Stats3Y03/3J04 Test 1 (Version 2)

Instructor: Mu He **Time: 7:00 - 8:15 P.M. May 22th, 2019**

First Name:______________________________

Last Name:______________________________

Student ID:______________________________

There are total 15 multiple choice questions for this test. Each question carries equal marks. All questions must be answered on the COMPUTER CARD with an HB PENCIL. You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Only the McMaster standard calculator Casio FX-991 MS or MS Plus is allowed.

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1. The CPU of a personal computer has a lifetime that is exponential distributed with a mean of six years. If you buy 3 CPUs, find the probability that exactly two of them fail after nine years.

(a) 0.3847856 (b) 0.4039951 (c) 0.3646744 (d) 0.4688617 (e) 0.1160342

2. An automated egg carton loader has a 2% probability of cracking an egg (for each egg loaded). On average, how many cartons (containing one dozen eggs) would you have to inspect in order to find 4 cartons with no cracked eggs?

(a) 5.90 (b) 5.10 (c) 6.23 (d) 7.22 (e) 6.87

3. Computer chips from a certain supplier have a 6% chance of containing a manufacturing defect. Suppose that we continue to test chips from this manufacturer until we find 10 that have no defects. Let X be the number of defective chips that we have found at the time of the 10th good chip. Find $P(X = 5)$.

(a) 0.00234244 (b) 0.001257738 (c) 0.00209623 (d) 0.001628485 (e)0.0008384919

4. A mechanical system consists of two components. The probability that the first component works is 0.95, and the probability that the second component works (independently of the first component) is 0.83. Let X be the number of working components. Find $P(X \geq 1)$.

(a) 0.9915 (b) 0.2030 (c) 0.2115 (d) 0.4060 (e) 0.6235

5. Suppose that a code (similar to a postal code) is of the form LDL DLD, where 'L' is an uppercase letter from A to N (i.e., 14 possible letters) and 'D' is a digit from 0 to 7. Suppose that such a code is randomly generated. Find the probability that the code starts with an 'A' and does not contain any 'B's.

(a) 0.07142857 (b) 0.06632653 (c)0.06158892 (d) 0.0683252 (e) 0.080656

6. A certain system can experience three different types of defects. Let Ai (i = 1,2,3) denote the event that the system has a defect of type i. Suppose that $P(A_1) = 0.31$, $P(A_2) = 0.4$, $P(A_3) = 0.34$, $P(A_1 \cup A_2) = 0.6$, $P(A_1 \cup A_3) = 0.62$, $P(A_2 \cup A_3) = 0.65$, $P(A_1 \cap A_2 \cap A_3) = 0.01$ Find the probability that the system has exactly 2 of the 3 types of defects.

(a) 0.21 (b) 0.23 (c) 0.2 (d) 0.24 (e) 0.22

7. Suppose that the random variable X has the following cumulative distribution function.

$$F(x) = \begin{cases} 0 \qquad x < 3 \\ \frac{c}{40}(x^2 - 9) \qquad 3 \leq x < 7 \\ 1 \qquad x \geq 7 \end{cases}$$

what is the variance for X?

(a) 27.73778 (b) 29 (c)29. 02387 (d) 1.262222 (e) 2.336767

8. How many possible combinations can the word **abracadabra** be rearranged?

(a) 332640 (b) 415800 (c) 831600 (d) 166320 (e) 83160

9. Suppose we have a deck of poker with 52 cards (No jokers). If we draw a 'hand' of 5 cards. What's the probability that I get a hand which contains 1 pair, 1 three of a kind.

(a) $\frac{13C2\times4C2\times4C3}{52C5}$ (b) $\frac{13P2\times4C2\times4C3}{52C5}$ (c) $\frac{52C4\times4C3\times48C3}{52C5}$ (d) $\frac{13C2\times8C5}{52C5}$ (e) $\frac{13C2\times4C2\times4C3\times4}{52C5}$

10. Suppose that a factory has a group of 105 electrical components, containing 20% default, if 15 pieces are randomly selected (**without replacement**). Let X denote the number of default items among the chosen ones. What is the variance of X?

(a) 1.94 (b) 2.40 (c) 16.80 (d) 2.08 (e) 2.35

11. According to an estimate, 50% of the people in the USA have at least one credit card. Suppose that a random sample of 30 people is selected. Use a suitable approximation to find the probability that fewer than 19 of them will have at least one credit card.

(a) 0.0507 (b) 0.9394 (c) 0.8997 (d) 0.0511 (e) 0.9495

12. Let X denote the vibratory stress (psi) on a wind turvine blade at a particular wind speed in a wind tunnel. Suppose the X has p.d.f.:

$$f(x) = \frac{x}{100}e^{-x^2/200}, \qquad x > 0$$

Then, 20% of time, the vibratory stress is less than what value?

(a) 6.68 (b) 21.56 (c) 17.94 (d) 5.12 (e) 8.92

13. On average, the shoppers across McMaster University have 2 customers per hour and assuming that for the next hour the number of customers denoted by X, follows a Poisson Distribution. Find the probability that at least two customers are there for the next two hours.

(a) 0.1353 (b) 0.4060 (c) 0.9817 (d) 0.9084 (e) 0.5940

14. The time it takes a supercomputer to perform a task is normally distributed with mean 10 milliseconds and standard deviation 4 milliseconds. What is the probability that it takes more than 18.2 milliseconds to perform the task?

(a) 0.9798 (b) 0.8456 (c) 0.0202 (d) 0.2236 (e) none of above

15. Let X be the lifetime of a bulb, which has an exponential distribution with mean 10 hours. What's the probability that it will last longer than 30 hours given that it has been used for 10 hours.

(a) 1 - exp(2) (b) exp(2) (c) exp(3) (d)1/10 (e) none of above

Test 1 Formula Sheet

Probability Rule:

Probability of Union (Two): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Probability of Union (Three): $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

Conditional Probability: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Independence: $P(A \cap B) = P(A) \times P(B)$ or $P(B|A) = P(B)$

Total Probability Rule: Suppose $E_1, E_2, ..., E_k$ are k exhaustive and mutually exclusive events, then $P(B) = P(B \cap E_1) + P(B \cap E_2) + ... + P(B \cap E_k) = P(B|E_1)P(E_1) + ... + P(B|E_k)P(E_k)$

Discrete R.V.:

Mean (Expected Value): $\mathbb{E}(X) = \mu = \sum_x xf(x)$

Variance: $\mathbb{V}(X) = \sigma^2 = \sum_x (x-\mu)^2 f(x) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$

C.D.F: $F(x) = P(X \leq x) = \sum_{y:y<x} f(x)$

Continuous R.V.:

Mean (Expected Value): $\mathbb{E}(X) = \mu = \int_{-\infty}^{\infty} xf(x)dx$

Variance: $\mathbb{V}(X) = \sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$

C.D.F: $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)dt$

Common Distributions:

1. Binomial Distribution (n,p):

$f(x) = \binom{n}{k} p^x (1-p)^{n-x}$, $x = 0, 1, 2, ..., n$, $\mathbb{E}(X) = np$, $V(X) = np(1-p)$

2. Hypergeometric Distribution (n,K,N):

$f(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$, $x = \max\{0, n+K-N\}$ to $\min\{K, n\}$, $\mathbb{E}(X) = np$, $V(X) = np(1-p)\frac{N-n}{N-1}$, where $p = \frac{K}{N}$

3. Geometric Distribution (p):

$f(x) = p(1-p)^{x-1}$, $x = 0, 1, 2, ...$, $\mathbb{E}(X) = \frac{1}{p}$, $\mathbb{V}(X) = \frac{1-p}{p^2}$

4. Negative Binomial Distribution (r,p):

$f(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}$, $x = 0, 1, 2, ...$, $\mathbb{E}(X) = \frac{r}{p}$, $\mathbb{V}(X) = \frac{r(1-p)}{p^2}$

5. Poisson Distribution (λ):

$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}$, $x = 0, 1, 2, ...$, $\mathbb{E}(X) = \lambda$, $\mathbb{V}(X) = \lambda$

6. Normal Distribution (μ, σ^2):

$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$, $-\infty < x < \infty$, $\mathbb{E}(X) = \mu$, $\mathbb{V}(X) = \sigma^2$

7. Exponential Distribution (λ):

$f(x) = \lambda e^{-\lambda x}$, $x \geq 0$, $\mathbb{E}(X) = \frac{1}{\lambda}$, $\mathbb{V}(X) = \frac{1}{\lambda^2}$

Approximation:

Normal Approximation to Binomial Distribution:

$$P(X \leq x) = P(X \leq x+0.5) \approx P(Z \leq \frac{x + 0.5 - np}{\sqrt{np(1-p)}}) \qquad P(X \geq x) = P(X \geq x-0.5) \approx P(Z \geq \frac{x - 0.5 - np}{\sqrt{np(1-p)}})$$

Standard Normal Probabilities

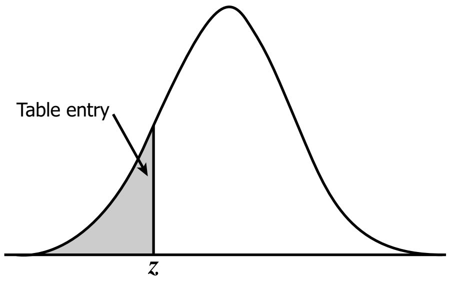

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

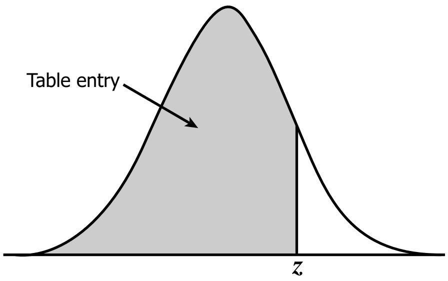

Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998