

#2.

≥ 6 Letters 10 integers.

A password of 6 character is randomly selected.

Find the probability that it consists only of letters or exactly 1 case 'm'.

A = consists only of letter

B = exactly 1 'm'.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\#A = 26^6$$

$$\square \overset{26}{\square} \square \overset{26}{\square} \square \overset{26}{\square} \square \overset{26}{\square}$$

$$\#B = 6 \cdot 35^5$$

$$\begin{matrix} \overset{35}{\square} \\ m \end{matrix} / \overset{35}{\square} / \overset{35}{\square} / \overset{35}{\square} / \overset{35}{\square} /$$

$$\# A \cap B = 6 \cdot 25^5$$

$$\begin{matrix} \overset{25}{\square} & \overset{25}{\square} & \overset{25}{\square} & \overset{25}{\square} & \overset{25}{\square} \\ / & \overset{25}{\square} & / \overset{25}{\square} & / \overset{25}{\square} & / \overset{25}{\square} \end{matrix} \\ m.$$

$$\# \text{ total} = 36^6$$

$$P(A \cup B) = \frac{\#A + \#B - \#A \cap B}{\# \text{ total}}$$

$$= 0.2598$$

$$\left. \begin{array}{ccccccc} m & \square & \square & \square & \square & \square & \square \\ \square & m & \square & \square & \square & \square & \square \\ \square & \square & m & \square & \square & \square & \square \\ \square & \square & \square & m & \square & \square & \square \\ \square & \square & \square & \square & m & \square & \square \\ \square & \square & \square & \square & \square & m & \square \end{array} \right\} \quad \downarrow$$

#11. Computer chips 7% chance of defect.

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Suppose we continue to test until we find 10 that have no defects.

X = # of defective chips that when we find 10th good chip.

Find $P(X=5)$

◻ ◻ ... ◻ ◻

Negative Binomial

$$n = X+10$$

$$r = 10$$

$$p = 0.93$$

$$P(X=x) = \binom{x+10-1}{q} 0.93^{10} 0.07^{(X+10)-10} \underset{\sim p.m.f.}{\sim}$$

$$x=5$$

$$= 0.0016285$$

#13. 85 conforming items
25 non-conforming items.

A sample of 10 is chosen.

Find the mean of non-conforming: - - -

Hypergeometric

$$' , k \backslash (N-k) '$$

Hypergeometric

$$N = 85 + 25 = 110$$

$$k = 25$$

$$n = 10$$

$$E(x) = \mu = n \cdot \frac{k}{N} = 10 \cdot \frac{25}{110} = \frac{25}{11} =$$

12. $P = 0.01$ cracking an egg.

Find, on average, how many cartons (containing one dozen eggs) would you have to inspect in order to find 3 cartons with no cracked eggs.

$p' = (1 - 0.01)^{12}$ # no cracked eggs for a carton.

$$r = 3.$$

Negative binomial.

$$E(x) = \mu = \frac{r}{p'} = \frac{3}{(1-0.01)^{12}} = 3.3845.$$

Chapter 5:

Two or More Random Variables

Example : 2 dice

X is fair dice.

X is fair dice.

Chapter 5: Y is unfair dice

$$P(Y=y) = \begin{cases} \frac{1}{6} & . 1 \\ \frac{1}{6} & . 2 \\ \frac{1}{6} & . 3 \\ \frac{1}{5} & . 4 \\ \frac{1}{5} & . 5 \\ \frac{1}{10} & . 6. \end{cases}$$

$X \& Y$ independent

$$P(X=x, Y=y) = \begin{cases} \frac{1}{6} \cdot \frac{1}{6} & x=1, y=1 \\ \frac{1}{6} \cdot \frac{1}{6} & x=1, y=2 \\ \vdots & \vdots \\ \frac{1}{6} \cdot \frac{1}{10} & x=6, y=6 \end{cases}$$

36 cases

[Joint] P.M.F of X, Y

The joint probability mass function:

X and Y Discrete R.V.

Then $f_{XY}(x,y) = P(X=x, Y=y)$

(1) $f_{XY}(x,y) \geq 0$

(2) $\sum_x \sum_y f_{XY}(x,y) = 1$

The joint probability density function:

X and Y Continuous R.V..

Then $f_{XY}(x, y)$.

(1) $f_{XY}(x, y) \geq 0$

(2) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{XY}(x, y) dx dy = 1$

(3) For any region B for two dimensional

$$P(x, y \in B) = \iint_B f_{XY}(x, y) dx dy$$

Example: X, Y are Continuous:

Joint p.d.f:

$$f(x, y) = Cx^2 + \frac{xy}{3} \quad 0 \leq x \leq 1, 0 \leq y \leq 2$$

(a) Find (C) .

$$\iint_{-\infty}^{\infty} f(x, y) dx dy = 1$$

$$\int_0^1 \int_0^2 Cx^2 + \frac{xy}{3} dy dx = 1$$

$$\int_0^1 \left(Cx^2 y + \frac{1}{3} \cdot \frac{1}{2} y^2 \right) \Big|_0^2 dx = 1$$

$$\int_0^1 2Cx^2 + \frac{2}{3}x dx = 1.$$

$$\left[C \cdot \frac{2}{3}x^3 + \frac{1}{3}x^2 \right] \Big|_0^1 = \frac{2}{3}C + \frac{1}{3} = 1$$

$$\therefore C = 1$$

$$\therefore C = 1$$

(b) Find $P(X+Y \geq 1)$.

$$P(Y \geq 1-x) = P(X+Y \geq 1)$$

$$X \in [0, 1] \quad 1-x \in [0, 1]$$

$$Y \in [0, 2] \quad Y \geq 1-x$$

$$\Rightarrow Y \in [1-x, 2]$$



$$P(X+Y \geq 1)$$

$$= P(Y \geq 1-x)$$

$$= \int_0^1 \int_{1-x}^2 \left(x^2 + \frac{xy}{3} \right) dy dx$$

$$= \int_0^1 \left[x^2y + \frac{x}{6}y^2 \right]_{1-x}^2 dx$$

$$= \int_0^1 \left(\frac{4}{3}x^2 + \frac{1}{2}x + \frac{5}{6}x^3 \right) dx$$

$$= \left(\frac{4}{9}x^3 + \frac{1}{4}x^2 + \frac{5}{24}x^4 \right) \Big|_0^1$$

$$= \frac{65}{72}$$

Example:

X : time until a computer server connects to your machine.

connects to your machine.

Υ : time until the server authorizes you as a valid user.

$$f_{X,Y}(x,y) = 6 \cdot 10^{-6} \exp(-0.001x - 0.002y)$$

for $\begin{cases} x < y \\ x > 0 \\ y > 0 \end{cases}$

$$\text{Find } P(X \leq 1000, Y \leq 2000)$$

$$P(X \leq 1000, Y \leq 2000)$$

$$= \int_0^{1000} \int_{\underline{x}}^{2000} 6 \cdot 10^{-6} \exp(-0.001x - 0.002y) dy dx$$

$$= 6 \cdot 10^{-6} \cdot \int_0^{1000} \left[\int_x^{2000} \exp(-0.002y) dy \right] \exp(-0.001x) dx$$

$$= 6 \cdot 10^{-6} \int_0^{1000} \left[\frac{\exp(-0.002y)}{-0.002} \Big|_x^{2000} \right] \exp(-0.001x) dx$$

$$= 6 \cdot 10^{-6} \int_0^{1000} \frac{\exp(-0.003x) - \exp(-4) \exp(-0.001x)}{0.002} dx$$

$$= 0.003 \left[\frac{\exp(-0.003x)}{0.003} - \frac{\exp(-4) \exp(-0.001x)}{-0.001} \right] \Big|_0^{1000}$$

$$= 1 - e^{-3} - 3(e^{-4} - e^{-5})$$

Marginal probability distribution:

$$f_{XY}(x, y) \Rightarrow f_X(x) (f(x)) \\ f_Y(y) (f(y))$$

Discrete one:

$$f_X(x) = \sum_y f_{XY}(x, y)$$

$$f_Y(y) = \sum_x f_{XY}(x, y)$$

Continuous

$$f_X(x) = \int f_{XY}(x, y) dy$$

$$f_Y(y) = \int f_{XY}(x, y) dx$$

Integral
over
points in
the range.

Example:

$$\underline{P(Y > 2000)}$$

$$f_{XY}(x, y) = 6 \cdot 10^{-6} e^{-(0.001x - 0.002y)}, x < y$$

$$f(y) = \int_{-\infty}^y f_{XY}(x, y) dx$$

$$f(y) = \int_0^y f_{XY}(x, y) dx$$

$$= \int_0^y 6 \cdot 10^{-6} \cdot e^{-0.001x - 0.002y} dx$$

$$= 6 \cdot 10^{-3} \underbrace{e^{-0.002y} [1 - e^{-0.001y}]}_{y > 0}, \text{ for}$$

$$P(Y > 2000) = \int_{2000}^{\infty} f(y) dy$$

$$= \int_{2000}^{\infty} 6 \cdot 10^{-3} [e^{-0.002y} - e^{-0.003y}] dy$$

$$= 6 \cdot 10^{-3} \left[\frac{e^{-0.002y}}{-0.002} - \frac{e^{-0.003y}}{-0.003} \right] \Big|_{2000}^{\infty}$$

$$= 0.05$$

$$P(Y > 2000) = \int_{2000}^{\infty} \int_0^y f_{XY}(x, y) dx dy$$

Conditional Distribution and Independent.

X, Y. Discrete R.V.

$$P_{Y|x}(y|x) = \frac{P(X=x \text{ & } Y=y)}{P(X=x)} \quad p(A|B) = \frac{P(A \cap B)}{P(B)}$$

Properties:

$$(1) P_{Y|x}(y|x) \geq 0$$

$$(2) \sum P_{Y|x}(y|x) = 1$$

$$\therefore \sum P(Y|X \cup J)^n = 1$$

$$(3) P(Y \in B | X=x) = \sum_B f_{Y|X}(y|x)$$

for any region B

Continuous X, Y R.V.

$$f_{Y|X}(y|x) = \frac{f_{XY}(x,y)}{f_X(x)}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Properties:

$$(1) f_{Y|X}(y|x) \geq 0$$

$$(2) \int f_{Y|X}(y|x) dy = 1$$

$$(3) P(Y \in B | X=x) = \int_B f_{Y|X}(y|x) dy$$

for any region B.

Independent:

$$\begin{cases} P(A \cap B) = P(A) \cdot P(B) \\ P(A|B) = P(A) \end{cases}$$

Discrete

$$(1) P(X=x, Y=y) = P(X=x) \cdot P(Y=y) \quad \text{for all } x, y.$$

$$(2) P_{Y|X}(y|x) = P(Y)$$

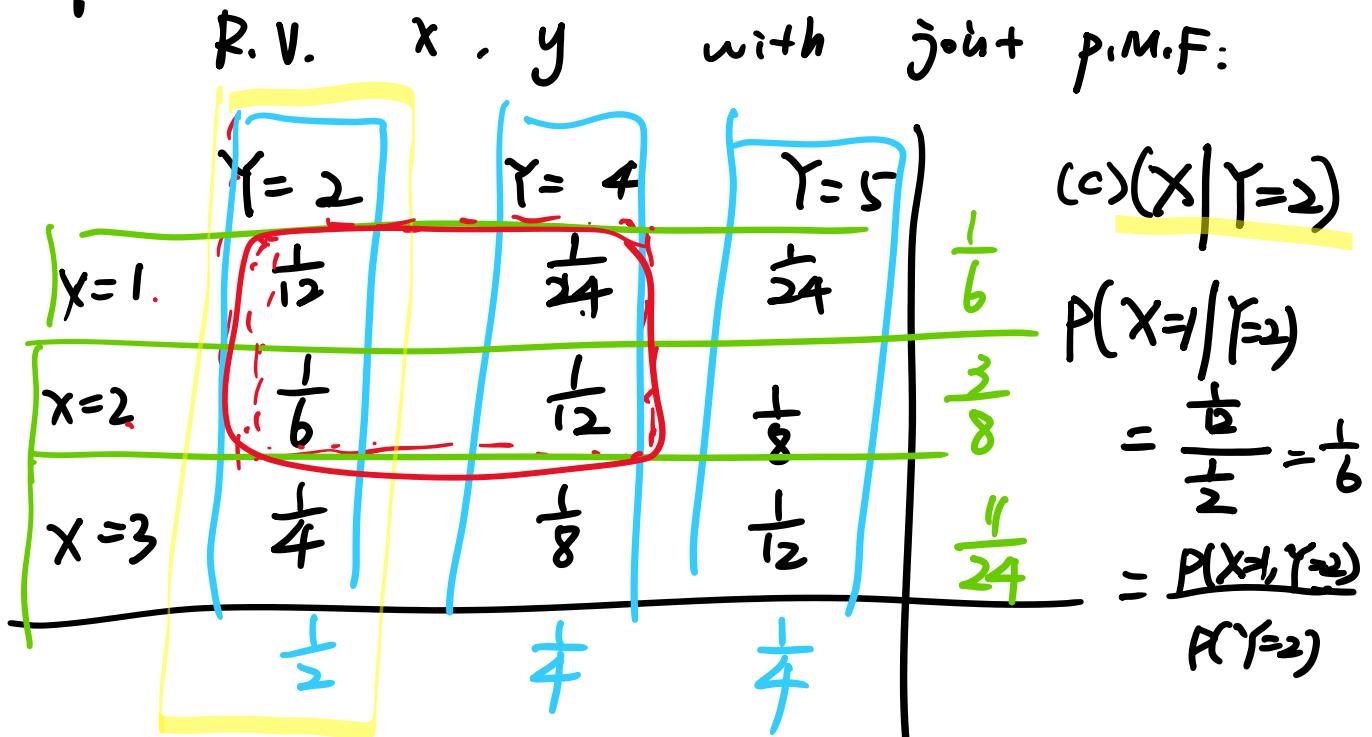
Continuous:

$$(1) f_{XY}(x,y) = f_X(x) \cdot f_Y(y)$$

$$\dots r \dots 1 \dots f \dots$$

$$(2) f_{Y|X}(y|x) = f_Y(y)$$

Example:



(a) Find $P(X \leq 2, Y \leq 4)$

$$P(X \leq 2, Y \leq 4) = \frac{3}{8}$$

(b) Find Marginal distribution for X, Y .

$$P(Y=y) = \begin{cases} \frac{1}{2} & Y=2 \\ \frac{1}{4} & Y=4 \\ \frac{1}{4} & Y=5 \end{cases}$$

$$P(X=x) = \begin{cases} \frac{1}{6} & x=1 \\ \frac{3}{8} & x=2 \\ \frac{11}{24} & x=3 \end{cases}$$

Example:

Joint pdf.

Example:

joint p.d.f.

$$f(x,y) = cxy \quad , \quad 0 \leq x \leq 1 \quad , \quad 0 \leq y \leq \sqrt{x}$$

(a) Find 'c'.

$$\iint f(x,y) dx dy = 1$$

$$\int_0^1 \int_0^{\sqrt{x}} cxy dy dx = 1$$

$$\int_0^1 \left(\frac{c}{2} x y^2 \Big|_0^{\sqrt{x}} \right) dx = 1$$

$$\int_0^1 \frac{c}{2} x^2 dx = 1$$

$$\frac{c}{6} x^3 \Big|_0^1 = 1$$

$$\therefore c = 6$$

(b) Find $f_X(x)$ and $f_Y(y)$.

$$\begin{aligned} f_X(x) &= \int_{-\infty}^{\infty} f_{XY}(x,y) dy \\ &= \int_0^{\sqrt{x}} 6xy dy \\ &= 3x^2 \quad , \quad 0 \leq x \leq 1 \end{aligned}$$

$$\begin{aligned} f_Y(y) &= \int_{-\infty}^{\infty} f_{XY}(x,y) dx \quad 0 \leq y \leq \sqrt{x} \\ &= \int_{y^2}^1 6xy dx \quad \Leftrightarrow y^2 \leq x \leq 1 \end{aligned}$$

$$= \int_{y^2}^1 6xy \, dx \quad \overbrace{\quad y = \sqrt{x} \quad} \\ = 3y(1-y^4) \quad . \quad 0 \leq y \leq 1$$

(c) Find $f_{X|Y}(x)$

$$f_{X|Y}(x) = \frac{f_{X,Y}(x,y)}{f_Y(y)} \quad \boxed{y \text{ is given.}}$$

$$= \frac{6xy}{3y(1-y^4)} \quad . \quad \boxed{y^2 \leq x \leq 1}$$

(d) Are X, Y independent.

$$f_{X|Y}(x) \neq f_X(x)$$

$\therefore X, Y$ are dependent.

$$(e) E(X|Y=y) \quad E(X|Y=y) = \int_{y^2}^1 x \cdot f_{X|Y}(x|y) \, dx$$

$$E(X|Y=y) = \int_{y^2}^1 x \cdot \frac{6xy}{3y(1-y^4)} \, dx$$

$$= \int_{y^2}^1 \frac{2x^2}{1-y^4} \, dx$$

$$= \frac{2}{3} \cdot \frac{1}{1-y^4} \cdot x^3 \Big|_{y^2}^1$$

$$; = \frac{2}{3} \frac{1-y^6}{1-y^4}; \quad y \text{ is given}$$

$$f(x) = \frac{1}{3} \frac{1-y^4}{1-y^4}, \quad y \text{ is given}$$

$$E(X|Y=2) = \frac{2}{3} \frac{1-2^4}{1-2^4}$$

$$E(X^2|Y=y) = \frac{1-y^8}{2(1-y^4)}$$

$$V(X|Y=y) = E(X^2|Y=y) - [E(X|Y=y)]^2$$

More General:

Suppose we have x_1, \dots, x_p R.V.
P dimensional.

Joint p.d.f.

$$(1) f_{x_1, \dots, x_p}(x_1, x_2, \dots, x_p) \geq 0$$

$$(2) \underbrace{\iiint \cdots \int}_{\# P} f_{x_1, \dots, x_p}(x_1, \dots, x_p) dx_1 \cdots dx_p = 1$$

(3) For any Region B.

$$P[(x_1, \dots, x_p) \in B] = \iint_B \cdots \int f_{x_1, \dots, x_p}(x_1, \dots, x_p) dx_1 \cdots dx_p$$

Conditional:

$$f(x_1, x_2, \dots, x_p)$$

$$f(x_1, x_2 \dots x_5)$$

$$f(x_1, x_2 | \underline{x_3, x_4, x_5}) = \frac{f(x_1, \dots, x_5)}{f(\underline{x_3, x_4, x_5})}$$

Independent

$$f_{x_1, \dots, x_p}(x_1, \dots, x_p) = f_{x_1}(x_1) \cdot f_{x_2}(x_2) \cdots f_{x_p}(x_p)$$

5.2 Covariance and Correlation

X, Y .

The covariance between R.V. X, Y .

denoted by $\text{Cov}(X, Y)$ or σ_{XY}

$$\begin{aligned} \sigma_{XY} &= \text{Cov}(X, Y) = E[(X - \mu_X)(Y - \mu_Y)] \\ &= \underline{\underline{E(XY)}} - \underline{\underline{E(X)E(Y)}} \end{aligned}$$

$$E[h(x, y)] = \begin{cases} \sum \sum h(x, y) f_{XY}(x, y) & (\text{Discrete}) \\ \iint h(x, y) f_{XY}(x, y) dx dy & (\text{Continuous}) \end{cases}$$

Correlation:

$$\rho_{XY}$$

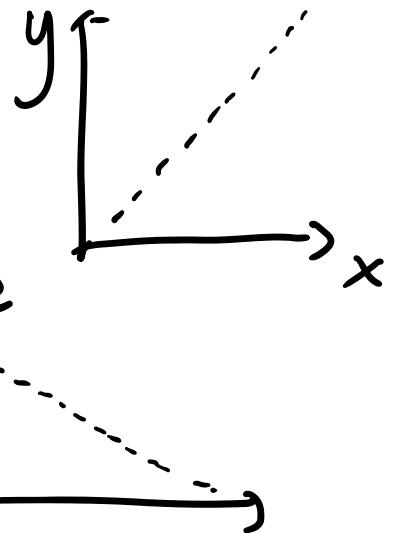
$$\text{Cov}(X, Y)$$

$\leftarrow \dots$

$$\rho_{xy} = \frac{\text{Cov}(X, Y)}{\sqrt{V(X) V(Y)}} = \frac{\sigma_{xy}}{\sigma_x \sigma_y}$$

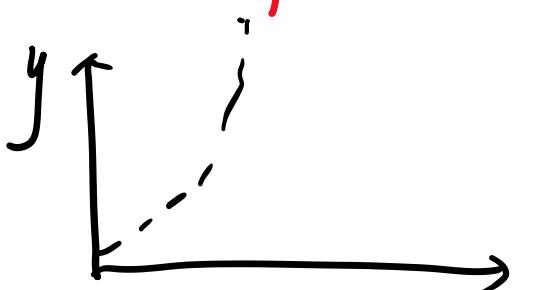
$$-1 \leq \rho_{xy} \leq 1$$

(1) $\rho_{xy} = 1$ x, y strictly on
 $\rho_{xy} = -1$ x, y a line



which is perfectly

linearly correlated



Correlation only measures
linear relationship

$$\rho_{xy} = 0 = \text{Cov}(X, Y)$$

$\Leftrightarrow X, Y$ are independent.

$1 > \rho_{xy} > 0$ positive linearly correlated

$0 > \rho_{xy} > -1$ negative linearly correlated.

Example:

joint p.d.f.

Joint p.d.f.

$$f(x, y) = \frac{1}{16}xy, \quad 0 \leq x \leq 2, \quad 0 \leq y \leq 4$$

Find $\text{Cov}(X, Y)$ and P_{XY} .

$$\text{Cov}(X, Y) = E(XY) - E(X)E(Y).$$

$$\begin{aligned} E(XY) &= \int_0^4 \int_0^2 xy \cdot f_{XY}(x, y) dx dy \\ &= \int_0^4 \int_0^2 \frac{1}{16}x^2y^2 dx dy \\ &= \frac{1}{16} \int_0^4 \left[\frac{1}{3}x^3y^2 \right]_0^2 dy \\ &= \frac{1}{16} \int_0^4 \frac{8}{3}y^2 dy \\ &= \frac{1}{16} \cdot \frac{8}{9}y^3 \Big|_0^4 \\ &= \frac{32}{9} \end{aligned}$$

$$E(X) = \int_0^4 \int_0^2 x \cdot f_{XY}(x, y) dx dy$$

$$\begin{aligned} E(X) &= \int x f_X(x) dx \\ f_X(x) &= \int f_{XY}(x, y) dy \end{aligned}$$

$$E(X) = \int \int f_{X,Y}(x,y) dx dy$$

$$= \frac{1}{16} \int_0^4 \int_0^2 x^2 y dx dy$$

$$= \frac{1}{16} \int_0^4 \frac{8}{3} y dy$$

$$= \frac{4}{3}$$

$$E(Y) = \int_0^4 \int_0^2 y f_{X,Y}(x,y) dx dy$$

$$= \frac{1}{16} \int_0^4 y^2 \cdot \frac{1}{2} x^2 \Big|_0^2 dy$$

$$= \frac{1}{8} \left(\frac{1}{3} y^3 \Big|_0^4 \right)$$

$$= \frac{8}{3}$$

$$\text{cov}(X, Y) = E(XY) - E(X) \cdot E(Y)$$

\Rightarrow

$\therefore X$ and Y are independent.

5.4 Linear Function of R.V.

shoppers 10p/hour

24 hours

$\square \quad \square \quad \square \quad \dots \quad - \quad \square$

$x_1 \quad x_2 \quad x_3 \quad \dots \quad - \quad x_{24}$

$x_1 \quad x_2 \quad x_3 \dots - \quad x_{24}$
 Identical and Independent. I.I.D

$$E(x_1 + x_2 + \dots + x_{24}) = E(x_1) + E(x_2) + \dots + E(x_{24})$$

$$= 24 \cdot E(x)$$

$$= 24 \cdot 10$$

$$= 240.$$

Linear Combination:

Given R.V. x_1, x_2, \dots, x_p and
 constants c_1, c_2, \dots, c_p .

$$Y = c_1 x_1 + c_2 x_2 + \dots + c_p x_p.$$

is a linear combination of x_1, \dots, x_p .

$$E(Y) = c_1 E(x_1) + c_2 E(x_2) + \dots + c_p E(x_p)$$

$$\left[c_1 \int x_1 f(x_1) dx_1 + c_2 \int x_2 f(x_2) dx_2 + \dots \right]$$

$$V(Y) = c_1^2 V(x_1) + c_2^2 V(x_2) + \dots + c_p^2 V(x_p)$$

$$+ \sum_{i < j} \sum c_i c_j \text{Cov}(x_i, x_j)$$

Independent $\Leftrightarrow \text{Cov}(x_i, x_j) = 0$

Example: $Y = 2x_1 + 3x_2 + 4x_3$

$$\therefore E(Y) = 2E(x_1) + 3E(x_2) + 4E(x_3)$$

$$V(Y) = 4V(x_1) + 9V(x_2) + 16V(x_3)$$

$$V(Y) = 4V(X_1) + 9V(X_2) + 16V(X_3) \\ + 2 \cdot 3 \operatorname{Cov}(X_1, X_2) + 2 \cdot 4 \operatorname{Cov}(X_1, X_3) \\ + 3 \cdot 4 \operatorname{Cov}(X_2, X_3)$$

Example : Geometric and Negative Binomial.

↓
Experiments until one success.

Negative Binomial

↓
Experiments until r success.

$X_1, \dots, X_p \sim$ Geometric.

$Y = X_1 + \dots + X_p \sim$ Negative Binomial
($r = p$)

X_1, \dots, X_p they I. I.D.

identical & independent distributed.

Hard to prove they are equal

could see $E(X), E(Y)$
 $V(X), V(Y)$

$$E(Y) = \underbrace{E(X_1)}_{\frac{1}{p}} + \underbrace{E(X_2)}_{\frac{1}{p}} + \dots + \underbrace{E(X_r)}_{\frac{1}{p}} \\ = \frac{1}{p} + \frac{1}{p} + \dots + \frac{1}{p}$$

$$= \frac{r}{P}$$

$$V(Y) = r \cdot V(X_1) + r \cdot V(X_2) + \dots + V(X_r)$$

$$= \frac{1-p}{p^2} + \dots + \frac{1-p}{p^2}$$

$$= \frac{r(1-p)}{p^2}$$

Example:

Shoppers sell 10 boxes of pop on average per day.

X denote per hour they sell # of boxes.

X follows Normal with $\mu=10$, $\sigma=5$.

(a) what's the probability that 20 days in total we sell more than 300 boxes?

X_1, \dots, X_{20}
 ↑ ↑
 1st Day 20th Day.

$$Y = X_1 + \dots + X_{20}$$

$$\underline{P(Y \geq 300)}$$

Standardizing

$$\bar{E}(Y) = \mu = E(X_1) + \dots + E(X_{20}) = 10 \cdot 20 = 200$$

$$V(Y) = \sigma^2 = 20 V(X) = 20 \cdot 5^2 = 500.$$

$$V(Y) = \sigma^2 = 20 \quad V(X) = 20 \cdot 5^2 = 500.$$

$$\sigma = \sqrt{500}$$

$$\begin{aligned} P(Y \geq 300) &= P\left(\frac{Y-\mu}{\sigma} \geq \frac{300-200}{\sqrt{500}}\right) \\ &= P(Z \geq 4.47) \\ &\stackrel{\text{Symmetric}}{=} P(Z \leq -4.47) \\ &= 0 \end{aligned}$$

Example:

20 Days.

$$X_1, \dots, X_{20} \sim \text{Normal}(\mu=10, \sigma=5).$$

What's the probability that on average more than 9 boxes are sold per day.

$$\bar{X} = \frac{X_1 + \dots + X_{20}}{20}$$

$$E(\bar{X}) = \underbrace{\frac{1}{20}E(X_1) + \frac{1}{20}E(X_2) + \dots + \frac{1}{20}E(X_{20})}_{20}$$

$$= \frac{20}{20} E(X_i) = E(X) = 10$$

$$V(\bar{X}) = V\left(\frac{1}{20}X_1 + \frac{1}{20}X_2 + \dots + \frac{1}{20}X_{20}\right)$$

$$= \frac{1}{400} V(X_1) + \frac{1}{400} V(X_2) + \dots + \frac{1}{400} V(X_{20}),$$

$$= \underbrace{\frac{1}{400} V(X_1) + \frac{1}{400} V(X_2) + \dots + \frac{1}{400} V(X_{20})}_{20}$$

$$= \frac{20}{400} V(X)$$

$$= \frac{1}{20} \cdot 5^2$$

$$= \frac{5}{4}$$

$$P(\bar{X} > 9) = P\left(Z > \frac{9 - 10}{\sqrt{\frac{5}{4}}}\right)$$

$$= P(Z > -0.89)$$

$$= P(Z < 0.89)$$

$$= 0.8133$$

Example:

$$X_1 . X_2$$

X_1, X_2 are independent identical.

$$f(x) = x^2(2x + \frac{3}{2}), \quad 0 < x < 1$$

Find $E(Y)$ where $Y = \frac{1}{X_1} + \frac{1}{X_2} + 3$.

$$E(Y) = E\left(\frac{1}{X_1}\right) + E\left(\frac{1}{X_2}\right) + 3$$

$$\neq \frac{1}{E(X)}$$

$$\boxed{E(h(x)) = \int h(x) f(x) dx}$$

∴ ANS

$$E\left(\frac{1}{X}\right) = \int_0^1 \frac{1}{x} \cdot f(x) dx \quad E\left(\frac{1}{X}\right) \neq \frac{1}{E(X)}$$

$$= \int_0^1 \frac{1}{x} \cdot x^2 \left(2x + \frac{3}{2}\right) dx$$
$$= x^2 + \frac{3}{2}x \Big|_0^1$$

$$= \frac{5}{2}$$

$$\therefore E(Y) = \frac{5}{2} + \frac{5}{2} + 3 = 8$$