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Practise Chap 4 for Mid

$$v_{avg} = \frac{\vec{dr}}{\Delta t}$$

velocity = instantaneous velocity

$$\vec{v} = \frac{d\vec{r}}{dt}$$

* The instantaneous velocity \vec{v} of a particle is always directed along the tangent to the particle's path at the particle's position.

$$\vec{v}_{avg} = \frac{\Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k}}{\Delta t}$$

$$\vec{v} = \frac{d}{dt} (x \hat{i} + y \hat{j} + z \hat{k})$$

$$a_{avg} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\vec{dv}}{\Delta t}$$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

* In projectile motion, $a_x = 0$ (always), $a_y = -g$ (always)

$$x - x_0 = (v_0 \cos \theta_0) t$$

$$y - y_0 = (v_0 \sin \theta_0) t - \frac{1}{2} g t^2$$

$$v_y = v_0 \sin \theta_0 - gt$$

$$v_y^2 = (v_0 \sin \theta_0)^2 - 2g(y - y_0)$$

The trajectory (path) of particle in projectile motion is parabolic

$$y = (\tan \theta_0) x - \frac{g x^2}{2(v_0 \cos \theta_0)^2} \quad (\text{if } x_0 \neq 0, y_0 = 0)$$

Particles horizontal range

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

$$T = \frac{2v_0 \sin \theta}{g}$$

Particles vertical Height (max)

$$H = \frac{v_0 y^2}{2g}$$

uniform circular motion

$$a = \frac{v^2}{r}$$

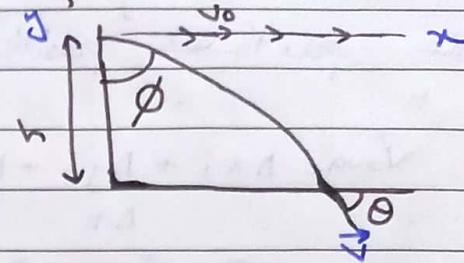
Period of revolution

$$T = \frac{2\pi r}{v}, T = \frac{t}{n}$$

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S.P 4.04) A rescue plane flies at $108 \text{ km/h} (= 30 \text{ m/s})$ at const height $h = 500\text{m}$ toward a point directly over a victim, where a rescue capsule is to land.

- (a) What should be the angle ϕ of the pilot's line of sight to the victim when the capsule release is made?
(b) As the capsule reaches the water, what is its velocity \vec{v} ?



We see that ϕ is given by

$$\phi = \tan^{-1}\left(\frac{x}{h}\right)$$

x is the horizontal coordinate of victim

$$\therefore h = 500\text{m}$$

$$x - x_0 = (v_0 \cos \theta_0) t \quad \text{(i)}$$

Finding t

-500 indicate
not capsule moves
stays at same height

$$y - y_0 = (v_0 \sin \theta_0) t - \frac{1}{2} g t^2$$
$$-500 = (55) (\sin 0^\circ) t - \frac{1}{2} (9.8) t^2$$
$$-500 = -4.9 t^2$$
$$t = 10.1 \text{ sec}$$

Put value in eq (i)

$$x - 0 = (55) (\cos 0^\circ) (10.1)$$

$$x = 555.5 \text{ m}$$

$$\phi = \tan^{-1}\left(\frac{555.5}{500}\right)$$

$$(a) \phi = 48.0^\circ$$

$$\vec{v}_x = (v_0 \cos \theta_0) t = 55 \text{ m/s} \text{ in direction}$$

$$\vec{v}_y = v_0 \sin \theta_0 - gt = -99 \text{ m/s}$$

$$\vec{v} = 55.0 \hat{i} - 99.0 \hat{j}$$

$$|\vec{v}| = 113 \text{ m/s}$$

$$\theta = -60.9^\circ$$

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$$x \rightarrow \text{area} \leftarrow a$$

UNIFORM CIRCULAR MOTION -

position vector (\vec{r}) & cent. acc (a_c)

\vec{r} & \vec{a}_c are always at $\theta = 180^\circ$

velocity (\vec{v}) & cent. acc (a_c)

\vec{v} & \vec{a}_c are always at $\theta = 90^\circ$

NEWTONS LAWS OF MOTION

i) If no net force then no change in the motion of the body.

ii) Reference frame is that in which Newton formulas of mechanics work. also called inertial frame of reference.

Direction

iii) Direction of Force = Direction of acceleration

Terminal Velocity:

when force of gravity = force of air resistance

then force balance out and

object $a=0$, travel at constant velocity.

Radial Acceleration

$$a_r = r \frac{d|\vec{v}|}{dt}$$

Tangential Acceleration

$$a_t = \frac{v^2}{r}$$

$$\text{Total } a = a_r + a_t$$

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ii) Friction is parallel to contact surface

Static friction

$$f_{us} \leq N = f_s$$

force of

Kinetic friction

$$\text{Dynamic } f_d = u_s N$$

$$u_k N = f_k$$

$$F_x = mg \sin \theta$$

$$F_y = n - mg \cos \theta$$

Elevator

$$\sum F_y = T - mg$$

$$a_y = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) g$$

$$T = \left(\frac{2m_1 m_2}{m_1 + m_2} \right) g$$

Special Case

if $m_1 = m_2$ then

$$a_y = 0, T = m_1 g$$

~~If~~ $m_2 \gg m_1$ then

$$a_y \approx g, T = 2m_1 g$$

Frictionless inclined pulley.

$$\sum F_x = 0$$

$$\sum F_x = m_2 g \sin \theta - T = m_2 a_x = m_2 a$$

$$\sum F_y$$

$$\sum F_y = T - m_1 g = m_1 a_y = m_1 a$$

$$\sum F_y = n - mg \cos \theta = 0$$

$$a = \frac{m_2 g \sin \theta - m_1 g}{m_1 + m_2}$$

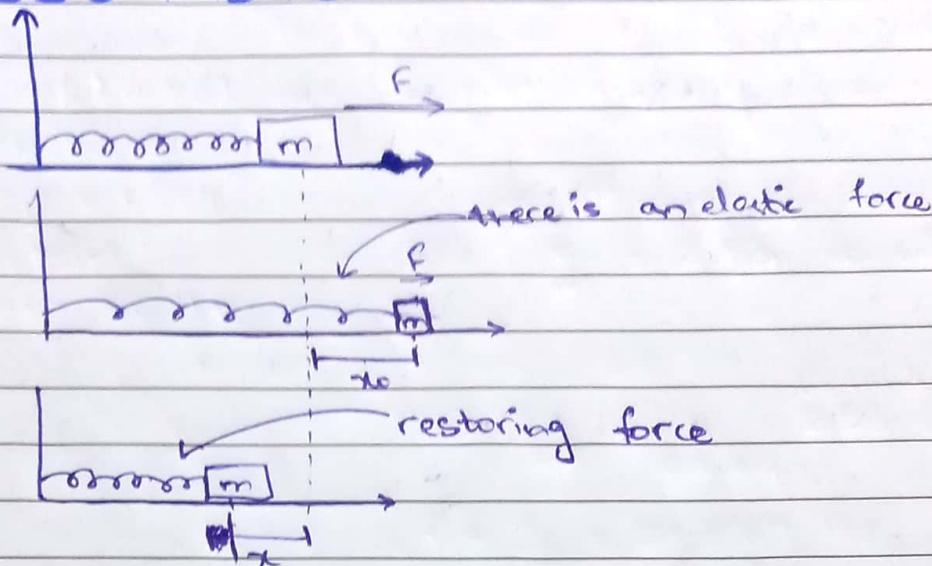
$$T = \frac{m_1 m_2 g (\sin \theta + 1)}{m_1 + m_2}$$

(i) An object can accelerate while travelling with constant speed.

(ii) It is possible to round a curve with a constant magnitude of acceleration.

WAVES & OSCILLATION

PERIODIC MOTION



$$\sum F = kx \quad (\text{Hooke's law})$$

$$\sum F = -kx \quad (\text{Restoring force})$$

$$\therefore \sum F = ma$$

$$ma = -kx$$

$$a = -\frac{k}{m}x$$

$$a = -(\text{constant})x$$

ad - x Simple harmonic motion

To & fro motion

• -ve sign shown always directed to mean position.

$$ma = -kx$$

$$m \ddot{x} = -kx$$

$$m \frac{d^2x}{dt^2} = -\frac{kx}{m}$$

differential equation of S.H.M

ordinary

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

second order b/c
second derivative

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rate of change of acceleration is called jerk

Imp pointr

if the equation is only dependent on 1 variable then it is ordinary differential eq.

if the equation is ~~not~~ dependent on more than one variable then it is partial differential eq.

Simple Harmonic Oscillation

i) Many problems involving mechanical vibrations at small amplitudes reduces to S.H.M or to a combination of such oscillators

second differential

ii) Above equation occurs in many problems in optics, electrical circuits and atomic physics.

$$x_m = \frac{x}{2}$$

$$x = A \cos(\omega t + \phi)$$

$$\omega = \frac{2\pi}{T}$$

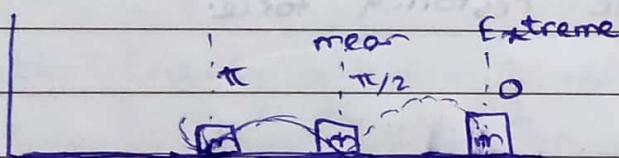
$$\omega = \sqrt{\frac{k}{m}}$$

$$\Sigma F = ma$$

$$\Sigma F = m \left(\frac{v^2}{r} \right)$$

$$F = m \left(\frac{r^2 \omega^2}{r} \right)$$

$$= m r \omega^2$$



$$\pi \quad \phi = \pi/2 \quad \theta = 0$$

one complete (extreme to extreme)
is called one revolution.

$$\Sigma F = -mx\omega^2$$

$$\Sigma F = -Kx$$

$$-mx\omega^2 = -Kx$$

$$\omega^2 = \frac{k}{m}$$

$$v = -Aw \sin(\omega t + \phi)$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$a = -Aw^2 \cos(\omega t + \phi)$$

$$f = \sqrt{f_1^2 + f_2^2}$$

$$a = -\omega^2 x$$

1) A block whose mass m is 680g is fastened to a spring whose spring constant K is 65N/m. The block is pulled a distance $x=11\text{cm}$ from its equilibrium position at $x=0$ on a frictionless surface & released from rest at $t=0$. (a) what are angular frequency, frequency & Time period

$$m = 680\text{g} = 0.68\text{kg}$$

$$K = 65 \text{ N/m}$$

$$x = 11\text{cm} = 0.11\text{m}$$

$$\omega = \frac{\text{on } 2\pi}{m} = \frac{65}{0.68} \text{ rad/sec}$$

$$0.643$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{0.643} = 9.77 \text{ sec}$$

$$\omega = ?$$

$$f = ?$$

$$T = ?$$

$$A = ?$$

$$f = \frac{1}{T} = 0.103 \text{ Hz}$$

$$0.11 = A \cos(\omega 0 + \phi)$$

$$x = A \cos(\omega t)$$

$$= A \cos(\omega t)$$

$$A = 0.11 \text{ m}$$

Condition for S.H.M.

i) System should have restoring force.

(ii) Inertia.

(iii) friction less.

Characteristics of Mass-Spring System

When a body is vibrating, its displacement from the mean position changes with time. The value of its distance from the mean position at any time is known as its instantaneous displacement.

The maximum value of displacement is known as its amplitude.

The angle $\theta = \omega t$ which specifies the displacement as well as the direction of the point executing SHM is known as

Energy Conservation in SHM:-

- i) If the friction effect are neglected, total mechanical energy of vibrating mass spring system remains constant.
- ii) velocity & position always changes and so kinetic & potential also change but their sum will be same always.

$$\therefore F = -Kx$$

$$\therefore \omega = \int F dx$$

$$\therefore U = -\omega$$

$$\text{so } U = - \int F dx$$

$$U = - \int -Kx dx$$

$$U = K \int x dx$$

$$U = K \frac{x^2}{2}$$

By putting displacement

$$U(t) = \frac{1}{2} K(x_m \cos(\omega t + \phi))^2$$

$$\text{Kinetic Energy} = K.E = \frac{1}{2} m v^2$$

Kinetic Energy is maximum if $x=0$ when the mass is at equilibrium position.

$$K(t) = \frac{1}{2} m (-\omega x_m \sin(\omega t + \phi))^2$$

$$K.E = \frac{1}{2} K(x_m^2 - x^2)$$

$$\therefore K = m \omega^2$$

$$K(t) = \frac{1}{2} m \omega^2 x_m^2 \sin^2(\omega t + \phi)$$

$$= \frac{1}{2} K x_m^2 \sin^2(\omega t + \phi)$$

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The energy is partly P.E and partly K.E.

$$E = P.E \text{ and } + K.E$$

$$E = \frac{1}{2} k x^2 = \text{constant}$$

Of the previous Question:

- (c) what is the max speed v_m of the oscillating block, and where is the block when it has this speed?
- (d) what is the magnitude of max acc a_m of the block?
- (e) what is the phase constant ϕ for the motion?
- (f) what is the displacement function $x(t)$ for the spring-block system?

$$(c) v_m = \omega x_m$$

$$= (9.78)(0.11)$$

$$= 1.1 \text{ m/s}$$

$$(d) a_m = \omega^2 x_m$$

$$= (9.78)^2 (0.11)$$

$$= 11 \text{ m/s}^2$$

$$(e) 1 = \cos \phi.$$

$$\therefore \phi = 0 \text{ rad}$$

$$(f) x(t) = x_m \cos(\omega t + \phi)$$

$$= (0.11) \cos(9.8 \times t + 0)$$

$$x(t) = 0.11 \cos(9.8t)$$

Q2) Many tall buildings have mass dampers, which are anti-sway devices to prevent them from oscillating in a wind.

The device might be a block oscillating at the end of a spring and on a lubricated track. If the building sways, say eastward, the block also moves eastward but delayed enough so that when it finally moves, the building is then moving back westward. Thus, the motion of the oscillator is out of step with the motion of the building.

Suppose that the block has mass $m = 2.72 \times 10^5 \text{ Kg}$ & is designed to oscillate frequency $f = 10.0 \text{ Hz}$ and with amplitude $x_m = 20.0 \text{ cm}$.

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- (a) What is the total mechanical energy E of the spring-block system?
- (b) What is the block's speed as it passes through the equilibrium point?

a) ~~Total~~ The mechanical Energy is constant throughout the motion. if $x = x_m$ then velocity $v = 0$.

$$K_E = m\omega^2 = m(2\pi f)^2$$

$$K = 1.073 \times 10^9 \text{ N/m}$$

$$\begin{aligned}\cancel{\text{Total}} E &= K + U \\ &= \frac{1}{2}mv^2 + \frac{1}{2}Kx^2 \\ &= 0 + 2.147 \times 10^7 \text{ J} \\ E &= 2.1 \times 10^7 \text{ J}\end{aligned}$$

b) When $x = 0$.

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}Kx^2$$

$$E = \frac{1}{2}(2.72 \times 10^5)v^2 + 0$$

$$v = 12.6 \text{ m/s}$$

Because E is kinetic energy, this is max speed.

DAMPED OSCILLATIONS

In a damped oscillations, the motion of the oscillator is reduced by an external force.

The liquid provides the external damping force F_d .

$$F_d = -bv$$

b is damping constant. SI unit kg/sec

Let assume that the gravitational force on a block is negligible relative to F_d & F_g .

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$$\therefore \text{f}_{\text{net}} = ma$$

then

$$-bx - Kx = ma$$

$$ma + bx + Kx = 0$$

$$\frac{md^2x}{dt^2} + b \frac{dx}{dt} + Kx = 0$$

The solution of this eq. is:-

$$x(t) = x_m e^{-bt/2m} \cos(\omega't + \phi)$$

The angular frequency is given by,

$$\omega' = \sqrt{\frac{K}{m} - \frac{b^2}{4m^2}}$$

$$\text{if } b=0 \text{ then } \omega = \sqrt{\frac{K}{m}}$$

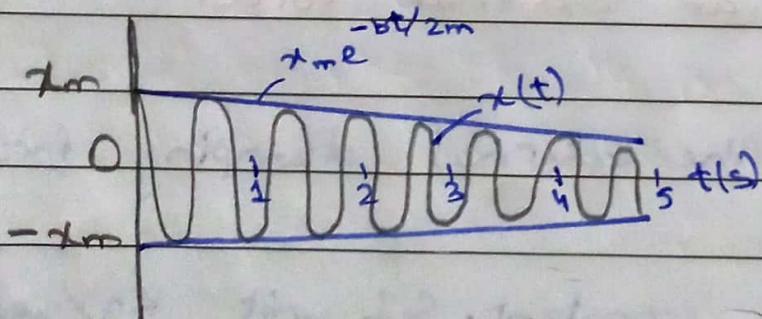
if b is small but not zero ($b \ll \sqrt{km}$) then
 $\omega' = \omega$

Damped Energy

If the oscillator is damped, the mechanical energy is not constant but decreases with time.

$$\checkmark \text{ no } 2m \times$$

$$E(t) \approx \frac{1}{2} Kx_m^2 e^{-bt/2m}$$



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Q) For the damped oscillator, $m=250\text{kg}$, $K=85\text{N/m}$,
and $b=70\text{g/s} = 0.07\text{kg/s}$

(a) What is the period of motion?

(b) How long does it take for the amplitude of the damped oscillations to drop to half its initial value?

(c) How long does it take for the mechanical energy to drop to one-half its initial value?

$$(a) \omega = \sqrt{\frac{K}{m}} = \sqrt{85 \times 0.25} \\ = 4.6 \text{ kg/s}$$

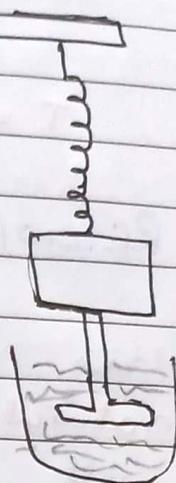
So;

$$b \ll \sqrt{Km}$$

means it is like undamped oscillation

$$T = 2\pi \sqrt{\frac{m}{K}} = 2\pi \sqrt{\frac{0.25}{85}}$$

$$T = 0.34 \text{ sec}$$



(b) The amplitude has the value x_m at $t=0$. Thus we must calculate to find the value of t .

$$x_m e^{-bt/2m} = \frac{1}{2} x_m \quad \left| \begin{array}{l} \ln(e^{-bt/2m}) = \ln(\frac{1}{2}) \\ -bt/2m = \ln(\frac{1}{2}) \end{array} \right.$$

put values

$$t = \frac{2m \ln(\frac{1}{2})}{-b} = -\frac{(2)(0.25) \ln(\frac{1}{2})}{(0.07)}$$

$$t = 5.0 \text{ sec}$$

(c) The mechanical energy at $t=0$ is $\frac{1}{2} K x_m^2$. The value of t would be

$$E = \frac{1}{2} K x_m^2 e^{-bt/2m} =$$

$$t = \frac{-m \ln(\frac{1}{2})}{b} = -\frac{(0.25) (\ln(\frac{1}{2}))}{0.07} = 2.5 \text{ sec}$$

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PENDULUM

$$T = 2\pi \sqrt{\frac{I}{K}} \quad (\text{torsion pendulum})$$

$$T = 2\pi \sqrt{\frac{L}{g}} \quad (\text{simple pendulum})$$

$$T = 2\pi \sqrt{\frac{I}{mgh}} \quad \begin{matrix} \leftarrow \\ \text{rotational inertia} \end{matrix} \quad (\text{physical pendulum})$$

RESONANCE

when a system is disturbed by a periodic driving force whose frequency is equal to the natural frequency (f_0) of the system, the system will oscillate with large amplitude.

Inelastic Collision

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$$

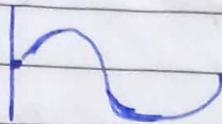
Parallel Axis Theorem

$$I = I_{\text{com}} + mh^2$$

~~Rotational inertia~~
~~Distance b/w pivot point & centre of mass is~~
 $= \frac{1}{2} L$
rotational inertia of the pendulum about a perpendicular axis through its centre of mass is $\frac{1}{12} m l^2$.

WAVE

wave is - pattern of motion of particles of the medium which transport disturbance without transporting the medium.



contains

It is sinusoidal graph because particle oscillate according to "sine function".

θ = angular displacement

ω = angular velocity (angular frequency)

t = time

$\theta = \omega t$

$$y(t) = \sin(\theta) = \sin(\omega t)$$

MECHANICAL WAVES

are disturbances ~~in~~ in matter that carry energy from one place to another through medium.

- Medium can be solid, liquid or gas.
- some waves travel without medium like Electromagnetic waves

TRANSVERSE WAVES

A wave that causes the medium to vibrate at right angles to the directions of the wave.

Travel in form of crest & trough.

* It travels in the form of crest & trough.

LONGITUDINAL WAVES

A wave in which the vibration of the medium is parallel to the direction of the wave travels.

A wave doesn't move the medium, it's just energy travelling through the medium.

* It travels in the form of compression & rarefaction.

LIGHT WAVES

Light is transverse wave but it is not mechanical wave.

- Energy is perpendicular to direction of motion.
- Moving photons creates electric & magnetic field.
- Light has both electric & magnetic fields at right angles and perpendicular to the direction of propagation of wave.

SOUND WAVES

Sound waves are longitudinal waves

travel

The denser the medium the faster sound will

The higher the temperature the faster sound travels

INFRA SONIC & ULTRASONIC

A healthy human can hear frequencies in the range of 20 Hz to 20,000 Hz

Below 20 Hz Infrasonic } human cannot
Above 20,000 Hz Ultrasonic } hear

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WAVE ON A STRING

we need function that gives the shape of the wave.

$$y = h(x, t)$$

y = transverse displacement

h = function (sine or cosine)

t = time

x = position along string

WAVE VARIABLES

Amplitude

Phase

$$y(x, t) = y_m \sin(Kx - \omega t)$$

Angular wave number
position

time

angular frequency

Calculate in radians

$$K = \frac{2\pi}{\lambda} \quad (\text{angular wave number})$$

λ (wave length)

$$\omega = \frac{2\pi}{T} \text{ or } 2\pi f$$

$$Kx - \omega t = \text{constant}$$

$$\frac{K dx}{dt} - \omega = 0$$

$$\frac{dx}{dt} = \frac{\omega}{K} = v$$

$$v = \frac{\omega}{K} = \frac{\gamma}{T} = \gamma f$$

Q) A wave travelling along a string is described by:

$$y(x, t) = 0.00327 \sin(72.1x - 2.72t)$$

in which the numerical constants are in SI units

(0.00327 m, 72.1 rad/m, 2.72 rad/s).

(a) What is the amplitude of this wave? wavelength, period, frequency, velocity, displacement y of the string at $x = 22.5\text{cm}$, and $t = 18.9\text{s}$?
 \downarrow
 0.225m

$$\text{Amplitude} \Rightarrow y_m = 0.00327 \text{ m} = 327 \text{ mm}$$

$$\text{wavelength} \Rightarrow \lambda = \frac{2\pi}{K \rightarrow 72.1} = 0.087 \text{ m} = 8.7 \text{ cm}$$

$$\text{Period} \Rightarrow T = \frac{2\pi}{\omega \rightarrow 2.72} = 2.31 \text{ sec}$$

$$\text{frequency} \Rightarrow f = \frac{1}{T \rightarrow 2.31} = 0.433 \text{ Hz}$$

velocity of wave $\Rightarrow v = \lambda f \Rightarrow 8.7 \times 0.433 = 3.76 \text{ m/s}$

displacement $\Rightarrow y = 0.00327 (\sin(72.1 \times \frac{x}{0.225} - 2.72 \times 18.9))$

$$= 0.00188 \text{ m} \quad \text{Calculate in radians}$$

$$= 0.2 \text{ mm}$$

THE WAVE EQUATION (FUNCTION)

A travelling wave is always in the following form

perpendicular dist \downarrow
horizontal dist \downarrow
wave function $y(x, t) = f(x \pm vt)$

If goes right then -ve
if goes left then +ve.

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \quad (\text{wave equation})$$

It is a linear partial difference equation, when y_1 and y_2 are solutions, any linear combination of y_1 & y_2 (like $ay_1 + by_2$) is also a solution.

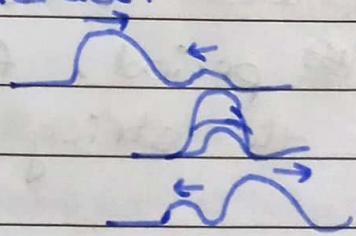
SUPERPOSITION OF WAVES

$$y' (x, t) = y_1 (x, t) + y_2 (x, t)$$

- i) Overlapping waves add to produce resultant.
- ii) They do not alter each other.
- iii) They interfere but do not interact.

INTERFERENCE

two sinusoidal with same amplitude & wavelength is same direction produce a resultant sinusoidal wave in that direction



$$y_1 (x, t) = y_m \sin (Kx - \omega t)$$

$$y_2 (x, t) = y_m \sin (Kx - \omega t + \phi)$$

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$$y'(x,t) = y_1(x,t) + y_2(x,t)$$
$$= y_m \sin(kx - \omega t) + y_m \sin(Kx - \omega t + \phi)$$

$$\underbrace{y'(x,t)}_{\text{Displacement}} = \underbrace{[2y_m \cos^2 \frac{1}{2}\phi]}_{\text{Magnitude gives Amplitude}} \underbrace{\sin(Kx - \omega t + \frac{1}{2}\phi)}_{\text{Oscillating term}}$$

In the preceding sample problem, $y = 1.92 \text{ mm}$ at $x = 22.5 \text{ cm}$ & $t = 18.9 \text{ sec}$.

(a) what is u (transverse velocity), transverse acceleration?

$$\therefore u = \frac{\partial y}{\partial t}$$

$$y(x,t) = y_m \sin(Kx - \omega t) \rightarrow (A)$$

Taking derivative

$$u = \frac{\partial y}{\partial t} = -\omega y_m \cos(Kx - \omega t)$$

$$u = (-2.72)(3.27) \cos(-35.18)$$

$$u = 7.2 \text{ mm/s}$$

transverse acceleration $a_y = \frac{\partial u}{\partial t} = -\omega^2 y_m \sin(Kx - \omega t)$

$$= -\omega^2 y \quad \therefore \text{by eq}(A)$$

$$= -(2.72)^2 (1.92)$$

$$a_y = -14.2 \text{ mm/s}^2$$

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Q3) Two identical sinusoidal waves, moving in the same direction along a stretched string, interfere with each other. The amplitude of y_m of each wave is 9.8 mm, and the phase difference ϕ between them is 100° .

(a) What is the amplitude y_m' of the resultant wave due to the interference, and what is the type of this interference?

(b) What phase difference, in radians and wavelength^{h.s.}, will give the resultant wave an amplitude of 4.9 mm?

$$\begin{aligned}y_m' &= |2y_m \cos \frac{1}{2}\phi| \\&= |2 \times 9.8 \times \cos\left(\frac{100}{2}\right)| \\&= 13 \text{ mm}\end{aligned}$$

Now;

$$\begin{aligned}y_m' &= |2y_m \cos \frac{1}{2}\phi| \\4.9 \text{ mm} &= 2 \times 9.8 \times \cos \frac{1}{2}\phi\end{aligned}$$

$$\begin{aligned}\phi &= 2 \cos^{-1} \left(\frac{4.9}{2 \times 9.8} \right) \\&= \pm 2.6 \text{ rad.}\end{aligned}$$

$$x = \frac{\phi}{2\pi}$$

$$= \frac{\pm 2.6}{2\pi}$$

$$= 0.42 \text{ wavelength}$$

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Q1) A pulse moving to the right along the x -axis is represented by the wave function

$$y(x,t) = \frac{2}{(x-3t)^2 + 1}$$

where x & y are measured in cm and t is measured in sec. Plot the wave function at $t=0$, $t=1\text{sec}$ & $t=2\text{sec}$

This function is of the form $y = f(x-vt)$, so $v = 3\text{ cm/s}$, The max value of the function

$$A = 2$$

Since; $y(x,t) = f(x-vt)$

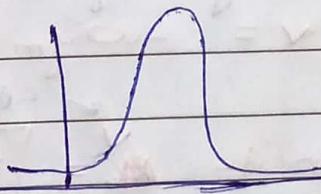
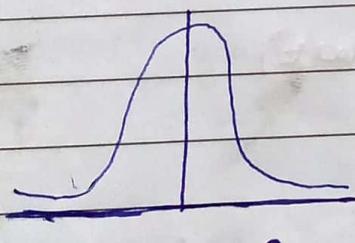
$$\therefore t=0$$

$$y(x,t) = \frac{2}{(x-3t)^2 + 1}$$

$$y(x,0) = \frac{2}{x^2 + 1} \quad \text{at } t=0$$

$$y(x,1) = \frac{2}{(x-3)^2 + 1} \quad \therefore t=1$$

$$y(x,2) = \frac{2}{(x-6)^2 + 1} \quad \therefore t=2$$



what if

$$y(x,t) = \frac{2}{(x+3t)^2 + 1}$$

The new feature is the plus sign in the denominator. This results in the same pulse with the same shape But to the left.

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★) What if $y(x,t) = \frac{4}{(x-3t)^2 + 1}$

The new feature is 4 in numerator rather than 2. This results in a pulse moving right but with twice height.

SINOSUIDAL WAVE

$$y(x,t) = A \sin\left[\frac{2\pi}{T}(x-vt)\right]$$

$$K = \frac{2\pi}{\lambda}, \quad \omega = \frac{2\pi}{T}, \quad v = \frac{\lambda}{T}$$

$$v = \lambda f$$

$$y = A \sin(Kx - \omega t)$$

$$v = \frac{\omega}{K}$$

$$y = A \sin(Kx - \omega t + \phi)$$

For element at certain location x , we find rate of change of y by taking derivative

$$v = -\omega y_m \cos(\omega Kx - \omega t)$$

← speed

Linear density $\mu = \frac{\text{Tension}}{\text{linear density}}$

$$f = \frac{nN}{2L}$$

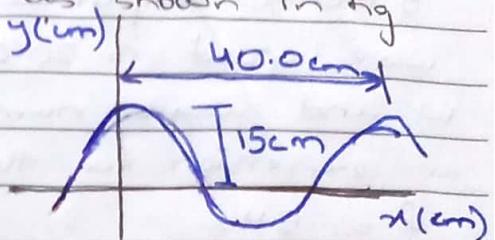
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Q2) A sinusoidal wave travelling in the positive x direction has an amplitude of 15.0 cm, a wavelength of 40.0 cm, and a frequency of 8.00 Hz. The vertical position of an element of the medium at $t=0$ and $x=0$ is also 15.0 cm as shown in fig below.

$$\text{wavelength} = \lambda = 40 \text{ cm}$$

$$\text{Amplitude} = A = 15 \text{ cm}$$

$$\text{wave function} \Rightarrow y = A \cos(Kx - \omega t)$$



(A) Find the wave number K , T , angular frequency ω , speed of wave v .

(B) Determine phase constant, and write general expression for the wave function.

$$(A) K = \frac{2\pi}{\lambda} = \frac{2\pi}{0.40} = 15.7 \text{ rad/m}$$

$$T = \frac{1}{f} = \frac{1}{8} = 0.125 \text{ sec}$$

$$\omega = 2\pi f = 2\pi \times 8 = 50.265 \text{ rad/sec}$$

$$v = \lambda f = 0.4 \times 8 = 3.2 \text{ m/sec}$$

(B) $y = A \sin(\omega t + \phi)$
 $y = A \sin(Kx - \omega t + \phi)$

Because $A = 15 \text{ cm}$ and because $y = 15 \text{ cm}$ at $x=0$ and $t=0$ then

$$15 = 15 \sin(0 - 0 + \phi)$$

$$15 = 15 \sin(\phi)$$

$$\phi = 90^\circ \text{ or } \pi/2$$

then

$$y = A \sin(Kx - \omega t + \frac{\pi}{2}) \Rightarrow A \cos(Kx - \omega t)$$

$$y = 15 \text{ cm} \cos(15\pi - 50\pi t)$$

Q3) The string shown is driven at a frequency of 5 Hz. The amplitude of the motion is 12 cm and the wave speed is 20.0 m/s. Determine the angular frequency ω and wave number K for this wave, and write an expression for the wave function.

$$f = 5 \text{ Hz}$$

$$v = \lambda f$$

$$\omega = 2\pi f$$

$$A = 12 \text{ cm}$$

$$20 = \lambda \times 5$$

$$= 2\pi \times 5$$

$$v = 20 \text{ m/s}$$

$$\lambda = 4 \text{ m}$$

$$= 31.41 \text{ rad/sec}$$

$$\omega = ?$$

$$K = ?$$

$$K = \frac{2\pi}{\lambda}$$

$$K = 1.57 \text{ rad/m}$$

$$y = A \sin(\omega t - Kx)$$

$$= (0.12) \sin(1.57x - 31.41t)$$

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ASSIGNMENT 2

Problem 1r

$$m = 1.75 \text{ kg}$$

$$x = 4 \cos(1.33t + \pi/5)$$

$$x_m = ? , f = ? , \omega = ? , T = ?$$

$$a = ? , v = ? , K = ?$$

$$\therefore x_m = x_m \cos(\omega t + \phi)$$

By comparing

$$x_m = 4 \text{ m}$$

$$\omega = 1.33 \text{ rad/sec}$$

$$T = \frac{2\pi}{\omega} = \frac{\omega}{2\pi}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$m\omega^2 = k$$

$$T = \frac{2\pi}{1.33} = \frac{1.33}{2\pi}$$

$$K = 1.75 \times 1.33^2$$

$$T = 4.72 \text{ sec}$$

$$f = 0.211 \text{ Hz}$$

$$K = 3.095 \text{ N/m}$$

$$v = -x_m \omega \sin(\omega t + \phi)$$

$$a = -x_m \omega^2 \cos(\omega t + \phi)$$

$$v = -4 \times 1.33 \sin(1.33t + \pi/5)$$

$$= -4 \times 1.33^2 \cos(1.33t + \pi/5)$$

$$v = -5.32 \sin(1.33t + \pi/5)$$

$$a = -7.07 \cos(1.33t + \pi/5)$$

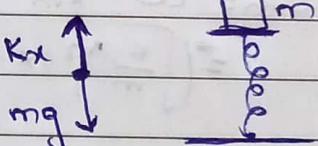
m/s

m/s²

Problem 2r

$$K = 600 \text{ N/m}$$

$$m = 6 \text{ kg}$$



$$F_s = Kx$$

$$F_g = mg$$

$$F_s > F_g$$

$$Kx > mg$$

$$x > \frac{mg}{K}$$

$$x > \frac{6 \times 9.8}{400}$$

$$x > 0.147 \text{ m}$$

When the block loses contact, its acceleration will be greater than or equal to 9.81 at a certain distance

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$$K = \frac{2\pi}{\tau} \Rightarrow \tau = \frac{2\pi}{K}$$

* The net charge is also called excess charge because charged object has an excess of either positive or negative charges.

* A tiny imbalance in either positive or negative charge on an object is the cause of static electricity

* The electrons in incubators are not free to move they are tightly bound inside atoms.

Coulomb constant

$$F = \frac{K q_1 q_2}{r^2}$$

$$K = 8.988 \times 10^9 \text{ Nm}^2/\text{C}^2$$

$$K = \frac{1}{4\pi\epsilon_0}$$

charges produced by rubbing are typically around a micro coulomb.

$$1 \mu C = 10^{-6} C$$

Permittivity constant of free space

$$\epsilon_0 = \frac{1}{4\pi K} = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

Quantization of charge

Any positive or negative charge q that can be detected can be written as

$$q = ne \quad n = \pm 1, \pm 2, \pm 3, \dots$$

in which e is elementary charge

$$e = 1.602 \times 10^{-19} C$$

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Q) Consider three point charges located at the corners of a right triangle as shown in fig. Find the resultant force exerted on q_3 .

$$q_1 = q_2 = 5.0 \mu C$$

$$q_3 = -2.0 \mu C$$

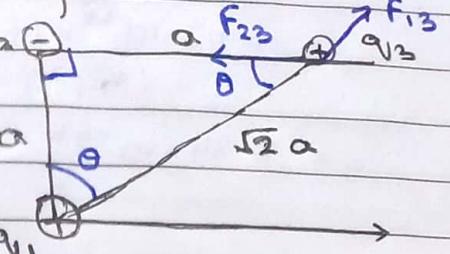
$$a = 0.10 m$$

Find magnitude & direction of $F_{\text{resultant}}$.

$$F_{x\text{ result}} = F_{13x} + F_{23x}$$

$$= 4.5 \cos 45 + 9 \cos 180$$

$$F_x = -1.1 i$$



$$F_{23} = \frac{k q_1 q_2}{r^2} = \frac{9 \times 10^9 \times 2 \mu \times 5 \mu}{(0.1)^2}$$

$$F_{23} = 9 N$$

$$F_{13} = \frac{9 \times 10^9 \times 5 \mu \times 5 \mu}{(\sqrt{2} \times 0.1)^2} = 4.5 N$$

$$F_y = F_{13y} + F_{23y}$$

$$= 4.5 \sin 45 + 9 \sin 180$$

$$F_y = 7.9$$

$$\lvert \vec{F} \rvert = 8 N$$

$$\theta = \tan^{-1} \left(\frac{7.9}{1.1} \right)$$

$$\theta = 82^\circ$$

F_x is -ve & F_y is +ve
lies in second Quad

$$\theta = 180 - 82$$

$$\theta = 98^\circ$$

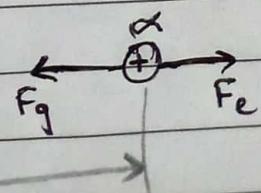
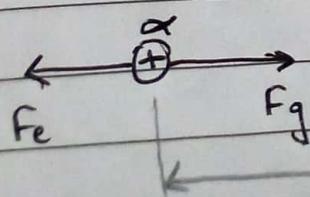
PRINCIPLES

- The algebraic sum of all the electric charges in any closed system is constant.
- The magnitude of charge of the electron or proton is a natural unit of charge.

Q) An ' α ' alpha particle is the nucleus of a helium atom.

It has mass $m = 6.64 \times 10^{-27} \text{ kg}$ and charge ~~$q = 2e$~~

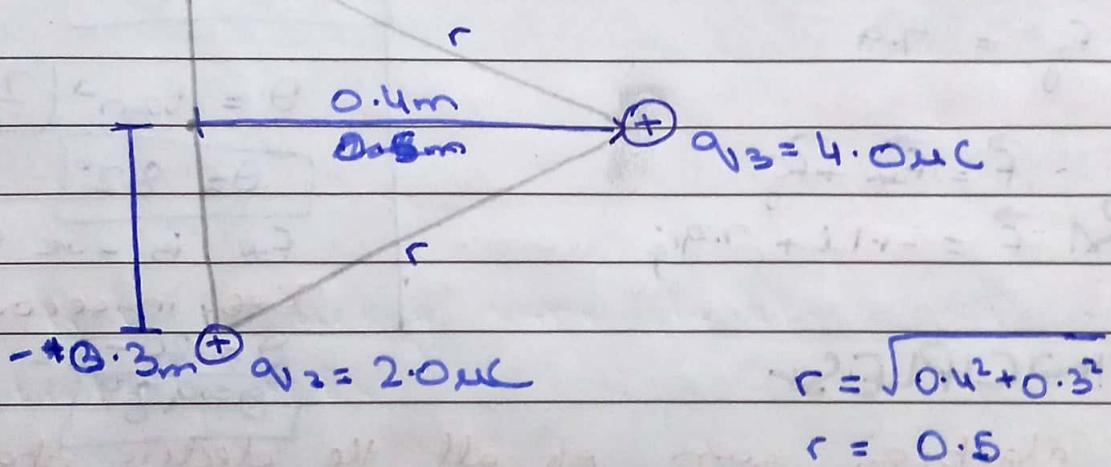
$q = +2e = 3.2 \times 10^{-19} \text{ C}$. Compare the force of the electric repulsion between two α particles with the force of gravitational attraction between them.



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$$F_g = \frac{Gm_1 m_2}{r^2} = \frac{Kq_1 q_2}{r^2} = F_e$$
$$= 6.67 \times 10^{-29} \times (6.64 \times 10^{-27})^2 = 8.988 \times 10^9 \times (3.2 \times 10^{-19})^2$$
$$(2 \times r^2) \quad r^2$$
$$= 2.94 \times 10^{-63} \quad = 9.2 \times 10^{-28}$$

Q) Two equal positive point charges $q_1 = q_2 = 2.0 \mu C$ are located at $x=0, y=0.3m$ & $x=0, y=-0.3m$, what are the magnitude & direction of the total (net) electric force that these charges exert on a third point charge $Q = 4.0 \mu C$ at $x=0.4m, y=0.3m$. $q_1 = 2.0 \mu C$



Since q_1 & q_2 are vertically at same distance with like charges so they cancels each other.
Only left x-axis

$$F_{net} = 2F_x$$

$$= 2 \frac{Kq_3^2}{r^2}$$

ELECTRIC FIELD

The electric field on a charged body is exerted by the electric field created by other charged bodies.

$$E = \frac{F_0}{q_0} \quad \begin{matrix} \text{Electric force per} \\ \text{unit charge} \end{matrix}$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{|q_0|}{r^2} \hat{r} \quad \begin{matrix} \leftarrow \text{unit vector} \end{matrix}$$

A point charge $q_0 = -8.0 \mu C$ is located at the origin.

Find the electric field vector at the field point $x=1.2 \text{ m}$, $y=-1.6 \text{ m}$

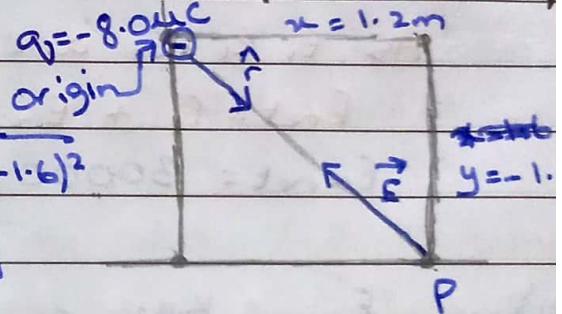
$$E = \frac{F_0}{q_0}$$

$$r = \sqrt{1.2^2 + (-1.6)^2}$$

$$r = 2 \text{ m}$$

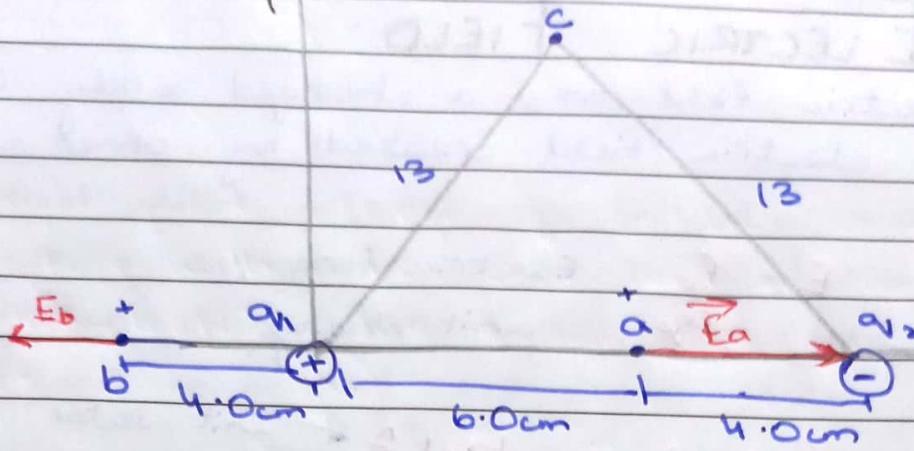
$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{|-8.0 \times 10^{-9}|}{2^2}$$

$$E = 17.98$$



* This combination of two charges with equal magnitude and opposite direction is called electric dipole.

Q) Point charges q_1 & q_2 of $+12\text{nC}$ & -12nC are placed 0.1m apart. Compute the electric field caused by q_1 , the field caused by q_2 , and the total field @ at point 'a' (b) at point 'b' and (c) at point 'c'.



$$\text{a) } E_1 = \frac{kq_1}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{(0.06)^2} = 30000 \text{ N/C}$$

$$E_2 = \frac{kq_2}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{(0.04)^2} = 67500 \text{ N/C}$$

As E_1 , E_1 , E_2 are in same direction (from left to right)
 $E_{\text{net}} = E_1 + E_2$ because q_1 repel a to q_2 .
 q_1 , q_2 attracts

$$E_{\text{net}} = 30000 + 67500 \Rightarrow 97500 \text{ N/C}$$

$$\text{b) } E_1 = \frac{kq_1}{r^2} = 67500 \text{ N/C}$$

$$E_2 = \frac{kq_2}{r^2} = \frac{9 \times 10^9 \times 12 \times 10^{-9}}{(0.04)^2} = 5510 \text{ N/C}$$

$$E_{\text{net}} = 67500 - 5510 = 61990 \text{ N/C}$$

ELECTRIC FLUX

$$\Phi_F = EA \cos\theta$$

N m²/C

Electric Flux for uniform E

if E & A are parallel
 $\Phi_F = EA$

if E & A are perpendicular
 $\Phi_F = 0^\circ$

They depend on by putting values on cos.

if Electric Field E is not uniform

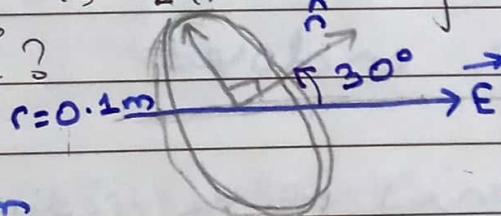
$$\begin{aligned}\Phi_F &= \int E \cos\phi dA \\ &= \int E_{\perp} dA \\ &= \int \vec{E} \cdot d\vec{A}\end{aligned}$$

(22.1)

Q) A disk with radius 0.10m is oriented with its normal unit vector \hat{n} at an angle of 30° to a uniform electric field \vec{E} with magnitude $2.0 \times 10^3 \text{ N/C}$. (a) what is Electric flux through disk.

(b) what is the flux through the disk if it is turned so that its normal is perpendicular to \vec{E} ?

(c) what is the flux through disk if Normal is parallel to \vec{E} ?



$$r = 0.10 \text{ m}$$

$$\theta = 30^\circ$$

$$E = 2.0 \times 10^3 \text{ N/C}$$

$$\Phi_F = A = \pi r^2$$

$$= \pi (0.10)^2$$

$$A = 0.0314 \text{ m}^2$$

$$\Phi_F = ?$$

$$\Phi_F = ? \text{ if } E \perp N$$

$$\Phi_F = ? \text{ if } E \parallel N$$

$$(a) \Phi_F = EA \cos 30^\circ$$

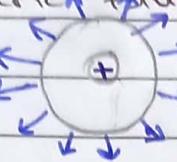
$$= 2.0 \times 10^3 \times 0.0314 \cos 30^\circ$$

$$\Phi_F = 54.38 \text{ m}^2/\text{C}$$

$$(b) \Phi_F = E A \cos(90^\circ) \\ = 0$$

$$(c) \Phi_F = EA \cos(0^\circ) \\ = 2.0 \times 10^3 \times 0.0314 \times 1 \\ = 62.8 \text{ Nm}^2/\text{C}$$

Q) A positive point charge $q = 3.0 \mu\text{C}$ is surrounded by a sphere with radius 0.2m centered on the charge. Find the electric flux through the sphere due to this charge.



Here N always parallel to E

$$\therefore E = \frac{F}{q}$$

$$= \frac{1}{4\pi\epsilon_0} \cdot \frac{(3 \times 10^{-6})}{(0.2)^2}$$

$$\Phi_F = EA$$

sphere Area formula

$$= \frac{K q}{r^2} \cdot 4\pi r^2$$

$$= \frac{q}{4\pi\epsilon_0} \cdot 4\pi$$

Gauss's Law

$$\Phi_F = EA = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2} (4\pi R^2)$$

$$= \frac{q}{\epsilon_0} \quad \text{charge enclosed}$$

$$\left. \begin{aligned} &= \frac{\pi q}{\epsilon_0} = \frac{3 \times 10^{-6}}{8.85 \times 10^{-12}} \\ &= 3.38 \times 10^5 \end{aligned} \right.$$

The flux is independent of the radius R of the sphere. It only depends on the charge enclosed by the sphere.

At Surface

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2}$$

Outside

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

$$E = \frac{1}{2\pi\epsilon_0} \cdot \frac{\sigma}{r} \quad \text{Field of an infinite line of charges}$$

(infinite line of charges)

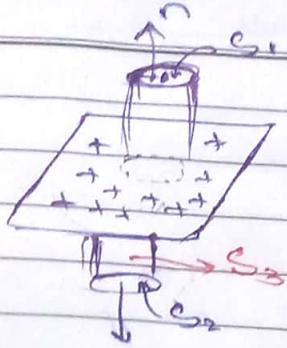
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Using Gauss's law

$$\Phi_E = \frac{q_{\text{enclosed}}}{\epsilon_0} \quad (\text{eqn A})$$

Electric Flux through S_1 ,

$$\Phi_1 = EA \cos 0^\circ = EA$$



flux through S_2

$$\Phi_2 = EA \cos 90^\circ = 0$$

flux through S_3

$$\Phi_3 = EA \cos 90^\circ = 0$$

total flux

$$\Phi = \Phi_1 + \Phi_2 + \Phi_3$$

$$= EA + EA + 0$$

$$\Phi_E = 2EA \quad (\text{eq B})$$

By comparing eq (A) & eq (B)

$$2EA = \frac{1}{\epsilon_0} q_V$$

$$\therefore \frac{q_V}{A} = \sigma$$

$$E = \frac{1}{2\epsilon_0} \frac{q_V}{A}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

$$E = E_1 + E_2$$

$$E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} \quad \text{if two opposite charge}$$

$$E = \frac{\sigma}{\epsilon_0}$$

Electric field near the surface of earth is 150 N/m.

Calculate electric charge density on earth and net charge on it.

$$E = 150 \text{ N/m}$$

$$\therefore E = \frac{\sigma}{\epsilon_0}$$

$$V = EA$$

$$\Phi_F = E (4\pi R^2)$$

$$\sigma = 150 \times 8.85 \times 10^{-12}$$

$$\sigma = 1.3275 \times 10^{-9} \text{ C/m}^2$$

$$E = \frac{1}{4\pi\epsilon_0} \frac{q_V}{r^2}$$

$$\therefore \sigma = \frac{q_V}{A}$$

$$= \frac{1}{\epsilon_0} \frac{q_V}{A}$$

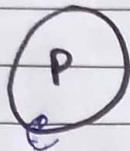
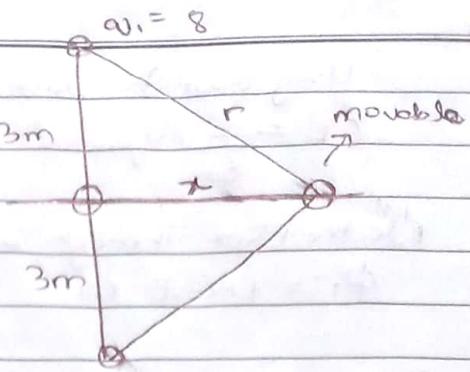
$$\sigma A = q_V$$

$$1.32 \times 10^{-9} \times 4\pi r^2 = q_V$$

$$E = \frac{\sigma}{\epsilon_0}$$

$$q_V = 1.32 \times 10^{-9} \times 4\pi (6.38 \times 10^6)^2 \approx \\ = 6.7 \times 10^5 \text{ C.}$$

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Pitch is the horizontal distance between two circles

$$P = T v_{\parallel}$$

~~× (Magnetic force outward) (Field Inward)
(Electron going inside)~~

~~○ (Magnetic force inward) (Field outward)
(Electron coming out)~~

When Magnetic force = Centripetal force

then

$$r = \frac{mv}{qB}$$

$$T = \frac{2\pi m}{qB}$$

thumb (Magnetic force direction)

pointer finger (current)

middle finger (Magnetic field)

~~if ○ Magnetic field (outward) (and electron)~~

~~if × - Magnetic field (Inward) (and proton)~~

field outward

$$x = A \cos(\omega t + \phi)$$

$$x_m = \frac{x}{2}$$

$$\omega = \frac{2\pi}{T}$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\omega = 2\pi f$$

$$v = \omega x$$

$$v = \frac{T}{\omega}$$

$$v = -A \omega \sin(\omega t + \phi)$$

$$v_m = \omega x_m$$

$$v = 0$$

$$a = -\omega^2 x$$

$$a = -x_m \omega^2 \cos(\omega t + \phi)$$

$$K = m \omega^2$$

$$K = \frac{2\pi}{T}$$

$$F = ma$$

$$F = -Kx$$

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$$E = \frac{1}{2} K x_m^2$$

$$P.E = \frac{1}{2} K x^2$$

$$K.E = \frac{1}{2} K (x_m^2 - x^2)$$

$$K.E = \frac{1}{2} m v^2$$

$$T = 2\pi \sqrt{\frac{m}{K}}$$

$$T = 2\pi \sqrt{\frac{I}{K}}$$

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$y(x,t) = y_m \sin(Kx - \omega t)$$

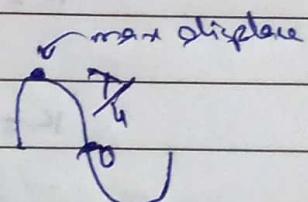
$$y(x,t) = f(x \pm vt)$$

$$y'(x,t) = [2y_m \cos(\frac{1}{2}\phi)]$$

$$y''(x,t) = x_m \sin \left[\frac{2\pi}{\lambda} (x - vt) \right]$$

$$v = -\omega y_m \cos(Kx - \omega t)$$

$$\text{max displacement to zero} = T/4$$



if wave in positive direction then

ω in eqn must be +ve

mostly 4 tick = 1π

$$V = \int \frac{\text{Tension}}{\mu} \, dl$$

μ ← linear density

$$V = \pi r^2 h \quad (\text{cylinder})$$

$$\frac{\mu_1}{\mu_2} = \frac{d_1^2}{d_2^2}$$

$$1 T = 2\pi$$

$$\Delta T = \frac{\phi}{2\pi} T$$

$$E = \frac{F}{q_{\text{vo}}}$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q_v}{r^2}$$

$$\Phi_f = EA \cos\theta \quad \text{Uniform}$$

$$\Phi_f = \int E_{\perp} dA \quad \Phi_{\text{net}} = \frac{q_{\text{enc}}}{\epsilon_0}$$

$$E = \frac{kq}{r^2}$$

$$E = \frac{\sigma}{\epsilon_0} \quad (\text{conducting surface})$$

$$E = \frac{\sigma}{2\epsilon_0} \quad (\text{sheet of charge})$$

$$\sigma = \frac{q}{A}$$

The total Electric Flux of a field through a closed surface is always zero.

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volume charge density $\rho = \frac{Q_{\text{enclosed}}}{V} \times \epsilon_0$

Field across electrons = $\frac{\text{Enclosed}}{\pi r^2}$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

when $r > R$
if $r > R$ then
 $R = R + \text{dist}$

$$E = \frac{Q}{4\pi\epsilon_0 r^2}$$

when $r < R$

$$E = \frac{\lambda}{2\pi\epsilon_0 r} \quad (\text{line of charge})$$

$$E = \frac{2\lambda}{r} \quad (\text{cylindrically symmetric charge distribution})$$

$$E = \frac{F}{Q} = \frac{Q}{\epsilon_0 A}$$

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

$$C = \frac{Q}{\Delta V} = \frac{\epsilon_0 A}{d}$$

$$C = \frac{4\pi\epsilon_0 ab}{b-a}$$

(3 Pages left of capacitor
sir notes)

$$C_{\text{eq}} = C_1 + C_2 \dots \quad (\text{for parallel})$$

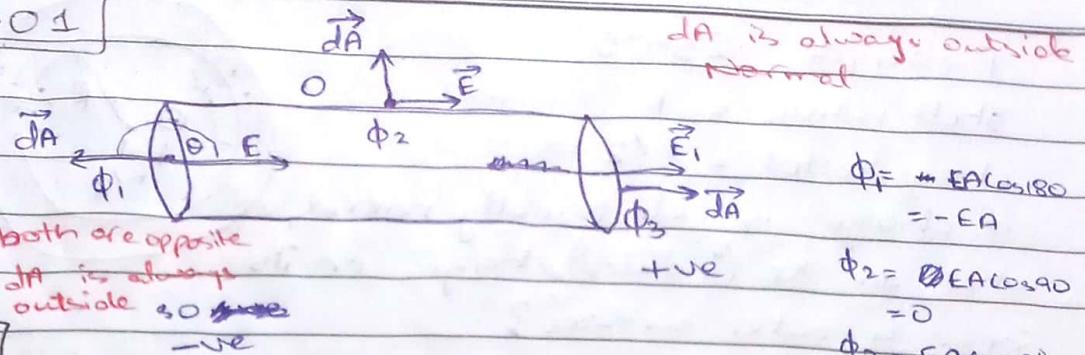
$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} \dots \quad (\text{for series})$$

$$\Delta V = \frac{\Delta V_0}{K} \rightarrow \text{dielectric const} \quad \left. \begin{array}{l} \text{dielectric} \\ \text{const} \end{array} \right\}$$

$$C = K C_0 = K \frac{\epsilon_0 A}{d} = K \frac{Q_0}{\Delta V_0}$$

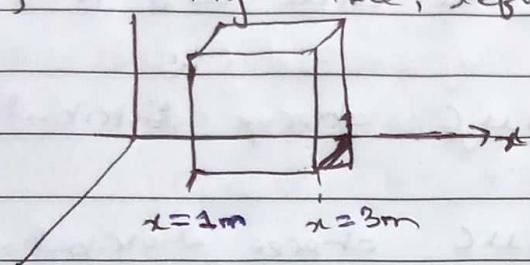
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23-01



23-02

A uniform electric field given by $\vec{E} = 3x\hat{i} + 4\hat{j}$ pierces the Gaussian cube shown. What is the electric flux through the right face, left face, and the top face?



RIGHT FACE

$$\int d\Phi = \int_{\text{right}} E \cdot dA$$

right $\therefore dA = dA \hat{i}$

$$\Phi = \int (3x\hat{i} + 4\hat{j}) (dA \hat{i})$$

$$= \int \underbrace{3x}_{\text{Here it is constant}} dA$$

$$= 3x \int dA$$

$$= 3x A$$

$$= 3(3)(4)$$

$$= 36 \text{ Nm}^2/\text{C}$$

LEFT FACE

$$\int d\Phi = \int_{\text{left}} E \cdot dA$$

left $dA = -dA \hat{i}$

$$\Phi = \int (3x\hat{i} + 4\hat{j}) (-dA \hat{i})$$

$$= - \int 3x dA$$

$$= -3(1)(24)$$

$$= -12 \text{ Nm}^2/\text{C}$$

TOP FACE

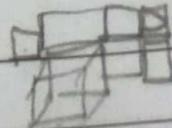
$$\text{top } \therefore dA = dA \hat{j}$$

$$\Phi = \int (3x\hat{i} + 4\hat{j}) dA \hat{j}$$

$$= \int 4 dA$$

$$= 4(2)^2$$

$$= 16 \text{ Nm}^2/\text{C}$$



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Example 23-05

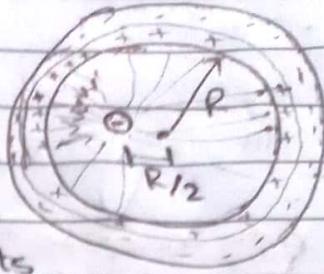
Shell radius = R

charge located = $R/2$

if charge is electrically neutral,
what are the (induced) charges on its
inner & outer surfaces?

what are the field pattern inside & outside the shell?

charge of particle = $-5 \mu C$



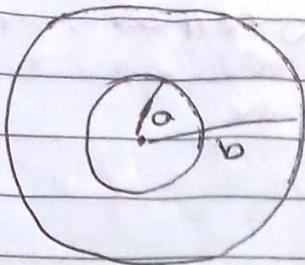
Since the charge was in the circle and it is $-5 \mu C$, so the

- * inner charge is $+5 \mu C$ charge distribution will be non-uniform
- * outer charge is $-5 \mu C$ charge distribution will non-uniform

The pattern of field is skewed because of the skew of the positive charge distribution.

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SPHERICAL



$$C = \frac{4\pi\epsilon_0 ab}{b-a}$$

$$= \frac{4\pi\epsilon_0 a}{1 - \frac{a}{b}}$$

For $b = \infty$ (for infinite surface area of radius)

$$C = \frac{4\pi\epsilon_0 a}{1 - \frac{a}{b}}$$

Isolated sphere as capacitor

- 1) The plates in a spherical capacitor have radii 38mm & 40mm. find it's capacitor.

$$\text{Given } C = \frac{4\pi \times 8.85 \times 10^{-12} \times 38 \times 10^{-3} \times 40 \times 10^{-3}}{40 \times 10^{-3} - 38 \times 10^{-3}}$$
$$= 8.45 \times 10^{-11} \text{ F}$$
$$= 8.45 \text{ PF}$$

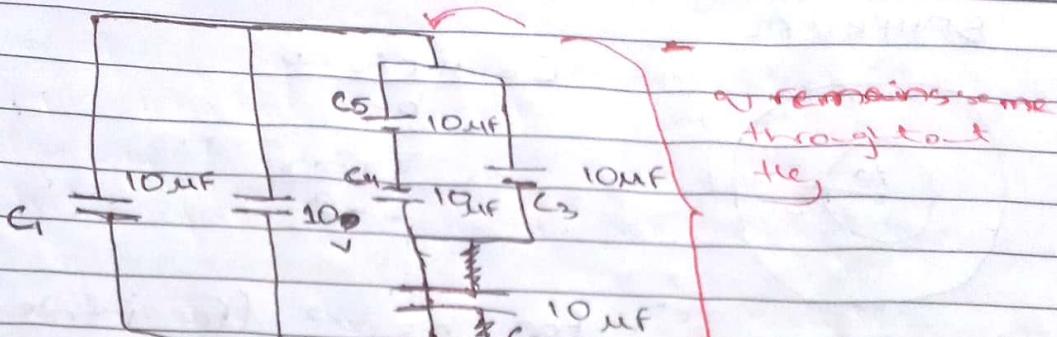
- 2) what area of the plates will be required to get same capacitance as above with same separation in case of parallel plates capacitor

$$\therefore C = \frac{\epsilon_0 A}{d} \rightarrow b-a$$

$$8.45 \times 10^{-11} = \frac{8.85 \times 10^{-12} \times A}{2 \times 10^{-3}}$$

$$A = 0.02 \text{ m}^2$$

Date _____



Find charge on each capacitor.

Charge on C_1

$$C_1 = \frac{q_1}{V_1} \Rightarrow q_1 = C_1 V_1 \Rightarrow q_1 = 10\mu F \times 10V \\ q_1 = 100\mu C$$

Charge on C_2

$$V_2 = C_2 V_2 ?$$

$$q_2 = 6\mu F \times 10$$

$$\boxed{q_2 = 60\mu F}$$

Charge on C_3

First find V_2 so

that we can minus with
- V to get V_3

~~$$q_2 = 60\mu F$$~~

$$V_2 = \frac{60\mu F}{10\mu F}$$

$$\boxed{V_2 = 6V}$$

$$V_3 = V - V_2$$

$$= 10 - 6$$

$$\boxed{V_3 = 4V}$$

C_4 & C_5 are in series

$$\frac{1}{C'} = \frac{1}{C_4} + \frac{1}{C_5}$$

$$\frac{1}{C'} = \frac{1}{10} + \frac{1}{10}$$

$$C' = 5\mu F$$

C' & C_3 are parallel

$$C'' = C' + C_3 = 5 + 10 = 15\mu F$$

C'' & C_2 are in series

$$\frac{1}{C''' } = \frac{1}{C'' } + \frac{1}{C_2}$$

$$\frac{1}{C''' } = \frac{1}{15} + \frac{1}{10}$$

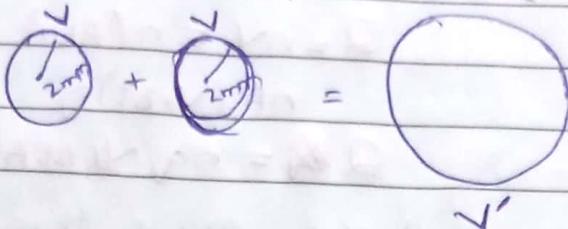
$$C''' = 6\mu F$$

$$q_3 = C_3 V_3 = 10 \mu F \times 4 \text{ V} \\ = 40 \mu F$$

$$q_5 = q_4 = C_4 V_4 = 6 \mu F \times 4 \text{ V} \\ = 24 \mu F$$

$$q_5 = q_M = 60 \mu F - 48 \mu F = 20 \mu F$$

Two spheres of mercury of radius 2.00 mm merge to form a drop. Find the capacitance of drop formed.



$$r' = (2r)^{1/3}$$

$$r' = 2^{1/3} r$$

$$\because V = \frac{4}{3} \pi r^3$$

$$V' = \frac{4}{3} \pi r'^3$$

$$V' = 2V$$

$$\frac{4}{3} \pi r^3 = 2 \left(\frac{4}{3} \pi r'^3 \right)$$

$$r'^3 = 2r^3$$

$$r' = 2^{1/3} (2 \times 10^{-3})$$

$$r' = 2.52 \times 10^{-3}$$

$$\because C = 4\pi \epsilon_0 r'$$

$$C' = 4 \times 3.14 \times 8.85 \times 10^{-12} \times$$

$$C' = 0.28 \text{ PF}$$

$$\epsilon_0 = 10^{-12}$$

Q) A parallel plate consist of 2 circular plates of 8.3 cm separation of 2.3 mm. find the charge when 110V is applied across it.

$$\because q = CV$$

$$\therefore C = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 (\pi r^2)}{d}$$

$$= \frac{8.885 \times 10^{-12} \times 3.14 \times (4.3 \times 10^{-3})^2}{2.8 \times 10^{-3}}$$

$$C = 8.3 \times 10^{-11} \mu F$$

$$q = CV \\ = 8.3 \times 10^{-11} \times 110$$

$$q = 9.16 \times 10^{-10} C$$

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CURRENT

= rate of change of charge
charge:

Positive charge goes from higher potential to lower potential (conventional current)

Later discovered:

Electron goes from lower potential to higher (Electron current)

$$I = \frac{dq}{dt}$$

$$n = \frac{Q}{\text{electron}} \quad V = \frac{\Delta x}{\Delta t}$$

$$V \Delta t = \Delta x$$

$$dq = Idt$$

$$Q = nV = n(ABz)$$

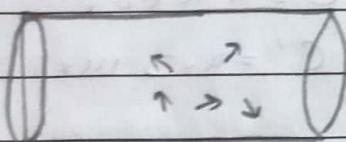
$$q = \int I dt$$

$$nA(Vz \Delta t) q$$

$$Q = nq \sqrt{Vz \Delta t A}$$

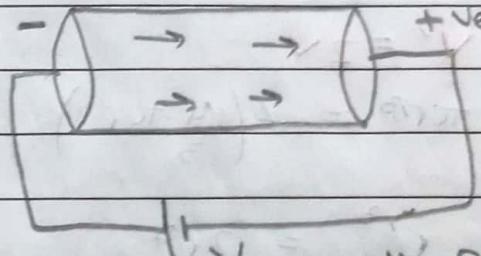
Current density = current of charge passing through perpendicularly Area.

$$\boxed{I = V \cdot J \cdot dA} \quad \boxed{I = \vec{J} \cdot d\vec{A}} \quad \boxed{J = \frac{I}{A}}$$



without pd

$$\text{so net } I = neVA$$



current density ~~diff~~
velocity

$$\vec{J} = (ne) \vec{V}_d \left(A/m^2 \right)$$

with pd
net charge > 0

$$J = \sigma E$$

Q) A uniform cylindrical wire has radius 2mm & current density $8.23 \times 10^5 N/m^2$. Find current through it ~~crossed~~ cross sectional area.

$$\begin{aligned} I &= \int \vec{J} \cdot d\vec{A} = JA \\ &= (8.23 \times 10^5) (\pi r^2) \\ &= (8.23 \times 10^5) (3.14 \times (2.0 \times 10^{-3})^2) \end{aligned}$$

$$\boxed{I = 10.3 A}$$

Resistance

$$I \propto V$$

$$I = (\text{const}) V$$

$$I = KV$$

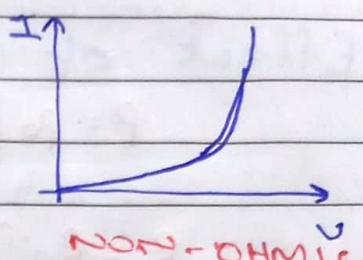
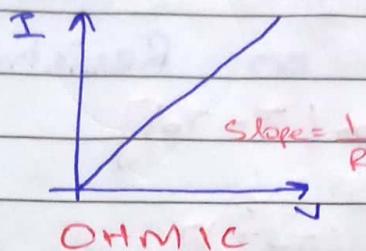
↳ conductance

$$\text{or } K = \frac{1}{R}$$

$$I = \frac{V}{R}$$

$$IR = V$$

$$R = \frac{\Delta V}{I}$$



Example 26.03

What is the drift speed of the conduction electrons in a copper wire with radius $r = 900\text{ }\mu\text{m}$ when it has a uniform current $i = 7\text{ mA}$? Assume that each copper atom contributes one conduction electron to the current & that the current density is uniform across the wire's cross section.

$$V_d = ?$$

$$r = 900\text{ }\mu\text{m}$$

$$i = 7\text{ mA}$$

$$J = neV_d \quad \text{(a)}$$

$$\therefore n = \frac{N_A \times \rho}{M}$$

$$\therefore I = JA$$

$$\frac{17 \times 10^{-3}}{\pi (900 \times 10^{-6})} = J$$

$$J = 6.68 \times 10^{-3}$$

$$n = \frac{6.02 \times 10^{23} \times 8.96 \times 10^3}{6.54 \times 10^{-3}}$$

$$n = 8.49 \times 10^{28} \text{ electrons/m}^3$$

put in eq (a)

$$6.68 \times 10^{-3} = 8.49 \times 10^{28} \times 10^6 \times 10^{-19} \times V_d$$

$$V_d = 4.9 \times 10^{-7}$$

Resistivity

- * It is a property of a material (every element has its constant resistivity).
- * Only affected by temp.

Resistance

Dependent on length, area, of material, & nature of material.

Effect of temp on Resistivity

$$\rho = \rho_0 [1 + \alpha (T - T_0)]$$

ρ = resistivity at some temp t in $^{\circ}\text{C}$

$$R = R_0 [1 + \alpha (T - T_0)]$$

R_0 = resistivity at some reference temp ($\approx 20^{\circ}\text{C}$)

α = temp coefficient of resistivity.

Electrical Energy & Power

The rate at which the ~~body~~ loses potential energy is going through the resistor is.

As charge moves from a to b its

$$\frac{\Delta U}{\Delta t} = \frac{\Delta q}{\Delta t} \cdot \Delta V$$

Electric Potential Energy ↑

$$\frac{\Delta U}{\Delta t} = I \cdot \Delta V$$

Chemical Potential Energy ↓

Note: The charge cannot build up at any point the current is the same everywhere in the circuit.

$$P = \frac{\Delta U}{\Delta t} = \frac{fd}{\Delta t} = \frac{q Ed}{\Delta t}$$

$$P = \frac{\Delta q V}{\Delta t} = \frac{\Delta q V}{\Delta t}$$

$$= IV$$

$$P = IV$$

$$P = I^2 R$$

$$P = \frac{V^2}{R}$$

Power dissipation

Refers to the rate of the conversion of energy from one form to another (loss or change in power).



* The stepup transformer increases voltage & decreases current thus less power dissipation.

1) Suppose that the material composing a ~~fuse~~ melts once the current density rises to ~~440 A/cm²~~ 440 A/cm². What diameter of cylinder wire should be used for the fuse to limit the current to 0.522 A?

$$J = 440 \text{ A/cm}^2$$

$$I = JA$$

$$= 440/100$$

$$0.552 = 4.4 \cdot A$$

$$J = 4.4 \text{ A}$$

$$A = 12.5$$

$$\pi r^2 = 12.5$$

$$r = 1.99 \text{ m}$$

2) How long does it take electrons to get from a car battery to the starting motor? current is ~~115 A~~ 115 A and e⁻ travel through copper wire with cross-sectional area 31.22 mm^2 & length 85.5 cm ($n = 8.49 \times 10^{28} \text{ m}^{-3}$)

$$\therefore I = ne \vec{v}_d A$$

$$\frac{115}{(8.49 \times 10^{28})(1.6 \times 10^{-19})(31.2 \times 10^{-3})^2} = \vec{J}_d$$

$$\pi r^2$$

Date _____

Whenever there is a flow of charge through some regions, current exist.

$$I_{av} = \frac{\Delta Q}{\Delta t} = nq \text{ drift speed} \times A$$

In electric conductors, the direction of the current is opposite the direction of flow of electrons.

$$\Delta V = EI = I \frac{R}{l}$$

$$J = \sigma E = \sigma \frac{\Delta V}{l}$$

When potential energy is neglected, the potential difference becomes equal to emf ϵ .

$$\Delta V = \epsilon$$

$$P = IE \quad (\text{emf})$$

Date _____

Q3) A fluid with a resistivity $9.4 \Omega \text{m}$ seeps into the space between the plates of a 110pF parallel plate air capacitor. When the space is completely filled, what is the resistance between them? ($\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$)

$$\therefore R = \frac{\rho A}{L}$$

$$\therefore C = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 A}{L}$$

$$R = \frac{\rho L}{A} \quad \text{---(i)}$$

$$\frac{1}{C} = \frac{L}{\epsilon_0 A}$$

(~~compute~~ put value of ϵ_0 in negli)

$$R = \frac{\rho E}{C}$$

$$\frac{\epsilon_0}{C} = \frac{L}{A} \quad \text{---(ii)}$$

$$R = \frac{9.4 \times 8.85 \times 10^{-12}}{110 \times 10^{-12}} = 0.76 \Omega$$

Q4) For a hypothetical electronic device, the potential difference V in volts, measured across the device, is related to the current ' i ' in mA by $V = 3.55i^2$

(a) find the resistance when current is 2.4 mA .

(b) At what value of the current is the resistance equal to 16Ω ?

$$V = 3.55i^2 \quad \text{---(i)}$$

$$(b) R = 3.55i$$

$$V = IR$$

$$16 = i$$

$$iR = 3.55i^2$$

$$3.55$$

$$R = 3.55i$$

$$i =$$

$$R = 3.55 \times 2.4 \times 10^{-3} \Omega$$

(a) R = 8.52Ω

Q5) A student's 9V, 7.5W portable radio was left on from 9.00pm until 3.00am. How much charge passed through the wires?

$$P = IV$$

$$q = Ixt$$

$$I = \frac{7.5}{9}$$

$$q = (0.833) 6$$

I = 0.833 A

q = 5 Amp/h

MAGNETIC FIELD

Perpendicular to electric field & direction of particle

$$F = q(\vec{v} \times \vec{B})$$

$$F = qvB \sin\theta$$

F_{m}

$$F = \frac{\theta}{t} LB \sin\theta$$

$$= \frac{\theta}{t} LB \sin\theta$$

$$f = ILB \sin\theta$$

$$\omega = \frac{v}{r} = 2\pi f = \frac{qvB}{m} = \omega$$

angular velocity

$$F_B = qvB = \frac{mv^2}{r}$$

radius $\rightarrow r = \frac{mv}{qB}$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi m}{qB} \rightarrow \text{Time period of a full circle}$$

$$F_c = F_B$$

$$\frac{mv^2}{r} = qvB$$

$$\frac{m}{q} = \frac{Br}{v}$$

$$v = \frac{E}{B}$$

$$\frac{m}{q} = \frac{Br}{\frac{E}{B_0}}$$

$$\frac{m}{q} = \frac{r B_0}{E}$$

→ The mass spectrometer

A magnetic field cannot change the speed of a particle.

~~off~~ =

~~off~~

$$qE = qvB$$

$$\because E = vB$$

$$v = \frac{E}{B}$$

← velocity selector

HALL EFFECT

$$\Delta V_H = E_{Hd} = vd B_d$$

Hall coefficient.

$$R_H = \frac{1}{nqA}$$

$$\Delta V_H = \frac{IBd}{nqA}$$

$$\Delta V_H = \frac{IB}{nqt} = R_H \frac{IB}{t}$$

• Permeability
of magnetic

Permeability of
free space

Paramagnetic $\mu_m > \mu_0$

Diamagnetic

$$\mu_m < \mu_0$$

$$B = \frac{\mu_0}{2\pi} \cdot \frac{I}{r}$$

Magnetic field

$$\mu_0 = 4\pi \times 10^{-7}$$

Permeability of free space

$$\int B \cdot ds = \mu_0 I$$

Amperes law.

SOLINOID

$$B = \frac{\mu_0 I}{L} \quad \text{for single loop}$$

$$B = \cancel{\mu_0 n} \mu_0 n I$$

$$B = \frac{\mu_0 N I}{L} \quad \text{For } N \text{ turns of loops}$$

$n = \frac{N}{l}$ number of turns per unit length

TOROID

$$\int \vec{B} \cdot d\vec{s} = B \int ds = B 2\pi r$$

$$B = \mu_0$$

$$B (2\pi r) = \mu_0 N I$$

$$B = \frac{\mu_0 N I}{2\pi r} \quad \text{Toroid, } N \text{ loops}$$

$$B = \mu_0 \quad \boxed{B = \frac{\mu_0 I}{2\pi r}} \quad \text{long straight wire}$$

Q31)

$$\text{Strands} = 125$$

$$R = 2.65 \mu\Omega$$

$$I_{\text{total}} = 0.75 \text{ A} \quad (\text{of the cable})$$

$$(a) I_{\text{each}} = \frac{0.75}{\text{all strands}} \frac{I_{\text{total}}}{125} = \frac{0.75}{125}$$

$$I_{\text{each}} = 6 \times 10^{-3} \text{ A}$$

$$(b) V = IR \approx 6 \times 10^{-3} \times 2.65 \times 10^{-6}$$

$$V = 1.59 \times 10^{-8} \text{ V}$$

$$(c) R = \frac{V}{I} = \frac{1.59 \times 10^{-8}}{0.75} = 2.12 \times 10^8 \Omega$$

3a*)

$$V = 120V$$

$$I = 10A$$

$$1 \text{ hot dog } \rightarrow E = 60 \times 10^3 \text{ J}$$

$$3 \text{ hot dog } E = 180 \times 10^3 \text{ J}$$

$$P = VI = 120 \times 10 = 1200$$

$$P = \frac{E}{t} \Rightarrow t = \frac{E}{P} = \frac{180 \times 10^3}{1200} = 150 \text{ sec}$$

$$\boxed{t = 25 \text{ minutes}}$$

Q1) A wire with a resistance of 6 Ohm is drawn out through a die so that its new length is three times its original length. Find the resistance of the longer wire, assuming that the resistivity & density of the material are unchanged.

Since ~~length~~ stretches so
Area decrease

$$R = 6 \Omega$$

$$\text{New length, } l' = 3l$$

$$l' = 3l$$

$$A' = \frac{1}{3} A$$

$$R = \frac{\rho L}{A}$$

$$R' = \frac{\rho l'}{A'}$$

$$R' = \frac{\rho 3l}{\frac{1}{3} A}$$

$$R' = 9 \frac{\rho l}{A}$$

$$R' = 9 R$$

$$R' = 9 \times 6$$

$$\boxed{R' = 54 \Omega}$$