Convexity

Hessian matrix.

$$H_f = \begin{bmatrix} f_{xx} & f_{yx} \\ f_{xy} & f_{yy} \end{bmatrix}$$

$$H_f = \begin{bmatrix} f_{xx} & f_{yx} & f_{zx} \\ f_{xy} & f_{yy} & f_{zy} \\ f_{xz} & f_{yz} & f_{zz} \end{bmatrix}$$

H_f is a positive definite, when the function is convex [$|H_f|$ all minor submatrices determinant >0

Semi-definate

D_1, D_2, D_3 either zero or positive

if $D_1, D_2, D_3 > 0$ → point is minimum

convex function

else if $D_1 < 0, D_2 > 0, D_3 < 0$ → maximum point

concave function.

else anyother condition will be lead to

$\boxed{\text{a saddle point}}$

(i) $f(x,y,z) = x^3 + 3xy + 3xz + y^3 + 3yz + z^3$

① critical Points

$f_x = 3x^2 + 3y + 3z \Rightarrow x^2 + y + z$

$f_y = 3x + 3y^2 + 3z \Rightarrow x + y^2 + z$

$f_z = 3y + 3x + 3z^2 = x + y + z^2$

$x^2 + y + z = 0$ — ① $x + y^2 + z = 0$ ② $x + y + z^2 = 0$ ③

$\boxed{\text{Point 1: } (0,0,0)}$

taking z from Eq ① putting in ②

$z = -x^2 - y$

taking x fu in ② & ③

$x = -z - y^2$

$$x + y^2 - x^2 - y = 0$$

$$x - y = x^2 - y^2$$

$$(x - y) = (x - y)(x + y)$$

$$(x + y)(x - y) - (x - y) = 0$$

$$(x - y)\left[(x + y) - 1\right] = 0$$

$$(x - y) = 0 \qquad x + y = 1$$

$$\boxed{x = y} \qquad \boxed{x + y = 1}$$

$$-z - y^2 + y + z^2 = 0$$

$$(y - z) = (y - z)(y + z)$$

$$y = z \qquad z + y = 1$$

$$x = y \qquad y = z$$

$$x = z$$

$$\boxed{x = y = z}$$

$$x + y = 0$$

$x^2 + x$

$x^2 + 2x = 0$

$x(x+2) = 0$

$\boxed{x = 0 \qquad x = -2}$

$\boxed{(0,0,0) \quad \& \quad (-2,-2,-2)}$

two points

$$H(f) = \begin{vmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{xy} & f_{yy} & f_{zy} \\ f_{xz} & f_{yz} & f_{zz} \end{vmatrix}$$

$$H(f) = \begin{vmatrix} 6x & 3 & 3 \\ 3 & 6y & 3 \\ 3 & 3 & 6z \end{vmatrix}$$

$\Rightarrow$ $H_f(0,0,0) = \begin{pmatrix} 0 & 3 & 3 \\ 3 & 0 & 3 \\ 3 & 3 & 0 \end{pmatrix}$

$D_1 = 0$ $\quad D_2 = -9$

$\boxed{0 \quad D_2 < -ve}$

saddle point

$\Rightarrow$ $H_f(-2,-2,-2) = \begin{pmatrix} -12 & 3 & 3 \\ 3 & -12 & 3 \\ 3 & 3 & -12 \end{pmatrix}$

$D_1 = -12 < 0$

$D_2 = \begin{vmatrix} -12 & 3 \\ & -12 \end{vmatrix}$

$$\begin{vmatrix} 3 & -12 \end{vmatrix}$$

$$144 - 9 = 135 > 0$$

$$D_3 = -12\begin{vmatrix} -12 & 3 \\ 3 & -12 \end{vmatrix} - 3\begin{vmatrix} 3 & 3 \\ 3 & -12 \end{vmatrix}$$

$$+3\begin{vmatrix} 3 & -12 \\ 3 & 3 \end{vmatrix}$$

$$= -12\,(135) \quad -3\,($$

$-ve$

$$\boxed{D_1 < 0 \quad D_2 > 0 \quad D_3 < 0}$$ — maximum point