Sample Test 2

May 15, 2018 4:27 AM

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Stats 3Y03/3J04

Sample Test Questions for Test #2

Name:____
(Last Name) (First Name)

Student Number:____________________ **Tutorial Number:**____________________

This test consists of 27 multiple choice questions worth 1 mark each (no part marks), and 1 question worth 1 mark (no part marks) on proper computer card filling. All questions must be answered on the COMPUTER CARD with an HB PENCIL. Marks will not be deducted for wrong answers (i.e., there is no penalty for guessing). You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Only the McMaster standard calculator Casio fx-991 is allowed.

1. The CPU of a personal computer has a lifetime that is exponentially distributed with a mean lifetime of six years. If you have owned your CPU for four years, what is the probability that it will fail within the next year?

(a) .5654 **(b)** .1535 **(c)** .4866 **(d)** .0788 **(e)** .3151

1. $f(x) = 6e^{-6x}$

$F(x) = 1 - e^{-6x}$

$P(x<5 \mid x>4) = \dfrac{P(4<x<5)}{P(x>4)}$

$= \dfrac{-e^{-6\cdot 5} + e^{-6\cdot 4}}{e^{-6\cdot 4}}$

$= 1 - e^{-6}$

$= P(x<1)$

memory less property

2. The CPU of a personal computer has a lifetime that is exponentially distributed with a mean lifetime of six years. 10% of CPUs last longer than how many years?

(a) .6322 **(b)** 27.63 **(c)** 9.65 **(d)** 8.41 **(e)** 13.82

2. 

$1 - e^{-6x} = 0.1$

$\therefore 0.9 = e^{-6x}$ $\quad x = -\dfrac{\ln 0.9}{6}$

3. The CPU of a personal computer has a lifetime that is exponentially distributed with a mean lifetime of six years. If you buy 10 CPUs, find the probability that they all last longer than seven years.

(a) 8.575×10^{-6} **(b)** .9760 **(c)** .0240 **(d)** .0734 **(e)** 2.840×10^{-4}

3. Binomial.

$p = P(x>7) = 1 - P(x<7) = e^{-6\cdot 7} = e^{-42}$

$\therefore \Rightarrow p^{10}$

4. The CPU of a personal computer has a lifetime that is exponentially distributed with a mean lifetime of six years. If a company buys 500 CPUs, find the approximate probability that between 100 and 150 of them (inclusive) last longer than 7 years.

(a) .3085 **(b)** .4317 **(c)** .3553 **(d)** .4629 **(e)** .5525

Normal Approximation (Binomial)

4. $P(100 < X < 150)$

$= P\left(\dfrac{99.5 - E(x)}{\sigma} < Z < \dfrac{150.5 - E(x)}{\sigma}\right)$

$E(x) = np$

$V(x) = 500\,p(1-p)$

5. The thickness of a laminated covering for a wood surface is normally distributed with a mean of 5 mm and a standard deviation of 0.2 mm. What is the probability that a laminated covering thickness is between 5.1 and 5.3 mm?

(a) .6247 **(b)** .7417 **(c)** .2417 **(d)** .3753 **(e)** .7583

5. $P(5.1 < X < 5.3)$

$= P\left(\dfrac{5.1-5}{0.2} < Z < \dfrac{5.3-5}{0.2}\right)$

$\underbrace{}_{P}$

6. The thickness of a laminated covering for a wood surface is normally distributed with a mean of 5 mm and a standard deviation of 0.2 mm. If a company buys 400 laminated coverings, find the approximate probability that at least 100 of them have a covering thickness between 5.1 and 5.3 mm.

(a) .6293 **(b)** .4315 **(c)** .5685 **(d)** .3707 **(e)** .2083

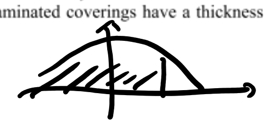
6. $N(5, 0.2)$

Binomial $(400, p)$

$\therefore P(X \geq 100) = P\left(Z > \dfrac{99.5 - \mu}{\sigma}\right)$

7. The thickness of a laminated covering for a wood surface is normally distributed with a mean of 5 mm and a standard deviation of 0.2 mm. 92% of laminated coverings have a thickness greater than what value?

(a) 1.410 **(b)** 4.718 **(c)** .282 **(d)** 5.282 **(e)** 6.410



7. $P\left(Z > \dfrac{x-5}{0.2}\right) = 0.92$

8. The thickness of a laminated covering for a wood surface is normally distributed with a mean of 5 mm and a standard deviation of 0.2 mm. If the company wants to keep the mean

8.

8. The thickness of a laminated covering for a wood surface is normally distributed with a mean of 5 mm and a standard deviation of 0.2 mm. If the company wants to keep the mean thickness at 5 mm, but wants to adjust the manufacturing process so that only 10% of the laminated coverings have a thickness less than 4.8 mm, what would the standard deviation have to be?

(a) .1563 **(b)** 1.28 **(c)** .7813 **(d)** .2387 **(e)** .9416

8. $p(X < 4.8) = 0.1$

$\Rightarrow p(Z < \frac{4.8-5}{\sigma}) = 0.1 \Rightarrow \sigma ?$

9. Suppose that X and Y have joint pdf

$$f(x,y) = cx(1+y), \qquad 0 \le x \le 1,\ 0 < y - x < 1$$

Find c.

(a) $\frac{11}{13}$ **(b)** $\frac{13}{11}$ **(c)** $\frac{12}{13}$ **(d)** 1 **(e)** $\frac{13}{12}$

$x < y < x+1$

9. $\int_0^1 \int_x^{x+1} cx + cxy \, dy \, dx$

$= \int_0^1 \left[cxy + \frac{c}{2} x y^2 \Big|_x^{x+1} \right] dx$

$= \int_0^1 \left[cx(x+1) + \frac{c}{2}x(x+1)^2 - cx^2 + \frac{c}{2}x^3 \right] dx$

$= \int_0^1 cx + cx^2 + \frac{c}{2}x \, dx$

$= \int_0^1 \frac{3}{2}cx + cx^2 \, dx$

$= \frac{3}{4}cx^2 + \frac{1}{3}cx^3 \Big|_0^1$

$= \frac{3}{4}c + \frac{1}{3}c = 1 \quad \therefore c = \frac{12}{13}$

10. Suppose that X and Y have joint pdf

$$f(x,y) = cx(1+y), \qquad 0 \le x \le 1,\ 0 < y - x < 1$$

Find $P(0 \le X \le \frac{1}{2}, 0 < Y < 1)$.

(a) .1274 **(b)** .0923 **(c)** .1145 **(d)** .1731 **(e)** .1363

10. $\int_0^{\frac{1}{2}} \int_0^1 \boxed{\frac{12}{13}} x(1+y) \, dy \, dx$

$= \frac{12}{13} \int_0^{\frac{1}{2}} \left[xy + \frac{1}{2}xy^2 \right] \Big|_0^1 dx$

$= \frac{12}{13} \cdot \int_0^{\frac{1}{2}} x + \frac{1}{2}x \, dx = \frac{12}{13} \left(\frac{3}{4}x^2 \Big|_0^{\frac{1}{2}} \right)$

$= 0.1731$

11. Suppose that X and Y have joint pdf

$$f(x,y) = cx(1+y), \qquad 0 \le x \le 1,\ 0 < y - x < 1$$

Find $P(Y \le \frac{3}{2})$.

(a) .7428 **(b)** .6212 **(c)** .7186 **(d)** .6866 **(e)** .5813

12. Suppose that X and Y have joint pdf

$$f(x,y) = cx(1+y), \qquad 0 \le x \le 1,\ 0 < y - x < 1$$

Find the marginal probability distribution of X.

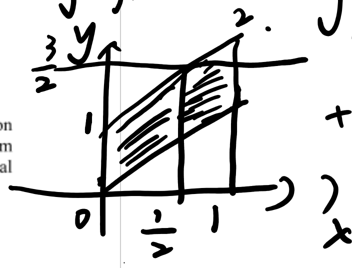
(a) $\frac{63}{52}x(2x+1),\ 0 \le x \le 1$ **(b)** $\frac{11}{6}x(x+3),\ 0 \le x \le 1$ **(c)** $\frac{12}{13}x(2x+3),\ 0 \le x \le 1$ **(d)** $\frac{6}{13}x(2x+3),\ 0 \le x \le 1$ **(e)** $\frac{12}{13}x(2x+1),\ 0 \le x \le 1$

12. $f(x) = \int_x^{x+1} \frac{12}{13}x + \frac{12}{13}xy \, dy$

$= \frac{12}{13}\left(\frac{3}{2}x + x^2\right)$

$y-1 < x < y$

11. $f(y) = \int_0^{\frac{1}{2}} \int_x^{x+1} \underbrace{\quad}_{x+xy} dy\,dx + \int_{\frac{1}{2}}^1 \int_x^{\frac{3}{2}} \ldots dy\,dx$



$= \frac{12}{13} \cdot \left\{ \int_0^{\frac{1}{2}} xy + \frac{1}{2}xy^2 \Big|_x^{x+1} dx + \int_{\frac{1}{2}}^1 xy + \frac{1}{2}xy^2 \Big|_x^{\frac{3}{2}} dx \right.$

$= \frac{12}{13} \left\{ \int_0^{\frac{1}{2}} \frac{3}{2}x + x^2 \, dx + \int_{\frac{1}{2}}^1 \frac{3}{2}x + \frac{9}{8}x - x^2 - \frac{1}{2}x^3 \right.$

13. Suppose that X and Y have joint pdf

$$f(x,y) = cx(1+y), \qquad 0 \le x \le 1,\ 0 < y - x < 1$$

Find the covariance between X and Y.

(a) .224850 **(b)** .329564 **(c)** .124859 **(d)** .315681 **(e)** .050493

$0 < x < 1.$

14. Adobe bricks for construction have a mean weight of 12.1 pounds, with standard deviation 1.1 pounds. Assume that the weights of adobe bricks are independent normal random variables. If these bricks are shipped in packages of 24, find the probability that the total weight of such a package is greater than 300 pounds.

(a) .9625 **(b)** .6406 **(c)** .0375 **(d)** .3594 **(e)** .5978

15. Adobe bricks for construction have a mean weight of 12.1 pounds, with standard deviation 1.1 pounds. Assume that the weights of adobe bricks are independent normal random variables. The company wants to ship the bricks in packages and guarantee that the average brick weight in a package is between 12.0 and 12.2 pounds. How many bricks should be put in each package so that 96% of all such packages will meet the guarantee?

(a) 1024 **(b)** 623 **(c)** 487 **(d)** 821 **(e)** 509

13. $E(XY) = \int_0^1 \int_x^{x+1} \frac{12}{13} x^2 y(1+y) \, dy \, dx$

$E(X) =$

$E(Y) =$

....

16. Marks on a test in a large introductory stats course are summarized in the Minitab output below. How many students passed the test (i.e., got a mark of 50% or higher)?

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```
Stem-and-Leaf Display: C2

Stem-and-leaf of C2  N  = ?
Leaf Unit = 1.0

 1     3   5
 25    4   2344555666777888888899999
 203   5   000000000111111222222222222222222222223333333333334444444444444444445+
(468)  6   00000000000000000000000000000000000000000000000000011111111111111111+
 654   7   00000000000000000000000000000000000000000000000000000111111111111+
 215   8   000000000000000000000000011111111111111111111111111111111112222222222+
 25    9   0000111123333444456677 8
 2     10  00
```

(a) 1300 **(b)** 1325 **(c)** 1122 **(d)** 1110 **(e)** 1295

17. (b) A data set with $n = 11$ observations has a sample mean of $\overline{x} = 15.3$. Suppose that one of the values in the data set 12.2 is now removed from the data set. What is the value of $\overline{x}$ for this new sample?

(a) 14.65 **(b)** 14.08 **(c)** 15.61 **(d)** 16.41 **(e)** 14.19

$$+ \int_{\frac{1}{2}}^{1} \frac{3}{2}x + \frac{9}{8}x - x^2 - \frac{1}{2}x^3 \, dx$$

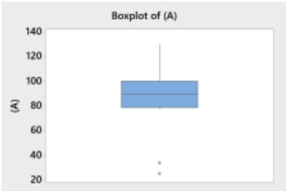
$$= \frac{12}{13}\left\{ \frac{3}{4}x^2 + \frac{1}{3}x^3 \Big|_0^{\frac{1}{2}} + \frac{21}{16}x^2 - \frac{1}{3}x^3 - \frac{1}{8}x^4 \Big|_{\frac{1}{2}}^{1} \right.$$

$$= \frac{12}{13}\left(\frac{3}{16} + \frac{1}{24} + \frac{21}{16} - \frac{1}{3} - \frac{1}{8} - \frac{21}{16}\cdot\frac{1}{4} + \frac{1}{3}\cdot\frac{1}{8} + \frac{1}{8}\cdot\frac{1}{16} \right)$$

$$= 0.7427885$$

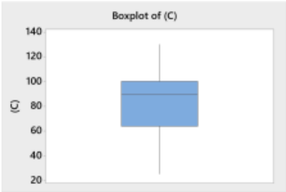
18. Construct a modified boxplot for the following data set.
34, 25, 91, 101, 130, 104, 88, 97, 87, 77, 93, 84

(a)

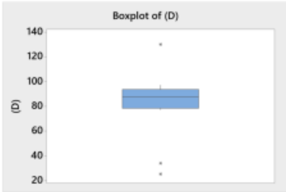


(b)

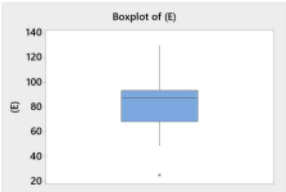
(c)



(d)



(e)



19. Suppose that a random variable X has the density function $f(x) = \frac{x}{8}$, $3 < x < 5$. A random sample of size 49 is selected from this distribution. Find the approximate probability that the sample mean is less than 4.2.

(a) .9236 **(b)** .8721 **(c)** .9597 **(d)** .8419 **(e)** .7787

20. The CPU of a personal computer has a lifetime that is exponentially distributed with a mean lifetime of six years. If a company purchases 32 CPUs, find the probability that the average lifetime greater than 6.2 years.

(a) .4247 **(b)** .4839 **(c)** .3563 **(d)** .4690 **(e)** .3224

21. Suppose that X is the number of observed "successes" in a sample of n observations where p is the probability of success on each observation and the observations are independent. Suppose that we use $\widehat{p}^2 = \frac{X^2}{n^2}$ as an estimator of p^2. Find the amount of bias in the estimator $\widehat{p}^2$.

(a) $\frac{p(1-p)}{n^2}$ **(b)** 0 **(c)** $\frac{p(1-p)}{n}$ **(d)** $\frac{p}{n^2}$ **(e)** $\frac{p}{n}$

22. Two different plasma etchers in a semiconductor factory have the same mean etch rate μ. However, machine 1 is newer than machine 2 and consequently has smaller variability in etch rate. We know that the variance of etch rate for machine 1 is σ_1^2 and for machine 2 is $\sigma_2^2 = \frac{1}{3}\sigma_1^2$. Suppose that we have a sample of n_1 independent observations on etch rate from machine 1 and n_2 independent observations on etch rate from machine 2, and that the samples are independent. Find the bias of the estimator $\widehat{\mu} = \frac{1}{3}\overline{X}_1 + \frac{2}{3}\overline{X}_2$.

(a) μ **(b)** 0 **(c)** $\frac{1}{3}\mu$ **(d)** $\frac{1}{6}\mu$ **(e)** $\frac{2}{3}\mu$

23. Two different plasma etchers in a semiconductor factory have the same mean etch rate μ. However, machine 1 is newer than machine 2 and consequently has smaller variability in etch rate. We know that the variance of etch rate for machine 1 is σ_1^2 and for machine 2 is $\sigma_2^2 = \frac{1}{3}\sigma_1^2$. Suppose that we have a sample of n_1 independent observations on etch rate from machine 1 and n_2 independent observations on etch rate from machine 2, and that the samples are independent. Find the standard error of the estimator $\widehat{\mu} = \frac{1}{3}\overline{X}_1 + \frac{2}{3}\overline{X}_2$.

(a) $\frac{\sigma_1}{3}\sqrt{\frac{3}{n_1} + \frac{4}{n_2}}$ **(b)** $\frac{\sigma_1}{3}\sqrt{\frac{4}{n_1} + \frac{3}{n_2}}$ **(c)** $\frac{\sigma_1}{3}\sqrt{\frac{3}{4n_1} + \frac{4}{3n_2}}$ **(d)** $\frac{\sigma_1}{3}\sqrt{\frac{1}{n_1} + \frac{3}{4n_2}}$
(e) $\frac{\sigma_1}{3}\sqrt{\frac{1}{n_1} + \frac{4}{3n_2}}$

24. A medical researcher wishes to estimate the percentage of females who take vitamins. He wishes to be 98% confident that the estimate is within 4 percentage points of the true proportion. What is the minimum sample size needed?

(a) 849 **(b)** 983 **(c)** 1697 **(d)** 1201 **(e)** 601

25. In order to estimate the average age of onset of a certain type of disease a researcher collects a sample of 15 people with the disease and produces the following 95% confidence interval `(83.9219, 90.6781)`. Find a 99% confidence interval for the average age based on the same data set.

(a) (85.91, 88.68) **(b)** (86.14, 88.45) **(c)** (82.61, 91.99) **(d)** (82.14, 92.45)
(e) (83.14, 91.45)

26. In order to estimate the proportion of people who are under the age of 21 that enter a particular hospital emergency room each week a researcher takes a sample and produces the following 90% confidence interval for the proportion of people that are under the age of 21 `(0.095970, 0.304030)`. What sample size was used?

(a) 25 **(b)** 50 **(c)** 35 **(d)** 40 **(e)** 45

27. In order to estimate the average weight of a certain breed of dog, a researcher takes a sample of 45 dogs of that breed and produces the following confidence interval `(26.6825, 29.5175)`. The sample standard deviation was 5.2535. What is the level of confidence?

(a) 99% **(b)** 93% **(c)** 95% **(d)** 96% **(e)** 98%

Answers

1. b **2.** e **3.** a **4.** a **5.** c **6.** d **7.** b **8.** a **9.** c **10.** a
11. a **12.** d **13.** e **14.** c **15.** e **16.** a **17.** c **18.** a **19.** a **20.** a
21. c **22.** b **23.** e **24.** a **25.** c **26.** d **27.** b

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