J-QURESHI

# SOLUTION MID-I

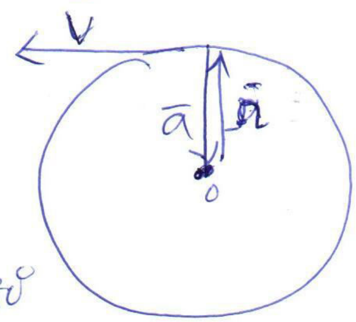
29SEPT2023

## CLO-1

### QUESTION-1: VECTORS

(a) $\bar{a} = (600\,m/s^2)\,\hat{i} + (-4\,m/s^2)\,\hat{j}$

$|\bar{r}| = 3\,m$




(i) $\bar{V}\cdot\bar{a} = va\cos 90^\circ$

$$\boxed{\bar{V}\cdot\bar{a} = va(0) = 0}$$

(ii) $\bar{r}\cdot\bar{a} = |r||a|\cos 180^\circ$

$= |r||a|(-1)$

$= -|r|\times|a|$

$= -(3)(|a|)$

$= -(3)\sqrt{(600)^2 + (-4)^2}$

$= -3\,(600.013)$

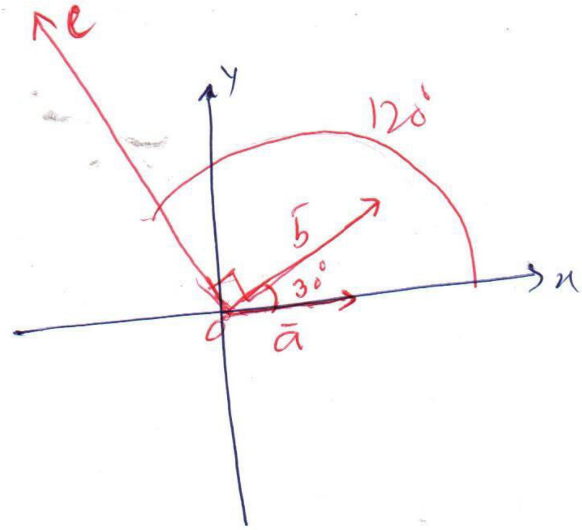
$$\boxed{\bar{r}\cdot\bar{a} = -1800.04}$$

(b)

$|\bar{a}| = 4\ m$

$|\bar{b}| = 6\ m$

$|\bar{c}| = 12\ m$




Suppose The Resultant vector is $\bar{D}$, So

$$\bar{D} = \bar{a} + \bar{b} + \bar{c}$$

Let us Find the components of the vector

$$\begin{cases} a_x = a \cos 0° = (4\ m)\, i \\ a_y = 0 \end{cases}$$

$$\begin{cases} b_x = b \cos 30° = 6(0.866) = (5.196)\, i \\ b_y = b \sin 30° = 6(0.5) = 3.00\, j \end{cases}$$

$c_x = 12 \cos 120° = 12\left(-\frac{1}{2}\right) = -6.00\, i$

$c_y = 12 \sin 120 = 12(0.866) = 10.39\, j$

So vector

$$\bar{D} = D_x + D_y$$
$$= [4 + 5.196 + (-6.00)]\, i + [0 + 3.00 j + 10.39 j]$$
$$= (3.196)\, i + [13.39]\, j$$

(2)

So Magnitude

$$|\bar{D}| = \sqrt{(3.196)^2 + (\cancel{13.39})^2 (13.39)^2}$$

$$= \sqrt{(32.65) + (179.29)}$$

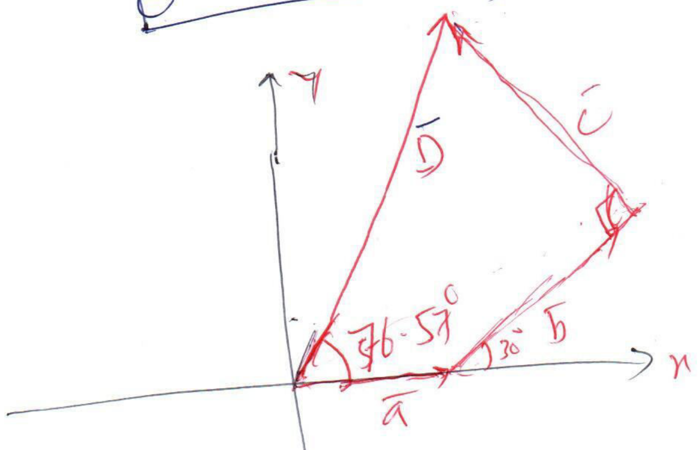
$$\boxed{|\bar{D}| = 14.558 \text{ m}}$$

Angle

$$\Theta = \tan^{-1}\left(\frac{13.39}{3.196}\right)$$

$$= \tan^{-1}(4.189)$$

$$\boxed{\Theta = 76.57^\circ}$$




Even if we draw the Resultant vector $\bar{D}$
on an exact graphical scale
by Head-to-Tail method.
i.e $a \to b \to c$ we still get the
Same vector $\bar{D}$ at an angle of $76.57^\circ$

(C).

Area of a Parallelogram when adjacent sides are given :

Area $= \bar{A} \times \bar{B}$

$= (3i - 2j + 4k) \times (-i - 4j + 2k)$

$= [(3i)\times(-i) + (3i)\times(-4j) + (3i)\times(2k)]$

$+ [(-2j)\times(-i) + (-2j)\times(-4j) + (-2j)\times(2k)]$

$+ [(4k)\times(-i) + (4k)\times(-4j) + (4k)\times(2k)]$

$= [0 - 12k - 6j - 2k + 0 - 4i \cancel{+} 4j + 16i - 0]$

$= (-4i + 16i - 6j - 4j - 12k - 2k)$

$\boxed{\text{Area} = 12i - 10j - 14k}$

Also Given with Matrix.

$$\bar{A} \times \bar{B} = \begin{vmatrix} i & j & k \\ 3 & -2 & 4 \\ -1 & -4 & 2 \end{vmatrix}$$

$= [(-2)(2) - (-4)(4)]\, i$

$- [(3)(2) - (-1)(4)]\, j$

$+ [(3)(-4) - (-2)(-1)]\, k$

$= \underline{12i - 10j - 14k}$ Same.

(4)

Magnitude of Area

$$= \sqrt{(12)^2 + (-10)^2 + (-14)^2}$$

$$= \sqrt{144 + 100 + 196}$$

$$= 20.970 \text{ m}^2$$

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CLO – 2

QUESTION-2: LINEAR MOTION.

(a)

$$V_i = 3i - 5j + 2k$$

$$V_f = -13i - 2j + 9k \qquad \Delta t = 4 \text{ sec}$$

$$a_{avg} = \frac{V_f - V_i}{\Delta t}$$

$$= \frac{(-13i - 2j + 9k) - (3i - 5j + 2k)}{4}$$

$$= \frac{-16i + 3j + 7k}{4}$$

(i) $\boxed{a_{avg} = 4i + 0.75j + 1.75k}$ in unit vector form

(ii) Magnitude $= \sqrt{(4)^2 + (0.75)^2 + (1.75)^2}$

$$= \sqrt{16 + 0.56 + 3.06}$$

$$\boxed{|a_{avg}| = 4.43 \text{ m/s}^2}$$

5

(iii)

$\bar{a}_{AV} = -4i + 0.75j + 1.75k$

$+Y$-axis $= +j$

So we take Dot Product of $\bar{a}_{AV}$ and $\bar{Y}$ axis

$\bar{a}_{AV} \cdot \bar{Y}_{ax} = |a_{AVE}||Y_{ax}| \cos\theta$

~~$\theta = \cos^{-1}($~~

$\cos\theta = \frac{\bar{a}_{AV} \cdot \bar{Y}_j}{|a_A||Y_{ax}|}$

Magnitude of $|\bar{Y}_j| = \frac{1}{\sqrt{3}} j$

As shown below

$= \frac{(-4i + 0.75j + 1.75k)\cdot(\frac{1}{\sqrt{3}} j)}{(4.43)(\frac{1}{\sqrt{3}})}$

~~0.75~~

Proof

If a unit vector $\hat{U} = \hat{U}_x + \hat{U}_y + \hat{U}_k$

are magnitude $|\hat{U}| = 1$

then magnitudes of its components would be

$U_x = \frac{1}{\sqrt{3}} i \quad U_y = \frac{1}{\sqrt{3}} j \;\&\; U_k = \frac{1}{\sqrt{3}} k$

such that $\hat{U} = (\frac{1}{\sqrt{3}})^2 + (\frac{1}{\sqrt{3}})^2 + (\frac{1}{\sqrt{3}})^2$

$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

$= \frac{1+1+1}{3} = \frac{3}{3} = 1$

$$\cos\theta = \frac{(0.75)(\sqrt{3})}{(4.43)(\frac{1}{\sqrt{3}})}$$

$$= \frac{0.4335}{2.651} = 0.16927$$

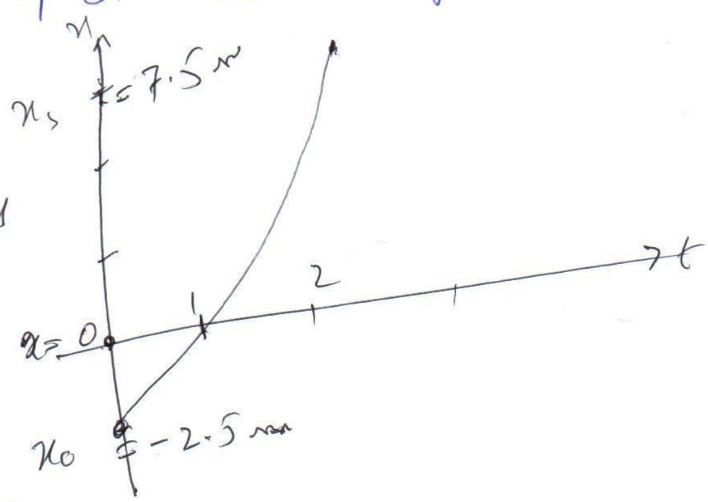
$$\theta = \cos^{-1}(0.16927)$$

$$\boxed{\theta = 80.255^{\circ}}$$

(b) (i) Free Falling objects are under the influence of Force of gravity only. Air resistance is ignored, on Earth.

(ii) All Free falling objects fall with the same rate of acceleration i.e. $g = 9.81\ m/s^2$ or $g = 10\ m/s^2$

(c)




From the graph we could see that if $x_s = 7.5\ m$ then $x_0 = -2.5$

So we will use the following eqn of Motion

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

(2)

First for time $t = 1$ sec

$$x_1 - x_0 = v_0 t + \frac{1}{2} a t^2$$

at $t = 0$ $x_0 = -2.5$, and at $t = 1$, $x = 0$

So

$$0 - (-2.5) = v_0 (1) + \frac{1}{2} a (1)^2$$

$$2.5 = v_0 + \frac{1}{2} a \quad \text{—— } (1)$$

Then for time $t = 2$ sec

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

at $t = 0$ $x_0 = -2.5$ and at $t = 2$ sec $x = 7.5$ m

So

$$(7.5) - (-2.5) = v_0 (2) + \frac{1}{2} a (2)^2$$

$$7.5 + 2.5 = 2v_0 + 2a$$

$$10 = 2v_0 + 2a \quad \text{—— } (2)$$

From (1) $v_0 = 2.5 - \frac{1}{2} a$

Putting this value in (2) we get

$$10 = 2(2.5 - \frac{1}{2} a) + 2a$$

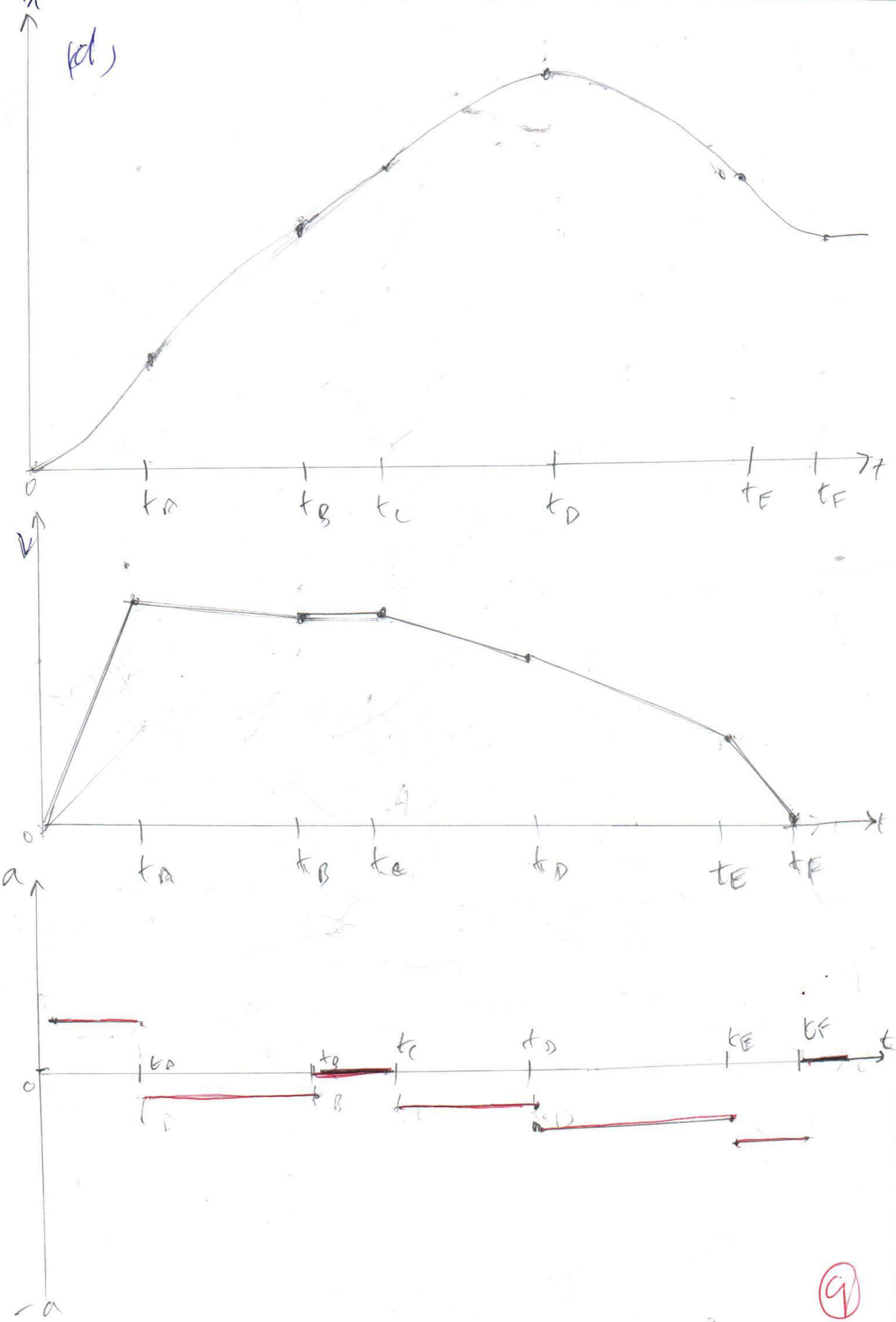
$$10 = 5 - a + 2a$$

(i) magnitude

$$2a = 5 \quad \boxed{a = 2.5 \text{ m/s}^2}$$

(ii) +ve value of $a$ shows that the acceleration vector points in the $+x$ direction.

(d)




9

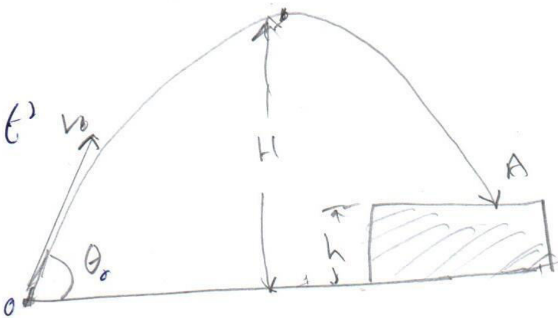
QUESTION 3 CLO – 3

PROJECTILE MOTION

$V_0 = 42\,m/s$ $\theta_0 = 70^\circ$ $t = 5.6\,s$

Let us use

$y - y_0 = V_0 \sin\theta_0 - \frac{1}{2} g t^2$




and let $y = h$ so

we have

$\cancel{y} = \cancel{y_0} + V_0 \sin\theta_0 t - \frac{1}{2} g t^2$

Let $y_0 = 0$

$h = 0 + 42 \sin 70^\circ t - \frac{1}{2}(9.8)(5.6)^2$

$h = 42 \times 0.94 \times 5.6 - \frac{1}{2}(9.8)(31.36)$

$h = 221.08 - 153.664$

(i) $h = 67.416\,m$

(ii) The Horizontal speed $V_0 \cos\theta_0$ remain constant

But vertical speed (Find)

$V_y = V_0 \sin\theta_0 - g t$

$= 42 \sin 70 - (9.8)(5.6$

$= 42 \times 0.94 - 54.88$

$= 39.48 - 54.88$

$= -15.4\,m/s$

$V_i = V_0 \cos 70^\circ = 42 \cos 70 = 42 \times 0.342$

$= 14.49\,m/s$ (10)

Hence the Final velocity when it hits Point A

$$V = \sqrt{V_i^2 + V_y^2}$$

$$= \sqrt{(14.49)^2 + (-15.4)^2}$$

$$= \sqrt{209 + 221.76}$$

$$= \sqrt{430.76}$$

$$\boxed{V = 20.75\ m/s}$$

iii) Max height $H = \dfrac{(V_0 \sin\theta)^2}{2g}$

$$H = \frac{V_0^2 \sin^2\theta}{2g}$$

$$= \frac{(42)^2 (\sin 70)^2}{2(9.8)}$$

$$= \frac{(1764)(0.94)^2}{19.6}$$

$$= \frac{(1764)(0.884)}{19.6}$$

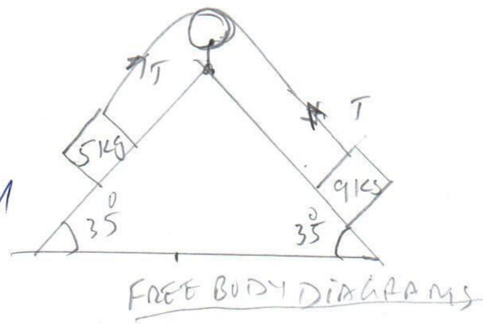
$$= \frac{1558.67}{19.6}$$

# CLO – 4

## QUESTION:- 4 FORCES

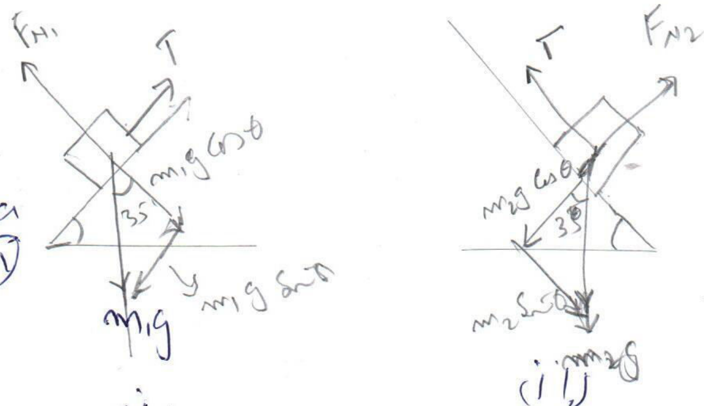
(a) $m_1 = 5\,kg$

$m_2 = 9\,kg$




FREE BODY DIAGRAMS

Now applying Newton's 2nd Law to Both of the FBDs




(i)

(i)

$T - m_1 g \sin\theta = m_1 a$ — (1)

$F_{N1} - m_1 g \cos\theta = 0$

(ii)

$m_2 g \sin\theta - T = m_2 a$ — (2)

$F_{N2} - m_2 g \cos\theta = 0$

Adding Eqn (1) and (2)

we get

$$\cancel{T} - m_1 g \sin\theta = m_1 a$$
$$-\cancel{T} + m_2 g \sin\theta = m_2 a$$

$$(m_2 - m_1) g \sin\theta = (m_1 + m_2) a$$

$$a = \frac{(m_2 - m_1) g \sin\theta}{m_1 + m_1} = 1.607\ m/s^2$$

$$\boxed{a = 1.607\ m/s}$$

(10)

Using Eqn ①

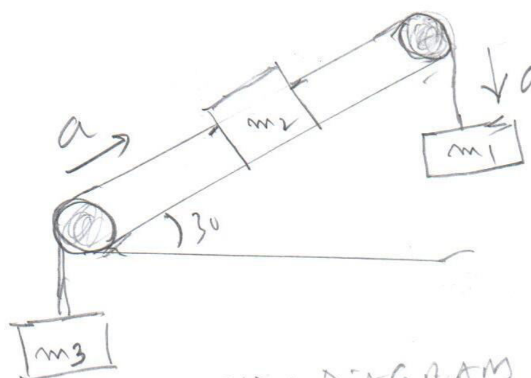
$$T = \frac{m_1 g \sin\theta + m_1 a}{}$$

$$= m_1 (g \sin\theta + a)$$

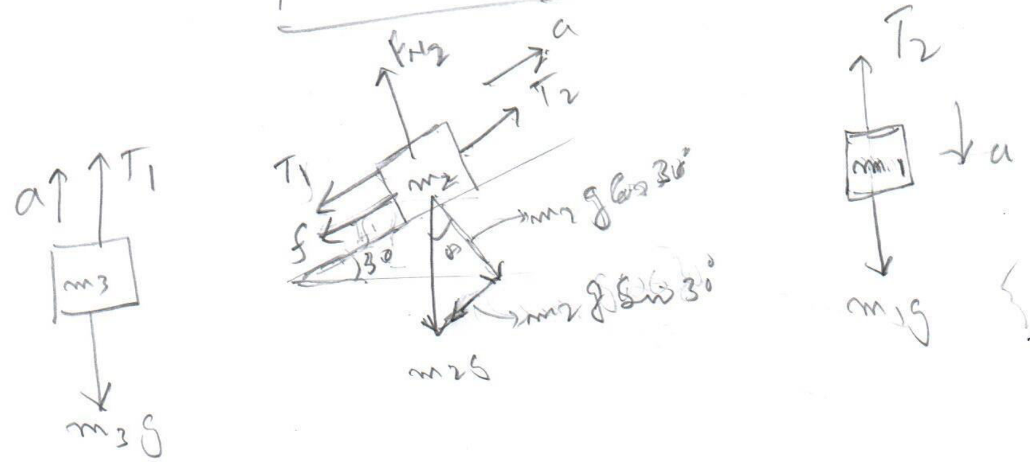
$$= 5(9.8 \sin 35^\circ + 1.607)$$

$$\boxed{T = 7.232\,N}$$

(b)




FREEBODY DIAGRAM



## Newton's LAW Eqns. FOR THREE MASSES

As shown in the Fig acceleration 'a' is same for all the three masses.

But
Tension for mass $m_1 = T_2$
Tension for mass $m_2 = T_1$ and $T_2$
Tension for mass $m_3 = T_1$

So the Eqn WOULD BE

FOR $m_1$

$$m_1 g - T_2 = m_1 a$$

FOR $m_2$

$$T_2 - T_1 - f - m_2 g \sin\theta = m_2 a$$

FOR $m_3$

$$T_1 - m_3 g = m_3 a$$

c

Newton's Laws affect all the sports and we will look how they impact the Soccer or Football sport.

## Newton's First Law

A Soccer Player understands that for the Ball to move a force must be applied on it, otherwise it will remain at rest.

## Newton's Second Law

A Soccer Player must understand the mass of the Ball as he or she kicks the ball. Player must think about the amount of acceleration produced by a certain amount of force needed for the Ball to reach its required destination.

## Newton's Third Law

As the Player applies the Force on the Ball, the ball pushes back the Player, so, he or she must optimize the force applied.

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