[MVC-QUIZ-01]

SOLUTION KEY

MVC-QUIZ-01.

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National University

Of Computer & Emerging Sciences, Karachi-Campus

Multivariable Calculus QUIZ-1 [Max Marks:10]

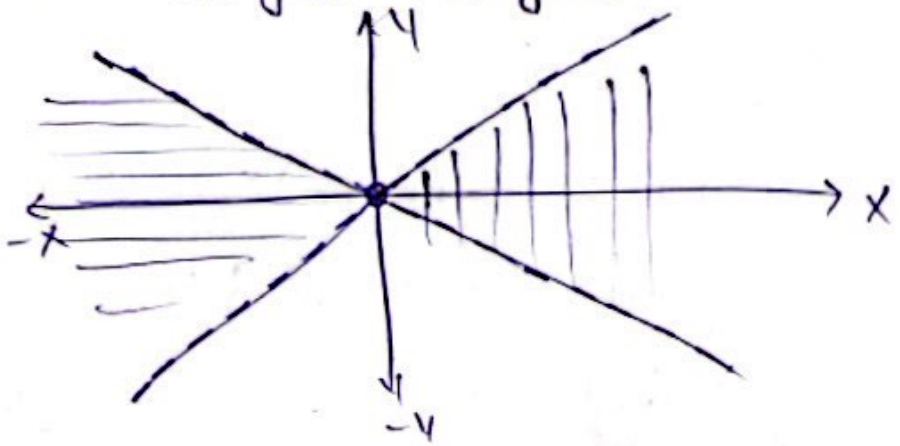
Instructor: Dr. Nazish Kanwal **Sections: BCS-2B, BCS-2F, BCY-2A, BCY-2B**

NAME: _____ **Roll No:** _____

Q#01 [3 marks] Find and sketch the domain of $f(x,y) = \frac{1}{\sqrt{x^2-y^2}}$.

$x^2 - y^2 > 0 \Rightarrow (x+y)(x-y) > 0 \rightarrow x+y>0 \ \& \ x-y>0$. 0

OR $x+y<0 \ \& \ x-y<0$



Q#02 [3 marks] Determine whether the limit $\lim_{(x,y)\to(0,0)} \frac{y^3-x^3}{y^2+x^2}$ exists if it does find the limits & prove that it is the limit, if not, explain why not?

Path $y = mx$.

$\lim_{x\to 0} \frac{x^3(m^3-1)}{x^2(m^2+1)} = \lim_{x\to 0} \frac{x(m^3-1)}{m^2+1} = 0$.

Path: along $y = kx^2$

$\lim_{x\to 0} \frac{x^3(k^3x^3-1)}{x^2(k^2x^2+1)} = \lim_{x\to 0} x\left(\frac{k^3x^3-1}{k^2x^2+1}\right) = 0$

Hence limit exist and.

$\lim_{(x,y)\to(0,0)} \frac{y^3-x^3}{y^2+x^2} = 0$.

ALTERNATIVELY

Put $x = r\cos\theta$, $y = r\sin\theta$

$$\lim_{r\to 0} r^4(\sin^3\theta - \cos^3\theta) = \lim_{r\to 0} r(\sin^3\theta - \cos^3\theta) = 0.$$

Q#03 [4 marks] Find $\frac{\partial z}{\partial x}$, for the implicit function $x^2 + z\sin(xyz) = 1$.

$$\frac{\partial}{\partial x}(x^2 + z\sin(xyz)) = \frac{\partial}{\partial x}(1)$$

$$2x + \frac{\partial z}{\partial x}\sin(xyz) + z\cos(xyz)\times\frac{\partial}{\partial x}(xyz) = 0$$

$$2x + \frac{\partial z}{\partial x}\sin(xyz) + z\cos(xyz)\left\{yz + xy\frac{\partial z}{\partial x}\right\} =$$

$$2x + yz^2\cos(xyz) + \frac{\partial z}{\partial x}\sin(xyz) + xyz\cos(xyz) =$$

$$\Rightarrow \boxed{\frac{\partial z}{\partial x} = -\frac{\{2x + yz^2\cos(xyz)\}}{\sin(xyz) + xyz\cos(xyz)}}$$

Ans.

Alternatively

$$\frac{\partial z}{\partial x} = -\frac{\partial f/\partial x}{\partial f/\partial z}, \quad f(x,y,z) = x^2 + z\sin(xyz)$$

$$\frac{\partial f}{\partial x} = 2x + yz^2\cos(xyz)$$

$$\frac{\partial f}{\partial z} = \sin(xyz) + z\times xy\times\cos(xyz)$$

So $\frac{\partial z}{\partial x} = -\frac{\{2x + yz^2\cos(xyz)\}}{\sin(xyz) + xyz\cos(xyz)}$ Ans.