CausalAI系列读书会

Causal Concepts and Graphical Models

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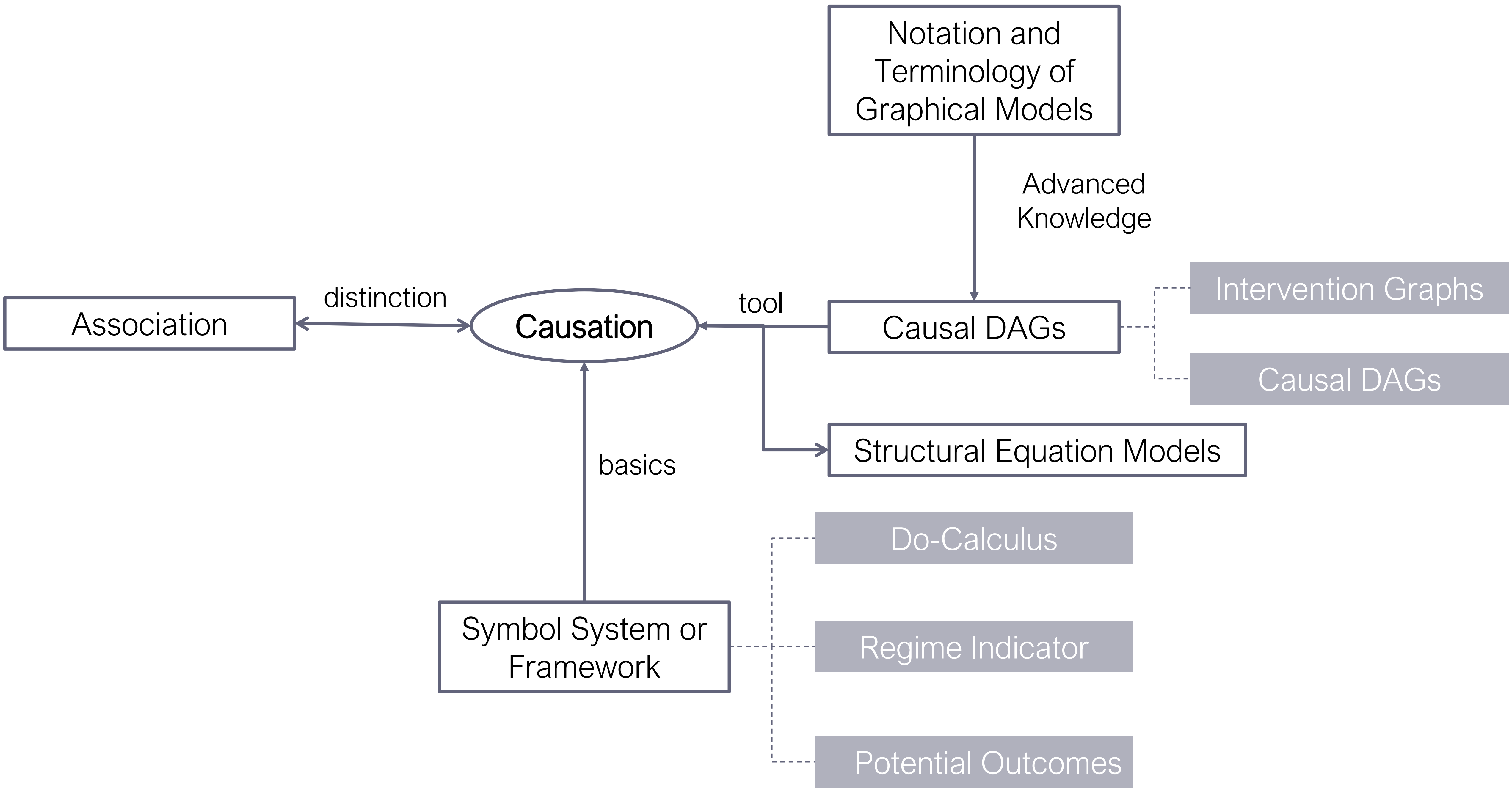
集智斑图
Swarma Pattern

Content



集智斑图 Swarma Pattern

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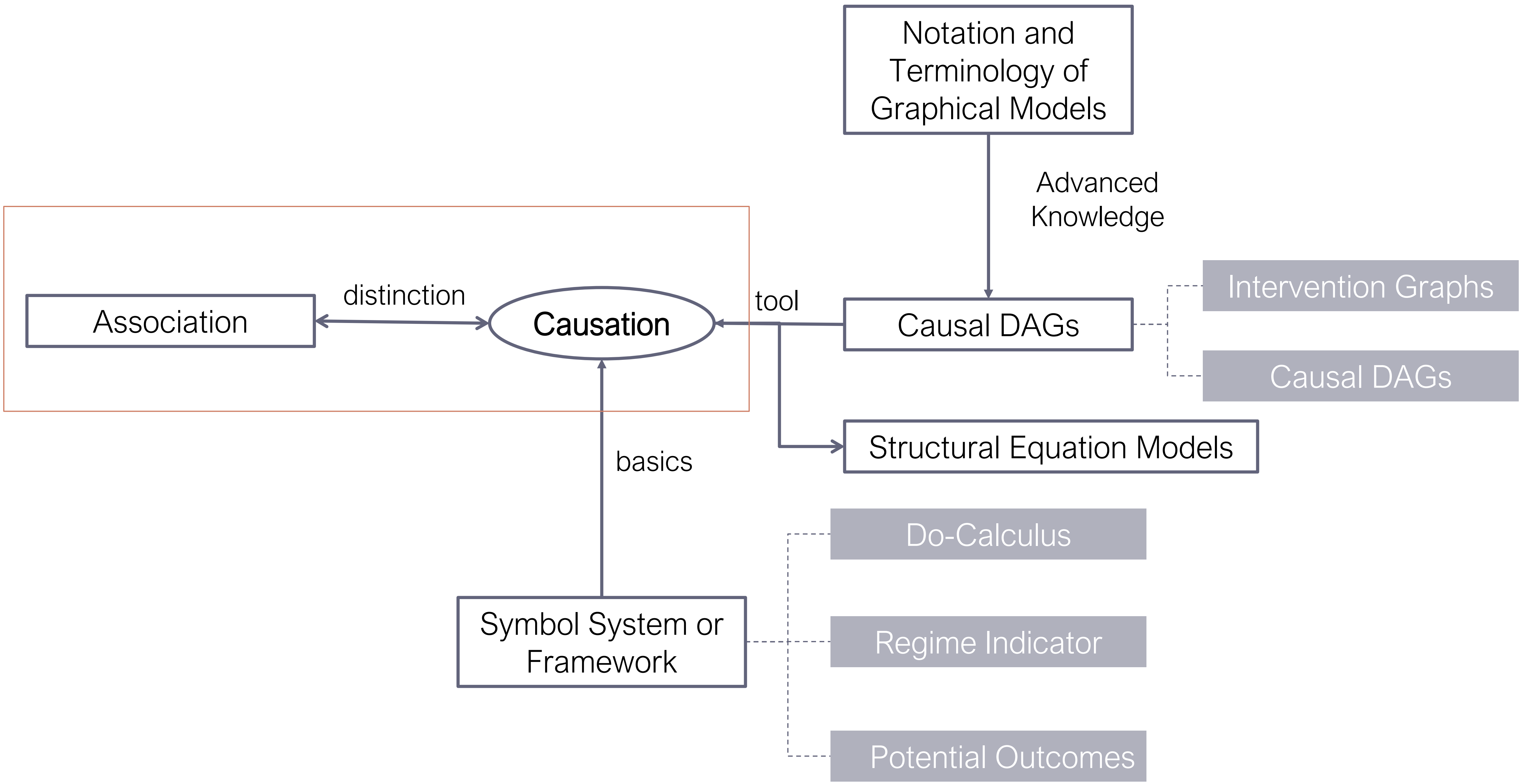

集智斑图 Swarma Pattern

Association versus Causation

- Traditional regression model : $p(y|x)$
  - Describe an association in the sense of how seeing different values of X to *predict* the value of Y

association

causation

- An explicit distinction between concepts of association and causation
  - While there can be an association between X and Y, a *manipulation* of X may not necessarily result in a corresponding change in Y.
  - X is causal for Y if some *manipulation* of X has an effect on Y

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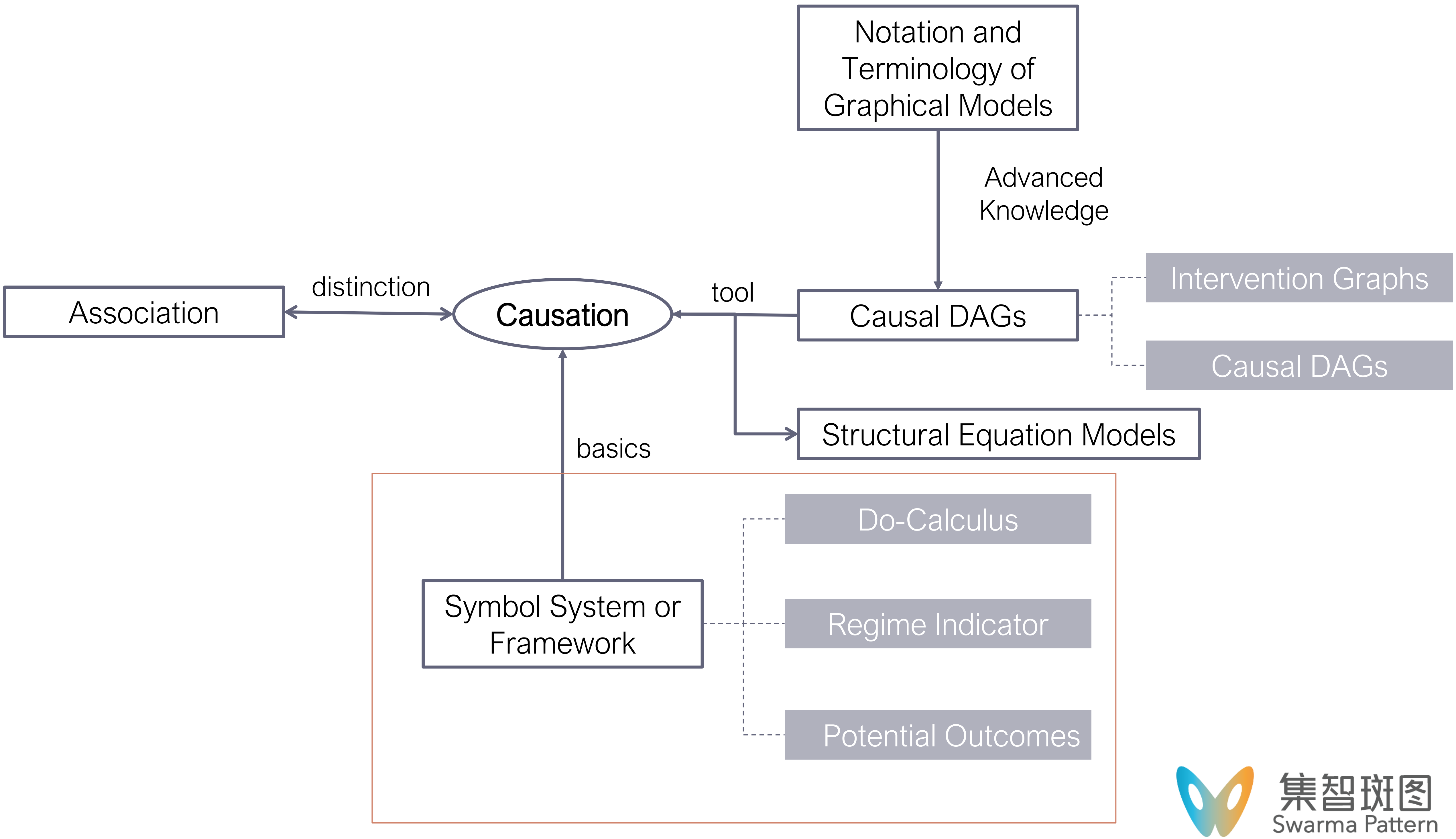

Notation and Terminology of Graphical Models
Advanced Knowledge
Association
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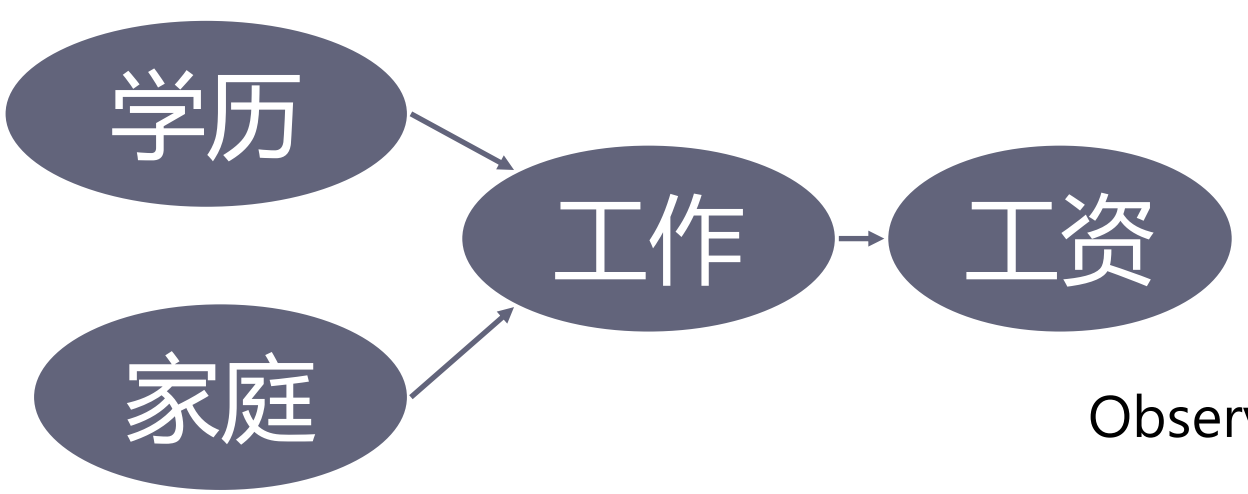
basics
Do-Calculus
Symbol System or Framework
Regime Indicator
Potential Outcomes

Symbol System

- **Do-calculus**
  - Observing $p\big(y; see(X = \tilde{x})\big) = p(y|\tilde{x})$
  - Intervening $p(y; do(X = \tilde{x}))$
  - the association is not the causation: $p\big(y; do(X = \tilde{x})\big) \neq p(y|\tilde{x})$
  - Do-calculus uses graphical rules to convert conditioning on doing into conditioning on seeing. And a $do\big(X_j = \tilde{x}_{\mathrm{j}}\big)$-intervention modularly replaces the factor $p(x_j|x_{pa(j)})$ by $I(x_j = \tilde{x}_{\mathrm{j}})$



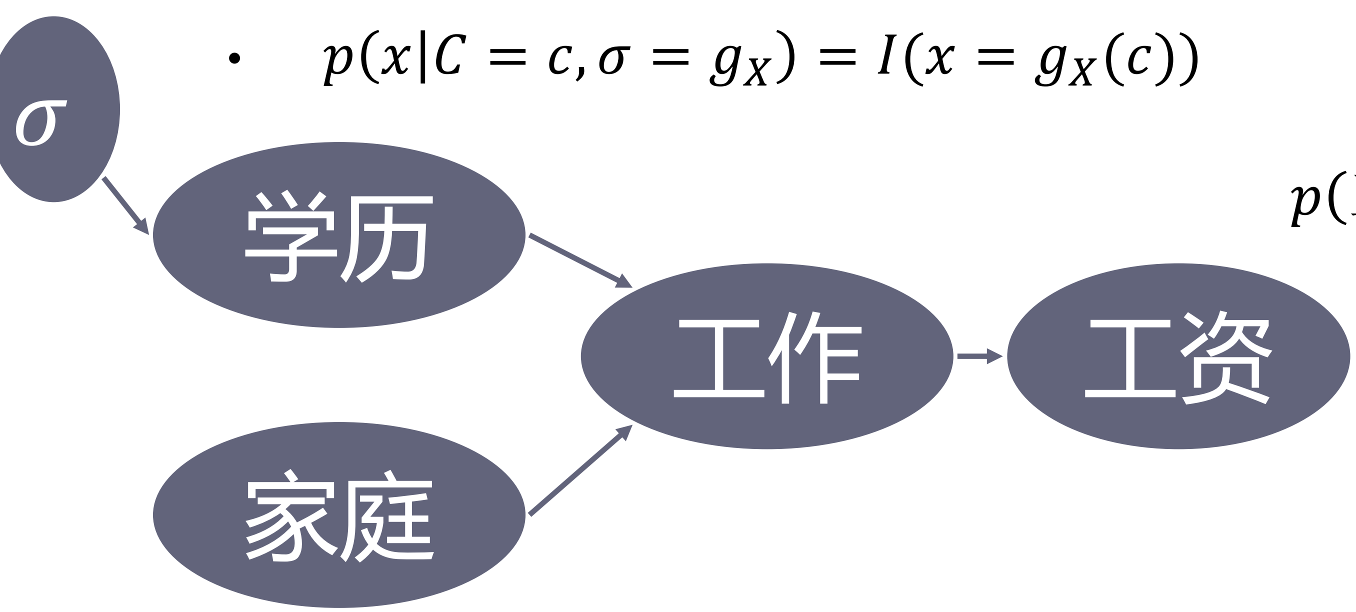
Observing $p\big(工作; see(学历 = 本科)\big) = p(工作|本科)$

Intervening $p(工作; do(学历 = 本科))$

集智斑图 Swarma Pattern

Symbol System

- **Regime Indicator**
- Regime is a specific value
  - Let σ be the indicator for the regime, taking value in S.
  - $p(x;\sigma = s) = p(x;s), s \in S$ —>The joint distribution of X under regime s.
  - $p(y,x;s) = p(x;s)p(y|x;s)$ seeing
  - Forcing X to be $\tilde{x}$, $p(x;\sigma = \tilde{x}) = I(x = \tilde{x})$
  - Association is not causation $p(y|\tilde{x};\sigma = \emptyset) \neq p(y;\sigma = \tilde{x})$
- Regime is conditional or dynamic interventions
  - $p(x|C = c, \sigma = g_X) = I(x = g_X(c))$



$$p(\text{工作},\text{学历};\text{本科}) = p(\text{学历};\text{本科})p(\text{工作}|\text{学历};\text{本科})$$

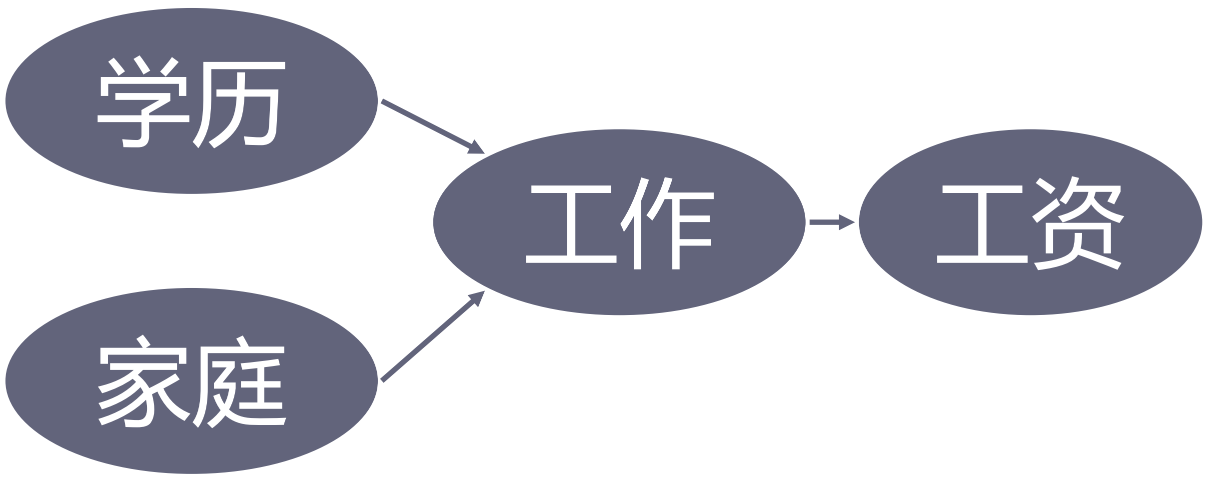
$$p(\text{学历};\text{本科}) = \mathrm{I}(\text{学历} = \text{本科})$$

$$p(\text{工作}|\text{学历};\sigma = \emptyset) \neq \mathrm{I}(\text{工作}|\text{学历} = \text{本科})$$

Symbol System

- **Potential Outcomes**
  - Goal: Considering some causal effect of X on an outcome Y.
  - Defining the potential outcome $Y(\tilde{x})$ to be the value of Y that if X is forced to $\tilde{x}$.
  - Association is not causation $p(Y(x) = y) \neq p(y|x)$
  - *Advantage1.* For this method is formulated at the level of variables instead of distributions, they can be used to express in individual causal effects as functions of $Y^i(\tilde{x})$ for different $\tilde{x}$.
  - *Advantage2.* Because of the same reason, it also allow to express "cross-world" independent such as $Y(\tilde{x}) \perp W(x')|(Z, X)$ for two different value $\tilde{x}, x'$.

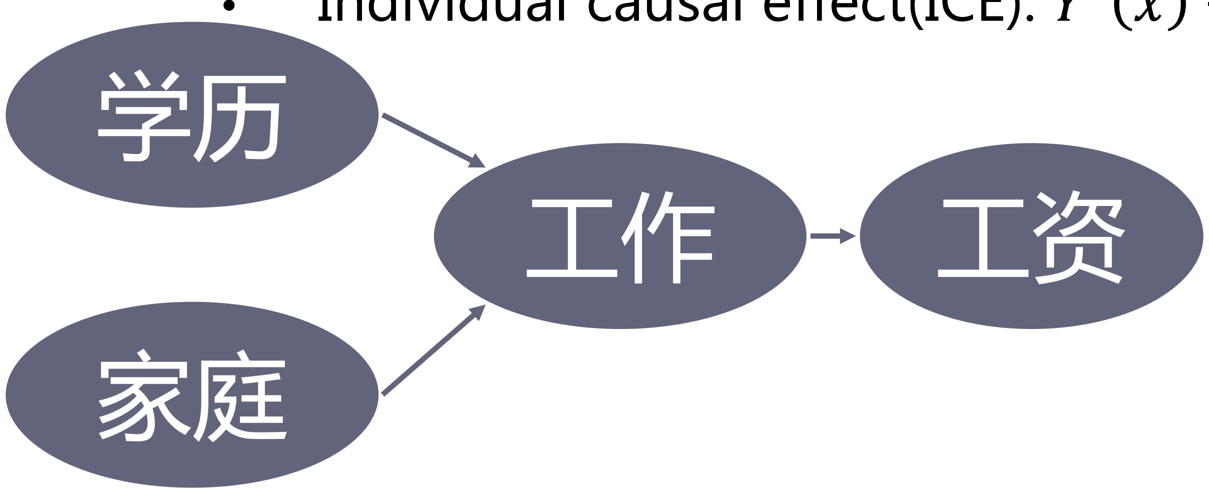
Symbol System



- 工作（本科）
- $p(\text{工作（本科）}=\text{程序员})\neq p(\text{工作}|\text{本科})$
- $\text{工资}(\text{高于}1\text{万})\perp\text{家庭}(\text{城市})|(\text{工作})$

Symbol System

- **Causal Effect**
  - X has a causal effect on Y if an intervention in the former affects the distribution of the latter.
    $$p(\text{工作}; do(\text{学历} = \text{本科})) \neq p(\text{工作}; do(X = \text{小学}))$$
  - Do-calculus: $p(y; do(X = \tilde{x})) \neq p(y; do(X = \tilde{x}'))$, if $\tilde{x} \neq \tilde{x}'$
  - Regime indicator: $p(y; \sigma = s) \neq p(y; \sigma = s')$, if $s, s' \in S$ denote two different interventions in X
    $$p(\text{工作}; \sigma = \text{本科}) \neq p(\text{工作}; \sigma = \text{小学})$$
  - Potential outcomes:
    $$E(\text{工作}; do(\text{学历} = \text{本科})) \neq E(\text{工作}; do(X = \text{小学}))$$
    - average causal effect(ACE) : $\mathrm{E}\big(Y; do(X = \tilde{x})\big) - E(Y; do(X = \tilde{x}'))$
    - Individual causal effect(ICE): $Y^i(\tilde{x}) - Y^i(\tilde{x}')$



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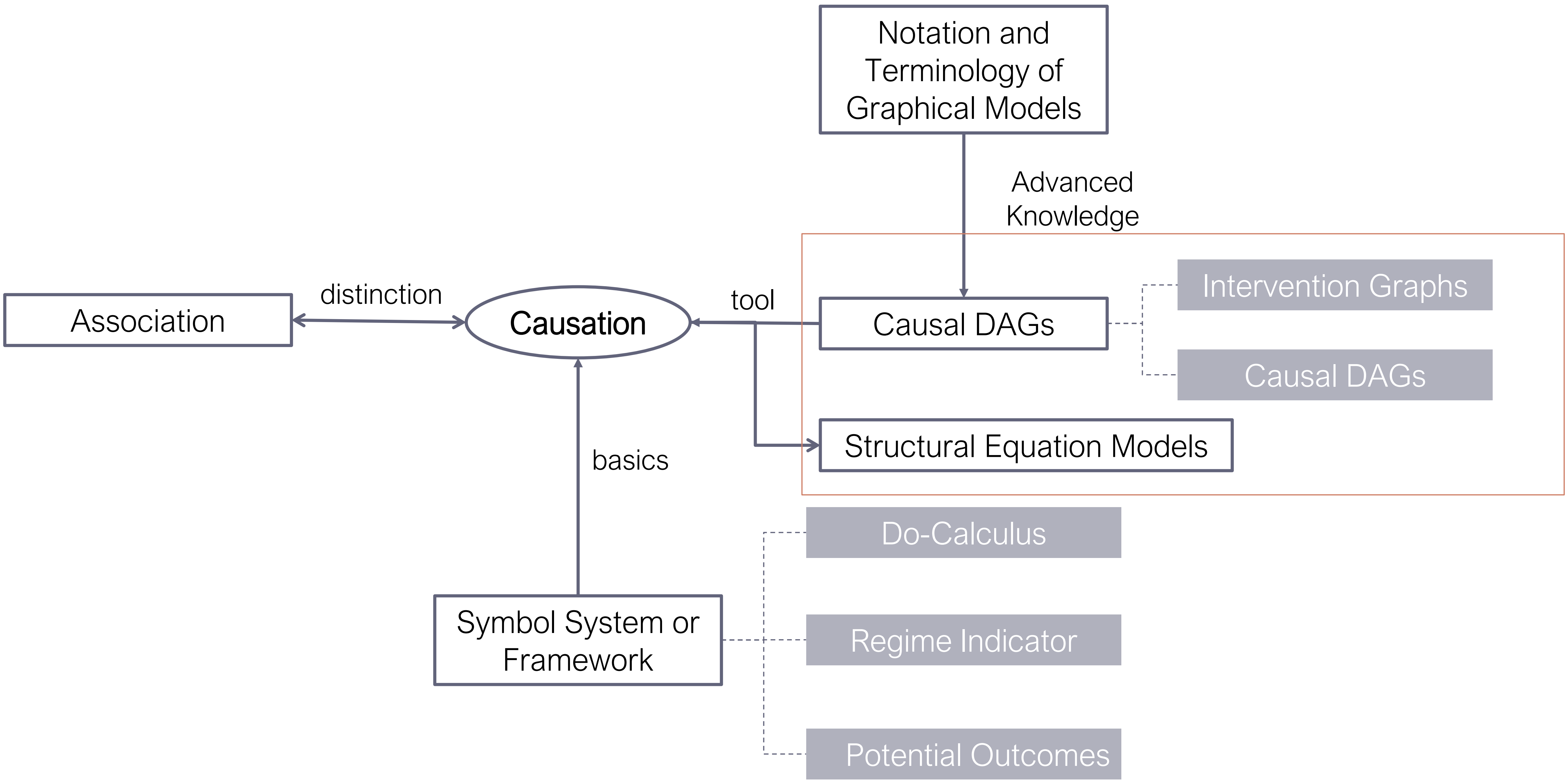

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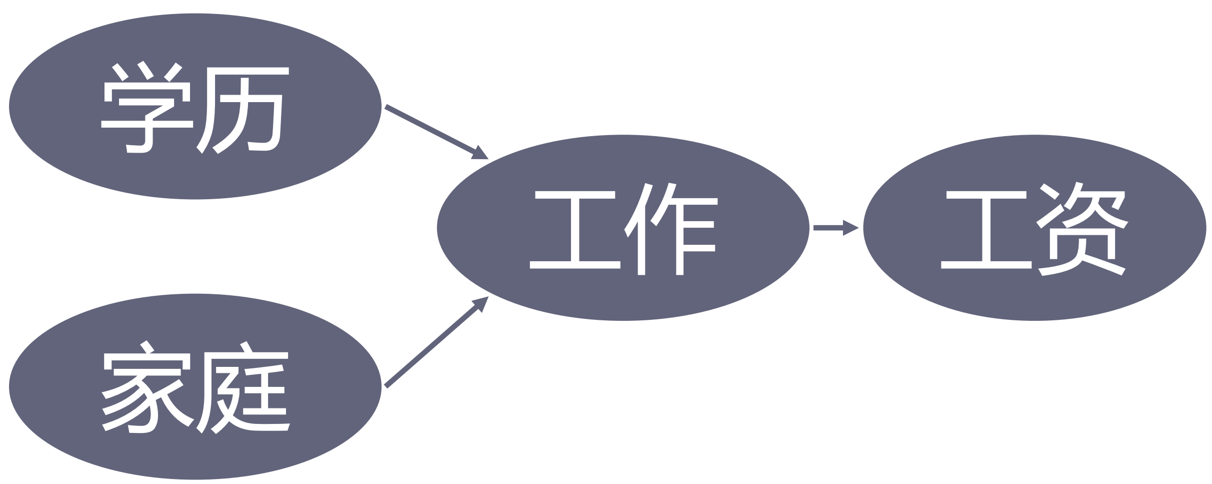
Notation and Terminology of Graphical Models

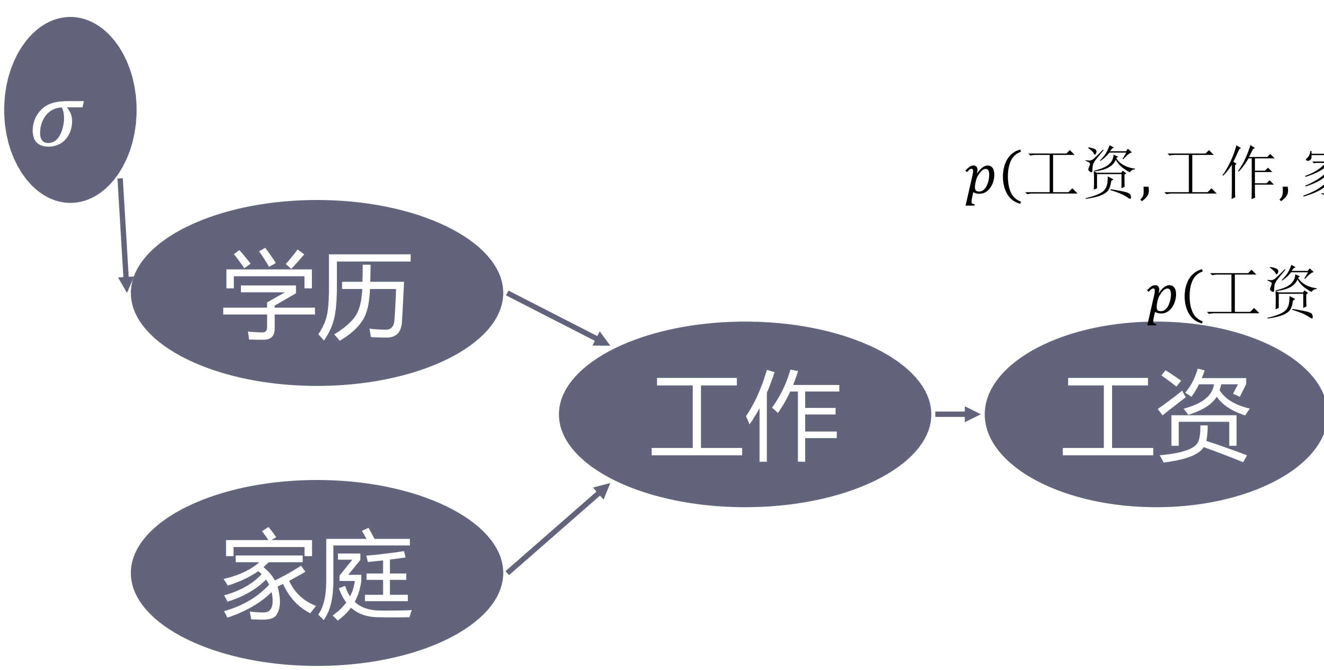


- If $A \subset V$, subvectors and induced subgraphs are denoted by X_A, G_A.
- Pa(工作): 学历、家庭
- Ch(工作): 工资
- De(学历): 工作、工资
- Nd(工资): 学历、家庭、工作
- $p(x) = \prod_{k \in \mathrm{V}} p(x_k | x_{pa(k)}) \Leftrightarrow X_k \perp X_{nd(k) \backslash \mathrm{pa}(k)} | X_{pa(k)}$
- $p(x) = p(\text{工资}|\text{工作})p(\text{工作}|\text{学历})p(\text{学历})p(\text{工作}|\text{家庭})p(\text{家庭})$

Causal DAGs

- **Intervention Graphs**

Definition 15.3.1 (Intervention DAG and Model). *Consider the random vector* $\mathbf{X} = (X_1, \ldots, X_K)$, *on vertices* $V = \{1, \ldots, K\}$ *of a DAG* $\mathcal{G}$. *Let* $\mathcal{S}$ *denote a set of regimes and let* $p(\cdot; \sigma = s), s \in \mathcal{S}$, *be the joint distributions under the respective regimes. The augmented DAG* $\mathcal{G}^\sigma = (V \cup \{\sigma\}, E^\sigma)$ *is called the* intervention DAG *for* $\mathbf{X}$ *under regimes* $\mathcal{S}$ *if it has the following properties:*
(i) the node σ *is a source node (has no incoming directed edges),*
(ii) each distribution $p(\mathbf{x}; s), s \in \mathcal{S}$, *factorizes according to* $\mathcal{G}$,
(iii) for disjoint $A, B \subset V$, *whenever* B *is d-separated from* σ *by* A *in* $\mathcal{G}^\sigma$ *we have:* $p(\mathbf{x}_B | \mathbf{x}_A; s) = p(\mathbf{x}_B | \mathbf{x}_A; s')$, *for all* $s \neq s'$. *This is denoted by* $\mathbf{X}_B \perp\!\!\!\perp \sigma | \mathbf{X}_A$.



$$p(工资,工作,家庭;s) = p(工资|工作;s)p(工作|学历;s)p(家庭;s)$$

$$p(工资|工作;小学) = p(工资|工作,本科;s'), for\ 小学 \neq 本科$$

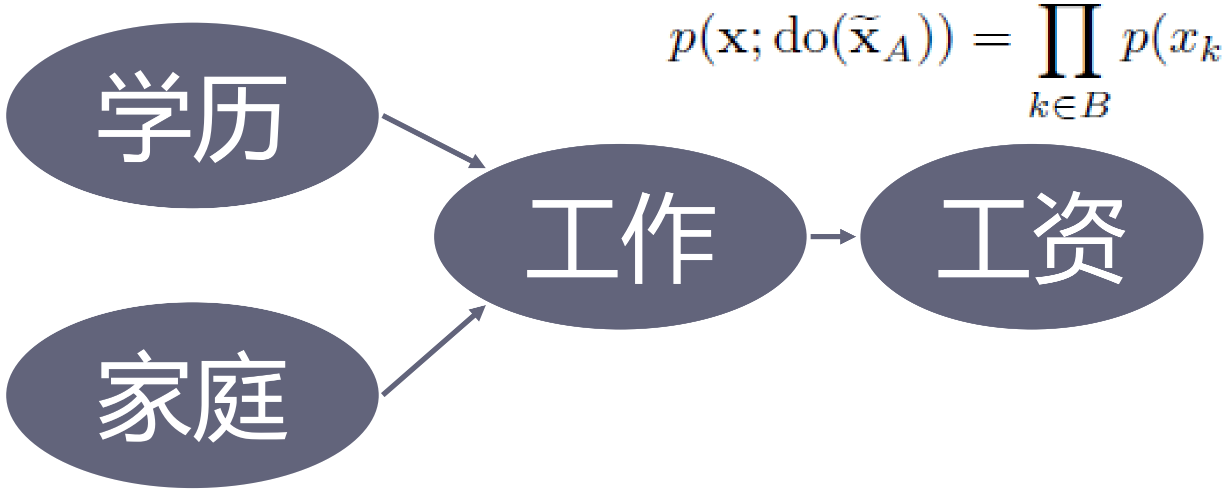
$$p(家庭;小学) = p(家庭;本科), for\ 小学 \neq 本科$$

Causal DAGs

- **Causal Graphs**

Definition 15.3.2 (Causal DAG). *Consider a DAG $\mathcal{G} = (V, E)$ and a random vector $\mathbf{X} = (X_1, \ldots, X_K)$ with distribution p. Then $\mathcal{G}$ is called a* causal DAG *for* $\mathbf{X}$ *if p satisfies the following:*
(i) p factorizes, and thus is Markov, according to $\mathcal{G}$, and
(ii) for any $A \subset V$ and any $\tilde{\mathbf{x}}_A, \mathbf{x}_B$ in the domains of $\mathbf{X}_A, \mathbf{X}_B$, where $B = V \backslash A$,

$$p(\mathbf{x}; \mathrm{do}(\tilde{\mathbf{x}}_A)) = \prod_{k \in B} p(x_k | x_{\mathrm{pa}(k)}) \prod_{j \in A} \mathbf{1}\{x_j = \tilde{x}_j\}. \tag{15.2}$$

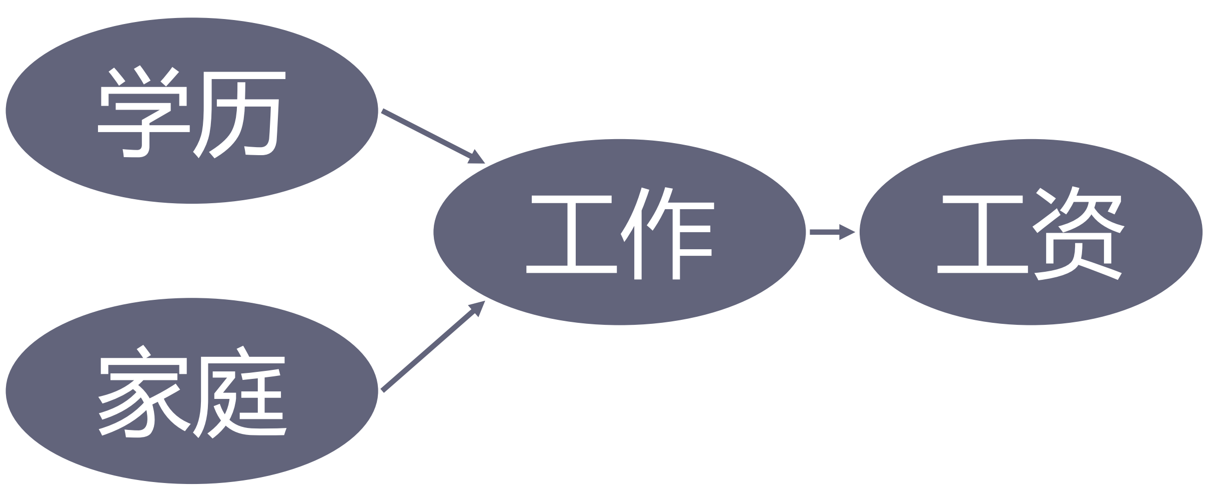


$$p(\text{工资}, \text{工作}, \text{家庭}, \text{学历}; \mathrm{do}(\text{小学})) = p(\text{工资}|\text{工作})p(\text{工作}|\text{学历}, \text{家庭})p(\text{家庭})I(\text{学历} = \text{小学})$$

集智斑图 Swarma Pattern

Structural Equation Method

- Assuming that X_k is a function of its graphical parents and possibly a random variables ϵ_k
- $X_k \coloneqq f_k(X_{pa(k)}, \epsilon_k)$
- Combining DAGs and with NPSEMs:
- NPSEMs allow to simply add equations for each intervention. The result is a system of equations that simultaneously describes what would happen if X_j was fixed at value $\widetilde{x_j}$ as well as at value $\widetilde{x_j}'$ etc.



- $工作 \coloneqq f_{工作}(学历, 家庭, \epsilon_k)$
- $工资 \coloneqq f_{工资}(工作, \epsilon_k)$

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Swarma Pattern

Comparison

- **Discussion**
  - The difference between the causal graph and Bayes