

Test 1 Formula Sheet

Probability Rule:

Probability of Union (Two): $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Probability of Union (Three): $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

Conditional Probability: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Independence: $P(A \cap B) = P(A) \times P(B)$ or $P(B|A) = P(B)$

Total Probability Rule: Suppose E_1, E_2, \dots, E_k are k exhaustive and mutually exclusive events, then $P(B) = P(B \cap E_1) + P(B \cap E_2) + \dots + P(B \cap E_k) = P(B|E_1)P(E_1) + \dots + P(B|E_k)P(E_k)$

Discrete R.V.:

Mean (Expected Value): $\mathbb{E}(X) = \mu = \sum_x x f(x)$

Variance: $\mathbb{V}(X) = \sigma^2 = \sum_x (x - \mu)^2 f(x) = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$

C.D.F: $F(x) = P(X \leq x) = \sum_{y: y \leq x} f(y)$

Continuous R.V.:

Mean (Expected Value): $\mathbb{E}(X) = \mu = \int_{-\infty}^{\infty} x f(x) dx$

Variance: $\mathbb{V}(X) = \sigma^2 = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx = \mathbb{E}(X^2) - (\mathbb{E}(X))^2$

C.D.F: $F(x) = P(X \leq x) = \int_{-\infty}^x f(t) dt$

Common Distributions:

1. Binomial Distribution (n,p):

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}, x = 0, 1, 2, \dots, n, \mathbb{E}(X) = np, V(X) = np(1-p)$$

2. Hypergeometric Distribution (n,K,N):

$$f(x) = \frac{\binom{K}{x} \binom{N-K}{n-x}}{\binom{N}{n}}, x = \max\{0, n+K-N\} \text{ to } \min\{K, n\}, \mathbb{E}(X) = np, V(X) = np(1-p) \frac{N-n}{N-1}, \text{ where } p = \frac{K}{N}$$

3. Geometric Distribution (p):

$$f(x) = p(1-p)^{x-1}, x = 0, 1, 2, \dots, \mathbb{E}(X) = \frac{1}{p}, \mathbb{V}(X) = \frac{1-p}{p^2}$$

4. Negative Binomial Distribution (r,p):

$$f(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}, x = 0, 1, 2, \dots, \mathbb{E}(X) = \frac{r}{p}, \mathbb{V}(X) = \frac{r(1-p)}{p^2}$$

5. Poisson Distribution (λ):

$$f(x) = \frac{\lambda^x e^{-\lambda}}{x!}, x = 0, 1, 2, \dots, \mathbb{E}(X) = \lambda, \mathbb{V}(X) = \lambda$$

6. Normal Distribution (μ, σ^2):

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}, -\infty < x < \infty, \mathbb{E}(X) = \mu, \mathbb{V}(X) = \sigma^2$$

7. Exponential Distribution (λ):

$$f(x) = \lambda e^{-\lambda x}, x \geq 0, \mathbb{E}(X) = \frac{1}{\lambda}, \mathbb{V}(X) = \frac{1}{\lambda^2}$$

Approximation:

Normal Approximation to Binomial Distribution:

$$P(X \leq x) = P(X \leq x+0.5) \approx P\left(Z \leq \frac{x+0.5-np}{\sqrt{np(1-p)}}\right) \quad P(X \geq x) = P(X \geq x-0.5) \approx P\left(Z \geq \frac{x-0.5-np}{\sqrt{np(1-p)}}\right)$$

Standard Normal Probabilities

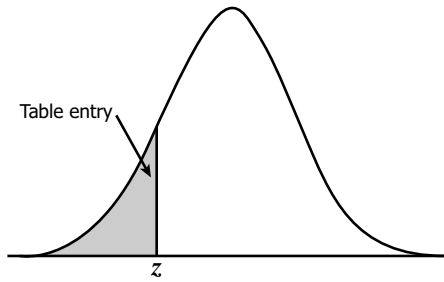


Table entry for z is the area under the standard normal curve to the left of z .

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
-3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
-3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
-3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
-3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
-2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
-2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
-2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
-2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
-2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
-2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
-2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
-2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
-2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
-2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
-1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
-1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
-1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
-1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
-1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
-1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
-1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
-1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
-1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
-1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
-0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
-0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
-0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
-0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
-0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
-0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
-0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
-0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
-0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
-0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

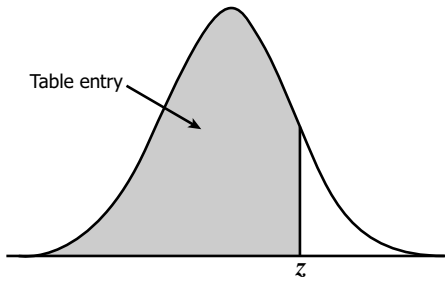


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