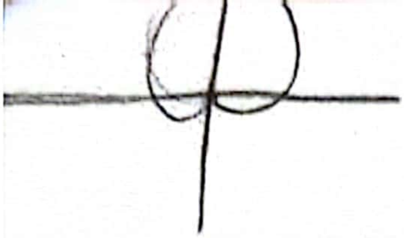
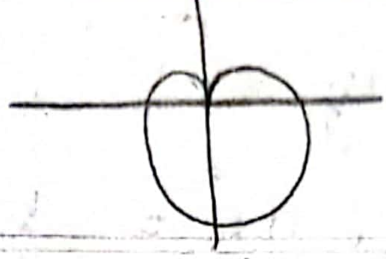


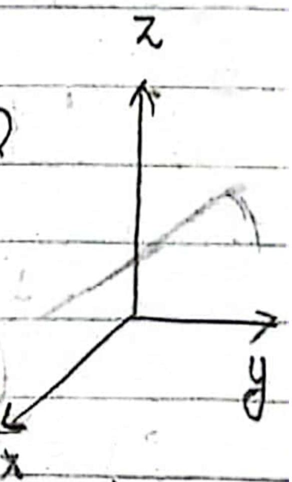


$r = a - a\sin\theta$

$r = a + a\sin\theta$

Ex # 14.4:

$$S = \iint_R \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\; dA$$



1) $y^2 + z^2 = 9$, that is above $0 \le x \le 2$
$-3 \le y \le 3$

$z^2 = 9 - y^2 \Rightarrow z = \sqrt{9 - y^2}$

(1) find z

$\frac{\partial z}{\partial x} = 0$, $\frac{\partial z}{\partial y} = \frac{1}{2}(9 - y^2)^{-1/2} \cdot (-2y)$

$\Rightarrow \frac{\partial z}{\partial y} = \frac{-y}{\sqrt{9 - y^2}}$

$$\boxed{\Rightarrow \left(\frac{\partial z}{\partial y}\right)^2 = \frac{y^2}{9 - y^2}}$$

$$\int_{-3}^{3}\int_{0}^{2} \sqrt{\frac{y^2}{9 - y^2} + 1}\; dx\, dy$$

$$\sqrt{\frac{y^2 + 9 - y^2}{9 - y^2}} \Rightarrow \frac{3}{\sqrt{9 - y^2}}$$

$$3\int_{-3}^{3}\int_{0}^{2} \frac{1}{\sqrt{9 - y^2}}\; dx\, dy$$

$$\Rightarrow 3\int_{-3}^{3} \frac{1}{\sqrt{9 - y^2}}\,[2 - 0]\, dy \Rightarrow 6\int_{-3}^{3} \frac{1}{\sqrt{9 - y^2}}\, dy$$

$\Rightarrow 6\int_{-3}^{3}\frac{1}{\sqrt{(3)^2-(y^2)}}\,dy \Rightarrow 6\int_{-3}^{3}\frac{1}{\sqrt{(3)^2\left[1-\left(\frac{y}{3}\right)^2\right]}}\,dy$

$\frac{\cancel{6}^{\,2}}{\cancel{3}_{\,1}}\int_{-3}^{3}\frac{1}{\sqrt{1-\left(\frac{y}{3}\right)^2}}\,dy$

$\Rightarrow 2\left[\sin^{-1}\left(\frac{y}{3}\right)\right]_{-3}^{3}$

$\Rightarrow 2\left[\sin^{-1}\left(\frac{3}{3}\right)-\sin^{-1}\left(\frac{-3}{3}\right)\right]$

$\Rightarrow 2\left[\sin^{-1}(1)+\sin^{-1}(1)\right] \Rightarrow \boxed{2\pi}$ → Answer

$\therefore \int\frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x)$

Q#2: The portion of plane $2x+2y+z=8$ in first octant.

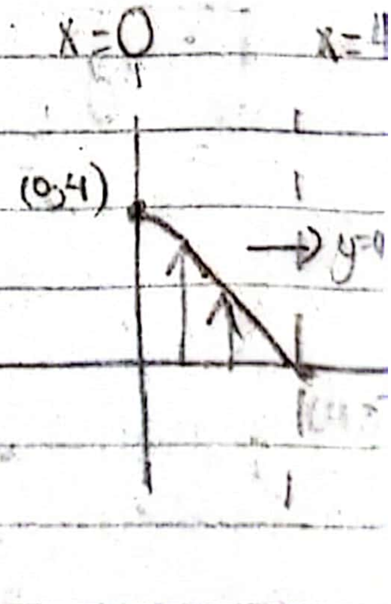
$z = 8-2x-2y$ — (1)

Put $z=0$ → xy plane

$0 = 8-2x-2y$

$+2y = 8-2x$

$\boxed{y = 4-x}$



$0 \le x \le 4$

$0 \le y \le 4-x$

$\int_0^4\int_0^{4-x} dy\,dx$

$\frac{\partial z}{\partial x} = -2$, $\frac{\partial z}{\partial y} = -2$

$$\int_0^4\int_0^{4-x}\sqrt{4+4+1}\,dy\,dx \Rightarrow 3\int_0^4\int_0^{4-x} dy\,dx$$

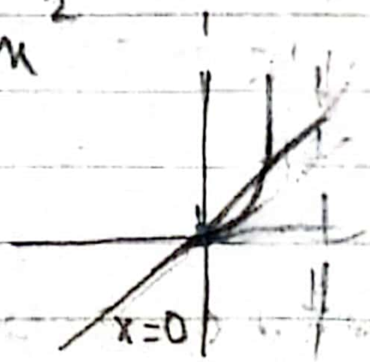
$$\Rightarrow 3\int_0^4 (4-x)\,dx \Rightarrow 3\int_0^4 4\,dx - 3\int_0^4 x\,dx$$

$$\Rightarrow 12\int_0^4 dx - \frac{3}{2}\left[x^2\right]_0^4$$

$$\Rightarrow 12[4-0] - \frac{3}{2}[16-0] \Rightarrow 48 - \frac{3}{2}[16]$$

$\Rightarrow \boxed{24}$ sq units.

$x = 1$



(3) $z^2 = 4x^2 + 4y^2$, $y = x$, $y = x^2$

$z = \sqrt{4x^2+4y^2}$

$$\int_0^1\int_{x^2}^{x} R\,dy\,dx$$

$x = x^2$

$x - x^2 = 0$

$x(1-x) =$

$x = 0$.

$1 - x = 0$

$x =$

$$\frac{\partial z}{\partial x} = \frac{1}{2}(4x^2+4y^2)^{-1/2}\cdot(8x) = \frac{4x}{\sqrt{4x^2+4y^2}}$$

$$\frac{16x^2}{4x^2+4y^2} \Rightarrow \frac{4x^2}{x^2+y^2} = \left(\frac{\partial z}{\partial x}\right)^2$$

$$\left(\frac{\partial z}{\partial y}\right)^2 = \frac{4y^2}{x^2+y^2}$$

$$\int_0^1\int_{x^2}^{x}\sqrt{\frac{4x^2}{x^2+y^2} + \frac{4y^2}{x^2+y^2} + 1}\;dy\,dx$$

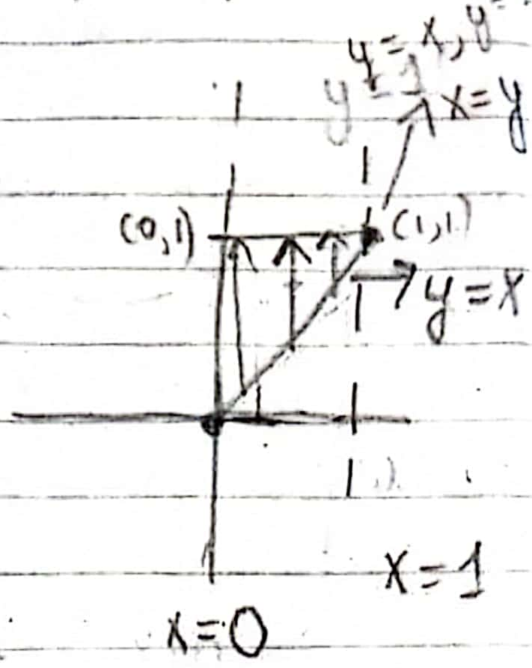
$$\int_0^1\int_{x^2}^{x}\sqrt{\frac{4x^2+4y^2+x^2+y^2}{x^2+y^2}}\,dy\,dx$$

$$\sqrt{\frac{5x^2+5y^2}{x^2+y^2}} \Rightarrow \sqrt{\frac{5(x^2+y^2)}{\cancel{x^2+y^2}}}$$

$$\sqrt{5}\int_0^1\int_{x^2}^{x} dy\,dx \to \text{solve.}$$

(4) $z = 2x + y^2$,

$$\int_0^1\int_x^1 dy\,dx$$



$$\frac{\partial z}{\partial x} = 2,\quad \frac{\partial z}{\partial y} = 2y$$

$$\left(\frac{\partial z}{\partial x}\right)^2 = 4,\quad \left(\frac{\partial z}{\partial y}\right)^2 = 4y^2$$

$$\int_0^1\int_0^y \sqrt{4y^2+4+1}\,dx\,dy \to \underline{\text{solve}}$$

(0,0)(1,1)

$m = 1$

$y = x + c$

when (0,0) →

$0 = 0 + c$

$\boxed{c = 0}$

when (1,1)

$1 = 1 + c$

$\boxed{c = 0}$

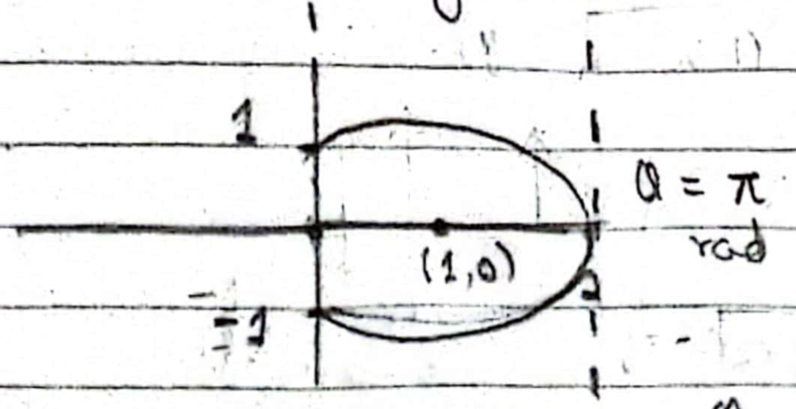
$y = x + 0$

$\boxed{y = x}$

S7-10 → Polar Cordinate

(51) $z = \sqrt{x^2+y^2}$, that lies $x^2+y^2 = 2x$

$$x^2 - 2x + y^2 = 0$$
$$(x)^2 - 2(x)(1) + (1)^2 + y^2 = (1)^2$$
$$(x-1)^2 + y^2 = 1$$



$x = 0$ $\quad x = 2\cos\theta$

$h = 1,\ k = 0$

radius $= 1$

$$r^2 = 2r\cos\theta$$
$$\underline{r = 2\cos\theta}$$

$$\frac{\partial z}{\partial x} = \frac{2x}{2\sqrt{x^2+y^2}} = \frac{x}{\sqrt{x^2+y^2}}, \quad \frac{\partial z}{\partial y} = \frac{y}{\sqrt{x^2+y^2}}$$

$$\left(\frac{\partial z}{\partial x}\right)^2 = \frac{x^2}{x^2+y^2}, \quad \left(\frac{\partial z}{\partial y}\right)^2 = \frac{y^2}{x^2+y^2}$$

$$\Rightarrow \iint_R \sqrt{\frac{x^2}{x^2+y^2} + \frac{y^2}{x^2+y^2} + 1}\ dA$$

$$\frac{x^2+y^2+x^2+y^2}{x^2+y^2} \Rightarrow \frac{2x^2+2y^2}{x^2+y^2}$$

$$\Rightarrow \iint_R \sqrt{2}\ dA$$

$$\sqrt{2}\iint_R dA \Rightarrow dA = r\,dr\,d\theta$$

$$\sqrt{2}\int_0^{\pi}\int_0^{2\cos\theta} r\,dr\,d\theta$$

$$\Rightarrow \frac{\sqrt{2}}{2}\int_0^{\pi}\left[r^2\right]_0^{2\cos\theta} d\theta$$

$$\frac{\sqrt{2}}{2}\int_0^{\pi} 4\cos^2\theta \, d\theta$$

$$\Rightarrow 2\sqrt{2}\int_0^{\pi}\cos^2\theta \, d\theta$$

$$2\sqrt{2}\int_0^{\pi} d\theta - 2\sqrt{2}\int_0^{\pi}\sin^2\theta \, d\theta$$

$$2\sqrt{2}[\pi] - 2\sqrt{2}\left[\frac{\theta}{2} - \frac{\sin(2\theta)}{4}\right]_0^{\pi}$$

$$2\sqrt{2}(\pi) - 2\sqrt{2}\left[\frac{\pi}{2} - \frac{\sin(2\pi)}{4} + \frac{\sin(0)}{4}\right]$$

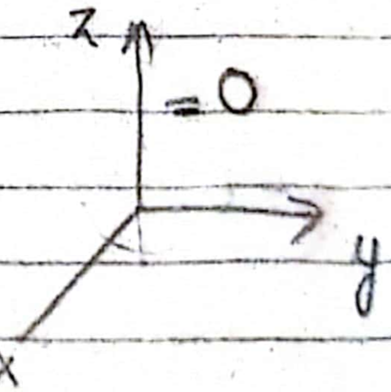
$$2\sqrt{2}(\pi) - \sqrt{2}(\pi) - 0 + 0$$

$$\Rightarrow \sqrt{2}\pi[2-1] = \boxed{\sqrt{2}\pi}$$

Answer.

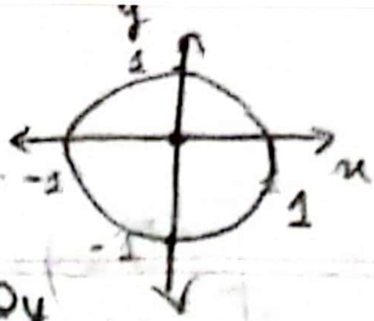
(6) $z = 1 - x^2 - y^2$, that is above xy plane

Above xy Plane:
At xy plane $z = 0$



$x^2 + y^2 = 1$ - (1) $\rightarrow$ $r = 1$

(0,0)



$z = 1 - x^2 - y^2$

$\frac{\partial z}{\partial x} = -2x$, $\frac{\partial z}{\partial y} = -2y$

$\left(\frac{\partial z}{\partial x}\right)^2 = 4x^2$ $\quad$ $\left(\frac{\partial z}{\partial y}\right)^2 = 4y^2$

$\Rightarrow \iint_R \sqrt{4x^2+4y^2+1}\; dA$ $\quad$ $\because r^2 = x^2 + y^2$

$\Rightarrow \iint_R \sqrt{4(x^2+y^2)+1}\; dA \Rightarrow \iint_R \sqrt{4r^2+1}\; dA$

$\Rightarrow \int_0^{2\pi}\int_0^1 \sqrt{4r^2+1}\; r\,dr\,d\theta$ $\rightarrow$ dA - (1)

$I = \int_0^1 \sqrt{4r^2+1}\,.\, r\,dr$

$I = \frac{1}{8}\int_0^1 \sqrt{4r^2+1}\,.\,8r\,dr \Rightarrow \frac{1}{8}\left[\frac{(4r^2+1)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1$

$\Rightarrow \frac{1}{\cancel{8}\,4}\cdot\frac{\cancel{2}}{3}\left[(4r^2+1)^{3/2}\right]_0^1$

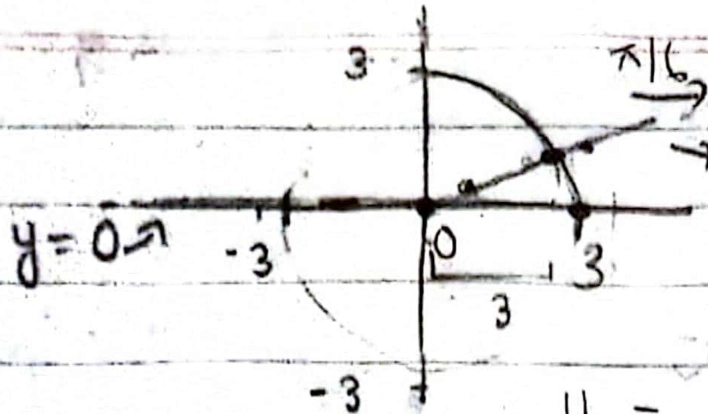
$\Rightarrow \frac{1}{12}\left[5^{3/2} - 1\right]$ $\rightarrow$ In (1)

$\frac{1}{12}\left[5^{3/2} - 1\right]\int_0^{2\pi} d\theta$

$\Rightarrow \frac{\pi(5^{3/2} - 1)}{6}$ Answer.

(7) $z = xy$, $y = x/\sqrt{3}$, $y = 0$ and

$x^2 + y^2 = 9 \to r = \pm 3$



$\pi/6 \to$ 1st quad

$\to y = x/\sqrt{3}$

$r \to 0 \to 3$

$$y = \frac{x}{\sqrt{3}}$$

$$\cancel{r}\sin\theta = \frac{\cancel{r}\cos\theta}{\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \pi/6$$

$$\theta \to 0 \to \pi/6$$

$$\frac{\partial z}{\partial x} = y , \quad \frac{\partial z}{\partial y} = x$$

$$\left(\frac{\partial z}{\partial x}\right)^2 = y^2 , \quad \left(\frac{\partial z}{\partial y}\right)^2 = x^2$$

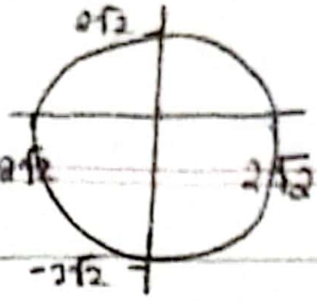
$$\iint_R \sqrt{y^2 + x^2 + 1}\; dA$$

$$\int_0^{\pi/6}\int_0^3 \sqrt{r^2+1}\; r\, dr\, d\theta$$

$\to$ now solve

(8) $2z = x^2 + y^2$ that is inside $x^2 + y^2 = 8$

$$z = \frac{x^2 + y^2}{2}$$



$$\frac{\partial z}{\partial x} = \frac{1}{2}[2x] = x, \quad \frac{\partial z}{\partial y} = y$$

$$\left(\frac{\partial z}{\partial x}\right)^2 = x^2, \quad \left(\frac{\partial z}{\partial y}\right)^2 = y^2$$

$r \to 0 \to 2\sqrt{2}$

$\theta \to 0 \to 2\pi$

whole

$$\iint_R \sqrt{x^2 + y^2 + 1}\, dA$$

$$\Rightarrow \int_0^{2\pi} \int_0^{2\sqrt{2}} \sqrt{r^2 + 1}\; r\, dr\, d\theta$$

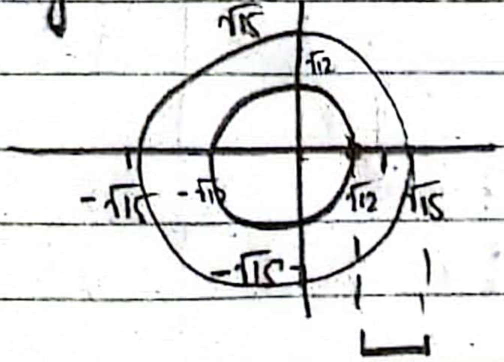
$\rightarrow$ solve

(9) $x^2 + y^2 + z^2 = 16$

when $z = 1$, $x^2 + y^2 = 15$, $z = 2$, $x^2 + y^2 = 12$

$$z = \sqrt{16 - x^2 + y^2}$$

It's between the circle so



$r \to \sqrt{12} \to \sqrt{15}$

$\theta \to 0 \to 2\pi$

$$\int_0^{2\pi} \int_{\sqrt{12}}^{\sqrt{15}} S\; r\, dr\, d\theta \quad (1)$$

$-1/2$

$$\Rightarrow \frac{\partial z}{\partial x} = \frac{1}{\cancel{2}}(16 - x^2 - y^2) \cdot (-\cancel{2}x$$

$$\frac{\partial z}{\partial x} = \frac{-x}{\sqrt{16 - x^2 - y^2}}$$

$$\frac{\partial z}{\partial y} = \frac{-y}{\sqrt{16 - x^2 - y^2}}$$

$$\left(\frac{\partial z}{\partial x}\right)^2 = \frac{x^2}{16 - x^2 - y^2}, \quad \left(\frac{\partial z}{\partial y}\right)^2 = \frac{y^2}{16 - x^2 - y^2}$$

$$\int_0^{2\pi}\int_{\sqrt{12}}^{\sqrt{15}} \sqrt{\frac{x^2}{16-x^2-y^2}+\frac{y^2}{16-x^2-y^2}+1}\; r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_{\sqrt{12}}^{\sqrt{15}} \sqrt{\frac{\cancel{x^2}+\cancel{y^2}+16-\cancel{x^2}-\cancel{y^2}}{16-x^2-y^2}}\; r\,dr\,d\theta$$

$$4\int_0^{2\pi}\int_{\sqrt{12}}^{\sqrt{15}} \frac{r}{\sqrt{16-r^2}}\,dr\,d\theta$$

$$\Rightarrow 4\int^{2\pi}\int^{\sqrt{15}}_{\sqrt{12}} \frac{r}{\sqrt{16-r^2}}\,dr\,d\theta$$

$$\Rightarrow 4\int_0^{2\pi}\int_{\sqrt{12}}^{\sqrt{15}} \left[(16-r^2)^{-1/2}\cdot r\,dr\right] d\theta$$

$$\Rightarrow \frac{-4}{\cancel{2}}\int_0^{2\pi}\left[\frac{(16-r^2)^{1/2}}{\frac{1}{\cancel{2}}}\right]_{\sqrt{12}}^{\sqrt{15}} d\theta$$

$$-4\int_0^{2\pi}(-1)\,d\theta \Rightarrow 4\int_0^{2\pi} d\theta$$

$$\Rightarrow 4\,[2\pi-0] = \boxed{8\pi}$$

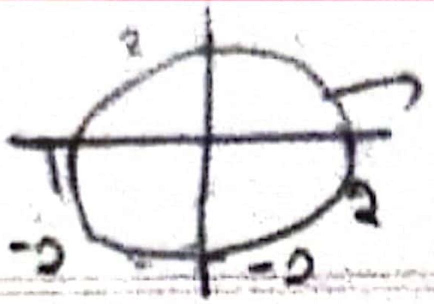
(10) $x^2+y^2+z^2=8$, $z=\sqrt{x^2+y^2}$

$z=\sqrt{8-x^2-y^2}$, $z=\sqrt{x^2+y^2}$

$8-x^2-y^2 = x^2+y^2$

$8 = 2x^2+2y^2$

$$\boxed{x^2+y^2=4}$$



Inside this thing

$r \Rightarrow 0 \to 2$

$\theta \to 0 \to 2\pi$

$$\int_0^{2\pi}\int_0^{2} S\, r\,dr\,d\theta \quad - ①$$

$$\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{8-x^2-y^2}}(-2x) = \frac{-x}{\sqrt{8-x^2-y^2}}, \quad \frac{\partial z}{\partial y} = \frac{-y}{\sqrt{8-x^2-}}$$

$$\left(\frac{\partial z}{\partial x}\right)^2 = \frac{x^2}{8-x^2-y^2}, \quad \left(\frac{\partial z}{\partial y}\right)^2 = \frac{y^2}{8-x^2-y^2}$$

$$\int_0^{2\pi}\int_0^{2} \sqrt{\frac{\cancel{x^2}+\cancel{y^2}+8-\cancel{x^2}-\cancel{y^2}}{8-x^2-y^2}}\; r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^{2} \sqrt{\frac{8}{8-r^2}}\; r\,dr\,d\theta \qquad \frac{1}{(8-r^2)^{1/2}}$$

$$2\sqrt{2}\int_0^{2\pi}\int_0^{2} (8-r^2)^{-1/2}\, r\,dr\,d\theta$$

$$-\frac{2\sqrt{2}}{\cancel{2}}\int_0^{2\pi}\left[2(8-r^2)^{1/2}\right]_0^{2} d\theta$$

$$-2\sqrt{2}\int_0^{2\pi}\left[(-0.828)\,d\theta\right]$$ Has ceb caddy

$\Rightarrow 2.819\,[2\pi - 0]\,\pi$

$\Rightarrow -\boxed{14.63\,\pi}$ Answer.