One Sample mean test: $H_0: \mu = \text{constant}$

$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$$

$$t_{n-1} = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

Two Sample mean test: $H_0: \mu_1 = \mu_2$

σ_1, σ_2 known

σ_1, σ_2 unknown assuming $\sigma_1^2 = \sigma_1^2$

σ_1, σ_2 unknown

More sampe mean test: $H_0: \mu_1 = \mu_2 = \mu_3 = \cdots \mu_a$

H_a: at least one of the mean is equal to others.

13.2 Analysis of Variance (F-test)

Example:

Level 1	level 2	level 3	level 4
7.5	5.8	5.9	6.2
6.2	7.3	6.2	6.8
6.9	8.2	5.8	5.7
7.4	7.1	4.7	4.9
9.2	7.8	8.3	6.2
8.3		7.2	7.1
7.6		6.2	5.8
			5.4
$n_1 = 7$	$n_2 = 5$	$n_3 = 7$	$n_4 = 8$

$y_{32} = 6.2$

$y_{14} = 7.4$

$y_{1.}$

$\bar{y}_{1.}$

① Hypothesis: $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

① Hypothesis: $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

H_a: at least of one mean is different.

② Test Stat:

Notation: a = # of treatment

$y_{i.}$ = total of ith troatment.

$\bar{y}_{i.}$ = mean of ith treatment.

$y_{..}$ = grand total

$\bar{y}_{..}$ = grand mean

N = # of total observations.

n_i = # of ith group's observations.

y_{ij} = jth observation in the ith group.

S_i^2 = sample variance of ith group.

Total sum o squares (SST)

$$SST = \sum_{i=1}^{a} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{..})^2 = \sum_{i=1}^{a} \sum_{j=1}^{n_i} y_{ij}^2 - \frac{y_{..}^2}{N}$$

Within groups or error sum of squares (SSE)

$$SSE = \sum_{i=1}^{a} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{i.})^2 = \sum_{i=1}^{a} (n_i - 1) S_i^2$$

Variation among groups or treatments sum of squares (SSTr)

$$SSTr = \sum_{i=1}^{a} n_i (\bar{y}_{i.} - \bar{y}_{..})^2 = \sum_{i=1}^{a} \frac{y_{i}^2}{n_i} - \frac{y_{..}^2}{N}$$

$$SST = SSE + SSTr$$

ANOVA Table - (F-test)

MS F

ANOVA Table - (F-test)

ANOVA	SS	DF	MS	F
Treatment	SS_{Tr}	$a-1$	$MSTr = \frac{SSTr}{a-1}$	$F = \frac{MSTr}{MSE}$
Error	SSE	$N-a$	$MSE = \frac{SSE}{N-a}$	
Total	SST	$N-1$		

Test Stat: $F_{a-1, N-a}$

Example: (Cont.)

$y_{1.} = 53.1$ $y_{2.} = 36.2$ $y_{3.} = 44.3$ $y_{4.} = 48.1$

$\bar{y}_{1.} = 7.59$ $\bar{y}_{2.} = 7.24$ $\bar{y}_{3.} = 6.33$ $\bar{y}_{4.} = 6.01$

$n_1 = 7$ $n_2 = 5$ $n_3 = 7$ $n_4 = 8$

$y_{..} = 181.7$ $\bar{y}_{..} = 6.73$ $N = 27$ $a = 3$

① Hypothesis: $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

H_a: at least one mean is different.

② Test Stat:

$$SSTr = \sum_{i=1}^{a} \frac{y_{i.}^2}{n_i} - \frac{y_{..}^2}{N} = 11.673$$

$$SSE = \sum_{i=1}^{a} (n_i - 1) S_i^2 = 20.3043$$

ANOVA Table

	SS	DF	MS	F
Treatment	11.673	$a-1=3$	3.891	$\frac{3.891}{0.8828} = 4.4$
Error	20.3043	$N-a = 27-4 = 23$	0.8828	
Total	Test Stat		Critical Value	

Total	Test Stat	Critical Value	

③ $F > F^*_{\alpha}$, reject H_0. (at $\alpha = 5\%$)

$|F = 4.4| > |F^*_{3,23,0.05} = 3.03|$

④ Conclusion; reject H_0. There is at least one of the mean is different.

13.2 Test for individual pairs of mean.

$_4C_2 = 6$.

1-2
1-3
1-4
2-3
2-4
3-4

Fisher's LSD Test:

① Hypothesis: $H_0: \mu_i = \mu_j$ for $i \neq j$ from 1 to a.

$H_a: \mu_i \neq \mu_j$

② Step 1: $\bar{y}_{i.} - \bar{y}_{j.}$

Step 2: $LSD = t_{n-a,\frac{\alpha}{2}} \cdot \sqrt{MSE\left(\frac{1}{n_i} + \frac{1}{n_j}\right)}$

from ANOVA

Compare $\bar{y}_{i.} - \bar{y}_{j.}$ and LSD.

Reject H_0 if $|\bar{y}_{i.} - \bar{y}_{j.}| > LSD$

Reject H_0 if $\underbrace{|\bar{y}_{i.} - \bar{y}_{j.}|} > \underbrace{LSD}$

Example (Cont.):

$$t_{n-a, \frac{\alpha}{2}} = t_{23}, \text{ two tail}, \alpha = 5\%.$$

$$= 2.069$$

Hypothesis	LSD	Differece	Conclusion
$H_0: \mu_1 = \mu_2$	1.138	0.35	✗
$H_0: \mu_1 = \mu_3$	1.039	1.26	✓ reject
$H_0: \mu_1 = \mu_4$	1.006	1.58	✓ reject
$H_0: \mu_2 = \mu_3$	1.138	0.91	✗
$H_0: \mu_2 = \mu_4$	1.108	1.23	✓ reject
$H_0: \mu_3 = \mu_4$	1.006	0.32.	✗

Confidence Interval, Fish's LSD:

$100(1-\alpha)\%$ C.I for $\mu_i - \mu_j$:

$$\bar{y}_{i.} - \bar{y}_{j.} \pm LSD$$

Note: (If zero is in this interval, do not reject H_0

is not .... , reject H_0)

Example: $\mu_1 - \mu_2$ 95% CI:

$$(7.59 - 7.24) \pm 1.138 = (-0.788, 1.488)$$

ANOVA'S Assumption:

1. Each population must be normal.
2. The population have equal Varianees.
3. The Sampes must be independent.
4. All the other factors must be consistent,
5. $\hat{e}_{ij} = y_{ij} - \bar{y}_{i.}$ (residual for ANOVA)

   $\hat{e}_{ij}$ follows a normal distribution.
6. The variance of residuals should be similar for groups.