MVC

Assignment #02

23K-2001

BCS 2J

Question#1:

a. $\int_1^2\int_4^6 \frac{x}{y^2}\,dx\,dy$

$\Rightarrow \int_1^2 \frac{1}{y^2}\left[\frac{x^2}{2}\right]_4^6 dy$

$\Rightarrow \int_1^2 \frac{1}{y^2}\left(\frac{36}{2}-\frac{16}{2}\right) dy$

$\Rightarrow 10\int_1^2 \frac{1}{y^2}\,dy$

$\Rightarrow 10\left[\frac{y^{-1}}{-1}\right]_1^2$

$\Rightarrow 10\left(\frac{-1}{2}+\frac{1}{1}\right)$

$\Rightarrow 10\left(\frac{1}{2}\right)$

$\Rightarrow 5$ Ans.

b. $\int\int (x^2+y^2)\,dx\,dy$

$\Rightarrow \int\left(\frac{x^3}{3}+y^2\frac{x}{1}\right)dy$

$\Rightarrow \frac{x^3}{3}y + \frac{x\,y^3}{3} + C$

$\Rightarrow \frac{xy(x^2+y^2)}{3} + C$ Ans.

c. $\int_0^1\int_1^2 \frac{xe^x}{y}\,dy\,dx$

$\Rightarrow \int_0^1 xe^x \left|\ln y\right|_1^2 dx$

$\Rightarrow \int_0^1 xe^x(\ln 2-\ln 1)\,dx$

$\Rightarrow \int_0^1 xe^x(\ln 2)\,dx$

$\Rightarrow$ Integrate by parts

$\Rightarrow \ln 2\left[xe^x-\int e^x\,dx\right]_0^1$

$\Rightarrow \ln 2\left[xe^x - e^x\right]_0^1$

$\Rightarrow \ln 2\left[(1e^1-e^1)-(0-e^0)\right]$

$\Rightarrow \ln 2\cdot(0+1)$

$\Rightarrow \ln 2$ Ans.

Question #2:

$P(L,K) = 70L^{0.6}K^{0.4}$

L = monthly labour hours
5000 ~ 6000

k = capital investment
(units of \$1000)
\$20 000 - \$30 000

$\therefore$ Output $= \frac{1}{A}\int_{20}^{30}\int_{5000}^{6000} 70L^{0.6}k^{0.4}\,dl\,dk$

$\because A = (6000-5000)(30-20)$
$A = 10000$

$$= \frac{1}{10000}\int_{20}^{30} 70k^{0.4}\left|\frac{L^{0.6+1}}{0.6+1}\right|_{5000}^{6000} dk$$

$$= \frac{1}{10000}\int_{20}^{30} 43.75k^{0.4}\left[(6000)^{1.6} - (5000)^{1.6}\right]dk$$

$$= \frac{12279250.57}{10000}\int_{20}^{30} k^{0.4}\,dk$$

$$= 1227.925057\left|\frac{k^{0.4+1}}{0.4+1}\right|_{20}^{30}$$

$$= \frac{1227.925057}{1.4}\left[(30)^{1.4} - (20)^{1.4}\right]$$

$$= \frac{1227.925057}{1.4}(50.65271)$$

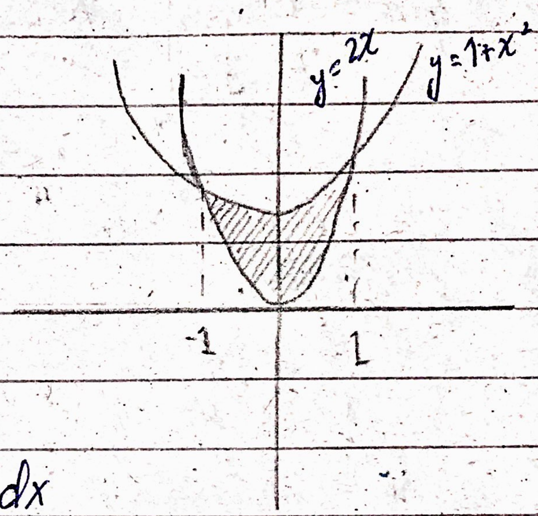
Output $= 44426.95558$ Ans.

Question #3

a. $\iint_D (x+2y)\,dA$, where D is region

$y = 2x^2$ & $y = 1+x^2$

$$\Rightarrow \int_{-1}^{1}\int_{2x^2}^{1+x^2} (x+2y)\,dy\,dx \qquad \Rightarrow x = -1, \quad x = 1$$



$$\Rightarrow \int_{-1}^{1}\left[x\,y\Big|_{2x^2}^{1+x^2} + 2\,\frac{y^2}{2}\Big|_{2x^2}^{1+x^2} \right] dx$$

$$\Rightarrow \int_{-1}^{1}\left(x(1+x^2) + (1+x^2)^2 - \left[x(2x^2) + (2x^2)^2\right]\right) dx$$

$$\Rightarrow \int_{-1}^{1} (x + x^3 + 1 + 2x^2 + x^4 - 2x^3 - 2x^4)\,dx$$

$$\Rightarrow \int_{-1}^{1} (x - x^3 + 1 + 2x^2 - 3x^4)\,dx$$

$$\Rightarrow \frac{x^2}{2}\Big|_{-1}^{1} - \frac{x^4}{4}\Big|_{-1}^{1} + \frac{x}{1}\Big|_{-1}^{1} + 2\,\frac{x^3}{3}\Big|_{-1}^{1} - 3\,\frac{x^5}{5}\Big|_{-1}^{1}$$

$$\Rightarrow 0 - 0 + [1-(-1)] + \frac{2}{3}\left[1-(-1)^3\right] - \frac{3}{5}\left[1-(-1)^5\right]$$

$$\Rightarrow 2 + \frac{2}{3}(2) - \frac{3}{5}(2)$$

$$\Rightarrow \frac{32}{15} \quad \text{Ans.}$$

b) $\int_0^4 \int_0^{\sqrt{y}} xy^2 \, dx\,dy$

$\Rightarrow \int_0^4 y^2 \left| \frac{x^2}{2} \right|_0^{\sqrt{y}} dy$

$\Rightarrow \frac{1}{2} \int_0^4 y^2 (y - 0)\, dy$

$\Rightarrow \frac{1}{2} \int_0^4 y^3 \, dy$

$\Rightarrow \frac{1}{2} \left| \frac{y^4}{4} \right|_0^4$

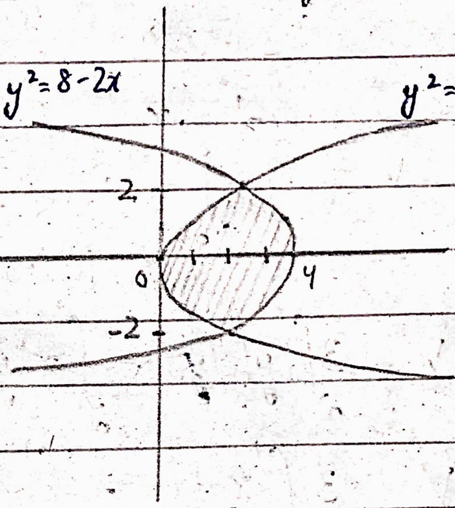
$\Rightarrow \frac{1}{8} (256 - 0)$

$\Rightarrow 32$ Ans.

c) $\iint_R (4 - y^2)\, dA$; region:

$y^2 = 2x$

$y^2 = 8 - 2x$



$y^2 = 8 - 2x$

$\frac{y^2}{2} = 4 - x$

$x = 4 - \frac{y^2}{2}$

$y^2 = 2x$

$x = \frac{y^2}{2}$

$\Rightarrow \int_{-2}^{2} \int_{y^2/2}^{4 - \frac{y^2}{2}} (4 - y^2)\, dx\,dy$

$\Rightarrow \int_{-2}^{2} 4 \left| x \right|_{y^2/2}^{4 - y^2/2} dy - \int_{-2}^{2} y^2 \left| x \right|_{y^2/2}^{4-y^2/2} dy$

$\Rightarrow \int_{-2}^{2} 4\left(4 - \frac{y^2}{2} - \frac{y^2}{2}\right) dy - \int_{-2}^{2} y^2 \left(4 - \frac{y^2}{2} - \frac{y^2}{2}\right) dy$

$\Rightarrow \int_{-2}^{2} 4(4 - y^2)\, dy - \int_{-2}^{2} y^2 (4 - y^2)\, dy$

$\Rightarrow 16 \left| y \right|_{-2}^{2} - 4 \left| \frac{y^3}{3} \right|_{-2}^{2} - 4 \left| \frac{y^3}{3} \right|_{-2}^{2} + \left| \frac{y^5}{5} \right|_{-2}^{2}$

$\Rightarrow 16(2+2) - \frac{4}{3}(2^3 + 2^3) - \frac{4}{3}(8+8) + \left(\frac{32+32}{5}\right)$

$\Rightarrow 64 - \frac{64}{3} - \frac{64}{3} + \frac{64}{5}$

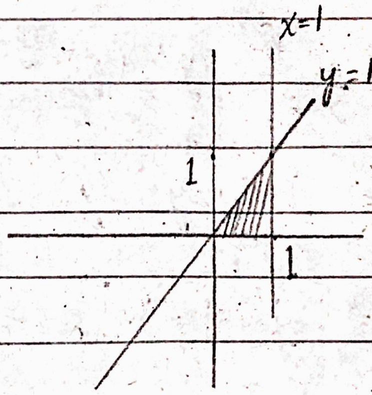
$\Rightarrow \frac{512}{15}$ Ans.

Question #4:

$V = ?$

base: xy-plane ; $y = x$, $x = 1$

top : plane $f(x,y) = 3 - x - y$



$$V = \int_0^1 \int_0^x (3 - x - y)\,dy\,dx$$

$$V = 3\int_0^1 |y|_0^x \cancel{dx} - \int_0^1 x|y|_0^x\,dx - \int_0^1 \left|\frac{y^2}{2}\right|_0^x dx$$

$$V = 3\int_0^1 x\,dx - \int_0^1 x^2 dx - \frac{1}{2}\int_0^1 x^2 dx$$

$y = x$, $y = 0$
$x = 1$, $x = 0$

$$V = 3\left|\frac{x^2}{2}\right|_0^1 - \left|\frac{x^3}{3}\right|_0^1 - \frac{1}{2}\left|\frac{x^3}{3}\right|_0^1$$

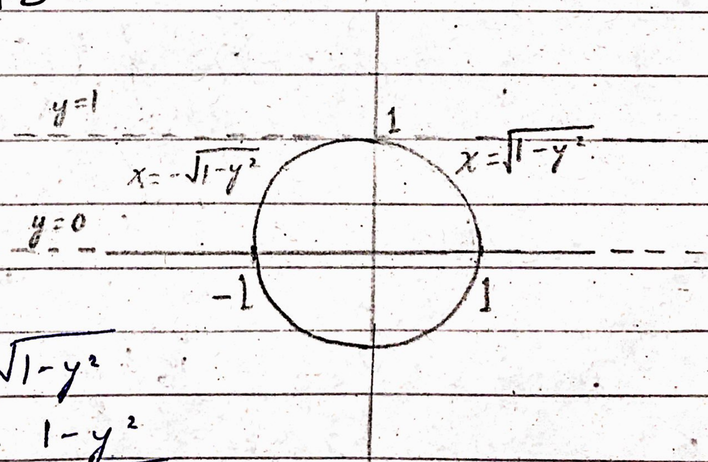
$$V = \frac{3}{2}(1-0) - \frac{1}{3}(1-0) - \frac{1}{6}(1-0)$$

$$V = \frac{3}{2} - \frac{1}{3} - \frac{1}{6}$$

$V = 1$ (units^3) Ans.

Question #5:

a. $$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y\,dx\,dy$$

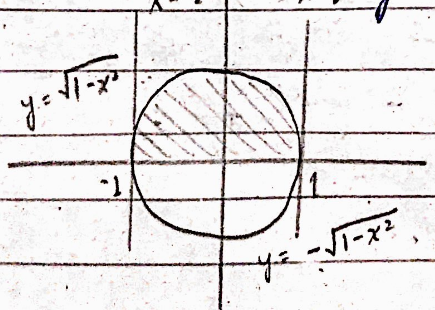


$$\Rightarrow \int_{-1}^{1} \int_0^{\sqrt{1-x^2}} 3y\,dy\,dx$$

$x = \sqrt{1-y^2}$

$x^2 = 1 - y^2$

Ans. $y = \sqrt{1-x^2}$, $y = -\sqrt{1-x^2}$



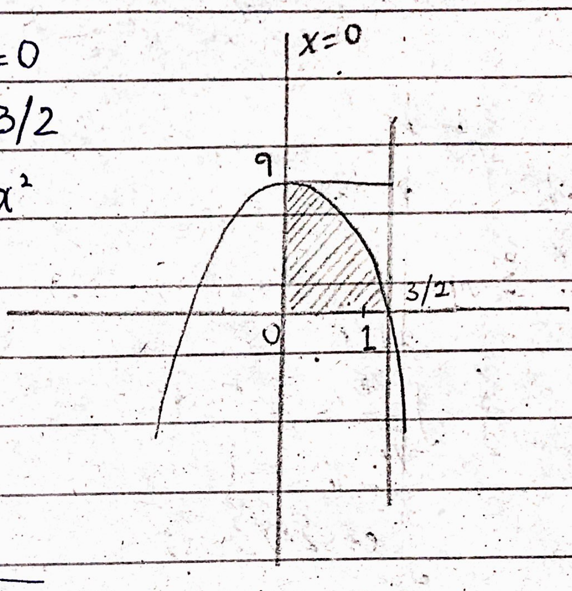
b) $\int_0^{3/2}\int_0^{9-4x^2} 16x\,dy\,dx$

$x=0$

$x=3/2$

$y=9-4x^2$

$y=0$



$y = 9-4x^2$

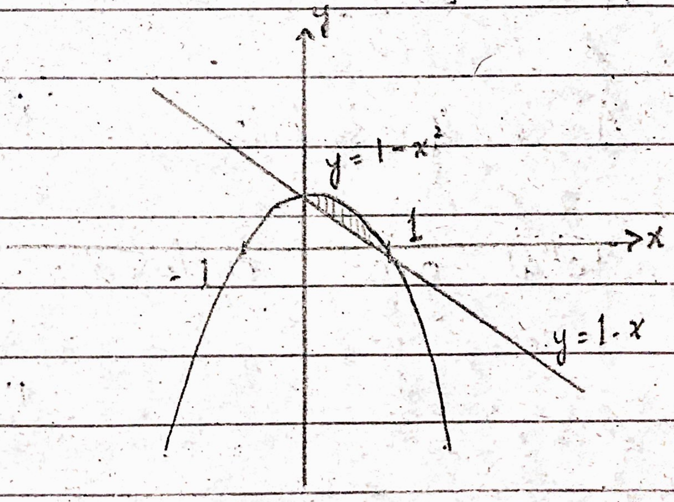
$4x^2 = 9-y$

$x = \frac{\sqrt{9-y}}{2}$

$\Rightarrow \int_0^9\int_0^{\sqrt{9-y}/2} 16x\,dx\,dy$

Ans.

c) $\int_0^1\int_{1-x}^{1-x^2} dy\,dx$



$y=1-x$

$x=1-y$

$\Rightarrow \int_0^1\int_{1-y}^{\sqrt{1-y}} dx\,dy$

$y = 1-x^2$

$x = \sqrt{1-y}$

Ans.

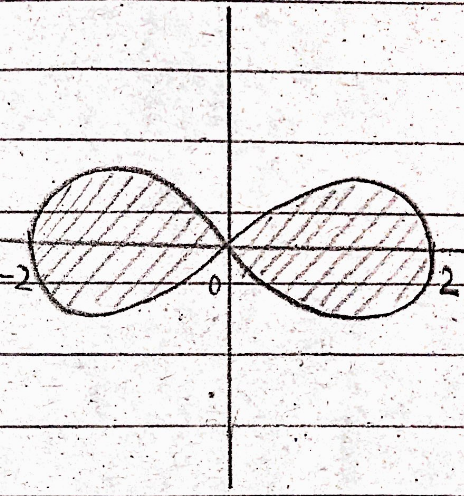
Question #6.

Area enclosed by lemniscate:

$$r^2 = 4\cos 2\theta$$

or

$$r = 2\sqrt{\cos 2\theta}$$



$$A = \frac{1}{2}\int_{-\pi/4}^{\pi/4} r^2 \, d\theta$$

$$A = \frac{1}{2}\int_{-\pi/4}^{\pi/4} 4\cos 2\theta \, d\theta$$

$$A = \int_{-\pi/4}^{\pi/4} \cos 2\theta \; 2d\theta$$

$$A = \Big[\sin 2\theta\Big]_{-\pi/4}^{\pi/4}$$

$$A = \sin 2\left(\frac{\pi}{4}\right) - \sin 2\left(\frac{-\pi}{4}\right)$$

$$A = \sin\frac{\pi}{2} + \sin\frac{\pi}{2}$$

$A = 2$ sq. units (Right half only)

Total area enclosed $= 2A$

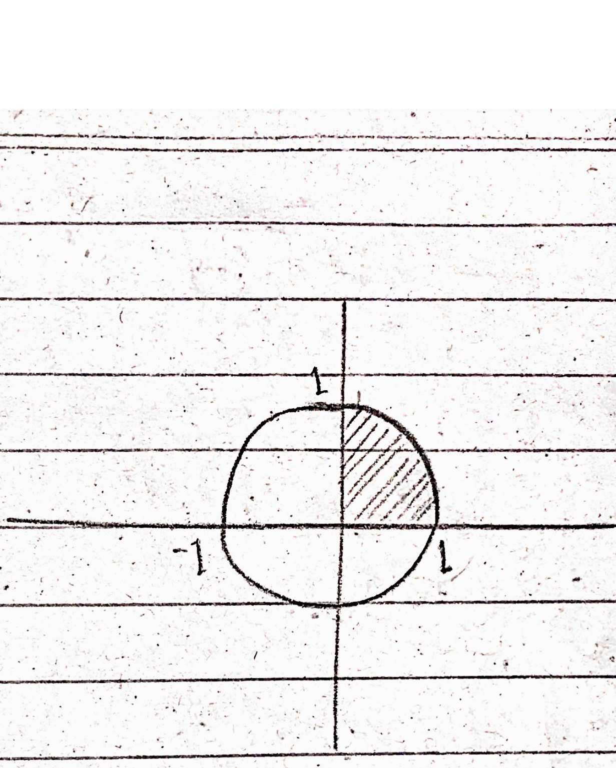
$= 2(2)$

$= 4$ sq. units

Ans.

Question #7

a. $$\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2)\,dx\,dy$$



$x^2+y^2=r^2$

$r=1$

$$\Rightarrow \int_0^{\pi/2} \int_0^1 (r^2)\, r\,dr\,d\theta$$

$$\Rightarrow \int_0^{\pi/2} \int_0^1 r^3\,dr\,d\theta$$

$$\Rightarrow \int_0^{\pi/2} \left[\frac{r^4}{4}\right]_0^1 d\theta$$

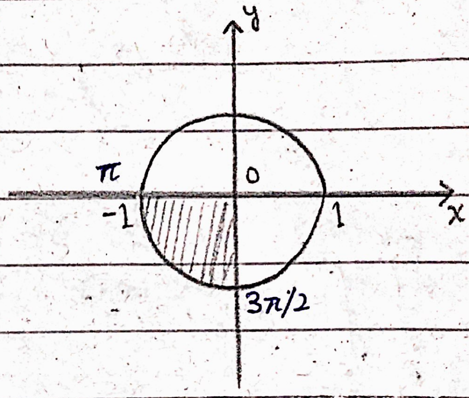
$$\Rightarrow \frac{1}{4}\int_0^{\pi/2} (1^4-0^4)\,d\theta$$

$$\Rightarrow \frac{1}{4}\int_0^{\pi/2} d\theta$$

$$\Rightarrow \frac{1}{4}\left[\theta\right]_0^{\pi/2}$$

$$\Rightarrow \frac{\pi}{8}$$ Ans.

b. $$\int_{-1}^{0}\int_{-\sqrt{1+x^2}}^{0} \frac{2}{1+\sqrt{x^2+y^2}}\, dy\, dx$$



$$\Rightarrow \int_{\pi}^{3\pi/2}\int_{0}^{1} \frac{2}{1+\sqrt{r^2}}\, r\, dr\, d\theta$$

$$\Rightarrow \int_{\pi}^{3\pi/2}\int_{0}^{1} \frac{2r}{1+r}\, dr\, d\theta$$

$$\begin{array}{r|l} 1+r & \;\; r \\ & \;\; 1 \\ & \underline{r+1} \\ & \;\; -1 \end{array}$$

$$\Rightarrow 2\int_{\pi}^{3\pi/2}\int_{0}^{1} \frac{r}{1+r}\, dr\, d\theta$$

$$\Rightarrow 2\int_{\pi}^{3\pi/2}\int_{0}^{1}\left(1-\frac{1}{1+r}\right) dr\, d\theta$$

$$\Rightarrow 2\int_{\pi}^{3\pi/2} [r]_0^1\, d\theta - 2\int_{\pi}^{3\pi/2} [\ln|1+r|]_0^1\, d\theta$$

$$\Rightarrow 2\int_{\pi}^{3\pi/2} d\theta - 2\int_{\pi}^{3\pi/2} \ln|2|\, d\theta$$

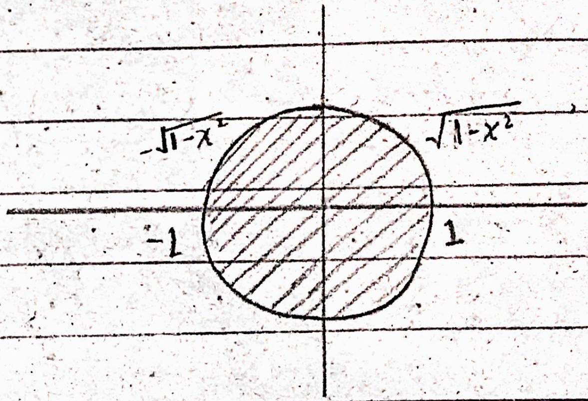
$$\Rightarrow 2[\theta]_{\pi}^{3\pi/2} - 2\ln 2\,[\theta]_{\pi}^{3\pi/2}$$

$$\Rightarrow 2\left(\frac{\pi}{2}\right) - 2\ln 2\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \pi - \pi\ln 2$$

$$\Rightarrow \pi(1-\ln 2)$$ Ans.

c. $$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2}\,dy\,dx$$



$x^2+y^2=r^2$

$r=1$

$$\Rightarrow \int_0^{2\pi}\int_0^{1} \frac{2}{(1+r^2)^2}\, r\,dr\,d\theta$$

$$\Rightarrow \int_0^{2\pi}\int_0^{1} (1+r^2)^{-2}\, 2r\,dr\,d\theta$$

$$\Rightarrow \int_0^{2\pi} \frac{\left[(1+r^2)^{-1}\right]_0^1}{-1}\, d\theta$$

$$\Rightarrow -\int_0^{2\pi} \left[(1+1^2)^{-1}-(1+0^2)^{-1}\right] d\theta$$

$$\Rightarrow -\int_0^{2\pi} \left(\frac{1}{2}-1\right) d\theta$$

$$\Rightarrow \frac{1}{2}\int_0^{2\pi} d\theta$$

$$\Rightarrow \left[\frac{\theta}{2}\right]_0^{2\pi}$$

$$\Rightarrow \frac{2\pi}{2}$$

$\Rightarrow \pi$ Ans.

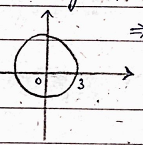
Question #8:

a. surface area:? portion of $z = x^2+y^2$ under $z=9$

$$\because A = \iint_D \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dxdy$$

$\because x^2+y^2=9$

$\Rightarrow r=3$



$$A = \iint_D \sqrt{1+(2x)^2+(2y)^2}\,dxdy$$

$$A = \iint_D \sqrt{1+4(x^2+y^2)}\,dxdy$$

$$A = \int_0^{2\pi}\int_0^3 \sqrt{1+4r^2}\;rdrd\theta$$

$$A = \int_0^{2\pi} \frac{1}{8}\left[\frac{(1+4r^2)^{3/2}}{3/2}\right]_0^3 d\theta$$

$$A = \frac{2}{3.8}\int_0^{2\pi}\left[(1+4\times 3^2)^{3/2}-(1+0)^{3/2}\right]d\theta$$

$$A = \frac{1}{12}\int_0^{2\pi}(37^{3/2}-1)\,d\theta$$

$$A = \frac{(37^{3/2}-1)}{12}\,[\theta]_0^{2\pi}$$

$$A = \frac{(37^{3/2}-1)(2\pi-0)}{12}$$

$$A = \frac{\pi}{6}(37^{3/2}-1)\ \text{units}^2 \quad \text{Ans.}$$

b. surface area:? $x^2+y^2+z=4$ above xy-plane

$z=0$

$\Rightarrow x^2+y^2=4$

$r=2$

$$\because A = \iint_D \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dxdy$$

$$A = \int_0^{2\pi}\int_0^2 \sqrt{1+(2x)^2+(2y)^2}\,dxd\theta$$

$$A = \int_0^{2\pi}\int_0^2 \sqrt{1+4(r^2)}\;rdrd\theta$$

$$A = \int_0^{2\pi} \frac{1}{8}\left[\frac{(1+4r^2)^{3/2}}{3/2}\right]_0^2 d\theta$$

$$A = \frac{1}{12}\int_0^{2\pi}\left(\left[1+4(4)\right]^{3/2}-\left[1+0\right]^{3/2}\right)d\theta$$

$$A = \frac{17^{3/2}}{12}\int_0^{2\pi} d\theta - \frac{1}{12}\int_0^{2\pi} d\theta$$

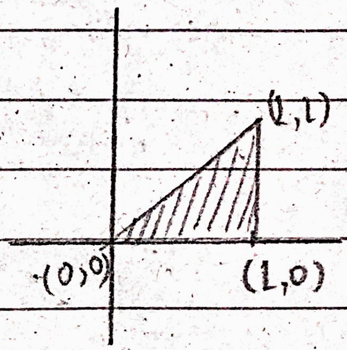
$$A = \frac{17^{3/2}}{12}[\theta]_0^{2\pi} - \frac{1}{12}[\theta]_0^{2\pi}$$

$$A = \frac{17^{3/2}}{12}(2\pi) - \frac{2\pi}{12}$$

$$A = \frac{\pi}{6}(17^{3/2}-1)\ \text{units}^2 \quad \text{Ans.}$$

c. surface area: ? $z = x^2 + 2y$, above triangle in xy-plane $(0,0), (1,0), (1,1)$

$$A = \iint_D \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dx\,dy$$



$$A = \int_0^1 \int_0^x \sqrt{1 + (2x)^2 + 2^2}\, dy\, dx$$

$$A = \int_0^1 \int_0^x \sqrt{5 + 4x^2}\, dy\, dx$$

$$A = \int_0^1 \sqrt{5 + 4x^2}\, \left| y \right|_0^x dx$$

$$A = \int_0^1 \sqrt{5 + 4x^2}\; x\, dx$$

$$A = \frac{1}{8} \frac{\left[(5 + 4x^2)^{3/2}\right]_0^1}{3/2}$$

$$A = \frac{1}{12}\left(\left[5 + 4(1)^2\right]^{3/2} - \left[5 + 4(0)\right]^{3/2}\right)$$

$$A = \frac{9^{3/2}}{12} - \frac{5^{3/2}}{12}$$

$$A = \frac{1}{12}\left(27 - 5\sqrt{5}\right) \quad \text{Ans.}$$

Question #9

a. $\int_0^{\sqrt{2}}\int_0^{\sqrt{2-x^2}}\int_{x^2+y^2}^{2} x\,dz\,dy\,dx$

$\Rightarrow \int_0^{\sqrt{2}}\int_0^{\sqrt{2-x^2}} x\left[z\right]_{x^2+y^2}^{2} dy\,dx$

$\Rightarrow \int_0^{\sqrt{2}}\int_0^{\sqrt{2-x^2}} x(2-x^2-y^2)\,dy\,dx$

$\Rightarrow \int_0^{\sqrt{2}}\left[2xy - x^3y - \frac{xy^3}{3}\right]_0^{\sqrt{2-x^2}} dx$

$\Rightarrow \int_0^{\sqrt{2}}\left[2x\sqrt{2-x^2} - x^3\sqrt{2-x^2} - \frac{x}{3}(2-x^2)^{3/2}\right]dx$

$\Rightarrow \int_0^{\sqrt{2}}\left[2x\sqrt{2-x^2} - x^3\sqrt{2-x^2} - \frac{(2x-x^3)(2-x^2)^{1/2}}{3}\right]dx$

$\Rightarrow \int_0^{\sqrt{2}} 2x\sqrt{2-x^2}\,dx - \int_0^{\sqrt{2}} x^3\sqrt{2-x^2}\,dx - \int_0^{\sqrt{2}}\frac{2x}{3}\sqrt{2-x^2}\,dx + \int_0^{\sqrt{2}}\frac{x^3}{3}\sqrt{2-x^2}\,dx$

$\Rightarrow \int_0^{\sqrt{2}}\frac{4x}{3}\sqrt{2-x^2}\,dx - \int_0^{\sqrt{2}} x^3\sqrt{2-x^2}\,dx + \int_0^{\sqrt{2}}\frac{x^3}{3}\sqrt{2-x^2}\,dx$

$\Rightarrow \frac{4}{3}\left(\frac{-1}{2}\right)\left[\frac{(2-x^2)^{3/2}}{3/2}\right]_0^{\sqrt{2}} - \frac{2}{3}\int_0^{\sqrt{2}} x^3\sqrt{2-x^2}\,dx$

let, $u = 2-x^2$

$du = -2x\,dx$

$\frac{-du}{2} = x\,dx$

$\Rightarrow \frac{-4}{6}\cdot\frac{2}{3}\left[(2-2)^{3/2} - (2-0)^{3/2}\right] + \frac{1}{3}\int_0^{\sqrt{2}} x^2\sqrt{2-x^2}(-2x\,dx)$

$\Rightarrow \frac{8\sqrt{2}}{9} + \frac{1}{3}\int_2^0 (2-u)\sqrt{u}\,du$

$u = 2-(\sqrt{2})^2$

$u = 0$

$u = 2-0^2$

$u = 2$

$\Rightarrow \frac{8\sqrt{2}}{9} + \frac{1(2)}{3}\left[\frac{u^{3/2}}{3/2}\right]_2^0 - \frac{1}{3}\left[\frac{u^{5/2}}{5/2}\right]_2^0$

$\Rightarrow \frac{8\sqrt{2}}{9} + \frac{4}{9}(-2)^{3/2} - \frac{2}{15}(-2)^{5/2}$

$\Rightarrow \frac{8\sqrt{2}}{9} - \frac{4\sqrt{8}}{9} + \frac{2\sqrt{32}}{15}$

$\Rightarrow \frac{8\sqrt{2}}{15}$ Ans.

b. $\int_0^3\int_0^2\int_0^1 (xyz)^2\,dx\,dy\,dz$

$$\Rightarrow \int_0^3\int_0^2\int_0^1 x^2y^2z^2\,dx\,dy\,dz$$

$$\Rightarrow \int_0^3\int_0^2 y^2z^2\left|\frac{x^3}{3}\right|_0^1 dy\,dz$$

$$\Rightarrow \int_0^3 \frac{1}{3}(1)\left|\frac{y^3}{3}\right|_0^2 z^2\,dz$$

$$\Rightarrow \frac{1}{9}\int_0^3 (8)\,z^2\,dz$$

$$\Rightarrow \frac{8}{9}\left|\frac{z^3}{3}\right|_0^3$$

$$\Rightarrow \frac{8}{27}(27-0)$$

$$\Rightarrow 8 \quad \text{Ans.}$$

c. $\int_0^{\pi/4}\int_0^{\ln\sec t}\int_{-\infty}^{2s} e^r\,dr\,ds\,dt$

$$\Rightarrow \int_0^{\pi/4}\int_0^{\ln\sec t}\left|e^r\right|_{-\infty}^{2s} ds\,dt$$

$$\Rightarrow \int_0^{\pi/4}\int_0^{\ln\sec t}(e^{2s}-e^{-\infty})\,ds\,dt$$

$$\Rightarrow \int_0^{\pi/4}\int_0^{\ln\sec t} e^{2s}\,ds\,dt$$

$$\Rightarrow \int_0^{\pi/4}\frac{1}{2}\left|e^{2s}\right|_0^{\ln\sec t} dt$$

$$\Rightarrow \frac{1}{2}\int_0^{\pi/4}(e^{2\ln\sec t}-e^0)\,dt$$

$$\Rightarrow \frac{1}{2}\int_0^{\pi/4}(e^{2\ln\sec t}-1)\,dt$$

$$\Rightarrow \frac{1}{2}\int_0^{\pi/4}(\sec^2 t-1)\,dt$$

$$\Rightarrow \frac{1}{2}\left|\tan t\right|_0^{\pi/4} - \frac{1}{2}\left|t\right|_0^{\pi/4}$$

$$\Rightarrow \frac{1}{2}\left(\tan\frac{\pi}{4}-0\right)-\frac{1}{2}\left(\frac{\pi}{4}-0\right)$$

$$\Rightarrow \frac{1}{2}(1) - \frac{\pi}{8}$$

$$\Rightarrow \frac{4-\pi}{8} \quad \text{Ans.}$$

d. $\iiint yz^2 \sin(xyz)\, dx\,dy\,dz$

$\Rightarrow \iiint z \sin(xyz)\, yz\, dx\,dy\,dz$

$\Rightarrow \iint z\left[-\cos(xyz)\right] dy\,dz$; let $xyz = u$

$du = xz\,dy$

$\frac{du}{x} = z\,dy$

$\Rightarrow -\iint \cos(xyz)\, z\,dy\,dz$

$\Rightarrow -\iint \cos u \; du/x\; dz$

$\Rightarrow -\frac{1}{x}\int \sin u \; dz$

let $xyz = w$

$dw = xy\,dz$

$\frac{dw}{xy} = dz$

$\Rightarrow -\frac{1}{x}\int \sin(xyz)\, dz$

$\Rightarrow -\frac{1}{x}\int \sin w \frac{dw}{xy}$

$\Rightarrow -\frac{1}{x^2y}(-\cos w) + C \rightarrow$ [Constants: $cyz + cz + c$]

$\Rightarrow \frac{\cos(xyz)}{x^2y} + C$ Ans.

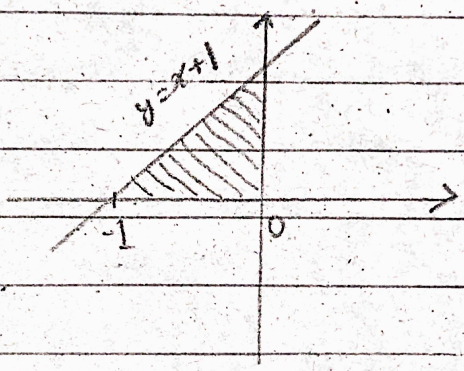
Question#10

a. tetrahedron, volume:?

$y=0,\ z=0,\ y=0,\ x=0$, and $y-x+z=1$

$\Rightarrow z = 1+x-y$

$$V = \int_{-1}^{0}\int_{0}^{x+1}\int_{0}^{1+x-y} dz\,dy\,dx$$



$$V = \int_{-1}^{0}\int_{0}^{x+1} \left| z \right|_{0}^{1+x-y} dy\,dx$$

$$V = \int_{-1}^{0}\int_{0}^{x+1} (1+x-y)\,dy\,dx$$

$$V = \int_{-1}^{0} \left| y \right|_{0}^{x+1} dx + \int_{-1}^{0} x \left| y \right|_{0}^{x+1} dx - \int_{-1}^{0} \left| \frac{y^2}{2} \right|_{0}^{x+1} dx$$

$$V = \int_{-1}^{0} (x+1)\,dx + \int_{-1}^{0} x(x+1)\,dx - \frac{1}{2}\int_{-1}^{0} (1+x)^2\,dx$$

$$V = \int_{-1}^{0} x\,dx + \int_{-1}^{0} dx + \int_{-1}^{0} x^2\,dx + \int_{-1}^{0} x\,dx - \frac{1}{2}\int_{-1}^{0} dx - \frac{1}{2}\int_{-1}^{0} 2x\,dx - \frac{1}{2}\int_{-1}^{0} x^2\,dx$$

$$V = \left|\frac{x^2}{2}\right|_{-1}^{0} + \left|x\right|_{-1}^{0} + \left|\frac{x^3}{3}\right|_{-1}^{0} + \left|\frac{x^2}{2}\right|_{-1}^{0} - \frac{1}{2}\left|x\right|_{-1}^{0} - \frac{1}{2}\left|x^2\right|_{-1}^{0} - \frac{1}{2}\left|\frac{x^3}{3}\right|_{-1}^{0}$$

$$V = \frac{(0-1)}{2} + [0-(-1)] + \frac{[0-(-1)]}{3} + \frac{(0-1)}{2} - \frac{1}{2}[0-(-1)] - \frac{1}{2}(0-1) - \frac{1}{6}(0+1)$$

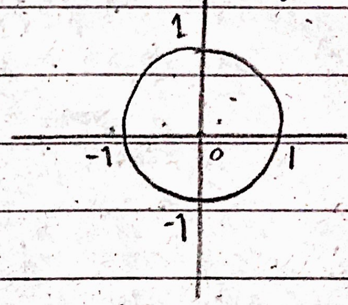
$$V = -\frac{1}{2} + 1 + \frac{1}{3} - \frac{1}{2} - \frac{1}{2} + \frac{1}{2} - \frac{1}{6}$$

$$V = \frac{1}{6} \text{ units}^3$$ Ans.

b. volume: ? xy-plane , cylinder: , plane:

$x^2+y^2=1$ $\Rightarrow r=1$

$x+y+z=3$ $\Rightarrow z=3-x-y$



$$V = \int_0^{2\pi}\int_0^{1}\int_0^{3-x-y} dz\, r\, dr\, d\theta$$

$$V = \int_0^{2\pi}\int_0^{1} |z|_0^{3-x-y}\, r\, dr\, d\theta$$

$$V = \int_0^{2\pi}\int_0^{1} (3-x-y)\, r\, dr\, d\theta$$

$$V = \int_0^{2\pi}\int_0^{1} (3 - r\cos\theta - r\sin\theta)\, r\, dr\, d\theta$$

$$V = \int_0^{2\pi} 3\left|\frac{r^2}{2}\right|_0^1 d\theta - \int_0^{2\pi} \cos\theta . \left|\frac{r^3}{3}\right|_0^1 d\theta - \int_0^{2\pi} \sin\theta . \left|\frac{r^3}{3}\right|_0^1 d\theta$$

$$V = \frac{3}{2}\int_0^{2\pi} d\theta - \frac{1}{3}\int_0^{2\pi} \cos\theta\, d\theta - \frac{1}{3}\int_0^{2\pi} \sin\theta\, d\theta$$

$$V = \frac{3}{2}\left|\theta\right|_0^{2\pi} - \frac{1}{3}\left|\sin\theta\right|_0^{2\pi} + \frac{1}{3}\left|\cos\theta\right|_0^{2\pi}$$

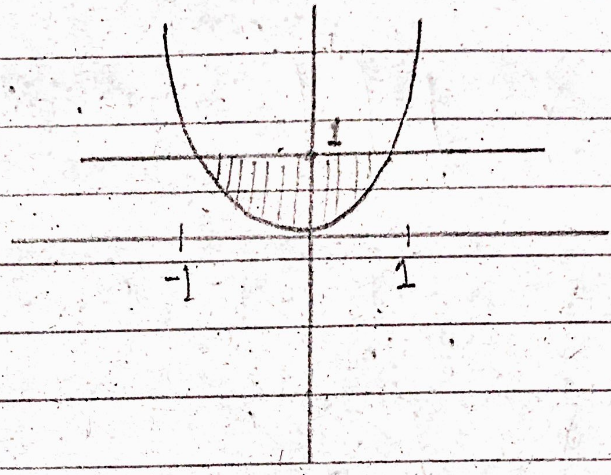
$$V = \frac{3}{2}(2\pi - 0) - \frac{1}{3}(\sin 2\pi - 0) + \frac{1}{3}(\cos 2\pi - \cos 0)$$

$$V = 3\pi - (0) + \frac{1}{3}(1-1)$$

$$V = 3\pi \text{ units}^3 \text{ Ans.}$$

c. volume : ? $\quad y+z=1, \quad y=x^2, \quad xy$-plane

$z = 1-y$



$$V = \int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} dz\,dy\,dx$$

$$V = \int_{-1}^{1}\int_{x^2}^{1} |z|_{0}^{1-y}\,dy\,dx$$

$$V = \int_{-1}^{1}\int_{x^2}^{1} (1-y)\,dy\,dx$$

$$V = \int_{-1}^{1} |y|_{x^2}^{1}\,dx - \int_{-1}^{1} \left|\frac{y^2}{2}\right|_{x^2}^{1} dx$$

$$V = \int_{-1}^{1} (1-x^2)\,dx - \frac{1}{2}\int_{-1}^{1}(1-x^4)\,dx$$

$$V = \int_{-1}^{1} dx - \int_{-1}^{1} x^2\,dx - \frac{1}{2}\int_{-1}^{1} dx + \frac{1}{2}\int_{-1}^{1} x^4\,dx$$

$$V = |x|_{-1}^{1} - \left|\frac{x^3}{3}\right|_{-1}^{1} - \frac{1}{2}|x|_{-1}^{1} + \frac{1}{2}\left|\frac{x^5}{5}\right|_{-1}^{1}$$

$$V = (1+1) - \frac{(1+1)}{3} - \frac{1}{2}(1+1) + \frac{1}{10}(1+1)$$

$$V = 2 - \frac{2}{3} - \frac{2}{2} + \frac{1}{10}(2)$$

$$V = \frac{8}{15} \text{ units}^3 \quad \text{Ans.}$$