Robotics Homework #4

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CSE 180

1) Sequence $\{R, R, B\}$, Likelihoods of $X = X_R$, $X = X_B$, $X = X_N$

$$P(x_i \mid z_1, z_2, z_3) = \left(\frac{P(z_1 \mid x_i)}{P(z_1)} \frac{P(z_2 \mid x_i)}{P(z_2)} \frac{P(z_3 \mid x_i)}{P(z_3)}\right) P(x_i)$$

$$P(X_R \mid R, R, B) = \eta P(R \mid X_R)^2 \cdot P(B \mid X_R) P(X_R)$$

$$P(X_B \mid R, R, B) = \eta P(R \mid X_B)^2 \cdot P(B \mid X_B) P(X_B)$$

$$P(X_N \mid R, R, B) = \eta P(R \mid X_N)^2 \cdot P(B \mid X_N) P(X_N)$$

$$= \left(\frac{P(R \mid X_R)}{P(R)}\right)^2 \cdot \left(\frac{P(B \mid X_R)}{P(B)}\right) P(X_R) = \eta (.8)^2 (.05) \cdot \frac{1}{3}$$

$$= .01056\,\eta$$

$$= \eta P(R \mid X_B)^2 \cdot P(B \mid X_B) \cdot P(X_B) = \eta (.2)^2 \cdot .6 \cdot 1/3$$

$$= .00792\,\eta$$

$$= \eta P(R \mid X_N)^2 \cdot P(B \mid X_N) P(X_N) = \eta (.2)^2 \cdot .1 \cdot .33$$

$$= .00132\,\eta$$

Normalizing:

$$.01056\,\eta + .00792\,\eta + .00132\,\eta = 1 \Rightarrow \eta = 50.505$$

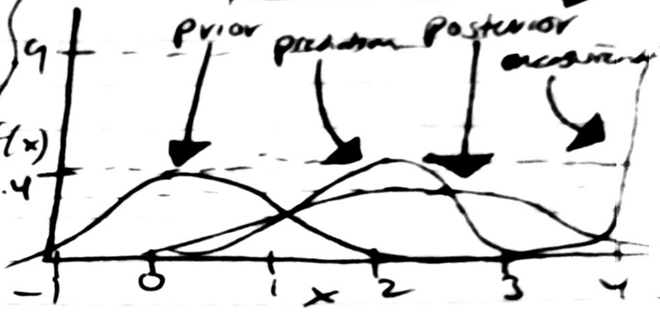
$$\boxed{\begin{aligned} P(X_R \mid R,R,B) &= 53.33\,\% \\ P(X_B \mid R,R,B) &= 40.00\,\% \\ P(X_N \mid R,R,B) &= 6.67\,\% \end{aligned}}$$

2) Unidimensional Kalman Filter

$$x_t = \overset{(A)}{x_{t-1}} + \overset{(B)}{2} u_t \qquad u = \text{input}$$
$$z_t = \overset{(H)}{2} x_t \qquad z = \text{observation}$$

Givens: $A = 1,\ B = 2,\ H = 2,\ X_0 \sim N(0,1),\ R = N(0,1),\ Q = N(0,$
$u_t = 2,\ z_t = 5$

Result $\boxed{\begin{aligned} \mu_t &= 2.5 \\ \Sigma_t &= 1.64 \end{aligned}}$



↑ DIAGRAM ↑

μ_{t-1} = mean of x_{t-1}
Σ_{t-1} = Covariance of x_{t-1}

$$\begin{aligned} x_{t-1} &= -2u_t + x_t \\ &= -2(2) + (\tfrac{1}{2} z_t) \\ &= -4 + 2.5 \\ &= -1.5 \end{aligned}$$

$\mu_{t-1} = -1.5 \qquad u_t = 2$
$\Sigma_{t-1} = 1 \qquad z_t = 5$

PREDICTION:

1. $$\begin{aligned} \bar{\mu}_t &= A_t \mu_{t-1} + B_t u_t \\ &= (1)(-1.5) + (2)(2) \\ &= 2.5 \end{aligned}$$
2. $$\begin{aligned} \bar{\Sigma}_t &= A_t \Sigma_{t-1} A_t^T + R_t \\ &= (1)(1)(1) + N(0,1) \\ &= 2 \end{aligned}$$

UPDATE:

1. $$\begin{aligned} K_t &= H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1} \\ &= 2(2 + .2)^{-1} \\ &= .91 \end{aligned}$$
2. $$\begin{aligned} \mu_t &= \bar{\mu}_t + K_t (z_t - H_t \bar{\mu}_t) \\ &= 2.5 + \cancel{.91(5 - 2 \cdot 2.5)} \rightarrow 0 \end{aligned}$$
3. $$\begin{aligned} \Sigma_t &= \bar{\Sigma}_t \cdot (1 - .91 \cdot 2) \\ &= 1.64 \end{aligned}$$