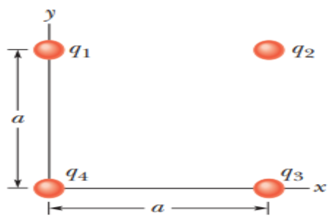
(1) n following Fig., the four particles form a square of edge length a = 5.00 cm and have charges q1 = +10.0 nC, q2 = -20.0 nC, q3 = +20.0 nC, and q4 = -10.0 nC. In unit vector notation, what net electric field do the particles produce at the square's center.



Soln:

EXPRESS Applying the superposition principle, the net electric field at the center of the square is

$$\vec{E} = \sum_{i=1}^{4} \vec{E}_i = \sum_{i=1}^{4} \frac{1}{4\pi\varepsilon_0} \frac{q_i}{r_i^2} \hat{\mathrm{r}}_i .$$

With $q_1 = +10$ nC, $q_2 = -20$ nC, $q_3 = +20$ nC, and $q_4 = -10$ nC, the x component of the electric field at the center of the square is given by, taking the signs of the charges into consideration,

$$E_x = \frac{1}{4\pi\varepsilon_0}\left[\frac{|q_1|}{(a/\sqrt{2})^2} + \frac{|q_2|}{(a/\sqrt{2})^2} - \frac{|q_3|}{(a/\sqrt{2})^2} - \frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45°$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}\left(|q_1| + |q_2| - |q_3| - |q_4|\right)\frac{1}{\sqrt{2}}.$$

Similarly, the y component of the electric field is

$$E_y = \frac{1}{4\pi\varepsilon_0}\left[-\frac{|q_1|}{(a/\sqrt{2})^2} + \frac{|q_2|}{(a/\sqrt{2})^2} + \frac{|q_3|}{(a/\sqrt{2})^2} - \frac{|q_4|}{(a/\sqrt{2})^2}\right]\cos 45°$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{1}{a^2/2}\left(-|q_1| + |q_2| + |q_3| - |q_4|\right)\frac{1}{\sqrt{2}}.$$

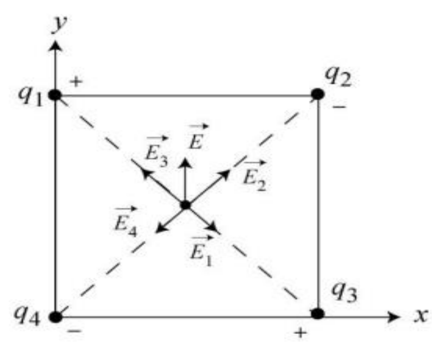
The magnitude of the net electric field is $E = \sqrt{E_x^2 + E_y^2}$.

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}}{a^2}\left(|q_1|+|q_2|-|q_3|-|q_4|\right) = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}}{a^2}\left(10\text{ nC}+20\text{ nC}-20\text{ nC}-10\text{ nC}\right)=0$$

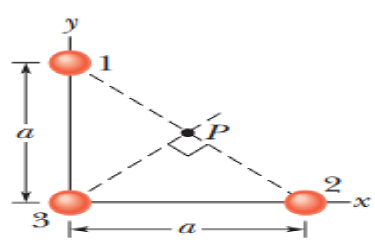
and

$$\begin{aligned}E_y &= \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}}{a^2}\left(-|q_1|+|q_2|+|q_3|-|q_4|\right) = \frac{1}{4\pi\varepsilon_0}\frac{\sqrt{2}}{a^2}\left(-10\text{ nC}+20\text{ nC}+20\text{ nC}-10\text{ nC}\right)\\ &= \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)(2.0\times10^{-8}\text{ C})\sqrt{2}}{(0.050\text{ m})^2}\\ &= 1.02\times10^5\text{ N/C.}\end{aligned}$$

Thus, the electric field at the center of the square is $\vec{E} = E_y\hat{\mathrm{j}} = (1.02\times10^5\text{ N/C})\hat{\mathrm{j}}$.



(2) In Fig. below three particles are fixed in place and have charges q1 = q2 = = e and q3 = +2e. Distance a = 6.00 mm. What are the (a) magnitude and (b) direction of the net electric field at point P due to the particles?



Soln:

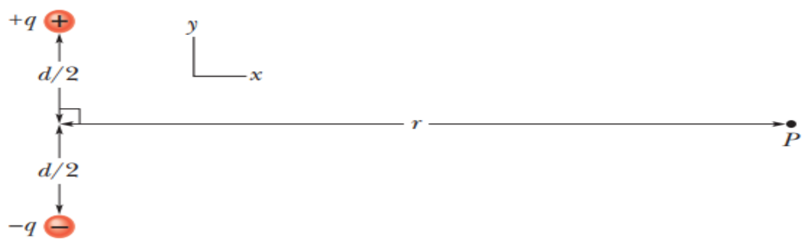
By symmetry we see that the contributions from the two charges q1 = q2 = +e cancel each other, and we simply compute magnitude of the field due to q3 = +2e.

(a) The magnitude of the net electric field is

$$|\vec{E}_{net}| = \frac{1}{4\pi\varepsilon_0}\frac{2e}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{2e}{(a/\sqrt{2})^2} = \frac{1}{4\pi\varepsilon_0}\frac{4e}{a^2}$$
$$= (8.99\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{4(1.60\times10^{-19}\ \mathrm{C})}{(6.00\times10^{-6}\ \mathrm{m})^2} = 160\ \mathrm{N/C}.$$

(b) This field points at 45.0°, counterclockwise from the x axis.

(3) Figure, below shows an electric dipole. What are the (a) magnitude and (b) direction (relative to the positive direction of the x axis) of the dipole's electric field at point P, located at distance r >>d ?



Soln:

(a) Consider the figure below. The magnitude of the net electric field at point P is

$$\left|\vec{E}_{net}\right| = 2E_1\sin\theta = 2\left[\frac{1}{4\pi\varepsilon_0}\frac{q}{(d/2)^2+r^2}\right]\frac{d/2}{\sqrt{(d/2)^2+r^2}} = \frac{1}{4\pi\varepsilon_0}\frac{qd}{\left[(d/2)^2+r^2\right]^{3/2}}$$

For $r \gg d$, we write $[(d/2)^2+r^2]^{3/2} \approx r^3$ so the expression above reduces to

$$|\vec{E}_{net}| \approx \frac{1}{4\pi\varepsilon_0}\frac{qd}{r^3}.$$

(b) From the figure, it is clear that the net electric field at point P points in the $-\hat{j}$ direction, or −90° from the $+x$ axis.