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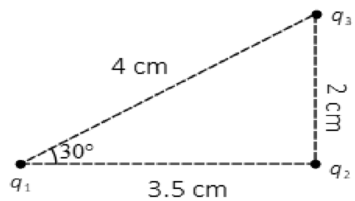
(1) Three-point charges, each of magnitude 3nC, sit at the corners of a right triangle as in the figure below. Find the direction and magnitude of the electric force on the charge *q*3.



Solution: First, find the individual forces acting on the desired charge. Next, the vector sum of those forces to find the net force on that charge. The magnitude of the electric force is given by Coulomb's law.

The magnitude of the force exerted by charge q_1 on charge q_3 is

$$F_{13} = k\frac{|q_1 q_3|}{d_{13}^2}$$

$$= \frac{(8.99 \times 10^9)(3 \times 10^{-9})^2}{4^2}$$

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$$= 5.06 \times 10^{-9} \quad \text{nC}$$

Since the charges have the same sign so they repel each other.

Recall that according to Coulomb's law, the electric force between two charges is along the line connecting them. In this case, the force is directed away from charge q_3 and makes an angle of 30° with the horizontal. Therefore, the force F_{13} has the following components

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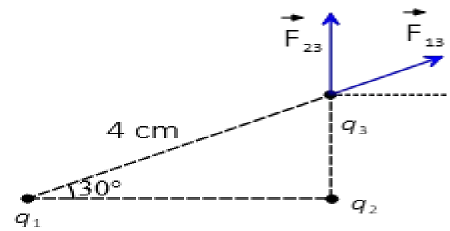
$$\begin{aligned}\vec{F}_{13} &= F_{13}\cos\theta\,\hat{i} + F_{13}\sin\theta\,\hat{j} \\ &= (5.06 \times 10^{-9})(\cos 30^\circ\,\hat{i} + \sin 30^\circ\,\hat{j}) \\ &= 4.38\hat{i} + 2.53\hat{j} \quad \text{nN}\end{aligned}$$



Similarly, the electric force $\vec{F}_{23}$ exerted on charge q_3 due to charge q_2 is along the y direction and away from charge q_3. Its magnitude is also found as

$$F_{23} = k\frac{|q_2 q_3|}{d_{23}^2}$$

$$= \frac{(8.99 \times 10^9)(3 \times 10^{-9})^2}{2^2}$$

$$= 20.2 \times 10^{-9} \quad \mathrm{nC}$$

Therefore, in component form is

$$\vec{F}_{23} = 20.2\,\hat{j} \quad \mathrm{nN}$$

Vector summing them get the net force on charge q_3 as below

$$\begin{aligned}\vec{F}_3 &= \vec{F}_{13} + \vec{F}_{23} \\ &= 4.38\hat{i} + 22.53\hat{j} \quad \mathrm{nN}\end{aligned}$$

The magnitude of the net force is determined from its components as below

$$F_3 = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{(4.38 \times 10^{-9})^2 + (22.53 \times 10^{-9})^2}$$

$$= 22.9 \times 10^{-9} \quad \mathrm{N}$$

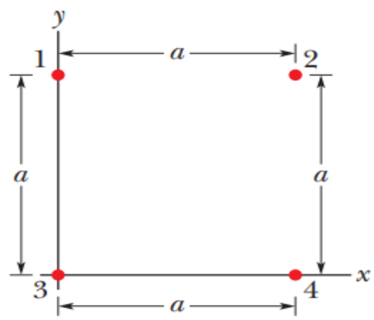
The net force makes an angle θ with the x axis whose value is found as below

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

$$= \tan^{-1}\left(\frac{22.53}{4.38}\right)$$

$$= 79^\circ$$

(2) In the following Fig., the particles have charges $q_1 = -q_2 = 100$ nC and $q_3 = -q_4 = 200$ nC, and distance a = 5.0 cm. What are the (a) x and (b) y components of the net electrostatic force on particle 3?



Soln:

$$\vec{F}_3 = \vec{F}_{31} + \vec{F}_{32} + \vec{F}_{34} = \frac{1}{4\pi\varepsilon_0}\left(-\frac{|q_3||q_1|}{a^2}\hat{j} + \frac{|q_3||q_2|}{(\sqrt{2}a)^2}(\cos 45°\hat{i} + \sin 45°\hat{j}) + \frac{|q_3||q_4|}{a^2}\hat{i}\right)$$

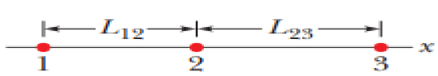
(a) Therefore, the *x*-component of the resultant force on q_3 is

$$F_{3x} = \frac{|q_3|}{4\pi\varepsilon_0 a^2}\left(\frac{|q_2|}{2\sqrt{2}} + |q_4|\right) = \left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{2\left(1.0\times10^{-7}\,\mathrm{C}\right)^2}{(0.050\ \mathrm{m})^2}\left(\frac{1}{2\sqrt{2}} + 2\right) = 0.17\ \mathrm{N}.$$

(b) Similarly, the *y*-component of the net force on q_3 is

$$F_{3y} = \frac{|q_3|}{4\pi\varepsilon_0 a^2}\left(-|q_1| + \frac{|q_2|}{2\sqrt{2}}\right) = \left(8.99\times10^9\,\mathrm{N\cdot m^2/C^2}\right)\frac{2\left(1.0\times10^{-7}\,\mathrm{C}\right)^2}{(0.050\ \mathrm{m})^2}\left(-1 + \frac{1}{2\sqrt{2}}\right)$$
$$= -0.046\ \mathrm{N}.$$

(3) In the following Fig. particles 1 and 2 are fixed in place, but particle 3 is free to move. If the net electrostatic force on particle 3 due to particles 1 and 2 is zero and $L_{23} = 2.00\,L_{12}$, what is the ratio q_1/q_2?



Soln:

Regarding the forces on q3 exerted by q1 and q2, one must “push” and the other must "pull" in order that the net force is zero; hence, q_1 and q_2 have opposite signs. For individual forces to cancel, their magnitudes must be equal:

$$k\frac{|q_1||q_3|}{(L_{12}+L_{23})^2} = k\frac{|q_2||q_3|}{(L_{23})^2}.$$

With $L_{23} = 2.00L_{12}$, the above expression simplifies to $\frac{|q_1|}{9} = \frac{|q_2|}{4}$. Therefore, $q_1 = -9q_2/4$, or $q_1/q_2 = -2.25$.