1 Remarks on the implementation

The representative sets for S_k on $[k]^t$ and $S_d^k \wr S_k$ on $([d] \times [k])^t$ we use in our paper differs slightly from the objects used in the implementation. Here we indicate the differences. In both cases, there are straightforward bijections. All notation is as in the paper, unless indicated otherwise. The section numbers are as in the most recent arXiv version (v4).

1.1 Remarks regarding the representative elements for the action of S_k on $[k]^t$ from Section 4.2

Remark 1.1. *To slightly simplify the definition of the vectors $u_{\tau,P}$, one can define the map $\psi_P : \mathbb{C}\widetilde{T}_{\lambda,\mu_r} \to V_P$ which maps a tableau $\tau \in \widetilde{T}_{\lambda,\mu_r}$ to the word $w \in P$ with $w(i) = \tau^{-1}(i+1)$ (for $i = 1,\dots,r$), where $\tau^{-1}(i+1)$ denotes the index of the box containing $i+1$. This is ϕ_P^{-1} but then interpreted as a map from $\mathbb{C}\tilde{T}_{\lambda,\mu_r} \to V_P$ instead of $M^{\mu_r} \to V_P$. Also, we let C_λ denote the column stabilizer of the shape λ, which consists of all permutations of $[k]$ that leave the columns of λ fixed. The group C_λ acts on a tableau τ by setting $c \cdot \tau(i) = \tau(c(i))$ for all $i \in [k]$. Then*

$$u_{\tau,P} = \sum_{\tau' \sim \tau} \sum_{c \in C_\lambda} \operatorname{sgn}(c)\psi_P(c \cdot \tau').$$

This formulation of the vectors $u_{\tau,P}$ is used in our implementation.

1.2 Remark regarding the representative elements for the action of $S_d \wr S_k$ on $([d] \times [k])^t$ from Section 6.3

Remark 1.2. *We mention a bijection (used in the code) between the set of pairs $(\boldsymbol{j},\boldsymbol{\tau})$ with $|\boldsymbol{j}^{-1}(a)| = |\Lambda^a|$ for $a \geq 2$, $\boldsymbol{\tau} \in \boldsymbol{T}^k_{\underline{\Lambda},\underline{\gamma}}$ and a set of tuples of semistandard tableaux that we define now. Let $\boldsymbol{T}^k_{\underline{\Lambda},r}$ be the set of tuples of semistandard young tableaux $\boldsymbol{\tau'} = (\tau'_1,\dots,\tau'_\ell)$ so that each τ'_a is of shape Λ^a and such that τ'_1 contains $k-r$ times symbol 1 and all tableaux in the tuple $\boldsymbol{\tau'}$ together contain 1 time symbols $2,\dots,r+1$ each. The bijection assigns to a pair $(\boldsymbol{j},\boldsymbol{\tau})$ the element $\boldsymbol{\tau'} \in \boldsymbol{T}^k_{\underline{\Lambda},r}$ for which τ'_1 is obtained from τ_1 by replacing each symbol $i+1$ by the ith smallest element of $\boldsymbol{j}^{-1}(1)$ plus 1. For $a \geq 2$, τ'_a is obtained from τ_a by replacing each symbol i by the ith smallest element of $\boldsymbol{j}^{-1}(a)$ plus 1.*

Remark 1.3. *Let us write, using the function ψ_P from Remark 1.1 and $\boldsymbol{\tau} \in \boldsymbol{T}^k_{\underline{\Lambda},r}$, $v_{\sigma_i,Q_i} := \psi_{Q_i}\left(\sum_{\sigma'_i \sim \sigma_i} \sum_{c_i \in C_{\nu_{j(i)}}} \operatorname{sgn}(c_i)(c_i \cdot \sigma'_i)\right)$ and $u_{\tau,P} := \psi_P\left(\bigotimes_{a=1}^{\ell} \sum_{\tau'_a \sim \tau_a} \sum_{d_a \in C_{\Lambda^a}} \operatorname{sgn}(d_a)(d_a \cdot \tau'_a)\right)$. Then $u_{(\boldsymbol{\sigma},\boldsymbol{\tau}),(P,\boldsymbol{Q})} = \operatorname{Perm}_P\left(\bigotimes_{i=1}^{r} v_{\sigma_i,Q_i}\right) \otimes u_{\tau,P}$, which is the description of the representative vectors we use in our implementation.*