REDUCTION FORMULAS:

$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x\sin x + \frac{n-1}{n}\int \cos^{n-2}x\,dx$$

$$\begin{aligned} u &= \cos^{n-1}x & dv &= \cos x\,dx \\ du &= (n-1)\cos^{n-2}x(-\sin x)\,dx & v &= \sin x \\ &= -(n-1)\cos^{n-2}x\sin x\,dx \end{aligned}$$

so that

$$\begin{aligned} \int \cos^n x\,dx = \int \cos^{n-1}x\cos x\,dx &= \int u\,dv = uv - \int v\,du \\ &= \cos^{n-1}x\sin x + (n-1)\int \sin^2 x\cos^{n-2}x\,dx \\ &= \cos^{n-1}x\sin x + (n-1)\int (1-\cos^2 x)\cos^{n-2}x\,dx \\ &= \cos^{n-1}x\sin x + (n-1)\int \cos^{n-2}x\,dx - (n-1)\int \cos^n x\,dx \end{aligned}$$

Moving the last term on the right to the left side yields

$$n\int \cos^n x\,dx = \cos^{n-1}x\sin x + (n-1)\int \cos^{n-2}x\,dx$$

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

$u = \sin^{n-1} x$, $dv = \sin x\,dx$, $du = (n-1)\sin^{n-2} x\cos x\,dx$, $v = -\cos x$; $\int \sin^n x\,dx = -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\cos^2 x\,dx = -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\,(1-\sin^2 x)dx = -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\,dx - (n-1)\int \sin^n x\,dx$, so $n\int \sin^n x\,dx = -\sin^{n-1} x\cos x + (n-1)\int \sin^{n-2} x\,dx$, and $\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x\cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$.