SOLUTION KEY

MVC - QUIZ 01

[MVC-QUIZ-01] February 26, 2024

National University

Of Computer & Emerging Sciences, Karachi-Campus

Multivariable Calculus QUIZ-1 [Max Marks:10]

Instructor: Dr. Nazish Kanwal **Sections: BCS-2B, BCS-2F, BCY-2A, BCY-2B**

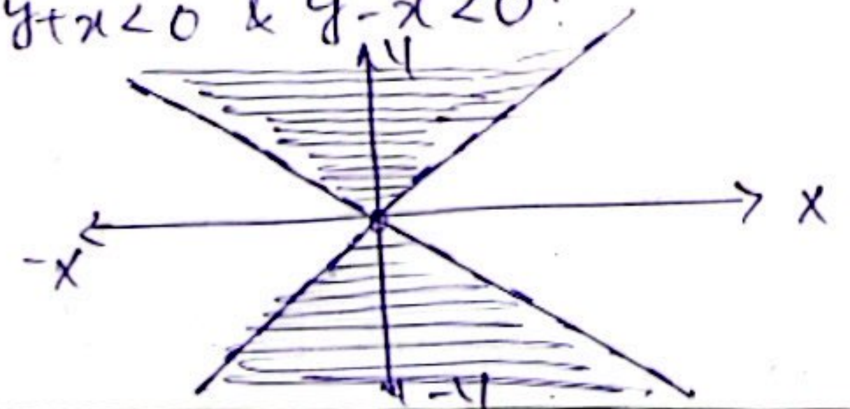
NAME:____________________ **Roll No:**________________

Q#01 [3 marks] Find and sketch the domain of $f(x,y) = \frac{1}{\sqrt{y^2-x^2}}$.

$y^2 - x^2 > 0 \Rightarrow (y+x)(y-x) > 0 \Rightarrow y+x > 0 \;\&\; y-x > 0$

OR

$y+x < 0 \;\&\; y-x < 0$:



Q#02 [3 marks] Determine whether the limit $\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x^2+y^2}$ exists if it does find the limit. If not, explain why not?

path along $y = mx$.

$$\lim_{x\to 0} \frac{x^3(1-m^3)}{x^2(1+m^2)} = \lim_{x\to 0} x\frac{(1-m^3)}{1+m^2} = 0.$$

path along $y = Kx^2$

$$\lim_{x\to 0} \frac{\cancel{x^3}(1-K^3x^3)}{\cancel{x^2}(1+K^2x^2)} = \lim_{x\to 0} x\frac{(1-K^3x^3)}{1+K^2x^2} = 0$$

Hence limit exist and

$$\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x^2+y^2} = 0.$$

Alternatively

Put $x = r\cos\theta$, $y = r\sin\theta$.

$$\lim_{r\to 0} \frac{r^{\cancel{3}}(\cos^3\theta - \sin^3\theta)}{\cancel{r^2}(1)} = \lim_{r\to 0} r(\cos^3\theta - \sin^3\theta)$$

$$= 0.$$

Q#03 [4 marks] Find $\frac{\partial z}{\partial y}$, for the implicit function $y^2 + z\cos(xyz) = 2$.

$$\frac{\partial}{\partial y}\{y^2 + z\cos(xyz)\} = \frac{\partial}{\partial y}(2)$$

$$\Rightarrow 2y + \frac{\partial z}{\partial y}\cos(xyz) + \{z \times -\sin(xyz) \times \frac{\partial}{\partial y}(xyz)\} = 0$$

$$\Rightarrow 2y + \frac{\partial z}{\partial y}\cos(xyz) - \{z\sin(xyz)\cdot\{xz + xy\frac{\partial z}{\partial y}\}\} = 0$$

$$\Rightarrow 2y + \frac{\partial z}{\partial y}\cos(xyz) - xz^2(\sin xyz) - xyz\sin(xyz)\frac{\partial z}{\partial y} = 0$$

$$\Rightarrow \boxed{\frac{\partial z}{\partial y} = \frac{-\{2y - xz^2\sin(xyz)\}}{\cos(xyz) - xyz\sin(xyz)}}$$

Alternatively

$$\frac{\partial z}{\partial y} = \frac{-\partial f/\partial y}{\partial f/\partial z}, \quad f(x,y,z) = y^2 + z\cos(xyz) - 2$$

$$\partial f/\partial y = 2y - xz^2\sin(xyz)$$

$$\partial f/\partial z = \cos(xyz) - xyz\sin(xyz)$$

So $$\frac{\partial z}{\partial y} = -\frac{\{2y - xz^2\sin(xyz)\}}{\cos(xyz) - xyz\sin(xyz)}$$ Ans.