Foundation for Advancement of Science & Technology

Exercise #14.1

1- $\int_0^1 \int_0^2 (x+3)\, dy\, dx$

$$\int_0^2 (x+3)\, dy$$

$$x\int_0^2 dy + 3\int_0^2 dy$$

$$xy + 3y\Big]_0^2$$

$$[x(2) + 6] - (0) = 2x + 6$$

$$\int_0^1 2x + 6\, dx$$

$$2\int_0^1 x\, dx + 6\int_0^1 dx$$

$$\frac{2x^2}{2} + 6x\Big]_0^1$$

$$[1 + 6] - [0 + 0] = 7$$

2. $\int_1^3 \int_{-1}^1 (2x - 4y)\, dy\, dx$

$$\int_{-1}^{8} (2x - 4y)\, dy$$

$$2x\int_{-1}^{1} dy - 4\int_{-1}^{1} y\,dy$$

$$2xy - \frac{4y^2}{2}\Big]_{-1}^{1}$$

$$2xy - 2y^2\Big]_{-1}^{1}$$

$$= [2x - 2] - [-2x - 2]$$

$$= 2x - 2 + 2x + 2$$

$$= 4x$$

$$\int_1^3 4x\,dx$$

$$4\int_1^3 x\,dx$$

$$\frac{4x^2}{2}\Big]_1^3 = 2x^2\Big]_1^3$$

$$2(9) - 2(1) = 18 - 2 = 16$$

3- $\int_2^4 \int_0^1 x^2 y \, dx \, dy$

$$\int_0^1 x^2 y \, dx$$

$$y \int_0^1 x^2 \, dx = y \frac{x^3}{3} \Big]_0^1$$

$$= \frac{y}{3} - 0 = \frac{y}{3}$$

$$\int_2^4 \frac{y}{3} \, dy$$

$$\frac{1}{3} \int_2^4 y \, dy = \frac{y^2}{6} \Big]_2^4$$

$$= \frac{16^{8}}{6_3} - \frac{4^{2}}{6_3} = \frac{8}{3} - \frac{2}{3} = \frac{6^{2}}{3_1} = 2$$

4- $\int_{-2}^{0} \int_{-1}^{2} (x^2 + y^2) \, dx \, dy$

$$\int_{-1}^{2} (x^2 + y^2) \, dx$$

$$\int_{-1}^{2} x^2\,dx + y^2 \int_{-1}^{2} dx$$

$$\frac{x^3}{3} + y^2 x \Big]_{-1}^{2}$$

$$\left[\frac{8}{3} + 2y^2\right] - \left[\frac{-1}{3} - y^2\right]$$

$$\frac{8}{3} + 2y^2 + \frac{1}{3} + y^2$$

$$\frac{9}{3} + 3y^2 \qquad = 3 + 3y^2$$

$$\int_{-2}^{0} 3 + 3y^2\,dy$$

$$3\int_{-2}^{0} dy + 3\int_{-2}^{0} y^2\,dy$$

$$3y + \frac{3y^3}{3}\Big]_{-2}^{0} \qquad = 3y + y^3\Big]_{-2}^{0}$$

$$0 - \left[-6 - 8\right] = 0 - (-14)$$

$$= 14$$

5. $\int_0^{\ln 3}\int_0^{\ln 2} e^{x+y}\,dy\,dx$

$$\int_0^{\ln 2} e^{x+y}\,dy = \frac{e^{x+y}}{1}\Big]_0^{\ln 2}$$

$$e^{x+\ln 2} - e^{x+0} = e^{x+\ln 2} - e^{x}$$

$$\int_0^{\ln 3} e^{x+\ln 2} - e^{x}\,dx$$

$$e^{x+\ln 2} - e^{x}\Big]_0^{\ln 3}$$

$$\left(e^{\ln 3+\ln 2} - e^{\ln 3}\right) - \left(e^{\ln 2} - 1\right)$$

$$3 - 1 = 2$$

6. $\int_0^2\int_0^1 y\sin x\,dy\,dx$

$$\int_0^1 y\sin x\,dy$$

$$\sin x\int_0^1 y\,dy = \frac{\sin x\, y^2}{2}\Big]_0^1$$

$$\frac{\sin x}{2} - 0 = \frac{\sin x}{2}$$

$$\int_0^2 \frac{\sin x}{2}\, dx$$

$$\frac{1}{2}\int_0^2 \sin x \cdot dx = \frac{-1}{2}\cos x \Big]_0^2$$

$$\frac{-1}{2}\cos 2 - \left(\frac{-1}{2}\cos 0\right) \Leftrightarrow$$

$$\frac{-1}{2}\cos 2 + \frac{1}{2} = \frac{1-\cos 2}{2}$$

7- $$\int_{-1}^{0}\int_{2}^{5} dx\, dy$$

$$\int_2^5 dx = x\Big]_2^5 = 5-2 = 3$$

$$\int_{-1}^{0} 3\, dy = 3\int_{-1}^{0} dy = 3y\Big]_{-1}^{0}$$

$$3(0) - (3)(-1) = 0 - (-3) = 3$$

8. $$\int_4^6\int_{-3}^{7} dy\, dx$$

$$\int_{-3}^{7} dy = y\Big]_{-3}^{7} = 7-(-3) = 10$$

$$\int_4^6 10\,dx = 10\int_4^6 dx$$

$$10x\Big]_4^6 = 60 - 40 = 20$$