Saturday, December 9, 2023 1:27 PM

$$\lim_{x\to a} f(x) = f(a)$$

$$f(a) = \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$$

MULTIVARIABLE CALCULUS

Continuity of Function of Several Variables.

A function $f(x,y)$ is said to be continuous at (x_0, y_0) if $f(x_0, y_0)$ is defined and

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = f(x_0, y_0)$$

Limit ✗

Value ✗

Limit of $f(x,y)$ is equal to value of $f(x,y)$ at point (x_0, y_0).

Ex# 13.2

Q10 $$\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{x^2+y^2} = \frac{0}{0}$$

Path

$x=0$

$y=0$

$x=y$

Sol Let $z = x^2+y^2$

$x\to 0,\ y\to 0 \Rightarrow z \to 0^+$

$$\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{x^2+y^2} = \lim_{z\to 0^+} \frac{1-\cos(z)}{z}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$$

we know that

$$\lim_{t\to 0} \frac{1-\cos t}{t} = 0$$

So, $$\lim_{(x,y)\to(0,0)} \frac{1-\cos(x^2+y^2)}{x^2+y^2} = 0$$

Ans.

Q14 $$\lim_{(x,y)\to(0,0)} \frac{x^4-16y^4}{x^2+4y^2} = \frac{0}{0}$$

Sol ... $x^4 - 16y^4$ ... $(x^2-4y^2)(x^2+4y^2)$

Sol $\lim_{(x,y)\to(0,0)} \dfrac{x^4-16y^4}{x^2+4y^2} = \lim_{(x,y)\to(0,0)} \dfrac{(x^2-4y^2)\cancel{(x^2+4y^2)}}{\cancel{(x^2+4y^2)}}$

$= 0$

Ans

Path
$x=0$ $\lim_{y\to 0} -4y^2 = 0$ Path $y=0$ $\lim_{x\to 0} x^2 = 0$

Path $y = mx$

$\lim_{x\to 0} \dfrac{x^4 - 16m^4x^4}{x^2+4m^2x^2} = \lim_{x\to 0} \dfrac{\cancel{x^4}^{\,2}(1-16m^4)}{\cancel{x^2}(1+4m^2)} = \lim_{x\to 0} x^2 \dfrac{(1-16)}{(\ )}$

Q19 $\lim_{(x,y,z)\to(0,0,0)} \dfrac{\sin(x^2+y^2+z^2)}{\sqrt{x^2+y^2+z^2}} = \dfrac{0}{0}$

$\to 0 \leftarrow$
$\lim_{x\to a} f(x)\, g(x)$

Sol Let $u = x^2+y^2+z^2$, $x\to 0$, $y\to 0$, $z\to 0 \Rightarrow u\to 0^+$

$\lim_{(x,y,z)\to(0,0,0)} \dfrac{\sin(x^2+y^2+z^2)}{\sqrt{x^2+y^2+z^2}} = \lim_{u\to 0^+} \dfrac{\sin u}{\sqrt{u}}$

" $= \lim_{u\to 0^+} \dfrac{\sin u}{\sqrt{u}} \times \dfrac{\sqrt{u}}{\sqrt{u}} = \lim_{u\to 0^+} \dfrac{\sin u}{u} \times \sqrt{u}$

" $= \lim_{u\to 0^+} \dfrac{\sin u}{u} \times \lim_{u\to 0^+} \sqrt{u}$

" $= 1 \times 0 = 0$ Ans

Q22 $\lim_{(x,y,z)\to(0,0,0)} \tan^{-1}\left[\dfrac{1}{x^2+y^2+z^2}\right]$

Sol Sub put $x=0$, $y=0$, $z=0$

$= \tan^{-1}\left(\dfrac{1}{0}\right) = \tan^{-1}(\infty) = \pi/2$ Ans

$= \tan^{-1}\left(\frac{1}{0}\right) = \tan^{-1}(\infty) = \pi/2$ Ans

Q15 $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2} = 0/0$

Case - I along y-axis: $x = 0$

$-y = 0$

$-y = x \Rightarrow 1/5$

$-y = -x \Rightarrow -1/5$

$y = 2x$

$y = mx$ | $x = 0$, $y = 0$, $y = x^2$

$$\lim_{y\to 0} \frac{0}{0+2y^2} = 0$$

Case - II $y = mx$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2} = \lim_{(x,mx)\to(0,0)} \frac{x \times mx}{3x^2+2m^2x^2}$$

$$= \lim_{x\to 0} \frac{m\cancel{x^2}}{\cancel{x^2}(3+2m^2)} = \lim_{x\to 0} \frac{m}{3+2m^2}$$

$$= \frac{m}{3+2m^2}$$

Here limit is depending upon m. So limit is different along two different lines. This implies limit does not exist.

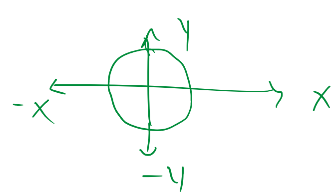
POLAR COORDINATES

$r = \sqrt{x^2+y^2}$

$r \geq 0$ OR $r \to 0^+$ if and only if $(x,y)\to(0,0)$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$x = r\cos\theta$, $y = r\sin\theta$ when circle is centered at origin.



$(x-h)^2 + (y-k)^2 = r^2$

Q26 $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+2y^2}}$

$\theta = 0^\circ$

$\theta = \pi/2$

Let $x = r\cos\theta$, $y = r\sin\theta$

$$xy = r^2 \cos\theta \sin\theta$$

$$x^2 + 2y^2 = r^2(\cos^2\theta + 2\sin^2\theta)$$

$$\sqrt{x^2+2y^2} = r\sqrt{\cos^2\theta + 2\sin^2\theta}$$

So

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+2y^2}} = \lim_{r\to 0^+} \frac{\cancel{r^2}\cos\theta\sin\theta}{\cancel{r}\sqrt{\cos^2\theta+2\sin^2\theta}} = \lim_{r\to 0} r\left(\frac{\cos\theta\sin\theta}{\sqrt{\quad}}\right)$$

$$= 0 \qquad \text{Ans}$$

$$\sqrt{\cos^2\theta + 2\sin^2\theta} \overset{?}{=} 0 \qquad \text{NO}$$

$$\sqrt{\underbrace{\cos^2\theta + \sin^2\theta}_{1} + \sin^2\theta} = \sqrt{1+\sin^2\theta} = \sqrt{1+\sin^2\theta} = 1$$

$$\frac{x^2y^2}{\sqrt{x^2+y^2}} = \frac{\cancel{r^4}\sin^2\theta\cos^2\theta}{\cancel{r}} = 0$$

Q34 $\lim_{(x,y)\to(0,0)} \frac{x^3y}{2x^6+y^2}$

Case - I $\quad y = mx$

Case - II : $\quad y = kx^2$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

Q19 $\lim_{(x,y,z)\to(0,0,0)} \frac{\sin(x^2+y^2+z^2)}{\sqrt{x^2+y^2+z^2}} = \frac{0}{0}$

Sol Let $u = x^2+y^2+z^2$, $x\to 0$, $y\to 0$, $z\to 0 \Rightarrow$

$u \to 0^+$

$\lim f(x) \times g(x)$

Sol Let $u = x^2+y^2+z^2$, $x \to 0$, $y \to 0$, ...

$u \to 0^+$

$$\lim_{(x,y,z)\to(0,0,0)} \frac{\sin(x^2+y^2+z^2)}{\sqrt{x^2+y^2+z^2}} = \lim_{u\to 0^+} \frac{\sin u}{\sqrt{u}}$$

$\lim_{x\to a} f(x)\times g(x)$

$\lim_{x\to a} f(x) \times \lim_{x\to a} g(x)$

$$'' = \lim_{u\to 0^+} \frac{\sin u}{\sqrt{u}} \times \frac{\sqrt{u}}{\sqrt{u}}$$

$$'' = \lim_{u\to 0^+} \frac{\sin u}{u} \times \sqrt{u}$$

$$'' = \lim_{u\to 0^+} \frac{\sin u}{u} \times \lim_{u\to 0^+} \sqrt{u}$$

$$'' = 1 \times 0 = 0$$

Ans.

Q22 $$\lim_{(x,y,z)\to(0,0,0)} \tan^{-1}\left[\frac{1}{x^2+y^2+z^2}\right]$$

Sol put $x=0$, $y=0$, $z=0$

$$\Rightarrow \tan^{-1}\left[\frac{1}{0}\right] = \tan^{-1}(\infty) = \pi/2$$

Ans.

Q15 $$\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2} = \frac{0}{0}$$

Sol Case-I : along y-axis: $x=0$

$$\lim_{y\to 0} \frac{0}{2y^2} = 0$$

Case - II : $y = mx$.

$$\lim_{(x,mx)\to(0,0)} \frac{x \times mx}{3x^2+2m^2x^2} = \lim_{x\to 0} \frac{m\cancel{x^2}}{\cancel{x^2}(3+2m^2)} = \frac{m}{3+2m^2}$$

$y=0 \leftarrow m=0 \Rightarrow 0$

$y=x \leftarrow m=1 \Rightarrow 1/5$

$y=-x \leftarrow m=-1 \Rightarrow -1/5$

$y=2x \leftarrow m=2 \Rightarrow 2/11$

... values of m, therefore

$(x, mx) \to (0,0)$ $\quad 3x^2+2m^2x^2$ $\quad x \to 0$ $\quad x(3+2m^2)$

Since limit depending upon the values of m, therefore limit is different for different values of m. This implies limit does not exist.

POLAR COORDINATE

$(x,y) \to (0,1)$

$r \to 1$

$r = \sqrt{x^2+y^2} \Rightarrow r \geq 0$

$r \to 0^+$ if and only if $(x,y) \to (0,0)$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$x = r\cos\theta$

$y = r\sin\theta$

y, $-x$, x, $-y$

when circle is centered at $(0,0)$

$(x-h)^2+(y-k)^2 = r^2$

$x-h = r\cos\theta$

$y-k = r\sin\theta$

$(x-1)^2 + y^2$

Q26 $\displaystyle \lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+2y^2}}$

Sol

Let $x = r\cos\theta$, $y = r\sin\theta$

$xy = r^2\cos\theta\sin\theta$

$x^2+2y^2 = r^2(\cos^2\theta + 2\sin^2\theta)$

$\sqrt{x^2+2y^2} = r\sqrt{\cos^2\theta+2\sin^2\theta}$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+2y^2}} = \lim_{r\to0^+} \frac{r^2\cos\theta\sin\theta}{r\sqrt{\cos^2\theta+2\sin^2\theta}}$$

$$= \lim_{r\to0^+} r \frac{\cos\theta\sin\theta}{\sqrt{\cos^2\theta+2\sin^2\theta}}$$

$\theta = 0$

$\theta = \pi/2$

$\cos^2\theta+\sin^2\theta + \sin^2\theta =$

$\sqrt{1+\sin^2\theta}$

$= 0$ Ans.

Q34 $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^3y}{2x^6+y^2}$

$y=0$: $\frac{x^3\times0}{2x^6+0} = \frac{0}{2x^6} = 0$

Case-II: $y = kx^2$

$$\lim_{(x,y)\to(0,0)} \frac{x^3 y}{2x^6+y^2}$$

Case-I : $y = mx$, $x \to 0 \Rightarrow y \to 0$

$$\lim_{x\to 0} \frac{x^3 \times mx}{2x^6 + m^2x^2}$$

$$\lim_{x\to 0} \frac{m x^{\cancel{4}2}}{\cancel{x^2}(2x^4+m^2)}$$

$$\lim_{x\to 0} \frac{m x^2}{2x^4+m^2} = 0$$

Hence proved

Case-II : $y = Kx^2$

$$\lim_{(x,mx)\to(0,0)} \frac{x^3 \times Kx^2}{2x^6+K^2x^4}$$

$$\lim_{kx\to 0} \frac{Kx^{\cancel{5}}}{\cancel{x^4}(2x^2+K^2)}$$

$$\lim_{x\to 0} \frac{Kx}{2x^2+K^2} = 0$$

Hence proved

Q35 (b)

$x = t^2,\ y = t,\ z = t$

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^4+z^4}$$

$x \to 0 \Rightarrow 0 = t^2 \Rightarrow t \to 0^+$

$y \to 0 \Rightarrow 0 = t \Rightarrow t \to 0$

$z \to 0 \Rightarrow 0 = t \Rightarrow t \to 0$

$x = t^2 - 1$

$0 = t^2 - 1$

$t^2 = 1 \Rightarrow t = \pm 1$ ✗

So

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^4+z^4} = \lim_{t\to 0^+} \frac{t^2 \times t \times t}{t^4+t^4+t^4} = \lim_{t\to 0^+} \frac{\cancel{t^4}}{3\cancel{t^4}}$$

$$= \lim_{t\to 0^+} \frac{1}{3} = 1/3$$

Ans

Date:- 07/02/2024

Q34 $$\lim_{(x,y)\to(0,0)} \frac{x^3 y}{2x^6+y^2}$$

Case-I $y = mx$

Case-II: $y = Kx^2$

$x \to 0 \Rightarrow y \to 0$

Case-I $y = mx$

$x \to 0 \Rightarrow y \to 0$

$$\lim_{x\to 0} \frac{x^3 \times mx}{2x^6 + m^2x^2}$$

$$\lim_{x\to 0} \frac{m x^{\cancel{4}2}}{\cancel{x^2}(2x^4+m^2)}$$

$$\lim_{x\to 0} \frac{m x^2}{2x^4+m^2} = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^3y}{2x^6+y^2} = 0$$

Case-II: $y = kx^2$

$x \to 0 \Rightarrow y \to 0$

$$\lim_{x\to 0} \frac{x^3 \times k\, x^2}{2x^6 + k^2x^4}$$

$$\lim_{x\to 0} \frac{k x^{\cancel{5}}}{\cancel{x^4}(2x^2+k^2)}$$

$$\lim_{x\to 0} \frac{k\, x}{2x^2+k^2} = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^3y}{2x^6+y^2} = 0$$

Ans

Hence $$\lim_{(x,y)\to(0,0)} \frac{x^3y}{2x^6+y^2} = 0$$

$x = r\cos\theta$
$y = r\sin\theta$

Q35 (b)

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^4+z^4}$$

$(1,0,0)$

$x \to 1$

$t^2 \to 1$

$t \to \pm 1$

Sol Let $x = t^2$, $y = t$, $z = t$

$x \to 0 \Rightarrow t^2 \to 0 \Rightarrow t \to 0$

$y \to 0 \Rightarrow t \to 0$

$z \to 0 \Rightarrow t \to 0$

Now
$$\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^4+z^4} = \lim_{t\to 0} \frac{t^2 \times t \times t}{t^4+t^4+t^4} = \lim_{t\to 0} \frac{t^4}{3t^4}$$

$$= \lim_{t\to 0} \frac{t^4}{3t^4} = 1/3$$

$$\Rightarrow \boxed{\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^4+z^4} = 1/3} \quad \text{— I}$$

but $\lim \quad \frac{xyz}{\quad} = 0$

but from part a

$$\boxed{\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^4+z^4} = 0} \quad \text{— (II)}$$

from eq (I) & eq (II), limit of function is different for different path. Therefore limit of this function when $(x,y,z)\to(0,0,0)$ does not exist.

$x=0$
$y=t \Rightarrow t\to 0$
$(x,y)\to(0,0)$

$y=x$
let $x=t$
$\Rightarrow y=t$

H.W

$$f(x,y) = \begin{cases} \frac{\sin x}{x}, & x \neq 1 \\ 1, & x \le 1 \end{cases} \quad x > 1$$

Q38

$$f(x,y) = \begin{cases} \frac{\sin(x^2+y^2)}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 1, & \text{if } (x,y) = (0,0) \end{cases}$$

Show that $f(x,y)$ is continuous at $(0,0)$

proof For continuity of $f(x,y)$, we need to show

$$\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0)$$

from given function we have $\boxed{f(0,0) = 1}$ — (1)

For Limit $f(x,y)$:-
$(x,y)\to(0,0)$

we have $f(x,y) = \frac{\sin(x^2+y^2)}{x^2+y^2}$ when $(x,y)\neq(0,0)$

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^2)}{x^2+y^2}$$

$$\lim_{t\to 0} \frac{1-\cos t}{t} = 0$$

$(x,y)\to(0,0)$ $(x,y)\to(0,0)$ x^2+y^2

Let $u = x^2+y^2$, $x\to 0$, $y\to 0 \Rightarrow u \to 0^+$

$$\boxed{\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{u\to 0^+} \frac{\sin u}{u} = 1} \quad (2)$$

$; \lim_{t\to 0} \frac{\sin t}{t} = 1$

From eq (1) & (2)

$$\lim_{(x,y)\to(0,0)} f(x,y) = 1 = f(0,0)$$

Hence $f(x,y)$ is continuous at $(0,0)$.

$$\lim_{(x,y)\to(0,0)} f(x,y) \text{ exist}$$

$$\neq f(0,0)$$

$f(0,0) \to$ not defined

Q39 $$\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}$$

Case-I along y-axis: $x=0$

$$\Rightarrow \lim_{y\to 0} \frac{0}{0+y^2} = 0$$

Case-II along x-axis: $y=0$

$$\lim_{x\to 0} \frac{x^2}{x^2+0} \Rightarrow \lim_{x\to 0} \frac{x^2}{x^2} \Rightarrow \lim_{x\to 0} 1 = 1$$

limit does not exist b/c on two different Path limit is different.

So this $f(x,y)$ does not have removable discontinuity

Date: 07/02/2024 MVC BCS-2B

Q34 (a) $\lim_{(x,y)\to(0,0)} \dfrac{x^3 y}{2x^6+y^2}$

Case – I : $y = mx$

$x \to 0 \Rightarrow y \to 0$

$$\lim_{(x,mx)\to(0,0)} \frac{x^3 \times mx}{2x^6 + m^2x^2}$$

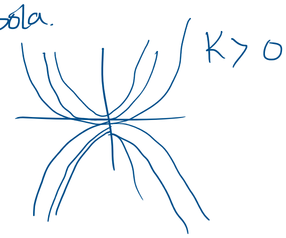
$$\lim_{x\to 0} \frac{m x^{\cancel{4}2}}{\cancel{x^2}(2x^4 + m^2)}$$

$$\lim_{x\to 0} \frac{m x^2}{2x^4+m^2} = 0$$

$$\Rightarrow \boxed{\lim_{(x,y)\to(0,0)} \frac{x^3 y}{2x^6+y^2} = 0}$$

Case – II : $y = kx^2$

Family of Parabola



$x \to 0 \Rightarrow y \to 0$

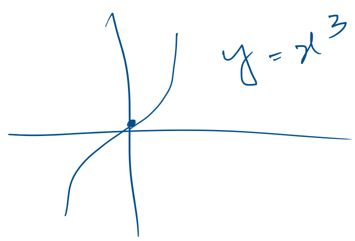
$$\lim_{x\to 0} \frac{x^3 \times kx^2}{2x^6 + k^2x^4}$$

$$\lim_{x\to 0} \frac{k x^{\cancel{5}}}{\cancel{x^4}(2x^2+k^2)}$$

$$\lim_{x\to 0} \frac{kx}{2x^2+k^2} = 0$$

$$\boxed{\lim_{(x,y)\to(0,0)} \frac{x^3 y}{2x^6+y^2} = 0}$$

Ans.



Q35 (b)

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^4+z^4}$$

$x \to 1$ (1, 0, 0)

$t^2 \to 1 \Rightarrow t \to \pm 1$

Sol Let $x = t^2$, $y = t$, $z = t$

$x \to 0 \Rightarrow t^2 \to 0 \Rightarrow t \to 0$

$y \to 0 \Rightarrow t \to 0 \Rightarrow t \to 0$

$z \to 0 \Rightarrow t \to 0$

Now,

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^4+z^4} = \lim_{t\to 0} \frac{t^2 \times t \times t}{t^4+t^4+t^4}$$

$$= \lim_{t\to 0} \frac{\cancel{t^4}}{3\cancel{t^4}}$$

$= \lim_{t\to 0} \frac{\quad}{3t^4}$

$$\boxed{\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^4+z^4} = \lim_{t\to 0} \frac{1}{3} = 1/3} \quad \text{— (1)}$$

but from part (a) we have

$$\boxed{\lim_{(x,y,z)\to(0,0,0)} \frac{xyz}{x^2+y^4+z^4} = 0} \quad \text{— (2)}$$

From eq (1) & (2) limit of $f(x,y)$ is different along different path. Therefore, limit of $f(x,y)$ does not exist when $(x,y)\to(0,0)$.

Q38

$$f(x,y) = \begin{cases} \dfrac{\sin(x^2+y^2)}{x^2+y^2}, & \text{if } (x,y)\neq(0,0) \\ 1, & \text{if } (x,y)=(0,0) \end{cases}$$

For continuity we need to check

$$\lim_{(x,y)\to(0,0)} f(x,y) = f(0,0)$$

For value of function: $f(0,0)$

we have $f(x,y) = 1$ when $(x,y) = (0,0)$

$\Rightarrow \boxed{f(0,0) = 1}$ — (I)

For $\lim_{(x,y)\to(0,0)} f(x,y)$:-

we have $f(x,y) = \dfrac{\sin(x^2+y^2)}{x^2+y^2}$ when $(x,y)\neq(0,0)$

So $\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{(x,y)\to(0,0)} \dfrac{\sin(x^2+y^2)}{x^2+y^2}$

Let $u = x^2+y^2$, $x\to 0$, $y\to 0 \Rightarrow u\to 0^+$

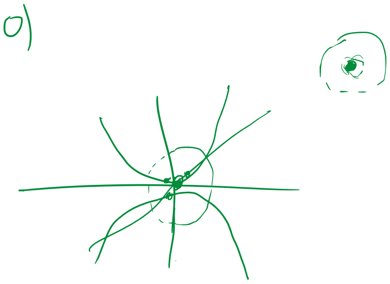
H.W

$$f(x) = \begin{cases} \dfrac{\sin x}{x}, & x\neq 0 \\ 2, & x=0 \end{cases}$$

$f(0) = 2$

$f(x) = \dfrac{\sin x}{x}$, $x\neq 0$

$f(1) =$



Hint

$\lim_{t\to 0} \dfrac{\sin t}{t} = 1$

$\lim \dfrac{1-\cos t}{} = 0$

Let $u = x^2+y^2$, $x \to 0$, $y \to 0 \Rightarrow u \to 0^+$

$$\lim_{t\to 0} \frac{1-\cos t}{t} = 0$$

$$\lim_{(x,y)\to(0,0)} f(x,y) = \lim_{u\to 0^+} \frac{\sin u}{u} = 1$$

$$\boxed{\lim_{(x,y)\to(0,0)} f(x,y) = 1} \quad \text{— II}$$

From eq (I) & (II) we get

$$\boxed{\lim_{(x,y)\to(0,0)} f(x,y) = 1 = f(0,0)}$$

This implies $f(x,y)$ is continuous at $(0,0)$

Ans.

Q39 Removable Discontinuity

let $f(x,y) = \dfrac{x^2}{x^2+y^2}$

First we check $\lim_{(x,y)\to(0,0)} f(x,y)$

Case-I along y-axis: $x=0$

$$\lim_{y\to 0} \frac{0}{0+y^2} = 0$$

Case-II along x-axis: $y=0$

$$\lim_{x\to 0} \frac{x^2}{x^2+0} \Rightarrow \lim_{x\to 0} \frac{x^2}{x^2} \Rightarrow \lim_{x\to 0} 1 = 1$$

Since limit is different along two different path

Therefore, limit does not exist.

Thus, $f(x,y)$ does not have removable discontinuity.

Q40 $\lim_{(x,y)\to(0,0)} f(x,y) = 0$ & $f(0,0) = -4$

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Obviously $\lim_{(x,y)\to(0,0)} f(x,y) \neq f(0,0)$

To remove discontinuity we us assume $f(0,0) = 0$ i.e.

$$f(x,y) = \begin{cases} x^2 + 7y^2, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

Ans.

$\lim_{x\to a} f(x)$ ✓

$f(a)$ ✓

$$f(x,y) = \begin{cases} \cancel{x^2 + 7y^2}, & \text{If } (x,y) \neq (0,0) \\ -4, & \text{If } (x,y) = (0,0) \end{cases}$$

$\lim_{(x,y)\to(0,0)} x^2 + 7y^2 = 0 \to$ limit exist

$f(0,0) = -4$

$f(x,y) = x^2 + 7y^2, \quad (x,y) \neq (0,0)$

$f(0,0) = ?$

$f(x,y) = 0$ If $(x,y) = (0,0)$