Robotics Homework #1
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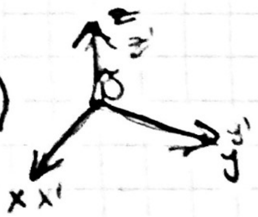
1) 

1) rotation of $\pi/4$ about y axis (fixed)

$\Rightarrow {}^{A}_{1}R = R_y(\pi/4)$

2) rotation of $\pi/2$ about x axis (fixed)

$\Rightarrow {}^{A}_{2}R = R_x(\pi/2) \cdot {}^{A}_{1}R$ ← premultiply

3) rotation of $\pi/6$ about z axis (moving)

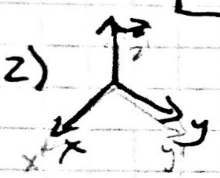
$\Rightarrow {}^{A}_{3}R = R_x(\pi/2) \cdot R_y(\pi/4) \cdot R_z(\pi/6)$ ← postmultiply

4) rotation of $\pi/3$ about x axis (fixed)

$\Rightarrow {}^{A}_{4}R = R_x(\pi/3) \cdot R_x(\pi/2) \cdot R_y(\pi/4) \cdot R_z(\pi/6)$

5) rotation of $\pi/3$ about y axis (moving)

$$\boxed{{}^{A}_{B}R = {}^{A}_{5}R = R_x(\pi/3)\, R_x(\pi/2)\, R_y(\pi/4)\, R_z(\pi/6)\, R_y(\pi/3)}$$

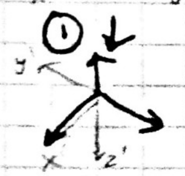
2) 

1) rotation of $\pi/2$ about x axis

${}^{A}_{1}R = R_x(\pi/2)$

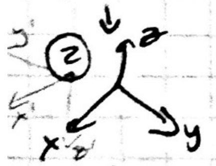
2) translate 3 units about y axis

${}^{A}_{2}T = {}^{A}_{1}R + [0,3,0]^T = R_x(\pi/2) + \begin{bmatrix}0\\3\\0\end{bmatrix}$



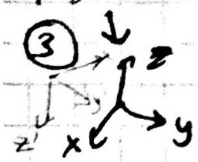
3) rotation of $\pi/2$ about z axis (fixed frame)

${}^{A}_{B}T = {}^{A}_{3}T = R_z(\pi/2)\, R_x(\pi/2) + \begin{bmatrix}0\\3\\0\end{bmatrix}$



$${}^{A}_{B}T = \begin{bmatrix} \left[R_z(\pi/2)\cdot R_x(\pi/2)\right] & \begin{matrix}0\\3\\0\end{matrix} \\ 0 \quad 0 \quad 0 & 1 \end{bmatrix}$$

$$R_x(\pi/2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\pi/2 & -\sin\pi/2 \\ 0 & \sin\pi/2 & \cos\pi/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$



$$R_z(\pi/2) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ \ldots \end{bmatrix}$$

$$\boxed{{}^{A}_{B}T = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$${}^{B}_{A}T = \boxed{{}^{A}_{B}T^T = \begin{bmatrix} 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

3) $R = \begin{bmatrix} 2(a^2+b^2)-1, & 2(bc-ad), & 2(bd+ac) \\ 2(bc+ad), & 2(a^2+c^2)-1, & 2(cd-ab) \\ 2(bd-ac), & 2(cd+ab), & 2(a^2+d^2)-1 \end{bmatrix}$, ~~prove~~ prove R is rotation matrix

to be a rotation matrix,

1) columnes have length 1.

2) columnes are mutually orthogonal (dot product = 0).

3) determinate is 1.

To prove ①: column #1 : $\sqrt{(2(a^2+b^2)-1)^2 + (2(bc+ad))^2 + (2(bd-ac))^2}$

given $\sqrt{a^2+b^2+c^2+d^2} = 1$

unity means $\sqrt{a^2+b^2+c^2+d^2} = 1$

$x = \sqrt{(2(a^2+b^2)-1)^2 + (2(bc+ad))^2 + (2(bd-ac))^2}$

(wolfram alpha

→ $\boxed{x = 1}$ is only soltion to equation. ✓

column #2: $\sqrt{(2(bc-ad))^2 + (2(a^2+c^2)-1)^2 + (2(cd+ab))^2}$

given $\sqrt{a^2+b^2+c^2+d^2} = 1$

$x = \sqrt{(2(bc-ad))^2 + (2(a^2+c^2)-1)^2 + (2(cd+ab))^2}$

$\boxed{x = 1}$ is only soltion ✓

column #3: $\sqrt{(2(bd+ac))^2 + (2(cd-ab))^2 + (2(a^2+d^2)-1)^2}$

given $\sqrt{a^2+b^2+c^2+d^2} = 1$

$x = \sqrt{(2(bd+ac))^2 + (2(cd-ab))^2 + (2(a^2+d^2)-1)^2}$

$\boxed{x = 1}$ is only solution ✓

∴ ① is true

to prove ②: 1·2 should be = 0

2.3 should be = 0

3·1 should be = 0

1 · 2 : $\{2(a^2+b^2)-1, 2(bc+ad), 2(bd-ac)\}$ dot $\{2(bc-ad), 2(a^2+c^2)-1, 2(cd+ab)\}$

$\Rightarrow 4bc(a^2+b^2+c^2+d^2-1) = x$, given $\sqrt{a^2+b^2+c^2+d^2} = 1$

$\boxed{x = 0}$ is only solution (wolfram Alpha). ✓

2·3 : $\{2(bc-ad), 2(a^2+c^2)-1, 2(cd+ab)\} \cdot \{2(bd+ac), 2(cd-ab), 2(a^2+d^2)-1\}$

$\Rightarrow 4a^2cd + 4b^2cd + c(4c^2d + d(4d^2-4)) = x$, $\sqrt{a^2+b^2+c^2+d^2} = 1$

$\boxed{x = 0}$ is only solution ✓

$3 \cdot 1 : \{2(bd+ac), 2(cd-ab), 2(a^2+d^2)-1\} \cdot \{2(a^2+b^2)-1, 2(bc+ad), 2(bd+ac)\}$

$= d(4a^2b+b(4b^2-4)+4bd^2)+4bc^2d = x, \sqrt{a^2+b^2+c^2+d^2}=1$

$\therefore$ $\boxed{x=0}$ is only solution

(2) holds true

to prove #3: determinant$(Q)=1 = \begin{vmatrix} 2(a^2+b^2)-1, & 2(bc+ad), & 2(bd-ac) \\ 2(bc-ad), & 2(a^2+c^2)-1, & 2(cd+ab) \\ 2(bd+ac), & 2(cd-ab), & 2(a^2+d^2)-1 \end{vmatrix}$

via wolfram: $(2a^2+2b^2+2c^2+2d^2-1)(4a^2+4a^2b^2+4a^2c^2+4a^2d^2 - 4a^2+1)$

$\Downarrow$

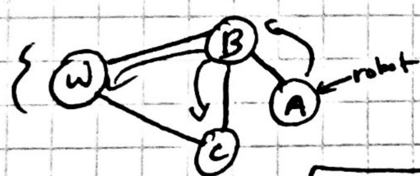
$= (2(1)-1)(4a^2+4a^2b^2+4a^2c^2+4a^2d^2-4a^2+1)$

$= (4a^2+4a^2b^2+4a^2c^2+4a^2d^2-4a^2+1)=x$

given $\sqrt{a^2+b^2+c^2+d^2}=1$

x is only $=1$, $\boxed{x=1}$ is only solution

$\boxed{\therefore 1,2,3 \text{ are true, } R \text{ is a rotation matrix}}$

4) 

transformation tree

1) ${}^{B}p = \boxed{{}^{B}_{A}T\,{}^{A}p}$

2) ${}^{C}p = \boxed{{}^{C}_{B}T\,{}^{B}_{A}T\,{}^{A}p \quad \text{where } {}^{C}_{B}T = {}^{B}_{C}T^{T}}$

3) ${}^{W}p = \boxed{{}^{W}_{B}T\,{}^{B}_{A}T\,{}^{A}p}$

5) $p = 1+2i-3k$

$q = 5+4j+2k$

Find product pq: $(1+2i-3k)(5+4j+2k)$

$5+4j+2k+10i+8ij+4ik-15k-12kj-6k^2$

$5+\cancel{4j}+2k+10i+8k-\cancel{4j}-15k+12i+6$

$11+22i+10k-15k$

$pq = \boxed{11+22i-5k}$

$|p|\cdot|q| = |pq| = \sqrt{11^2+22^2+(-5)^2+0^2} = \sqrt{630} = \boxed{3\sqrt{70} = 25.0998}$

$= 25.0998$