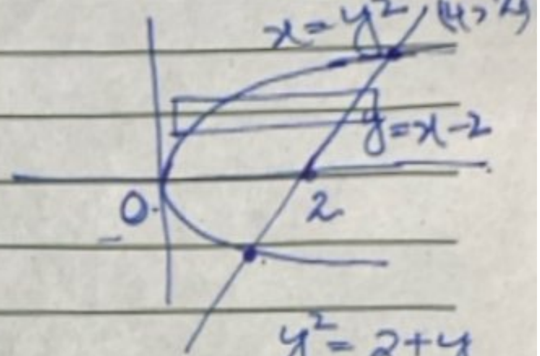
MVC - SOLUTION KEY OF QUIZ-03
Type - II

Q#01 Solution:



$y^2 \le x \le 2+y$ ~~$y \le x \le 2$~~.

$-1 \le y \le 2$.

$y^2 = 2+y$

$y^2-y-2=0$

$(y-2)(y+1)=0$

$$\iint_R y\,dA = \int_{-1}^{2}\int_{y^2}^{2+y} y\,dx\,dy$$

$$= \int_{-1}^{2} [x.y]_{y^2}^{2+y}\,dy$$

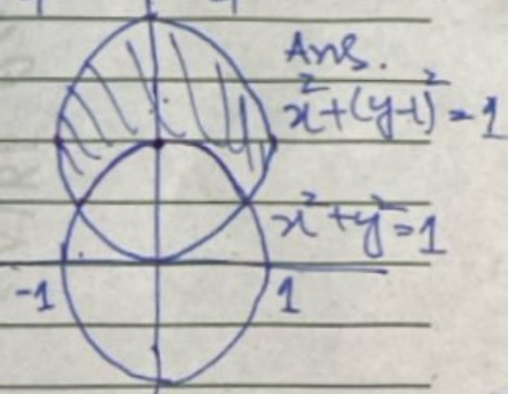
$$= \int_{-1}^{2} \{(2+y)y - y^3\}\,dy$$

$$= \left[\frac{2y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4}\right]_{-1}^{2}$$

$$= (4-1) + \frac{1}{3}(8+1) - \frac{1}{4}(16-1)$$

$$\iint_R y\,dA = 3+3-15/4 = \frac{24-15}{4} = \frac{9}{4}$$

Question #02



$x^2+y^2=1 \Rightarrow \boxed{r=1}$ ——— (i)

$x^2+(y-1)^2=1 \Rightarrow \boxed{r=2\sin\theta}$ — (ii)

on equating (1) & (2)

$1 = 2\sin\theta \Rightarrow \theta = \sin^{-1}(1/2) = \pi/6$

$\pi/6 \le \theta \le 5\pi/6$.

$$A = \int_{\pi/6}^{5\pi/6}\int_{1}^{2\sin\theta} r\,dr\,d\theta = \int_{\pi/6}^{5\pi/6}\left[\frac{r^2}{2}\right]_1^{2\sin\theta} d\theta$$

$$A = \frac{1}{2}\int_{\pi/6}^{5\pi/6} (4\sin^2\theta - 1)\, d\theta = \frac{1}{2}\int_{\pi/6}^{5\pi/6} \left[\cancel{4}^{\,2}\left(\frac{1-\cos 2\theta}{\cancel{2}}\right) - 1\right] d\theta$$

$$A = \frac{1}{2}\left[2\theta - \cancel{2}\frac{\sin 2\theta}{\cancel{2}} - \theta\right]_{\pi/6}^{5\pi/6}$$

$$= \frac{1}{2}\left\{\left(\frac{5\pi}{6} - \pi/6\right) - \left(\sin \frac{10\pi}{\cancel{6}_{3}} - \sin \frac{2\pi}{\cancel{6}_{3}}\right)\right\}$$

$$= \frac{1}{2}\left\{\frac{4\pi}{6} - \left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right)\right\}$$

$$A = \frac{1}{2}\left(\frac{2\pi}{3} + \sqrt{3}\right) \quad \text{Ans.}$$

Question # 03

$$\int_0^3 \int_{y/2}^{y} f(x,y)\, dx\, dy + \int_3^4 \int_{y/2}^{6-y} f(x,y)\, dx\, dy$$

when $0 \le y \le 3$, $\frac{y}{2} \le x \le y$

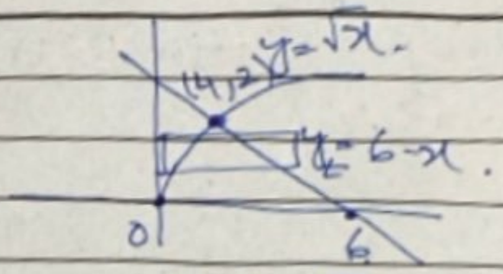
when $3 \le y \le 4$, $y/2 \le x \le 6-y$

MVC.
Solution KEY OF
QUIZ-03 Type-1

Continuation Sheet ______ of ______

Q#01 Solution.

$$\iint_R xy\, dA$$



$y^2 \le x \le 6-y$

$0 \le y \le 2$

$6-x=\sqrt{x} \Rightarrow$

$(6-x)^2 = x$

$x^2-12x+36=x$

$x^2-13x+36=0$

$(x-4)(x-9)=0$

$$\iint_R xy\, dA = \int_0^2 \int_{y^2}^{6-y} xy\, dx\, dy$$

$$= \int_0^2 \left[\frac{x^2 y}{2}\right]_{y^2}^{6-y} dy$$

$$= \frac{1}{2}\int_0^2 \{(6-y)^2 y - y^5\} dy$$

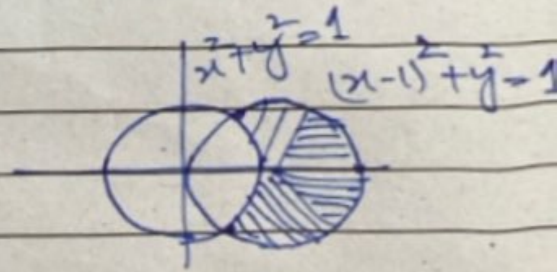
$$= \frac{1}{2}\left[\frac{36y^2}{2} - \frac{12y^3}{3} + \frac{y^4}{4} - \frac{y^6}{6}\right]_0^2$$

$$= \frac{1}{2}\left\{18(2-0) - 4(8-0) + \frac{1}{4}(16-0) - \frac{1}{6}(64)\right\}$$

$$= \frac{1}{2}\left\{36 - 32 + 4 - \frac{32}{3}\right\}$$

$$\iint_R xy\, dA = \frac{1}{2}\left(8 - \frac{32}{3}\right) = \frac{1}{2}\left(-\frac{8}{3}\right) = -\frac{4}{3}$$

Q#02



$x^2+y^2=1 \Rightarrow \boxed{r=1}$ —— ①

$(x-1)^2+y^2=1 \Rightarrow \boxed{r=2\cos\theta}$ —— ②

equating ① & ②

$2\cos\theta = 1 \Rightarrow \theta = \cos^{-1}(1/2) = \pi/3$

$5\pi/3 \le \theta \le \pi/3$

$$A = \int_{5\pi/3}^{\pi/3}\int_1^{2\cos\theta} r\, dr\, d\theta = \int_{5\pi/3}^{\pi/3}\left[\frac{r^2}{2}\right]_1^{2\cos\theta} d\theta$$

$$A = \frac{1}{2}\int_{5\pi/3}^{\pi/3} (4\cos^2\theta - 1)\, d\theta = \frac{1}{2}\int_{5\pi/3}^{\pi/3} \left\{ \cancel{4}^{2}\left(\frac{1+\cos 2\theta}{\cancel{2}}\right) - 1 \right\} d\theta$$

$$= \frac{1}{2}\left[2\theta + \cancel{2}\frac{\sin 2\theta}{\cancel{2}} - \theta \right]_{5\pi/3}^{\pi/3}$$

$$= \frac{1}{2}\left\{ \left(\pi/3 - 5\pi/3\right) + \sin\left(\frac{2\pi}{3}\right) - \sin\left(\frac{10\pi}{3}\right) \right\}$$

$$= \frac{1}{2}\left\{ -\frac{2\pi}{3} + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \right\}$$

$$\boxed{A = -\pi/3}$$

Ans.

Q #03

$$\int_{-1}^{0}\int_{-1}^{y-y^3} f(x,y)\, dx\, dy + \int_{0}^{1}\int_{\sqrt{y}-1}^{y-y^3} f(x,y)\, dx\, dy$$

OR

when $-1 \le y \le 0$, $-1 \le x \le y - y^3$

when $0 \le y \le 1$, $\sqrt{y} - 1 \le x \le y - y^3$