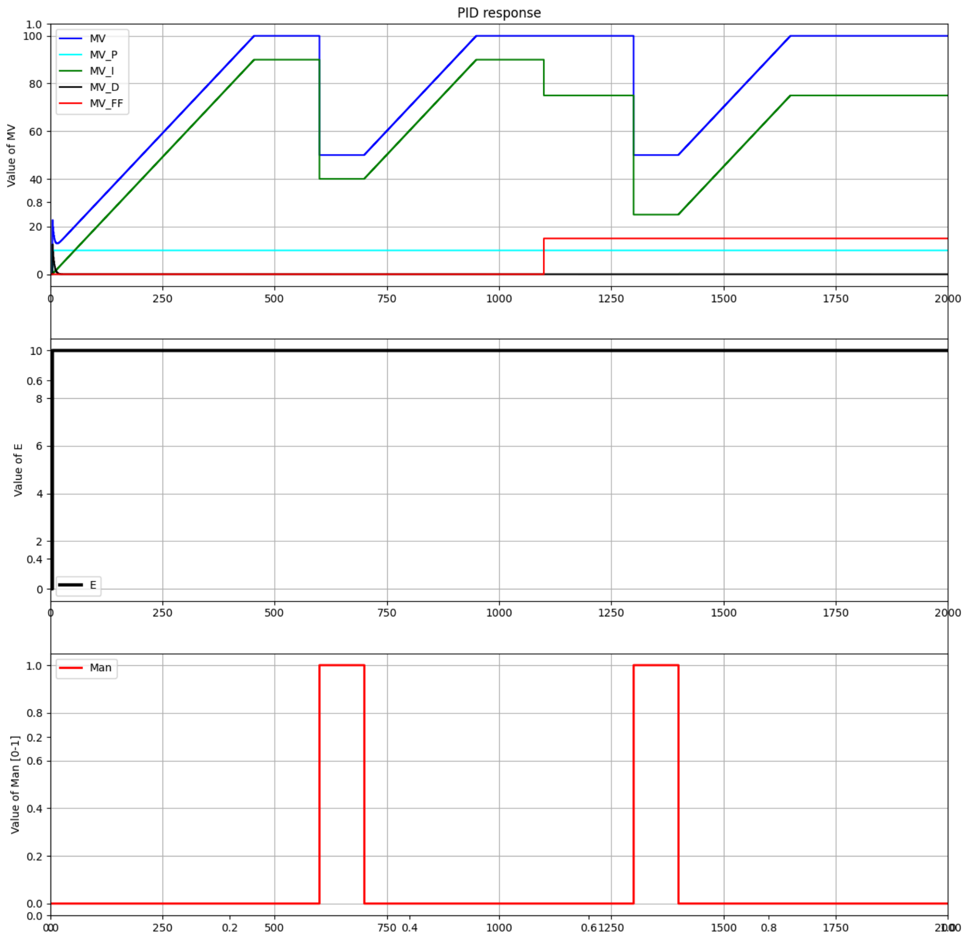


Figure 15: Effects of an Error on the component of a PID regulator.

Observations:

- The proportional term (cyan) responded immediately to the step change in error. It will stay high until the error is corrected (E set back to 0).
- The integral term (green) responded slowly, gradually increasing the MV over time until MV reached its max.
- The derivative term (black) responded quickly to changes in the error rate, adding a corrective signal to the MV. It came back down as the system's response came into play, decreasing the error changing rate back to 0.
- The MV (blue) saturated at its maximum value. The regulator is now applying the maximum correction it can currently deliver.
- As MV_FF increase, the integral actions decrease because the total amount of correction needed to eliminate the error is reduced[3]. This might be desirable to rely less on the integral term and limit the risk of wind-up.

[3]See slide p.198 : $\mathrm{MV} = \mathrm{MV}_P + \mathrm{MV}_I + \mathrm{MV}_D + \mathrm{MV}_{\mathrm{FF}}$. Here the $\mathrm{MV}_{\mathrm{FF}}$ is added to the equation since we added a Feedforward structure to our PID controller compared to the slides.

6.1 Influence of Parameters :

- **Increasing K_p (Proportionnal Gain) :** *output value is proportional to the current error.*
  - Increases P-Action.
  - Decreases static error (SSE) : A higher K_c makes the controller react more significantly to the current error, driving the system output closer to the desired setpoint and reducing steady-state error.[4]
  - Decreases phase margin (stability) : A stronger proportional action can lead to a more aggressive response, causing oscillations and reducing the system's stability (by decreasing the phase margin).
  - Increases reaction time : as seen in class, this is one of the only reasons we increase K_p, as it could otherwise cause the system to become unstable very fast by becoming oversensitive to noise and respond too aggressively.
  - Increases overshoot.[4]
- **Increasing T_i (Integral Time Constant) :**

sum of the instantaneous error over time and gives the accumulated offset that should have been corrected previously. since the integral term responds to accumulated errors from the past, it can cause the present value to overshoot the setpoint value.[4]

- Decreases Integral Action : A higher T_i allows the controller to integrate the error over a longer period. This helps eliminate offset (persistent difference between setpoint and actual output) but can also **slow down** the system's response.
- Increases the overshoot.[4]
- Increases settling time.[4]
- Decreases/eliminate SSE.[4]
- **Increasing T_d (Derivative Time Constant) :** *Rate of change of the error.*
  - Decreases overshoot.[4]
  - Decrease Settling time.[4]
  - Theoretically no effect on SSE.[4]
  - Increases reaction time : A stronger derivative term helps the controller anticipate changes in the error and react faster to them, potentially improving response time. However, too high a value can lead to instability.
- **T_s Sampling Period :**

time between each measurement made by the controller. Affects the resolution. Value imposed at 1.

[4]See references : 1, 2, 3

7 Simulation of a PID controller with Feedforward

7.1 Overview

As seen in class (see Figure 16), our system is made out of :

- PID regulator (red path),
- our Feedforward structure with gain, Lead, Lag and delay implementation (green path)
- a linear desription of TCLab (blue path) :

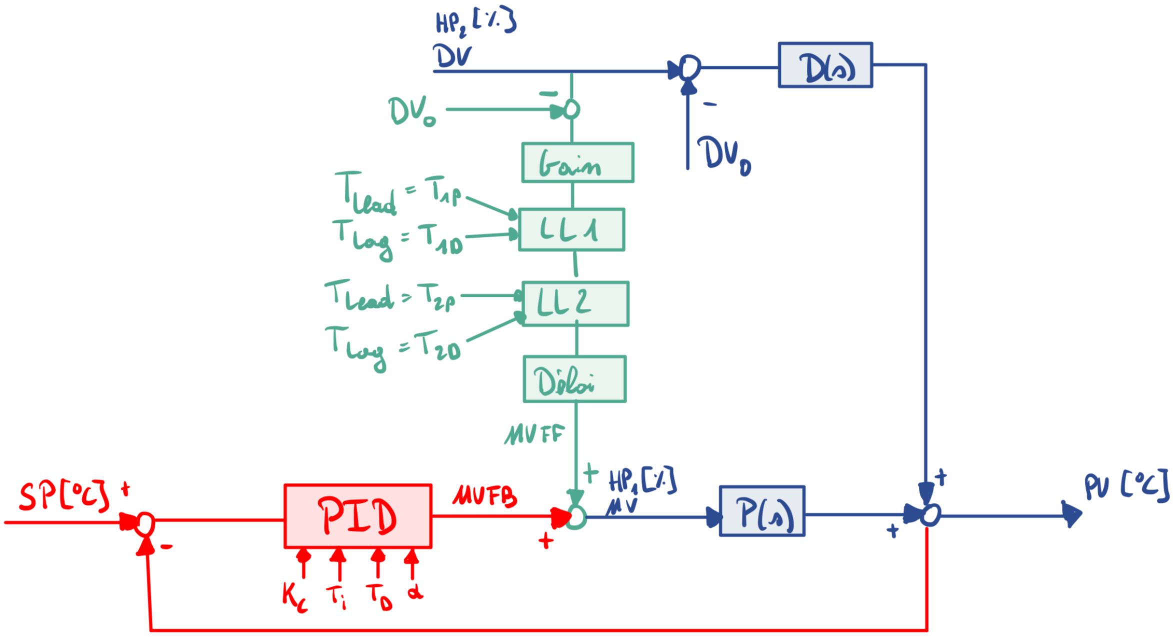


Figure 16: Closed-loop scheme.

- Perturbation (SOPDT) : D(s)

$$D(s) = \frac{K_p e^{-\theta D_s}}{(T_1 D_s + 1)(T_2 D_s + 1)}$$

- Process (SOPDT) : P(s)

$$P(s) = \frac{K_p e^{-\theta P_s}}{(T_1 p_s + 1)(T_2 p_s + 1)}$$

7.2 System Parameters

Working point : $\mathbf{MV_0}$ and $\mathbf{DV_0}$ = 50%

	Value	Comment
$\mathbf{PV_0}$:	58°C.	Value derived from our TCLab experiment (Section 9.1) from when PV reached its Steady-state and $\mathrm{MV}_0, \mathrm{PV}_0 = 50\%$.
$\boldsymbol{K_c}, \boldsymbol{T_i}$ **et** $\boldsymbol{T_d}$:	see code[5] (1.5, 209 and 7.69)	values obtained from IMCTuning, we also had to divide Kc by 3 to prevent saturation.[6]
γ :	0.6	Regulates the system response to disturbances or setpoint changes. However, a very low gamma (like 0.2) can lead to instability as T_{CLP} would be 5 faster than T_{Lead}. We chose a less aggressive value.

	Value	Comment
α :	1	Limit the high-frequency gain of the derivative action.[7]

For the influence analysis of α and γ, refer to the Annex.

7.3 Simulations

The goal of the next simulations is to emphasize the effect of adding a Feedforward structure on Open and Closed-loops systems.

7.3.1 Simple open-loop without feedforward

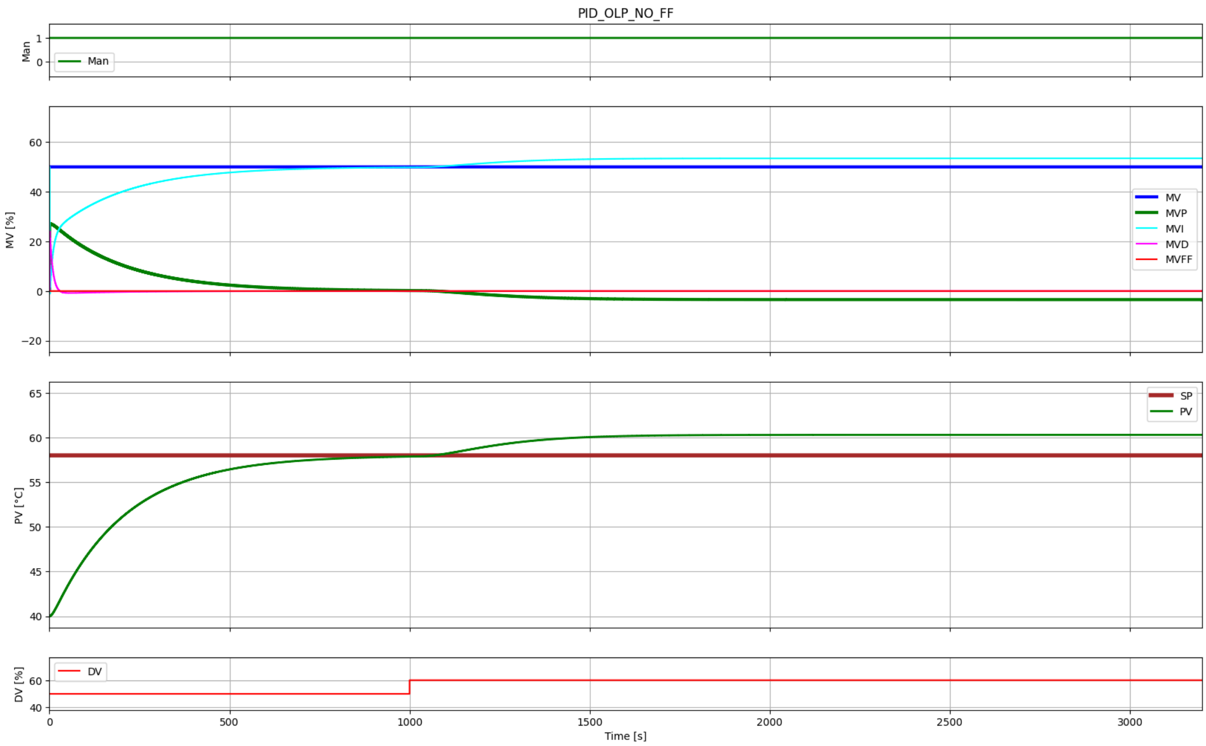


Figure 17: Simple open-loop simulation without feedforward.

This simulation demonstrates the limitations of open-loop systems. While the process variable (PV) initially reached the desired setpoint, the introduction of a disturbance (DV) caused the PV to significantly deviate. This is because open-loop systems lack the ability to adjust their output based on the measured process variable. In the next simulation, we will explore how adding a feedforward structure can address this limitation by anticipating disturbances and providing a pre-calculated corrective action.

[5]https://github.com/Muten-Roshi-Sama/TCLab_PID_Controller_FeedForward

[6]see Annex 10.2

[7]See annex 10.3

7.3.2 Open-loop with feedforward

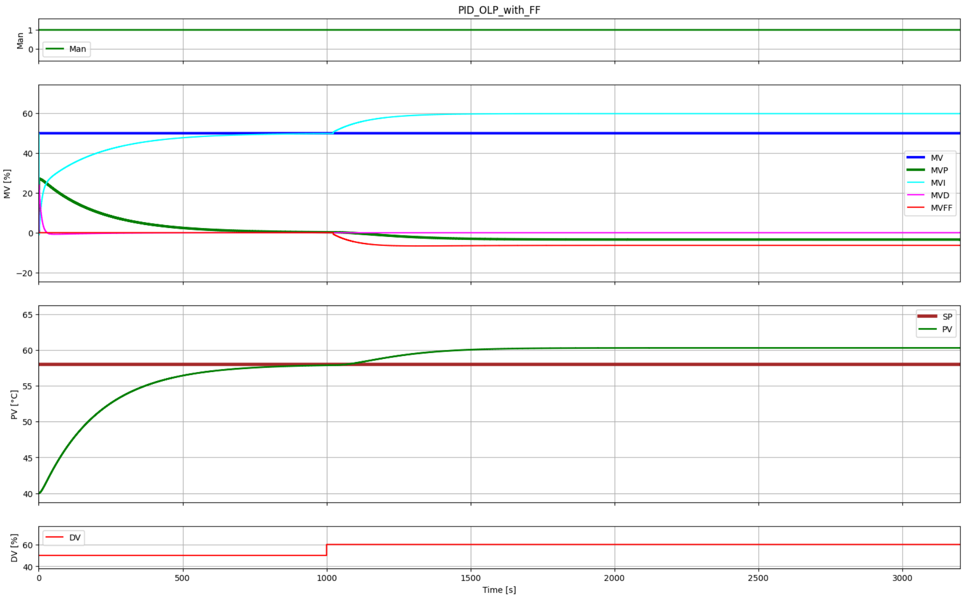


Figure 18: Open-loop simulation with feedforward.

- Feedforward (red curve) is now activated (t=1000).
- The integral action is now greater.
- Unfortunately PV does not seem to be any closer to its Setpoint. Which is an unexpected result.

7.3.3 Closed-loop without feedforward

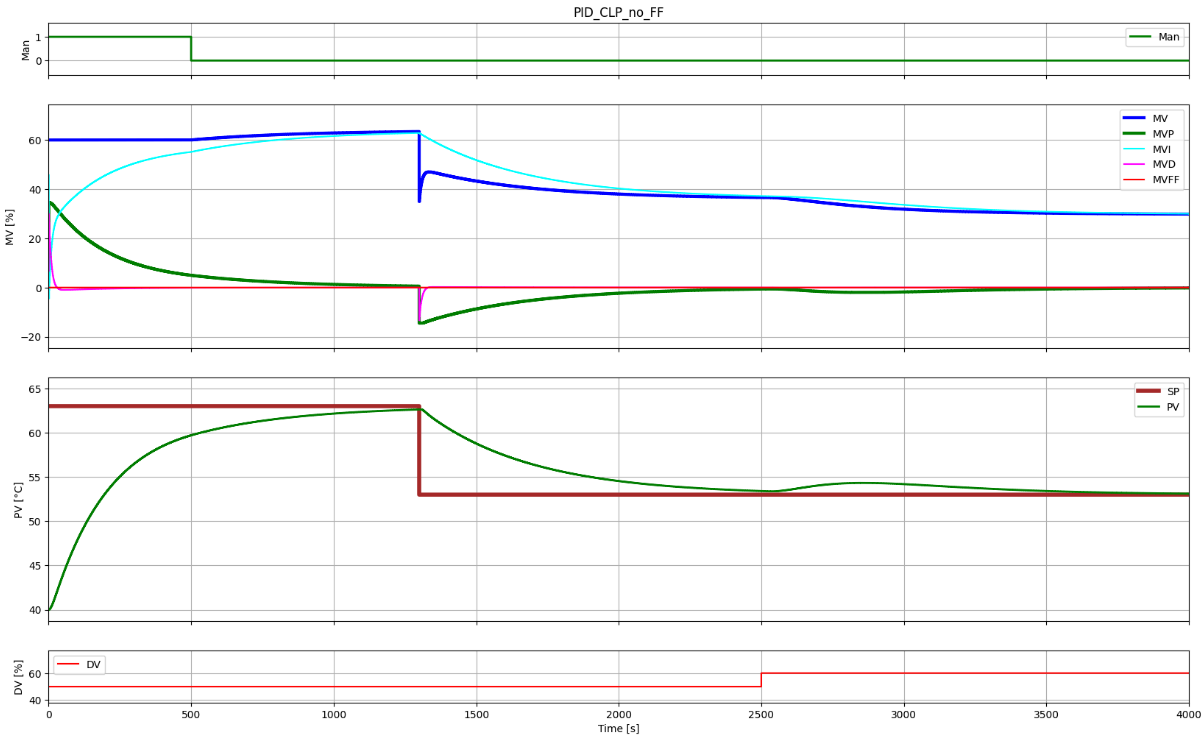


Figure 19: Closed-loop simulation without feedforward.

- We introduce a change in SP since the system is now able to handle it (closed-loop).

- Without feedforward, any change in DV will have an immediate and lasting effect on PV before being clumsily corrected by PID, we see that the proportionnal and integral action try to correct the error caused by the disturbance but are unable to do so in a reasonable amount of time (+800s).

7.3.4 Closed-loop with feedforward

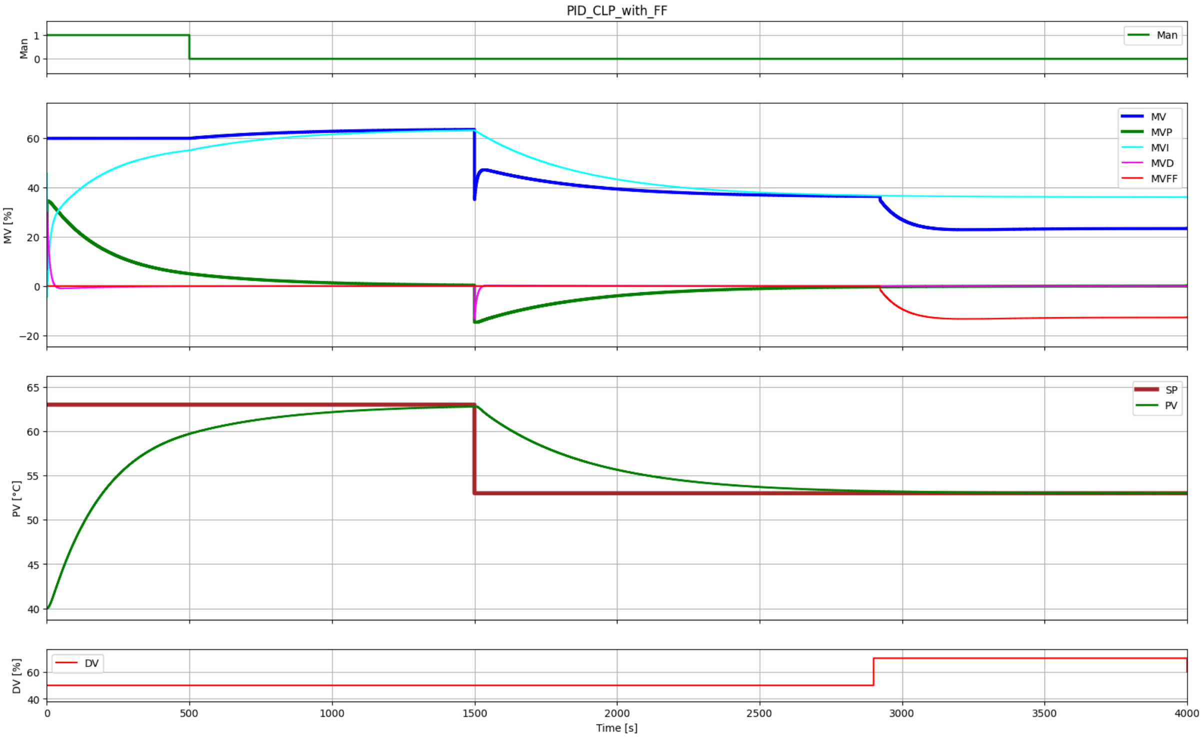


Figure 20: Closed-loop simulation with feedforward.

- Any disturbance (DV) is now completely anticipated and corrected even before it takes effect. The Feedforward is such that PV isn't even slightly bothered by it.
- We even introduced double the disturbance of the previous simulations, the feedforward handled those without any problem.

7.3.5 Conclusions :

Overall, these simulations highlight the clear benefits of using feedforward control in conjunction with closed-loop control. While open-loop with feedforward might work in some scenarios, it requires careful design and consideration of unmodeled dynamics like disturbances. Closed-loop control with feedforward offers a more robust solution for handling disturbances and reducing errors while improving system's overall performance.

8 Experimentation on TCLab

8.1 Open-loop without feedforward

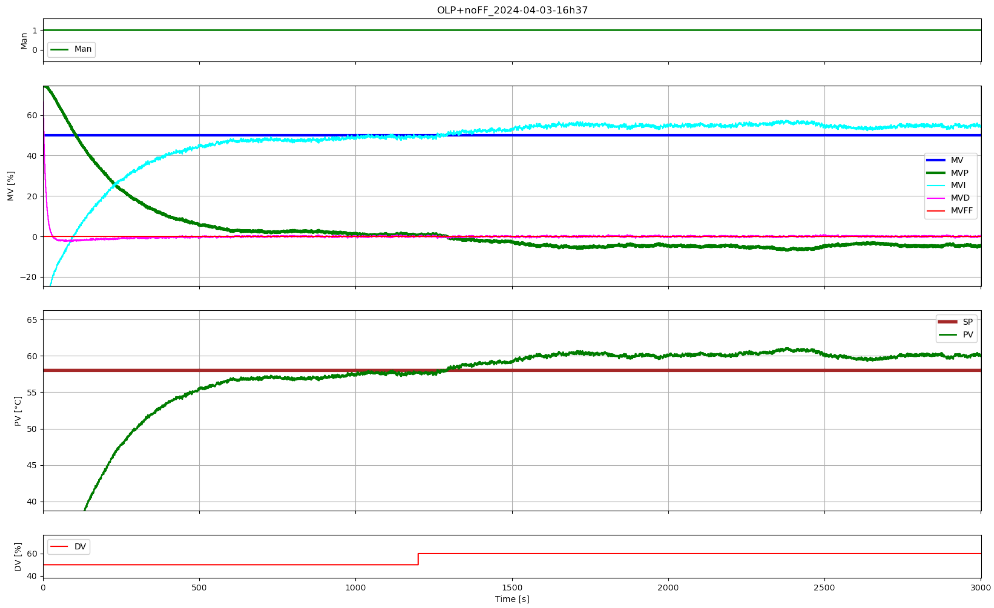


Figure 21: Experiment OLP + noFF.

8.2 Open-loop with feedforward

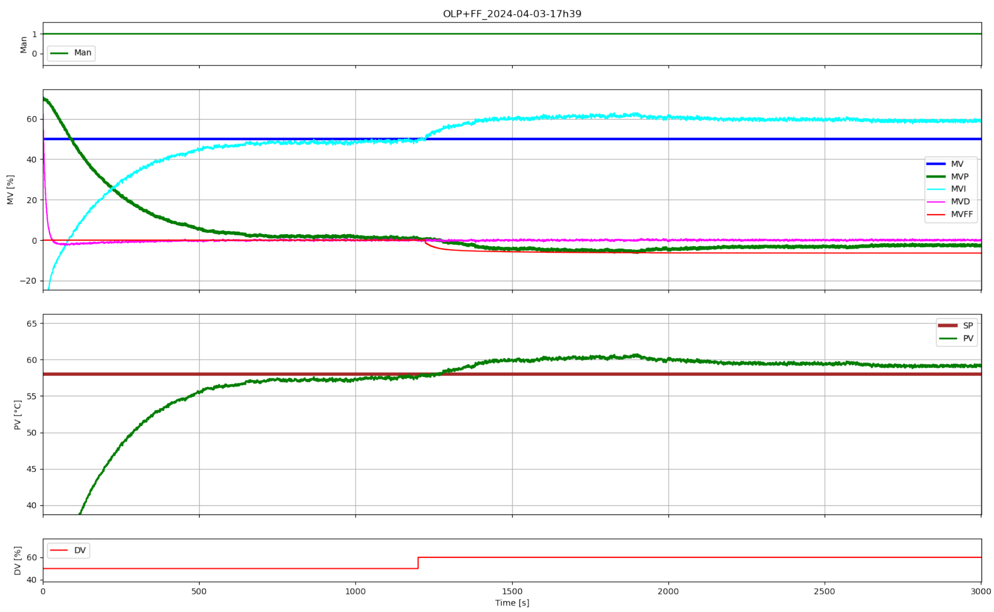


Figure 22: Experiment OLP + FF.

8.3 Closed-loop without feedforward

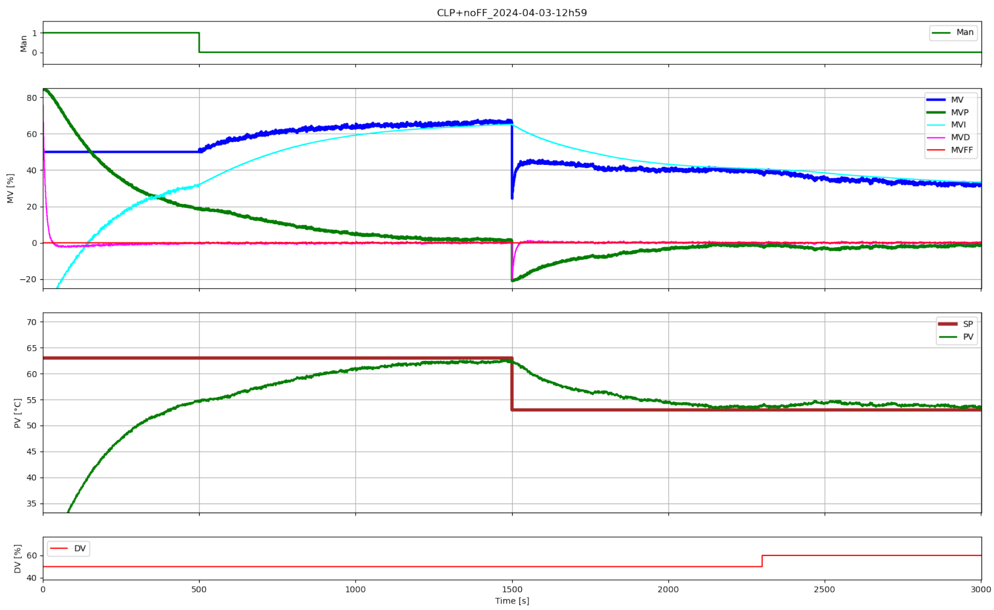


Figure 23: Experiment CLP + noFF.

8.4 Closed-loop with feedforward

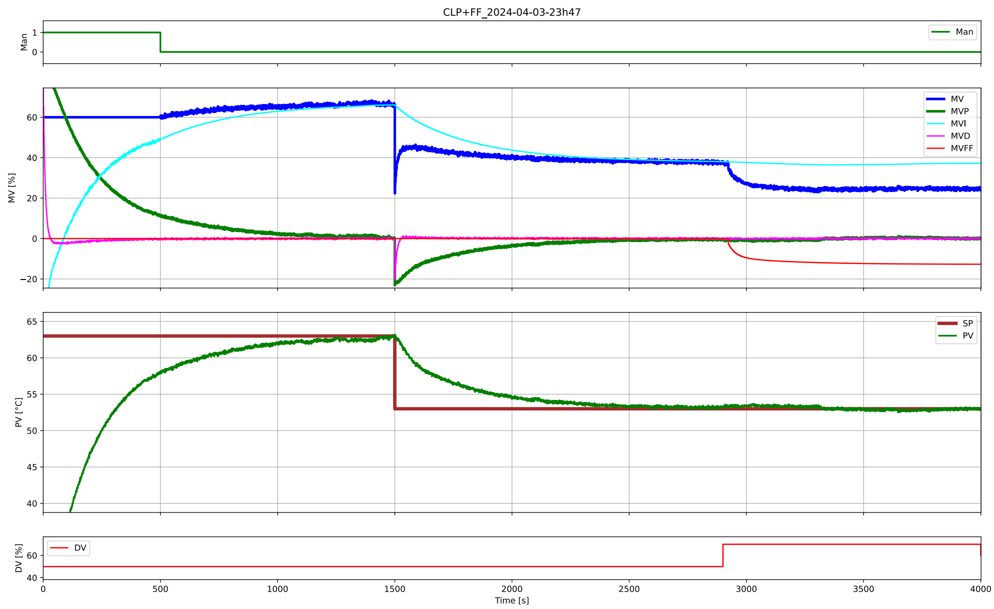


Figure 24: Experiment CLP + FF.

8.5 Conclusions on our experimentations:

Our experimental results strongly support the observations made in the simulations. The Closed-Loop with Feedforward (CLP+FF) model consistently outperformed other models in achieving the desired system response, especially when the model tuning was accurate.

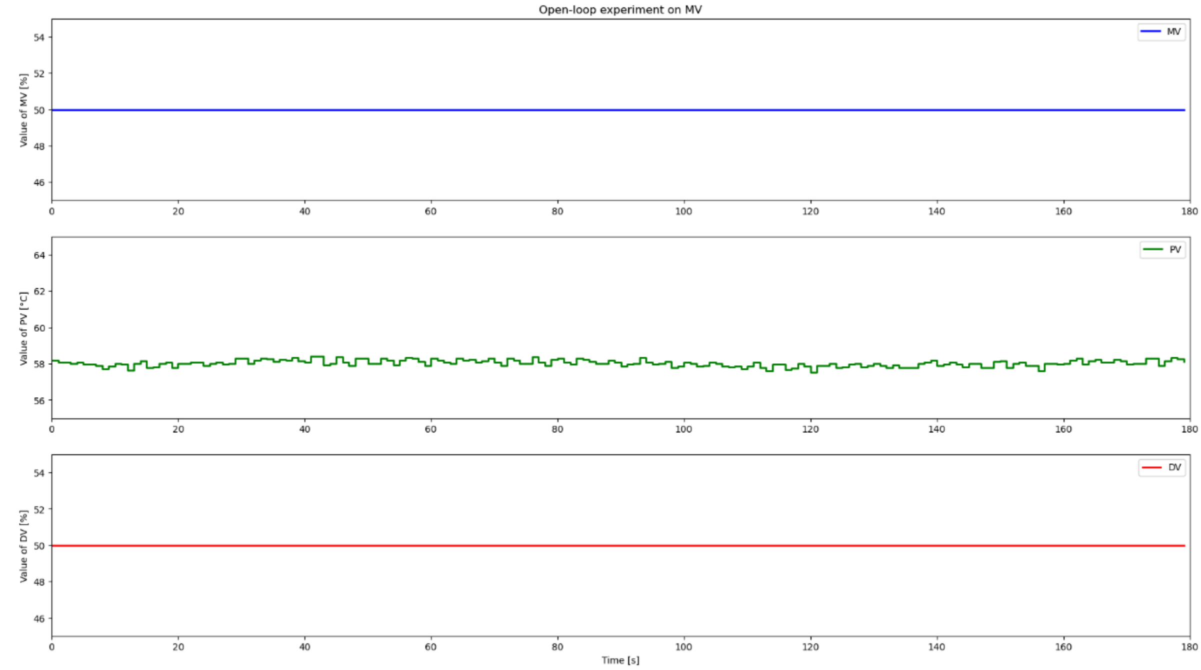
However, there's always room for improvement:

- <u>Numerical Tangent Approximation:</u> Replacing the current tangent approximation with a numerical method for calculating the time constants, gain,… etc can provide more precise values.
- <u>Kc Tuning:</u> The need to significantly reduce the Kc value obtained from the IMCTuning function suggests potential shortcomings in the tuning process. Using an alternative tuning method or improving the IMCTuning function itself could lead to smoother control outputs without compromising our Kc value.

By implementing these improvements, we can achieve even greater accuracy and robustness in the CLP+FF model, further improving its advantage in maintaining system stability and performance in the presence of disturbances.

9 Annexes

9.1 Obtaining PV_0




```
PV [°C]: 58.12, SlopePV [°C/s]: -0.05898161326660353
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Figure 25: Experience en boucle ouverte afin d'obtenir PV_0

9.2 Saturated PID controller with Kc = 4.5

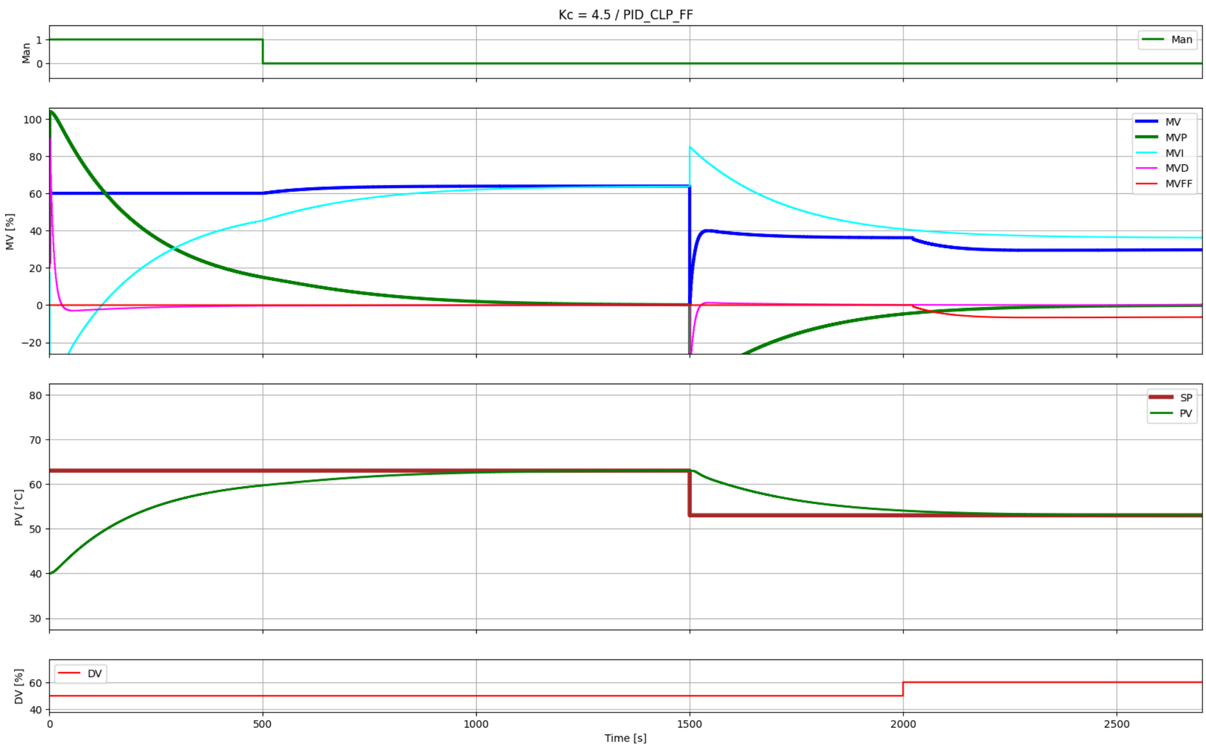

Figure 26: Saturated response with Kc=4.5 being too high. We prefer to use Kc/3.

- The proportionnal action is here too aggressive, we see it overhoots the change is SP and even goes below −40 graduations.
- It has one of the fastest settling time for PV to reach SP. Which might be interesting in some cases.

9.3 Influence of alpha

Some noise cause high-frequency variations that can be misinterpreted as rapid changes in the error, leading to unwanted corrections by DV, that is where α comes into play and act as a low-pass filter.

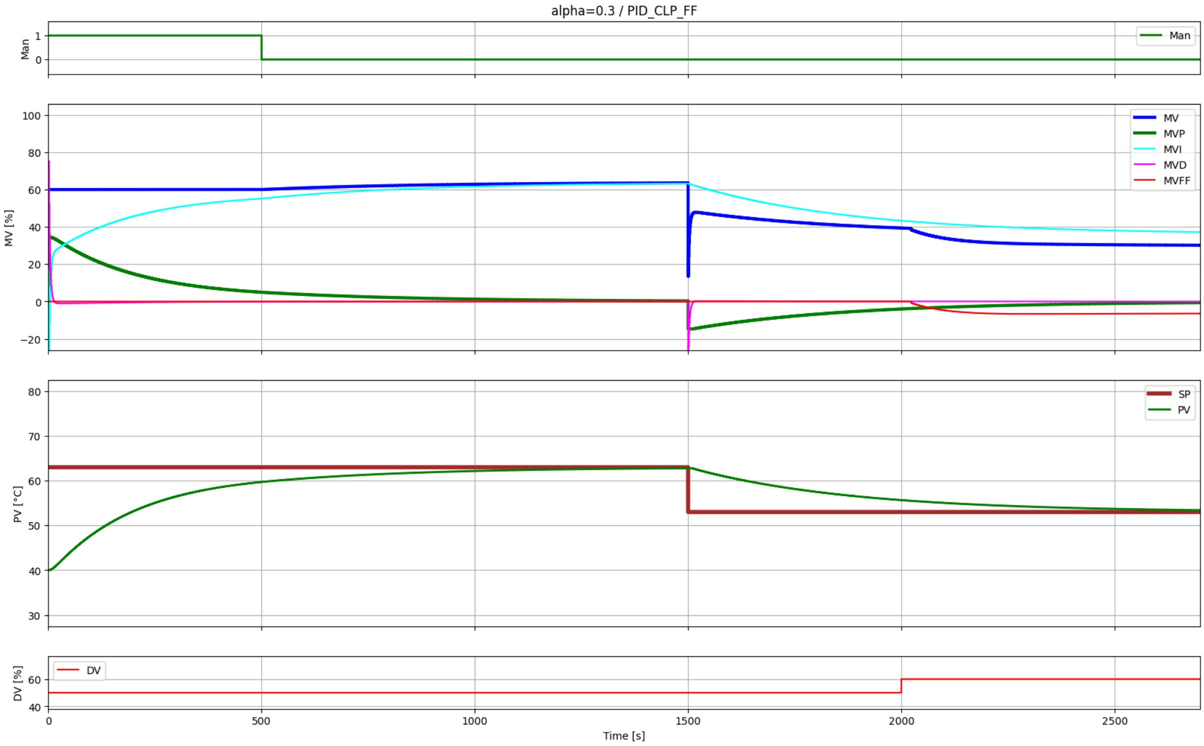

Figure 27: Effects of a very low value of alpha.

Here alpha has a clear effect on MV_D :

- at the beginning of the simulation the derivative actions climbs up way higher,
- we observe the same happenning at t=1500. This choice of alpha=0.3 makes the controller react quite aggressively to any change and noise. For any lower value of alpha, MV will saturate.
- Our best curves resulted in alpha = 1.0, no significant noise being present in the simulation. During experimentations, it would probably be helpful to use alpha=0.8 to help reduce noise.

9.4 Influence of gamma

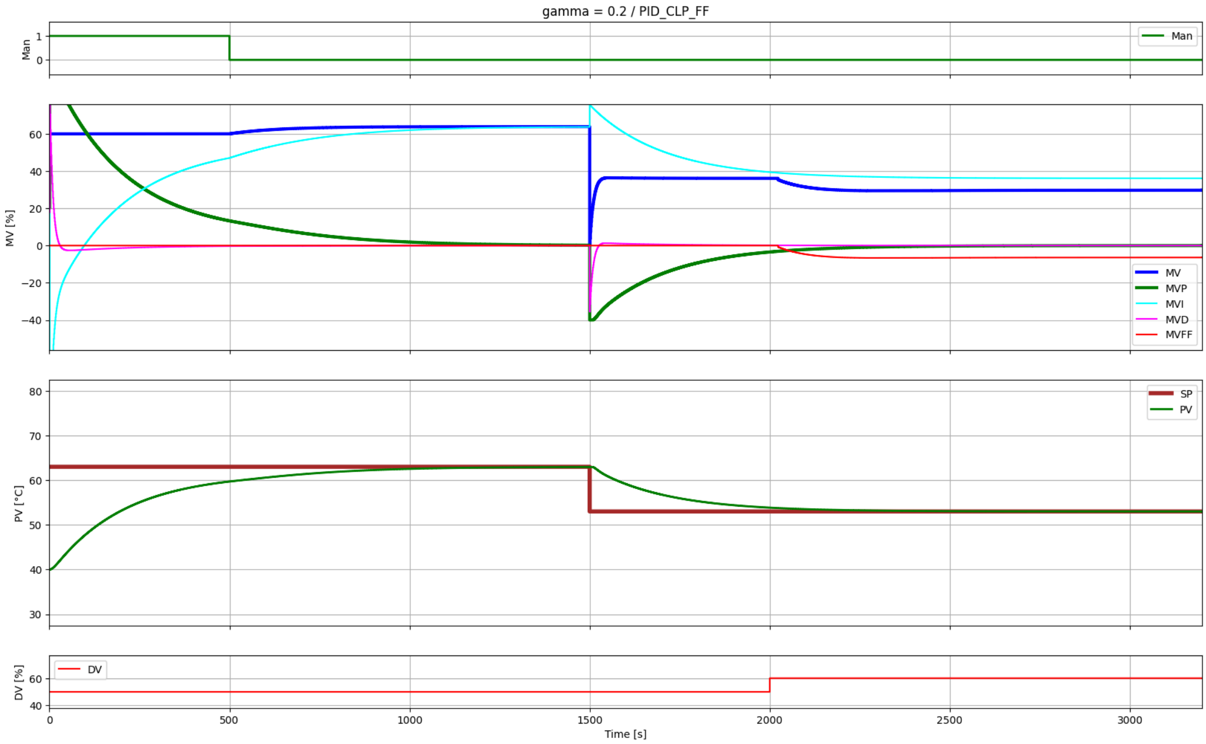

Figure 28: Effects of a very low value of gamma.

- at $\gamma = 0.2$ the response is the most aggressive.
- All three components of the PID controller tend to overshoot and spike-up/down[8].
- PV reaches the SP significantly faster than $\gamma = 0.6$ (at 2400s compared to 2800s).
- It has one of the fastest settling time for PV to reach SP. Which might be interesting in some cases.

[8]refer to chapter 10.1

Figure 29: Effects of a very high value of gamma.

- at $\gamma = 1.0$ the response is the slowest. This means the derivative and proportionnal components overshoot significantly less when subjected to a change in SP.
- this also means that PV reaches SP painfully slow (at least more than 1700s compared to 900s for $\gamma = 0.2$ and 1300s for $\gamma = 0.6$).

10 Sources and references (under APA norms) :

[1] : Wikipedia "Proportional–integral–derivative controller" consulted the 04/03/2024 on : https://en.wikipedia.org/wiki/Proportional%E2%80%93integral%E2%80%93derivative_controller

[2] : Unknown author () "Introduction: PID Controller Design" consulted the 04/03/2024 on : https://ctms.engin.umich.edu/CTMS/index.php?example=Introduction&section=ControlPID

[3] : Tim Wescott, FLIR Systems (2000) "PID-without-a-PhD" consulted the 04/03/2024 on : http://manuals.chudov.com/Servo-Tuning/PID-without-a-PhD.pdf

[4] : Pedro Ney Stroski (2019) "PID controller tuning: Ziegler-Nichols methods" consulted the 04/03/2024 on : https://www.electricalelibrary.com/en/2019/11/06/pid-controller-tuning-ziegler-nichols-methods/

10.1 Questions for Further Investigation :

- Zieghler-Nichols $\frac{T_D}{T_I}$ ratio is recommended to always be inferior to 0.25 (or even 0.16)[9]. But our Closed-loop PID has a ratio of 0.507 and 0.66 for T_1d and T_2d respectively. What impact has this on our model ? Is this ratio even applicable to our studied case ? [4]

- How and why does gamma affect the PID components ? How would our model react to values of $\gamma > 1$?

[9]see Control Theory and applications(v.2024) slide 181 : Adjust PID action.