Lecture 1

May 6, 2019 4:34 PM

## 2.1 Sample Space and Event.

Sample space is a set of all possibles outcomes of an experiment : S

$S = \{1, 2, 3, 4, 5, 6\}$ ⇒ Sample Space of rolling a dice

set is a collection of distinct obejects.

1. Distinct $S = \{\underline{1}, \underline{1}, 2\}$
2. No order $S = \{1, 2\} = \{2, 1\}$
3. Certain $S =$

Event : A event "E", "A" "B" is a subset of the sample space.

$E = \{1, 3\}$ $p(E) = \frac{2}{6} = \frac{\text{\# of element Event}}{\text{\# ~~of~~ element sample space}}$.

| | | |
|---|---|---|
| $A \cap B$ | intersection | and & |
| $A \cup B$ | union | or |
| $A' / A^c$ | Complement | not |

Example: $A = \{1, 2\}$

$B = \{2, 3, 4, 5\}$

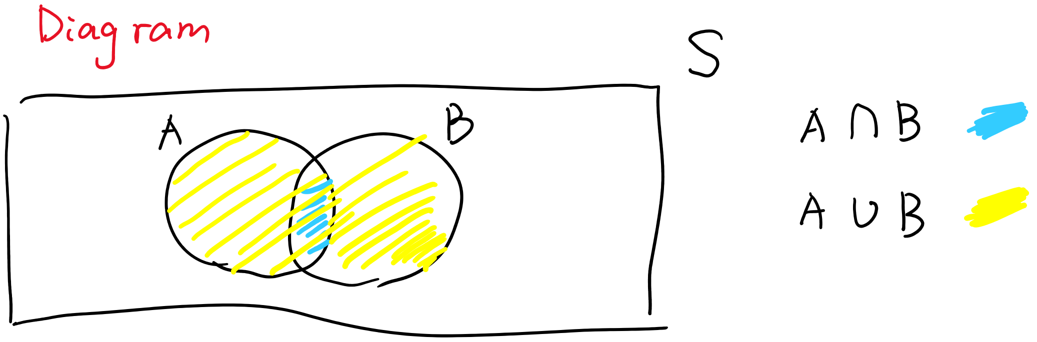
$S = \{1, 2, 3, 4, 5, 6\}$. $p(S) = 1$

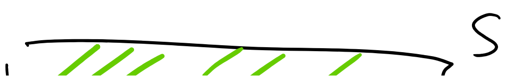
$A \cup B = \{1, 2, 3, 4, 5\}$

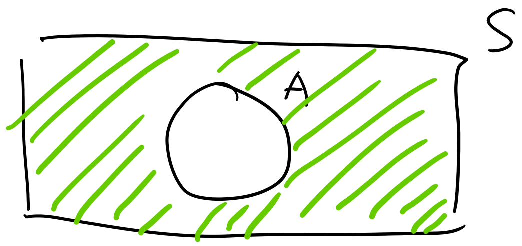
$A \cap B = \{2\}$

$A' = \{3, 4, 5, 6\}$.

Venn Diagram






$A \,;\, A^c$

Example:

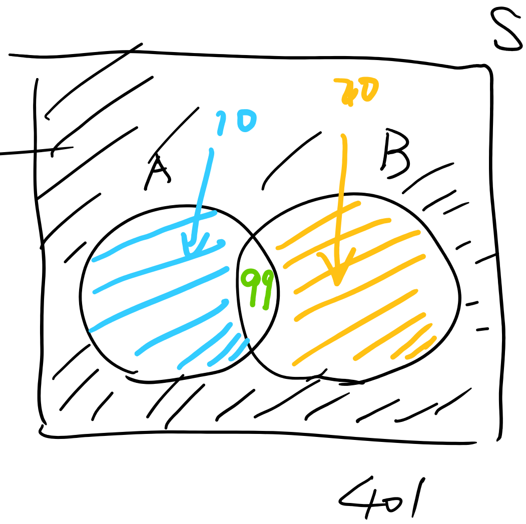
$A$ = Patient with Disease

$A'$ = Patient without Disease.

$B$ = Test positive

$B'$ = Test Negative.

| | $A$ | $A'$ |
|---|---|---|
| $B$ | 99 $A \cap B$ | 20 |
| $B'$ | 10 $A \cap B'$ | 401 |




## Counting Technoligies:

### Multiplication Rule:

If there are $k$ steps of an experiment.

for each step we have $n_1, n_2, n_3 \ldots n_k$ choices.

Then, the total # of outcomes is $n_1 \cdot n_2 \cdots n_k$.

Example: web design. $k=3$

$\underbrace{4 \text{ colours}}_{n_1}$, $\underbrace{3 \text{ fonts}}_{n_2}$, $\underbrace{3 \text{ positions of figures}}_{n_3}$.

$\# = 4 \cdot 3 \cdot 3 = 36$

Example: A license plate consists of 4 letters followed by 3 digits.

(a) How many license plate are possible?

$26 \cdot 26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10$

(b) what if there is no repetitions?

$$\underline{26}\cdot\underline{25}\cdot\underline{24}\cdot\underline{23}\cdot\underline{10}\cdot\underline{9}\cdot\underline{8}$$

(C) How many license start with double 'A'?

$$\underline{A}\quad\underline{A}$$

$$\underline{1}\cdot\underline{1}\quad\underline{26}\quad\underline{26}\quad\underline{10}\quad\underline{10}\quad\underline{10}$$

**Factorials: line staff**

Example: 4 guest, seat in a row, how many ways can they be seated?

$$\underline{4}\cdot\underline{3}\cdot\underline{2}\cdot\underline{1} = \underline{4!}$$

**Permutation: Choose with order.**

Example: 10 guest, 4 seats.

$$\underline{10}\ \underline{9}\ \underline{8}\ \underline{7} = \frac{10!}{(10-4)!} = P^{10}_4 = {}_{10}P_4$$

n choose r with order

$$nPr = \frac{n!}{(n-r)!}$$

**Combination: Choose without order.**

Example: Choose 4 people out of 6 to take 3Y.

$$6C4 = \frac{6!}{4!\,2!}$$

$$nCr = \frac{n!}{r!\,(n-r)!}$$

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$$\underbrace{nCr}\cdot\underbrace{r!} = \underbrace{nPr} \Leftarrow \text{choose with order}$$

Choose without order

order then

---

Example: 4 apples, 2 oranges.

Select 3 fruit with no restrictions:

$$_6C_3$$

Select 3 fruit and make a list to eat.

$$_6P_3$$

Example: 15 men and 10 women,

(a) what if you are selecting 7 men and 5 women.

$$\boxed{_{15}C_7} \times \boxed{_{10}C_5}$$

(b) what if selecting 8 but at least one men is selected?

$$\underset{\text{total}}{_{25}C_8} - \underset{\text{no man}}{\underline{_{10}C_8}}$$

Compliment rule.

Permutation for similar things:

Hamilton $\Rightarrow$ $8!$

$\underline{8,7,6,\ldots,1}$

Engineer $\Rightarrow$ 3e 2n

$$\frac{8!}{3!\,2!}$$

~~8!~~

The number of ordered arrangements of $n = n_1 + \cdots n_r$ objects of which $n_1$ are alike, $n_2$ alike ..... is

$$\frac{n!}{n_1!\,n_2!\cdots n_r!}$$

apple $\frac{5!}{2!}$

2.2 Axioms of Probabilities.

$$P(A) = \frac{\#\text{ of elements in } A}{\ldots} \Leftarrow$$

$$p(A) = \frac{\#\text{ of elements in } A}{\#\text{ of element is } S} \Leftarrow$$

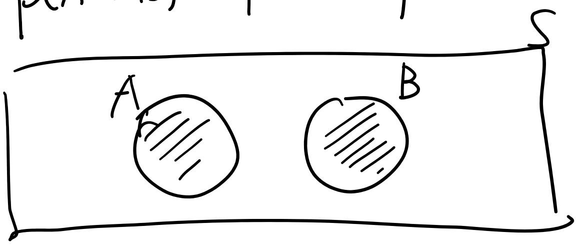
1. $p(S) = 1$
2. $0 \le p(A) \le 1$
3. $A \cap B = \phi$ empty set $\phi$ no elements

$\phi' = S$

$p(\phi) = 0$

$A \cap B = \phi \iff$ A and B are mutually exclusive.

$p(A \cup B) = p(A) + p(B)$



Example: post code

L8S 4L8

If Letters : A to N, 14 letters

digits : 1 to 8, 8 digits.

what's the probability that code has no repeat letters.

$A =$

$S =$ no restrictions.

$$\# A = \underbrace{14}_{\text{letters}} \cdot \boxed{8} \cdot \underbrace{13}_{\text{letter}} \cdot \boxed{8} \cdot \underbrace{12}_{\text{letter}} \cdot \boxed{8}$$

$\# S = 14 \cdot 8 \cdot 14 \cdot 8 \cdot 14 \cdot 8$

$$p(A) = \frac{\# A}{\# S}$$

$p(A) = \frac{}{\# S}$

Example: Birthday Example

365 364 363 362 361
360 359 358 357 356

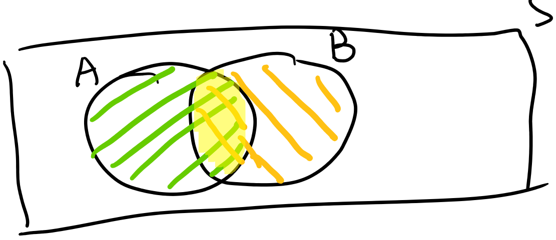
$p(\text{no same birthday})$

$$= \frac{365 \cdot 364 \cdot 363 \cdots 166}{\underbrace{365 \cdot 365 \cdots 365}_{200}} = \frac{}{365^{200}}$$

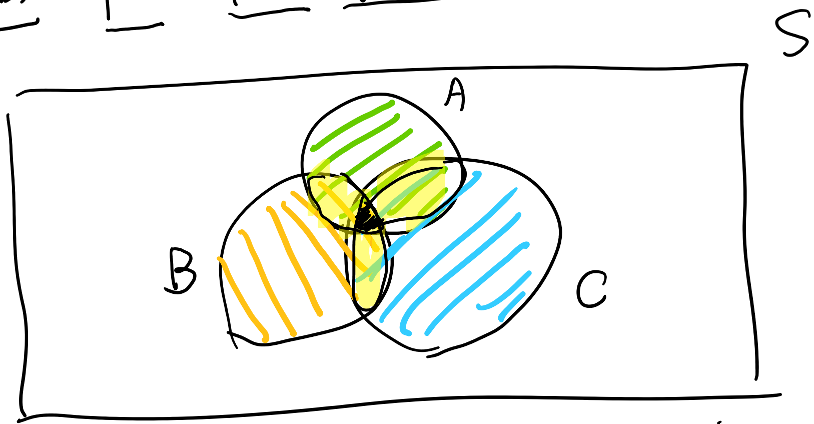
$1 - p(\text{no same birthday}) = p(\text{at least one pair or more})$

50 people probability: 97.04%

2.3 Addition Rules




$\underline{p(A \cup B)} = \underline{p(A)} + \underline{p(B)} - \underline{p(A \cap B)} \Rightarrow$ addition rule.




$p(A \cup B \cup C) = p(A) + p(B) + p(C) \Leftarrow +3\blacktriangle$
$- p(A \cap B) - p(B \cap C) - p(A \cap C) \quad -3\blacktriangle$
$\underline{+ p(A \cap B \cap C)}$

Example: Code question:

Find the probability that code either starts with "A" or ends with an even digits.

ends with an even digits.

L8S 1A1 14 letter 8 digits

Events: B = The post code starts with "A"

C = The post code ends with an even digit

$p(B\cup C) = p(B) + p(C) - p(B\cap C)$

$\#B = \boxed{1} \cdot 8 \cdot 14 \cdot 8 \cdot 14 \cdot 8$

$\# C = 14 \cdot 8 \cdot 14 \cdot 8 \cdot 14 \cdot \boxed{4}$

$\# B\cap C = \boxed{1} \cdot 8 \cdot 14 \cdot 8 \cdot 14 \cdot \boxed{4}$

$\#\text{ total} = 14 \cdot 8 \cdot 14 \cdot 8 \cdot 14 \cdot 8$

$$p(B\cup C) = \frac{\#B + \#C - \#B\cap C}{\#\text{ total}} =$$

2.4 Conditional Probability:

A, B

The probability of A given that B has happened is denoted by $p(A|B)$

$$p(A|B) = \frac{p(A\cap B)}{p(B)}, \quad p(B|A) = \frac{p(A\cap B)}{p(A)}$$

not same.

Example:

| | Pass | Fail | |
|---|---|---|---|
| C01 | A∩B 212 | 41 | 254 B |
| C02 | 287 | 18 | 305 |
| | 500 A | 59 | 559 |

whats the probability if a student is pass given

A

(a) what's the probability if a student is pass given that he is from section 1.

$$p(A|B) = \frac{p(A \cap B)}{p(B)} = \frac{213/559}{254/559}$$

(b) what's the probability if a student is from section 1 given he has passed.

$$p(B|A) = \frac{p(A \cap B)}{p(A)} = \frac{213/559}{500/559}$$

Example:

A word is randomly generated using letters $\overbrace{AABBBCCCC}^{9}$

Find the probability that the two A' occurs first given that a B occurs last.

C&D

$AA \underbrace{\_\ \_\ \_\ \_\ \_\ \_}_{6} B$

2Bs and 4Cs

C = two A occur first

D = a B occurs last.

$$p(C|D) = \frac{p(C \cap D)}{p(D)} = \frac{\frac{6!}{2!\,4!}}{\frac{8!}{2!\,2!\,4!}}$$

C&D

D

D

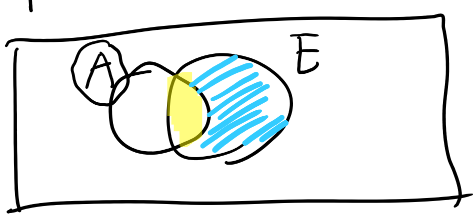
$\underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_\ \_}_{8} B$

2A 2B 4C

2.5 Multiplication and Total Probabilites Rules:

$$p(A|B) = \frac{p(A \cap B)}{p(B)}$$

$$\Longleftrightarrow \quad p(A \cap B) = p(A|B) \cdot p(B) = p(B|A) \cdot p(A)$$




$\Rightarrow$ generlize it

$$p(E) = \underline{p(A \cap E)} + \underline{p(A' \cap E)}$$

Example: If two people are seleted from a group of 8

Example: If two people are seleted from a group of 8 men and 5 women, find the probability that

(a) the $\underbrace{\text{first one is a man}}_{A}$ and $\underbrace{\text{the second one is a woman}}_{B}$

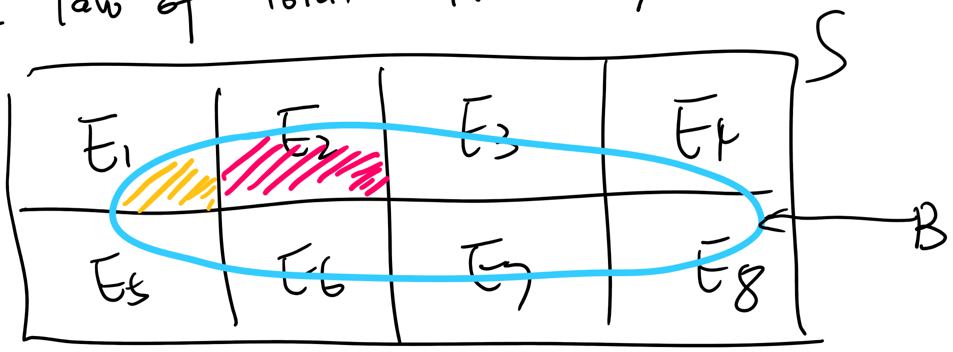
$$P(A \cap B) = P(A) \cdot P(B|A)$$
$$= \frac{8}{8+5} \cdot \frac{5}{7+5}$$

(b) what's the probability that the second is woman.

E
- first one is a man & second is woman
- first one is a woman & second is woman

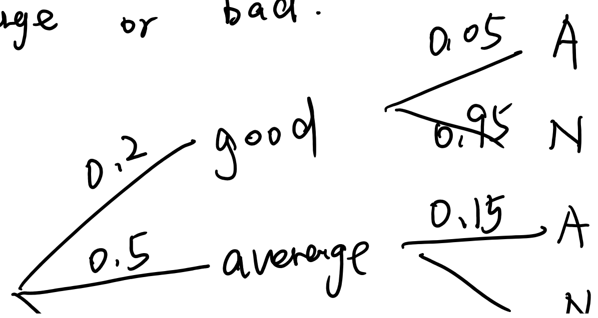
$$P(E) = P(A \cap E) + P(A' \cap E)$$
$$= \frac{8}{13} \cdot \frac{5}{12} + \frac{5}{13} \cdot \frac{4}{12}$$

The law of Total Probability:



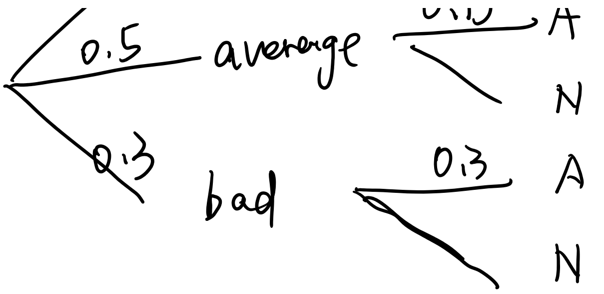

$$P(B) = P(B \cap E_1) + P(B \cap E_2) \cdots \quad P(B \cap E_8)$$

Example: An insurance company classfies people as good averge or bad.




$P(A | \text{good}) = 0.05$

$P(\text{good}) = 0.2$




what proportion of people have accident in a year

$$p(A) = \underline{p(A \cap good)} + p(A \cap average) + p(A \cap bad)$$
$$= p(A|good) \cdot p(good) + p(A|average) \cdot p(average) + \text{''}$$
$$= 0.2 \cdot 0.05 + 0.5 \cdot 0.15 + 0.3 \cdot 0.3.$$

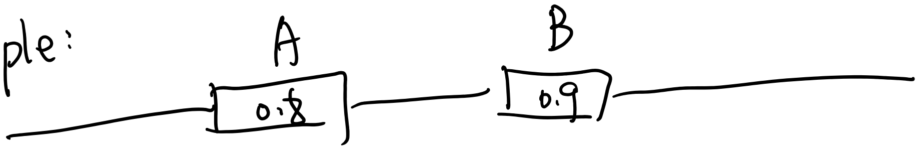
## 2.6 Independence:

Two events are independent if

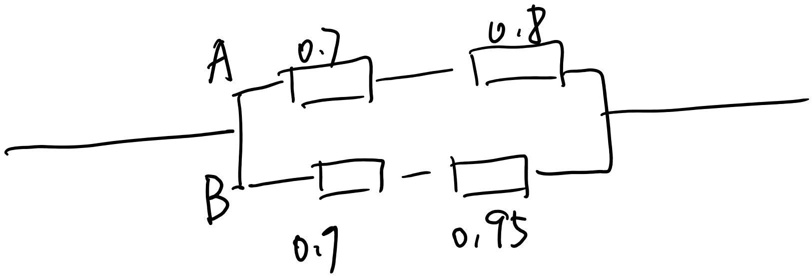
$\underline{p(A|B) = p(A)}$ or $\underline{p(B|A) = p(B)}$ or $\underline{p(A \cap B) = p(A) \cdot p(B)}$

Circuit.

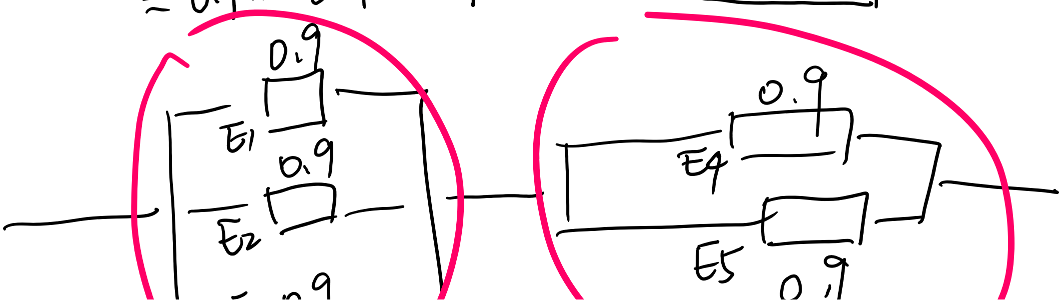
Example:

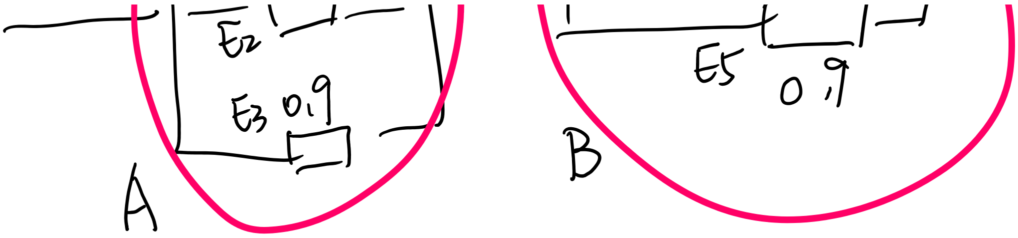



$$p(A \cap B) = p(A) \cdot p(B) = 0.72$$




$$p(A \cup B) = p(A) + p(B) - p(A \cap B)$$
$$= 0.7 \times 0.8 + 0.9 \cdot 0.95 - \boxed{0.7 \cdot 0.8} \cdot \boxed{0.9 \cdot 0.95}$$






$$p(A \cap B) = p(A)\cdot p(B)$$

$$p(A) = p(E_1 \cup E_2 \cup E_3) = p(E_1) + p(E_2) + p(E_3) - p(E_1 \cap E_2) - p(E_2 \cap E_3) - p(E_3 \cap E_1) + p(E_1 \cap E_2 \cap E_3)$$

$$= 0.9 + 0.9 + 0.9 - 0.9^2 - 0.9^2 - 0.9^2 + 0.9^3$$

$$p(B) = p(E_4 \cup E_5) = 0.9 + 0.9 - 0.9^2$$

Example: $A_1, A_2, A_3$ different types of default.

$p(A_1) = 0.39$ $\quad p(A_1 \cup A_3) = 0.64$

$p(A_2) = 0.36$ $\quad p(A_2 \cup A_3) = 0.69$

$p(A_3) = 0.39$ $\quad p(A_1 \cup A_2) = 0.61$

$p(A_1 \cap A_2 \cap A_3) = 0.04$

(a) exactly 2 of 3 types of defaults happen.


$= \underline{p(A_1 \cap A_2)} + \underline{p(A_1 \cap A_3)} + \underline{p(A_2 \cap A_3)} - \underline{3}\, p(A_1 \cap A_2 \cap A_3)$

(b) the probabity that $\underline{\quad A_1 \quad}$ given

not happening

(b) the probabity that $A_1$ given that $\underbrace{A_2, A_3 \text{ are not happening}}_{C =}$

$$P(A_1 | C) = \frac{P(A_1 \cap C)}{P(C)} =$$

$$P(C) = P(A_2' \cap A_3')$$

$$= 1 - P(A_2 \cup A_3)$$

$$= 1 - 0.69$$

$$P(A_1 \cap C) = P(A_1 \cap A_2' \cap A_3')$$

$$= \underbrace{0.39}_{P(A_1)} - 0.1 - 0.1 - 0.04$$