

Course Code: MT - 1003	Course Name: Calculus and Analytical Geometry
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Student Roll No: 23K-2032	Section No: BCY-1A

Instructions:

- Attempt all questions. There are 03 Questions and 02 pages.
- Solve the paper according to the sequence given in the question paper.
- Graphical Calculator is not allowed.
- Return the question paper with the answer copy.

Time: 60 minutes

Max Marks: 30

Question 01:

CLO1

[2.5 + 2.5 = 5]

- a) Find three smallest consecutive whole numbers x , $x+1$ and $x+2$ such that the difference between one-fourth of the largest and one fifth of the smallest is at least 3. According to given statements equation inequalities is

$$\frac{x+2}{4} - \frac{x}{5} \geq 3$$

$$x \geq 50$$

$$50, 51, 52$$

- b) Solve the inequality for value of x , and also draw the solution on real line.

$$\frac{3}{|2x-3|} < 4$$

$$x > \frac{15}{8} (1.875), x < \frac{9}{8} (1.125)$$

$$(-\infty, \frac{9}{8}) \cup (\frac{15}{8}, +\infty)$$

$$[4 + 3 + 3 = 10]$$

Question 02:

CLO2

- a) Express the function in piecewise form without using absolute values. Then find inverse of piecewise function.

$$f^{-1}(x) = \begin{cases} x+3 & x < -6 \\ x-3 & x \geq -6 \end{cases}$$

$$f(x) = |x+3| + 2x$$

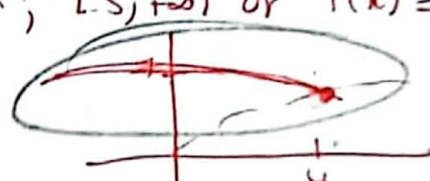
$$f(x) = \begin{cases} x-3 & x < -3 \\ 3x+3 & x \geq -3 \end{cases}$$

- b) Find domain and range of the function $f(x) = \sqrt{4-x} + 5$ and also Sketch the graph of $f(x)$.

$$D: (-\infty, 4] \text{ or } x \leq 4$$

$$R: [5, +\infty) \text{ or } f(x) \geq 5$$

- c) If $f(x) = \frac{1}{x^2+1}$ and $g(x) = \sqrt{x-1}$ then

Find $f \circ g$.Write Domain of $f \circ g$. $f \circ g$ is odd or even?[1, +\infty)
odd**Question 03:**

CLO3

[6 + 2 + 5 + 2 = 15]

- a) Find the derivative $\frac{dy}{dx}$ of the following

i. $\cos(x^2 + 2y) + e^y = x$

$$\frac{dy}{dx} = \frac{1 + 2x \sin(x^2 + 2y)}{-2 \sin(x^2 + 2y) + e^y}$$

ii. $y = \ln(\sec 2x + \tan 2x)^{\frac{1}{2}}$

$$\frac{dy}{dx} = \sec 2x$$

- b) If $f(2) = -8$, $f'(2) = 3$, $g(2) = 17$ and $g'(2) = -8$, Find the value of $(fg)'(2)$.

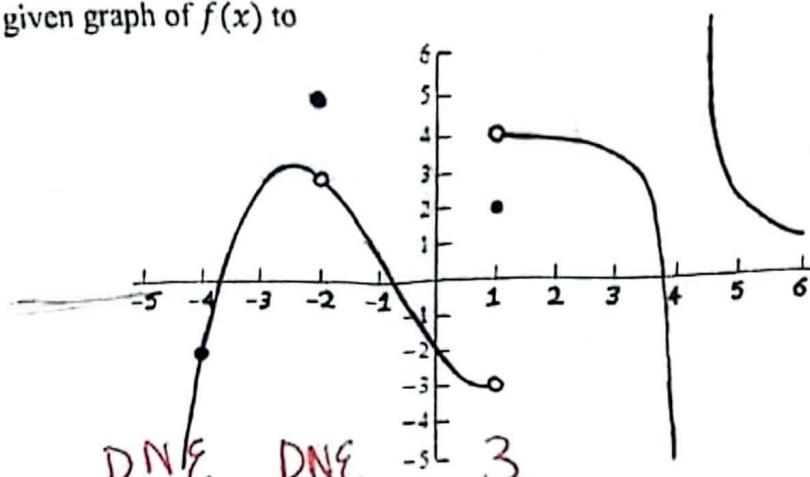
$$(f \cdot g)'(2) = f'(2) \cdot g(2) + g'(2) \cdot f(2)$$

$$= (3)(17) + (-8)(-8)$$

$$= 115$$

$$\begin{aligned} x+3 &< -3 \\ x &< -6 \\ x-3 &< -3 \\ x &< 0 \end{aligned}$$

c) Use the given graph of $f(x)$ to



i. Find $\lim_{x \rightarrow -1} f(x)$, $\lim_{x \rightarrow -2} f(x)$ and $\lim_{x \rightarrow 1} f(x)$.

ii. List all the open intervals in which graph is continuous?

$(-\infty, -2) \cup (-2, 1) \cup (1, 4) \cup (4, \infty)$

d) The candy factory sells candy by the pounds, charging 1.50 per pound for quantities up to and including 20 pounds. Above 20 pounds, the candy factory charges 1.25 per pounds for the entire quantity, plus a quantity surcharges k . If x represents the number of pounds the price function is

$$p(x) = \begin{cases} 1.50x, & \text{for } x \leq 20. \\ 1.25x + k, & \text{for } x > 20. \end{cases}$$

Find k such that the price function $p(x)$ is continuous at $x = 20$.

$k = 5$

The End

Q: 3 (a) Derivatives = 3 + 3'

(b) $(f \cdot g)'(2) = 2$

(c) i = 3
ii = 2

(d) Value of $k = 2$

1 July 2022
30 June 2022

Q: 1 (a) value of $x = 1.5$
value of $x+1, x+2 = 1$

(b) Solve for $x = 2$
No. line = 0.5

Q: 2 (a) Piecewise Fun = 2
Inverse Func = 2

(b) Domain, Range, Gra = 3

(c) $f \circ f$, Domain of $f \circ g$, even/odd = 3