DAYSTAR UNIVERSITY

SCHOOL OF SCIENCE, ENGINEERING AND HEALTH
DEPARTMENT OF SCIENCE AND ENGINEERING

MATH 312A: LINEAR ALGEBRA

FINAL EXAMINATION

JANUARY TRISEMESTER, 2023

INSTRUCTIONS TO CANDIDATES

i) *Answer question ONE and any other TWO questions*
ii). *Symbols have their usual meaning.*
iii). *Mobile phones are not allowed in the examination hall*
vi). *The maximum possible points that can be earned in this paper is 60.*

QUESTION ONE (COMPULSORY – 24 MARKS)

a. Classify the following equations as either linear or non-linear **2mks**

i. $(\sin\pi)x + y = 2$

ii. $e^{-2}x + 5y = 8$.

b. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -7 & 1 \\ -1 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 & -1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$.

i. Find $(A-B)^T$ and its trace. **2mks**

ii. Find the minor of A_{12} and the cofactor of B_{23}. **2mks**

c. Given $\hat{u} = \langle 1,2,1 \rangle$ *and* $\langle 0,2,-2 \rangle$ find:

i. $\|\hat{u} + \hat{v}\|$ **2mks**

ii. $\hat{u} \times \hat{v}$ **3mks**

d. Use matrix operations to rewrite the matrix in Echelon form: $\begin{bmatrix} 3 & 1 & -1 & 2 \\ 1 & -1 & 2 & 2 \\ 1 & 1 & 1 & 3 \end{bmatrix}$ **4mks**

e. Use Cramer's Rule to solve:
$$\begin{array}{rcrcrcr} 5x & + & 3y & + & z & = & -13 \\ 2x & - & 4y & - & z & = & -8 \\ -3x & + & y & + & 2z & = & 13 \end{array}$$
5mks

f. Determine whether the set $S=\{(1,-5,4),(11,6,-1),(2,3,5)\}$ spans $\mathbb{R}^3$. **4mks**

QUESTION TWO (18MKS)

a. Consider matrix $A=\begin{bmatrix} 3 & 3 \\ 4 & 2 \end{bmatrix}$

i. Find all the Eigen values of A. **4mks**

ii. Find the Eigen vectors corresponding to the **second** Eigen value. **3mks**

b. Consider matrix $A=\begin{bmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{bmatrix}$

i. Find the $Adj(A)$. **5mks**

ii. Use the $Adj(A)$ found above to evaluate A^{-1}.

iii. Use A^{-1} found above to decode the cryptogram below: **6mks**

$-2 \quad 2 \quad 5 \quad 39 \quad -53 \quad -72 -6 \quad -9 \quad 93 \quad 4 -12 \quad 27$

$31 \quad -49 \quad -16 \quad 19 \quad -24 \quad -46 \quad -8 \quad -7 \quad 99$

QUESTION THREE (18MKS)

a. Use elimination method to solve:
$$\begin{array}{rcrcrcr} x & - & y & + & 2z & = & 1 \\ 3x & + & y & - & z & = & 13 \\ 2x & - & 4y & + & 3z & = & 6 \end{array}$$
5mks

b. Determine whether the vectors $v_1=\langle 2,0,1\rangle$, $v_2=\langle 2,-1,1\rangle$, $v_3\langle 4,2,0\rangle$, form a linearly dependent set or a linearly independent set. **6mks**

c. Use Gauss-Jordan to solve:
$$\begin{array}{rcrcrcrcr} x & + & 2y & - & 3z & + & w & = & -21 \\ 2x & - & 3y & + & z & - & 2w & = & 15 \\ x & - & 4y & - & 2z & + & 3w & = & -3 \\ 3x & & & + & 2z & - & w & = & 7 \end{array}$$
7mks

QUESTION FOUR (18MKS)

a. Given $\hat{u}=\langle 4,0,-3,5\rangle$ *and* $\langle 0,2,5,4\rangle$ Evaluate

i. $(\hat{u}\cdot\hat{v})\hat{v}$ **2mks**

ii. $\hat{u}\cdot(5\hat{v})$ **2mks**

b. Find the basis for the row space for the matrix $A=\begin{bmatrix} 1 & 2 \\ -4 & 3 \\ 6 & 1 \end{bmatrix}$. **3mks**

c. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 1 & 2 \\ 1 & 4 & 0 & 3 \\ -2 & 3 & 0 & 2 \\ 1 & 2 & 6 & 1 \end{bmatrix}$

i. Use matrix operations to rewrite the matrix in Row-Reduced Echelon form. **5mks**

ii. Find the null space and the nullity of A. **4mks**

iii. Find the rank of A. **2mks**

QUESTION FIVE (18MKS)

a. Classify the following statements as either true or false: **2mks**

i. The cofactor C_{22} of a given matrix is always a positive number.

ii. The inverse of matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is $A^{-1} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

b. Given the system:
$$\begin{aligned} -7x &+ 2y + z = 7 \\ 2x &- 3y + 5z = -28 \\ 4x & - 3z = 8 \end{aligned}$$

i. Use Basket method to determine $|A|$. **2mks**

ii. Using the $adj(A)$ evaluate A^{-1}. **3mks**

iii. Use the inverse found in (ii) above to solve the system of linear equation above. **3mks**

c. A mixture of 6 gallons of chemical A, 8 gallons of chemical B, 13 gallons of chemical C is required to kill a destructive crop insect. Commercial spray X contains 1, 2 and 2 parts respectively, of these chemicals. Commercial spray Y contains only chemical C. Commercial spray Z contains chemicals A, B, and C in an equal amounts. Form a system of linear equations and find out how much of each type of the commercial spray is needed to get the desired mixture? **8mks**

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QUESTION ONE (COMPULSORY – 24 MARKS)

a. Determine the order of the following matrices: **2mks**

i. $\begin{bmatrix} 6 & 2 & -5 & 8 & 0 \end{bmatrix}$

ii. $[3]$

b. State whether the following statements are true or false: **2mks**

i. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 4 & -2 \\ 1 & 1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$, the operation AB is not possible.

ii. The matrix $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is in reduced row-echelon form.

c. Given that $A=\begin{bmatrix}-1 & 5 & 0\\ -2 & 3 & 4\\ -3 & -8 & -7\\ -3 & 0 & 9\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0 & 1\\ 1 & -1 & 2\\ -2 & 3 & -9\end{bmatrix}$, compute AB^T if it is possible.

3mks

d. Use cofactor expansion across the third column to compute $\det(A)$ given that:

$A=\begin{bmatrix}2 & 7 & 1\\ -2 & 4 & 0\\ 2 & -2 & 0\end{bmatrix}$. 2mks

e. Find the norm of the vector $\hat{u}=\langle 2,-2,3\rangle$ and $\hat{v}=\langle 1,-3,4\rangle$. 2mks

f. Given that $\hat{A}=2\hat{i}-\hat{j}+\hat{k},\ \hat{B}=\hat{i}+\hat{j}+2\hat{k}$, Find the angle between $\hat{A}$ *and* $\hat{B}$. 3mks

g. Given the matrix $A=\begin{bmatrix}3 & 0\\ 8 & -1\end{bmatrix}$, find all the Eigen values and the Eigen vectors corresponding to the **second** Eigen value. 5mks

h. Determine whether the vectors $v_1=\langle 1,-5,4\rangle$, $v_2=\langle 11,6,-1\rangle$, $v_3\langle 2,3,5\rangle$, spans $\mathbb{R}^3$. **5mks**

QUESTION TWO (18MKS)

a. Show whether the vectors $\hat{u}=\langle 11,-2,3\rangle$ *and* $\hat{v}=\langle 3,5,-1\rangle$ are perpendicular. 2mks

b. Given the vectors $\hat{u}=\langle 1,2,-1\rangle$ and $\hat{v}=\langle 6,4,2\rangle$ in R^3, show that $\hat{w}=\langle 4,-1,8\rangle$ is not a linear combination of $\hat{u}$ *and* $\hat{v}$. 4mks

c. Use elimination method to solve the following system of linear equations: 6mks

$$\begin{aligned} 2x - y + 9z &= -8\\ -x - 3y + 4z &= -15\\ 5x + 2y - z &= 17 \end{aligned}$$

d. Use Gauss-Jordan to solve: 6mks

$$\begin{aligned} 3x_1 - x_2 + 5x_3 - 2x_4 &= -44\\ x_1 + 6x_2 + 4x_3 - x_4 &= 1\\ 5x_1 - x_2 + x_3 + 3x_4 &= -15\\ 4x_2 - x_3 - 8x_4 &= 58 \end{aligned}$$

QUESTION THREE (18MKS)

a. Given $\hat{u} = \langle 1, -2, 0 \rangle$ and $v = \langle 3, -2, -1 \rangle$, find $(2\hat{u} - \hat{v}) \times (\hat{u} + 3\hat{v})$ **6mks**

b. Show whether the following sets of vectors in R^3 $\hat{v}_1 = \langle -3, 0, 4 \rangle$, $\hat{v}_2 = \langle 5, -1, 2 \rangle$ *and* $\hat{v}_3 = \langle 1, 1, 3 \rangle$ are linearly dependent or linearly independent. **5mks**

a. Given the system:
$$\begin{array}{rcrcrcr} -x & + & y & + & 2z & = & 1 \\ 2x & + & 3y & + & z & = & -2 \\ 5x & + & 4y & + & 2z & = & 4 \end{array}$$

i. Use Basket method to determine $|A|$. **2mks**

ii. Using the $adj(A)$ evaluate A^{-1}. **3mks**

iii. Use the inverse found in (ii) above to solve the system of linear equation above. **2mks**

QUESTION FOUR (18MKS)

a. Use Cramer's Rule to solve the following systems if **possible**: **7mks**

i.
$$\begin{array}{rcrcr} x & + & y & = & 3 \\ 2x & + & 2y & = & 6 \end{array}$$

ii.
$$\begin{array}{rcrcrcr} -2x & + & 3y & - & 5z & = & -11 \\ 4x & - & y & + & z & = & -3 \\ -x & - & 4y & + & 6z & = & 15 \end{array}$$

b. Given that : $A = \begin{bmatrix} -5 & 4 & -3 \\ 10 & -7 & 6 \\ 8 & -6 & 5 \end{bmatrix}$

i. Use Gauss-Jordan to find A^{-1} **5mks**

ii. Use the A^{-1} evaluated above to decode the cryptogram below: **6mks**

$$\begin{array}{cccccccc} -5 & 11 & -2 & 370 & -265 & 225 & -57 & 48 \\ -33 & 32 & -15 & 20 & 245 & -171 & 147 & \end{array}$$

QUESTION FIVE (18MKS)

a. Find the basis for the null space of the matrix: $A=\begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$. **8mks**

b. Given that matrix $A=\begin{bmatrix} 2 & 5 & -3 & -4 & 8 \\ 4 & 7 & -4 & -3 & 9 \\ 6 & 9 & -5 & 2 & 4 \\ 0 & -9 & 6 & 5 & -6 \end{bmatrix}$. Find the rank (A) and the nullity of (A) and verify that the $rank(A)+nullity(A)=n$ (where n is the number of columns of A of the matrix: **10mks**