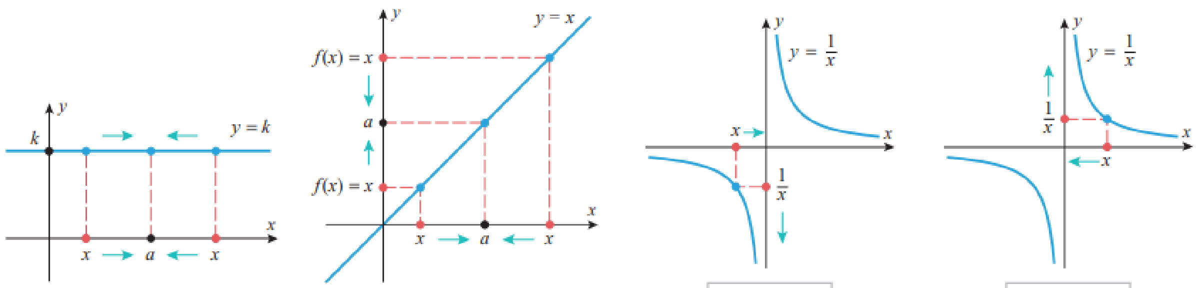
# One Sided Limit

**1.2.1 THEOREM** *Let a and k be real numbers.*

(a) $\lim_{x \to a} k = k$ (b) $\lim_{x \to a} x = a$ (c) $\lim_{x \to 0^-} \frac{1}{x} = -\infty$ (d) $\lim_{x \to 0^+} \frac{1}{x} = +\infty$




$\lim_{x \to a} k = k$ $\lim_{x \to a} x = a$ $\lim_{x \to 0^-} \frac{1}{x} = -\infty$ $\lim_{x \to 0^+} \frac{1}{x} = +\infty$

**1.2.2 THEOREM** *Let a be a real number, and suppose that*

$$\lim_{x \to a} f(x) = L_1 \quad \text{and} \quad \lim_{x \to a} g(x) = L_2$$

*That is, the limits exist and have values $L_1$ and $L_2$, respectively. Then:*

(a) $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) = L_1 + L_2$

(b) $\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) = L_1 - L_2$

(c) $\lim_{x \to a} [f(x)g(x)] = \left(\lim_{x \to a} f(x)\right)\left(\lim_{x \to a} g(x)\right) = L_1 L_2$

(d) $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{L_1}{L_2}$, *provided* $L_2 \neq 0$

(e) $\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} = \sqrt[n]{L_1}$, *provided* $L_1 > 0$ *if n is even.*

*Moreover, these statements are also true for the one-sided limits as $x \to a^-$ or as $x \to a^+$.*

**Question:** Find the limits.

**21.** $\lim_{y \to 6^+} \dfrac{y+6}{y^2-36}$

**Answer:**

The limit is $+\infty$.

**Question:** Find the limits.

**25.** $\lim_{x \to 4^-} \dfrac{3-x}{x^2-2x-8}$

**Answer:**

The limit is $+\infty$.

**Question:** Find the limits.

**28.** $\lim_{x \to 3^-} \dfrac{1}{|x-3|}$

**Answer:**

The limit is $+\infty$.

**Question:** Find the limits.

**31.** Let

$$f(x) = \begin{cases} x-1, & x \leq 3 \\ 3x-7, & x > 3 \end{cases}$$

Find

(a) $\lim_{x \to 3^-} f(x)$ (b) $\lim_{x \to 3^+} f(x)$ (c) $\lim_{x \to 3} f(x)$.

**Answer:**

**(a)** 2 **(b)** 2 **(c)** 2