Boolean Algebra

and

Logic Simplification

Chapter 4

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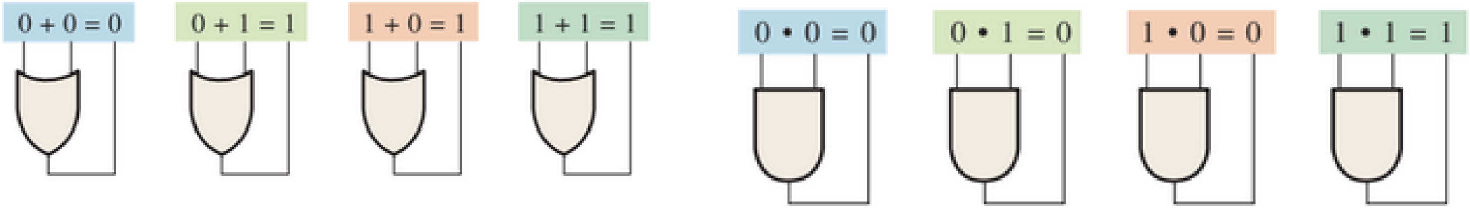
Boolean Algebra

Boolean algebra is the mathematics of digital logic.

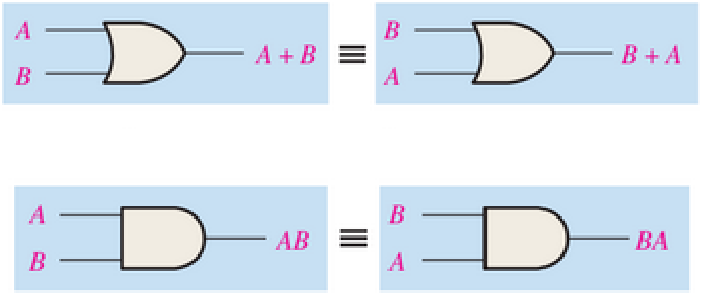


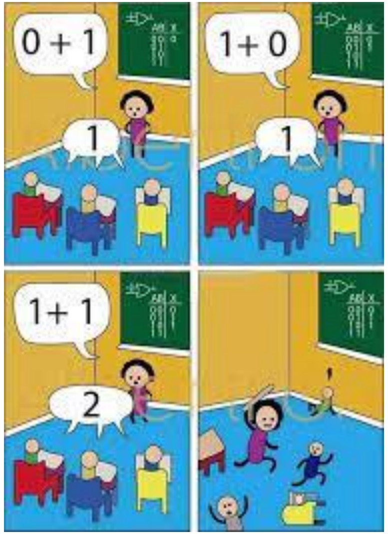
Determine the values of A, B, and C that make the sum term A\` + B\` + C equal to 0.

Determine the values of A, B, and C that make the product term AB\`C equal to 1.

Laws of Boolean Algebra

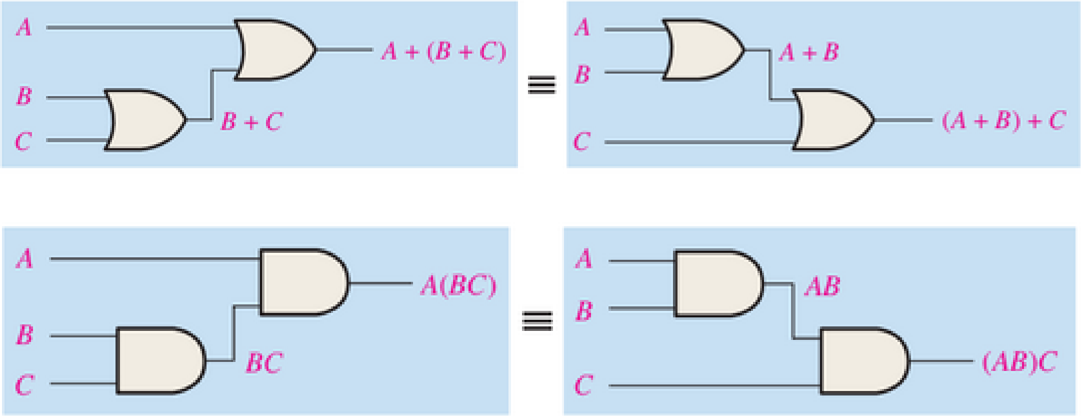
Commutative Laws





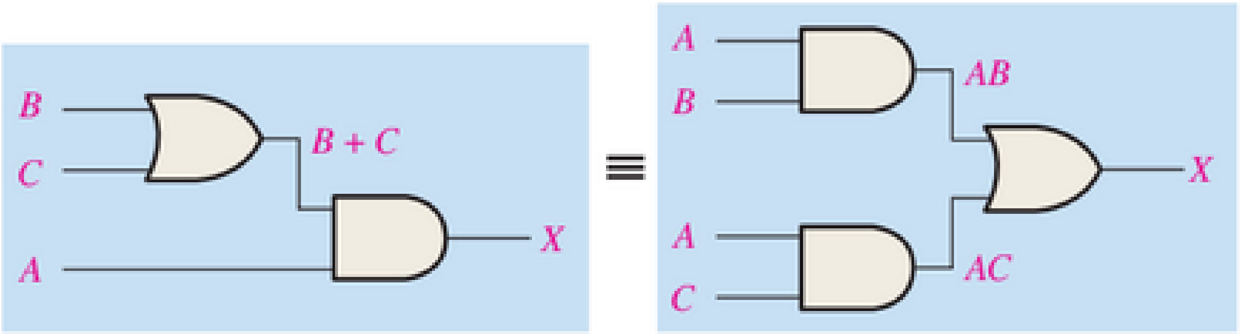
Laws of Boolean Algebra

Associative Laws



Laws of Boolean Algebra

Distributive Laws



$X = A(B + C)$ $\quad$ $X = AB + AC$

Rules of Boolean Algebra

TABLE 4–1

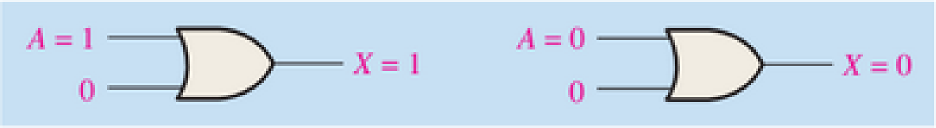
Basic rules of Boolean algebra.

1. $A + 0 = A$	**7.** $A \cdot A = A$
2. $A + 1 = 1$	**8.** $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	**9.** $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	**10.** $A + AB = A$
5. $A + A = A$	**11.** $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	**12.** $(A + B)(A + C) = A + BC$

A, B, or C can represent a single variable or a combination of variables.

Rules of Boolean Algebra

Rule 1: A + 0 = A



$X = A + 0 = A$

Rule 2: A + 1 = 1

$X = A + 1 = 1$

Rules of Boolean Algebra

Rule 3: A.0 = 0

$X = A \cdot 0 = 0$

Rule 4: A.1 = 1

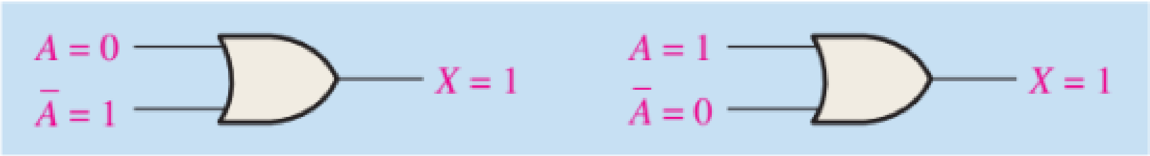
$X = A \cdot 1 = A$

Rules of Boolean Algebra

Rule 5: A + A = A

$X = A + A = A$

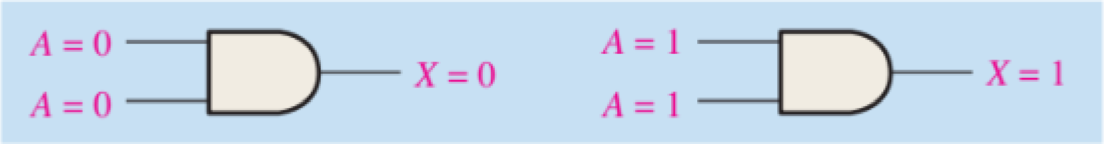
Rule 6: A + A` = 1



$X = A + \bar{A} = 1$

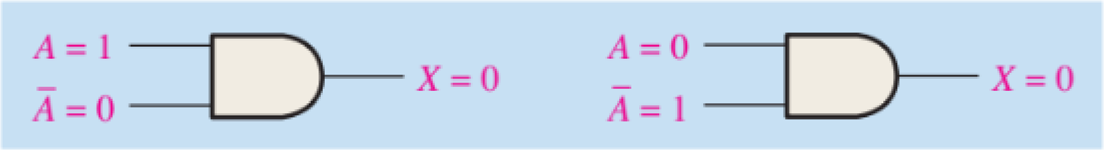
Rules of Boolean Algebra

Rule 7: A.A = A



$X = A \cdot A = A$

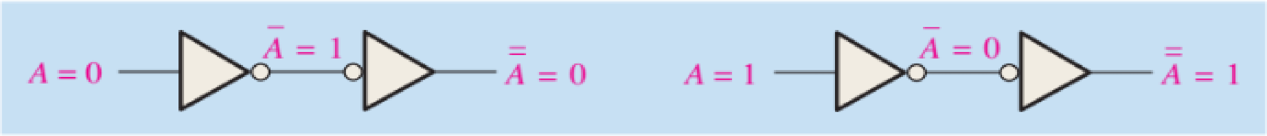
Rule 8: A.A` = 0



$X = A \cdot \overline{A} = 0$

Rules of Boolean Algebra

Rule 9: $\bar{\bar{A}} = A$



$$\bar{\bar{A}} = A$$

Rule 10: A + AB = A

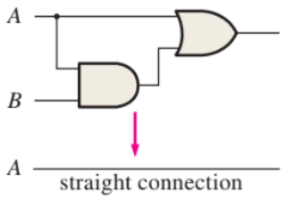
$A + AB = A \cdot 1 + AB = A(1 + B)$	Factoring (distributive law)
$= A \cdot 1$	Rule 2: $(1 + B) = 1$
$= A$	Rule 4: $A \cdot 1 = A$

TABLE 4–2

Rule 10: $A + AB = A$. Open file T04-02 to verify.

A	B	AB	$A + AB$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

equal



Rules of Boolean Algebra

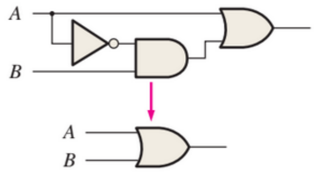
Rule 11: A+A`B = A + B

TABLE 4–3

Rule 11: $A + \overline{A}B = A + B$. Open file T04-03 to verify.

A	B	$\overline{A}B$	$A + \overline{A}B$	$A + B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

equal



$$
\begin{aligned}
A + \overline{A}B &= (A + AB) + \overline{A}B && \text{Rule 10: } A = A + AB \\
&= (AA + AB) + \overline{A}B && \text{Rule 7: } A = AA \\
&= AA + AB + A\overline{A} + \overline{A}B && \text{Rule 8: adding } A\overline{A} = 0 \\
&= (A + \overline{A})(A + B) && \text{Factoring} \\
&= 1 \cdot (A + B) && \text{Rule 6: } A + \overline{A} = 1 \\
&= A + B && \text{Rule 4: drop the 1}
\end{aligned}
$$

Rules of Boolean Algebra

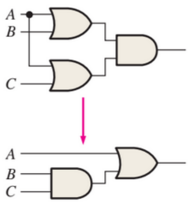
Rule 12:(A+B)(A+C)= A+BC

TABLE 4–4

Rule 12: $(A + B)(A + C) = A + BC$. Open file T04-04 to verify.

A	B	C	$A+B$	$A+C$	$(A+B)(A+C)$	BC	$A+BC$
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

equal



$$
\begin{aligned}
(A + B)(A + C) &= AA + AC + AB + BC && \text{Distributive law} \\
&= A + AC + AB + BC && \text{Rule 7: } AA = A \\
&= A(1 + C) + AB + BC && \text{Factoring (distributive law)} \\
&= A \cdot 1 + AB + BC && \text{Rule 2: } 1 + C = 1 \\
&= A(1 + B) + BC && \text{Factoring (distributive law)} \\
&= A \cdot 1 + BC && \text{Rule 2: } 1 + B = 1 \\
&= A + BC && \text{Rule 4: } A \cdot 1 = A
\end{aligned}
$$

Practice Exercise

(a) $\overline{\overline{AB + CD}} + \overline{EF} = AB + CD + \overline{EF}$

(c) $A(BC + BC) + AC = A(BC) + AC$

(e) $A\overline{B} + A\overline{B}C = A\overline{B}$

Practice Exercise

(b) $A\overline{A}B + AB\overline{C} + AB\overline{B} = AB\overline{C}$

(d) $AB(C + \overline{C}) + AC = AB + AC$

(f) $ABC + \overline{AB} + \overline{ABC}D = ABC + \overline{AB} + D$

Practice Exercise

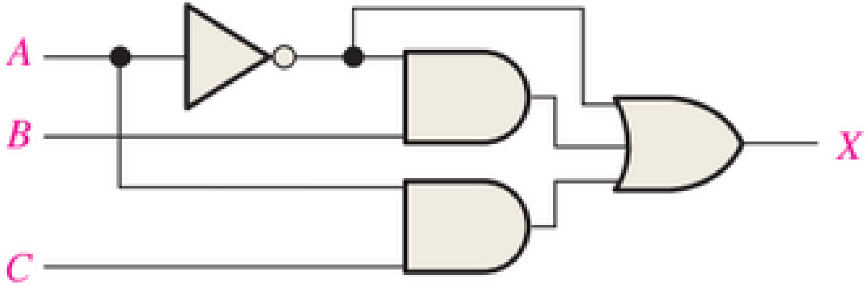
Using Boolean algebra techniques, simplify the following expressions as much as possible:

(a) $A(A + B)$ (b) $A(A' + AB)$ (c) $BC + B'C$

(d) $A(A + A'B)$ (e) $AB'C + A'BC + A'B'C$

Practice Exercise

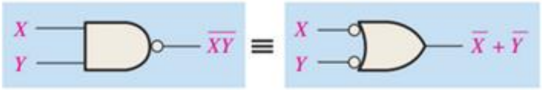
Write the output expression for circuit and simplify it if possible.



De Morgan's Theorem

The complement of a product of variables is equal to the sum of the complements of the variables.

$$\overline{XY} = \overline{X} + \overline{Y}$$

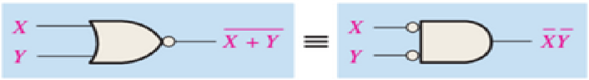


NAND Negative-OR

Inputs X	Inputs Y	Output $\overline{XY}$	Output $\overline{X} + \overline{Y}$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

The complement of a sum of variables is equal to the product of the complements of the variables.

$$\overline{X + Y} = \overline{X}\,\overline{Y}$$



NOR Negative-AND

Inputs X	Inputs Y	Output $\overline{X + Y}$	Output $\overline{X}\,\overline{Y}$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

Practice Exercise

Apply DeMorgan's theorem to the expression $\overline{\overline{X} + \overline{Y} + \overline{Z}}$.

Apply DeMorgan's theorem to the expression $\overline{\overline{W}\,\overline{X}\,\overline{Y}\,\overline{Z}}$.

Practice Exercise

EXAMPLE 4–6

Apply DeMorgan's theorems to each expression:

(a) $\overline{(\overline{A + B}) + \overline{C}}$

(b) $\overline{(\overline{A} + B) + CD}$

(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}}$

Practice Exercise

EXAMPLE 4–6

Apply DeMorgan's theorems to each expression:

(a) $\overline{\overline{(A + B)} + \overline{C}}$

(b) $\overline{(\overline{A} + B) + CD}$

(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}}$

Solution

(a) $\overline{\overline{(A + B)} + \overline{C}} = (\overline{\overline{A + B}})\overline{\overline{C}} = (A + B)C$

(b) $\overline{(\overline{A} + B) + CD} = (\overline{\overline{A} + B})\overline{CD} = (\overline{\overline{A}}\,\overline{B})(\overline{C} + \overline{D}) = A\overline{B}(\overline{C} + \overline{D})$

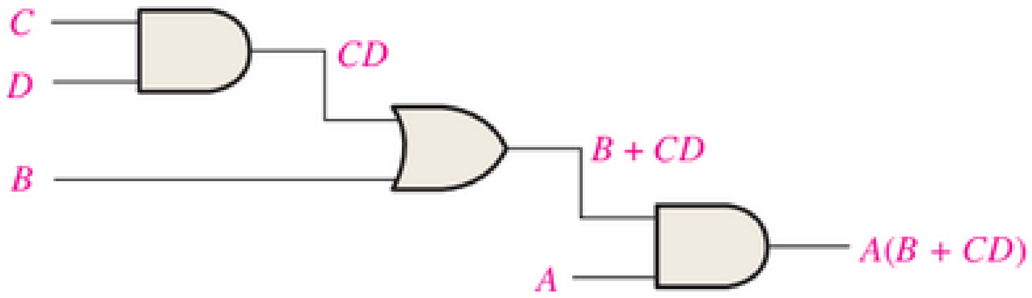
(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}} = \overline{((A + B)\overline{C}\,\overline{D})}(\overline{E + \overline{F}}) = (\overline{A}\,\overline{B} + C + D)\overline{E}F$

Related Problem

Apply DeMorgan's theorems to the expression $\overline{\overline{A}B(C + \overline{D}) + E}$.

Boolean Analysis of Logic Circuits,

Boolean Expression for a Logic Circuit



Boolean Analysis of Logic Circuits,

Constructing a Truth Table for a Logic Circuit

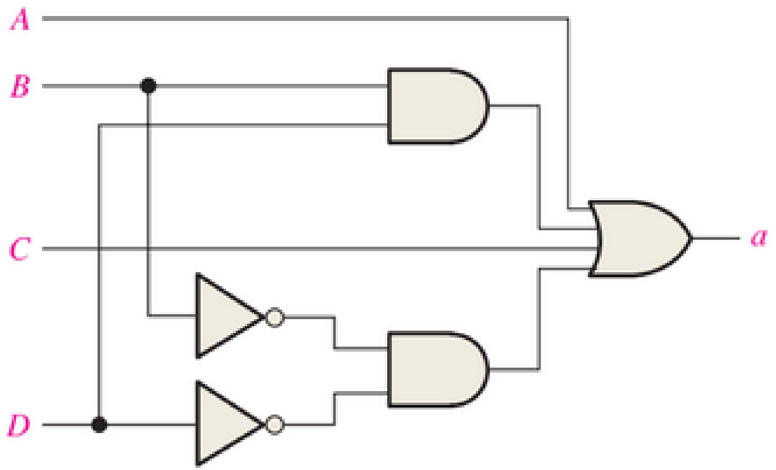
TABLE 4–5

Truth table for the logic circuit in Figure 4–18.

Inputs				Output
A	B	C	D	$A(B + CD)$
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Practice Exercise

Write a Boolean expression for the logic circuit and construct the truth table.



Logic Simplification Using Boolean Algebra,

Using Boolean algebra techniques, simplify this expression:

$$AB + A(B + C) + B(B + C)$$

Related Problem

Simplify the Boolean expression $A\overline{B} + A(\overline{B + C}) + B(\overline{B + C})$.

Logic Simplification Using Boolean Algebra,

Using Boolean algebra techniques, simplify this expression:

$$AB + A(B + C) + B(B + C) \quad == \quad B + AC$$

Related Problem

Simplify the Boolean expression $A\overline{B} + A(\overline{B + C}) + B(\overline{B + C})$. $==$ $A\overline{B}$

Logic Simplification Using Boolean Algebra,

EXAMPLE 4–10

Simplify the following Boolean expression:

$$[A\overline{B}(C + BD) + \overline{A}\,\overline{B}]C$$

EXAMPLE 4–11

Simplify the following Boolean expression:

$$\overline{A}BC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + ABC$$

EXAMPLE 4–12

Simplify the following Boolean expression:

$$\overline{AB + AC} + \overline{A}\,\overline{B}C$$

Logic Simplification Using Boolean Algebra,

EXAMPLE 4–10

Simplify the following Boolean expression:

$$[A\overline{B}(C + BD) + \overline{A}\,\overline{B}]C$$

$\overline{B}C$

EXAMPLE 4–11

Simplify the following Boolean expression:

$$\overline{A}BC + A\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + ABC$$

$BC + A\overline{B} + \overline{B}\,\overline{C}$

EXAMPLE 4–12

Simplify the following Boolean expression:

$$\overline{AB + AC} + \overline{A}\,\overline{B}C$$

$\overline{A} + \overline{B}\,\overline{C}$

Logic Simplification Using Boolean Algebra,

Related Problem

Simplify the Boolean expression $[AB(C + \overline{BD}) + \overline{AB}]CD$.

Related Problem

Simplify the Boolean expression $AB\overline{C} + \overline{A}\,\overline{B}C + \overline{A}BC + \overline{A}\,\overline{B}\,\overline{C}$.

Related Problem

Simplify the Boolean expression $\overline{AB} + \overline{AC} + \overline{A}\,\overline{B}\,\overline{C}$.

Practice Exercises

20. Using Boolean algebra, simplify the following expressions:

(a) $(\overline{A} + B)(A + C)$

(b) $A\overline{B} + A\overline{B}C + A\overline{B}CD + A\overline{B}CDE$

(c) $BC + \overline{BCD} + B$

(d) $(B + \overline{B})(BC + BC\overline{D})$

(e) $BC + (\overline{B} + \overline{C})D + BC$

21. Using Boolean algebra, simplify the following expressions:

(a) $CE + C(E + F) + \overline{E}(E + G)$

(b) $\overline{B}\,\overline{C}D + (\overline{B + C + D}) + \overline{B}\,\overline{C}\,\overline{D}E$

(c) $(C + CD)(C + \overline{C}D)(C + E)$

(d) $BCDE + BC(\overline{DE}) + (\overline{BC})DE$

(e) $BCD[BC + \overline{D}(CD + BD)]$

Standard Forms of Boolean Expressions

All Boolean expressions, regardless of their form, can be converted into either of two standard forms:

- The sum-of-products form
- The product-of-sums form

The sum-of-products form

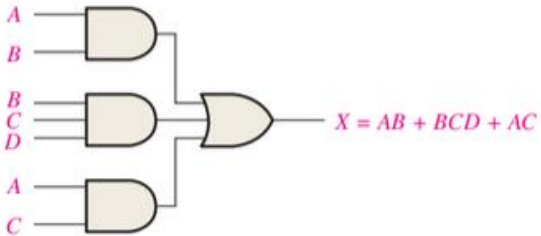


FIGURE 4–22 Implementation of the SOP expression $AB + BCD + AC$.

$AB + ABC$

$ABC + CDE + \overline{B}C\overline{D}$

$\overline{A}B + \overline{A}B\overline{C} + AC$

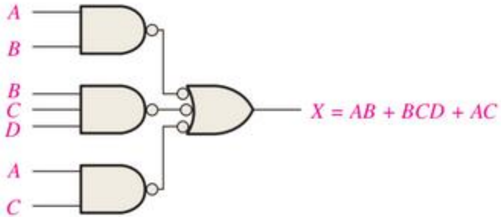


FIGURE 4–23 This NAND/NAND implementation is equivalent to the AND/OR in Figure 4–22.

The sum-of-products form

EXAMPLE 4–14

Convert each of the following Boolean expressions to SOP form:

(a) $AB + B(CD + EF)$ **(b)** $(A + B)(B + C + D)$ **(c)** $\overline{(\overline{A + B}) + C}$

The sum-of-products form

EXAMPLE 4–14

Convert each of the following Boolean expressions to SOP form:

(a) $AB + B(CD + EF)$ **(b)** $(A + B)(B + C + D)$ **(c)** $\overline{(\overline{A + B}) + C}$

Solution

(a) $AB + B(CD + EF) = AB + BCD + BEF$

(b) $(A + B)(B + C + D) = AB + AC + AD + BB + BC + BD$

(c) $\overline{(\overline{A + B}) + C} = (\overline{\overline{A + B}})\overline{C} = (A + B)\overline{C} = A\overline{C} + B\overline{C}$

Related Problem

Convert $\overline{A}B\overline{C} + (A + \overline{B})(B + \overline{C} + A\overline{B})$ to SOP form.

Standard SOP form

A standard SOP expression is one in which all the variables in the domain appear in each product term in the expression.

EXAMPLE 4–15

Convert the following Boolean expression into standard SOP form:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$

Standard SOP form

EXAMPLE 4–15

Convert the following Boolean expression into standard SOP form:

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D$$

$$A\overline{B}C + \overline{A}\,\overline{B} + AB\overline{C}D = A\overline{B}CD + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + AB\overline{C}D$$

Related Problem

Convert the expression $W\overline{X}Y + \overline{X}Y\overline{Z} + WX\overline{Y}$ to standard SOP form.

Binary Representation of Standard Product Term

EXAMPLE 4–16

Determine the binary values for which the following standard SOP expression is equal to 1:

$$ABCD + A\overline{B}\,\overline{C}D + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$$

Construct the Truth table for given SOP expression.

Binary Representation of Standard Product Term

Related Problem

Determine the binary values for which the following SOP expression is equal to 1:

$$\overline{X}YZ + X\overline{Y}Z + XY\overline{Z} + \overline{X}Y\overline{Z} + XYZ$$

Is this a standard SOP expression?

Construct the Truth table for given SOP expression.

Practice Exercises

24. Convert the following expressions to sum-of-product (SOP) forms:

(a) $BC + DE(B\overline{C} + DE)$ **(b)** $BC(\overline{C}\,\overline{D} + CE)$ **(c)** $B + C[BD + (C + \overline{D})E]$

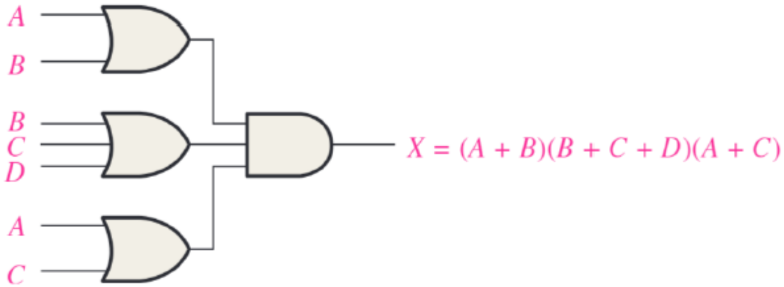
26. Convert each SOP expression in Problem 24 to standard SOP form.

32. Develop a truth table for each of the following standard SOP expressions:

(a) $A\overline{B}C\overline{D} + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$

(b) $WXYZ + \overline{W}X\overline{Y}Z + W\overline{X}Y\overline{Z} + \overline{W}\,\overline{X}YZ + WX\overline{Y}\,\overline{Z}$

The Product-of-Sums form



$$(\overline{A} + B)(A + \overline{B} + C)$$
$$(\overline{A} + \overline{B} + \overline{C})(C + \overline{D} + E)(\overline{B} + C + D)$$
$$(A + B)(A + \overline{B} + C)(\overline{A} + C)$$

A *standard POS expression* is one in which *all* the variables in the domain appear in each sum term in the expression. For example,

$$(\overline{A} + \overline{B} + \overline{C} + \overline{D})(A + \overline{B} + C + D)(A + B + \overline{C} + D)$$

Standard POS form

EXAMPLE 4–17

Convert the following Boolean expression into standard POS form:

$$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

Rule 8: A.A` = 0

Rule 12:(A+B)(A+C)= A+BC

Standard POS form

EXAMPLE 4–17

Convert the following Boolean expression into standard POS form:

$$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

$$(A + \overline{B} + C)(\overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D) =$$
$$(A + \overline{B} + C + D)(A + \overline{B} + C + \overline{D})(A + \overline{B} + C + \overline{D})(\overline{A} + \overline{B} + C + \overline{D})(A + \overline{B} + \overline{C} + D)$$

Binary Representation of Standard Sum Term

EXAMPLE 4–18

Determine the binary values of the variables for which the following standard POS expression is equal to 0:

$$(A + B + C + D)(A + \overline{B} + \overline{C} + D)(\overline{A} + \overline{B} + \overline{C} + \overline{D})$$

Construct the Truth table for given POS expression.

Converting Standard SOP to Standard POS

EXAMPLE 4–19

Convert the following SOP expression to an equivalent POS expression:

$$\overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + ABC$$

Converting Standard SOP to Standard POS

EXAMPLE 4–19

Convert the following SOP expression to an equivalent POS expression:

$$\overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + ABC$$

Solution

The evaluation is as follows:

$$000 + 010 + 011 + 101 + 111$$

Since there are three variables in the domain of this expression, there are a total of eight (2^3) possible combinations. The SOP expression contains five of these combinations, so the POS must contain the other three which are 001, 100, and 110. Remember, these are the binary values that make the sum term 0. The equivalent POS expression is

$$(A + B + \overline{C})(\overline{A} + B + C)(\overline{A} + \overline{B} + C)$$

SOP and POS from a Truth Table

From the truth table in Table determine the standard SOP and POS expression.

TABLE 4–8

Inputs			Output
A	B	C	X
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

SOP and POS from a Truth Table

From the truth table in Table determine the standard SOP and POS expression.

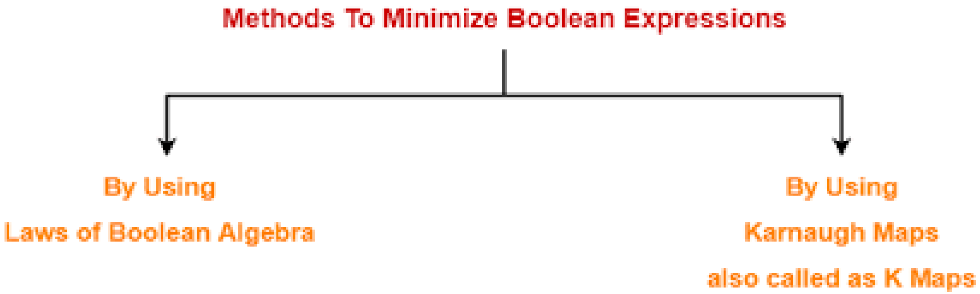
Minterm = $\Sigma(3,4,6,7)$

Maxterm = $\Pi(0,1,2,5)$

TABLE 4–8

Inputs			Output
A	*B*	*C*	*X*
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

The Karnaugh Map



K Map is an array of cells.

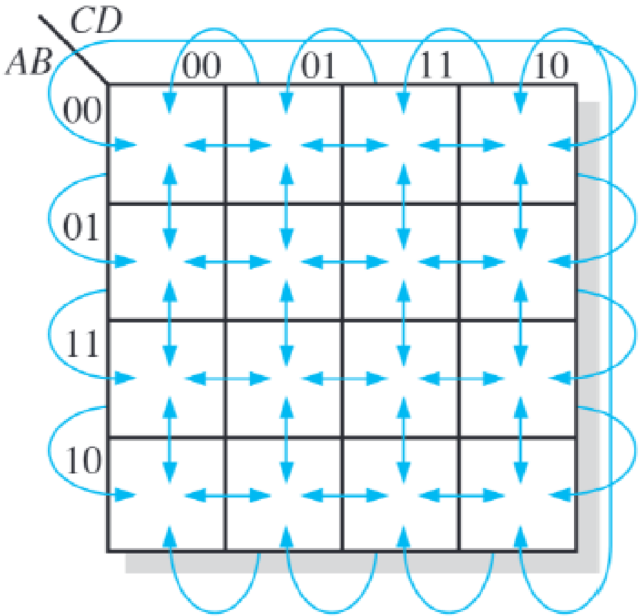
AB \ C	0	1
00		
01		
11		
10		

AB \ C	0	1
00	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$
01	$\bar{A}B\bar{C}$	$\bar{A}BC$
11	$AB\bar{C}$	ABC
10	$A\bar{B}\bar{C}$	$A\bar{B}C$

AB \ CD	00	01	11	10
00				
01				
11				
10				

AB \ CD	00	01	11	10
00	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}C\bar{D}$
01	$\bar{A}B\bar{C}\bar{D}$	$\bar{A}B\bar{C}D$	$\bar{A}BCD$	$\bar{A}BC\bar{D}$
11	$AB\bar{C}\bar{D}$	$AB\bar{C}D$	$ABCD$	$ABC\bar{D}$
10	$A\bar{B}\bar{C}\bar{D}$	$A\bar{B}\bar{C}D$	$A\bar{B}CD$	$A\bar{B}C\bar{D}$

Adjacent Cells in K Map



K-Map SOP Minimization

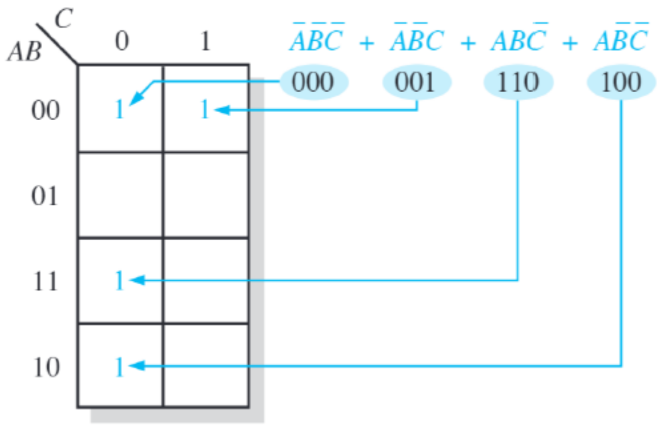


FIGURE 4–28 Example of mapping a standard SOP expression.

K-Map SOP Minimization

EXAMPLE 4–23

Map the following standard SOP expression on a Karnaugh map:

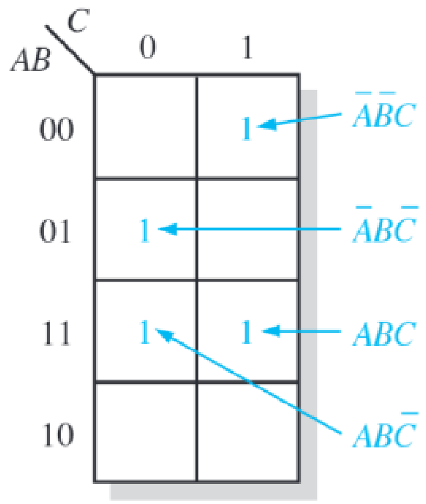
$$\overline{A}\,\overline{B}C + \overline{A}B\overline{C} + AB\overline{C} + ABC$$

K-Map SOP Minimization

EXAMPLE 4–23

Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}\,\overline{B}C + \overline{A}B\overline{C} + AB\overline{C} + ABC$$



K-Map SOP Minimization

EXAMPLE 4–24

Map the following standard SOP expression on a Karnaugh map:

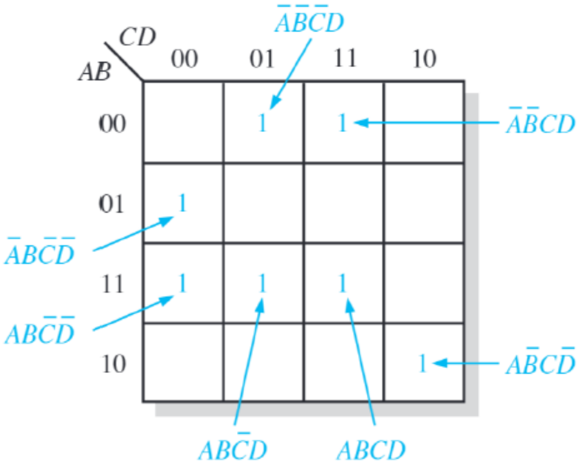
$$\overline{A}\,\overline{B}CD + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}C\overline{D}$$

K-Map SOP Minimization

EXAMPLE 4–24

Map the following standard SOP expression on a Karnaugh map:

$$\overline{A}\,\overline{B}CD + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}D + ABCD + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}C\overline{D}$$



Numerical Expansion

EXAMPLE 4–25

Map the following SOP expression on a Karnaugh map: $\overline{A} + A\overline{B} + AB\overline{C}$.

$\overline{A}$	$+ A\overline{B}$	$+ AB\overline{C}$
000	100	110
001	101	
010		
011		

$AB \backslash C$	0	1
00	1	1
01	1	1
11	1	
10	1	1

Numerical Expansion

EXAMPLE 4–26

Map the following SOP expression on a Karnaugh map:

$$\overline{B}\,\overline{C} + A\overline{B} + AB\overline{C} + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}CD$$

Numerical Expansion

EXAMPLE 4–26

Map the following SOP expression on a Karnaugh map:

$$\overline{B}\,\overline{C} + A\overline{B} + AB\overline{C} + A\overline{B}C\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + A\overline{B}CD$$

$\overline{B}\,\overline{C}$	+	$A\overline{B}$	+	$AB\overline{C}$	+	$A\overline{B}C\overline{D}$	+	$\overline{A}\,\overline{B}\,\overline{C}D$	+	$A\overline{B}CD$
0000		1000		1100		1010		0001		1011
0001		1001		1101						
1000		1010								
1001		1011								

Minterm Solution of K Map

Grouping the 1s

1. A group must contain either 1, 2, 4, 8, or 16 cells, which are all powers of two. In the case of a 3-variable map, $2^3 = 8$ cells is the maximum group.
2. Each cell in a group must be adjacent to one or more cells in that same group, but all cells in the group do not have to be adjacent to each other.
3. Always include the largest possible number of 1s in a group in accordance with rule 1.
4. Each 1 on the map must be included in at least one group. The 1s already in a group can be included in another group as long as the overlapping groups include noncommon 1s.

Minterm Solution of K Map

EXAMPLE 4–27

Group the 1s in each of the Karnaugh maps in Figure 4–33.

AB \ C	0	1
00	1	
01		1
11	1	1
10		

(a)

AB \ C	0	1
00	1	1
01	1	
11		1
10	1	1

(b)

AB \ CD	00	01	11	10
00	1	1		
01	1	1	1	1
11				
10		1	1	

(c)

AB \ CD	00	01	11	10
00	1			1
01	1	1		1
11	1	1		1
10	1		1	1

(d)

FIGURE 4–33

Minterm Solution of K Map

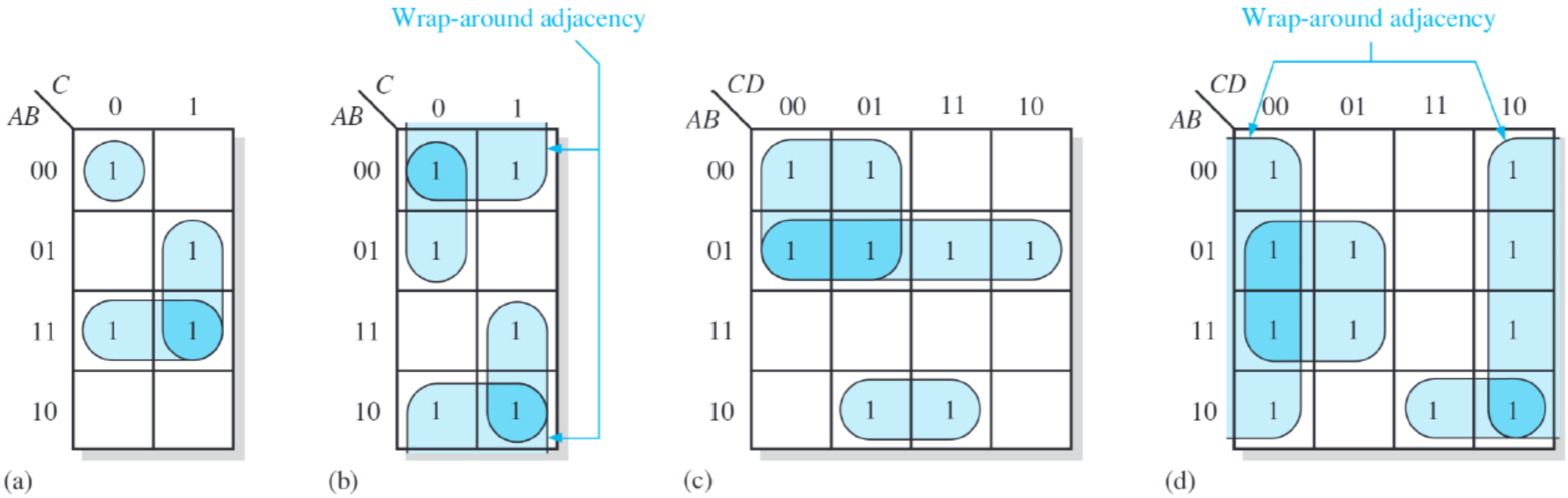


FIGURE 4–34

Minterm Solution of K Map

Determining the Equation

1. Group the cells that have 1s. Each group of cells containing 1s creates one product term composed of all variables that occur in only one form (either uncomplemented or complemented) within the group. Variables that occur both uncomplemented and complemented within the group are eliminated. These are called *contradictory variables*.
2. Determine the minimum product term for each group.
   - **(a)** For a 3-variable map:
     - **(1)** A 1-cell group yields a 3-variable product term
     - **(2)** A 2-cell group yields a 2-variable product term
     - **(3)** A 4-cell group yields a 1-variable term
     - **(4)** An 8-cell group yields a value of 1 for the expression
   - **(b)** For a 4-variable map:
     - **(1)** A 1-cell group yields a 4-variable product term
     - **(2)** A 2-cell group yields a 3-variable product term
     - **(3)** A 4-cell group yields a 2-variable product term
     - **(4)** An 8-cell group yields a 1-variable term
     - **(5)** A 16-cell group yields a value of 1 for the expression
3. When all the minimum product terms are derived from the Karnaugh map, they are summed to form the minimum SOP expression.

Minterm Solution of K Map

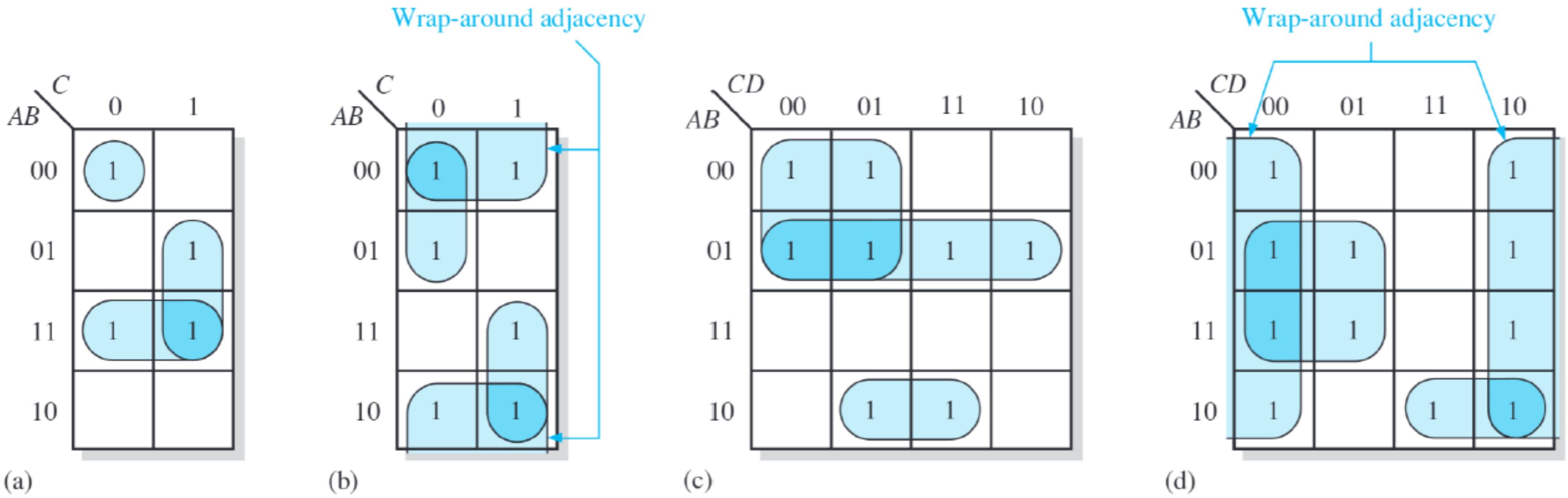
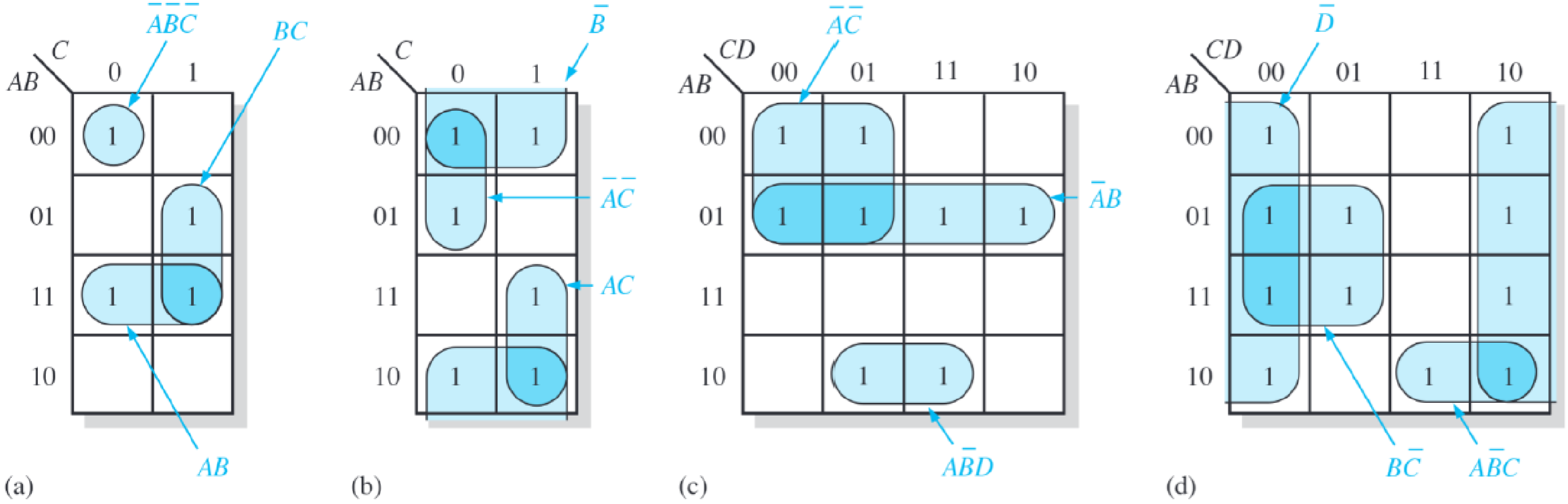


FIGURE 4–34

Minterm Solution of K Map



Minterm Solution of K Map

EXAMPLE 4–31

Use a Karnaugh map to minimize the following SOP expression:

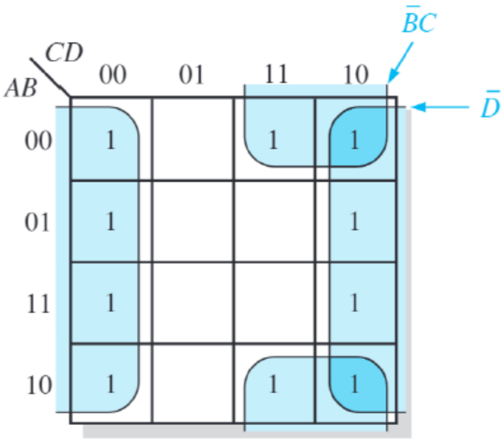
$$\overline{B}\,\overline{C}\,\overline{D} + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + A\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}BC\overline{D} + ABC\overline{D} + A\overline{B}C\overline{D}$$

Minterm Solution of K Map

EXAMPLE 4–31

Use a Karnaugh map to minimize the following SOP expression:

$$\overline{B}\,\overline{C}\,\overline{D} + \overline{A}B\overline{C}\,\overline{D} + AB\overline{C}\,\overline{D} + \overline{A}\,\overline{B}CD + A\overline{B}CD + \overline{A}\,\overline{B}C\overline{D} + \overline{A}BC\overline{D} + ABC\overline{D} + A\overline{B}C\overline{D}$$



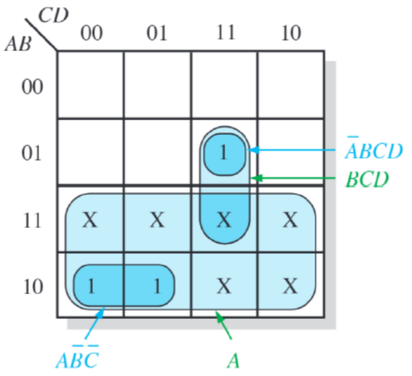
$$\overline{D} + \overline{B}C$$

Don't Care Conditions

Inputs				Output
A	*B*	*C*	*D*	*Y*
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Don't cares

(a) Truth table



(b) Without "don't cares" $Y = A\bar{B}\bar{C} + \bar{A}BCD$
With "don't cares" $Y = A + BCD$

FIGURE 4–40 Example of the use of "don't care" conditions to simplify an expression.

K-Map POS Minimization

EXAMPLE 4–33

Map the following standard POS expression on a Karnaugh map:

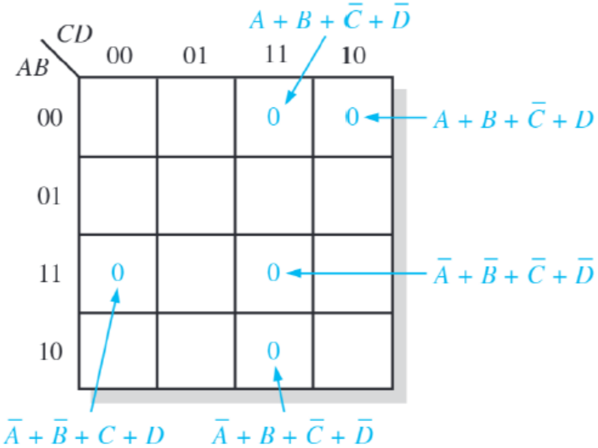
$$(\overline{A} + \overline{B} + C + D)(\overline{A} + B + \overline{C} + \overline{D})(A + B + \overline{C} + D)(\overline{A} + \overline{B} + \overline{C} + \overline{D})(A + B + \overline{C} + \overline{D})$$

K-Map POS Minimization

EXAMPLE 4–33

Map the following standard POS expression on a Karnaugh map:

$$(\overline{A} + \overline{B} + C + D)(\overline{A} + B + \overline{C} + \overline{D})(A + B + \overline{C} + D)(\overline{A} + \overline{B} + \overline{C} + \overline{D})(A + B + \overline{C} + \overline{D})$$



K-Map POS Minimization

EXAMPLE 4–34

Use a Karnaugh map to minimize the following standard POS expression:

$$(A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + \overline{B} + C)$$

Also, derive the equivalent SOP expression.

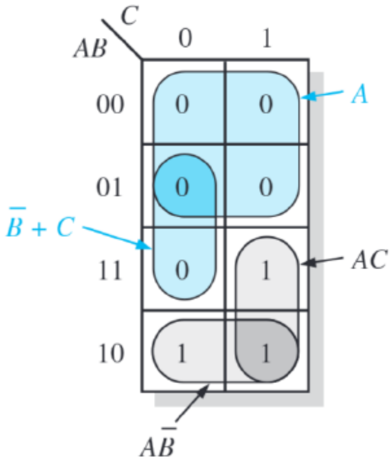
K-Map POS Minimization

EXAMPLE 4–34

Use a Karnaugh map to minimize the following standard POS expression:

$$(A + B + C)(A + B + \overline{C})(A + \overline{B} + C)(A + \overline{B} + \overline{C})(\overline{A} + \overline{B} + C)$$

Also, derive the equivalent SOP expression.



$$AC + A\overline{B} = A(\overline{B} + C)$$

K-Map POS Minimization

EXAMPLE 4–35

Use a Karnaugh map to minimize the following POS expression:

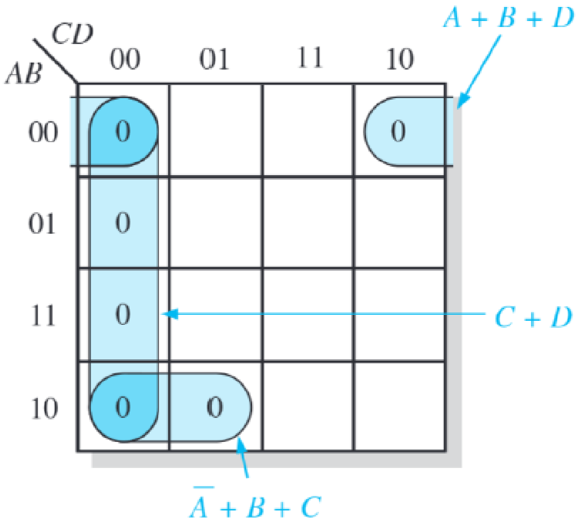
$$(B + C + D)(A + B + \overline{C} + D)(\overline{A} + B + C + \overline{D})(A + \overline{B} + C + D)(\overline{A} + \overline{B} + C + D)$$

K-Map POS Minimization

EXAMPLE 4–35

Use a Karnaugh map to minimize the following POS expression:

$$(B + C + D)(A + B + \overline{C} + D)(\overline{A} + B + C + \overline{D})(A + \overline{B} + C + D)(\overline{A} + \overline{B} + C + D)$$



Converting Between SOP and POS

EXAMPLE 4–36

Using a Karnaugh map, convert the following standard POS expression into a minimum POS expression, a standard SOP expression, and a minimum SOP expression.

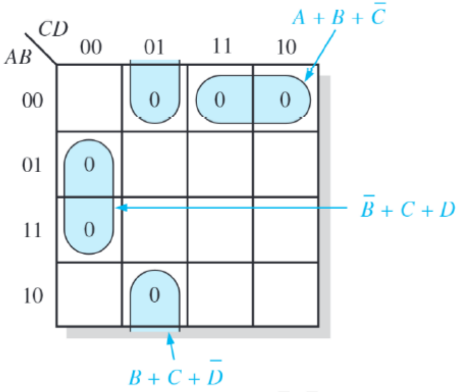
$$(\overline{A} + \overline{B} + C + D)(A + \overline{B} + C + D)(A + B + C + \overline{D})(A + B + \overline{C} + \overline{D})(\overline{A} + B + C + \overline{D})(A + B + \overline{C} + D)$$

Converting Between SOP and POS

EXAMPLE 4–36

Using a Karnaugh map, convert the following standard POS expression into a minimum POS expression, a standard SOP expression, and a minimum SOP expression.

$$(\overline{A} + \overline{B} + C + D)(A + \overline{B} + C + D)(A + B + C + \overline{D})(A + B + \overline{C} + \overline{D})(\overline{A} + B + C + \overline{D})(A + B + \overline{C} + D)$$



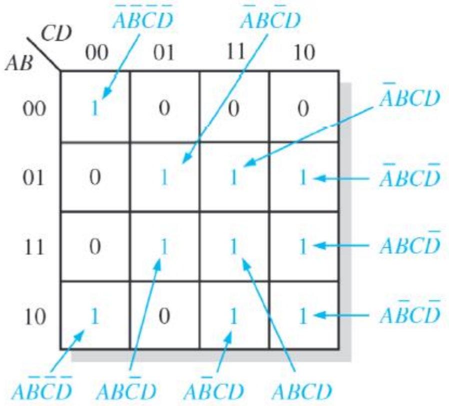
(a) Minimum POS: $(A + B + \overline{C})(\overline{B} + C + D)(B + C + \overline{D})$

Converting Between SOP and POS

EXAMPLE 4–36

Using a Karnaugh map, convert the following standard POS expression into a minimum POS expression, a standard SOP expression, and a minimum SOP expression.

$$(\bar{A} + \bar{B} + C + D)(A + \bar{B} + C + D)(A + B + C + \bar{D})(A + B + \bar{C} + \bar{D})(\bar{A} + B + C + \bar{D})(A + B + \bar{C} + D)$$



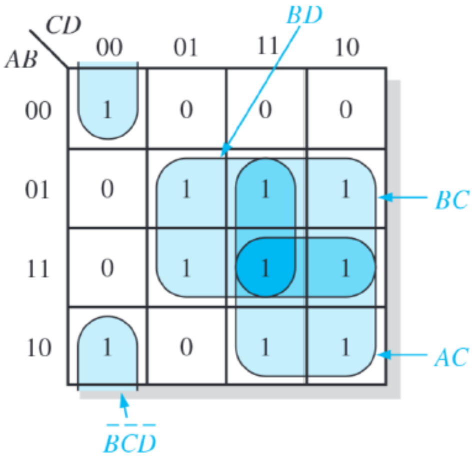
(b) Standard SOP:
$\bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BCD + \bar{A}BC\bar{D} + ABC\bar{D} + A\bar{B}C\bar{D} +$
$A\bar{B}\bar{C}\bar{D} + AB\bar{C}D + A\bar{B}CD + ABCD$

Converting Between SOP and POS

EXAMPLE 4–36

Using a Karnaugh map, convert the following standard POS expression into a minimum POS expression, a standard SOP expression, and a minimum SOP expression.

$$(\overline{A} + \overline{B} + C + D)(A + \overline{B} + C + D)(A + B + C + \overline{D})(A + B + \overline{C} + \overline{D})(\overline{A} + B + C + \overline{D})(A + B + \overline{C} + D)$$



(c) Minimum SOP: $AC + BC + BD + \overline{B}\overline{C}\overline{D}$

Practice Exercises

Section 4–9 Karnaugh Map SOP Minimization

40. Use a Karnaugh map to find the minimum SOP form for each expression:

(a) $\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + A\overline{B}C$ **(b)** $AC(\overline{B} + C)$

(c) $\overline{A}(BC + B\overline{C}) + A(BC + B\overline{C})$ **(d)** $\overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C} + \overline{A}B\overline{C} + AB\overline{C}$

41. Use a Karnaugh map to simplify each expression to a minimum SOP form:

(a) $\overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + \overline{A}BC + AB\overline{C}$ **(b)** $AC[\overline{B} + B(B + \overline{C})]$

(c) $DE\overline{F} + \overline{D}E\overline{F} + \overline{D}\,\overline{E}\,\overline{F}$

42. Expand each expression to a standard SOP form:

(a) $AB + A\overline{B}C + ABC$ **(b)** $A + BC$

(c) $A\overline{B}\,\overline{C}D + AC\overline{D} + B\overline{C}D + \overline{A}BC\overline{D}$ **(d)** $A\overline{B} + A\overline{B}\,\overline{C}D + CD + B\overline{C}D + ABCD$

43. Minimize each expression in Problem 42 with a Karnaugh map.

44. Use a Karnaugh map to reduce each expression to a minimum SOP form:

(a) $A + B\overline{C} + CD$

(b) $\overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}D + ABCD + ABC\overline{D}$

(c) $\overline{A}B(\overline{C}\,\overline{D} + \overline{C}D) + AB(\overline{C}\,\overline{D} + \overline{C}D) + A\overline{B}\,\overline{C}D$

(d) $(\overline{A}\,\overline{B} + A\overline{B})(CD + C\overline{D})$

(e) $\overline{A}\,\overline{B} + A\overline{B} + \overline{C}\,\overline{D} + C\overline{D}$

Practice Exercises

Section 4–10 Karnaugh Map POS Minimization

48. Use a Karnaugh map to find the minimum POS for each expression:

(a) $(A + B + C)(\overline{A} + \overline{B} + \overline{C})(A + \overline{B} + C)$

(b) $(X + \overline{Y})(\overline{X} + Z)(X + \overline{Y} + \overline{Z})(\overline{X} + \overline{Y} + Z)$

(c) $A(B + \overline{C})(\overline{A} + C)(A + \overline{B} + C)(\overline{A} + B + \overline{C})$

49. Use a Karnaugh map to simplify each expression to minimum POS form:

(a) $(A + \overline{B} + C + \overline{D})(\overline{A} + B + \overline{C} + D)(\overline{A} + \overline{B} + \overline{C} + \overline{D})$

(b) $(X + \overline{Y})(W + \overline{Z})(\overline{X} + \overline{Y} + \overline{Z})(W + X + Y + Z)$

50. For the function specified in Table 4–16, determine the minimum POS expression using a Karnaugh map.

51. Determine the minimum POS expression for the function in Table 4–17.

52. Convert each of the following POS expressions to minimum SOP expressions using a Karnaugh map:

(a) $(A + \overline{B})(A + \overline{C})(\overline{A} + \overline{B} + C)$

(b) $(\overline{A} + B)(\overline{A} + \overline{B} + \overline{C})(B + \overline{C} + D)(A + \overline{B} + C + \overline{D})$