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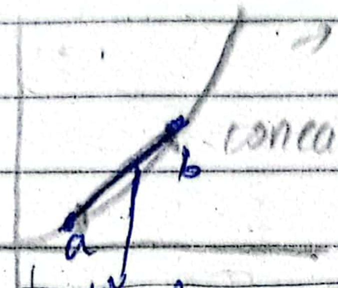
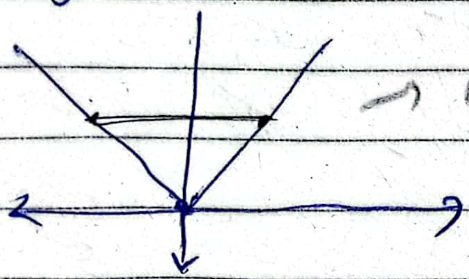
CONVEX OPTIMIZATION



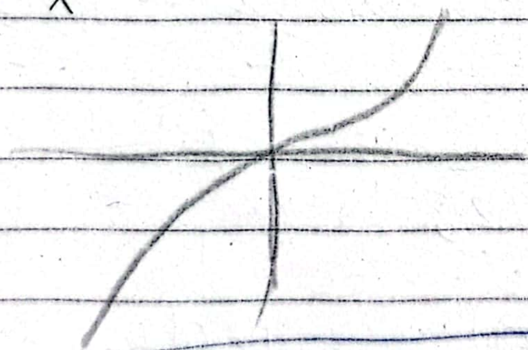
* for convex functions : this line must be on or above the function.

* minimum point can be found only in concave up functions.

$f(x) = |x|$



$f(x) = x^3$



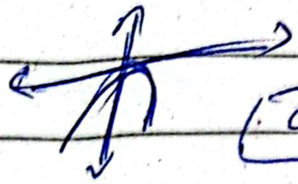
$\boxed{[0,\infty)}$ convex

$\boxed{(-\infty, 0]}$ concave

$[0, -\infty)$

Signature

$f(x) = -x^2$

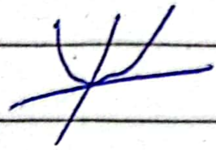


$\boxed{\text{concave}}$

if $+x^2$

then convex

i.e.



* to check if the function is convex.

① $f(tx_1, (1-t)x_2) <=$
$t\,f(x_1) + (1-t)f(x_2)$.

② **Hessian matrix:**

(i) if matrix is positive definite → convex
local minimum

(ii) if negative definite → concave.
local maximum

(iii) indefinite → saddle pt.

$$
\begin{array}{cc}
 & \text{for } n \\
3\times3 & \begin{vmatrix}
\frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\
\frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f_2}{\partial x_2 \partial x_2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\
\frac{\partial^2 f}{\partial x_3 \partial x_1} & \frac{\partial^2 f}{\partial x_3 \partial x_2} & \frac{\partial^2 f}{\partial x_3^2} & \frac{\partial^2 f}{\partial x_3 \partial x_n}
\end{vmatrix}
\end{array}
$$

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① +ve definite

Determinants

D_1 $1\times1 > 0$

D_2 $2\times2 > 0$

D_3 $3\times3 > 0$

then (x_1, x_2, x_3) is local minimum
relative

② -ve definite

D_1 1×1 < 0 (-ve)

D_2 2×2 > 0 (+ve)

D_3 3×3 < 0 (-ve)

* alternate signs but starting with negative.

then (x_1, x_2, x_3) is local/relative maxima.

③ indefinite

fails above 2 conditions

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$f(x,y,z) = x^4 + y^4 + z^4 + x^2 + y^2 + z^2$

$f_x = 4x^3 + 2x \Rightarrow 0$

$f_y = 4y^3 + 2y \Rightarrow 0$

$f_z = 4z^3 + 2z \Rightarrow 0$

$2x(2x^2 + 1) = 0 \quad \rightarrow x = 0$

$\cancel{x^2 = -1/2} \quad x^2 = -1/2$

$2y(2y^2 + 1) = 0 \quad \rightarrow y = 0$

$2z\underbrace{(2z^2 + 1)}_{\text{complex no.}} = 0 \quad \rightarrow z = 0$

$(0, 0, 0)$

$$\begin{vmatrix} 12x^2+2 & 0 & 0 \\ 0 & 12y^2+2 & 0 \\ 0 & 0 & 12z^2+2 \end{vmatrix}_{3\times 3}$$

$D_1 = 12x^2 + 2 \quad > 0$

$D_2 = (12x^2+2)(12y^2+2) - 0 \quad > 0$

$D_3 = (12x^2+2)(12y^2+2)(12z^2+2) > 0 \quad \because$ diagonal matrix.

this is +ve definite

* convex

* $(0,0,0)$ relative minima.

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Q) find all local ext and classify the

$$f(x,y) = x^3 - 3xy + y^3$$

$f_x = 3x^2 - 3y \rightarrow 0$

$f_y = 3y^2 - 3x \rightarrow 0$

$y = x^2 \leftarrow ①$

$x = y^2 \leftarrow ②$

$y = y^4$

$y^4 - y = 0$

$y(y^3 - 1) = 0$

$\boxed{y=0}$ $\quad$ $\boxed{y=1}$

$\downarrow$ $\quad$ $\downarrow$

$\boxed{x=0}$ $\quad$ $\boxed{x=1}$

$(0,0)$ & $(1,1)$

$$\begin{vmatrix} 6x & -3 \\ -3 & 6y \end{vmatrix} \qquad \begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{vmatrix}$$

$D_1 = 6x$

~~$D_2 = 18x + 18y$~~

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$D_2 = 36xy - 9$

at $(0,0)$

$D_1 = 0$

$D_2 = -9$

$(0,0)$ = saddle point.
(indefinite)

at $(1,1)$

$D_1 = 6 \quad (>0)$

$D_2 = 27 \quad (>0)$

$(1,1)$ = relative minima.
[+ve definite]