Stats3Y03/3J04 Test 2 (Version 3)

Instructor: Mu He **Time: 7:00 - 8:00 P.M. June 5th, 2019**

First Name:____

Last Name:____

Student ID:____

There are total 15 multiple choice questions for this test. Each question carries equal marks. All questions must be answered on the COMPUTER CARD with an HB PENCIL. You are responsible for ensuring that your copy of the test is complete. Bring any discrepancy to the attention of the invigilator. Only the McMaster standard calculator Casio FX-991 MS or MS Plus is allowed.

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1. Consider the following data set $n = 8$,

$$12.5, 15.7, 18.3, 25.1, 31.2, 47.5, 55.22, 78.01$$

If we were to construct a normal probability plot for this data set, which of the following would be one of the points on the plot?

(a)(-1.15, 12.5)   (b) (-0.87, 31.2)   (c) (-1.38,12.5)   (d) (-0.97, 47.5)   (e) (-1.53,12.5)

2. Suppose that we wish to estimate a population proportion p in such a way that the resulting 95% confidence interval has width of at most 0.04 no matter what the value of p or its estimate $\hat{p}$. Then the minimum sample size required is

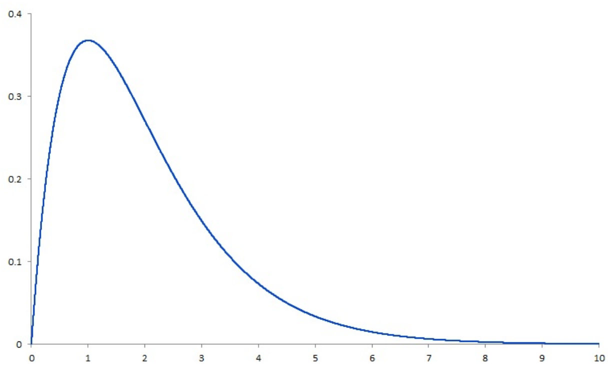
(a) 1068   (b) 2401   (c) 9604   (d) 1225   (e) 601

3. Suppose that we measure the systolic blood pressure of n = 15 random Type I Diabetes patients. A 95% Student's t confidence interval for the population mean blood pressure is (115.87, 122.73). The sample mean and standard deviation are

(a) x = 115.87, s = 1.60.   (b) x = 119.30, s = 1.60.   (c) x = 119.30, s = 6.19.

(d) x = 119.30, s = 6.98.   (e) Impossible to determine based on this information.

4. The following graph has the property:



(a) right-skewed, mean < median < mode

(b) right-skewed, mean > median > mode

(c) left-skewed, mean < median < mode

(c) left-skewed, mean > median > mode

(e) can not tell.

5. Intelligence tests are often calibrated to come from a normal distribution with **known** population standard deviation $\sigma = 10$. Suppose that a sample of n = 25 individuals are tested and their mean score is $\bar{x} = 106$ then a 99% interval for the mean μ is

(a) 102.08, 109.92

(b) 101.348, 110.652

(c) 98.16, 113.84

(d) 100.426, 111.574

(e) 100.848, 111.152

6. Two random variables X and Y are independently distributed if all of the following conditions hold, with the exception of

(a) $P(X|Y) = P(X)$

(b) Knowing the value of one of the variables provides no information about the other.

(c) The conditional distribution of Y given X equals the marginal distribution of Y.

(d) They are mutually exclusive.

(e) $Cov(X, Y) = 0$

7. A randomly selected sample of 400 students at a university with 15-week semesters was asked whether or not they think the semester should be shortened to 14 weeks (with longer classes). Forty-six percent (46%) of the 400 students surveyed answered 'yes'. Which one of the following statements about the number 46% is correct?

(a) It is a sample statistic. (b) It is a population parameter. (c) It is a margin of error.

(d) It is a standard error. (e) It is a significance level.

8. The phrase 95% confidence in an election poll means that

(a) the results are true for 95% of the population of adults

(b) 95% of the population falls within the margin of error we announce

(c) We are 95% certain that this interval will contain population parameter.

(d) the probability is 0.95 that a randomly chosen adult falls in the margin of error

(e) 95% of samples will not have the same stated statistic as the current sample

9. Adobe bricks for construction have a mean weight of 12.1 pounds, with standard deviation 1.1 pounds. Assume that the weights of adobe bricks are independent normal random variables. The company wants to ship the bricks in packages and guarantee that the average brick weight in a package is between 12.0 and 12.2 pounds. How many bricks should be put in each package so that 93% of all such packages will meet the guarantee?

(a) 397 (b) 371 (c) 642 (d) 821 (e) 509

10. An 1868 paper by German physician Carl Wunderlich reported, based on over a million body temperature readings, that healthy adult body temperatures are approximately normally distributed with mean 98.6 degrees Farenheit and standard deviation 0.6. In a random sample of 65 healthy adults, find the probability that the average body temperature is between 98.44 and 98.72.

(a) 0.9684 (b) 0.8926 (c) 0.9245 (d) 0.9305 (e) 0.9608

11. A McMaster student has just handed you a report where the mean and standard deviation of a set of lab results are stated as 53.5 and 3.0, but the original data is not in the report. There is a note saying there was an error and all the individual values need to have 100 added to them. You need to correct the reported values. What is the new mean and standard deviation?

(a) Need all the data points to recalculate (b) 53.5 and 3.0. No change (c) 153.5 and 103.0 (d) 153.5 and 3.0 (e) None of above

12. Suppose that X and Y have joint pdf

$$f(x, y) = x + y, \qquad 0 \le x \le 1, 0 < y < 1$$

Find $Cov(X, Y)$

(a) $\frac{7}{12}$ (b) $-\frac{1}{144}$ (c) $\frac{1}{3}$ (d) $\frac{49}{144}$ (e) $\frac{5}{6}$

13. Suppose that the random variables X and Y have the following joint probability density function.

$$f(x, y) = ce^{-4x-8y}, \quad 0 < y < x.$$

Find the value of c.

(a) 90 (b) 36 (c) 72 (d) 48 (e) 22

14. Find IQR of the following data set

2 6.5 7.9 9.2 10.0 10.8 12.0 12.5 14.5 14.9 21.9

(a) 6.6 (b) 5.3 (c) 11.4 (d) 7.0 (e) 2.2

15. Two different plasma etchers in a semiconductor factory have the same mean etch rate .. However, machine 1 is newer than machine 2 and consequently has smaller variability in etch rate. We know that the variance of etch rate for machine 1 is σ_1^2 and for machine 2 is $\sigma_2^2 = \frac{1}{2}\sigma_1^2$. Suppose that we have a sample of n_1 independent observations on etch rate from machine 1 and n_2 independent observations on etch rate from machine 2, and that the samples are independent. Find the standard error of the estimator. $\mu = \frac{1}{3}\bar{X}_1 + \frac{2}{3}\bar{X}_2$

(a) $\frac{\sigma_1}{3}\sqrt{\frac{3}{n_1} + \frac{4}{n_2}}$ (b) $\frac{\sigma_1}{3}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ (c) $\frac{\sigma_1}{3}\sqrt{\frac{3}{n_1} + \frac{1}{n_2}}$ (d) $\frac{\sigma_1}{3}\sqrt{\frac{3}{n_1} + \frac{4}{3n_2}}$ (e) None of above

Test 2 Formula Sheet

Continuous R.V.:

Mean (Expected Value): $\mathbb{E}(X) = \mu = \int_{-\infty}^{\infty} xf(x)dx$

Variance: $\mathbb{V}(X) = \sigma^2 = \int_{-\infty}^{\infty}(x-\mu)^2 f(x)dx = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$

C.D.F: $F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)dt$

Common Distributions:

Normal Distribution (μ, σ^2):

$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$, $-\infty < x < \infty$

$\mathbb{E}(X) = \mu$, $\mathbb{V}(X) = \sigma^2$

Exponential Distribution (λ):

$f(x) = \lambda e^{-\lambda x}$, $x \geq 0$

$F(x) = P(X \leq x) = 1 - e^{-\lambda x}$

$\mathbb{E}(X) = \frac{1}{\lambda}$, $\mathbb{V}(X) = \frac{1}{\lambda^2}$

Marginal Distributions: $f_X(x) = \int f_{XY}(x,y)dy$, $f_Y(y) = \int f_{XY}(x,y)dx$

Correlation: $\rho_{XY} = \frac{Cov(X,Y)}{\sqrt{\mathbb{V}(X)\mathbb{V}(Y)}} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$

Standizing: $Z = \frac{X-\mu}{\sigma}$

Sample Mean: $\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n}$

Sample Variance: $s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} = \frac{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}{n-1}$

Q_1: The (n+1)/4 th number in the data set.

Q_3: The 3(n+1)/4 th number in the data set.

Outliers: $Q_1 - 1.5IQR, Q_3 + 1.5IQR$

Normal Probability Plot: $\Phi(z_j) = \frac{j-0.5}{n}$, $j = 1, 2, ..., n$

Central Limit Theorem Formula: $z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

z-Confidence Interval for the Mean: $\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$

t-Confidence Interval for the Mean: $\bar{x} \pm t_{\alpha/2,n-1} \frac{s}{\sqrt{n}}$

Confidence Interval for a Proportion: $\hat{p} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Sample Size (Mean): $n = (\frac{z_{\alpha/2}\sigma}{E})^2$

Sample Size (Proportion): $n = (\frac{z_{\alpha/2}}{E})^2 p(1-p)$

Sample Size (Proportion, not specified): $n = (\frac{z_{\alpha/2}}{E})^2 0.5^2$

Standard Normal Probabilities

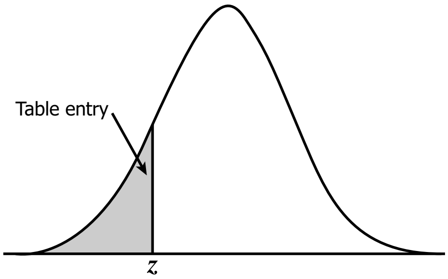


Table entry for z is the area under the standard normal curve to the left of z.

z	**.00**	**.01**	**.02**	**.03**	**.04**	**.05**	**.06**	**.07**	**.08**	**.09**
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

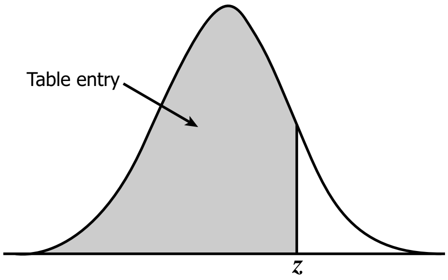


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

t Table

cum. prob	$t_{.50}$	$t_{.75}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$
one-tail	**0.50**	**0.25**	**0.20**	**0.15**	**0.10**	**0.05**	**0.025**	**0.01**	**0.005**	**0.001**	**0.0005**
two-tails	**1.00**	**0.50**	**0.40**	**0.30**	**0.20**	**0.10**	**0.05**	**0.02**	**0.01**	**0.002**	**0.001**
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
40	0.000	0.681	0.851	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
60	0.000	0.679	0.848	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
80	0.000	0.678	0.846	1.043	1.292	1.664	1.990	2.374	2.639	3.195	3.416
100	0.000	0.677	0.845	1.042	1.290	1.660	1.984	2.364	2.626	3.174	3.390
1000	0.000	0.675	0.842	1.037	1.282	1.646	1.962	2.330	2.581	3.098	3.300
z	0.000	0.674	0.842	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291
	0%	50%	60%	70%	80%	90%	95%	98%	99%	99.8%	99.9%
					Confidence Level						