LE and nonlinearity

I worked some result like "LE correlation with maps", but when organizing the contents, I found that LE can actually be computed under different approximations and I just add some new content about it

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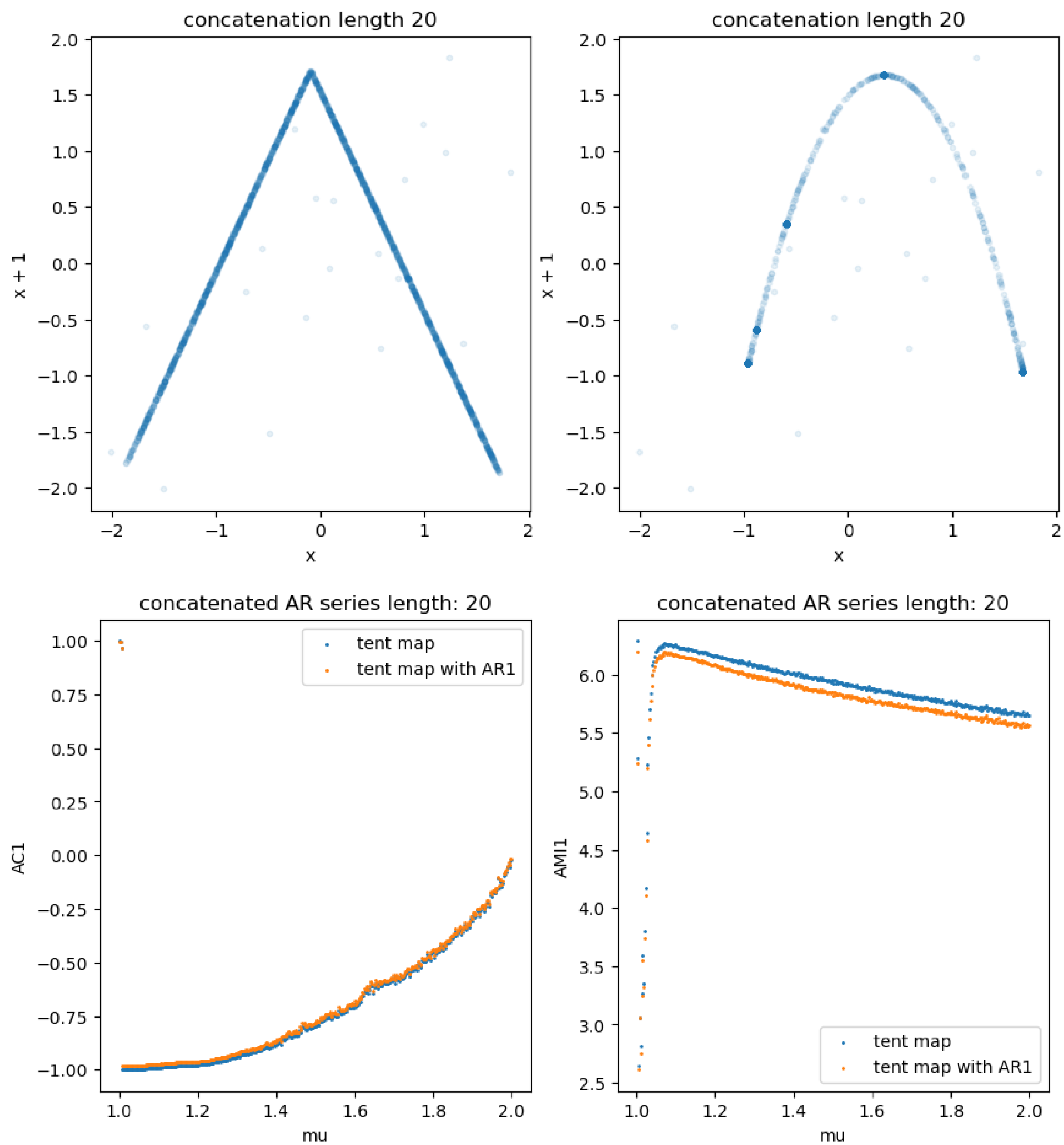
In the litereview, I noticed that a comment on what Lyapunov exponent does "**Make sure you clearly define and distinguish two related concepts: (i) nonlinearity, and (ii) chaos. Is one a subset of another? What are the minimal conditions for a system to display chaos?**".

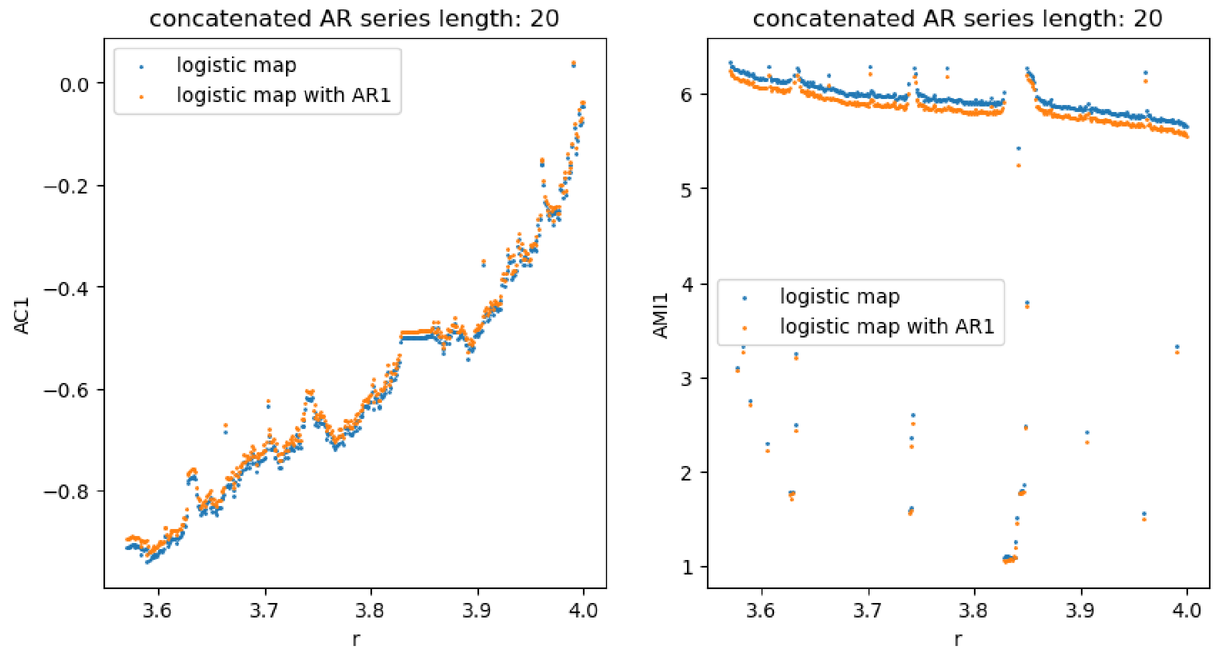
To my understanding, nonlinearity presents when a time series could not fit in a linear model. That is, for a linear model, no combinations of parameters can be used to reproduce a nonlinear time series.
And chaos for a time series, means infinite and unpredictable paths (no periodic cycles).

In this case I prefer to say that "chaos" is a subset of "nonlinearity", nonlinear systems can exhibit chaos. Since when chaos onsets, it definitely couldn't fit into a linear model and a nonlinear model may be periodic.
Therefore I think when chaos is detected (LE > 0), it would be safe to that there is a nonlinear structure.

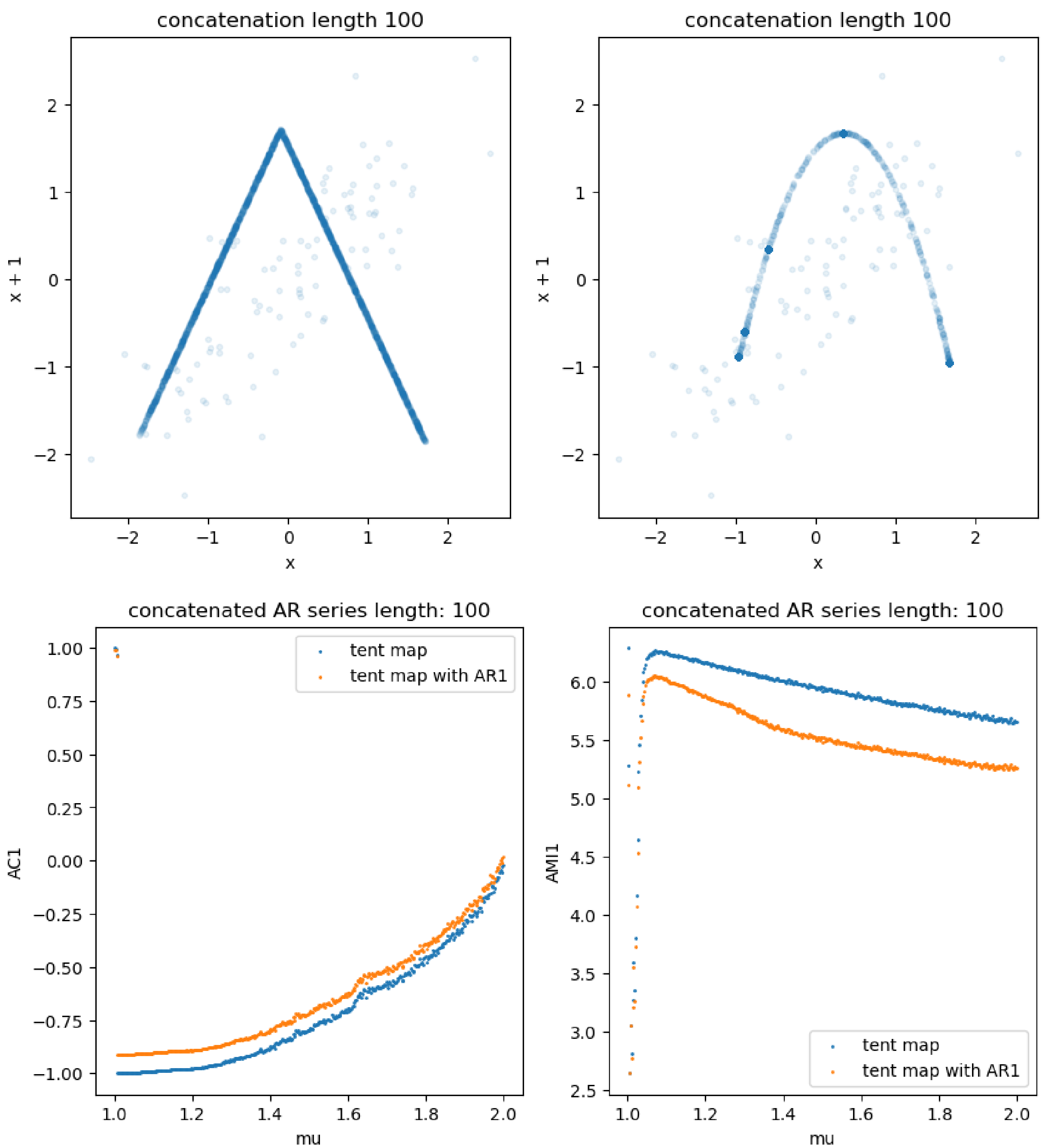
I searched and did a lot reading on degree of degree of nonlinearity.
The most relevant one I have found is https://doi.org/10.1080/00207720110121105 .
The author controls the number of nonlinear terms included in a model used for a Duffing Ueda system.
I tried to concatenate a linear segment to the end of each nonlinear series, control the length of concatenated AR series. (both z-scored)



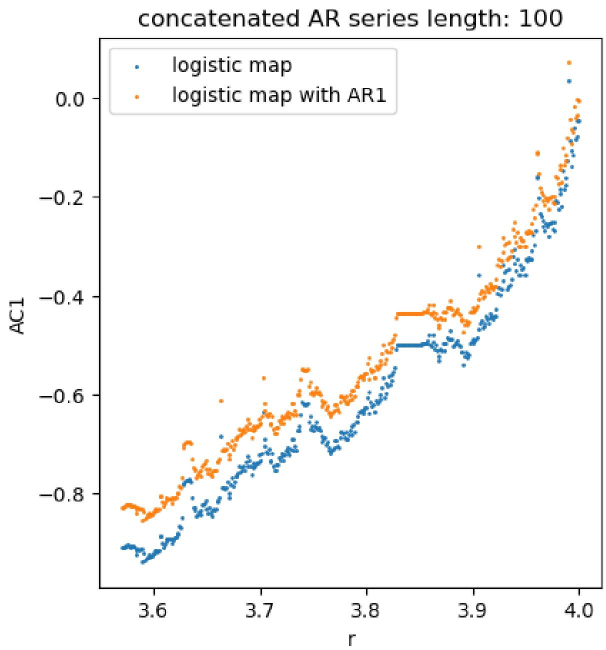
concatenation length 20
x + 1
x
concatenation length 20
x + 1
x
concatenated AR series length: 20
AC1
mu
tent map
tent map with AR1
concatenated AR series length: 20
AMI1
mu
tent map
tent map with AR1

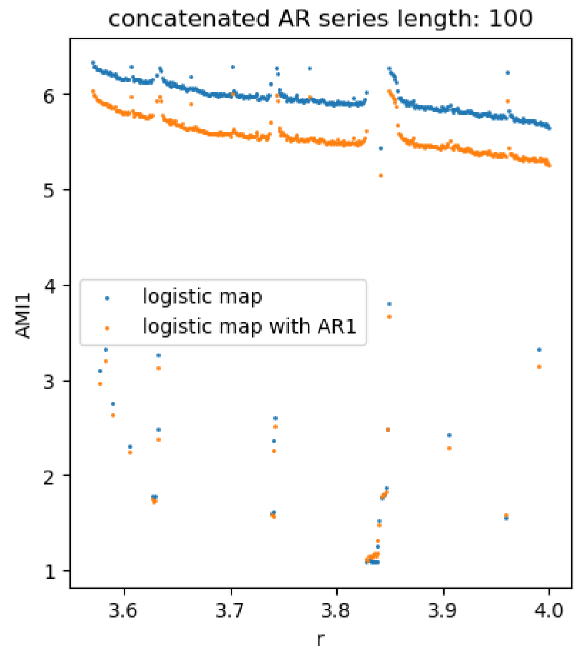


not surprisingly, there's a tiny shift up for AC part and shift down for AMI part



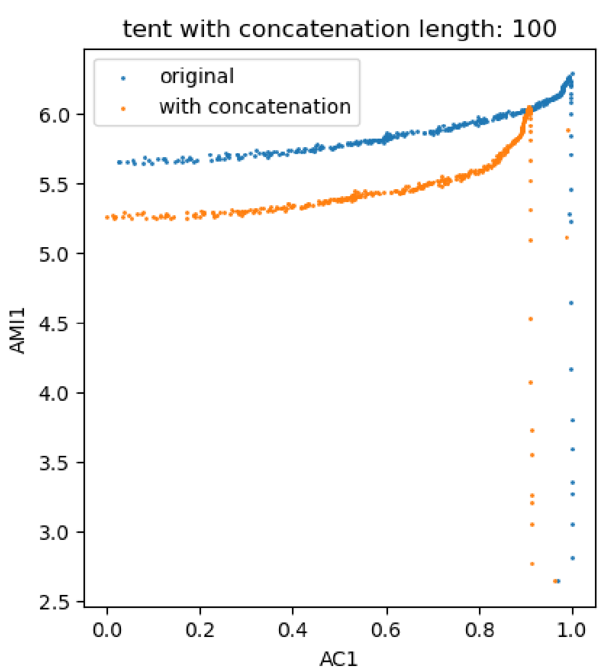
concatenation length 100
x + 1
x
concatenation length 100
x + 1
x
concatenated AR series length: 100
AC1
mu
tent map
tent map with AR1
concatenated AR series length: 100
AMI1
mu
tent map
tent map with AR1

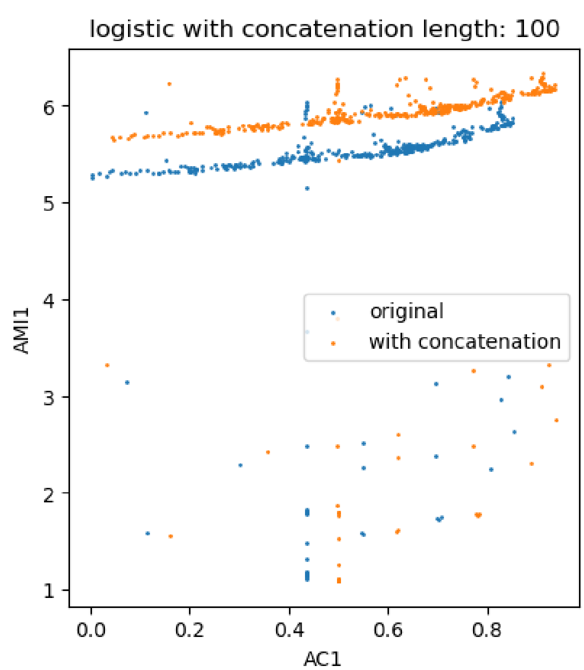




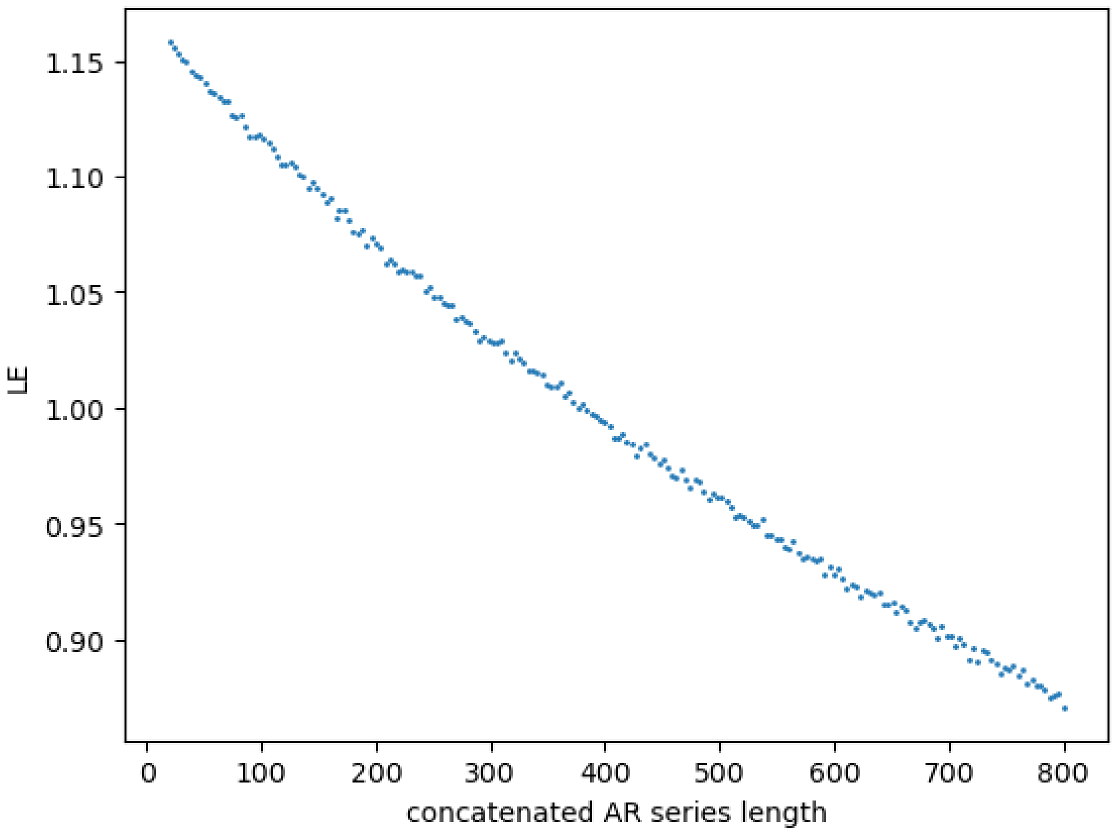
And shift more for larger length

As for AC AMI relation, for each AC1, the corresponding AMI2 drops and it results an overall drop in AC-AMI plot.





And obtain this



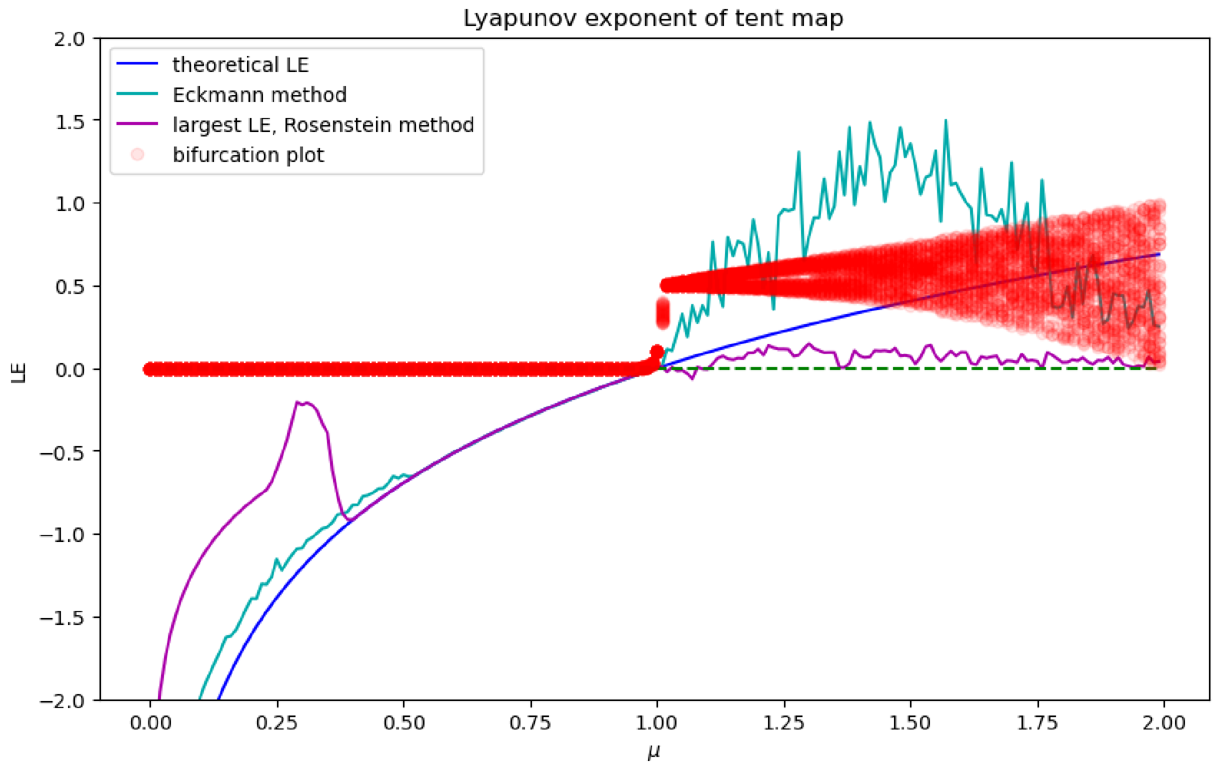
Under the assumption: the more linear part added in the series, the more nonlinear the series would be

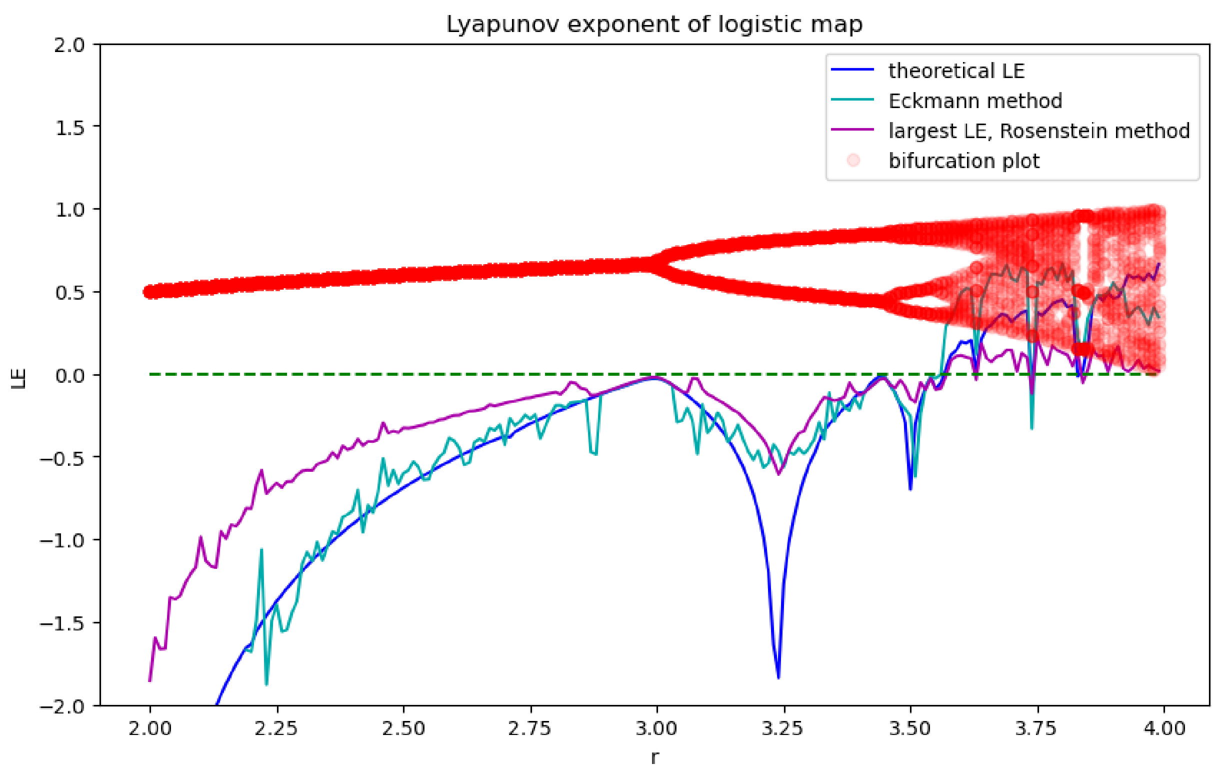
LE can be an indicator to degree of nonlinearity

The Lyapunov exponent describes the rate of separation of two infinitesimally close trajectories of a dynamical system in phase space.
Two options for LE:

- using the algorithm of Eckmann et al. for a general LE apporixmation
    - Manfred Füllsack, "Lyapunov exponent",
        - http://systems-sciences.uni-graz.at/etextbook/sw2/lyapunov.html
    - Steve SIU, Lyapunov Exponents Toolbox (LET),
        - http://www.mathworks.com/matlabcentral/fileexchange/233-let/content/LET/findlyap.m
    - Rainer Hegger, Holger Kantz, and Thomas Schreiber, TISEAN,
        - http://www.mpipks-dresden.mpg.de/~tisean/Tisean_3.0.1/index.html
- using the algorithm of Rosenstein et al. for largest LE

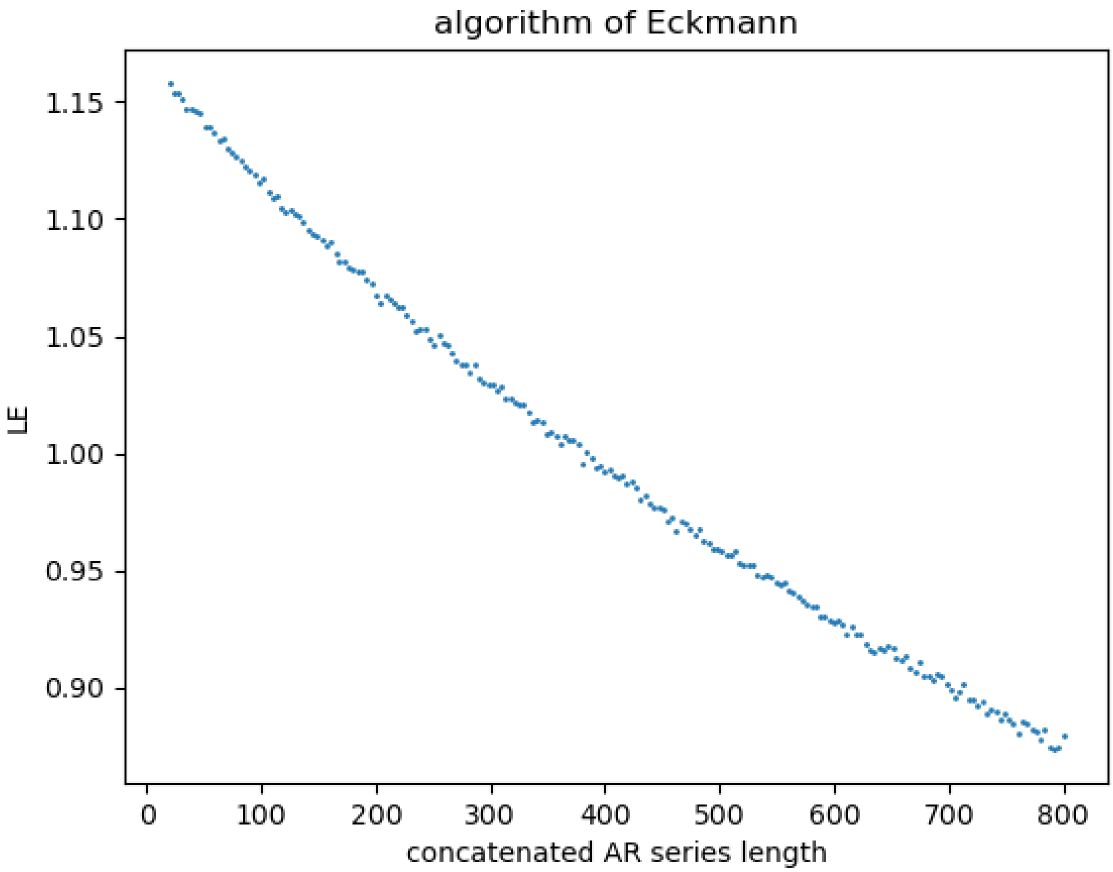
- vectorize with each lag, X_i = [x_i, x_(i+lag), x_(i+2*lag), ..., x_(i+(emb_dim-1) * lag)], then find the closest neighbor X_j using the euclidean distance. Follow the trajectories from X_i and X_j in time in a chaotic system the distances between X_(i+k) and X_(j+k) denoted as d_i(k) can be modeled as a power law d_i(k) = c * e^(lambda * k) where lambda is a good approximation of the highest Lyapunov exponent
- Rosenstein, Michael T., et al. “A Practical Method for Calculating Largest Lyapunov Exponents from Small Data Sets.” *Physica D: Nonlinear Phenomena*, vol. 65, no. 1-2, May 1993, pp. 117–134, https://doi.org/10.1016/0167-2789(93)90009-p. Accessed 1 Sept. 2019.
- mirwais, "Largest Lyapunov Exponent with Rosenstein's Algorithm",
  - http://www.mathworks.com/matlabcentral/fileexchange/38424-largest-lyapunov-exponent-with-rosenstein-s-algorithm
- Shapour Mohammadi, "LYAPROSEN: MATLAB function to calculate Lyapunov exponent"
  - https://ideas.repec.org/c/boc/bocode/t741502.html



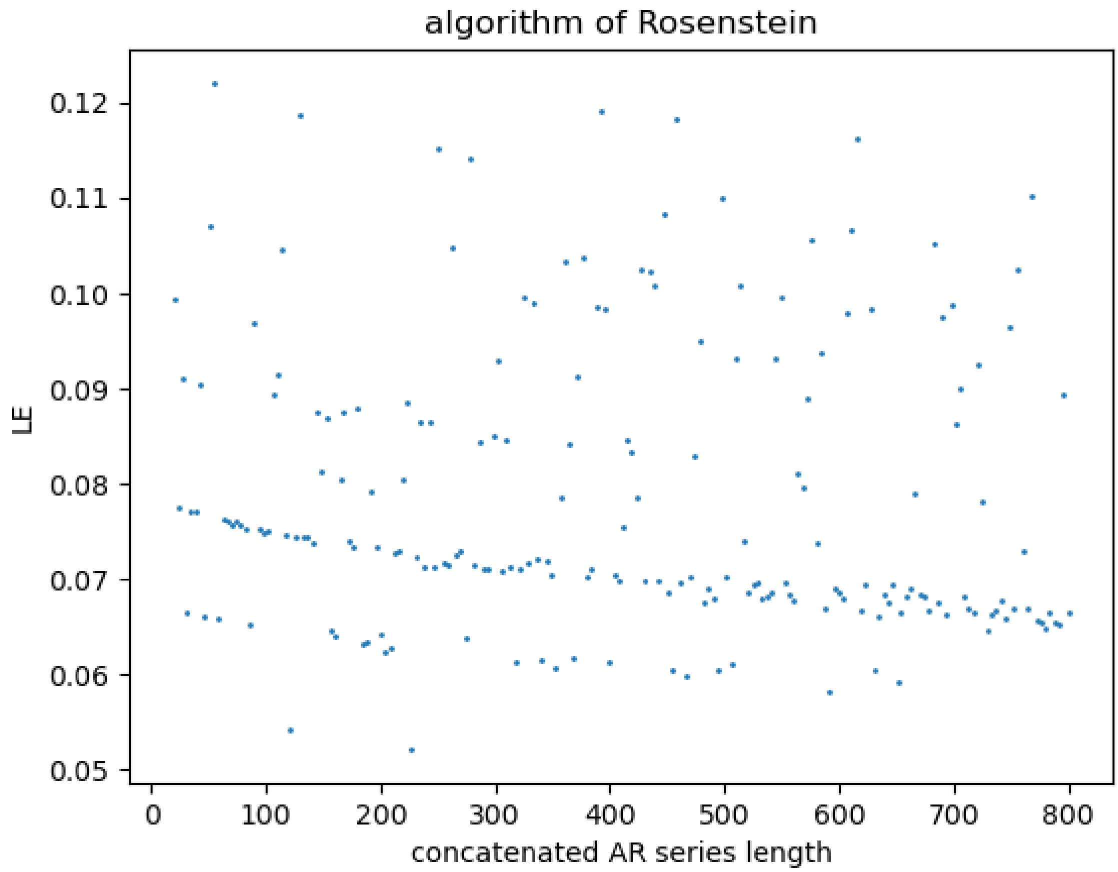


Actually both two of the approximations not recovering good enough for the theoretical LE

And with both method



algorithm of Eckmann
LE
1.15
1.10
1.05
1.00
0.95
0.90
0
100
200
300
400
500
600
700
800
concatenated AR series length



Rosenstein's method is about 5 times faster that Eckmann's (5.3it/s vs 1.5it/s), however it fluctuates much more