# Homework #3

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CSE 180

1) $\Omega = \{1,2,3,4,5,6\}$, $A = \{\emptyset, \Omega, \{1,2\}, \{3,4,5,6\}\}$,
$P(\emptyset) = 0$, $P(\Omega) = 1$, $P(\{1,2\}) = .1$, $P(\{3,4,5,6\}) = 0.9$

Yes, $(\Omega, A, P)$ is a valid probability space

- for each event, $P(\text{event}) \geq 0$ (Given)
- $P(\Omega) = 1$
- $\{1,2\} \not= \{3,4,5,6\}$ is a valid partition because $P(\{1,2\}) + P(\{3,4,5,6\}) = 1$
- $\{\emptyset\} \not= \{\Omega\}$ is a valid partition because $P(\emptyset) + P(\Omega) = 1$.

2) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ $A = \{\emptyset, \Omega,$

$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12,$
black
$\{2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22,$
$26, 28, 30, 32, 34, 36\}$

- $P(\%3==0 \mid \text{black})$

- $P(\%3==0 \,||\, \%4==0 \mid \text{Range}(1,1\ldots$

$$\frac{P(\%3==0 \cap \text{black})}{P(\text{black})} = \frac{P(\{6,12,18,24,30,36\})}{18/37}$$

$$= \frac{6/37}{18/37} = \frac{6}{18} = \boxed{1/3}$$

$$P(\%3==0 \,||\, \%4==0 \mid \text{Range}(1,12)) = \frac{P(\{3,4,6,8,9,12\})}{P(\text{Range}(1,12))} = \frac{6/37}{12/37} = \boxed{\frac{1}{2}}$$

3) Bayes Rules

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$A$ = robot at door
$\bar{A}$ = robot not at door
$B$ = Sensor at door
$\bar{B}$ = Sensor not at door

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$P(\bar{B}|A) = 0.1$
$P(B|\bar{A}) = 0.2$

$P(A) = .5, P(\bar{A}) = .5$
(assume equal prob)

$$P(B) = P(B|A)P(A) + P(B|\bar{A})P(\bar{A})$$

$$P(B|A) = 1 - P(\bar{B}|A) = 0.9$$

$$P(B) = (0.9)(0.5) + (0.2)(0.5) = 0.55$$
$$0.45 \quad + \quad 0.10$$

All together:

$$P(A|B) = \frac{(0.9)(0.5)}{(0.55)} = 0.\overline{81}$$

$$\boxed{P(A|B) = .818}$$