Calculus and Analytical Geometry (MT1003)

Date: November 6th, 2024
Course Instructors
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Sessional-II Exam

Total Time: 1 Hour
Total Marks: 30
Total Questions: 03
Semester: Fall-2024
Campus: Karachi
Dept: Computer Science

Student Name	Roll No	Section	Student Signature

Note: Attempt all parts of a question at once.

*CLO # 4: Apply derivatives and integrals for solving different problems arising in daily life.*

Q:1   [6+4]

(a) For the function defined by $f(x) = \frac{x^2-3}{x-2}$, find:

   i. The increasing and decreasing intervals of the function.
   ii. The critical points and identify them as stationary and non-stationary.
   iii. The inflection point/s, if any.
   iv. The intervals in which the function is concave up and concave down.
   v. Relative extrema, if any.

(b) An air traffic controller spots two planes at the same altitude converging on a point as they fly at right angles to each other. One plane is 150 miles from the point moving at 450 miles per hour. The other plane is 200 miles from the point moving at 600 miles per hour. At what rate is the distance between the planes decreasing?
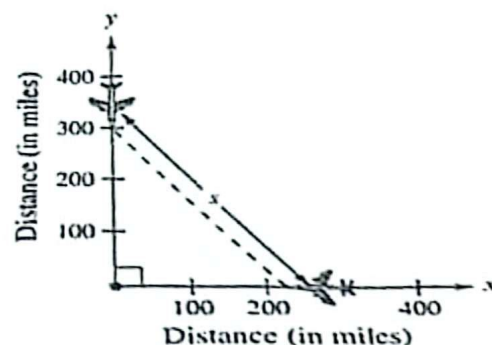
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CLO # 3: *Express the ideas of rate of change, derivatives and anti-derivatives using the concept of Limits and Continuity.*

Q:2 [3+5+2]

(a) Find $\frac{dy}{dx}$ for the following expression:

$$x^2\cos y + \sin 2y = xy$$

(b) Evaluate the Limits:

i. $\lim\limits_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$

ii. $\lim\limits_{x\to +\infty}[x - \ln(1 + 2e^x)]$

(c) Verify the hypothesis of the Mean value theorem are satisfied on the given interval, and find all values of c in that interval that satisfy the conclusion of the theorem.

$$f(x) = x + 3\cos x \quad : \quad [-\pi, \pi]$$

CLO # 4: *Apply derivatives and integrals for solving different problems arising in daily life.*

Q:3 [3+3+4]

Integrate the following integrals by appropriate techniques:

i. $\displaystyle\int_{\frac{\pi^2}{36}}^{\frac{\pi^2}{4}} \frac{\cos\sqrt{\theta}}{\sqrt{\theta}\sin\sqrt{\theta}}\, d\theta$

ii. $\displaystyle\int x\tan^2 x\, dx$

iii. $\displaystyle\int \frac{1}{(x^2+2x+3)^{\frac{3}{2}}}\, dx$

BEST OF LUCK