Ch # 15

(Q17 – 28)

Q1

- Divergance $= \nabla \cdot F = \dfrac{\partial f}{\partial x} + \dfrac{\partial g}{\partial y} + \dfrac{\partial h}{\partial z}$

- Curl $= \nabla \times F = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f & g & h \end{vmatrix}$

- laplacian Eq: $\nabla \cdot \nabla$ or $\nabla^2 = 0$

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = 0$$

Q17) $F(x, y, z) = x^2 i - 2j + yz k$

- Divergance $(\nabla \cdot F) = \dfrac{\partial f}{\partial x} + \dfrac{\partial g}{\partial y} + \dfrac{\partial h}{\partial z}$

$$= 2x - 0 + y \rightarrow 2x + y$$

- Curl $(\nabla \times F) = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f & g & h \end{vmatrix} = \begin{vmatrix} i & j & k \\ \partial x & 0 & y \\ x^2 & -2 & yz \end{vmatrix}$

$$\cancel{= [0(yz) - (-2y)]i - [2x(yz) - x^2 y]j + [2x(-2) - x^2(0)]k}$$

$$\cancel{= 2y i}$$

$$= \left[\frac{\partial}{\partial y}(yz) - \frac{\partial}{\partial z}(-2)\right]i - \left[\frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial z}(x^2)\right]j$$

$$+ \left[\frac{\partial}{\partial x}(-2) - \frac{\partial}{\partial y}(x^2)\right]k$$

$(z-0)i - (0-0)j + (0+0)z$

$\Rightarrow zi$

18) $F(x,y,z) = xz^3 i + 2y^4 z^2 j + 5z^2 y k$

. Divergance $= z^3(1) + 2z^2(4y^3) + 5y(2z)$

$= z^3 + 28y^3z^2 + 10yz$

. Curl $= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f & g & h \end{vmatrix} = \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xz^3 & 2y^4z^2 & 5z^2y \end{vmatrix}$

$= \left\{\frac{\partial}{\partial y}(5z^2y) - \frac{\partial}{\partial z}(2y^4z^2)\right\} - \left\{\frac{\partial}{\partial x}(5z^2y) - \frac{\partial}{\partial z}(xz^3)\right\}j + \left\{\frac{\partial}{\partial x}(2y^4z^2) - \frac{\partial}{\partial y}(xz^3)\right\}k$

$= (5z^2 - 4y^4z)i - (0 - 3xz^2)j + (\overset{4y^4x}{\cancel{0}} - 0)k$

19) $7y^3z^2 i - 8x^2z^5 j - 3xy^4 k$

. Divergance $= 0 - 0 - 0 = 0$

. Curl $= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 7y^3z^2 & -8x^2z^5 & -3xy^4 \end{vmatrix} = i\left(\frac{\partial}{\partial y}(-3xy^4) - \frac{\partial}{\partial z}(-8x^2z^5)\right)$

$- j\left(\frac{\partial}{\partial x}(-3xy^4) - \frac{\partial}{\partial z}(7y^3z^2)\right) + k\left(\frac{\partial}{\partial x}(-8x^2z^5) - \frac{\partial}{\partial y}(7y^3z^2)\right)$

$= (-12xy^3 + 40x^2z^4)i - (-3y^4 - 14y^3z)j + (-16xz^5 - 21y^2z^2)k$

$= (40x^2z^4 - 12xy^3)i + (3y^4 + 14y^3z)j - (16xz^5 + 21y^2z^2)k$

Q20) $F(x,y,z) = e^{xy} i - \cos y\, j + \sin^2 z\, k$

• Divergance $= e^{xy}(y) + \sin y + 2\sin z(\cos z)(1)$

$= y e^{xy} + \sin y + 2\sin z \cos z$

• Curl $= \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^{xy} & \cos y & \sin^2 z \end{vmatrix}$

$= \left(\frac{\partial}{\partial y}(\sin^2 z) - \frac{\partial}{\partial z}(\cos y)\right)i - \left(\frac{\partial}{\partial x}(\sin^2 z) - \frac{\partial}{\partial z}(e^{xy})\right)j + \left(\frac{\partial}{\partial x}(\cos y) - \frac{\partial}{\partial y}(e^{x\cdot}\right.$

$= (0-0)i - (0-0)j + (0 - e^{xy}(x))k \rightarrow -xe^{xy}k$

Q21) $\frac{1}{\sqrt{x^2+y^2+z^2}}(xi + yj + zk) \rightarrow x(x^2+y^2+z^2)^{-1/2}$

• Divergance $= x\left[-\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2x)\right] + 1(x^2+y^2+z^2)^{-1/2}$

$= y\left[-\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2y)\right] + 1(x^2+y^2+z^2)^{-1/2}$

$= z\left[-\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2z)\right] + 1(x^2+y^2+z^2)^{-1/2}$

$= \frac{\cancel{2}x^2}{\cancel{2}(x^2+y^2+z^2)^{3/2}} + \frac{\cancel{2}y^2}{\cancel{2}(x^2+y^2+z^2)^{3/2}} + \frac{\cancel{2}z^2}{\cancel{2}(x^2+y^2+z^2)^{3/2}}$

$+ \frac{1}{(x^2+y^2+z^2)^{1/2}} + \frac{1}{(x^2+y^2+z^2)^{1/2}} + \frac{1}{(x^2+y^2+z^2)^{1/2}}$

$\frac{x^2+y^2+z^2+x^2+y^2+z^2}{(x^2+y^2+z^2)\sqrt{x^2+y^2+z^2}} \Rightarrow \frac{2\cancel{(x^2+y^2+z^2)}}{\cancel{(x^2+y^2+z^2)}\sqrt{x^2+y^2+z^2}}$

Curl: $\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \frac{x}{\sqrt{x^2+y^2+z^2}} & \frac{y}{\sqrt{x^2+y^2+z^2}} & \frac{z}{\sqrt{x^2+y^2+z^2}} \end{vmatrix} \Rightarrow$ Ez but lengthy

Q22) $F(x,y,z) = \ln(x)i + e^{xyz}j + \tan^{-1}\left(\frac{z}{x}\right)k$

· Divergance: $\partial \frac{1}{x} + e^{xyz}(xz) + \frac{1}{1+\frac{z^2}{x^2}}\left(\frac{1}{x}\right)$

$$= \frac{1}{x} + xze^{xyz} + \frac{x}{x^2+z^2}$$

· Curl: $\begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \ln x & e^{xyz} & \tan^{-1}\left(\frac{z}{x}\right) \end{vmatrix} = \left\{\frac{\partial}{\partial y}\left(\tan^{-1}\left(\frac{z}{x}\right)\right) - \frac{\partial}{\partial z}\left(e^{xyz}\right)\right\}i -$

$\left\{\frac{\partial}{\partial x}\left(\tan^{-1}\left(\frac{z}{x}\right)\right) - \frac{\partial}{\partial z}(\ln x)\right\}j +$

$\left\{\frac{\partial}{\partial x}\left(e^{xyz}\right) - \frac{\partial}{\partial y}(\ln x)\right\}k$

$$= \cancel{\frac{1(x^2)}{x^2+z^2}}\left(0 - xye^{xyz}\right)i - \left\{\frac{x^2}{x^2+z^2}\left(-x^{-2}z\right) - 0\right\}j + yze^{xyz}k$$

$$= -xye^{xyz}i - \frac{1}{1+\frac{z^2}{x^2}}\left(\frac{-z}{x^2}\right)j + k\left(yze^{xyz}\right)$$

Q23) $f(x,y,z) = 2xi + j + 4yk$ } $\nabla \cdot (F \times G)$
$g(x,y,z) = xi + yj - zk$ }

$$F \times G = \begin{vmatrix} i & j & k \\ 2x & 1 & 4y \\ x & y & -z \end{vmatrix} = i(-z - 4y^2) - j(-2xz - 4yx) + k(2xy - x)$$
$$= 0 + 4x(1) + 0 = 4x$$

Q24) $f(x,y,z) = yzi + xzj + xyk$
$g(x,y,z) = xyj + xyzk$

$$F \times G = \begin{vmatrix} i & j & k \\ yz & xz & xy \\ 0 & xy & xyz \end{vmatrix} = (x^2yz^2 - x^2y^2)i - (xy^2z^2 - 0)j + (y^2xz)k$$
$$= (2xyz^2 - 2xy^2)\hat{i} - \cancel{2yxz^2} + xy^2 = -xy^2$$

Q25) $f(x,y,z) = \sin(x)i + \cos(x-y)j + zk$

$$\nabla \times F \begin{vmatrix} i & j & k \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \sin x & \cos(x-y) & z \end{vmatrix} = \left(\frac{\partial}{\partial y}(z) - \frac{\partial}{\partial z}(\cos(x-y))\right)i - \left(\frac{\partial}{\partial x}(z) - \frac{\partial}{\partial z}(\sin x)\right)j + \left(\frac{\partial}{\partial x}(\cos(x-y)) - \frac{\partial}{\partial y}(\sin x)\right)$$

$$= (0-0)i - (0-0)j + (-\sin(x-y) - 0)k$$
$$= -\sin(x-y)k = 0$$

Q26) $f(x,y,z) = e^{xz} i + 3xe^{y} j - e^{yz} K$

$$\nabla \times F = \begin{vmatrix} i & j & K \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^{xz} & 3xe^{y} & -e^{yz} \end{vmatrix} = \left(\frac{\partial}{\partial y}(-e^{yz}) - \frac{\partial}{\partial z}(3e^{y}x)\right) i - \left(\frac{\partial}{\partial x}(-e^{yz}) - \frac{\partial}{\partial z}(e^{xz})\right) j + \left(\frac{\partial}{\partial x}(3e^{y}x) - \frac{\partial}{\partial y}(e^{xz})\right) K$$

$$= (-ze^{yz} - 0) i + (0 + e^{xz}x) j + \frac{\partial}{\partial z}(3e^{y})$$

$$= 0 + 0 + 0$$

Q27) $f(x,y,z) = xyj + xyzK$

$$\nabla \times F = \begin{vmatrix} i & j & K \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & xy & xyz \end{vmatrix} = \left\{\frac{\partial}{\partial y}(xyz) - \frac{\partial}{\partial z}(xy)\right\} i - \left\{\frac{\partial}{\partial x}(xyz) - 0\right\} j + \left\{\frac{\partial}{\partial x}(xy) - 0\right\} K$$

$$= \cancel{xy}\ xz i - yz j + y k$$

$$\nabla \times (\nabla \times F) = \begin{vmatrix} i & j & k \\ \cancel{xz}\, \partial/\partial x & -\cancel{yz}\, \partial/\partial y & \cancel{y}\, \partial/\partial z \\ xz & -yz & y \end{vmatrix} \left\{\frac{\partial}{\partial y}(y) - \frac{\partial}{\partial z}(-yz)\right\} i - \left\{\frac{\partial}{\partial x}(y) - \frac{\partial}{\partial z}(xz)\right\} j + \left\{\frac{\partial}{\partial x}(yz) - \frac{\partial}{\partial y}(xz)\right\} K$$

$$= (1 - (-y)) i - (0 - x) j + (0 - 0) K$$

$$= (1 + y) i + xj$$