$9-20 \ (f_x \cdot dx + f_y \cdot dy)$

$26 \ 21-26 \ (f_x(\Delta x) + f_y(\Delta y))$ compare $\left(P \text{ subs}, Q \text{ subs and } \right)$

$33-40 \ \left(f(x_0, y_0) + f_x(x_0, y_0)(x-x_0) + f_y(x_0, y_0)(y-y_0) \right)$

\# Ex. 13.4 (9-26, 33-40) : $\dfrac{|L(Q) - f(Q)|}{|PQ|}$

Q:09) $z = 7x - 2y \quad \rightarrow \quad z = 7dx - 2dy$

$dx = 7$

10) $z = e^{xy} \rightarrow e^{xy}(x)(1) \cdot dy + e^{xy}(y)(1) \cdot dx$

11) $z = x^3 y^2 \rightarrow 3x^2 y^2 \cdot dx + 2y(x^3) \cdot dy$

12) $z = 5x^2 y^5 - 2x + 4y + 7$

$(y^5(10x) - 2 + 0 + 0) dx + (25y^4 x^2 - 0 + 4 + 0) dy$

13) $z = \tan^{-1} xy$

$\left[y(1) \dfrac{1}{1 + x^2 y^2} \right] \cdot dx + \left[x(1) \dfrac{1}{1 + x^2 y^2} \right] \cdot dy$

14) $z = e^{-3x} \cos 6y$

$\left[\cos 6y \ e^{-3x}(-3) \right] dx + \left[e^{-3x} (-6 \sin 6y) \right] \cdot dy$

15) $w = 8x - 3y + 4z$

$8 \cdot dx - 3 \cdot dy + 4 \cdot dz$

16) $w = e^{xyz}$

$e^{xyz}(xy(1)) \cdot dz + e^{xyz}(yz(1)) \cdot dx + e^{xyz}(xz(1)) \cdot dy$

17) $w = x^3 y^2 z$

$3x^2 y^2 z \cdot dx + x^3 2y \cdot z \cdot dy + x^3 y^2(1) \cdot dz$

18) $w = 4x^2 y^3 z^7 - 3xy + z + 5$

$\left[4y^3 z^7 (2x) - 3y(1) \right] \cdot dx + \left[4x^2 z^7(3y^2) - 3x \right] \cdot dy + \left[4x^2 y^3 (7 \times z^6) + 1 \right] dz$

$= \left[8y^3 z^7 - 3y \right] \cdot dx + \left[12x^2 z^7 y^2 - 3x \right] dy + \left[28x^2 y^3 z^6 + 1 \right]$

19) $w = \tan^{-1}(xyz)$

$$y_z(1)\frac{\tan 1}{1+x^2y^2z^2}\cdot dx + \frac{xz}{1+x^2y^2z^2}\cdot dy + xy\frac{1}{1+x^2y^2z^2}\cdot dz$$

20) $w = x^{1/2} + y^{1/2} + z^{1/2}$

$$\frac{1}{2}x^{-1/2}\cdot dx + \frac{1}{2}y^{-1/2}\cdot dy + \frac{1}{2}z^{-1/2}\cdot dz$$

21) $f(x,y) = x^2 + 2xy - 4x$, $P(1,2)$, $Q(1.01, 2.04)$

$f_x = 2x + 2y - 4$

$f_y = 0 + 2x + 0$

$dw = (2x + 2y - 4)dx + (2x)dy$

$\quad = (2(1)+2(2)-4)(0.01) + 2(1)(0.04)$

$\quad = 0.1$

$\cdot P \to (1)^2 + 2(1)(2) - 4(1) = -1$

$\cdot Q \to (1.01)^2 + 2(1.01)(2.04) - 4(1.01) = 1.1009$

$\quad \cdot \Delta f = -1.1009 + 1 \Rightarrow 0.1009$

22) $x^{1/3}y^{1/2}$, $P(8,9)$, $Q(7.78, 9.03)$

$f_x = y^{1/2}\frac{1}{3}x^{-2/3}$

$f_y = x^{1/3}\frac{1}{2}y^{-1/2}$

$\cdot dw = \frac{1}{3}(8)^{-2/3}(9)^{1/2}(-0.22) + \frac{1}{2}(9)^{-1/2}(8)^{1/3}$ $\overset{(0.03)}{}$

$\Rightarrow -0.045$

$\cdot P \to (8)^{1/3}(9)^{1/2} = 6$

$\cdot Q \to (7.78)^{1/3}(9.03)^{1/2} = 5.9543$

$\left.\begin{array}{l}= 6 \\ = 5.9543\end{array}\right\}$ $6 - 5.954 = 0.0457$ $\overset{0.045613}{}$

23) $\frac{xyz}{x+y+z}$ ^{v}, $P(-1,-2,4)$, $Q(-1.04, -1.98, 3.97)$

$\cdot f_x = \dfrac{(1)(yz) - xyz(1)}{\cancel{}}$ $\dfrac{(x+y+z)(yz) - (xyz)(1)}{(x+y+z)^2}$

$\cdot f_z = \dfrac{(x+y+z)(xy) - (xyz)(1)}{(x+y+z)^2}$

$\cdot f_y = \dfrac{(x+y+z)(xz) - (xyz)(1)}{(x+y+z)^2}$

$$df = \frac{yz(y+z)}{(x+y+z)^2} dx + \frac{xz(x+z)}{(x+y+z)^2} + dy + \frac{xy(x+y)}{(x+y+z)^2} \cdot dz$$

$$=$$

33) $f(x,y) = \dfrac{1}{\sqrt{x^2+y^2}}$, $P(4,3)$, $Q(3.92, 3.01)$

· $f_x = \dfrac{-x}{(\sqrt{x^2+y^2})^3}$, $f_y = \dfrac{-y}{(\sqrt{x^2+y^2})^3}$

· L at $P = f_x(x_0, y_0) \to \dfrac{4}{\frac{6}{5}}$, $f_y(x_0, y_0) = \dfrac{3}{\frac{5^3}{5}}$

$\qquad\qquad\qquad = -0.032 \qquad\qquad = -0.024$

· $f(x_0, y_0) \Rightarrow \dfrac{1}{5}$

$$L = \frac{1}{5} - 0.032\left(x - \frac{4}{5}\right) - 0.024\left(y - \frac{3}{5}\right)$$

b) $\dfrac{|L(Q) - f(Q)|}{|PQ|}$

· $L(Q) = \dfrac{1}{5} - 0.032(3.92 - 4) - 0.024(3.01 - 3) \Rightarrow 0.20232$

· $f(Q) = \dfrac{1}{\sqrt{(3.92)^2 + (3.01)^2}} \to 0.20233\,42382$

· $|PQ| = \sqrt{(0.08)^2 + (0.01)^2} \Rightarrow 0.08062257748$

$$\frac{|0.20232 - 0.2023342382|}{0.08062257748} = 0.00017660^3$$

3) $f(x,y) = x^{0.5} y^{0.3}$ $P(1,1), Q(1.05, 0.97)$

$L(Q) = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$

$f(x_0, y_0) = 1$

if $f_x = y^{0.3} x^{0.5} (0.5 x^{-0.5}) \rightarrow f_x(x_0, y_0) = 0.5$

$f_y = x^{0.5} (0.3 y^{-0.7}) \rightarrow f_y(x_0, y_0) = 0.3$

$L(Q) = 1 + 0.5(x - 1) + 0.3(y - 1)$

b) $\dfrac{|L(Q) - f(Q)|}{|PQ|} \rightarrow$

$L(Q) = 1 + 0.5(1.05 - 1) + 0.3(0.97 - 1) \Rightarrow 1.016$

$f(Q) = (1.05)^{0.5} (0.97)^{0.3} = 1.015374307$

$|L(Q) - f(Q)| \Rightarrow 0.0006256930436$

$|PQ| = \sqrt{(0.05)^2 + (0.03)^2} \Rightarrow 0.05830951895$

$\Rightarrow 0.0107$

4) $f(x,y) = x \sin y$; $P(0,0), Q(0.003, 0.004)$

$f_x = \sin y (1) \rightarrow f_x(x_0, y_0) = \sin(0) \rightarrow 0$

$f_y = x \cos y (1) \rightarrow f_y(x_0, y_0) = 0$

$f(x_0, y_0) = 0$

$L(Q) = 0 + 0 + 0 \Rightarrow 0$

b) $f(Q) = (0.003) \sin(0.004) \rightarrow -0.0000000209443$

$|PQ| = \sqrt{0.003^2 + (0.004)^2} \Rightarrow 0.005$

8.36) $f(x,y) = \ln(xy)$, $P(1,2)$, $Q(1.01, 2.02)$

- $f_x = \frac{1}{xy}(y) \to \frac{1}{x} \to f_x(x_0,y_0) = \frac{1}{1}$

- $f_y = \frac{1}{xy}(x) \to \frac{1}{y} \to f_y(x_0,y_0) = \frac{1}{2}$

- $f(x,y) = \ln(1 \times 2) \to \ln 2$
- $\ln 2 + 1(x-1) + \frac{1}{2}(y-2)$

b) $L(Q) = \ln 2 + 0.01 + \frac{1}{2}(0.02)$

$f(Q) = \ln(1.01 \times 2.02) \Rightarrow 0.7130478423$

$L Q - f Q = 0.00009933825995$

$|PQ| = \sqrt{0.02^2 + 0.01^2} \ne 0.0044425$

\# Ex: 9.4 (At a glance)

- (9-20), find differential of a func

$dz = f_x\, dx + f_y\, dy$

- (21-26) total diff to approx change in P to Q, compare with actual

① $f_x(\Delta x) + f_y(\Delta y)$

② $f_P(x,y)$, $f_Q(x_0,y_0)$ subtract & compare

- (33-40) local linear $\overset{ab}{\circ}$, compare error at point Q with distance b/w P and Q

a) ① $f_x(x_0,y_0) \to$ coordinates of Q } $L Q = f(x_0,y_0)$
   ② $f_y(x_0,y_0) \to$ ⁴ $+ f_x(x_0-y_0)$
   ③ $f(x_0,y_0) \to$ ⁴   ⁴   ⁴ $+ f_y(x_0,y_0)$

b) $\dfrac{L(Q)}{f(Q)}$ } $\dfrac{L(Q) - f(Q)}{|PQ|}$