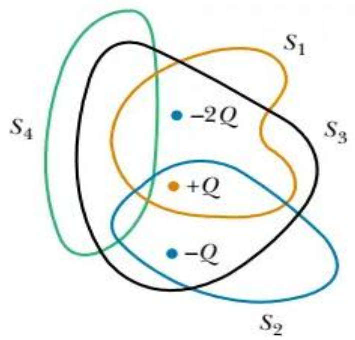
(1) In the figure below, a configuration of four closed surfaces and three charges of **-** 2Q, +Q**, and** -Q **is shown**. What is the electric flux through each surface?



Solution: Electric flux Φ_E or the number of field lines passing through a given closed surface is found by Gauss's law as below

$$\Phi_E = \frac{Q_{enclosed}}{\epsilon_0}$$

As you can see, all that matters is the presence of an electric charge and a closed surface surrounding it.

There are two charges inside the orange closed surface that give a total charge of $Q_{enclosed} = -2Q + Q = -Q$. Dividing this net charge by ϵ_0, we get the electric flux through the orange surface

$$\phi_{orange} = \frac{Q_{enc}}{\epsilon_0} = \frac{-Q}{\epsilon_0}$$

In a similar manner, the flux through other colored closed surfaces are computed as below

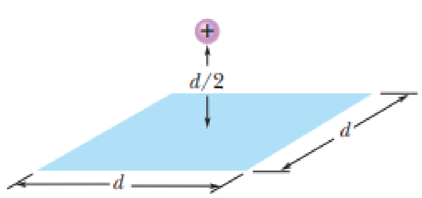
$$\begin{aligned}\phi_{blue} &= \frac{Q_{enc}}{\epsilon_0} \\ &= \frac{-Q+Q}{\epsilon_0} = \boxed{0}\end{aligned}$$

$$\begin{aligned}\phi_{black} &= \frac{Q_{enc}}{\epsilon_0} \\ &= \frac{-2Q+Q-Q}{\epsilon_0} \\ &= \boxed{-\frac{2Q}{\epsilon_0}}\end{aligned}$$

No electric flux passes through the green closed surface because it does not surround any charge.

$$\phi_{green} = \frac{Q_{inside}}{\epsilon_0} = \boxed{0}$$

(2) In the Fig, a proton is at a distance d/2 directly above the center of a square of side d. What is the magnitude of the electric flux through the square? (Hint: Think of the square as one face of a cube with edge d.)

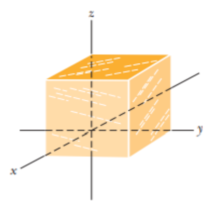


Solution:

To exploit the symmetry of the situation, we imagine a closed Gaussian surface in the shape of a cube, of edge length d, with a proton of charge q = + 1.6 x 10^{-19} C situated at the inside center of the cube. The cube has six faces, and we expect an equal amount of flux through each face. The total amount of flux is $\Phi_{net} = q/\varepsilon_0$, and we conclude that the flux through the square is one-sixth of that. Thus,

$$\Phi = \frac{q}{6\varepsilon_0} = \frac{1.6\times10^{-19}\ \text{C}}{6(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 3.01\times10^{-9}\ \text{N}\cdot\text{m}^2/\text{C}.$$

(3) At each point on the surface of the cube shown in the following Fig., the electric field is parallel to the z axis. The length of each edge of the cube is 3.0 m. On the top face of the cube the field is E = -34k N/C and on the bottom face it is E = + 20k N/C. Determine the net charge contained within the cube.



Solution:
There is no flux through the sides, so we have two "inward" contributions to the flux, one from the top (of magnitude [$(34)(3.0)^2$] and one from the bottom of magnitude[$(20)(3.0)^2$] With "inward" flux being negative, the result is $\Phi = -486$ N·m^2 /C. Gauss' law then leads to

Magnitude of Flux from top (inward) = ϕ_t = E. A = $(34)\,(3)^2$ = 306 N. m^2/C

Magnitude of Flux from bottom (inward) = ϕb = E.A = $(20)\,(3)^2$ = 180 N. m^2/C

So, Total Inwards E – field = 306+ 180 = 486 N.m^2
Applying Gauss's Law

$q_{enc} = \epsilon_0\, \Phi = (8.85 \times 10^{-12}\ C^2 / N.\ m^2)(486\ N.\ m^2 /C) = 4.3 \times 10^{-9}\ C$

As both the E- field are going inward, so there must be a –ve charge present inside the Gaussian cube.