MVC - QUIZ #02 SOLUTION KEY - 01

Q#01 (3 marks)

$\nabla f(0,0) = 2i + \frac{3}{2}j$

The Direction derivative is maximum in the direction of ∇f.

So $u = \frac{\nabla f(0,0)}{\|\nabla f(0,0)\|} = \frac{2i + 3/2\, j}{5/2} = \frac{4}{5}i + \frac{3}{5}j$ $\boxed{2\text{ mark}}$

and its maximum value is $\|\nabla f(0,0)\| = \frac{5}{2}$. $\boxed{1\text{ mark}}$

Q#02 (4 marks)

Cone: $z^2 = x^2 + y^2$, $P(4,2,0)$.

Let (x,y,z) be the point on cone. The distance of (x,y,z) & $(4,2,0)$ is given as.

$D = \sqrt{(x-4)^2 + (y-2)^2 + z^2} \Rightarrow D^2 = (x-4)^2 + (y-2)^2 + z^2$ $\boxed{1\text{ mark}}$

Replace z^2 by $x^2 + y^2$.

$D^2 = (x-4)^2 + (y-2)^2 + x^2 + y^2 \rightarrow$ Minimized this for closest point

Let $g(x,y) = D^2 = (x-4)^2 + (y-2)^2 + x^2 + y^2$

$g_x = 2(x-4) + 2x = 4x - 8$, $g_y = 2(y-2) + 2y = 4y - 4$

Put $g_x = 0$ & $g_y = 0$.

$4x - 8 = 0 \Rightarrow \boxed{x = 2}$, $4y - 4 = 0 \Rightarrow \boxed{y = 1}$

The only critical point is $(2,1) \rightarrow \boxed{2\text{ mark}}$

$g_{xx} = 4$, $g_{yy} = 4$, $g_{xy} = 0$.

So $(2,1)$ is absolute minimum.

So the point on the cone is:

$z^2 = (2)^2 + (1)^2 = 5 \Rightarrow z = \pm\sqrt{5}$

$(2, 1, \pm\sqrt{5})$ is closest to $P(4,2,0)$

$\boxed{1\text{ mark}}$

Ans.

SOLUTION KEY-01

Q#03 (3 marks)

Surface: $x^2y^3z^4+xyz=2$, $P(2,1,-1)$

the equation of tangent plane is given as

$$F_x(x_0,y_0,z_0)(x-x_0)+F_y(x_0,y_0,z_0)(y-y_0)+$$
$$F_z(x_0,y_0,z_0)(z-z_0)=0 \quad \text{--- (I)}$$

let $F(x,y,z)=x^2y^3z^4+xyz+2$.

$F_x=2xy^3z^4+yz \Rightarrow F_x(2,1,-1)=4-1=3$

$F_y=3x^2y^2z^4+xz \Rightarrow F_y(2,1,-1)=12-2=10$.

$F_z=4x^2y^3z^3+xy \Rightarrow F_z(2,1,-1)=\underbrace{-16+2=-14}_{\boxed{2\text{ mark}}}$

(I) $\Rightarrow 3(x-2)+10(y-1)-14(z+1)=0$.

$\Rightarrow \boxed{3x+10y-14z-30=0} \longrightarrow \boxed{1\text{ mark}}$

Ans.

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MVC - QUIZ #02 SOLUTION KEY - 02

Q#01 (3 marks)

$\nabla f(0,0) = 2i - \frac{3}{2} j$

The directional derivative is minimum in the direction opposite to ∇f.

So

$$u = -\frac{\nabla f(0,0)}{\|\nabla f(0,0)\|} = -\frac{(2i - 3/2 j)}{5/2} = -\frac{4}{5} i + \frac{3}{5} j$$

$\boxed{2 \text{ mark}}$

and its minimum value is $-\|\nabla f(0,0)\| = -5/2$

$\boxed{1 \text{ mark}}$

Q#02 (4 marks)

Surface: $y^2 = 9 + xz$, $P(0,0,0)$

Let (x, y, z) be the point on surface. The distance of (x,y,z) & $(0,0,0)$ is given as

$D = \sqrt{x^2+y^2+z^2} \Rightarrow D^2 = x^2+y^2+z^2$ $\boxed{1 \text{ mark}}$

Replace y^2 by $9+xz$

$D^2 = x^2+9+xz+z^2 \rightarrow$ Minimized this for closest point.

Let $g(x,z) = x^2+9+xz+z^2$

$g_x = 2x+z$, $g_z = x+2z$

Put $g_x = 0$ & $g_z = 0$.

$2x+z=0$ — ①, $x+2z=0$ — ②.

$\Rightarrow \boxed{z=2x}$ & ② $\Rightarrow$ $x+4x-0 \Rightarrow \boxed{x=0}$, ① $\Rightarrow \boxed{z=0}$

The only critical point is $(0,0)$.

$\boxed{2 \text{ mark}}$

$g_{xx} = 1$, $g_{zz} = 2$, $g_{xz} = 1$.

$D = \{g_{xx}\, g_{zz}\} - \{g_{xz}\}^2 = 2-1 = 1 > 0$

& $g_{xx} - 1 > 0 \Rightarrow (0,0)$ is minimum point

So the point on the surface : $y^2 = 9 + xz$ is

$y^2 = 9 + 0 \Rightarrow \boxed{y = \pm 3}$

$(0, \pm 3, 0)$ is the point on the surface closest to origin. —— $\boxed{1 \text{ mark}}$

Q#03 (3 marks)

Surface: $x^3y^2z^4 - 2xyz = 12$, $P(2, 1, -1)$.

The equation of tangent plane is given as.

$$F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0 \longrightarrow (I).$$

Let $F(x, y, z) = x^3y^2z^4 - 2xyz - 12$

$F_x = 3x^2y^2z^4 - 2yz \Rightarrow F_x(2, 1, -1) = 12 + 2 = 14$

$\therefore F_y = 2x^3yz^4 - 2xz \Rightarrow F_y(2, 1, -1) = 16 + 4 = 20$

$\therefore F_z = 4x^3y^2z^3 - 2xy \Rightarrow F_z(2, 1, -1) = -32 - 4 = -36$ $\longrightarrow \boxed{2 \text{ mark}}$

$(I) \Rightarrow 14(x - 2) + 20(y - 1) - 36(z + 1) = 0$

$\Rightarrow 14x + 20y - 36z - 84 = 0.$

OR.

$7x + 10y - 18z - 42 = 0 \longrightarrow \boxed{1 \text{ mark}}$

Ans.