$$f'(x_0) = \lim_{x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

$$dz = f_x(x,y)\,dx + f_y(x,y)\,dy$$

Total differential

$dx = \Delta x = x_2 - x_1$ | $dy = \Delta y = y_2 - y_1$

$dz \approx \Delta z$

Approx change — Actual change

$\because \frac{dz}{dx} = f_x$

$\therefore dz = f_x dx$

$f(x) = f(x_0) - f'(x_0)(x - x_0)$

$y = y_0 - y'(x - x_0)$

LOCAL LINEAR APPROXIMATION

$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

$L(x,y,z) = f(x_0,y_0,z_0) + f_x(x_0,y_0,z_0)(x-x_0) + f_y(x_0,y_0,z_0)(y-y_0) + f_z(x_0,y_0,z_0)(z-z_0)$

13.6 **Directional Derivatives**

$\nabla f(x,y,z) = f_x\hat{i} + f_y\hat{j} + f_z\hat{k}$

$D_u f(x,y,z) = \nabla f \cdot u = f_x u_1 + f_y u_2 + f_z u_3$

$a = P - Q$ & $u = \frac{a}{|a|}$

$u = \cos\theta\, i + \sin\theta\, j$

unit vectors

$* D = f_{xx} f_{yy} - (f_{xy})^2$ 13.8

- if $D>0$ && $f_{xx}(x_0,y_0)>0 \Rightarrow$ r. minima.
- if $D>0$, $f_{xx}(x_0,y_0)<0 \Rightarrow$ r. maxima.
- if $D<0 \Rightarrow$ saddle point
- if $D=0$, inconclusive

r = relative

second derivative test

LAGRANGE MULTIPLIERS 13.9

$* \boxed{\nabla f = \lambda \nabla g}$

14.1 ch

Type II Region $\int_c^d \int_a^b f(x,y)\,dx\,dy$.

Type-I region $\int_a^b \int_c^d f(x,y)\,dy\,dx$

property: $f_{xy} = f_{yx}$ *depends on scenarios

14

LIATE priority.

* Tabular → AE or A.T

$\boxed{\int u\,dv = u \cdot v - \int v\,du}$ ①

- $\int \sin^n x\,dx = \frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x\,dx$
- $\int \cos^n x\,dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x\,dx$

① $\int e^x \cos x\,dx$

Let $u = \cos x$ $\int dv = e^x$

$\frac{du}{dx} = -\sin x$ $v = e^x$

$du = -\sin x\,dx$

14.3

$r = 1 - \cos\theta$ (cardiod)

$r = \sin 2\theta$ (rose)

$r = \sqrt{\sin 2\theta}$ (lemniscates)

$r = \sin n\theta$; if n is odd then n-petals; if n is even then 2n petals will be drawn.

Surface area (14.4)

$$\iint_R \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\, dA$$

or $\iint_R \sqrt{f_x^2 + f_y^2 + 1}\, dA$

two point form

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

1. $\int \sqrt{x^2-a^2}\,dx = \frac{1}{2}x\sqrt{x^2-a^2} - \frac{1}{2}a^2 \ln\left(x+\sqrt{x^2-a^2}\right)+c$

2. $\int \sqrt{x^2+a^2}\,dx = \frac{1}{2}x\sqrt{x^2+a^2} + \frac{1}{2}a^2 \ln\left(x+\sqrt{x^2+a^2}\right)+c$

3. $\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2 \sin^{-1} x/a + c$

4. $\int \frac{dx}{\sqrt{a^2-x^2}}\,dx = a.\sin^{-1}\frac{x}{a} + c$

5. $\int \frac{dx}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right)+c$

6. $\int \frac{dx}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)+c$

Date

⑦ $$\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln\left(\frac{a-x}{a+x}\right) + C$$

⑧ $$\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\left(\frac{x-a}{x+a}\right) + c$$

⑨ $$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

<u>Special case:-</u>

⑩ $$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a} \sec^{-1} \frac{x}{a} + C$$

Date

TRIGONOMETRIC FUNCTIONS

① $\int \sin x \, dx = -\cos x + C$

② $\int \cos x \, dx = \sin x + C$

③ $\int \sec^2 x \, dx = \tan x + C$

④ $\int \operatorname{cosec}^2 x \, dx = -\cot x + C$

⑤ $\int \sec x \tan x \, dx = \sec x + C$

⑥ $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + C$

⑦ $\int \tan x \, dx = \ln(\sec x) + C$

⑧ $\int \cot x \, dx = \ln(\sin x) + C$

⑨ $\int \sec x \, dx = \ln(\sec x + \tan x) + C$

OR $\ln \tan\left(\frac{x}{2} + \frac{\pi}{4}\right) + C$ (FOR MCQ)

⑩ $\int \operatorname{cosec} x \, dx = \ln(\operatorname{cosec} x - \cot x) + C$

OR $\ln \tan \frac{x}{2} + C$ (FOR MCQ)

$$\boxed{(a-b-c)^2 = a^2+b^2+c^2-2ab+2bc-2ca}$$

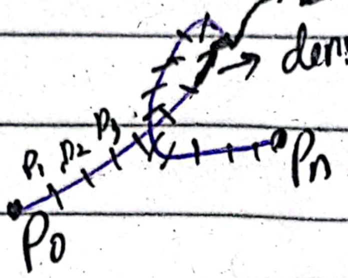
NOTE: used in Q# 38, 39 of Ex# 13.B

$$(a+b+c)^2 = a^2+b^2+c^2+2ab+2bc+2ca$$

8/5/24

15.2 LINE INTEGRALS

* to calculate the mass of thin wire we use the line integral.



$$\boxed{\Delta M_k = f(x_k^*, y_k^*)\Delta S_k}$$

$$M = \lim_{n\to\infty} \sum_{k=1}^{n}$$

$$\Delta M_k = \lim_{n\to\infty} \sum_{k=1}^{n} f(x_k^*, y_k^*)\,\Delta S_k$$

$$M = \int_C f(x,y)\,ds = \int_a^b f(r(t))\,\|r'(t)\|\,dt$$

$$M = \int_C F\cdot dr = \int_a^b F(r(t)) \cdot r'(t)\,dt$$

dot

r = vector valued function.

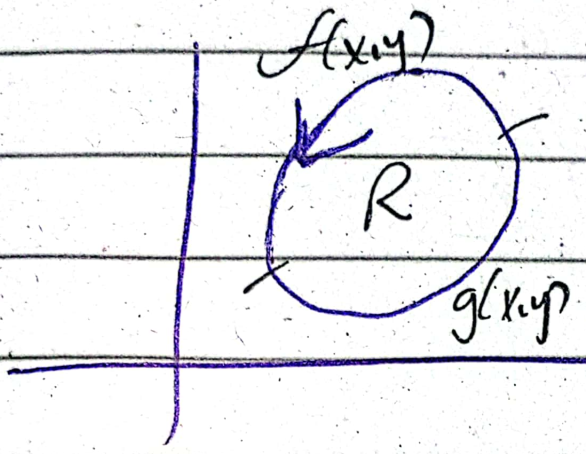
$\rightarrow r(t) = x(t)i + y(t)j + z(t)k$

ture ______

15.4

Date 10/5/24

GREEN'S THEOREM



~~not a fun~~

+ not a function.

* it's a piecewise function. i.e. combo of 2 functions

$$\oint_C f(x,y)\,dx + g(x,y)\,dy = \iint_R \left(\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y} \right) dA$$

NOTE:

when the function is counter-clockwise.