Projectile Motion:-

Projectile: A body moving freely under the action of gravity, having x and y-components of velocity is called projectile.

Diagram
→ Derivation] Book.
→ Components
→ Numericals

Trajectory of Projectile:-

For horizontal motion; $x = (V_{0x})(t)$

$$x = (V_0\cos\theta)t$$

$$t = \frac{x}{V_0\cos\theta}$$

For vertical motion;

$$y = V_{0y}t + \frac{1}{2}a_y t^2$$

$$= (V_0\sin\theta)t + \frac{1}{2}(-g)t^2$$

$$= (V_0\sin\theta)\left(\frac{x}{V_0\cos\theta}\right) - \frac{1}{2}g\frac{x^2}{V_0^2\cos^2\theta}$$

$$y = (\tan\theta)x - \frac{1}{2}\left(\frac{g}{V_0^2\cos^2\theta}\right)x^2 \quad (1)$$

Here, $\tan\theta$, g, V_0^2, $\cos^2\theta$ are constant.

$$\boxed{y = bx - cx^2}$$

It, Eq. of Parabola. Trajectory of projectile is parabolic.

Diff. eq (1) w.r.t "x"

$$\frac{dy}{dx} = \tan\theta - \frac{1}{2}\frac{g}{V_0^2\cos^2\theta}(2x)$$

$\because \frac{dy}{dx} = 0$; at max. point

$$\tan\theta = \frac{gx}{V_0^2\cos^2\theta}$$

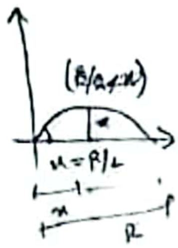
Standard Eq: $y^2 = 4ax$, $x^2 = 4ay$

General Eq: $y = a(x-h)^2 + k$
$x = a(y-k)^2 + h$

$$x = \left(\frac{\sin\theta}{\cos\theta}\right)\frac{V_0^2\cos^2\theta}{g}$$

$$x = \frac{V_0^2\cos\theta\sin\theta}{g}$$

$\because 2x = R$
$R = x/2$
$x = R/2$



$$\frac{R}{2} = \frac{V_0^2\cos\theta\sin\theta}{g}$$

$$R = \frac{V_0^2\, 2\cos\theta\sin\theta}{g}$$

$$\boxed{R = \frac{V_0^2\sin 2\theta}{g}}$$

Motion with constant Acceleration:

1) $V_x = V_{0x} + a_x t$
2) $x = x_0 + V_{0x}t + \frac{1}{2}a_x t^2$
3) $V_x^2 = V_{0x}^2 + 2a_x(x - x_0)$
4) $x - x_0 = \left(\frac{V_{0x} + V_x}{2}\right)t$; doesn't involve acceleration

Newton's Laws of Motion:-

Newton's first law of Motion:-

"Every object continues in its state of rest, or of uniform velocity in a straight line, as long as no net force acts on it."

Q1: A person driving a car on a straight testing road at a constant

Q2: In a film, spaceship is in vacuum, when it engine dies. As a result spaceship slows down and stop. What does Newton's law say about this event?

- Aristotle (300 B
- Abu Ali Sena (980-1037) No body begin to move or come to rest of itself
- Galileo
- Newton (160

Contact force
Long range force

Newton's Second law of Motion:

The acceleration of an object is directly proportional to the net force acting on it and is inversely proportional to the object's mass. The direction of the acceleration is in the direction of the net force acting on the object.

$$\vec{a} = \frac{\Sigma F}{m}$$; Force as an action capable of accelerating an object.

Q1: A Jumbo jet cruise moves at a constant velocity of 1000km/h, when the thrusting force of its engine is a constant 100,000 N. What is the acceleration of the jet? What is the force of air resistance on the jet

$x = (V_{0x})t = (9.0)(0.5) = 4.5m$

$y = -\frac{1}{2}gt^2 = -\frac{g}{2} \quad -1.2m$

Stored Job $\boxed{V_{x0} = 28.2 m/s}$ **HP LaserJet M507** **Page 1**

$\theta = 5$

$V_x = 9.0 m/s$ $\quad v = \sqrt{}$

$V_y = -gt = -4.9 m/s$

Device Information

Engine Cycles: 18339
Service ID: 31098
Product Serial Number: PHBKM993TD
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Job Folders | Job Name | Job Type | Copy Count

Inertial Reference Frames:

$\therefore a = 0$

Non-Inertial / Accelerating Reference Frame:

$\therefore a \neq 0$

Freely Falling bodies:-

*) Example of motion with (nearly) constant acceleration is a body falling under the influence of Earth's gravitational attraction.

*) Such motion has held attention of philosophers since ancient times.

*) 4th century, Aristotle thought that heavy bodies fall faster than light bodies, in proportion to their weight.

*) 19th century later, Galileo argued that a body should fall with downward acceleration that is constant and indep. of weight.

*) Experiments, shows that if the effects of the air can be ignored; Galileo is right.

*) Same downward acceleration regardless of their size or weight; distance of the fall is small compared with radius of the earth, if ignore small effects due to Earth's rotation; acceleration is constant.

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The idealize motion that results under all of these assumptions is called "free-fall," Although it includes rising as well as falling motion.

$g = 9.8 m/s^2$

*) The magnitude of the acceleration due to gravity is a positive quantity, g. The acceleration of a body in free-fall is always downward.

$a_y = -g$
$= -9.8 m/s^2$

Motion with Constant Acceleration:

•) Falling body has constant acceleration if the effects of the air are not important, body sliding on an incline surface etc.

*) $V_x = V_{0x} + a_x t$

*) $x = x_0 + V_{0x}t + \frac{1}{2}a_x t^2$

*) $V_x^2 = (V_{0x})^2 + 2a(x - x_0)$

Q#01: A one-euro coin is dropped from the leaning Tower of Pisa and falls freely from rest. What are its position and velocity after 1.0 sec, 2.0 sec, and 3.0 sec?

Q#02: The super hero Green Lantern steps from the top of the building. He falls freely from rest to the ground, falling half the total distance to the ground during the last 1.00 sec of his fall. What is "h" of building.

$h = -\frac{1}{2}gt^2$ — ①

$\frac{h}{2} = -\frac{1}{2}g(t-1)^2$ — ②

$\div$ ② by ①

$t = ?$ $\quad t = 2 + \sqrt{2}$ s

English (United States)

Motion In a Circle:-

Uniform Circular Motion:

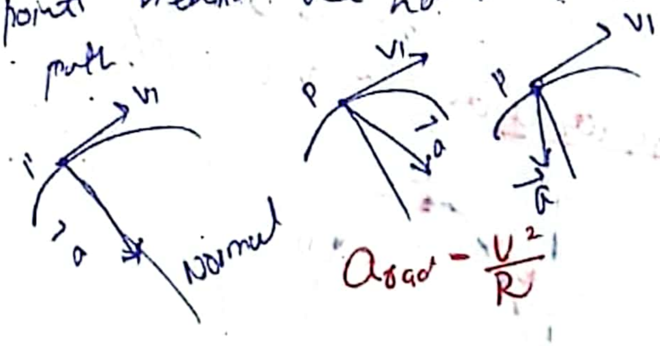
When the particle moves in a circle with constant speed, the motion is called uniform circular motion.

Along curved path:

1) If the speed is constant, $\vec{a}$ is perpendicular to the path and to $\vec{v}$, and points towards concave side of path.

2) If the speed is increasing, $\vec{a}$ points ahead of the normal to the path (because in addition, $\vec{a}$ points ahead of the normal to perpendicular components in the direction of $\vec{v}$ is also present.

3) If speed is decreasing, $\vec{a}$ points behind the normal to the path.



$$a_{rad} = \frac{v^2}{R}$$

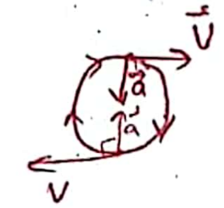
In uniform circular motion, the magnitude 'a' of the instantaneous acceleration is equal to the square of the speed 'v' divided by the radius 'R' of the circle. It's direction $\perp$ to $\vec{v}$ and inward along the radius. Because the acceleration is always directed toward the centre of the circle, it is sometimes called centripetal acceleration.

$$\therefore v = \frac{2\pi R}{T}$$

$$\omega = \frac{2\pi}{T}$$

$$v = \frac{2\pi r}{T}$$

$$a_{rad} = \frac{4\pi^2 R}{T^2}$$



*) At later time 't', the object has changed its velocity, but the speed has not changed.

→ speed not changing, but velocity vector is changing; there must be acceleration.

→ $|a| = \frac{v^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2$; magnitude of a_c.

→ a_c is $\propto$ r; ω is same for entire motion.

→ If disc was rotating, you were at the centre of the disc, the 'a_c' would be zero bcz $\boxed{r=0}$.

→ Some thing that must responsible for the change in velocity, and that some thing, I will call 'Push' or 'Pull' i.e. Force.

→ Planets go around the sun; It must be gravity; It must be sun that is pulling on the planets.

Projectile.

Ans: ⓐ

$v_{0x} = v_0\cos\theta = 22.2\,m/s$

$v_{0y} = v_0\sin\theta = 29.6\,m/s$

$x = (v_{0x})t = 44.4\,m$

$y = v_{0y}t - \frac{1}{2}gt^2 = 39.6\,m$

$v_x = v_{0x} = 22.2\,m/s$

$v_y = v_{0y} - gt = 10.0\,m/s$

$v = \sqrt{v_x^2 + v_y^2} = 24.4\,m/s.$

ⓑ $v_y = v_{0y} - gt_1$

$0 = v_{0y} - gt$ (at high point)

$t_1 = 3.02\,s$

$h = v_{0y}t_1 - \frac{1}{2}gt_1^2$

$h = 44.7\,m$

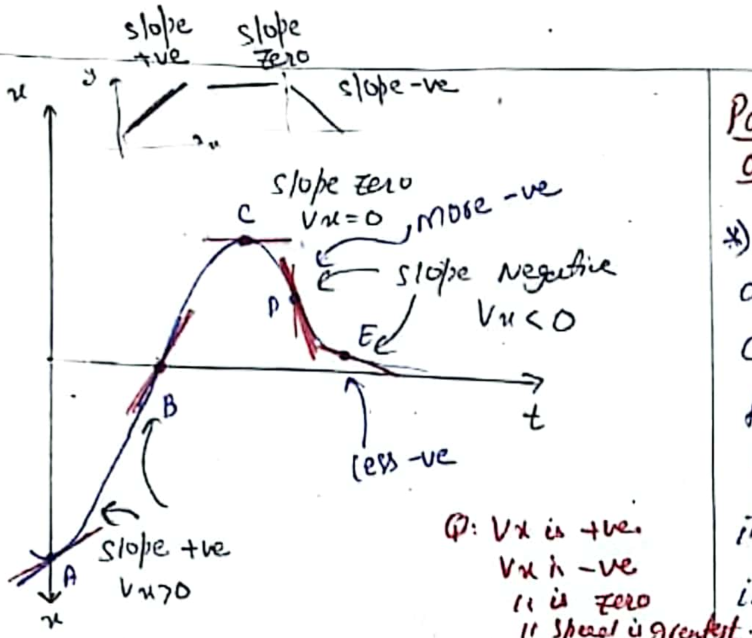
ⓒ $y = v_{0y}t - \frac{1}{2}gt^2$

$0 = v_{0y}t - \frac{1}{2}gt^2$

$t_1 = 0, \quad t_2 = 6.04\,s$

$R = (v_{0x})t_2 = 134\,m$

$v_y = v_{0y} - gt_2$



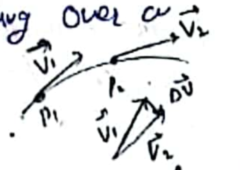
The steeper the slope (+ve or -ve) of an object's x-t graph, the greater is the object's speed in the positive or negative x-direction.

Direction of a_{avg} may or may not be parallel to either $\vec{v_1}$ or $\vec{v_2}$, In general it is not.

Acceleration:- (Average):-

When particle's velocity changes, the particle is said to to undergo "acceleration". For motion along x-axis, the average acceleration a_{avg} over a time interval Δt is,

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}.$$



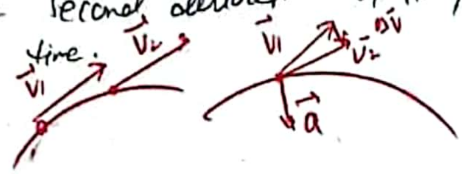
∴ a_{avg} direction as $\vec{\Delta v}$

Instantaneous Acceleration:-

Acceleration of a particle at any instant is the rate at which its velocity is changing at that instant.

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

Acceleration of the particle at any instant is the second derivative of its position $x(t)$ w.r.t time.



Parallel and Perpendicular component of Acceleration:-

*) Whenever there is changing velocity, acceleration is produced. Velocity can be changed by one of following ways.

i) changing magnitude only.
ii) " direction only.
iii) Changing both (mag. and direction)

*) Motion of Earth w.r.t Sun = 30 km/s
*) Spinning speed of Earth.

*) Velocity. tells → how fast and in what direction.

*) Acceleration describe → how velocity changes with time; tells speed and direction of motion are changing.

Acceleration from v-t graph:-