National University of Computer & Emerging Sciences, Karachi
Fall-2023 FAST School of Computing
Mid-Term - II Exam
7th November 2023, 10:00 AM - 11:00 AM

Course Code: MT-1003	Course Name: Calculus and Analytical Geometry
Instructors Name: Ms. Urooj / Ms. Alishba Tariq / Ms.Fareeha sultan / Mr. Nadeem Khan / Mr. Mairaj Ahmed	
Student Roll No:	Section No:

Instructions:

- Attempt all questions. There are **03 Questions and 02 pages.**
- Solve the paper according to the sequence given in the question paper.
- Graphical Calculator is not allowed.
- Return the question paper with the answer copy.

Time: 60 minutes **Max Marks: 30**

Question 01: **[CLO-4]** **[5]**

Answer the following.

a. Rolle's Theorem is used to find the zeros of a function. (True/False)

b. $\lim_{x\to 0^+} sinxlnx$

I) 0 II) $+\infty$ III) -1 IV) $\frac{\pi}{2}$ V) 2

c. For which value of x the function $f(x) = x^3 - 8$ on [3,7] satisfies the conditions of mean value theorem.

I) 4.509 II) 3.512 III) 8.888 IV) 5.132 V) 6.285

d. $\lim_{t\to 0} \frac{te^t}{1-e^t}$

I) 0 II) $-\infty$ III) -1 IV) $\frac{-1}{e}$ V) $-e$

e. The rational function $f(x) = \frac{3-x^2}{x^3}$, has

I) a stationary point at $x = 1$

II) a stationary point at $x = -1$

III) two stationary points at approximately $x = -1.723$ and $x = 1.723$

IV) three stationary points at approximately $x = 0, x = -1.723$ and $x = 1.723$

V) no stationary points

Question 02: **[CLO-3]** **[5+5]**

Evaluate the following integrals:

a. $\int_1^4 \frac{\ln(x)}{\sqrt{x}}\,dx$

b. $\int \frac{\sqrt{1+4x^2}}{x}\,dx$

Question 03: **[CLO-4]** **[5+5+5]**

a. Use the definition of area under the curve with x_k^* as the midpoint of the subinterval to find the area under the curve $y = 4x - x^2$ over the interval $[0, 4]$. [2+2.5+0.5]

b. If $f(x) = \frac{1}{2}x^{\frac{4}{3}} - 2x^{\frac{1}{3}}$, Find Derivation (1)

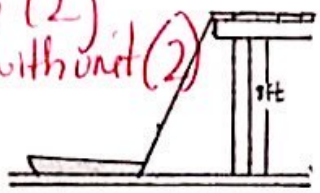
I. All critical points. (1)

II. Intervals in which function is increasing and decreasing. (2)

III. Relative extrema. (1)

c. A pulley is on the edge of a dock, 8 *ft* above the water level. (See the figure below.) A rope is being used to pull a boat. The rope is attached to the boat at water level. The rope is being pulled at the rate of 1 *ft* per second. Find the rate at which the boat is approaching the dock at the instant the boat is 4 *ft* from the dock.

Data (1)
Correct eq & derivative (2)
Calculation & Ans without unit (2)



Δx, $f(\bar{x}_k)$ ⟶ 2 marks
Correct Summation & Limit ⇒ 2.5 marks
Correct Answer = 0.5 mark

$$\frac{1}{2} \cdot \frac{4}{3} x^{\frac{4}{3}-1} - 2 \cdot \frac{1}{3} x^{\frac{1}{3}-1}$$

$$\frac{2}{3} x^{-1/3} \Rightarrow \frac{2}{3} x^{-2/3}$$