Ex : 7.6

Formula's :-

1) $\sin x = \frac{2u}{1+u^2}$

2) $\cos x = \frac{1-u^2}{1+u^2}$

3) $\tan x = \frac{2u}{1-u^2}$

4) $\sin(x/2) = \frac{u}{\sqrt{1+u^2}}$

5) $\cos(x/2) = \frac{1}{\sqrt{1+u^2}}$

6) $dx = \frac{2\,du}{1+u^2}$

7) $u = \tan(x/2)$

Q1) $\int \frac{dx}{1-\sin x+\cos x}$

sol:

$$\int \frac{dx}{1-\left(\frac{2u}{1+u^2}\right)+\left(\frac{1-u^2}{1+u^2}\right)}$$

$$\int \frac{2\,du}{\frac{\cancel{1+u^2}-2u+1-\cancel{u^2}}{\cancel{1+u^2}}} \cdot \frac{1}{\cancel{1+u^2}}$$

$$\int \frac{\cancel{2}\,du}{\cancel{2}(1-u)}$$

$$\int \frac{du}{(1-u)}$$

$$-\ln(1-u)+c = -\ln[1-\tan(x/2)]+c$$

Q63) $\int \frac{dx}{1+\sin x+\cos x}$

$$\int \frac{2\,du}{1+\frac{2u}{1+u^2}+\frac{1-u^2}{1+u^2}} \cdot \frac{1}{1+u^2}$$

$$\int \frac{2\,du}{\frac{1+\cancel{u^2}+2u+1-\cancel{u^2}}{\cancel{1+u^2}}} \cdot \frac{1}{\cancel{1+u^2}}$$

$$\int \frac{du}{1+u} = \ln[1+\tan(x/2)]$$

$$\begin{array}{c|c} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

Q66) $\int \frac{dx}{2+\sin x}$

$\int \frac{2du}{2+\frac{2u}{1+u^2}} \cdot \frac{1}{1+u^2}$

$\int \frac{2du}{2+2u^2+2u}$

$\int \frac{du}{u^2+u+1}$

$\int \frac{du}{u^2} + \int \frac{du}{u} + \int du$

$-\frac{1}{u} + \ln[u] + u + c$

$\frac{-1}{\tan(x/2)} + \ln[\tan(x/2)] + \tan(x/2) + c$

Q67) $\int \frac{d\theta}{1-\cos\theta}$

$\int \frac{2du}{1-\left(\frac{1-u^2}{1+u^2}\right)} \cdot \frac{1}{1+u^2}$

$\int \frac{2du}{\frac{\cancel{1}+u^2-\cancel{1}+u^2}{\cancel{1+u^2}}} \cdot \frac{1}{\cancel{1+u^2}}$

$\int \frac{\cancel{2}du}{\cancel{2}u^2} = \int u^{-2}du$

$-\frac{1}{u} = \frac{-1}{\tan(x/2)} + c$

Q68) $\int \frac{dx}{4\sin x - 3\cos x}$

Sol:

$\int \frac{2du}{4\left(\frac{2u}{1+u^2}\right)-3\left(\frac{1-u^2}{1+u^2}\right)} \cdot \frac{1}{1+u^2}$

$\int \frac{2du}{\frac{8u-3+3u^2}{\cancel{1+u^2}}} \cdot \frac{1}{\cancel{1+u^2}}$

$\int \frac{2du}{3u^2-8u+3}$

$\frac{-2}{3u} - \frac{1}{4}\ln|u| + \frac{2}{3}u + c$

$\frac{-2}{3(\tan x/2)} - \frac{1}{4}\ln|\tan(x/2)| + \frac{2}{3}(\tan x/2) + c$

Q69) $\int \frac{dx}{\sin x + \tan x}$

Sol:

$$\int \frac{2du}{\frac{2u}{1+u^2} + \frac{2u}{1-u^2}} \cdot \frac{1}{1+u^2}$$

$$\int \frac{2du}{\frac{2u - \cancel{2u^3} + 2u + \cancel{2u^3}}{(\cancel{1+u^2})(1-u^2)}} \cdot \frac{1}{\cancel{1+u^2}}$$

$$\int \frac{\cancel{2}du}{\frac{4u}{1-u^2}}$$

$$\int \frac{(1-u^2)\,du}{2u}$$

$$\frac{1}{2}\int \frac{du}{u} - \frac{1}{2}\int u\,du$$

$$\frac{1}{2}\ln|u| - \frac{1}{4}u^2 + c$$

$$\frac{1}{2}\ln(\tan x/2) - \frac{1}{4}\tan^2(x/2) + c$$

Q70) $\int \frac{\sin x}{\sin x + \tan x}\,dx$

Sol:

$$\int \frac{\left(\frac{2u}{1+u^2}\right)}{\frac{2u}{1+u^2} + \frac{2u}{1-u^2}}\,dx$$

$$\int \frac{\frac{2u}{\cancel{1+u^2}}}{\frac{2u - \cancel{2u^3} + 2u + \cancel{2u^3}}{(\cancel{1+u^2})(1-u^2)}}\,dx$$

$$\int \frac{\cancel{2u}(1-u^2)}{\cancel{4u}}\,dx$$

$$\int \frac{(1-u^2)}{2}\,dx$$

$$\int \frac{\cancel{2}(1-u^2)\,du}{\cancel{2}(1+u^2)}$$

$$\cancel{\int \frac{1\,du}{1+u^2} - \int \frac{-u^2\,du}{1+u^2}}$$

$$\cancel{u - \frac{1}{u} + \frac{u^3}{3} + u + c}$$

$$\cancel{3u^2 - 3 + u^4 + 3u^2 + c}$$

$$\begin{array}{r} 1 \\ 1+x^2 \overline{) \; x^2} \\ \underline{\cancel{x^2}} \\ 1 \end{array}$$

$$\int \frac{1}{1+u^2}\,du - \int \frac{-u^2}{1+u^2}\,du$$

$$\tan^{-1}(u) - \int 1 - \frac{1}{1+x^2}\,du$$

$$\tan^{-1}(u) - \int du + \int \frac{1}{1+u^2}\,du$$

$$2\tan^{-1}(u) - u + c$$

$$2\tan^{-1}(\tan(x/2)) - \tan x/2 + c$$

$$x - \tan(x/2) + c$$

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Ex 7.8