SOLUTION KEY Assignment #02

Question #01

Part(a)

$$\int_1^2 \int_4^6 \frac{x}{y^2}\,dx\,dy = \int_1^2 \left[\frac{x^2}{2y^2}\right]_4^6 dy$$

$$= \frac{1}{2}\int_1^2 \frac{1}{y^2}(36-24)\,dy$$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{12}^{6} \times y^{-1}}{-1}\Bigg]_1^2$$

$$= -6\left(\frac{1}{2} - 1\right) = 3$$

Ans.

Part (b)

$$\int_c^d \int_a^b x^2 + y^2\,dx\,dy = \int_c^d \left[\frac{x^3}{3} + y^2 x\right]_a^b dy$$

$$= \int_c^d \left\{\left(\frac{b^3-a^3}{3}\right) + y^2(b-a)\right\}dy$$

$$= \left[\frac{b^3-a^3}{3}y + \frac{y^3}{3}(b-a)\right]_c^d$$

$$= \frac{1}{3}\left\{(b^3-a^3)(d-c) + (d^3-c^3)(b-a)\right\}$$

Ans

Part (c)

$$\int_0^1\int_1^2 \frac{x e^x}{y}\, dy\, dx = \int_0^1 \left[x e^x \ln y\right]_1^2 dx$$

$$= \int_0^1 x e^x (\ln 2 - \cancel{\ln 1}^{\,0})\, dx.$$

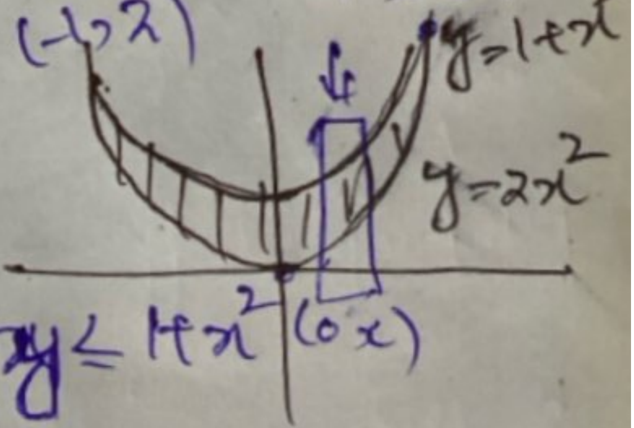
$$= \ln 2 \left[x e^x - e^x\right]_0^1$$

$$= \ln 2 \cdot (\cancel{e} - \cancel{e} - 0 + 1)$$

$$= \ln 2$$

Ans (b?)

Question #03



$$\iint_D (x + 2y)\, dA$$

D: $-1 \le x \le 1$

~~$0 \le y \le 2$~~, $2x^2 \le y \le 1+x^2$

$$\int_{-1}^{1}\int_{2x^2}^{1+x^2} (x+2y)\, dy\, dx.$$

$$\int_{-1}^{1} \left[xy + y^2\right]_{2x^2}^{1+x^2} dx = \int_{-1}^{1} \left[x(1+x^2) + (1+x^2)^2 - 2x^3 - 4x^4\right] dx$$

$$\int_{-1}^{1} (x + x^3 + 1 + 2x^2 + x^4 - 2x^3 - 4x^4)\, dx$$

$$= \left[\cancel{\frac{x^2}{2}}^{\to 0} - \cancel{\frac{x^4}{4}}^{\to 0} + x + \frac{2x^3}{3} - \frac{3x^5}{5}\right]_{-1}^{1}$$

$$= \left(1 + \frac{2}{3} - \frac{3}{5}\right) - \left(-1 - \frac{2}{3} + 3/5\right)$$

$$= 2 + \frac{4}{3} - \frac{6}{5} = \frac{30+20-18}{15} = \frac{32}{15}$$

Ans

<u>Part (b)</u>

$$\int_0^4 \int_0^{\sqrt{y}} xy^2 \, dx\, dy$$

$$= \int_0^4 \left[\frac{x^2 y^2}{2}\right]_0^{\sqrt{y}} dy$$

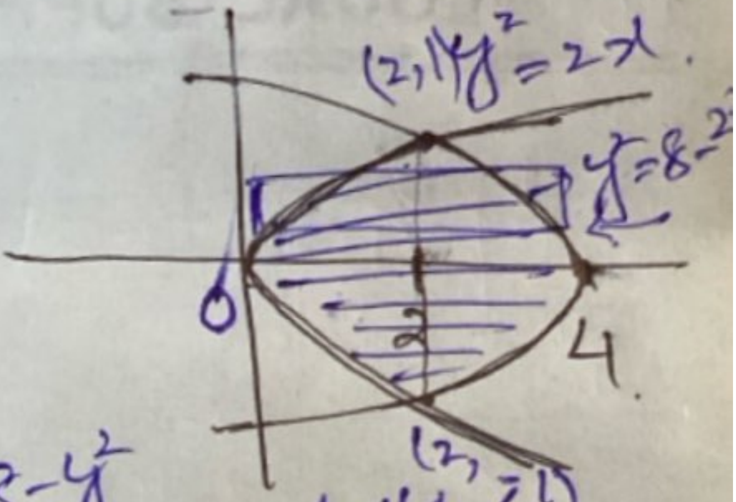
$$= \frac{1}{2}\int_0^4 y^3\, dy$$

$$= \frac{1}{2} \times \frac{y^4}{4}\Big]_0^4$$

$$= \frac{1}{8} \times 256 = 32 \quad \text{Ans}$$

Part (c)

$$\iint_R (4-y^2)\,dA$$



$\frac{y^2}{2} \le x \le \frac{8-y^2}{2}$, $-1 \le y \le 1$

$$\int_{-1}^{1}\int_{y^2/2}^{\frac{8-y^2}{2}} (4-y^2)\,dx\,dy$$

$$= \int_{-1}^{1} \left[(4-y^2)x\right]_{y^2/2}^{\frac{8-y^2}{2}} dy$$

$$= \int_{-1}^{1}(4-y^2)\left(\frac{8-y^2}{2}-\frac{y^2}{2}\right)dy$$

$$\int_{-1}^{1}(16-4y^2)\,dy$$

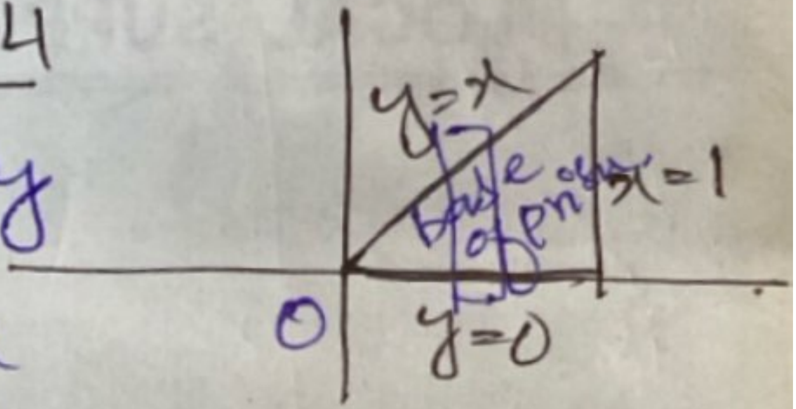
$$= \left[16y - 4\frac{y^3}{3}\right]_{-1}^{1}$$

$$= 16(1+1) - \frac{4}{3}(1+1)$$

$$= 32 - \frac{8}{3} = \frac{88}{3}$$

Ans

Question # 04



$f(x,y) = 3 - x - y$

$0 \le y \le x$

$0 \le x \le 1$

$$V = \int_0^1 \int_0^x (3 - x - y)\, dy\, dx.$$

$$= \int_0^1 \left[3y - xy - \frac{y^2}{2}\right]_0^x dx$$

$$= \int_0^1 \left(3x - x^2 - \frac{x^2}{2} - 0\right) dx.$$

$$= \left[\frac{3x^2}{2} - \frac{3}{2} \times \frac{x^3}{\cancel{3}}\right]_0^1$$

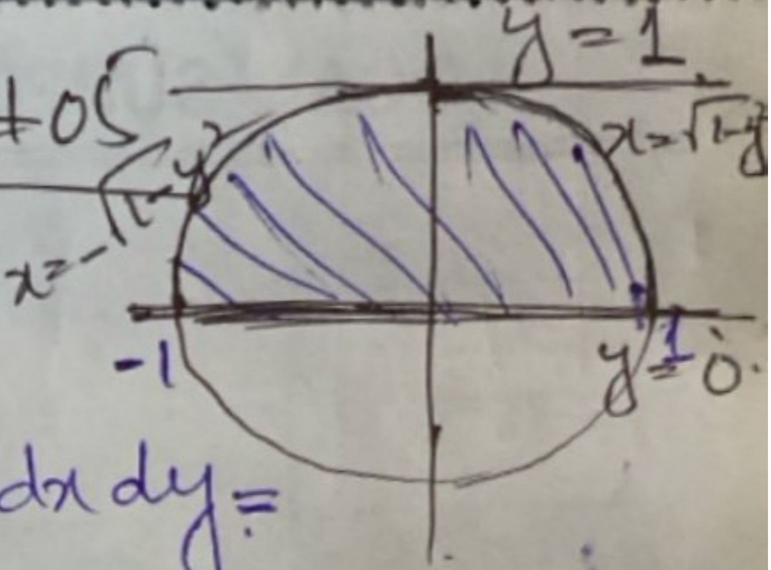
$$= \frac{3}{2}(1-0) - \frac{1}{2}(1-0)$$

$$V. = \frac{3}{2} - \frac{1}{2} = 1$$

Ans.

Question #05

Part (a)



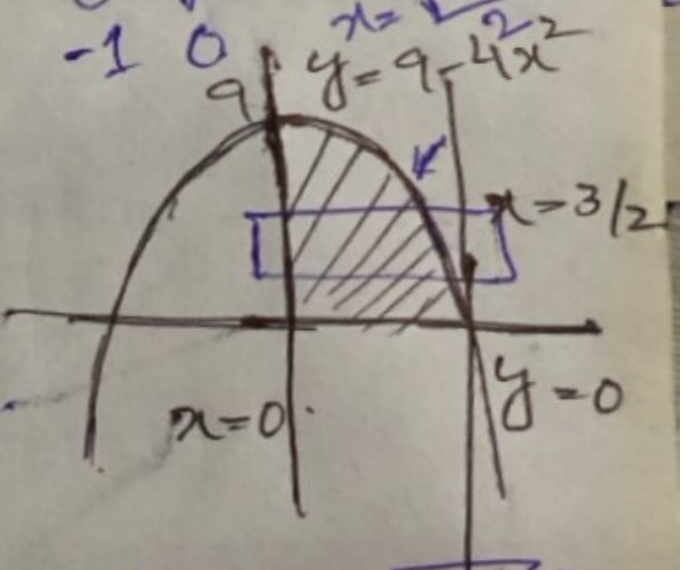
$$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y\,dx\,dy =$$

Sol

$$-1 \le x \le 1$$
$$0 \le y \le \sqrt{1-x^2}$$

$$\int_0^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3y\,dx\,dy = \int_{-1}^{1} \int_0^{\sqrt{1-x^2}} 3y\,dy\,dx.$$

Part (b)



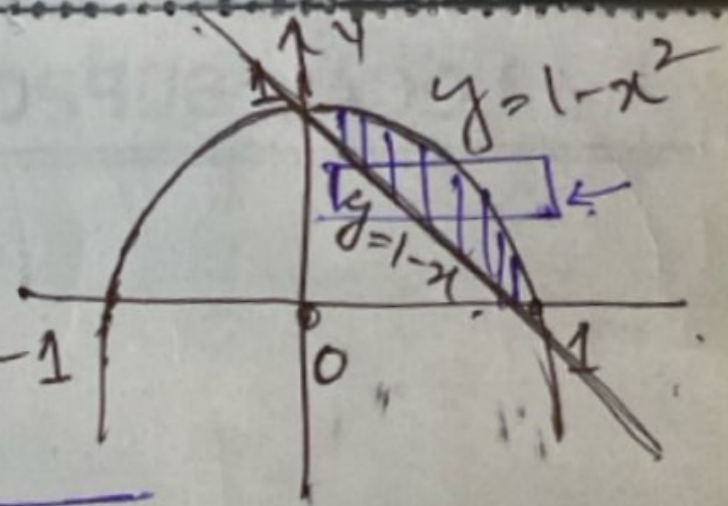
$$\int_0^{3/2} \int_0^{9-4x^2} 16x\,dy\,dx.$$

Sol $\quad 0 \le y \le 9, \quad 0 \le x \le \sqrt{\frac{9-y}{2}}$

$$\int_0^{3/2} \int_0^{9-4x^2} 16x\,dy\,dx = \int_0^9 \int_0^{\sqrt{\frac{9-y}{2}}} 16x\,dx\,dy$$

Part (C)

$$\int_0^1 \int_{1-x}^{1-x^2} dy\,dx.$$



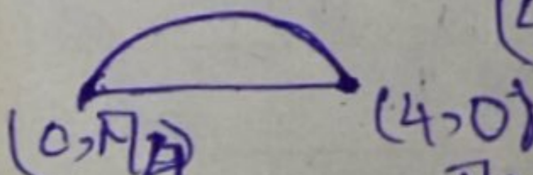
Sol $1-y \le x \le \sqrt{1-y}$

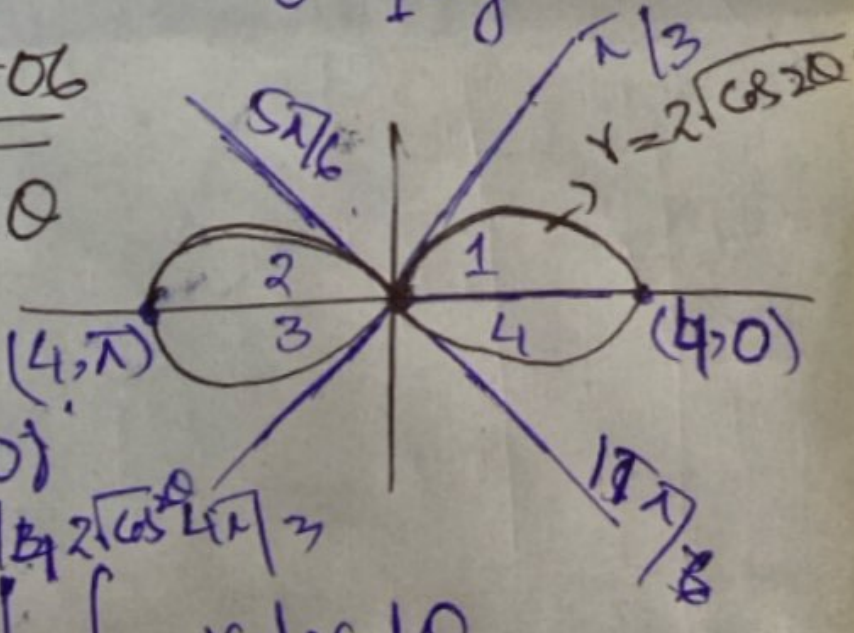
$0 \le y \le 1$

$$\int_0^1 \int_{1-x}^{1-x^2} dy\,dx = \int_0^1 \int_{1-y}^{\sqrt{1-y}} dx\,dy$$

Question #06

$r^2 = 4\cos 2\theta$





$$\text{area of } R = 4\int_0^{\pi/4} \int_0^{2\sqrt{\cos 2\theta}} r\,dr\,d\theta$$

$$\text{area of } R = 4\int_0^{\pi/4}\int_0^{2\sqrt{\cos 2\theta}} r\,dr\,d\theta$$

$$= \overset{2}{\cancel{4}}\int_0^{\pi/4}\left[\frac{r^2}{\cancel{2}}\right]_0^{2\sqrt{\cos 2\theta}} d\theta$$

$$= 2\int_0^{\pi/4}(4\cos 2\theta - 0)\,d\theta$$

$$= 8\int_0^{\pi/4}\cos 2\theta\,d\theta$$

$$= 4\cancel{8}\left[\frac{\sin 2\theta}{\cancel{2}}\right]_0^{\pi/4}$$

$$= 4\left[\sin\frac{\pi}{2} - \sin(0)\right]$$

$$= 4(1-0)$$

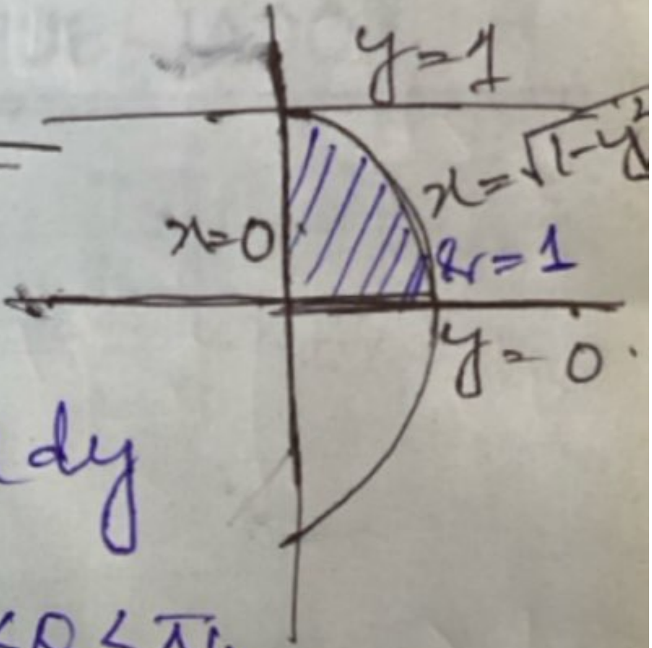
$$= 4$$

Ans

Question #07

Part (a)

$$\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2)\,dx\,dy$$



$0 \le r \le 1$, $0 \le \theta \le \pi/2$

$$\int_0^1 \int_0^{\sqrt{1-y^2}} (x^2+y^2)\,dx\,dy = \int_0^1 \int_0^{\pi/2} r^2 \times r\,dr\,d\theta$$

$$= \int_0^{\pi/2} \left[\frac{r^3}{3}\right]_0^1 d\theta.$$

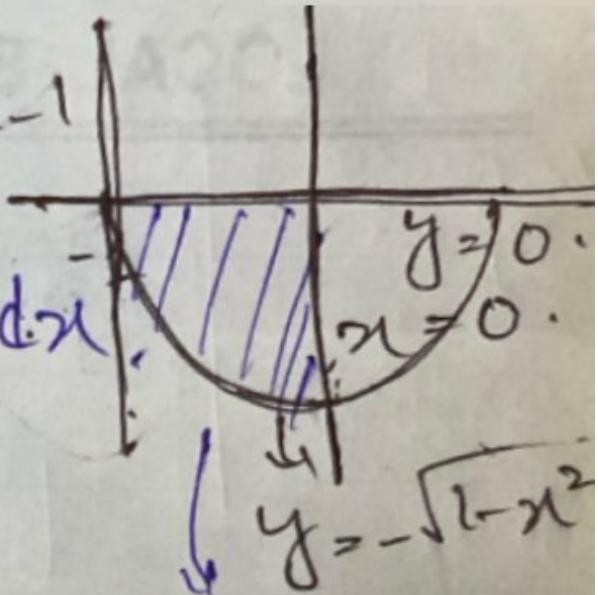
$$= \frac{1}{3}\int_0^{\pi/2} d\theta.$$

$$= \left[\frac{\theta}{3}\right]_0^{\pi/2} = \frac{\pi}{6}$$

Ans

Part (b)

$$\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0} \frac{2}{1+\sqrt{x^2+y^2}}\,dy\,dx$$



Sol

$0 \le r \le 1$, $\pi \le \theta \le 3\pi/2$ $\quad r=1$

$$\int_{-1}^{0}\int_{-\sqrt{1-x^2}}^{0}\frac{2}{1+\sqrt{x^2+y^2}}\,dy\,dx = \int_{\pi}^{3\pi/2}\int_{0}^{1} \frac{2}{(1+r)}\times r\,dr\,d\theta$$

$$= 2\int_{\pi}^{3\pi/2}\int_{0}^{1} \left[1-\frac{1}{1+r}\right]dr\,d\theta$$

$$=2\int_{\pi}^{3\pi/2}\left[r-\ln(1+r)\right]_{0}^{1} d\theta$$

$$=2\int_{\pi}^{3\pi/2}(1-\ln 2 - 0)\,d\theta$$

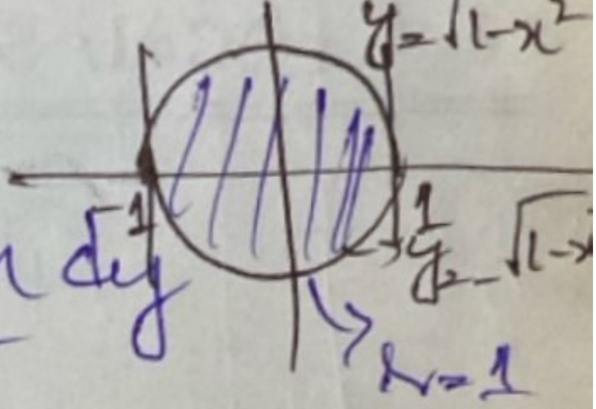
$$=\left[2(1-\ln 2)\,\theta\right]_{\pi}^{3\pi/2}$$

$$=2(1-\ln 2)\left(\frac{3\pi}{2}-\pi\right)$$

$$=(1-\ln 2)\pi$$ Ans

Part (C)

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1+x^2}} \frac{2}{(1+x^2+y^2)^2}\,dx\,dy$$



$0 \le r \le 1$, $0 \le \theta \le 2\pi$

$$= \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1+x^2}} \frac{2}{(1+x^2+y^2)^2}\,dx\,dy = \int_{0}^{2\pi}\int_{0}^{1} \frac{2r\,dr\,d\theta}{(1+r^2)^2}$$

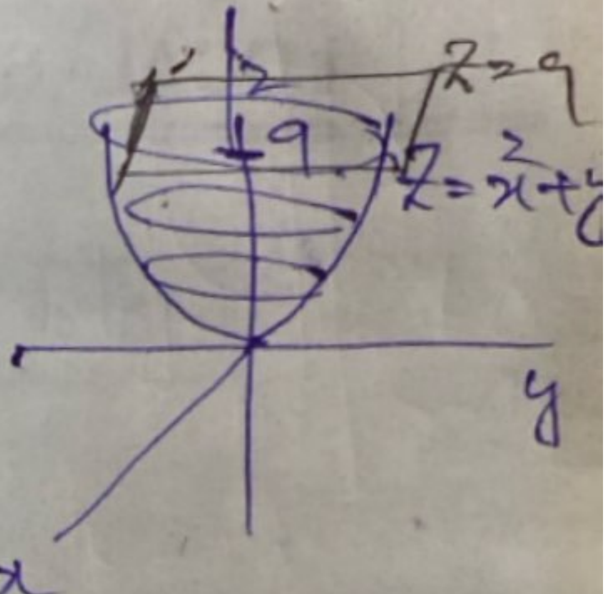
$$= \int_{0}^{2\pi}\left[\frac{-1}{(1+r^2)}\right]_{0}^{1} d\theta$$

$$= +\int_{0}^{2\pi}\left(\frac{-1}{2}+1\right)d\theta.$$

$$= \left[\frac{\theta}{2}\right]_{0}^{2\pi}$$

$$= \pi$$ Ans.

Question # 08



Part (a)

So $z = x^2+y^2 \Rightarrow$

$9 = x^2+y^2$

a circle of radius 3.

$$S = \iint_R \sqrt{\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2+1}\; dA$$

$$= \iint_R \sqrt{4x^2+4y^2+1}\; dA$$

$$= \int_0^{2\pi}\int_0^{3} \sqrt{4r^2+1}\; r\,dr\,d\theta$$

$$= \frac{1}{8}\int_0^{2\pi} \left[(4r^2+1)^{3/2}\times\frac{2}{3}\right]_0^{3} d\theta$$

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$$\cancel{= \frac{1}{12}\int_0^{2\pi} (5\sqrt{5}-1)\,d\theta}$$

$$\cancel{= \frac{(5\sqrt{5}-1)}{12}\,\theta\Big]_0^{2\pi} = \frac{(5\sqrt{5}-1)\pi}{6}}$$

$$= \frac{1}{12}\int_0^{2\pi} (37\sqrt{37}-1)\, d\theta.$$

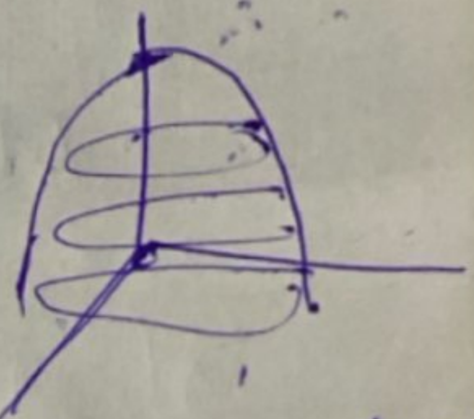
$$= \left(\frac{37\sqrt{37}-1}{12}\right)\left[\theta\right]_0^{2\pi}$$

$$= \frac{(37\sqrt{37}-1)\pi}{6} \quad \text{Ans.}$$

Part (b)

$$x^2+y^2+z=4 \Rightarrow z = 4-(x^2+y^2)$$

$z = 4-(x^2+y^2)$ lies above xy planes is the region bounded by



$4 = x^2+y^2 \rightarrow$ circle of radius 2.

so

$$S = \iint_R \sqrt{\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2+1}\; dA$$

$$S = \iint \sqrt{4x^2+4y^2+1}\; dA$$

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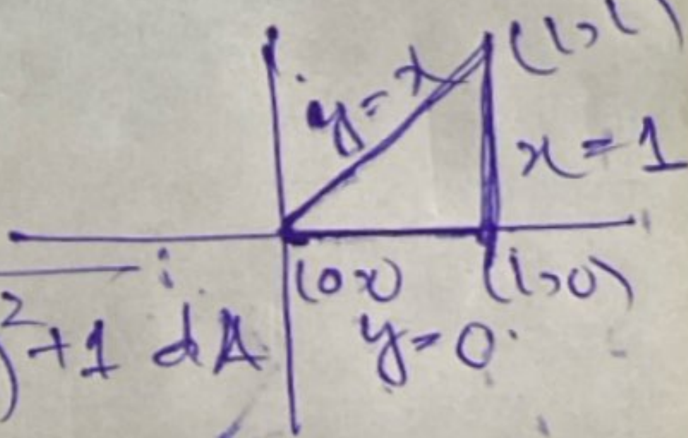
$$S=\int_0^{2\pi}\int_0^{2}\sqrt{4r^2+1}\; r\,dr\,d\theta$$

$$=\frac{1}{8}\int_0^{2\pi}\left[(4r^2+1)^{3/2}\times\frac{2}{3}\right]_0^2 d\theta$$

$$=\frac{1}{12}\int_0^{2\pi}(17\sqrt{17}-1)\,d\theta$$

$$=\frac{(17\sqrt{17}-1)}{12}\,[\theta]_0^{2\pi}$$

$$=\frac{(17\sqrt{17}-1)\pi}{6} \quad \text{Ans.}$$



Part C

$$S=\iint_R \sqrt{\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2+1}\; dA$$

$$0\le y\le x\,,\quad 0\le x\le 1$$

$$S=\int_0^1\int_0^x \sqrt{4x^2+4+1}\;.\,dy\,dx.$$

$$=\int_0^1\left[\sqrt{4x^2+5}\;\;y\right]_0^x dx$$

$$=\int_0^1 x\sqrt{4x^2+5}\;dx$$

$$=\frac{1}{\cancel{8}\,4}\left[(4x^2+5)^{3/2}\times\frac{\cancel{2}}{3}\right]_0^1$$

$$=\frac{1}{12}\left\{(9)^{3/2}-(5)^{3/2}\right\}$$

$$=\frac{1}{12}\left(27-5\sqrt{5}\right)$$

Ans.

Question #09

Part (a)

$$\int_0^{\sqrt{2}}\int_0^{\sqrt{2-x^2}}\int_{x^2+y^2}^{2} x\,dz\,dy\,dx.$$

$$\int_0^{\sqrt{2}}\int_0^{\sqrt{2-x^2}} \left[x z\right]_{x^2+y^2}^{2} dy\,dx.$$

$$\int_0^{\sqrt{2}}\int_0^{\sqrt{2-x^2}} (2x - x^3 - xy^2)\,dy\,dx$$

$$\int_0^{\sqrt{2}} \left[2xy - x^3y - \frac{xy^3}{3}\right]_0^{\sqrt{2-x^2}} dx.$$

$$\int_0^{\sqrt{2}} \left\{(2x - x^3)\sqrt{2-x^2} - \frac{x}{3}(2-x^2)\sqrt{2-x^2}\right\} dx$$

$$\int_0^{2} \left\{x(2-x^2)\sqrt{2-x^2} - \frac{x}{3}(2-x^2)\sqrt{2-x^2}\right\} dx$$

$$\int_0^{2} \frac{2x}{3}\,\underbrace{(2-x^2)}_{u}\,\underbrace{\sqrt{2-x^2}}_{u}\,dx \qquad du$$

$$= \frac{}{}\left[-\frac{1}{3}\left(2-x^2\right)^{5/2} \times \frac{2}{5}\right]_0^{\sqrt{2}}$$

$$= \frac{-2}{15}\left[0-(2)^{5/2}\right].$$

$$= \frac{2}{15} \times 4\sqrt{2} = \frac{8\sqrt{2}}{5} \text{ Ans}$$

Part (b)

$$\int_0^3 \int_0^2 \int_0^1 (xyz)^2 \, dx\, dy\, dz$$

$$= \int_0^3 \int_0^2 \left[\frac{x^3}{3} y^2 z^2\right]_0^1 dy\, dz$$

$$= \frac{1}{3} \int_0^3 \left[\frac{y^3}{3} z^2\right]_0^2 dz$$

$$= \frac{8}{9} \int_0^3 z^2 \, dz$$

$$= \frac{8}{9} \times \left[\frac{z^3}{3}\right]_0^3 = \frac{8}{27} \times 27 = 8$$

Ans

Part (c)

$$\int_0^{\pi/4}\int_0^{\ln \sec t}\int_{-\infty}^{2s} e^{r}\,dr\,ds\,dt$$

$$\int_0^{\pi/4}\int_0^{\ln \sec t}\left[e^{r}\right]_{-\infty}^{2s} ds\,dt$$

$$\int_0^{\pi/4}\int_0^{\ln \sec t}\left(e^{2s}-\cancel{e^{-\infty}}\right)ds\,dt$$

$$\int_0^{\pi/4}\left[\frac{e^{2s}}{2}\right]_0^{\ln \sec t} dt$$

$$\frac{1}{2}\int_0^{\pi/4}\left(e^{2\ln \sec t}-e^{0}\right)dt$$

$$\frac{1}{2}\int_0^{\pi/4}\left(\sec^2 t-1\right)dt$$

$$\frac{1}{2}\left[\tan t - t\right]_0^{\pi/4}$$

$$=\frac{1}{2}\left(\tan \pi/4 - \pi/4 - 0\right)=\frac{1}{2}\left(1-\pi/4\right)$$

Ans

Question #10

Part (a)

Bounded by $y=0$, $z=0$, $x=0$ & $y-x+z=1$,

$0 \le z \le 1+x-y$
$0 \le y \le 1+x$
$0 \le x \le 1$

$$V = \iiint_G dV$$

$$V = \int_0^1 \int_0^{1+x} \int_0^{1+x-y} dz\,dy\,dx$$

$$V = \int_0^1 \int_0^{1+x} [z]_0^{1+x-y}\,dy\,dx$$

$$V = \int_0^1 \int_0^{1+x} (1+x-y)\,dy\,dx$$

$$= \int_0^1 \left[y(1+x) - \frac{y^2}{2}\right]_0^{1+x} dx$$

$$= \int_0^1 \frac{(1+x)^2}{2}\,dx = \left.\frac{(1+x)^3}{6}\right]_0^1$$

$$= 8/6 = 4/3 \quad \text{Ans}$$

Part (b)

xy-plane $\Rightarrow z=0$

$x^2+y^2=1$, $x+y+z=3 \Rightarrow$

$z=3-x-y$.

$$V=\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{0}^{3-x-y} dz\,dy\,dx.$$

or

$$V=\iint_R\int_0^{3-x-y} dz\,dA$$

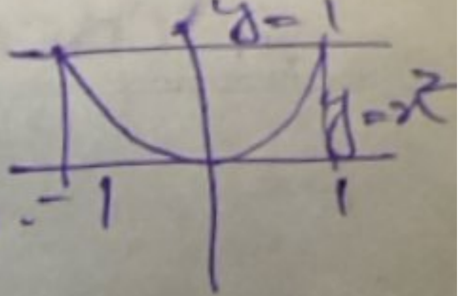
$$=\iint_R (3-x-y)\,dA$$

$$=\int_0^{2\pi}\int_0^1 \{3-r(\cos\theta-\sin\theta)\}\,r\,dr\,d\theta$$

Complete it

And

Part (c)



xy plane $\Rightarrow z=0$

$y+z=1 \Rightarrow z=1-y$

$y=x^2$

when $z=0$ we have

$1-y=0 \Rightarrow y=1$

$\Rightarrow x^2 \le y \le 1$ and

$-1 \le x \le 1$

So

$$V=\int_{-1}^{1}\int_{x^2}^{1}\int_{0}^{1-y} dz\,dy\,dx.$$

$$=\int_{-1}^{1}\int_{x^2}^{1}(1-y)\,dy\,dx$$

$$=\int_{-1}^{1}\left[y-\frac{y^2}{2}\right]_{x^2}^{1}dx$$

$$=\int_{-1}^{1}\left(\frac{1}{2}-x^2+\frac{x^4}{4}\right)dx.$$

$$=2\left[\frac{x}{2}-\frac{x^3}{3}+\frac{x^5}{20}\right]_0^1=2\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{20}\right)$$