Date:

Ex 13.6 Directional Derivatives

$D_u f(x,y,z) = \nabla f \cdot u$

$\nabla f(x,y) = f_x i + f_y j$

$\nabla f(x,y,z) = f_x i + f_y j + f_z k$

$u = u_1 i + u_2 j$

$u = u_1 i + u_2 j + u_3 k$

$D_u f(x,y,z) = f_x u_1 + f_y u_2 + f_z u_3$

3. $f(x,y) = \ln(1+x^2+y)$; $P(0,0)$

$$u = \underbrace{-\frac{1}{\sqrt{10}}}_{u_1} i \underbrace{- \frac{3}{\sqrt{10}}}_{u_2} j$$

$$D_u f(0,0) = \frac{2x}{1+x^2+y}\left(-\frac{1}{\sqrt{10}}\right) + \frac{1}{1+x^2+y}\left(-\frac{3}{\sqrt{10}}\right)$$

Since $P(0,0)$, = substitute $(0,0)$

$$= -\frac{3}{\sqrt{10}}$$

Date: ______________

13. $f(x,y) = \tan^{-1}(y/x)$; $P(-2,2)$; $a = \cancel{5i - 2j}$ $-i-j$

$$u = \frac{a}{\|a\|} = \frac{-i-j}{\sqrt{2}} = -\frac{1}{\sqrt{2}}i - \frac{1}{\sqrt{2}}j$$

$$D_u f(-2,2) = \frac{-y/x^2}{1+\frac{y^2}{x^2}}\left(-\frac{1}{\sqrt{2}}\right) + \frac{1/x}{1+\frac{y^2}{x^2}}\left(-\frac{1}{\sqrt{2}}\right)$$

$$= \frac{-2/(-2)^2}{1+\frac{2^2}{(-2)^2}}\left(-\frac{1}{\sqrt{2}}\right) + \frac{1/-2}{1+\frac{2^2}{(-2)^2}}\left(-\frac{1}{\sqrt{2}}\right)$$

$$= \frac{1}{2\sqrt{2}}$$

21. $f(x,y) = \tan(2x+y)$; $P(\pi/6, \pi/3)$; $\theta = \frac{7\pi}{4}$

$u = \cos\theta i + \sin\theta j$ // When θ given, $u = \cos\theta i + \sin\theta j$

$$u = \cos\left(\frac{7\pi}{4}\right) i + \sin\left(\frac{7\pi}{4}\right) j$$

$$u = \frac{1}{\sqrt{2}} i - \frac{1}{\sqrt{2}} j$$

$$D_u f(\pi/6, \pi/3) = 2\sec^2(2x+y)\left(\frac{1}{\sqrt{2}}\right) + \sec^2(2x+y)\left(-\frac{1}{\sqrt{2}}\right)$$

$$= \sqrt{2}\sec^2\left(2\left(\frac{\pi}{6}\right) + \frac{\pi}{3}\right) - \frac{1}{\sqrt{2}}\sec^2\left(\frac{2}{3}\pi\right)$$

$$= 2\sqrt{2}$$

Date:

23. $f(x,y) = \frac{x}{x+y}$ at $P(1,0)$ in direction $Q(-1,-1)$

$\vec{PQ} = \cancel{\vec{P}-\vec{Q}} = \vec{Q}-\vec{P} = -2i - j = a$

$u = \frac{a}{\|a\|} = \frac{-2}{\sqrt{\cancel{2^2}(-2)^2+(-1)^2}} i - \frac{1}{\sqrt{5}} j$

$= -\frac{2}{\sqrt{5}} i - \frac{1}{\sqrt{5}} j$

$D_u f(1,0) = \frac{(x+y)(1) - x(1+0)}{(x+y)^2}\left(\frac{-2}{\sqrt{5}}\right) + \frac{(x+y)(0) - x(0+1)}{(x+y)^2}\left(\frac{-1}{\sqrt{5}}\right)$

$= \frac{\cancel{x}(1+0)(1) - 1(1)}{(1+0)^2}\left(\frac{-2}{\sqrt{5}}\right) + \frac{\cancel{(1+0)} - 1(0+1)}{(1+0)^2}\left(\frac{-1}{\sqrt{5}}\right)$

$= 0 + \cancel{\frac{2\sqrt{5}}{5} = \frac{2\sqrt{5}}{5}}\ \frac{\sqrt{5}}{5} = \frac{\sqrt{5}}{5} = \frac{1}{\sqrt{5}}$

25. $f(x,y) = \sqrt{xy}\, e^{y}$; $P(1,1)$ in direction negative y-axis

Since negative y-axis, $Q(1,-1)$

$\cancel{\vec{PQ} = \vec{Q} - \vec{P}}\ \vec{PQ} = 0i + (-1-1)j = -2j$

$u = \frac{PQ}{|PQ|} = \frac{-2}{\sqrt{(-2)^2}} j = -j$

MIGHTY PAPER PRODUCT

Date:

~~28.~~ 29. $D_u f(1,2) = -5$, $D_v f(1,2) = 10$

$u = \frac{3}{5} i - \frac{4}{5} j$, $v = \frac{4}{5} i + \frac{3}{5} j$

$D_u f = f_x u_1 + f_y u_2$

$-5 = f_x \left(\frac{3}{5}\right) - f_y \left(4/5\right) \longrightarrow (1)$

$D_v f = f_x (4/5) + f_y (3/5)$

$10 = f_x (4/5) + f_y (3/5) \longrightarrow (2)$

Date: ____________

30.

$a = \overrightarrow{PQ} = \overrightarrow{Q} - \overrightarrow{P} = i + 2j$

$u_x = \frac{1}{\sqrt{5}} i + \frac{2}{\sqrt{5}} j$

$D_u f(-5,1) = f_x(-5,1)\,u_x + f_y(-5,1)\,u_y$

$= -3\left(\frac{1}{\sqrt{5}}\right) + 2\left(\frac{2}{\sqrt{5}}\right)$

$= \frac{\sqrt{5}}{5} = \frac{1}{\sqrt{5}}$

40. $w = e^{-5x} \sec x^2yz$

$\nabla w = f_x i + f_y j + f_z k$

$= [-5e^{-5x} \sec x^2yz + e^{-5x}(\sec x^2yz \tan x^2yz)(2xyz)]i$
$+ [e^{-5x} \sec(x^2yz) \tan x^2yz\,(x^2z)]j + [e^{-5x} \sec x^2yz \tan x^2yz(x^2y)]k$

$\cancel{= [-5e\; 2xyze^{-5x} \sec(x^2yz) \tan(x^2yz)]i + [x^2ze^{-5x} \sec(x^2yz)\tan(x^2yz)}$
$\cancel{[x^2ye^{-5x}}$

$= [2xyze^{-5x} \sec(x^2yz)\tan(x^2yz) - 5e^{-5x} \sec x^2yz]i +$
$[x^2ze^{-5x} \sec(x^2yz)\tan(x^2yz)]j + [x^2ye^{-5x} \sec(x^2yz)\tan(x^2yz)]k$

41. $f(x,y) = 5x^2 + y^4$; $(4,2)$

$\nabla f = 10xi + 4y^3 j$

$\nabla f(4,2) = 40i + 32j$ (gradient of f at specified points)

Date: ______________

53. $f(x,y) = 4x^3y^2$; $P(-1,1)$

$\nabla f = f_x i + f_y j$

$= 12x^2y^2 i + 8x^3y j$

$\nabla f(-1,1) = 12i - 8j$

$u = \dfrac{\nabla f}{\|\nabla f\|} = \dfrac{12}{4\sqrt{13}} i - \dfrac{8}{4\sqrt{13}} j$

$u = \dfrac{3}{\sqrt{13}} i - \dfrac{2}{\sqrt{13}} j$

$D_u f(-1,1) = f_x u_1 + f_y u_2$

$= 12\left(\dfrac{3}{\sqrt{13}}\right) + (-8)\left(-\dfrac{2}{\sqrt{13}}\right) = 4\sqrt{13}$

66. $f(x,y,z) = 4e^{xy}\cos z$; $P(0, 1, \pi/4)$

$\nabla f = f_x i + f_y j + f_z k$

$= 4ye^{xy}\cos z\, i + 4xe^{xy}\cos z\, j - 4e^{xy}\sin z\, k$

$= \dfrac{4}{\sqrt{2}} i - 0j - \dfrac{4}{\sqrt{2}} k = 2\sqrt{2} i - 0j - 2\sqrt{2} k$

$u = -\dfrac{\nabla f}{\|\nabla f\|} = -\left(\dfrac{2\sqrt{2}}{4} i - 0j - \dfrac{2\sqrt{2}}{4} k\right)$ // $-u$ since f decreasing

$= -\dfrac{1}{\sqrt{2}} i + 0j + \dfrac{1}{\sqrt{2}} k$

$D_u f(0,1,\pi/4) = f_x u_1 + f_y u_2 + f_z u_3$

$= 2\sqrt{2}\left(-\dfrac{1}{\sqrt{2}}\right) \cancel{+} - 0(+0) - \dfrac{\cancel{4}\ 2\sqrt{2}}{\cancel{\sqrt{2}}}\left(\dfrac{1}{\sqrt{2}}\right) = -4$

Date:

63. $f(x,y) = \cos(3x-y)$; $P(\pi/6, \pi/4)$

$\nabla f = f_x i + f_y j$

$= -\sin(3x-y)(3)\, i + (-\sin(3x-y)(-1))\, j$

$= -3\sin(3x-y)\, i + \sin(3x-y)\, j$

$\nabla f(\pi/6, \pi/4) = -\frac{3\sqrt{2}}{2} i + \frac{\sqrt{2}}{2} j$

$u = \left(\frac{\nabla f}{\|\nabla f\|}\right) = +\frac{3\sqrt{10}}{10} i \mp \frac{\sqrt{10}}{10} j$

Date:

Ex 13.7: Tangent Planes and Normal Vectors

$$ax+by+cz+d=0$$
$$n = \langle a, b, c \rangle$$

$P(x_0, y_0, z_0)$

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

$$f(x,y,z) \quad , \quad P(x_0, y_0, z_0)$$

normal vector $= \nabla f(x_0, y_0, z_0) = f_x i + f_y j + f_z k$

$$n = \langle f_x, f_y, f_z \rangle$$

$$f_x(x-x_0)+f_y(y-y_0)+f_z(z-z_0)=0$$

Parametric Equations of Line:

$$\left.\begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \\ z &= z_0 + ct \end{aligned}\right\} \begin{aligned} &x_0 + f_x t \\ &y_0 + f_y t \\ &z_0 + f_z t \end{aligned}$$

Date:

3. $x^2+y^2+z^2=25$; $P(-3,0,4)$

$\nabla f = 2x\,i + 2y\,j + 2z\,k$

$\nabla f(-3,0,4) = -6i + 0j + 8k$

$n = \langle -6, 0, 8 \rangle$

$-6(x+3) + 0(y-0) + 8(z-4) = 0$

$-6x - 18 + 8z - 32 = 0$

$-6x + 8z \cancel{=50} - 50 = 0$ (Equation of plane ~~for~~ tangent at point P)

$x = -3 - 6t$

$y = 0 + 0t$

$z = 4 + 8t$

10. $z = \ln\sqrt{x^2+y^2}$; $P(-1,0,0)$

$\ln\sqrt{x^2+y^2} - z = 0$

$\nabla f = \left[\frac{1}{\sqrt{x^2+y^2}}\left(\frac{1}{2}\frac{(2x)}{\sqrt{x^2+y^2}}\right)\right]i \cancel{= \frac{x}{x^2+y^2}} + \left[\frac{1}{\sqrt{x^2+y^2}}\left(\frac{1}{2}\frac{(2y)}{\sqrt{x^2+y^2}}\right)\right]j + (-)k$

$\nabla f = \frac{x}{x^2+y^2}\,i + \frac{y}{x^2+y^2}\,j - k$

$\nabla f(-1,0,0) = -i \cancel{k} - k$

$n = \langle -1, 0, -1 \rangle$

$-1(x+1) + 0(y-0) - 1(z-0) = 0$

$\cancel{-x-1=0}$ $-x - z - 1 = 0$

$x = -1 - 1t$

$z = -t$

Date:

Ex #13.8: Absolute Extremas

1. (a) $f(x,y) = (x-2)^2 + (y+1)^2$

minima $= 0 = (2,-1)$

maxima $= \infty$ (no absolute max.)

Range $= [0, \infty)$

(b) $f(x,y) = 1 - x^2 - y^2$

Maxima $= 1 = (0,0)$

Minima $= -\infty$ (no absolute min.)

Range $(-\infty, 1]$

(c) $f(x,y) = x + 2y - 5$

Range $(-\infty, \infty)$ (No absolute extremas)

2. (a) $f(x,y) = 1 - (x+1)^2 - (y-5)^2$

Maxima $= 1$ $(-1, 5)$ $(-\infty, 1]$

Minima $= -\infty$ (D.N.E.)

(b) $f(x,y) = e^{xy}$

$(-\infty, \infty)$ Extremas D.N.E.

(c) $f(x,y) = x^2 - y^2$

~~minima = 0~~ $(-\infty, \infty)$ Extremas D.N.E.

Date: ____________

Ex 13.8: Relative Extrema

$z = f(x, y)$

$$\left.\begin{array}{l} f_x = 0 \\ f_y = 0 \end{array}\right\} \text{Critical Points}$$

Second Derivative Test:

$D = f_{xx} f_{yy} - f_{xy} f_{yx}$

$\quad = f_{xx} f_{yy} - (f_{xy})^2 \qquad (f_{xy} = f_{yx})$

1. $D > 0$ && $f_{xx}(x_0, y_0) > 0$ THEN relative minima at (x_0, y_0)
2. $D > 0$ && $f_{xx}(x_0, y_0) < 0$ THEN relative maxima at (x_0, y_0)
3. $D < 0$, THEN Saddle Point
4. $D = 0$ THEN no conclusion

Q: $f(x, y) = 4xy - x^4 - y^4$

$f_x = 4y - 4x^3 = 0$

$\quad y = x^3 \rightarrow 1$

$f_y = 4x - 4y^3 = 0$

$\quad x = y^3 \rightarrow 2$

$f_{xx} = -12x^2$

$f_{yy} = -12y^2$

$f_{xy} = 4$

~~$D = (12$~~

$y = x^3$

$y = (y^3)^3$

$y = y^9$

$y^9 - y = 0$

$y(y^8 - 1) = 0$

$y = 0, \pm 1$

$x = y^3$

$x = (x^3)^3$

$x = x^9$

$x = 0, \pm 1$

$(0,0), (1,1), (-1,-1)$

(x_0, y_0)	f_{xx}	f_{yy}	f_{xy}	D	Result
(0,0)	0	0	4	-16	Saddle Point
(1,1)	-12	-12	4	128	rel. max.
(-1,-1)	-12	-12	4	128	rel. max.

Date: ____________

12. $f(x,y) = xy - x^3 - y^2$

$f_x = y - 3x^2 = 0$

$y = 3x^2$

$f_y = x - 2y = 0$

$x = 2y$

$y = 3(2y)^2$

$y = 3(4y^2)$

$y = 12y^2$

$12y^2 - y = 0$

$y(12y-1) = 0$

$y = 0, \ 1/12$

$x = 2(3x^2)$

$x = 6x^2$

$6x^2 - x = 0$

$x(6x-1) = 0$

$x = 0, \ 1/6$

$(0,0), \ (1/6, 1/12)$

$f_{xx} = -6x$

$f_{yy} = -2$

$f_{xy} = 1$

(x_0, y_0)	f_{xx}	f_{yy}	f_{xy}	D	Result
$(0,0)$	0	-2	1	-1	saddle
$(1/6, 1/12)$	-1	-2	1	1	rel. max

14. $f(x,y) = xe^y$

$f_x = e^y = 0$

$y \ln e = \ln 0$

$y = \ln 0$ (Invalid)

$f_y = xe^y = 0$

$x = 0, \quad e^y = 0$

$y = \ln 0$ (Invalid)

No critical points thus no ~~se~~ rel. extremas

17. $f(x,y) = e^x \sin y$

$f_x = e^x \sin y = 0$

$e^x = 0, \quad \sin y = 0$

$x \ln e = \ln 0$ (D.N.E.)

$f_y = e^x \cos y$

$e^x = 0$ (D.N.E.), $\cos y = 0$

$\sin y = \cos y = 0$ (D.N.E.)

Critical points D.N.E. thus rel. extremas D.N.E

Date: ____________

Ex #13.9: Lagrange Multipliers

5. $f(x,y) = xy$; $4x^2+8y^2=16$

$\nabla f = \lambda \nabla g$

$yi + xj = \lambda(8x i + 16y j)$

Compare i components

$yi = 8x\lambda i$

$\lambda = y/8x$

$xj = 16y\lambda j$

$\lambda = x/16y$

$\frac{y}{8x} = \frac{x}{16y}$

$16y^2 = 8x^2$

$2y^2 = x^2$

$x = \pm\sqrt{2}\, y$

$4x^2+8y^2=16$

As $x^2 = 2y^2$

$4(2y^2)+8y^2=16$

$8y^2+8y^2=16$

$y^2=1$

$y=\pm 1$

$x=\sqrt{2}\,y$

When $y=\pm 1$, $x=\pm\sqrt{2}$

$x=-\sqrt{2}\,y$

When $y=\pm1$, $x=\pm\sqrt{2}$

(x_0, y_0)	$f(x,y)$
$(\sqrt{2}, 1)$	$\sqrt{2}$
$(\sqrt{2}, -1)$	$-\sqrt{2}$
$(-\sqrt{2}, 1)$	$-\sqrt{2}$
$(-\sqrt{2}, -1)$	$\sqrt{2}$

Substituting (x_0, y_0) into $f(x,y)$

Date:

7. $f(x,y) = 4x^3 + y^2 ; \; 2x^2 + y^2 = 1$

$\nabla f = \lambda \nabla g$

$12x^2 i + 2y j = \lambda(4x i + 2y j)$

$12x^2 = 4x\lambda$

$\cancel{\lambda = 3x}$ $12x^2 - 4x\lambda$

$4x(3x - \lambda) = 0$

$4x = 0 \qquad 3x - \lambda = 0$

$x = 0 \qquad x = \lambda/3$

$2y = 2y\lambda$

$\cancel{\lambda = 1}$ $\therefore 2y - 2y\lambda = 0$

$2y(1-\lambda) = 0$

$2y = 0 \qquad 1 - \lambda = 0$

$y = 0 \qquad \lambda = 1$

As $\lambda = 1$, $x = 1/3$

$2x^2 + y^2 = 1$

When $y = 0$,

$x = \pm 1/\sqrt{2}$, $\left(\pm\frac{1}{\sqrt{2}}, 0\right)$

When $x = 0$,

$y = \pm 1$ $(0, \pm 1)$

When $x = 1/3$

$y = \pm \sqrt{7}/3$, $\left(\frac{1}{3}, \pm\frac{\sqrt{7}}{3}\right)$

(x_0, y_0)	$f(x,y)$
$\left(\frac{1}{\sqrt{2}}, 0\right)$	$\sqrt{2}$
$\left(-\frac{1}{\sqrt{2}}, 0\right)$	$-\sqrt{2}$
$(0, 1)$	1
$(0, -1)$	1
$\left(\frac{1}{3}, \frac{\sqrt{7}}{3}\right)$	$25/27$
$\left(\frac{1}{3}, -\frac{\sqrt{7}}{3}\right)$	$25/27$

Max $= \sqrt{2} = \left(\frac{1}{\sqrt{2}}, 0\right)$

Min $= -\sqrt{2} = \left(-\frac{1}{\sqrt{2}}, 0\right)$

Date:

20 - $f(x, y, z) = 3x + 6y + 2z$; $2x^2 + 4y^2 + z^2 = 70$

$\nabla f = \lambda \nabla g$

$3i + 6j + 2k = \lambda(4xi + 8yj + 2zk)$

$3 = 4x\lambda$

$\lambda = 3/4x$

$6 = 8y\lambda$

$\lambda = \frac{3}{4y}$

$2 = 2z\lambda$

$\lambda = 1/z$

$\frac{\cancel{3}}{\cancel{4}x} = \frac{\cancel{3}}{\cancel{4}y}$

$y = x$

$\frac{3}{4y} = \frac{1}{z}$

$z = \frac{4y}{3}$

$2x^2 + 4y^2 + z^2 = 70$

As $x = y$ and $z = \frac{4y}{3}$,

$2y^2 + 4y^2 + \left(\frac{4y}{3}\right)^2 = 70$

$6y^2 + \frac{16y^2}{9} = 70$

$\frac{70}{9} y^2 = 70$

$y = \pm 3$

When $y = 3$,

$x = 3$

$z = 4$

When $y = -3$,

$x = -3$

$z = -4$

(x_0, y_0, z_0)	$f(x, y, z)$	
$(3, 3, 4)$	35	Max
$(-3, -3, -4)$	-35	Min

Date: ____________

11. $f(x,y,z) = xyz;\ x^2+y^2+z^2=1$

$\nabla f = \cancel{\nabla g}\ \lambda \nabla g$

$yzi + xzj + xyk = \lambda(2xi + 2yj + 2zk)$

$yz = 2x\lambda$

$\lambda = \frac{yz}{2x}$

$xz = 2y\lambda$

$\lambda = \frac{xz}{2y}$

$xy = 2z\lambda$

$\lambda = \frac{xy}{2z}$

$\frac{y\cancel{z}}{2x} = \frac{x\cancel{z}}{2y}$

$\cancel{2y^2 = 2x^2}$

$\cancel{y^2 = x^2}$

$\cancel{y=x}\quad \cancel{y = \pm x}$

$\cancel{\frac{xz}{2y} = \frac{xy}{2z}}$

$\cancel{2z^2 = 2y^2}$

$\cancel{z=y}\quad \cancel{z = \pm y}$

$\frac{yz}{2x} = \frac{xy}{2z}$

$2z^2y = 2x^2y$

$z^2y - x^2y = 0$

$y(z^2-x^2)=0$

$y=0 \qquad z^2-x^2=0$

$z = \pm x$

$y^2z = x^2z$

$y^2z - x^2z = 0$

$z(y^2-x^2)=0$

$z=0 \qquad y^2-x^2=0$

$y^2=x^2$

$y = \pm x$

$\cancel{x = \pm y}$

$\frac{xz}{2y} = \frac{xy}{2z}$

$2xz^2 = 2xy^2$

$xz^2 - xy^2 = 0$

$x(z^2-y^2)=0$

$x=0 \qquad z^2-y^2=0$

$y = \pm z\quad \cancel{z = \pm y}$

$x^2=z^2$

As $x=y$ and $z=y$

$y^2+y^2+y^2=1$

$y = \pm\frac{1}{\sqrt3}$

As $y^2=x^2$ and $z^2=x^2$,

$x^2+(x^2)+(x^2)=1$

$x^2 = 1/3$

$x = \pm 1/\sqrt3$

(x_0, y_0, z_0)	$f(x,y,z)$
$(-\frac{1}{\sqrt3}, -\frac{1}{\sqrt3}, -\frac{1}{\sqrt3})$	$-\sqrt3/9$
$(-1/\sqrt3, -1/\sqrt3, 1/\sqrt3)$	$\sqrt3/9$
$(-1/\sqrt3, 1/\sqrt3, -1/\sqrt3)$	$\sqrt3/9$
$(1/\sqrt3, -1/\sqrt3, -1/\sqrt3)$	$\sqrt3/9$
$(-1/\sqrt3, 1/\sqrt3, 1/\sqrt3)$	$-\sqrt3/9$
$(1/\sqrt3, 1/\sqrt3, -1/\sqrt3)$	$-\sqrt3/9$
$(1/\sqrt3, -1/\sqrt3, 1/\sqrt3)$	$-\sqrt3/9$
$(1/\sqrt3, 1/\sqrt3, 1/\sqrt3)$	$\sqrt3/9$

Date: ______

Gradient Descent Algorithm:

$f(x) = f(x_1, x_2) = 4x_1^2 + 3x_1x_2 + 2.5x_2^2 - 5.5x_1 - 4x_2$

$x_{n+1} = x_n \mp \alpha f'(x_n)$ n starts from 0

$x_1 = x_0 \mp \alpha f'(x_0)$

Initial Value Given

$x_0 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} \begin{matrix} x_1 \\ x_2 \end{matrix}$

Step Size

$\alpha = 0.135$, (Values are a guess, they should be within domain ~~and~~ of $f(x_1, x_2)$ and when plugged in $f'(x)$, $f'(x)$ should not be 0)

$f'(x) = \begin{bmatrix} 8x_1 + 3x_2 - 5.5 \\ 3x_1 + 5x_2 - 4 \end{bmatrix} \begin{matrix} \partial f/\partial x_1 \\ \partial f/\partial x_2 \end{matrix}$

$x_1 = x_0 \mp \alpha f'(x_0)$

$x_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix} - 0.135 \begin{bmatrix} 8x_1 + 3x_2 - 5.5 \\ 3x_1 + 5x_2 - 4 \end{bmatrix}$

$x_1 = \begin{bmatrix} -0.2275 \\ 0.38 \end{bmatrix} \begin{matrix} x_1 \\ x_2 \end{matrix}$ } place these in $f(x_1, x_2)$ to find $f(x_1)$

$f(x_1) = 0.0399$

Usually found till values converging or x_3, x_4

$x_2 = x_1 - \alpha f'(x_1)$

$= \begin{bmatrix} -0.2275 \\ 0.38 \end{bmatrix} - 0.135 \begin{bmatrix} 8x_1 + 3x_2 - 5.5 \\ 3x_1 + 5x_2 - 4 \end{bmatrix}$

$x_2 = \begin{bmatrix} 0.6068 \\ 0.7556 \end{bmatrix}$

$f(x_2) = -2.0842$

Date: ____________

$x_3 = x_2 - \alpha f'(x_2)$

$= \begin{bmatrix} 0.6068 \\ 0.7556 \end{bmatrix} - 0.135 \begin{bmatrix} 8(0.6068) + 3(0.7556) - 5.5 \\ 3(0.6068) + 5(0.7556) - 4 \end{bmatrix}$

$= \begin{bmatrix} 0.3879 \\ 0.5398 \end{bmatrix}$

$f(x_3) = \cancel{-2.7342}\ -2.3342$

$x_4 = x_3 - \alpha f'(x_3)$

$= \begin{bmatrix} 0.3879 \\ 0.5398 \end{bmatrix} - 0.135 \begin{bmatrix} 8(0.3879) + 3(0.5398) - 5.5 \\ 3(0.3879) + 5(0.5398) - 4 \end{bmatrix}$

$= \begin{bmatrix} 0.4928 \\ 0.5583 \end{bmatrix}$

$f(x_4) = -2.3676$

Date: ____________

Ex #24.1: Double Integrals

$f(x,y) - a \le x \le b, \ c \le y \le d$

If limits constant

$$\int_c^d \left[\int_a^b f(x,y)\,dx\right]dy = \int_a^b\left[\int_c^d f(x,y)\,dy\right]dx$$

Type I　　　　　Type II

1. $\int_0^1 \int_0^2 (x+3)\,dy\,dx$

First integrate w.r.t. y,

$\int_0^1 (x+3)y \Big|_0^2 dx$

$\int_0^1 2(x+3)\,dx$

$\int_0^1 2x+6\,dx$

$\frac{2x^2}{2} + 6x \Big|_0^1$

$x^2 + 6x \Big|_0^1$

7

9. $\int_0^1 \int_0^1 x(xy+1)^{-2}\,dy\,dx$

$\int (f(x))^n f'(x)\,dx = \frac{(f(x))^{n+1}}{n+1}$

$\int_0^1 \frac{(xy+1)^{-1}}{\cancel{x}\,-1} \cancel{x} \Big|_0^1 dx$

$\int_0^1 -\left(\frac{1}{x+1} - 1\right)dx$

$-\ln(x+1) + x \Big|_0^1$

$-\ln(2) + 1$

$1 - \ln 2$

$-\frac{1}{\cancel{x}x(1)+1} - \left(-\frac{1}{x(0)+1}\right)$

$1 - \frac{1}{x+1}$

$-[\ln(1+1) + 1 - \ln(0+1) + 0]$

$-[\ln 2 + 1 - \ln 1 + 0]$

$-(\ln 2 + 1)$

$R = \{(x,y) : 0 \le x \le 1, 0 \le y \le 1\}$
Rectangular Region R provides limits
Date: ____________

11. $\int_0^{\ln 2} \int_0^1 xye^{y^2x}\, dy\, dx$

$u = y^2x$

$\frac{du}{2} = xy\, dy$

$\frac{1}{2}\int_0^{\ln 2} \left[\int_0^1 \cancel{e^x}\, e^u\, \cancel{dx}\, du\right] dx$

$\frac{1}{2}\int_0^{\ln 2} e^{xy^2}\Big|_0^1 dx$

$e^{x(1)^2} - e^{x(0)}$

$\frac{1}{2}\int_0^{\ln 2} (e^x - 1)\, dx$

$\frac{1}{2}(e^x - x)\Big|_0^{\ln 2}$

$\frac{1}{2}\left[(2 - \ln 2) - (1 - 0)\right]$

$\frac{1}{2}(1 - \ln 2)$

14. $\int_0^1 \frac{1}{2}\int_0^1 2xy\,(x^2+y^2+1)^{1/2}\, dx\, dy$

$\frac{1}{2}\int_0^1 y\frac{(x^2+y^2+1)^{3/2}}{3/2}\Big|_0^1 dy$

$\frac{1}{2}\int_0^1 2y(y^2+2)^{1/2} - 2y(y^2+1)^{1/2}\, dy$

$\frac{1}{2}\left(\frac{(y^2+2)^{3/2}}{3/2} - \frac{(y^2+1)^{3/2}}{3/2}\right)\Big|_0^1$

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