MVC - Task

23K-2001
BCS 2J
Date:

Q1. Determine whether $f(x,y)$ has a removable discontinuity at $(0,0)$

a. $$f(x,y) = \frac{x^2}{x^2+y^2}$$

At $x=0 \rightarrow$

$$\lim_{x\to 0} f(0,y) = \frac{0^2}{0^2+y^2} = 0$$

At $y=0 \rightarrow$

$$\lim_{y\to 0} f(x,0) = \frac{x^2}{x^2+0} = 1$$

At $y=x \rightarrow$

$$\lim_{y\to x} f(x,x) = \frac{x^2}{x^2+x^2} = \frac{1}{2}$$

Since different values,
hence;
limit does not exist

at $f(0,0) \rightarrow$

$$f(0,0) = \frac{0^2}{0^2+0^2} = \frac{0}{0} \text{ (indeterminate form)}$$

$\rightarrow$ Discontinuity is non-removable!

Ans.

6. $f(x) = \begin{cases} x^2+7y^2, & \text{if } (x,y) \neq (0,0) \\ -4, & \text{if } (x,y) = (0,0) \end{cases}$

$\lim_{x,y \to 0,0} f(x,y) = 0^2 + 7(0)^2$

$\lim_{x,y \to 0,0} f(x,y) = 0$

But,

$\lim_{(x,y) \to 0,0} f(x,y) \neq f(0,0)$

$0 \neq -4$

Hence, Limit exists and the discontinuity is removable.

Q2. $\nabla . (\nabla \times F)$: ?

$F(x,y,z) = e^{xz} i + 3xe^y j - e^{yz} k$

$\because \nabla \times F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^{xz} & 3xe^y & -e^{yz} \end{vmatrix}$

$= i\left[\frac{\partial}{\partial y}(-e^{yz}) - \frac{\partial}{\partial z}(3xe^y)\right] - j\left[\frac{\partial}{\partial x}(-e^{yz}) - \frac{\partial}{\partial z} e^{xz}\right] + k\left[\frac{\partial}{\partial x} 3xe^y - \frac{\partial}{\partial y} e^{xz}\right]$

$= i\left[-ze^{yz} - 0\right] - j\left[0 - xe^{xz}\right] + k\left[3e^y - 0\right]$

$= -ze^{yz} i + xe^{xz} j + 3e^y k$

URBANE PAPER PRODUCT

$\because \nabla . (\nabla \times F) = \frac{\partial}{\partial x}(-ze^{yz}) + \frac{\partial}{\partial y}(xe^{xz}) + \frac{\partial}{\partial z}(3e^y)$

$= 0 + 0 + 0$

$\nabla . (\nabla \times F) = 0$ Ans.

Q3. $\nabla \times (\nabla \times F)$: ?

$F(x,y,z) = xy j + yxz k$

$\because \nabla \times F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & xy & xyz \end{vmatrix}$

$= i\left[\frac{\partial}{\partial y}(xyz) - \frac{\partial}{\partial z}(xy)\right] - j\left[\frac{\partial}{\partial x}(xyz) - \frac{\partial}{\partial z}(0)\right]$

$+ k\left[\frac{\partial}{\partial x}(xy) - \frac{\partial}{\partial y}(0)\right]$

$= i[xz - 0] - j[yz - 0] + k[y - 0]$

$\nabla \times F = xzi - yzj + ky$

$\nabla \times (\nabla \times F) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xz & -yz & y \end{vmatrix}$

$= i\left[\frac{\partial}{\partial y}(y) - \frac{\partial}{\partial z}(-yz)\right] - j\left[\frac{\partial}{\partial x} y - \frac{\partial}{\partial z}(xz)\right] + k\left[\frac{\partial}{\partial x}(-yz) - \frac{\partial}{\partial y}(xz)\right]$

$= i[1 + y] - j[0 - x] + k[0 - 0]$

$= (1+y)i + xj$

Ans.