National ...
of Computer & Emerging Sciences

Course Code & Title: Calculus

Roll No: ______ Section: ______ Student's Signature: ______ Date: ______

Tick (✓) Extra Sheet No: 1 2 3 4 5 Invigilator's Signature: ______

MID – II

Q.1 $f(x) = \dfrac{x^2-3}{x-2}$

Sol:-

$$f'(x) = \frac{(x-1)(x-3)}{(x-2)^2}$$

$$f''(x) = \frac{2}{(x-2)^3}$$

C.P:-

$f'(x) = 0$ & $f'(x) =$ undef

$(x-1)(x-3) = 0$ $x-2 = 0$

$x = 1, 3$ $x = 2$ (not in domain)

$+$ | $-$ | $-$ | $+$

1 2 3 R

Increasing $(-\infty, 1) \cup (3, +\infty)$

Decreasing $(1, 2) \cup (2, 3)$

Inflection Point:- & $f''(x) = \text{undef}$

$f''(x) = 0$ $\qquad$ $x - 2 = 0$

$2 \neq 0$ $\qquad$ $x = 2$

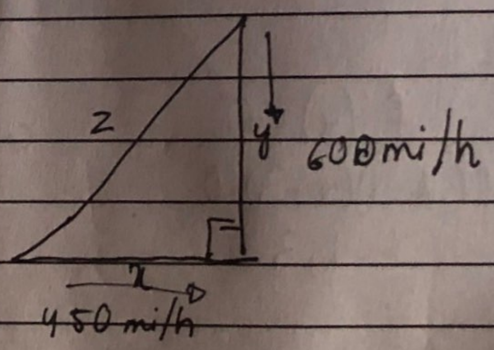
$-\infty \quad \ominus \quad 2 \quad \oplus \quad +\infty$

Concave up: $(2, +\infty)$

Concave down:- $(-\infty, 2)$

Inflection Point:- (none)

Q: 1b



$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$\because$ $x = 150mi$ $\quad \frac{dx}{dt} = -450 mi/h$

$y = 200mi$ $\quad \frac{dy}{dt} = -600 mi/hr$

$z = 250$ mil (By ~~ph~~ pythagor's Theorem)

$$\frac{dz}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{z}$$

$$= \frac{(-450)(150) + (-600)(200)}{250}$$

$$= -750 \text{ mi/hr}$$

Distance b/w planes is decreasing by 750 mi/hr when their distance ~~to~~ to same point is 150 & 200 mil.

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Q# 2(a)

$$x^2\cos y + \sin 2y = xy$$

$$\frac{d}{dx}\left(x^2\cos y + \sin 2y\right) = \frac{d}{dx}(xy)$$

$$2x\cos y - x^2\sin y \cdot y' + 2\cos 2y \cdot y' = y + xy'$$

$$2x\cos y - y = xy' + x^2\sin y \cdot y' - 2\cos 2y \cdot y'$$

$$\boxed{y' = \frac{2x\cos y - y}{x + x^2\sin y - 2\cos 2y}}$$

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2(b)

(i) $\lim_{x\to 0} \left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$; (1^∞)

Let

$\ln y = \ln\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$

$\ln y = \frac{1}{x^2} \ln\left(\frac{\tan x}{x}\right)$

$\lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{1}{x^2} \ln\left(\frac{\tan x}{x}\right)$

$= \lim_{x\to 0} \frac{\ln\left(\frac{\tan x}{x}\right)}{x^2}$; $\left(\frac{0}{0}\right)$

$= \lim_{x\to 0} \frac{\frac{1}{\left(\frac{\tan x}{x}\right)} \cdot \left(\frac{x\sec^2 x - \tan x}{x^2}\right)}{2x}$

$= \frac{\lim_{x\to 0}\left(\frac{x}{\tan x}\right) \cdot \lim_{x\to 0}\left(\frac{x\sec^2 x - \tan x}{x^2}\right)}{2x}$

$= 1 \cdot \lim_{x\to 0}\left(\frac{x\sec^2 x - \tan x}{2x^3}\right)$; $\left(\frac{0}{0}\right)$

$=$

$$= \lim_{x \to 0} \left[\frac{\cancel{\sec^2 x} + x(2\sec^2 x \tan x) - \cancel{\sec^2 x}}{\cancel{6} x^2} \right]$$

$$= \lim_{x \to 0} \frac{\sec^2 x \tan x}{3x}$$

$$= \lim_{x \to 0} \frac{\sec^2 x}{3} \cdot \underbrace{\lim_{x \to 0} \frac{\tan x}{x}}_{1}$$

$$= \frac{1}{3} \cdot 1$$

$$\ln y = \frac{1}{3}$$

$$\boxed{y = e^{1/3}}$$

Sectio

Q/Part No.

Rough Work

b ii

$$\lim_{x\to\infty}\left(x-\ln(1+2e^x)\right)$$

$$=\lim_{x\to\infty}\left[\ln e^x-\ln(1+2e^x)\right]$$

$$=\lim_{x\to\infty}\ln\left(\frac{e^x}{1+2e^x}\right) \quad ; \left(\frac{\infty}{\infty}\right)$$

$$=\ln\lim_{x\to\infty}\left(\frac{\cancel{e^x}}{2\cancel{e^x}}\right)$$

$$=\ln\left(\frac{1}{2}\right)$$

$$=\ln 1-\ln 2$$

$$=0-\ln 2$$

$$\boxed{=-0.6931}$$

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2c

$f(x) = x + 3\cos x \quad ; \quad [-\pi, \pi]$

$f'(c) = \dfrac{f(b) - f(a)}{b - a}$

$f'(x) = 1 - 3\sin x$

$f'(c) = 1 - 3\sin c$

$f(\pi) = \pi + 3\cos\pi = \pi - 3$

$f(-\pi) = -\pi + 3\cos(-\pi) = -\pi - 3$

$1 - 3\sin c = \dfrac{\pi \cancel{-3} + \pi + \cancel{3}}{\pi + \pi}$

$1 - 3\sin c = \dfrac{\cancel{2\pi}}{\cancel{2\pi}}$

$1 \; 3\sin c = 1$

$-3\sin c = 0$

$c = \sin^{-1}(0)$

$\boxed{c = 0}$

Q # 3

(i) $\displaystyle\int_{\pi^2/36}^{\pi^2/4} \frac{\cos\sqrt{\theta}}{\sqrt{\theta}\,\sin\sqrt{\theta}}\,d\theta$

Sol:-

Let $u = \sqrt{\theta}$

$du = \dfrac{1}{2\sqrt{\theta}}\,d\theta$

$2du = \dfrac{d\theta}{\sqrt{\theta}}$

$= 2\displaystyle\int_{\pi^2/36}^{\pi^2/4} \frac{\cos u}{\sin u}\,du$

$= 2\left[\ln \sin u\right]_{\pi^2/36}^{\pi^2/4}$

$= 2\left[\ln \sin\sqrt{\dfrac{\pi^2}{4}} - \ln \sin\sqrt{\dfrac{\pi^2}{36}}\right]$

$= 2\left[\ln 1 - \ln\left(\dfrac{1}{2}\right)\right]$

$= 2\left[0 - (\cancel{\ln 1} - \ln 2)\right]$

$= 2\ln 2$

$\boxed{= \ln 4}$

Q: $\displaystyle\int \underset{u}{x}\,\underset{v}{\tan^2 x}\,dx$

$= x\displaystyle\int \tan^2 x\,dx - \int 1 \int \tan^2 x\,dx$

$= x\displaystyle\int (\sec^2 x - 1)\,dx - \int 1 \int (\sec^2 x - 1)\,dx$

$= x(\tan x - x) - \displaystyle\int (\tan x - x)\,dx$

$\boxed{= x(\tan x - x) - \ln \sec x + \dfrac{x^2}{2} + C}$

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Rough Work

(iii) $\int \frac{1}{(x^2+2x+3)^{3/2}}\,dx$

Sol:-

$$\int \frac{1}{[(x+1)^2+2]^{3/2}}\,dx$$

Let $u = x+1$

$du = dx$

$$\int \frac{1}{(u^2+2)^{3/2}}\,du$$

Let $u = \sqrt{2}\tan\theta$

$du = \sqrt{2}\sec^2\theta\,d\theta$

$$\int \frac{\sqrt{2}\sec^2\theta\,d\theta}{(2\tan^2\theta+2)^{3/2}}$$

$$= \frac{1}{2^{3/2}}\int \frac{\sqrt{2}\sec^2\theta\,d\theta}{(\tan^2\theta+1)^{3/2}}$$

$$= \frac{\sqrt{2}}{\sqrt{8}}\int \frac{\sec^2\theta}{(\sec^2\theta)^{3/2}}\,d\theta$$

$$= \frac{1}{2}\int \frac{1}{\sec\theta}\,d\theta$$

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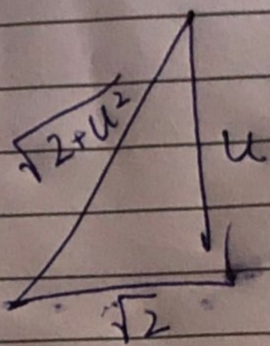
$$= \frac{1}{2}\int \cos\theta \, d\theta$$

$$= \frac{1}{2}\sin\theta + C$$

$$= \frac{1}{2}\frac{u}{\sqrt{2+u^2}} + C$$



$$= \frac{1}{2}\frac{x+1}{\sqrt{2+(x+1)^2}} + C$$

$$= \frac{1}{2}\frac{x+1}{\sqrt{2+x^2+2x+1}} + C$$

$$\boxed{= \frac{1}{2}\frac{x+1}{\sqrt{x^2+2x+3}} + C}$$