Tangent equation:

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

or

$$f_x(x-x_0) + f_y(y-y_0) + f_z(z-z_0) = 0$$

Normal line :

$$x = (P) + f_x(t)$$
$$y = (P) + f_y(t)$$
$$z = (P) + f_z(t)$$

3. $x^2 + y^2 + z^2 = 25$; $P(-3, 0, 4)$

$$f(x, y, z) = x^2 + y^2 + z^2 \qquad \text{-- ignore constant}$$

$$\nabla f(x, y, z) = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$$

$$\nabla f(-3, 0, 4) = 2(-3)i + 2(0)j + 2(4)k$$
$$= -6i + 8k$$

→ use formula :

$$= -6(x+3) + 0(y-0) + 8(z-4)$$

$$= -6x - 18 + 0 + 8z - 24$$

$$0 = -6x - 18 + 8z - 24$$

$$-6x + 8z - 42 = 0$$

$$6x - 8z + 42 = 0 \longrightarrow \text{tangent plane}$$

normal :

$$x = (-3) + (-6)t \qquad y = 0 + 0t \qquad z = 4 + 8t$$
$$= -3 - 6t \qquad\qquad y = 0$$

4. $x^2 y - 4z^2 = -7$; $P(-3, 1, -2)$

step #01 : Find $\nabla f (x, y, z)$

$$f(x, y, z) = x^2 y - 4z^2 \qquad\qquad \text{ignoring constant}$$
$$\nabla f (x, y, z) = 2xy i + x^2 j - 8zk$$
$$\nabla f (-3, 1, -2) = 2(-3)(1)i + (-3)^2 j - 8(-2) k$$
$$= -6i + 9j + 16k$$

step #02 . use tangent formula :

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$
$$(-6)(x + 3) + 9(y - 1) + 16(z + 2) = 0$$
$$-6x - 18 + 9y - 9 + 16z + 32 = 0$$
$$-6x + 9y + 16z - 27 + 32 = 0$$
$$-6x + 9y + 16z + 5 = 0$$

or
$$6x - 9y - 16z - 5 = 0$$

step #03   use normal formula.

$$x = x_0 + (f_x)t \longrightarrow x = -3 - 6t$$
$$y = y_0 + (f_y)t \longrightarrow y = 1 + 9t$$
$$z = z_0 + (f_z)t \longrightarrow z = -2 + 16t$$

5. $x^2 - xyz = 56$; $P(-4, 5, 2)$

step #01   Find $\nabla f(x, y, z)$

$$f(x, y, z) = x^2 - xyz \qquad \text{ignoring constant}$$
$$\nabla f(x, y, z) = (2x - yz)i + (-xz)j + (-xy)k$$

$$= (2x - yz)i - xzj - xyk$$
$$\nabla f(-4, 5, 2) = (-8 - 10)i - (-8)j - (-20)k$$
$$= -18i + 8j + 20k$$

step #02 , use tangent formula

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$
$$-18(x + 4) + 8(y - 5) + 20(z - 2) = 0$$
$$-18x - 72 + 8y - 40 + 20z - 40 = 0$$

$$-18x + 8y + 20z - 152 = 0$$
$$18x - 8y - 20z + 152 = 0$$

step #03   normal formula

$$x = x_0 + (f_x)t$$
$$x = -4 - 18t$$

$$y = y_0 + (f_y)t$$
$$y = 5 + 8t$$

$$z = z_0 + (f_z)t$$
$$z = 2 + 20t$$