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Exercise # 13.8

$$D(x,y) = f_{xx}(x,y) \cdot f_{yy}(x,y) - (f_{xy})^2$$

1- $D(a,b) > 0$, $f_{xx}(a,b) > 0$, $(a,b) \xrightarrow{\text{gives}}$ relative minima

2. $D(a,b) > 0$, $f_{xx}(a,b) < 0$, $(a,b) \xrightarrow{\text{gives}}$ relative maxima

3. $D(a,b) < 0$, (a,b) is a saddle point

4- $D(a,b) = 0 \longrightarrow$ inconclusive.

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9. $f(x,y) = y^2 + xy + 3y + 2x + 3$

step#01: find partial derivatives

$f_x = y + 2$

$f_y = 2y + x + 3$

step #02: find critical points

$f_x = 0$, $f_y = 0$

critical point $(1,-2)$

$y + 2 = 0$

$\boxed{y = -2}$

$2(-2) + x + 3 = 0$

$-4 + x + 3 = 0$

$\boxed{x = 1}$

step # 03 : use formula:

$$D(x,y) = f_{xx} \cdot f_{yy} - (f_{xy})^2$$

$$= (0)(2) - (1)^2$$

$$D(x,y) = -1$$

$D(x,y) < 0$ hence it is a saddle point

10. $f(x,y) = x^2 + xy - 2y - 2x + 1$

step #01: find partial derivatives

$f_x = 2x + y - 2$

$f_y = x - 2$

step # 02 : find critical point

$f_x = 0$, $f_y = 0$

$2x + y - 2 = 0$

$2(2) + y - 2 = 0$

$4 + y - 2 = 0$

$\boxed{y = -2}$

$x - 2 = 0$

$\boxed{x = 2}$

critical point $(2,-2)$

step # 03 : formula :

$$D(x,y) = f_{xx} \cdot f_{yy} - (f_{xy})^2$$

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$= (2)(0) - (1)^2$

$= -1$

$D(x,y) < 0$ hence it is a saddle point.

11. $f(x,y) = x^2 + xy + y^2 - 3x$

step #01 : Find partial derivatives:

$f_x = 2x + y - 3$

$f_y = x + 2y$

step # 02 : find critical point

$f_x = 0$, $f_y = 0$

$2x + y - 3 = 0$ $\quad\quad$ $x + 2y = 0$

$2x + y = 3$

$\times 2$: $4x + 2y = 6$

$$\begin{array}{r} 4x + \cancel{2y} = 6 \\ \pm\, x \pm \cancel{2y} = 0 \\ \hline 3x = 6 \\ \boxed{x = 2} \end{array}$$

critical point $(2,-1)$

$2 + 2y = 0$

$2y = -2$

$\boxed{y = -1}$

Step # 03 : Use formula:

$$D(x,y) = f_{xx} \cdot f_{yy} - (f_{xy})^2$$
$$= (2)(2) - 1$$
$$= 4 - 1 = 3$$

$$\left.\begin{array}{l} D(x,y) = 3 > 0 \\ f_{xx}(x,y) = 2 > 0 \end{array}\right\} \text{relative minima}$$

Step #04: put critical point in question.

critical point $(2,-1)$

$$f(x,y) = x^2 + xy + y^2 - 3x$$
$$f(2,-1) = 4 + (2)(-1) + (-1)^2 - 3(2)$$
$$= 4 - 2 + 1 - 6$$
$$= 5 - 8 = -3$$

relative minima at -3.

12. $f(x,y) = xy - x^3 - y^2$

Step #01: Find partial derivative

$$f_x = y - 3x^2$$
$$f_y = x - 2y$$

Step # 02 : Find critical points

$f_x = 0 \quad , \quad f_y = 0$

$y - 3x^2 = 0$

$y = 3x^2$

$x - 2y = 0$

$x - 2(3x^2) = 0$

$x - 6x^2 = 0$

$x(1 - 6x) = 0$

when $x = 0$

$\boxed{y = 0}$

$\boxed{x = 0}$

$1 - 6x = 0$

$1 = 6x$

$\boxed{x = \frac{1}{6}}$

when $x = \frac{1}{6}$

$y = \cancel{3}^{1}\left(\frac{1}{\cancel{36}_{12}}\right)$

$\boxed{y = \frac{1}{12}}$

critical points $(0,0)$ and $\left(\frac{1}{6}, \frac{1}{12}\right)$

Step # 03 : Use formula.

$D(x,y) = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$= (-6x) \cdot (-2) - (1)^2$

$= 12x - 1$

$D(0,0) = -1 < 0$ saddle point

$D\left(\frac{1}{6}, \frac{1}{12}\right) = \cancel{12}^{2}\left(\frac{1}{\cancel{6}_{1}}\right) - 1 = 1 > 0$

$f_{xx}\left(\frac{1}{6}, \frac{1}{12}\right) = -\cancel{6}\left(\frac{1}{\cancel{6}}\right) = -1 < 0$ relative maxima

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step #04 : put critical point in question

critical point $\left(\frac{1}{6}, \frac{1}{12}\right)$

$$f\left(\frac{1}{6}, \frac{1}{12}\right) = xy - x^3 - y^2$$

$$= \left(\frac{1}{6}\right)\left(\frac{1}{12}\right) - \left(\frac{1}{6}\right)^3 - \left(\frac{1}{12}\right)^2$$

$$= \frac{1}{72} - \frac{1}{216} - \frac{1}{144}$$

$$= 0.0138 - 0.0046 - 0.0069$$

$$= 0.0023$$

Result : relative maxima at 0.0023