

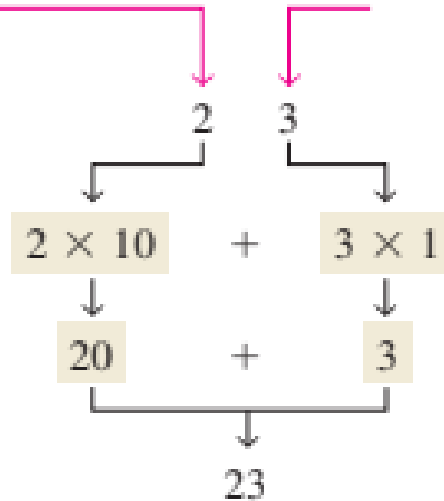
# NUMBER SYSTEMS

**Sumaiyah Zahid**

# DECIMAL NUMBER SYSTEM

The digit 2 has a weight of 10 in this position.

The digit 3 has a weight of 1 in this position.



# DECIMAL NUMBER SYSTEM

## EXAMPLE 2-2

Express the decimal number 568.23 as a sum of the values of each digit.

### Solution

The whole number digit 5 has a weight of 100, which is  $10^2$ , the digit 6 has a weight of 10, which is  $10^1$ , the digit 8 has a weight of 1, which is  $10^0$ , the fractional digit 2 has a weight of 0.1, which is  $10^{-1}$ , and the fractional digit 3 has a weight of 0.01, which is  $10^{-2}$ .

$$\begin{aligned} 568.23 &= (5 \times 10^2) + (6 \times 10^1) + (8 \times 10^0) + (2 \times 10^{-1}) + (3 \times 10^{-2}) \\ &= (5 \times 100) + (6 \times 10) + (8 \times 1) + (2 \times 0.1) + (3 \times 0.01) \\ &= \quad \mathbf{500} \quad + \quad \mathbf{60} \quad + \quad \mathbf{8} \quad + \quad \mathbf{0.2} \quad + \quad \mathbf{0.03} \end{aligned}$$

### Related Problem

Determine the value of each digit in 67.924.

# BINARY NUMBER SYSTEM

There are two digits in the binary number system, 1 and 0.

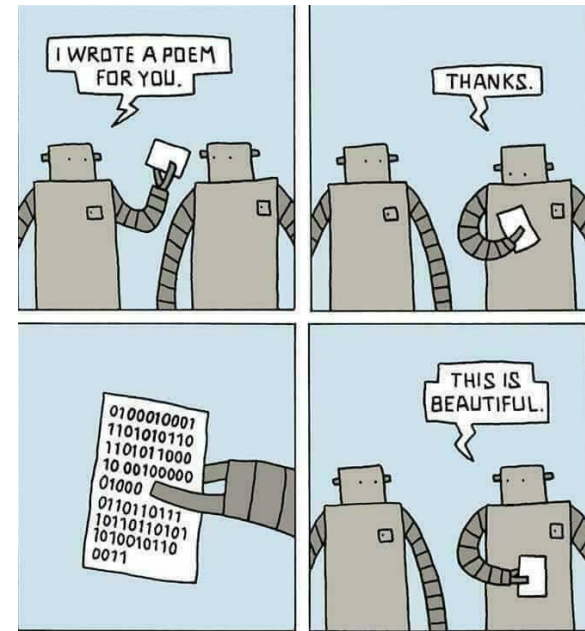
Each digit is called a bit.

**Binary + Digit = Bit**

There are only 10  
types of people  
in the world:  
Those who understand binary  
and those who don't.

# BINARY COUNTING

With  $n$  bits you can count, 0 to  $2^n - 1$ .



# BINARY COUNTING

**TABLE 2-1**

Decimal Number	Binary Number			
0	0	0	0	0
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
12	1	1	0	0
13	1	1	0	1
14	1	1	1	0
15	1	1	1	1

**TABLE 2-2**

Binary weights.

Positive Powers of Two (Whole Numbers)									Negative Powers of Two (Fractional Number)					
$2^8$	$2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$	$2^{-1}$	$2^{-2}$	$2^{-3}$	$2^{-4}$	$2^{-5}$	$2^{-6}$
256	128	64	32	16	8	4	2	1	1/2	1/4	1/8	1/16	1/32	1/64
									0.5	0.25	0.125	0.625	0.03125	0.015625

**InfoNote**

Processors use binary numbers to select memory locations. Each location is assigned a unique number called an *address*. Some microprocessors, for example, have 32 address lines which can select  $2^{32}$  (4,294,967,296) unique locations.

# BINARY TO DECIMAL CONVERSION

Convert the binary whole number 1101101 to decimal.



# BINARY TO DECIMAL CONVERSION

Convert the binary whole number 1101101 to decimal.

## EXAMPLE 2-3

Convert the binary whole number 1101101 to decimal.

### Solution

Determine the weight of each bit that is a 1, and then find the sum of the weights to get the decimal number.

$$\begin{array}{rcll} \text{Weight:} & 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ \text{Binary number:} & 1 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1101101 & = & 2^6 & + & 2^5 & + & 2^3 & + & 2^2 & + & 2^0 \\ & = & 64 & + & 32 & + & 8 & + & 4 & + & 1 & = & \mathbf{109} \end{array}$$

### Related Problem

Convert the binary number 10010001 to decimal.

# BINARY TO DECIMAL CONVERSION

Convert the fractional binary number 0.1011 to decimal.

# BINARY TO DECIMAL CONVERSION

Convert the fractional binary number 0.1011 to decimal.

## EXAMPLE 2-4

Convert the fractional binary number 0.1011 to decimal.

### Solution

Determine the weight of each bit that is a 1, and then sum the weights to get the decimal fraction.

$$\begin{array}{rcccc} \text{Weight:} & 2^{-1} & 2^{-2} & 2^{-3} & 2^{-4} \\ \text{Binary number:} & 0 & 1 & 0 & 1 \\ 0.1011 & = 2^{-1} + 2^{-3} + 2^{-4} \\ & = 0.5 + 0.125 + 0.0625 = \mathbf{0.6875} \end{array}$$

### Related Problem

Convert the binary number 10.111 to decimal.

# PRACTICE QUESTIONS

1. What is the largest decimal number that can be represented in binary with eight bits?
2. Determine the weight of the 1 in the binary number 10000.
3. Convert the binary number 10111101.011 to decimal.



# DECIMAL TO BINARY CONVERSION

Sum-of-Weights Method:

$$\begin{array}{cccc} 2^3 & 2^2 & 2^1 & 2^0 \\ 1 & 0 & 0 & 1 \end{array} \quad \text{Binary number for decimal 9}$$

Repeated Division Method:

Decimal to Binary

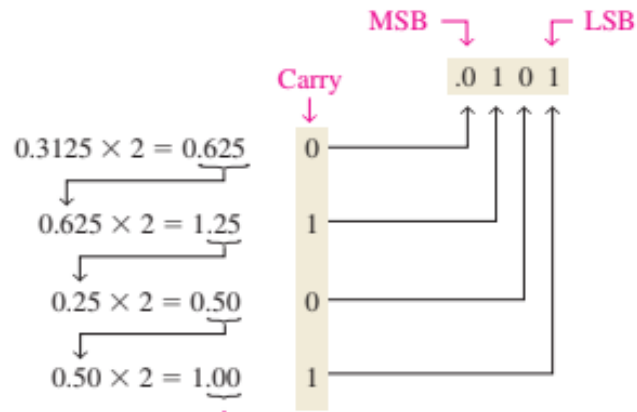
Remainder

$$(47)_{10} = (101111)_2$$

# DECIMAL FRACTIONS TO BINARY CONVERSION

Sum-of-Weights Method:  $0.625 = 0.5 + 0.125 = 2^{-1} + 2^{-3} = 0.101$

Repeated Multiplication Method:



# PRACTICE QUESTIONS

1. Convert each decimal number to binary by using the sum-of-weights method:

(a) 23 (b) 57 (c) 45.5

1. Convert each decimal number to binary by using the repeated division-by-2 method (repeated multiplication-by-2 for fractions):

(a) 14 (b) 21 (c) 0.375



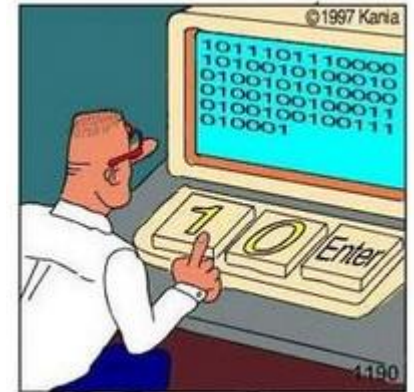
# BINARY ARITHMETIC

## Binary Addition

$0 + 0 = 0$	Sum of 0 with a carry of 0
$0 + 1 = 1$	Sum of 1 with a carry of 0
$1 + 0 = 1$	Sum of 1 with a carry of 0
$1 + 1 = 10$	Sum of 0 with a carry of 1

Add the following binary numbers:

- (a)  $11 + 11$       (b)  $100 + 10$   
(c)  $111 + 11$       (d)  $110 + 100$



Real programmers code in binary.



# BINARY ARITHMETIC

Add the following binary numbers:

- (a)  $11 + 11$       (b)  $100 + 10$   
(c)  $111 + 11$     (d)  $110 + 100$

## Solution

The equivalent decimal addition is also shown for reference.

$$\begin{array}{r} \text{(a)} \quad 11 \quad 3 \\ + 11 \quad + 3 \\ \hline 110 \quad 6 \end{array}$$

$$\begin{array}{r} \text{(b)} \quad 100 \quad 4 \\ + 10 \quad + 2 \\ \hline 110 \quad 6 \end{array}$$

$$\begin{array}{r} \text{(c)} \quad 111 \quad 7 \\ + 11 \quad + 3 \\ \hline 1010 \quad 10 \end{array}$$

$$\begin{array}{r} \text{(d)} \quad 110 \quad 6 \\ + 100 \quad + 4 \\ \hline 1010 \quad 10 \end{array}$$

## Related Problem

Add 1111 and 1100.

# BINARY ARITHMETIC

## Binary Subtraction

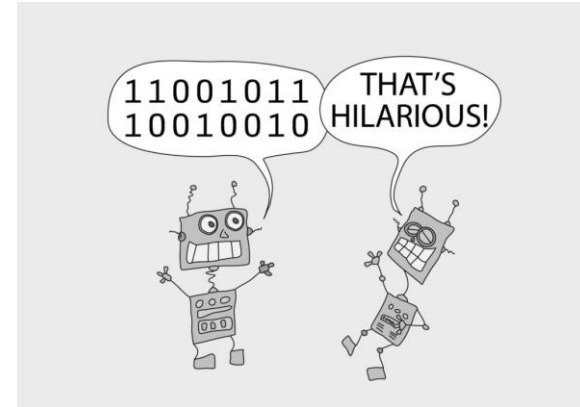
$$0 - 0 = 0$$

$$1 - 1 = 0$$

$$1 - 0 = 1$$

$$10 - 1 = 1 \quad 0 - 1 \text{ with a borrow of } 1$$

Subtract 011 from 101



# BINARY ARITHMETIC

## Binary Multiplication

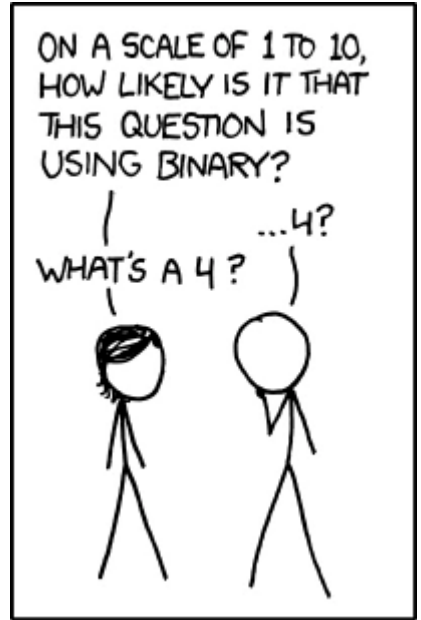
$$0 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 0 = 0$$

$$1 \times 1 = 1$$

Multiply 101 with 111



# BINARY ARITHMETIC

Binary Multiplication

$$0 \times 0 = 0$$

$$0 \times 1 = 0$$

$$1 \times 0 = 0$$

$$1 \times 1 = 1$$

Multiply 101 with 111

# BINARY ARITHMETIC

Binary Division

Divide 110 by 10

# PRACTICE QUESTIONS

1. Perform the following binary additions:

(a)  $1101 + 1010$  (b)  $10111 + 01101$

2. Perform the following binary subtractions:

(a)  $1101 - 0100$  (b)  $1001 - 0111$

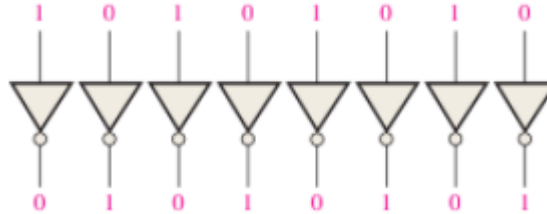
3. Perform the indicated binary operations:

(a)  $110 * 111$  (b)  $1100 \div 011$



# COMPLEMENT OF BINARY NUMBER

1's Complement



2's complement = (1's complement) + 1

$$\begin{array}{r} 10110010 \\ 01001101 \\ + \quad \quad 1 \\ \hline \mathbf{01001110} \end{array}$$

Binary number  
1's complement  
Add 1  
2's complement

# SIGNED NUMBER

## Sign Magnitude Form

The left-most bit in a signed binary number is the sign bit, which tells you whether the number is positive or negative.

0 -> positive number

1 -> negative number

e.g:

+25=00011001

-25=10011001

### EXAMPLE 2-15

Determine the decimal value of this signed binary number expressed in sign-magnitude: 10010101.

#### Solution

The seven magnitude bits and their powers-of-two weights are as follows:

$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
0	0	1	0	1	0	1

Summing the weights where there are 1s,

$$16 + 4 + 1 = 21$$

The sign bit is 1; therefore, the decimal number is **-21**.

#### Related Problem

Determine the decimal value of the sign-magnitude number 01110111.



# SIGNED NUMBER

## 1's Complement Form

+ve number is same as sign magnitude form

-ve number is the 1's complement of the corresponding positive number

e.g:

+25=00011001

-25=11100110

### EXAMPLE 2-16

Determine the decimal values of the signed binary numbers expressed in 1's complement:

- (a) 00010111      (b) 11101000

#### Solution

- (a) The bits and their powers-of-two weights for the positive number are as follows:

$-2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
0	0	0	1	0	1	1	1

Summing the weights where there are 1s,

$$16 + 4 + 2 + 1 = +23$$

# SIGNED NUMBER

## 2's Complement Form

+ve number is same as sign magnitude form

-ve number is the 2's complement of the corresponding positive number

e.g:

+25=00011001

-25=11100111

### EXAMPLE 2-17

Determine the decimal values of the signed binary numbers expressed in 2's complement:

- (a) 01010110      (b) 10101010

#### Solution

- (a) The bits and their powers-of-two weights for the positive number are as follows:

$-2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
0	1	0	1	0	1	1	0

Summing the weights where there are 1s,

$$64 + 16 + 4 + 2 = \mathbf{+86}$$

- (b) The bits and their powers-of-two weights for the negative number are as follows. Notice that the negative sign bit has a weight of  $-2^7 = -128$ .

$-2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
1	0	1	0	1	0	1	0

Summing the weights where there are 1s,

$$-128 + 32 + 8 + 2 = \mathbf{-86}$$

#### Related Problem

Determine the decimal value of the 2's complement number 11010111.

# PRACTICE QUESTIONS

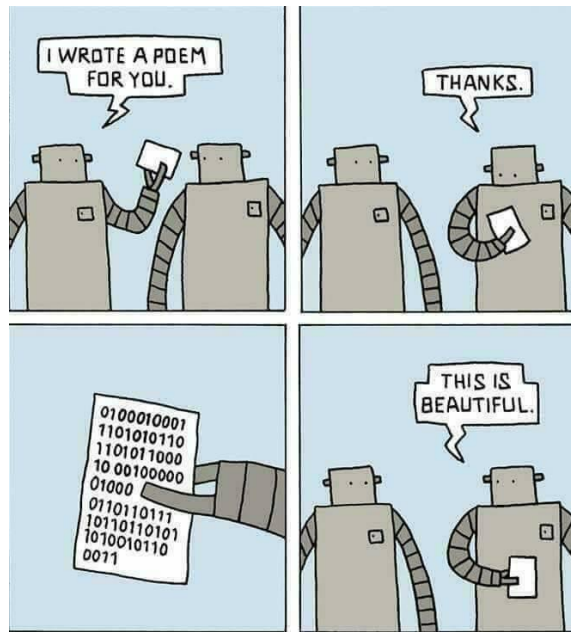
1. Express the decimal number +9 as an 8-bit binary number in the sign-magnitude system.
2. Express the decimal number 233 as an 8-bit binary number in the 1's complement system.
3. Express the decimal number 246 as an 8-bit binary number in the 2's complement system.



# BINARY COUNTING FOR SIGNED NUMBERS

With  $n$  bits you can count,

$$-2^{(n-1)} \text{ to } 2^{(n-1)} - 1$$



# ADDITION WITH SIGNED NUMBERS

1. Both numbers positive
2. Positive number with magnitude larger than negative number
3. Negative number with magnitude larger than positive number
4. Both numbers negative

# ADDITION WITH SIGNED NUMBERS

1 Both numbers positive:

$$\begin{array}{r} 00000111 \quad 7 \\ + 00000100 \quad + 4 \\ \hline 00001011 \quad 11 \end{array}$$

The sum is positive and is therefore in true (uncomplemented) binary.

**Positive number with magnitude larger than negative number:**

$$\begin{array}{r} 00001111 \quad 15 \\ + 11111010 \quad + -6 \\ \hline \text{Discard carry} \longrightarrow 1 \quad 00001001 \quad 9 \end{array}$$

The final carry bit is discarded. The sum is positive and therefore in true (uncomplemented) binary.

# ADDITION WITH SIGNED NUMBERS

3. **Negative number with magnitude larger than positive number:**

$$\begin{array}{r} 00010000 \\ + 11101000 \\ \hline 11111000 \end{array} \qquad \begin{array}{r} 16 \\ + -24 \\ \hline -8 \end{array}$$

The sum is negative and therefore in 2's complement form.

4. **Both numbers negative:**


$$\begin{array}{r} 11111011 \\ + 11110111 \\ \hline 11110010 \end{array} \qquad \begin{array}{r} -5 \\ + -9 \\ \hline -14 \end{array}$$


Discard carry  $\longrightarrow$  1


The final carry bit is discarded. The sum is negative and therefore in 2's complement form.

# OVERFLOW CONDITION

	01111101	125
	+ 00111010	+ 58
	<hr/>	<hr/>
	10110111	183

Sign incorrect 

Magnitude incorrect 





# SUBTRACTION WITH SIGNED NUMBERS

To subtract two signed numbers, take the 2's complement of the subtrahend and add. Discard any final carry bit.

(a)  $00001000 - 00000011$  (b)  $00001100 - 11110111$

# SUBTRACTION WITH SIGNED NUMBERS

## EXAMPLE 2-20

Perform each of the following subtractions of the signed numbers:

- (a)  $00001000 - 00000011$       (b)  $00001100 - 11110111$   
(c)  $11100111 - 00010011$       (d)  $10001000 - 11100010$

### Solution

Like in other examples, the equivalent decimal subtractions are given for reference.

- (a) In this case,  $8 - 3 = 8 + (-3) = 5$ .

	00001000	Minuend (+8)
	+ 1111101	2's complement of subtrahend (-3)
Discard carry →	<u>1 00000101</u>	Difference (+5)

- (b) In this case,  $12 - (-9) = 12 + 9 = 21$ .

	00001100	Minuend (+12)
	+ 00001001	2's complement of subtrahend (+9)
	<u>00010101</u>	Difference (+21)

# MULTIPLICATION WITH SIGNED NUMBERS

- **If the signs are the same, the product is positive.**
  - **If the signs are different, the product is negative.**
1. Determine if the signs of the multiplicand and multiplier are the same or different. This determines what the sign of the product will be.
  2. Change any negative number to true (uncomplemented) form.
  3. Multiply both numbers.
  4. If the sign bit that was determined in step 1 is negative, take the 2's complement of the product. If positive, leave the product in true form. Attach the sign bit to the product.

# MULTIPLICATION WITH SIGNED NUMBERS

Multiply the signed binary numbers: 01010011 (multiplicand)  
and 11000101 (multiplier).

# MULTIPLICATION WITH SIGNED NUMBERS

Multiply the signed binary numbers: 01010011 (multiplicand)  
and 11000101 (multiplier).

Attach the sign bit



**1 011001101111**

# DIVISION WITH SIGNED NUMBERS

- **If the signs are the same, the result is positive.**
  - **If the signs are different, the result is negative.**
1. Determine if the signs of the divisor and dividend are the same or different. This determines what the sign of the result will be.
  2. Change any negative number to true (uncomplemented) form.
  3. Divide both numbers.
  4. If the sign bit that was determined in step 1 is negative, take the 2's complement of the product. If positive, leave the product in true form. Attach the sign bit to the product.

# DIVISION WITH SIGNED NUMBERS

Divide the signed binary numbers:

00001001 / 11111101

# PRACTICE QUESTIONS

1. Add the signed numbers 00100001 and 10111100.

1. Subtract the signed numbers 00110010 from 01110111.

1. Multiply 01111111 by 00000101.

1. Divide 00110000 by 00001100.





# HEXADECIMAL NUMBER SYSTEM

Counting in Hexadecimal

**TABLE 2-3**

Decimal	Binary	Hexadecimal
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

# BINARY TO HEXADECIMAL

Convert the following binary numbers to hexadecimal:

(a) 1100101001010111 (b) 111111000101101001

# BINARY TO HEXADECIMAL

## EXAMPLE 2-24

Convert the following binary numbers to hexadecimal:

- (a) 1100101001010111      (b) 111111000101101001

### Solution

$$\begin{array}{lcl} \text{(a)} & \begin{array}{cccc} \overbrace{1100} & \overbrace{1010} & \overbrace{0101} & \overbrace{0111} \\ \downarrow & \downarrow & \downarrow & \downarrow \\ C & A & 5 & 7 \end{array} & = \mathbf{CA57}_{16} \\ \text{(b)} & \begin{array}{ccccc} \overbrace{0011} & \overbrace{1111} & \overbrace{1000} & \overbrace{1011} & \overbrace{01001} \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 3 & F & 1 & 6 & 9 \end{array} & = \mathbf{3F169}_{16} \end{array}$$

Two zeros have been added in part (b) to complete a 4-bit group at the left.

### Related Problem

Convert the binary number 1001111011110011100 to hexadecimal.

# HEXADECIMAL TO BINARY

Determine the binary numbers for the following hexadecimal numbers:

(a) 10A416 (b) CF8E16 (c) 974216

# HEXADECIMAL TO BINARY

## EXAMPLE 2-25

Determine the binary numbers for the following hexadecimal numbers:

- (a)  $10A4_{16}$       (b)  $CF8E_{16}$       (c)  $9742_{16}$

### Solution

(a)  $\begin{array}{cccc} 1 & 0 & A & 4 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 1000010100100 \end{array}$

(b)  $\begin{array}{cccc} C & F & 8 & E \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 1100111110001110 \end{array}$

(c)  $\begin{array}{cccc} 9 & 7 & 4 & 2 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 1001011101000010 \end{array}$

In part (a), the MSB is understood to have three zeros preceding it, thus forming a 4-bit group.

### Related Problem

Convert the hexadecimal number  $6BD3$  to binary.

# HEXADECIMAL TO DECIMAL

Convert the following hexadecimal numbers to decimal:

(a) 1C16 (b) A8516

# HEXADECIMAL TO DECIMAL

## EXAMPLE 2-26

Convert the following hexadecimal numbers to decimal:

- (a)  $1C_{16}$       (b)  $A85_{16}$

### Solution

Remember, convert the hexadecimal number to binary first, then to decimal.

(a)

$$\begin{array}{cc} 1 & C \\ \downarrow & \downarrow \\ \overbrace{0001} & \overbrace{1100} \end{array} = 2^4 + 2^3 + 2^2 = 16 + 8 + 4 = \mathbf{28}_{10}$$

(b)

$$\begin{array}{ccc} A & 8 & 5 \\ \downarrow & \downarrow & \downarrow \\ \overbrace{1010} & \overbrace{1000} & \overbrace{0101} \end{array} = 2^{11} + 2^9 + 2^7 + 2^2 + 2^0 = 2048 + 512 + 128 + 4 + 1 = \mathbf{2693}_{10}$$

### Related Problem

Convert the hexadecimal number 6BD to decimal.

# HEXADECIMAL TO DECIMAL

## EXAMPLE 2-27

Convert the following hexadecimal numbers to decimal:

- (a)  $E5_{16}$       (b)  $B2F8_{16}$

### Solution

Recall from Table 2-3 that letters A through F represent decimal numbers 10 through 15, respectively.

$$(a) \ E5_{16} = (E \times 16) + (5 \times 1) = (14 \times 16) + (5 \times 1) = 224 + 5 = \mathbf{229}_{10}$$

$$\begin{aligned}(b) \ B2F8_{16} &= (B \times 4096) + (2 \times 256) + (F \times 16) + (8 \times 1) \\ &= (11 \times 4096) + (2 \times 256) + (15 \times 16) + (8 \times 1) \\ &= \quad 45,056 \quad + \quad 512 \quad + \quad 240 \quad + \quad 8 \quad = \mathbf{45,816}_{10}\end{aligned}$$

### Related Problem

Convert  $60A_{16}$  to decimal.



# DECIMAL TO HEXADECIMAL

Convert the decimal number 650 to hexadecimal by repeated division by 16.

# HEXADECIMAL ADDITION

Add the following hexadecimal numbers:

(a)  $23_{16} + 16_{16}$     (b)  $58_{16} + 22_{16}$

# HEXADECIMAL SUBTRACTION

Subtract the following hexadecimal numbers:

(a)  $84_{16} - 2A_{16}$       (b)  $C3_{16} - 0B_{16}$

# PRACTICE QUESTIONS

1. Convert the following binary numbers to hexadecimal: (a) 10110011 (b) 11001110100
1. Convert the following hexadecimal numbers to binary: (a) 57 (b) 3A5 (c) F80B
1. Convert 9B3016 to decimal.
1. Convert the decimal number 573 to hexadecimal.
1. Add the following hexadecimal numbers directly:  
(a) 18 + 34 (b) 3F + 2A
2. Subtract the following hexadecimal numbers: (a) 75  
- 21 (b) 94 - 5C



# OCTAL NUMBER SYSTEM

## Counting in Octal

**TABLE 2-4**

Octal/binary conversion.

Octal Digit	0	1	2	3	4	5	6	7
Binary	000	001	010	011	100	101	110	111

# DECIMAL TO OCTAL AND OCTAL TO DECIMAL

Convert the following decimal numbers to octal:

(a) 359

Convert the following octal numbers to decimal:

(a) 2374

# DECIMAL TO OCTAL AND OCTAL TO DECIMAL

Convert the following decimal numbers to octal:

(a) 359 -----> 547 in octal

Convert the following octal numbers to decimal:

(a) 2374 -----> 1276 in decimal

# BINARY TO OCTAL AND OCTAL TO BINARY

Convert the following binary numbers to octal:

(a) 110101 (b) 101111001

Convert the following octal numbers to binary:

(a) 140 (b) 7526



# BINARY TO OCTAL AND OCTAL TO BINARY

Convert the following binary numbers to octal:

(a) 110101 (b) 101111001

(a)  $\begin{array}{cc} 110101 \\ \downarrow \downarrow \\ 6 \quad 5 = 65_8 \end{array}$

(b)  $\begin{array}{ccc} 101111001 \\ \downarrow \downarrow \downarrow \\ 5 \quad 7 \quad 1 = 571_8 \end{array}$

Convert the following octal numbers to binary:

(c) 140 (d) 7526

(c)  $\begin{array}{ccc} 1 & 4 & 0 \\ \downarrow & \downarrow & \downarrow \\ 001 & 100 & 000 \\ \hline 001100000 \end{array}$

(d)  $\begin{array}{cccc} 7 & 5 & 2 & 6 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 111 & 101 & 010 & 110 \\ \hline 111101010110 \end{array}$

# PRACTICE QUESTIONS

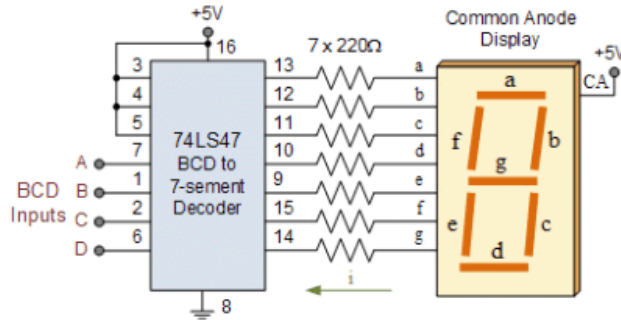


1. Convert the following octal numbers to decimal:  
(a) 73 (b) 125
2. Convert the following decimal numbers to octal:  
(a) 9810 (b) 16310
3. Convert the following octal numbers to binary:  
(a) 468 (b) 7238 (c) 56248
4. Convert the following binary numbers to octal:  
(a) 110101111 (b) 1001100010 (c) 10111111001

# BINARY CODED DECIMAL

- 8421 BCD Code
- Invalid Codes
- Digital clocks
- Digital thermometers
- Digital meters

## What is Binary Coded Decimal?



Decimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001



**Electrical 4 U**

# DECIMAL TO BCD AND BCD TO DECIMAL

Convert each of the following decimal numbers to BCD:

(a) 35 (b) 98 (c) 170 (d) 2469

Convert each of the following BCD codes to decimal:

(a) 10000110 (b) 001101010001 (c) 1001010001110000

# DECIMAL TO BCD AND BCD TO DECIMAL

## EXAMPLE 2-33

Convert each of the following decimal numbers to BCD:

- (a) 35      (b) 98      (c) 170      (d) 2469

### Solution

- (a)  $\begin{array}{cc} 3 & 5 \\ \downarrow & \downarrow \\ \overbrace{00110101} \end{array}$
- (b)  $\begin{array}{cc} 9 & 8 \\ \downarrow & \downarrow \\ \overbrace{10011000} \end{array}$
- (c)  $\begin{array}{ccc} 1 & 7 & 0 \\ \downarrow & \downarrow & \downarrow \\ \overbrace{000101110000} \end{array}$
- (d)  $\begin{array}{cccc} 2 & 4 & 6 & 9 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \overbrace{0010010001101001} \end{array}$

### Related Problem

Convert the decimal number 9673 to BCD.

# DECIMAL TO BCD AND BCD TO DECIMAL

## EXAMPLE 2-34

Convert each of the following BCD codes to decimal:

- (a) 10000110      (b) 001101010001      (c) 1001010001110000

### Solution

- (a)  $\begin{array}{c} 10000110 \\ \downarrow \quad \downarrow \\ 8 \quad 6 \end{array}$       (b)  $\begin{array}{c} 001101010001 \\ \downarrow \quad \downarrow \quad \downarrow \\ 3 \quad 5 \quad 1 \end{array}$       (c)  $\begin{array}{c} 1001010001110000 \\ \downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ 9 \quad 4 \quad 7 \quad 0 \end{array}$

### Related Problem

Convert the BCD code 10000010001001110110 to decimal.

# BCD ADDITION

Add the following BCD numbers:

(a)  $0011 + 0100$  (b)  $00100011 + 00010101$

(c)  $10000110 + 00010011$  (d)  $010001010000 + 010000010111$

# BCD ADDITION

## EXAMPLE 2-35

Add the following BCD numbers:

(a)  $0011 + 0100$

(b)  $00100011 + 00010101$

(c)  $10000110 + 00010011$

(d)  $010001010000 + 010000010111$

### Solution

The decimal number additions are shown for comparison.

(a) 
$$\begin{array}{r} 0011 \quad 3 \\ + 0100 \quad + 4 \\ \hline 0111 \quad 7 \end{array}$$

(b) 
$$\begin{array}{r} 0010 \quad 0011 \quad 23 \\ + 0001 \quad 0101 \quad + 15 \\ \hline 0011 \quad 1000 \quad 38 \end{array}$$

(c) 
$$\begin{array}{r} 1000 \quad 0110 \quad 86 \\ + 0001 \quad 0011 \quad + 13 \\ \hline 1001 \quad 1001 \quad 99 \end{array}$$

(d) 
$$\begin{array}{r} 0100 \quad 0101 \quad 0000 \quad 450 \\ + 0100 \quad 0001 \quad 0111 \quad + 417 \\ \hline 1000 \quad 0110 \quad 0111 \quad 867 \end{array}$$

Note that in each case the sum in any 4-bit column does not exceed 9, and the results are valid BCD numbers.



# BCD ADDITION

Add the following BCD numbers:

(a)  $1001 + 0100$  (b)  $1001 + 1001$

(c)  $00010110 + 00010101$  (d)  $01100111 + 01010011$

# BCD ADDITION

## Solution

The decimal number additions are shown for comparison.

(a)

1001	9
+ 0100	+4
1101	13
+ 0110	
<u>0001</u> <u>0011</u>	
↓        ↓	
1        3	

Invalid BCD number ( $>9$ )  
Add 6  
Valid BCD number

(b)

1001	9
+ 1001	+ 9
1 0010	18
+ 0110	
<u>0001</u> <u>1000</u>	
↓        ↓	
1        8	

Invalid because of carry  
Add 6  
Valid BCD number

# BCD ADDITION

(c)

$$\begin{array}{r} 0001 \quad 0110 \\ + 0001 \quad 0101 \\ \hline 0010 \quad 1011 \end{array}$$

$$+ 0110$$

$$\begin{array}{r} \overline{0011} \quad \overline{0001} \\ \downarrow \quad \downarrow \\ 3 \quad 1 \end{array}$$

Right group is invalid ( $>9$ ),  
left group is valid.

Add 6 to invalid code. Add  
carry, 0001, to next group.

Valid BCD number

$$\begin{array}{r} 16 \\ + 15 \\ \hline 31 \end{array}$$

(d)

$$\begin{array}{r} 0110 \quad 0111 \\ + 0101 \quad 0011 \\ \hline 1011 \quad 1010 \\ + 0110 \quad + 0110 \\ \hline \overline{0001} \quad \overline{0010} \quad \overline{0000} \\ \downarrow \quad \downarrow \quad \downarrow \\ 1 \quad 2 \quad 0 \end{array}$$

Both groups are invalid ( $>9$ )

Add 6 to both groups

Valid BCD number

$$\begin{array}{r} 67 \\ + 53 \\ \hline 120 \end{array}$$

# PRACTICE QUESTIONS

1. Convert the following decimal numbers to BCD:

(a) 6 (b) 15 (c) 273 (d) 849

1. What decimal numbers are represented by each BCD code?

(a) 10001001 (b) 001001111000

(c) 000101010111



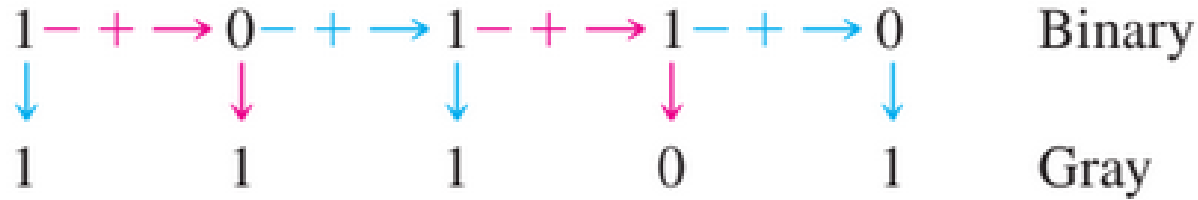
# GRAY CODE

**TABLE 2-6**

Four-bit Gray code.

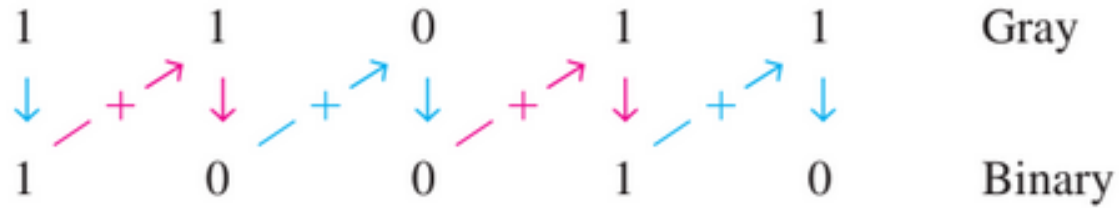
Decimal	Binary	Gray Code	Decimal	Binary	Gray Code
0	0000	0000	8	1000	1100
1	0001	0001	9	1001	1101
2	0010	0011	10	1010	1111
3	0011	0010	11	1011	1110
4	0100	0110	12	1100	1010
5	0101	0111	13	1101	1011
6	0110	0101	14	1110	1001
7	0111	0100	15	1111	1000

# BINARY TO GRAY CODE



(a) Convert the binary number 11000110 to Gray code.

# GRAY TO BINARY CODE



(b) Convert the Gray code 10101111 to binary.