

Ex : 14.2

$$Q1) \int_0^1 \int_{x^2}^x xy^2 dy dx \rightarrow \frac{y^3}{3} x \Big|_{x^2}^x \cdot dx \Big|_0^1 \rightarrow \frac{1}{3} (x^4 - x^7) \cdot dx$$

$$= \frac{1}{3} \int x^4 \cdot dx - \frac{1}{3} \int x^7 \cdot dx$$

$$= \frac{1}{3} \frac{x^5}{5} - \frac{1}{3} \frac{x^8}{8} \Rightarrow \left| \frac{1}{15} x^5 - \frac{1}{24} x^8 \right|_0^1 \Rightarrow \frac{1}{15} - \frac{1}{24} = \frac{1}{40}$$

$$Q2) \int_1^{3/2} \int_y^{3-y} \frac{1}{y^2} \cdot dx dy \rightarrow \int_y^{3-y} \frac{1}{y^2} \cdot dx \rightarrow \frac{1}{y^2} (3-y) - \frac{1}{y^2} (y)$$

$$= \frac{1}{y^2} (3y - y^2 - y^2)$$

$$= \int_1^{3/2} \frac{3y - 2y^2}{y^2} \cdot dy = \int_1^{3/2} \left( \frac{3}{y} - 2y \right) \cdot dy$$

$$= \left| \frac{3}{2} y^2 - 2 \frac{y^3}{3} \right|_1^{3/2} \Rightarrow \frac{3}{2} \left( \frac{3}{2} \right)^2 - \frac{2}{3} \left( \frac{3}{2} \right)^3 - \left( \frac{3}{2} (1)^2 - \frac{2}{3} (1)^3 \right)$$

$$Q3) \int_0^3 \int_0^{\sqrt{9-y^2}} y dx dy \rightarrow \int_0^3 y x \Big|_0^{\sqrt{9-y^2}} \cdot dy \Rightarrow y (9-y^2)^{1/2}$$

$$\frac{1}{2} \int_0^3 y (9-y^2)^{1/2} \cdot dy \quad (2) \rightarrow \frac{1}{2} \frac{(9-y^2)^{3/2+1}}{3/2+1} \rightarrow \frac{2}{3} (9-y^2)^{3/2} \Big|_0^3$$

$$\frac{1}{2} \left[ \frac{2}{3} (9-9)^{3/2} - \frac{2}{3} (9-0)^{3/2} \right] \Rightarrow \frac{2}{3} \frac{(9)^{3/2}}{2} \rightarrow \frac{2}{3} \cdot \frac{27}{2} \rightarrow 9$$

$$Q4) \int_{1/4}^1 \int_{x^2}^x \left( \frac{x}{y} \right)^{1/2} \cdot dy \cdot dx \rightarrow \int_{x^2}^x x y^{-1/2} \cdot dy \rightarrow x \ln y \Big|_{x^2}^x$$

$$= \left( x y^{-1/2} \right)^{1/2} \cdot dy \rightarrow \int_{x^2}^x x^{1/2} y^{-1/2} \cdot dy \rightarrow x^{1/2} \frac{y^{1/2}}{1/2} \Big|_{x^2}^x$$

$$2x^{1/2} x^{1/2} - 2x^{1/2} x \Rightarrow 2x - 2x^{3/2}$$

$$-\frac{1}{x} \cos\left(\frac{y}{x}\right) \Big|_0 \rightarrow -\frac{1}{x} \cos\left(\frac{y}{x}\right) - \frac{1}{x} \cos(x^2)$$

$$= \frac{2x^2}{2} - \frac{2x^{5/2}}{5/2} \Rightarrow x^2 - \frac{4}{5} x^{5/2} \Rightarrow 1 - \frac{4}{5} - \left(\frac{1}{4}\right)^2 + \frac{4}{5} \left(\frac{1}{4}\right)^{5/2}$$

$$\Rightarrow \frac{13}{80}$$

$$Q5) \int_{\sqrt{\pi}}^{\sqrt{2\pi}} \int_0^{x^3} \sin(y/x) \cdot dy \cdot dx$$

$$u = \frac{y}{x} \rightarrow \frac{dy}{dy} = \frac{1}{x} \rightarrow \frac{du}{dy} = \frac{1}{x} \rightarrow dy = x du$$

$$\int \int x \sin(u) \cdot du \cdot dx \rightarrow \int x \sin(u) \cdot \frac{1}{x} \cdot dx$$

$$- \int x \cos u \Big|_0^{x^3} \rightarrow -x \cos\left(\frac{y}{x}\right) \Big|_0^{x^3} \rightarrow -x \cos\left(\frac{x^3}{x}\right) + x$$

$$- \frac{1}{2} \int x \cos(x^2) + \int x \rightarrow -\frac{1}{2} \sin(x^2) + \frac{x^2}{2} \Big|_{\sqrt{\pi}}^{\sqrt{2\pi}}$$

$$= -\frac{1}{2} \sin(2\pi) + \frac{2\pi}{2} + \frac{1}{2} \sin(\pi) - \frac{\pi}{2} = \frac{\pi}{2}$$

$$Q6) \int_{-1}^1 \int_{-x^2}^{x^2} (x^2 - y) dy dx$$

$$\left| x^2 y - \frac{y^2}{2} \right|_{-x^2}^{x^2} \rightarrow \frac{1}{2} (x^4 - x^4) = x^4 - \frac{x^4}{2} + x^4 + \frac{x^4}{2}$$

$$= 2x^4$$

$$2 \int_{-1}^1 x^4 \rightarrow \frac{2x^5}{5} \Big|_{-1}^1 \rightarrow \frac{2(1)}{5} - \frac{2(-1)}{5} = \frac{4}{5}$$

$$Q7) \int_0^1 \int_0^x y \sqrt{x^2 - y^2} \cdot dy \cdot dx \rightarrow -\frac{1}{2} \frac{(x^2 - y^2)^{3/2}}{3/2} \Big|_0^x$$

$$-\frac{1}{3} (x^2 - y^2)^{3/2} \Big|_0^x \rightarrow -\frac{1}{3} (0)^{3/2} + \frac{1}{3} (x^2)^{3/2} = \frac{1}{3} x^3$$

$$\int_{\text{const}}^{\text{const}} \int_{f(x)}^{f(x)} \cdot dx dy$$

$$\frac{1}{3} \int_0^1 x^3 \cdot dx \rightarrow \frac{x^4}{4(3)} \Big|_0^1 \rightarrow \frac{x^4}{12} \Big|_0^1 \rightarrow \frac{1}{12}$$

$$\int_1^8 \int_0^{y^2} y^2 e^{x/y^2} \cdot dx dy \rightarrow e^{xy^{-2}}$$

$$xy^{-2} = v \rightarrow \frac{dv}{dx} = y^{-2}(1) \rightarrow dv = y^{-2} \cdot dx$$

$$\int_1^8 \int_0^{y^2} e^{x/y^2} \cdot \frac{dv}{y^{-2}} \cdot dy \rightarrow y^2 e^v \cdot dv \cdot dy$$

$$y^2 e^v \rightarrow y^2 e^{x/y^2} \Big|_0^{y^2} \rightarrow y^2 e^{y^4} - y^2 e^0$$

$$= y^2 e^{y^4} - y^2(1)$$

$$\int_1^8 y^2 e - \int_1^8 y^2 \Rightarrow e \left[ \frac{y^3}{3} \right]_1^8 - \left[ \frac{y^3}{3} \right]_1^8 \Rightarrow \frac{e}{3} \left( \frac{8}{3} \right) - \frac{8}{3} - \frac{e}{3} + \frac{1}{3}$$

$$\Rightarrow \frac{8e}{3} - \frac{e}{3} - \frac{8}{3} + \frac{1}{3} = \frac{7e}{3} - \frac{7}{3}$$

$$\Rightarrow \frac{7(e-1)}{3}$$

$$8.9) a) \int_0^2 \int_0^{x^2} \cdot dy dx$$

$$b) \int_0^4 \int_{\sqrt{y}}^2 \cdot dx dy$$

$$9.10) a) \int_0^1 \int_{x^2}^{\sqrt{x}} \cdot dy dx$$

$$b) \int_0^1 \int_{y^2}^{\sqrt{y}} \cdot dx dy$$

$$x^2 = \sqrt{x}$$

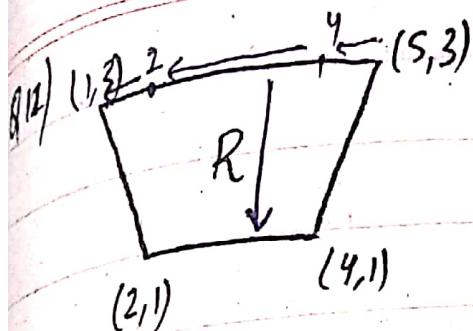
$$x^4 = x \rightarrow x^4 - x = 0$$

$$x^3(x-1) \rightarrow x = 0, 1, -1$$

$$y = x^2$$

$$x = \sqrt{y}$$





$$a) \int_1^2 \int_{-2x+5}^3 dy dx + \int_2^4 \int_1^3 dy dx + \int_4^5 \int_{2x-7}^3 dy dx + \text{[scribble]}$$

①  $(1,3), (2,1) \rightarrow y-3 = -2(x-1)$

$$m = \frac{3-1}{1-2} = \frac{2}{-1}$$

$$y = -2x + 5$$

② Ez tha

③  $(5,3), (4,1) \rightarrow y-1 = 2(x-4)$

$$m = \frac{1-3}{4-5} = \frac{-2}{-1} = 2$$

$$y = 2x - 7$$

Q12)

b)  $\int_1^3 \int_{y-5/2}^{y+7/2} dx dy$

①  $y = -2x + 5$

$$\frac{y-5}{-2} = x$$

②  $y = 2x - 7$

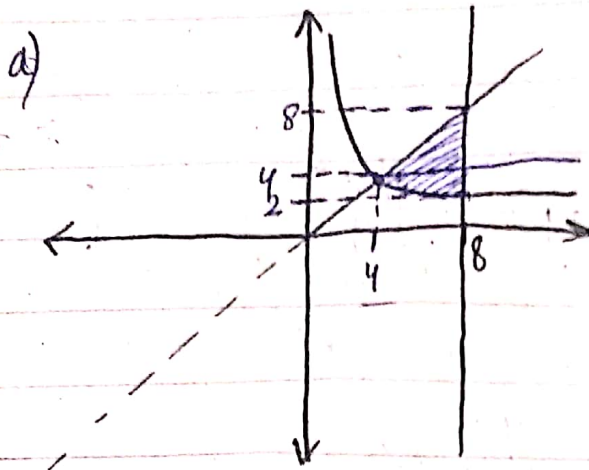
$$\frac{y+7}{2} = x$$

Q12) a)  $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} dy dx$

b)  $\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} dx dy$

3

Q15)  $\iint x^2 \cdot dA$ ,  $y = \frac{16}{x}$ ,  $y = x$ ,  $x = 8$



$$y = \frac{16}{8} \rightarrow 2$$

$$x^2 = 16 \rightarrow \pm 4$$

$$y = 8$$

a)  $\int_4^8 \int_{16/x}^x x^2 \cdot dy dx \rightarrow x^2 y \Big|_{16/x}^x \rightarrow \int x^3 - 16x$

$$\cdot \left( \frac{x^4}{4} - \frac{16x^2}{2} \right) \Big|_4^8 \Rightarrow \frac{(8)^4}{4} - 8(8)^2 - \frac{(4)^4}{4} + 8(4)^2$$

$$= 576$$

b)  $\int_2^4 \int_{16/y}^8 x^2 \cdot dx dy + \int_4^8 \int_y^8 x^2 \cdot dx dy$

$$\cdot \frac{x^3}{3} \Big|_{16/y}^8 \rightarrow \frac{512}{3} - \frac{4096}{3y^3}$$

$$\cdot \int_2^4 \frac{512}{3} \cdot dy - \frac{4096}{3} \int_2^4 y^{-3} \cdot dy \Rightarrow \frac{512}{3} y - \frac{4096}{3} \frac{y^{-2}}{-2}$$

$$\frac{512}{3} (4-2) - \frac{4096}{-6} \left( \frac{1}{16} - \frac{1}{4} \right) \Rightarrow \frac{640}{3}$$

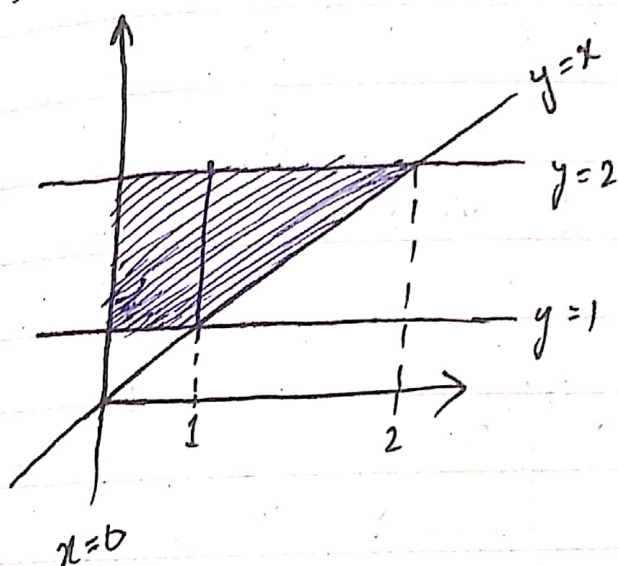
$$\left. \frac{x^3}{3} \right|_4^8 \rightarrow \frac{512}{3} - \frac{64}{3}$$

$$\int \frac{512}{3} \cdot dy - \frac{1}{3} \int y^3 \cdot dy = \left( \frac{512}{3} y - \frac{1}{3} \frac{y^4}{4} \right) \Big|_4^8$$

$$\frac{512}{3} (8-4) - \frac{1}{12} (4096 - 256) \Rightarrow \frac{2048}{3} - 320 \Rightarrow \frac{1088}{3}$$

$$\Rightarrow \frac{640}{3} + \frac{1088}{3} \Rightarrow 576$$

816)  $\iint xy^2 dA$ ,  $y=1$ ,  $y=2$ ,  $x=0$ ,  $y=x$



$$d) \iint xy^2 \cdot dy dx \rightarrow \int_0^1 \int_1^2 xy^2 \cdot dy dx + \int_1^2 \int_x^2 xy^2 \cdot dy dx$$

$$\left. \frac{xy^3}{3} \right|_1^2 \Rightarrow \frac{8x}{3} - \frac{x}{3} \Rightarrow \int_0^1 \frac{7}{3} x \cdot dx \rightarrow \frac{7x^2}{3 \cdot 2} + \frac{7}{6} (1-0) = \frac{7}{6}$$

$$\left. \frac{xy^3}{3} \right|_x^2 \rightarrow \frac{x(8)}{3} - \frac{x^4}{3} \Rightarrow \int_1^2 \frac{8}{3} x - \frac{x^4}{3}$$

$$= \frac{8x^2}{3 \cdot 2} - \frac{1}{3} \frac{x^5}{5} = \frac{8}{3} - \frac{1}{3} \Rightarrow \frac{7}{3}$$



$$= \frac{8}{3} \frac{x^2}{2} - \frac{1}{3} \frac{x^5}{5} \Rightarrow \frac{8}{6} x^2 - \frac{1}{15} x^5 \Rightarrow \frac{4}{3} x^2 - \frac{1}{15} x^5$$

$$\frac{4}{3} (2^2 - 1) - \frac{1}{15} (2^5 - 1) \Rightarrow \frac{4(3)}{3} - \frac{1}{15} (32 - 1)$$

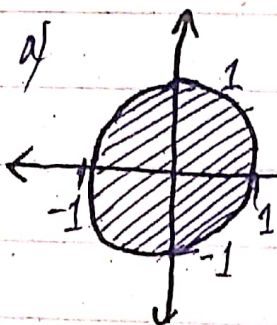
$$\Rightarrow \frac{29}{15}$$

$$\frac{29}{15} + \frac{1}{6} = \frac{31}{10}$$

$$b) \int_1^2 \int_0^y x y^2 \cdot dx dy \Rightarrow y^2 \frac{x^2}{2} \Big|_0^y = \frac{y^4}{2} - 0$$

$$\frac{1}{2} \frac{y^5}{5} \Rightarrow \frac{1}{10} y^5 \Big|_1^2 = \frac{32}{10} - \frac{1}{10} \Rightarrow \frac{31}{10}$$

$$Q17) \iint (3x - 2y) \cdot dA, x^2 + y^2 = 1$$



$$\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (3x - 2y) \cdot dy dx \Rightarrow \frac{-2y^2}{2}$$

$$-y^2 \Big|_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \Rightarrow -(1-x^2) + (1-x^2) \Rightarrow \cancel{1} + x^2 + \cancel{1} - x^2 = 0$$

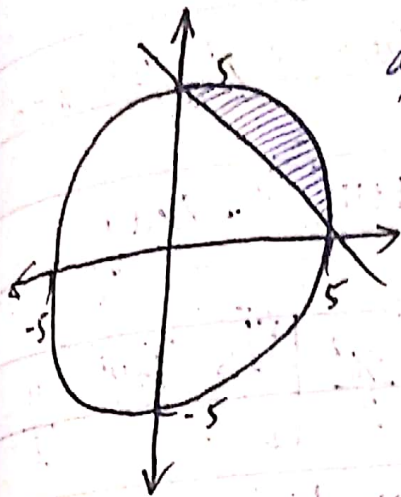
$$b) \int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} (3x - 2y) \cdot dx dy \Rightarrow \frac{3x^2}{2}$$

$$\frac{3}{2} (1-y^2)^2 - \frac{3}{2} (1-y^2)^2 = 0$$

$$-\frac{25}{2} + \frac{10x}{2} + \frac{x^2}{2} + \frac{25}{2} - \frac{x^2}{2}$$

$$5x - x^2$$

818)  $\iint_{x+y=5} y \, dA$ ,  $x^2 + y^2 = 25$  (only 1<sup>st</sup> Quadrant) &



$$a) \int_0^5 \int_{5-x}^{\sqrt{25-x^2}} y \cdot dy \, dx \rightarrow \frac{y^2}{2} \Big|_{5-x}^{\sqrt{25-x^2}}$$

$$\frac{1}{2} (25-x^2) - \frac{1}{2} (5-x)^2$$

$$= \frac{25}{2} - \frac{x^2}{2} - \frac{25}{2} + \frac{10x}{2} - \frac{x^2}{2}$$

$$= 5x - x^2$$

$$\frac{25}{2} - \frac{x^2}{2} - \frac{25}{2} + \frac{10x}{2} - \frac{x^2}{2} \Rightarrow 5x - x^2$$

$$10x - \frac{1}{2} \frac{x^3}{3} + \frac{1}{2} \frac{x^3}{2} \rightarrow \left( 10x - \frac{1}{6} x^3 + \frac{1}{4} x^3 \right) \Big|_0^5$$

$$10(5) - \frac{1}{6} (5)^3 + \frac{1}{4} (5)^3 = 0$$

$$\frac{25}{2} x - \frac{x^3}{6} - \frac{1}{26} (5-x)^3 \Big|_0^5 \Rightarrow \frac{25}{2} (5) - \frac{(5)^3}{6} - \frac{1}{6} (0) - 0 - 0$$

$$+ \frac{1}{6} (5)^3$$

$$b) \int_0^5 5x \cdot dx - \int_0^5 x^2 \cdot dx$$

$$\frac{5x^2}{2} - \frac{x^3}{3} \Big|_0^5 \Rightarrow \frac{5}{2} (5-0)^2 - \frac{(5-0)^3}{3} \Rightarrow \frac{125}{2} - \frac{125}{3} = \frac{125}{6}$$

$$b) \int_0^5 \int_{\sqrt{25-y^2}}^{5-y} y \cdot dx \, dy \rightarrow yx \Big|_{\sqrt{25-y^2}}^{5-y} \Rightarrow y(\sqrt{25-y^2} - 5 + y)$$

$$-\frac{1}{2} \int y(25-y^2)^{1/2} dy - 5 \int y \, dy + \int y^2 \, dy$$

$$\frac{1}{2} \frac{(25-y^2)^{3/2}}{3/2} - 5 \frac{y^2}{2} + \frac{y^3}{3} \Rightarrow \frac{(25-y^2)^{3/2}}{3} - \frac{5y^2}{2} + \frac{y^3}{3} \Big|_0^5$$

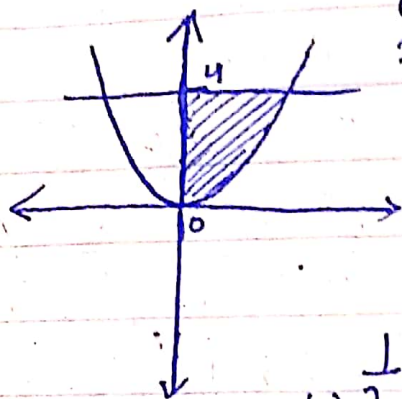


$$= 0 - \frac{25}{2} + \frac{125}{3} - \frac{125}{3} - 0 + 0 \Rightarrow$$

$$= 0 - \frac{125}{2} + \frac{125}{3} + \frac{125}{3} + 0 + 0 \Rightarrow \frac{125}{6}$$

Q19)  $\iint x(1+y^2)^{-1/2} dA$ , first Quadrant  $y=x^2$ ,  $y=4$ ,  $x=0$

$$y=x^2 \Rightarrow x=\sqrt{y} \quad \int_0^4 \int_0^{\sqrt{y}} x(1+y^2)^{-1/2} dx dy$$



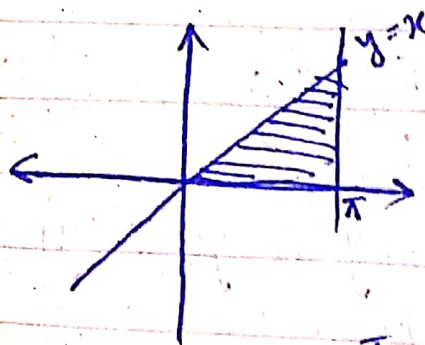
$$(1+y^2)^{-1/2} \frac{x^2}{2} \Big|_0^{\sqrt{y}} \Rightarrow (1+y^2)^{-1/2} \frac{y}{2} - 0$$

$$(2) \frac{1}{2} \int_0^4 y (1+y^2)^{-1/2} dy \Rightarrow \frac{1}{4} \frac{(1+y^2)^{1/2}}{1/2}$$

$$= \frac{1}{2} (1+y^2)^{1/2} \Big|_0^4 = \frac{1}{2} (1+16)^{1/2} - \frac{1}{2} (1)$$

$$= \frac{\sqrt{17} - 1}{2}$$

Q20)  $\iint x \cos y dA$ ,  $y=x$ ,  $y=0$ ,  $x=\pi$



$$\int_0^{\pi} \int_0^x x \cos y \cdot dy dx$$

$$x \int_0^x \cos y \cdot dy \rightarrow x \sin y \Big|_0^x \rightarrow x \sin x - 0$$

$$\int_0^{\pi} x \sin x \cdot dx$$

$\downarrow$     $\downarrow$   
 $u$     $dv$

$$u=x \quad \int dv = \int \cos x$$

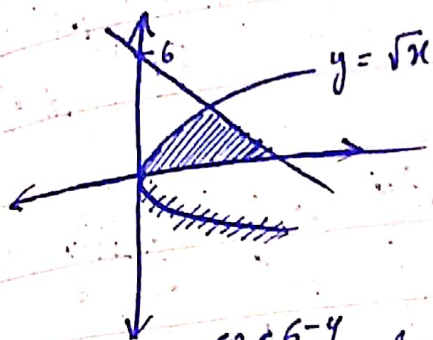
$$\frac{du}{dx} = 1 \quad v = -\cos x$$

$$w- \int v \cdot dv \Rightarrow (x)(-\cos x) + \int \cos x \cdot dx$$

$$(-x \cos x + \sin x) \Big|_0^{\pi} \Rightarrow -\pi \cos \pi + \sin \pi + 0 - 0$$

$$= -\pi(-1) + 0 = 1 + \pi$$

Q21)  $\iint xy \, dA$ ,  $y = \sqrt{x}$ ,  $y = 6-x$ ,  $y=0$



$$\bullet \quad 6-x = \sqrt{x}$$

$$(6-x)^2 = x \Rightarrow 36 - 2(6)x + x^2 =$$

$$\therefore = 36 - 12x + x^2 = x$$

$$x=4, x=9$$

$$y=2, y=3$$

$$= \int_0^2 \int_{y^2}^{6-y} xy \cdot dx dy$$

$$\bullet \quad y \int x \cdot dx \rightarrow y \frac{x^2}{2} \Big|_{y^2}^{6-y} = y \frac{(6-y)^2}{2} - y \frac{(y^2)^2}{2}$$

$$\frac{y}{2} (6-y)^2 - \frac{y^5}{2}$$

$$\bullet \quad \frac{y}{2} (36 - 12y + y^2) \Rightarrow 18y - 6y^2 + \frac{1}{2}y^3 - \frac{y^5}{2}$$

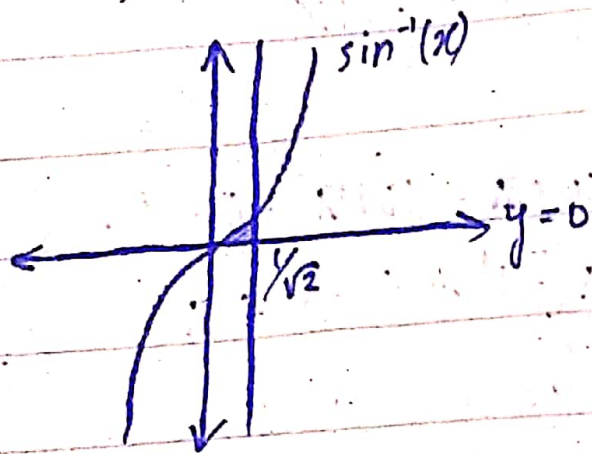
$$\bullet \quad 18 \int y \cdot dy - 6 \int y^2 \cdot dy + \frac{1}{2} \int y^3 - \frac{1}{2} \int y^5$$

$$= 18 \frac{y^2}{2} - 6 \frac{y^3}{3} + \frac{1}{2} \frac{y^4}{4} - \frac{1}{2} \frac{y^6}{6} \Rightarrow 9y^2 - 2y^3 + \frac{1}{8}y^4 - \frac{1}{12}y^6$$

$$= 9(2-0)^2 - 2(2-0)^3 + \frac{1}{8}(2-0)^4 - \frac{1}{12}(2-0)^6 = \frac{50}{3}$$



Q22)  $\iint x \, dA$ ,  $y = \sin^{-1}(x)$ ,  $x = 1/\sqrt{2}$ ,  $y = 0$



$$\int_0^{1/\sqrt{2}} \int_0^{\sin^{-1}(x)} x \cdot dy \, dx$$

$$xy \Big|_0^{\sin^{-1}(x)} \rightarrow x \sin^{-1}(x)$$

OR

$$\int_0^{\pi/4} \int_{\sin y}^{1/\sqrt{2}} x \cdot dx \, dy$$

$$\therefore \sin^{-1}(1/\sqrt{2}) = \pi/4$$

$$\sin^{-1}(1/\sqrt{2}) = \pi/4$$

$$\frac{x^2}{2} \Big|_{\sin y}^{1/\sqrt{2}} \rightarrow \frac{(1/2)}{2} - \frac{\sin^2 y}{2} \Rightarrow \frac{1}{4} - \frac{1}{2} \sin^2 y$$

$$\frac{1}{4} \int dy = \frac{1}{2} \int \sin^2 y \rightarrow \left( \frac{1}{4} y + \frac{1}{2} \cos y \right) \Big|_0^{\pi/4} \Rightarrow \frac{1}{4} \left( \frac{\pi}{4} \right) + \frac{1}{2} \left( \cos \frac{\pi}{4} \right) - 0 - \frac{1}{2} \cos(0)$$

$$= \frac{\pi}{16} + \frac{1}{4} - \frac{1}{2} = \frac{\pi}{16} - \frac{1}{4}$$

$$= \frac{\pi}{8} + \frac{1}{4} \sqrt{2} - \frac{1}{2} (1)$$

$$\frac{1}{4} \left( \frac{1}{2} - 2 \sin^2 y \right) \rightarrow \frac{1}{4} \cos(2y) \cdot dy$$

$$\frac{1}{4} \int_0^{\pi/4} \cos(2y) \cdot dy \rightarrow \frac{1}{4} \sin(2y) \Big|_0^{\pi/4} \Rightarrow \frac{1}{8} (1) - 0 = \frac{1}{8}$$



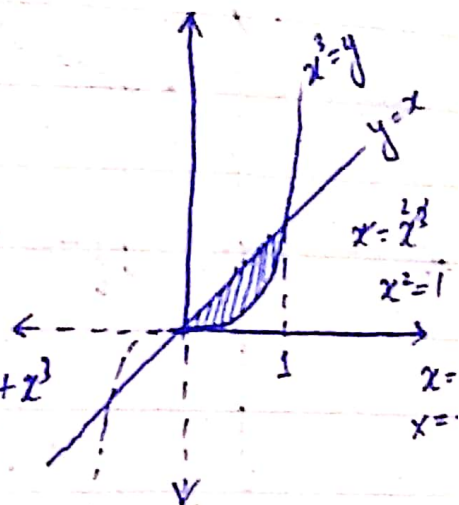
Q23)  $\iint (x-1) \cdot dA$ ,  $y=x$ ,  $y=x^2$ , in first Quadrant

$$\int_0^1 \int_{x^2}^x (x-1) \cdot dy dx \rightarrow \frac{x^2}{2} - x$$

$$\left. \frac{(x^2-1)}{2} \right|_{x^2}^x \Rightarrow \frac{x^2}{2}$$

$$\left. (xy - y) \right|_{x^2}^x \Rightarrow x(x) - x(x^2) - x(x^3) + x^3$$

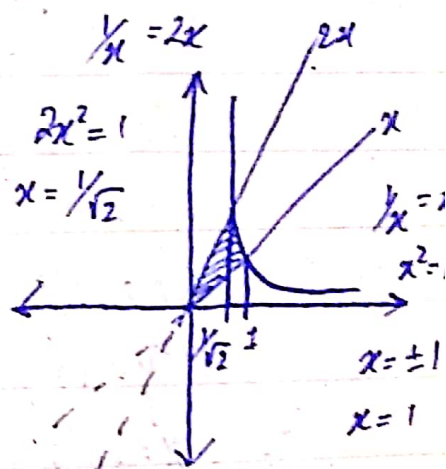
$$= -x + x^2 - x^4 + x^3 \Big|_0^1$$



$$\left. -\frac{x^2}{2} + \frac{x^3}{3} - \frac{x^5}{5} + \frac{x^4}{4} \right|_0^1 \Rightarrow -\frac{1}{2} + \frac{1}{3} - \frac{1}{5} + \frac{1}{4} = -\frac{1}{60}$$

Q24)  $\iint x^2 \cdot dA$ ,  $xy=1$ ,  $y=x$ ,  $y=2x$

$$\int_{1/\sqrt{2}}^{1/2} \int_{2x}^{2x^2} x^2 \cdot dy dx + \int_{1/\sqrt{2}}^1 \int_x^{1/x} x^2 \cdot dy dx$$



$$\left. \frac{x^3}{3} \right|_{2x}^{2x^2} \Rightarrow \frac{(2x^2)^3}{3} - x^2(2x) - x^2(x)$$

$$\Rightarrow \frac{8x^6}{3} - 2x^3 - x^3 \Rightarrow \frac{8x^6}{3} - 3x^3$$

$$\int x^3 \cdot dx \rightarrow \frac{x^4}{4} \Big|_0^{1/\sqrt{2}} \rightarrow \frac{(1/\sqrt{2})^4}{4} \rightarrow \frac{1}{16}$$

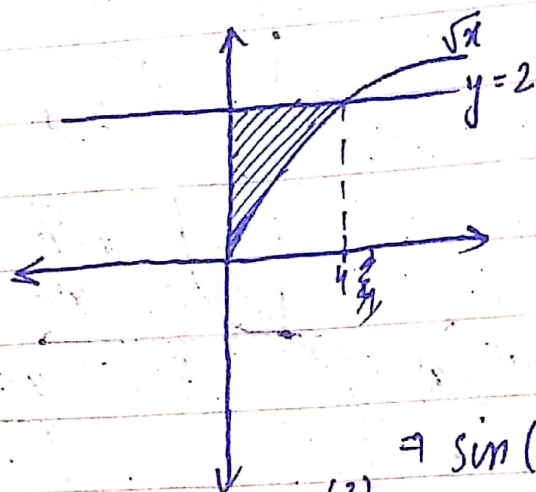
$$\left. x^2 y \right|_x^{1/x} \rightarrow x^2 \left( \frac{1}{x} \right) - x^2 x \rightarrow x - x^3$$

$$\int_{1/\sqrt{2}}^1 x \cdot dx - \int_{1/\sqrt{2}}^1 x^3 \cdot dx \rightarrow \left. \left( \frac{x^2}{2} - \frac{x^4}{4} \right) \right|_{1/\sqrt{2}}^1 \Rightarrow \frac{(1 - 1/\sqrt{2})^2}{2} - \frac{(1 - 1/\sqrt{2})^4}{4}$$

$$\frac{1}{2} - \frac{1}{4} - \frac{1/2}{2} + \frac{1/4}{4} = \frac{1}{16}$$

$$\frac{3 - 2\sqrt{2}}{4} \rightarrow \frac{1}{16} + \frac{1}{16} = \frac{1}{8}$$

Q25)  $\iint \sin(y^3) \cdot dA$   $y = \sqrt{x}$ ,  $y = 2$ ,  $x = 0$



$x = \sqrt{y} \rightarrow x = y^2$   
 $\int_0^2 \int_0^{y^2} \sin(y^3) \cdot dx dy$

$\int_0^2 \sin(y^3) \cdot dx \rightarrow \sin(y^3) x \Big|_0^{y^2}$

$\Rightarrow \sin(y^3)(y^2) - 0 \Rightarrow y^2 \sin(y^3)$   
 $\frac{1}{3} \int_0^2 y^2 \sin(y^3) \cdot dy \rightarrow \frac{1}{3} (-\cos(y^3))$

$= \frac{1}{3} (-\cos(8) + \cos(0)) \rightarrow \frac{1 - \cos(8)}{3}$

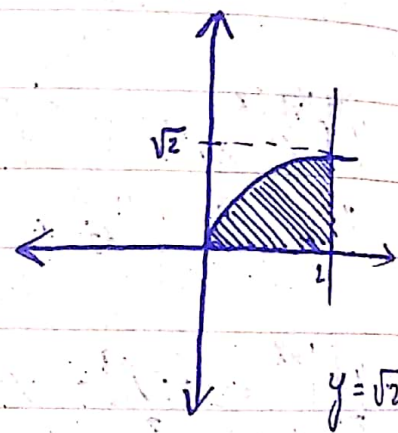
Q47)  $\int_0^2 \int_0^{\sqrt{x}} f(x, y) \cdot dy dx$

• Extracting eq from limits:

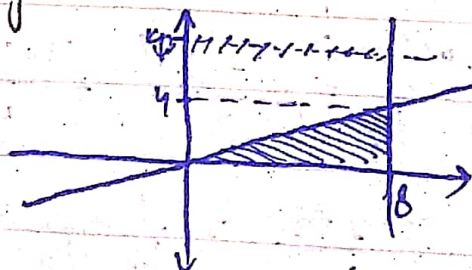
①  $y = \sqrt{x}$ ,  $y = 0$

②  $x = 2$ ,  $x = 0$

$\int_0^2 \int_0^{\sqrt{x}} f(x, y) \cdot dx dy$



Q48)  $\int_{2y}^8 \int_0^8 f(x, y) \cdot dx dy$

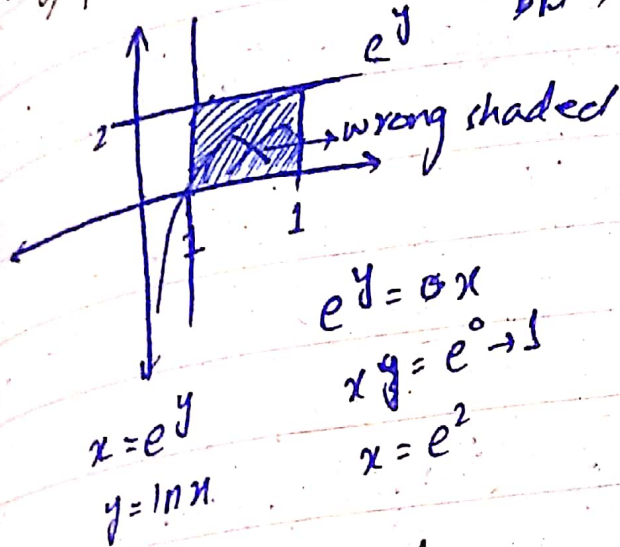


$x = 8$ ,  $x = 2y$

$y = \frac{x}{2}$   
 $= \int_0^8 \int_0^{x/2} dy dx$

$$Q49) \int_0^2 \int_1^{e^y} f(x, y) \cdot dx dy$$

$$\text{OR} \int_{\ln x}^2 \int_1^{e^y} f(x, y) \cdot dy dx$$

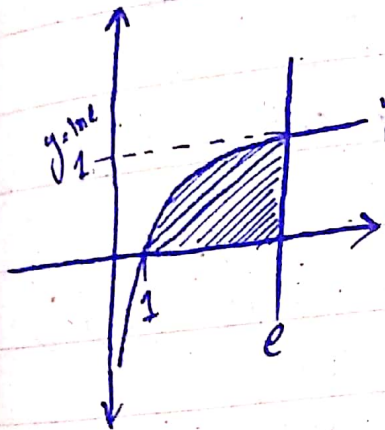


$$e^y = e^x$$

$$x y = e^0 \rightarrow 1$$

$$x = e^2$$

$$Q50) \int_1^e \int_0^{\ln x} f(x, y) dy dx$$



$$\int_0^1 \int_{e^y}^e dy x dy$$