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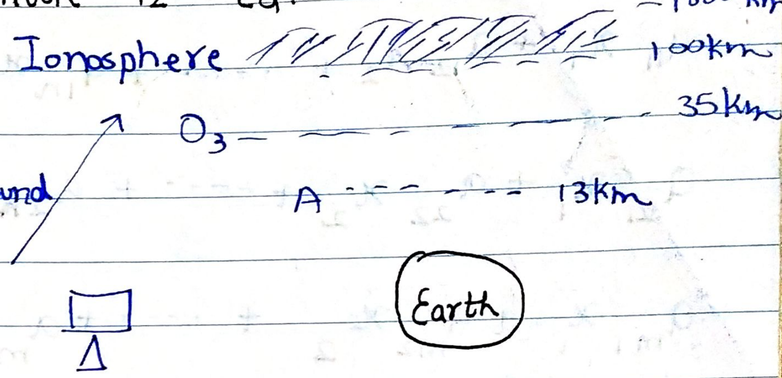
Linear Algebra (MT-1004)

Book: Elementary L.A, Harvard Anton 12^{th} Ed.

Teacher: Dr. Khusro Mian



$$\left.\begin{aligned} x+y&=1 \\ x+y+z&=4 \end{aligned}\right\} \text{Linear}$$

$x^2+y^2=1$ Non-Linear Eq

Delay is called Plasma Turbulance

Linear Eq: Every system of linear Eq has zero, one or infinitely many solution. There are no other possibilities.

Ex 1

$x+y=4$

$3x+3y=6$

$\hookrightarrow x+y=2$

$0=-6$ No Solution

$2x+y=6$

$x-y=1$

$(x,y) = (7/3, 4/3)$

one sol

$4x-2y=1$

$16x-8y=4$

$4x-2y=1$ parallel

$0=0$ Infinitely many sol

Parametric Eq

$4x-2y=1$

$x=\frac{1}{4}+\frac{y}{2}$

Let $y=t$

$t=0,1,2,3$

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Augmented Matrix

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_n$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & & a_{2n} & b_2 \\ \vdots & \vdots & & \vdots & \vdots \\ a_{m1} & a_{m2} & & a_{mn} & b_n \end{bmatrix}$$

Ex1

$$x_1 + x_2 + 2x_3 = 9$$
$$2x_1 + 4x_2 - 3x_3 = 1$$
$$3x_1 + 6x_2 - 5x_3 = 0$$

Augmented Matrix

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 2 & 4 & -3 & 1 \\ 3 & 6 & -5 & 0 \end{bmatrix}$$

Q) Find Augmented Matrix

$$2x_2 - 3x_4 + x_5 = 0$$
$$-3x_1 - x_2 + x_3 = -1$$
$$6x_1 + 2x_2 - x_3 + 3x_4 - 3x_5 = 6$$

$$\begin{bmatrix} 0 & 2 & -3 & 0 & 1 & 0 \\ -3 & -1 & 1 & 0 & 0 & -1 \\ 6 & 2 & -1 & 2 & -3 & 6 \end{bmatrix}$$

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Q) Find the system eq Ex 1.1

$$\begin{bmatrix} 3 & 0 & 1 & -4 & 3 \\ -4 & 0 & 4 & 1 & -3 \\ -1 & 3 & 0 & -2 & -9 \\ 0 & 0 & 0 & -1 & -2 \end{bmatrix}$$

$$\begin{aligned} 3x_1 + x_3 - 4x_4 &= 3 \\ -4x_1 + 4x_3 + x_4 &= -3 \\ -x_1 + 3x_2 - 2x_4 &= -9 \\ -x_4 &= -2 \end{aligned}$$

Q4) In each part, determine whether given 3-tuple is a set of linear system.

$2x_1 - 4x_2 - x_3 = 1$ — ①
$x_1 - 3x_2 + x_3 = 1$ — ②
$3x_1 - 5x_2 - 3x_3 = 1$ — ③

Yes → A $(3, 1, 1)$
B $(3, -1, 1)$
C $(13, 5, 2)$
Yes D $(\frac{13}{2}, \frac{5}{2}, 2)$
Yes E $(17, 7, 5)$

Q5) $3x - 2y = 4$
ⓐ $6x - 4y = 9 \xrightarrow{\div 2} 3x - 2y = 9/2$ No Solution

ⓑ $2x - 4y = 1$
$4x - 8y = 2 \xrightarrow{\div 2} 2x - 4y = 1$ Infinety many Sol

ⓒ $x - 2y = 0$
$x - 4y = 8$ One Sol $(-8, -4)$

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Echelon & Reduced Echelon form

$$EF\ \begin{bmatrix} 0 & 1 & -2 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & -2 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}\ NEF$$

3rd row have 2 zero < 2nd row

$$EF\ \begin{bmatrix} 1 & 2 & -3 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & -2 & 4 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}\ NEF$$

* Each Row should have equal or more zero's while moving down for being Echelon.
* Number of zero's are counted before leading number (1).

Reduced Echelon Form

$$\begin{bmatrix} 0 & 1 & \boxed{0} & 4 \\ 0 & \boxed{0} & 1 & 2 \\ 0 & \boxed{0} & \boxed{0} & 0 \end{bmatrix}\ EF$$

$$\begin{bmatrix} 1 & 2 & \boxed{0} & \boxed{0} \\ \boxed{0} & 0 & 1 & \boxed{0} \\ \boxed{0} & 0 & \boxed{0} & 1 \end{bmatrix}\ EF$$

$$\begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 1 & 3 & -1 \\ 0 & 0 & 1 & 2 \end{bmatrix}\ NEF$$

$$\begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\ NEF$$

Q) Solve the system eq.

$$x_1 - 3x_2 + 4x_3 = 7$$
$$x_2 + 2x_3 = 2$$
$$\boxed{x_3 = 5}$$

1) $\begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

$\boxed{x_2 = -8}$ One Solution

$\boxed{x_1 = -37}$

2) $x_1 + 8x_3 - 5x_4 = 6$ ①
$x_2 + 4x_3 - 9x_4 = 3$ ②
$x_3 + x_4 = 2$ ③

2) $\begin{bmatrix} 1 & 0 & 8 & -5 & 6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$

Applying Parametric (Arbitory)

③ → $x_3 = 2 - x_4$

$\boxed{x_4 = t}$

$\boxed{x_3 = 2 - t}$

② → $x_2 = 3 - 4x_3 + 9x_4$
$= 3 - 4(2-t) + 9t$
$\boxed{x_2 = -5 + 13t}$

Infinitely Many Solution

① → $x_1 = 6 - 8x_3 + 5x_4$
$= 6 - 8(2-t) + 5t$
$\boxed{x_1 = -10 + 13t}$

Ex1.2

Q1) Solve the system eq by Gauss-Jordan elimination [First Convert to Reduced Echelon form]

$$x_1 + x_2 + 2x_3 = 8$$
$$-x_1 - 2x_2 + 3x_3 = 1$$
$$3x_1 - 7x_2 + 4x_3 = 10$$

$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array} \begin{bmatrix} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{bmatrix}$$

$R_1 + \underline{R_2}$

$$\begin{array}{cccc} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ \hline 0 & -1 & 5 & 9 \end{array}$$

$-3R_1 + \underline{R_3}$

$$\begin{array}{cccc} -3 & -3 & -6 & -24 \\ 3 & -7 & 4 & 10 \\ \hline 0 & -10 & -2 & -14 \end{array}$$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{bmatrix}$$

$\underline{-R_2}$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & -10 & -2 & -14 \end{bmatrix}$$

$10R_2 + R_3$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -52 & -104 \end{bmatrix}$$

$\left(-\frac{1}{52}\right) R_3 \rightarrow$

$$\begin{bmatrix} 1 & \boxed{1} & \boxed{2} & 8 \\ 0 & 1 & \boxed{-5} & -9 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$5R_3 + R_2$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$\cancel{-2R_3 + R_1}$

$$\cancel{\begin{bmatrix} 0 & 0 & -2 & -4 \\ 1 & 1 & 2 & 8 \\ \hline 1 & 1 & 0 & 4 \end{bmatrix}}$$

$-2R_3 + R_1$

$$\begin{array}{cccc} 0 & 0 & -2 & -4 \\ 1 & 1 & 2 & 8 \\ \hline 1 & 1 & 0 & 4 \end{array}$$

$$\begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$-R_2 + R_1$

This is Reduced Echelon Form $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

$x_1 = 3$
$x_2 = 1$
$x_3 = 2$

Q2)

$$2x_1 + x_2 + 3x_3 = 0$$
$$x_1 + 2x_2 = 0$$
$$x_2 + x_3 = 0$$

$$\begin{bmatrix} 2 & 1 & 3 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$\cancel{R_1 - R_2}$ $-R_2 + R_1$

$$\begin{bmatrix} 1 & -1 & 3 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$R_1 - R_2$

$$\begin{bmatrix} 1 & -1 & 3 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$-\frac{1}{3} R_2$

$$\begin{bmatrix} 1 & -1 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

$R_2 - R_3$

$$\begin{bmatrix} 1 & -1 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -2 & 0 \end{bmatrix}$$

$-\frac{1}{2} R_3$

$$\begin{bmatrix} 1 & \boxed{-1} & \boxed{3} & 0 \\ 0 & 1 & \boxed{-1} & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Echelon form

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For R. E

$$\begin{bmatrix} 1 & -1 & 3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_2 + R_1$

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$-2R_3 + R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$x_1 = 0$
$x_2 = 0$
$x_3 = 0$

R. Echelon form

Q3) $3x_1 + x_2 + x_3 + x_4 = 0$
$5x_1 - x_2 + x_3 - x_4 = 0$

$$\begin{bmatrix} 3 & 1 & 1 & 1 & 0 \\ 5 & -1 & 1 & -1 & 0 \end{bmatrix}$$

Teacher

$$\begin{bmatrix} 3 & 1 & 1 & 1 & 0 \\ 5 & -1 & 1 & -1 & 0 \end{bmatrix}$$

$\frac{1}{3}R_1$ $$\begin{bmatrix} 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 5 & -1 & 1 & -1 & 0 \end{bmatrix}$$

$-5R_1 + R_2$

$$\begin{bmatrix} 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & -8/3 & -2/3 & -8/3 & 0 \end{bmatrix}$$

$(-3/8)\,R_2$

$$\begin{bmatrix} 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 \\ 0 & 1 & \frac{1}{4} & 1 & 0 \end{bmatrix}$$

$-\frac{1}{3}R_2 + R_1$

$$\begin{bmatrix} 1 & 0 & \frac{1}{4} & 0 & 0 \\ 0 & 1 & \frac{1}{4} & 1 & 0 \end{bmatrix}$$

Ex1.3

Q1) $A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 4 & 2 \\ 3 & 1 & 5 \end{bmatrix}$

$D = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}$, $E = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$

a) $\frac{1}{2} C^T - \frac{1}{4} A$

$$\frac{1}{2}\begin{bmatrix} 1 & 3 \\ 4 & 1 \\ 2 & 5 \end{bmatrix} - \frac{1}{4}\begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} \frac{1}{2} - \frac{3}{4} & \frac{3}{2} - 0 \\ 2 + \frac{1}{4} & \frac{1}{2} - \frac{1}{2} \\ 1 - \frac{1}{4} & \frac{5}{2} - \frac{1}{4} \end{bmatrix}$$

b) $= (D - E)^T$

$$\begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix} - \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} -5 & 4 & -1 \\ 0 & -1 & -1 \\ -1 & 1 & 1 \end{bmatrix}$$

After Transpose

$$\begin{bmatrix} -5 & 0 & -1 \\ 4 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

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ⓒ AB & BA

AB

$$\begin{bmatrix} \boxed{3 \quad 0} \\ -1 \quad 2 \\ 1 \quad 1 \end{bmatrix} \cdot \begin{bmatrix} \boxed{\begin{matrix}4\\0\end{matrix}} & -1 \\ & 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 12+0 & -3+0 \\ -4+0 & 1+4 \\ 4+0 & -1+2 \end{bmatrix}$$

$$\underbrace{3\times 2 \qquad 2\times 2}_{\text{should be same}} \qquad\qquad 3\times 2$$

BA

$$\begin{bmatrix} 4 & -1 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix} = \text{Not possible}$$

$$\underbrace{2\times 2 \qquad 3\times 2}_{\text{Diff, hence not possible}}$$

Q2) Find matrix A, x & b to express Linear system $Ax=b$

$$2x_1 - 3x_2 + 5x_3 = 7$$
$$9x_1 - x_2 + x_3 = -1$$
$$x_1 + 5x_2 + 4x_3 = 0$$

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 9 & -1 & 1 \\ 1 & 5 & 4 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad b = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix}$$

Q3) Express the matrix Eq

$$\begin{bmatrix} 5 & 6 & -7 \\ -1 & -2 & 3 \\ 0 & 4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}$$

$$5x_1 + 6x_2 - 7x_3 = 2$$
$$-x_1 - 2x_2 + 3x_3 = 0$$
$$4x_2 - x_3 = 3$$

Q4) $K = ?$

$$\underset{1\times 3}{\begin{bmatrix} K & 1 & 1 \end{bmatrix}} \underset{3\times 3}{\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 2 \\ 0 & 2 & -3 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} K \\ 1 \\ 1 \end{bmatrix}} = 0$$

$$\underset{1\times 3}{\begin{bmatrix} K+1+0 & K+0+2 & 2-3 \end{bmatrix}} \underset{3\times 1}{\begin{bmatrix} K \\ 1 \\ 1 \end{bmatrix}} = 0$$

$$\left[(K+1)K + (K+2)(1) + (-1)(1) \right] = 0$$

$$K^2 + K + K + 2 - 1 = 0$$
$$K^2 + 2K + 1 = 0$$
$$(K+1)^2 = 0$$
$$\boxed{K = -1}$$

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Ex 1.5

Q1) $A^{-1} = ?$ $\qquad A^{-1} = \dfrac{Adj\,(A)}{|A|}$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$[\,I \mid A^{-1}\,]$ Raw operation

$-2R_1 + R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array}\right]$$

$-R_1 + R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array}\right]$$

$2R_2 + R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & -1 & -5 & 2 & 1 \end{array}\right]$$

~~$(-1)R_3$~~ $(-1)R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -2 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$3R_3 + R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$-3R_3 + R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 0 & -14 & 6 & 3 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$-2R_2 + R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -48 & 16 & 9 \\ 0 & 1 & 0 & 13 & -5 & -3 \\ 0 & 0 & 1 & 5 & -2 & -1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -48 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$$ Ans

Q1) $A^{-1} = ?$

$$A = \begin{bmatrix} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 5 \end{bmatrix} \quad \begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}$$

~~$R_3 + R_1$~~ $2R_1 - R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & 8 & 9 & 2 & -1 & 0 \\ -1 & 2 & 5 & 0 & 0 & 1 \end{array}\right]$$

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$R_2/8$

$$\left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & 1 & 9/8 & 1/4 & -1/8 & 0 \\ -1 & 2 & 5 & 0 & 0 & 1 \end{array}\right]$$

$R_1 + R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & 1 & 9/8 & 1/4 & -1/8 & 0 \\ 0 & 8 & 9 & 1 & 0 & 1 \end{array}\right]$$

$8R_2 - R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 6 & 4 & 1 & 0 & 0 \\ 0 & 1 & 9/8 & 1/4 & -1/8 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 \end{array}\right]$$

A^{-1} Does Not Exist

Matrix is not invertible

Q2) $A^{-1} = ?$

$$A = \left[\begin{array}{ccc} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{array}\right] \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\Bigg]$$

$R_1 - R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & -1 \end{array}\right]$$

$R_2 + R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 2 & 1 & 1 & -1 \end{array}\right]$$

$R_3/2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array}\right]$$

Echelon
Epsilon

$-R_3 + R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & 1/2 & 1/2 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array}\right]$$

$-R_3 + R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 1/2 \\ 0 & 1 & 0 & -1/2 & 1/2 & 1/2 \\ 0 & 0 & 1 & 1/2 & 1/2 & -1/2 \end{array}\right]$$

Q3) $A^{-1} = ?$

$$\left[\begin{array}{ccc|ccc} -1 & 3 & -4 & 1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ -4 & 2 & -9 & 0 & 0 & 1 \end{array}\right]$$

$-R_1$

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 2 & 4 & 1 & 0 & 1 & 0 \\ -4 & 2 & -9 & 0 & 0 & 1 \end{array}\right]$$

$R_3 + 2R_2$

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 10 & -7 & 0 & 2 & 1 \\ -4 & 2 & -9 & 0 & 0 & 1 \end{array}\right]$$

$R_2/10$

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -7/10 & 0 & 1/5 & 1/10 \\ -4 & 2 & -9 & 0 & 0 & 1 \end{array}\right]$$

$4R_1 + R_3$

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -7/10 & 0 & 1/5 & 1/10 \\ 0 & -10 & 7 & -4 & 0 & 1 \end{array}\right]$$

$10R_2 + R_3$

$$\left[\begin{array}{ccc|ccc} 1 & -3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -7/10 & 0 & 1/5 & 1/10 \\ 0 & 0 & 0 & -4 & 2 & 2 \end{array}\right]$$

A^{-1} Does Not Exist

Matrix is not invertible.

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Q) Solve the system by inverting the coefficient matrix & using $Ax=b$

$-x-2y-3z=0$

$w+x+4y+4z=7$

$w+3x+7y+9z=4$

$-w-2x-4y-6z=6$

$Ax=b$

$X=A^{-1}b$

$X=bA^{-1}$ ✗

$$X=\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix}, \quad A=\begin{bmatrix} 0 & -1 & -2 & -3 \\ 1 & 1 & 4 & 4 \\ 1 & 3 & 7 & 9 \\ -1 & -2 & -4 & -6 \end{bmatrix} \quad b=\begin{bmatrix} 0 \\ 7 \\ 4 \\ 6 \end{bmatrix}$$

$[A \mid I]$

Raw operation.

$[I \mid A^{-1}]$

$$\left[\begin{array}{cccc|cccc} 0 & -1 & -2 & -3 & 1 & 0 & 0 & 0 \\ 1 & 1 & 4 & 4 & 0 & 1 & 0 & 0 \\ 1 & 3 & 7 & 9 & 0 & 0 & 1 & 0 \\ -1 & -2 & -4 & -6 & 0 & 0 & 0 & 1 \end{array}\right]$$

R_2+R_1

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 4 & 4 & 0 & 1 & 0 & 0 \\ 1 & 3 & 7 & 9 & 0 & 0 & 1 & 0 \\ -1 & -2 & -4 & -6 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_4 + R_2$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & -2 & 0 & 1 & 0 & 1 \\ 1 & 3 & 7 & 9 & 0 & 0 & 1 & 0 \\ -1 & -2 & -4 & -6 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_2/-1$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & -1 & 0 & -1 \\ 1 & 3 & 7 & 9 & 0 & 0 & 1 & 0 \\ -1 & -2 & -4 & -6 & 0 & 0 & 0 & 1 \end{array}\right]$$

$\cancel{R_3 + R_3}$ $R_4 + R_3$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & -1 & 0 & -1 \\ 0 & 1 & 3 & 3 & 0 & 0 & 1 & 1 \\ -1 & -2 & -4 & -6 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_2 - R_3$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & -1 & 0 & -1 \\ 0 & 0 & -3 & -1 & 0 & -1 & -1 & -2 \\ -1 & -2 & -4 & -6 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_3/-3$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1/3 & 0 & 1/3 & 1/3 & 2/3 \\ -1 & -2 & -4 & -6 & 0 & 0 & 0 & 1 \end{array}\right]$$

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$R_1 + R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1/3 & 0 & 1/3 & 1/3 & 2/3 \\ 0 & -2 & -2 & -5 & 1 & 1 & 0 & 1 \end{array}\right]$$

$2R_2 + R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1/3 & 0 & 1/3 & 1/3 & 2/3 \\ 0 & 0 & -2 & -1 & 1 & -1 & 0 & -1 \end{array}\right]$$

$2R_3 + R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1/3 & 0 & 1/3 & 1/3 & 2/3 \\ 0 & 0 & 0 & -1/3 & 1 & -1/3 & 2/3 & 1/3 \end{array}\right]$$

$-3R_4$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1/3 & 0 & 1/3 & 1/3 & 2/3 \\ 0 & 0 & 0 & 1 & -3 & 1 & -2 & -1 \end{array}\right]$$

$\cancel{R_4} > \cancel{R_3}$ $-R_4/3 + R_3$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & -1 & 0 & -1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & -3 & 1 & -2 & -1 \end{array}\right]$$

$-2R_4 + R_2$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & +6 & -3 & 4 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & -3 & 1 & -2 & -1 \end{array}\right]$$

$-R_4 + R_1$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 2 & 0 & 4 & 0 & 2 & 1 \\ 0 & 1 & 0 & 0 & 6 & -3 & 4 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & -3 & 1 & -2 & -1 \end{array}\right]$$

$-2R_3 + R_1$

$$\left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 6 & -3 & 4 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & -3 & 1 & -2 & -1 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 2 & 0 & 0 & -1 \\ 6 & -3 & 4 & 1 \\ 1 & 0 & 1 & 1 \\ -3 & 1 & -2 & -1 \end{bmatrix}$$

$X = A^{-1} . b$

$$\begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & -1 \\ 6 & -3 & 4 & 1 \\ 1 & 0 & 1 & 1 \\ -3 & 1 & 2 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 7 \\ 4 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} -6 \\ 1 \\ 10 \\ -7 \end{bmatrix}$$

Q2) Solve the linear System By Reduced from Echelon form

$$x_1 - 5x_2 = b_1$$
$$3x_1 + 2x_2 = b_2$$

① $b_1 = 1$, $b_2 = 4$

② $b_1 = -2$, $b_2 = 5$

$$\left[\begin{array}{cc|c|c} 1 & -5 & 1 & -2 \\ 3 & 2 & 4 & 5 \end{array}\right]$$

$-3R_1 + R_2$

$$\left[\begin{array}{cc|c|c} 1 & -5 & 1 & -2 \\ 0 & 17 & 1 & 11 \end{array}\right]$$

$\frac{1}{17} R_2$

$$\left[\begin{array}{cc|c|c} 1 & -5 & 1 & -2 \\ 0 & 1 & 1/17 & 11/17 \end{array}\right]$$

$5R_2 + R_1$

$$\left[\begin{array}{cc|c|c} 1 & 0 & 22/17 & 21/17 \\ 0 & 1 & 1/17 & 11/17 \end{array}\right]$$

① $x_1 = 22/17$

$x_2 = 1/17$

② $x_1 = 21/17$

$x_2 = 11/17$

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Q) Solve the matrix Eq for x.

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix} \cdot X = \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}$$

$Ax = b$

$$X = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix}^{-1} = \left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 2 & 3 & 0 & 0 & 1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

Reduced Raw Echelon form

$2R_1 - R_2$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & -5 & 2 & 2 & -1 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$R_2/-5$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2/5 & -2/5 & 1/5 & 0 \\ 0 & 2 & -1 & 0 & 0 & 1 \end{array}\right]$$

$-2R_2 + R_3$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2/5 & -2/5 & 1/5 & 0 \\ 0 & 0 & -1/5 & 4/5 & -2/5 & 1 \end{array}\right]$$

$-5R_3$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -2/5 & -2/5 & 1/5 & 0 \\ 0 & 0 & 1 & -4 & 2 & -5 \end{array}\right]$$

Echelon

$\frac{2}{5}R_3 + R_2$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 & 1 & -2 \\ 0 & 0 & 1 & -4 & 2 & -5 \end{array}\right]$$

$-R_3 + R_1$

$$\left[\begin{array}{ccc|ccc} 1 & -1 & 0 & 5 & -2 & 5 \\ 0 & 1 & 0 & -2 & 1 & -2 \\ 0 & 0 & 1 & -4 & 2 & -5 \end{array}\right]$$

$R_2 + R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1 & 3 \\ 0 & 1 & 0 & -2 & 1 & -2 \\ 0 & 0 & 1 & -4 & 2 & -5 \end{array}\right]$$

$$X = \begin{bmatrix} 3 & -1 & 3 \\ -2 & 1 & -2 \\ -4 & 2 & -5 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}$$

$$X = \begin{bmatrix} 6+(-4)+9 & -3+0+15 & 15+3-21 & 21+0+6 & 24-1+3 \\ -4+4-6 & 2+0-10 & -10-3+14 & -14+0-4 & -16+1-2 \\ -8+8-15 & 4+0-25 & \cancel{-25}-20-6+35 & -28-10 & -32+2-5 \end{bmatrix}$$

$$X = \begin{bmatrix} 11 & 12 & -3 & 27 & 26 \\ -6 & -8 & +1 & -18 & -17 \\ -15 & -21 & 9 & -38 & -35 \end{bmatrix}$$

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Diagonal Matrix :

i) $\begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix}$

ii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$D = \begin{bmatrix} d_1 & 0 & 0 & 0 \\ 0 & d_2 & 0 & 0 \\ 0 & 0 & d_3 & 0 \\ 0 & 0 & 0 & d_4 \end{bmatrix}$$

In General nxn Diagonal Matrix

Invertible Diagonal Matrix

$$D^{-1} = \begin{bmatrix} 1/d_1 & 0 & 0 \\ 0 & 1/d_2 & 0 \\ 0 & 0 & 1/d_3 \end{bmatrix}$$

Power of Diagonal Matrix

$$D^K = \begin{bmatrix} d_1^K & 0_k & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3^K \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$A^{-1} = ?$, $A^3 = ?$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & 1/2 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} (1)^3 & 0 & 0 \\ 0 & (-3)^3 & 0 \\ 0 & 0 & (2)^3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & -27 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

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Symmetric Matrix

Ex 1.7

A sq matrix is said to be symmetric if

$A = A^T$

$$A = \begin{bmatrix} 1 & 4 & 5 \\ 4 & -3 & 0 \\ 5 & 0 & 7 \end{bmatrix}$$

Q1) Find A^{-1}, A^{-3}, A^{K}

$$A = \begin{bmatrix} -6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -1/6 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/5 \end{bmatrix}, \quad A^{-3} = \begin{bmatrix} -1/216 & 0 & 0 \\ 0 & 1/27 & 0 \\ 0 & 0 & 1/125 \end{bmatrix}$$

$$A^{K} = \begin{bmatrix} (-6)^K & 0 & 0 \\ 0 & 3^K & 0 \\ 0 & 0 & 5^K \end{bmatrix}$$

Q2) Determine by the inspection whether the matrix is invertable.

a) $\begin{bmatrix} 0 & 6 & -1 \\ 0 & 7 & -4 \\ 0 & 0 & -2 \end{bmatrix}$ Non Inv

b) $\begin{bmatrix} -1 & 2 & 4 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ Invertible [Inverse possible]

Non Zero All values

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Q4) Find all values of the unknown constant for which A is symmetric.

ⓐ $A = \begin{bmatrix} 4 & -3 \\ a+5 & -1 \end{bmatrix}$

A is Symmetric

$A = A^T$

$a_{ij} = a_{ji}$

$a+5 = -3$

$\boxed{a = -8}$

ⓑ $A = \begin{bmatrix} 2 & a-2b+2c & 2a+b+c \\ 3 & 5 & a+c \\ 0 & -2 & 7 \end{bmatrix}$

A is Symmetric

$A = A^T$, $a_{ij} = a_{ji}$

$a - 2b + 2c = 3$

$2a + b + c = 0$

$a + c = -2$

By Gauss Jordan Elimination
Reduced Echelon Form
Augmented Matrix

$$\begin{bmatrix} 1 & -2 & 2 & 3 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 1 & -2 \end{bmatrix}$$

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$a = 11$

$b = -9$

$c = -13$

$-2R_3 + R_2$

$$\begin{bmatrix} 1 & -2 & 2 & 3 \\ 0 & 1 & -1 & 4 \\ 1 & 0 & 1 & -2 \end{bmatrix}$$

$-R_1 + R_3$

$$\begin{bmatrix} 1 & -2 & 2 & 3 \\ 0 & 1 & -1 & 4 \\ 0 & 2 & -1 & -5 \end{bmatrix}$$

$-2R_2 + R_3$

$$\begin{bmatrix} 1 & -2 & 2 & 3 \\ 0 & 1 & -1 & 4 \\ 0 & 0 & 1 & -13 \end{bmatrix}$$

$R_3 + R_2$

$$\begin{bmatrix} 1 & -2 & 2 & 3 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & -13 \end{bmatrix}$$

$-2R_3 + R_1$

$$\begin{bmatrix} 1 & -2 & 0 & \cancel{3}29 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & -13 \end{bmatrix}$$

$2R_2 + R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 11 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & -13 \end{bmatrix}$$

$x_1 = 11$

$x_2 = -9$

$x_3 = -13$

Q5) Find the values of x for which A is invertible

$$A = \begin{bmatrix} x-1 & x^2 & x^4 \\ 0 & x+2 & x^3 \\ 0 & 0 & x-4 \end{bmatrix}$$

$x-1 \neq 0 \rightarrow x \neq 1$

$x+2 \neq 0 \qquad x \neq -2$

$x-4 \neq 0 \qquad x \neq 4$

$$\begin{bmatrix} x-y-1 & x^2 & x^4 \\ 0 & y-x+2 & x^3 \\ 0 & 0 & y+x-4 \end{bmatrix}$$

$x-y-1 \neq 0$

$y-x+2 \neq 0$

$y+x-4 \neq 0$

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Linear Tranformation   Ex1.8

$T : R^n \rightarrow R^m$

If T is a function with domain R^n & codomain R^m, then we say that T is   from R^n to R^m

T: variables → Equation
T: Domain → Codomain

Q1) Find the domain & codomain of the tranformation

$T_A(x) = Ax$

a)
$$w_1 = 4x_1 + 5x_2$$
$$w_2 = x_1 - 8x_2$$

$T : R^2 \rightarrow R^2$

Domain = 2   Co Domain = 2

b)
$$w_1 = 5x_1 - 7x_2$$
$$w_2 = 6x_1 + x_2$$
$$w_3 = 2x_1 + 3x_2$$

$T : R^2 \rightarrow R^3$

Domain = 2   Codomain = 3

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Q2) a)

$$\begin{bmatrix} 3 & 1 & 2 \\ 6 & 7 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$T : R^3 \to R^2$

Q3) a) $T(x_1, x_2) = (2x_1 - x_2, x_1 + x_2)$

$R^2 \to R^2$

Q4) Find the standard Matrix.

$w_1 = 2x_1 - 3x_2 + x_3$

$w_2 = 3x_1 + 5x_2 - x_3$

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 2 & -3 & 1 \\ 3 & 5 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

St. Matrix $\begin{bmatrix} 2 & -3 & 1 \\ 3 & 5 & -1 \end{bmatrix}$

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b) $w_1 = x_1$

$w_2 = x_1 + x_2$

$w_3 = x_1 + x_2 + x_3$

$w_4 = x_1 + x_2 + x_3 + x_4$

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

S.t Matrix $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

Q5) $T(x_1, x_2) = (x_2, -x_1, x_1 + 3x_2, x_1 - x_2)$

$$T \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -x_1 \\ x_1 + 3x_2 \\ x_1 - x_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 1 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

St. Matrix $\begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 1 & 3 \\ 1 & -1 \end{bmatrix}$

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Q4) Find the St. Matrix

$T: R^3 \to R^3$

$w_1 = 3x_1 + 5x_2 - x_3$

$w_2 = 4x_1 - x_2 + x_3$

$w_3 = 3x_1 + 2x_2 - x_3$

& then compute $T(-1,2,4)$ by directly substituting in the eq & then by matrix multiplication.

$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

St. Matrix $\begin{bmatrix} 3 & 5 & -1 \\ 4 & -1 & 1 \\ 3 & 2 & -1 \end{bmatrix}$

$$T(x) = T\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} \boxed{3 \quad 5 \quad -1} \\ 4 \quad -1 \quad 1 \\ 3 \quad 2 \quad -1 \end{bmatrix} \boxed{\begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}}$$

$$= \begin{bmatrix} -3 + 10 - 4 \\ \\ \\ \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ -2 \\ -3 \end{bmatrix}$$

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Q7) $T : R^4 - R^2$

$w_1 = 2x_1 + 3x_2 - 5x_3 - x_4$

$w_2 = x_1 - 5x_2 + 2x_3 - 3x_4$

Find the St. Matrix & then compute $T(1, -1, 2, 4)$

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = T(x) = T\begin{bmatrix} 1 \\ -1 \\ 2 \\ 4 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & 3 & -5 & -1 \\ 1 & -5 & 2 & -3 \end{bmatrix}}_{\text{St. Matrix}} \begin{bmatrix} 1 \\ -1 \\ 2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} -15 \\ -2 \end{bmatrix}$$

Q8) a) $T(x_1, x_2) = (-x_1 + x_2, x_2)$; $x = (-1, 4)$

Find St. Matrix & use it to compute $T(x)$

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_1 + x_2 \\ x_2 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \to -1 \\ x_2 \to 4 \end{bmatrix}$$

St. Matrix

$$T\begin{bmatrix} -1 \\ 4 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

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$T(x_1, x_2, x_3) = (2x_1 - x_2 + x_3, x_2 + x_3, 0)$; $x = (2, 1, -3)$

St. Matrix $\begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

$$T(x) = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ +1 \\ -3 \end{bmatrix}$$

$$T(x) = \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}$$

Q9) The Image of the St. Matrix basis vectors for R^3 are given $T: R^3 \to R^3$

Find St. Matrix & $T(x)$.

$T(e_1) = \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}$, $T(e_2) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $T(e_3) = \begin{bmatrix} 4 \\ -3 \\ -1 \end{bmatrix}$, $x = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

St. Matrix $[\, T(e_1), T(e_2), T(e_3) \,]$

$$= \begin{bmatrix} 1 & 0 & 4 \\ 3 & 0 & -3 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$$

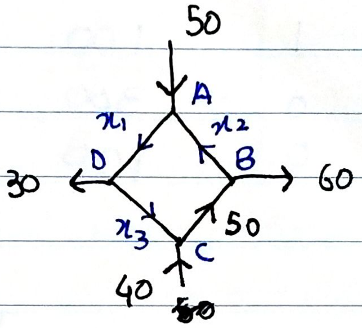
$$T(x) = \begin{bmatrix} 3 \\ 6 \\ 1 \end{bmatrix}$$

Applications of Linear System   Ex1.10

Network Analysis

Polynomial Interpolation

Q1) Find the flow rates & direction.

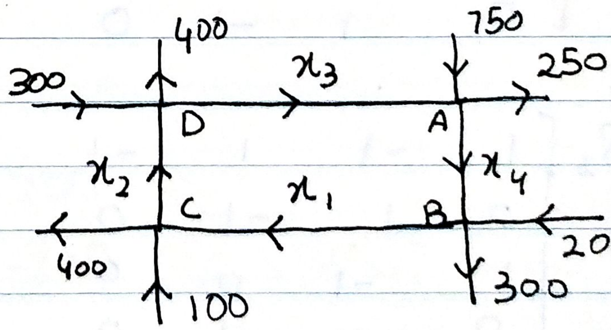


Node	In	Out	
A	$50 + x_2$	$= x_1$	$\rightarrow$ $x_1 - x_2 = 50$
B	50	$= 60 + x_2$	$\rightarrow$ $x_2 = -10$
C	$40 + x_3$	$= 50$	$\rightarrow$ $x_3 = 10$
D	x_1	$= 30 + x_3$	$\rightarrow$ $\cancel{x_4}$ $x_1 - x_3 = 30$ ↳ $x_1 = 40$

Q3) a) Set up a linear system.

b) Solve the system for unknown flow rates

c) If flow from A to B must be reduced, what is min. flow reqd to keep traffic flowing on all roads?



Node	In	Out
A	$x_3 + 750$	$= 250 + x_4$
B	$x_4 + 200$	$= x_1 + 300$
C	$x_1 + 100$	$= x_2 + 400$
D	$x_2 + 300$	$= x_3 + 400$

$$x_3 - x_4 = -500$$
$$-x_1 + x_4 = 100$$
$$x_1 - x_2 = 300$$
$$x_2 - x_3 = 100$$

$$\begin{bmatrix} 0 & 0 & 1 & -1 & -500 \\ -1 & 0 & 0 & 1 & 100 \\ 1 & -1 & 0 & 0 & 300 \\ 0 & 1 & -1 & 0 & 100 \end{bmatrix}$$

$R_3 + R_1$

$$\begin{bmatrix} 1 & -1 & 1 & -1 & -200 \\ -1 & 0 & 0 & 1 & 100 \\ 1 & -1 & 0 & 0 & 300 \\ 0 & 1 & -1 & 0 & 100 \end{bmatrix}$$

$R_1 + R_2$

$$\begin{bmatrix} 1 & -1 & 1 & -1 & -200 \\ 0 & -1 & 1 & 0 & -100 \\ 1 & -1 & 0 & 0 & 300 \\ 0 & 1 & -1 & 0 & 100 \end{bmatrix}$$

$-R_2$

$$\begin{bmatrix} 1 & -1 & 1 & -1 & -200 \\ 0 & 1 & -1 & 0 & 100 \\ 1 & -1 & 0 & 0 & 300 \\ 0 & 1 & -1 & 0 & 100 \end{bmatrix}$$

$R_1 - R_3$

$$\begin{bmatrix} 1 & -1 & 1 & -1 & -200 \\ 0 & 1 & -1 & 0 & 100 \\ 0 & 0 & 1 & -1 & -500 \\ 0 & 1 & -1 & 0 & 100 \end{bmatrix}$$

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~~$R_1 + R_4$~~ $R_2 - R_4$

$$\begin{bmatrix} 1 & -1 & 1 & -1 & -200 \\ 0 & 1 & -1 & 0 & 100 \\ 0 & 0 & 1 & -1 & -500 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 - x_2 + x_3 - x_4 = -200$

$x_2 - x_3 = 100$

$+x_3 - x_4 = -500$

Let $x_4 = t$

$x_3 - t = -500$

$\boxed{x_3 = t - 500}$

$x_2 - x_3 = 100$

$x_2 - (t - 500) = 100$

$\boxed{x_2 = t - 400}$

$x_1 - x_2 + x_3 - x_4 = -200$

$x_1 - (t-400) + (t-500) - (t) = -200$

$x_1 \cancel{=} -t + t - t + 400 - 500 = -200$

$\boxed{x_1 = t - 100}$

c) $x_4 = t$ -

if $x_4 = 500$, then

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· DATA Array / ~~BYTE~~ 10k, 20k, 30 Ex 1.10

Polynomial Interpolation

$y = a_0 + a_1 x + a_2 x^2 + \cdots + a_{n-1} x^{n-1}$

Q13) Find the quadratic polynomial whose graph passes through the points (1,1), (2,2) & (3,5)

At $(1,1) = (x_1, y_1)$

$1 = a_0 + a_1(1) + a_2(1)^2$

$1 = a_0 + a_1 + a_2$ — ①

At (2,2)

$2 = a_0 + a_1(2) + a_2(2)^2$

$2 = a_0 + 2a_1 + 4a_2$ — ②

At (3,5)

$5 = a_0 + a_1(3) + a_2(3)^2$

$5 = a_0 + 3a_1 + 9a_2$ — ③

Augmented Matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 2 \\ 1 & 3 & 9 & 5 \end{bmatrix}$$

By Gauss Jordan Elimination method (Reduced Echelon form)

$R_2 - \cancel{R_1}$

$$\begin{bmatrix} 0 & 1 & 3 & 1 \\ 1 & 2 & 4 & 2 \\ 1 & 3 & 9 & 5 \end{bmatrix}$$

$-R_1 + R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 1 & 3 & 9 & 5 \end{bmatrix}$$

$R_1 - R_3$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & -2 & -8 & -4 \end{bmatrix}$$

$2R_2 + R_3$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & -2 & -2 \end{bmatrix}$$

$R_3/-2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$ Echelon form

$-3R_3 + R_2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$-R_3 + R_1$ $\qquad\qquad\qquad\qquad$ $-R_2 + R_1$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

FAST
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$$\begin{bmatrix} a_0 = 2 \\ a_1 = -2 \\ a_2 = 1 \end{bmatrix}$$

$y = a_0 + a_1 x + a_2 x^2$

$\boxed{y = 2 - 2x + x^2}$

Q14) $(1,1)$, $(2,2)$, $(3,5)$, $(0,2)$

Find the Cubic Eq

$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$

At $(1,1)$

$\cancel{y=}\ 1 = a_0 + a_1 + a_2 + a_3$

At $(2,2)$

$2 = a_0 + \cancel{2x} + 2a_1 + 4a_2 + 8a_3$

At $(3,5)$

$5 = a_0 + 3a_1 + 9a_2 + 27a_3$

At $(0,2)$

$2 = a_0 + 0 + 0 + 0$

Augmented Form

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 & 2 \\ 1 & 3 & 9 & 27 & 5 \\ 1 & 0 & 0 & 0 & 2 \end{bmatrix}$$

Swap R_1 and R_4

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 1 & 2 & 4 & 8 & 2 \\ 1 & 3 & 9 & 27 & 5 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$R_1 - R_2$ $\qquad\qquad$ $R_2/-2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & -2 & -4 & -8 & 0 \\ 1 & 3 & 9 & 27 & 5 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 4 & 0 \\ 1 & 3 & 9 & 27 & 5 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$R_1 - R_3$ $\qquad\qquad$ $3R_2 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 4 & 0 \\ 0 & -3 & -9 & -27 & -3 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 4 & 0 \\ 0 & 0 & -3 & -15 & -3 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$R_3/-3$ $\qquad\qquad$ $R_1 - R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 4 & 0 \\ 0 & 0 & 1 & 5 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 4 & 0 \\ 0 & 0 & 1 & 5 & 1 \\ 0 & -1 & -1 & -1 & 1 \end{bmatrix}$$

$R_2 + R_4$ $\qquad\qquad$ $R_3 - R_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 4 & 0 \\ 0 & 0 & 1 & 5 & 1 \\ 0 & 0 & 1 & 3 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 4 & 0 \\ 0 & 0 & 1 & 5 & 1 \\ 0 & 0 & 0 & 2 & 0 \end{bmatrix}$$

$R_4/2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 4 & 0 \\ 0 & 0 & 1 & 5 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \rightarrow \text{Echelon form}$$

$-5R_4 + R_3$ $\qquad\qquad$ $-4R_4 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 4 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$-2R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & \cancel{0}\,-2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} a_0 = 2 \\ a_1 = -2 \\ a_2 = 1 \\ a_3 = 0 \end{bmatrix}$$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$y = 2 - 2x + x^2$$

Ex 1.8

Table 1

Reflection about x-axis $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

about y-axis $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

Reflection about $y = x$
x-y plane. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

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Q1) U

Chpt # 2

Determinant

Q1) Using Row Reduction to Evaluate determinant

$$A = \begin{bmatrix} 0 & 1 & 5 \\ 3 & -6 & 9 \\ 2 & 6 & 1 \end{bmatrix}$$

$$0 - 1[3 - 9(2)] + 5(18 + 12)$$

det(A)
or
$$|A| = 165$$

Row Reduction

$$A = \begin{bmatrix} 0 & 1 & 5 \\ 3 & -6 & 9 \\ 2 & 6 & 1 \end{bmatrix}$$

Interchange R_1 & R_2

$$= (-1)\begin{bmatrix} 3 & -6 & 9 \\ 0 & 1 & 5 \\ 2 & 6 & 1 \end{bmatrix}$$

$$= (-1)(3)\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 2 & 6 & 1 \end{bmatrix}$$

$$= -2R_1 + R_3$$

$$= -3\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 10 & -5 \end{bmatrix}$$

$-10R_2+R_3$

$$= -3\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & -55 \end{bmatrix}$$

$$= -3(-55)\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= 165\,(1)(1)(1)$$

$$= 165$$

q2) $\begin{bmatrix} 3 & -6 & 9 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix}$

$$= 3\begin{bmatrix} 1 & -2 & 3 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix}$$

$2R+R_2$

$$= 3\begin{bmatrix} 1 & -2 & 3 \\ 0 & 3 & 4 \\ 0 & 1 & 5 \end{bmatrix}$$

$$= 3(3)\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 4/3 \\ 0 & 1 & 5 \end{bmatrix}$$

$-R_2+R_3$

$$9\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 4/3 \\ 0 & 0 & +11/3 \end{bmatrix}$$

$$= 9\left(\frac{+11}{3}\right)\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 4/3 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= +33$$

Q2) Row operation & Cofactor Expansion
Find the determinant.

By Cofactor Expansion

$$\begin{bmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

Interchange R_1 to R_2

$$= (-1)\begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & 1 & 3 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$-2R_1 + R_2$

$$= (-1)\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$$= (-1)\begin{bmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & 2 & 3 \end{bmatrix}$$

$-2R_1 + R_2$

$$= (-1)\begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

$-R_1 + R_2$

$$= (-1)\begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$$

$$= (-1)\begin{bmatrix} -1 & 2 \\ 1 & 4 \end{bmatrix}$$

$$= (-1)\,[-4-2]$$

$= 6$ Ans

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q3)

$$A = \begin{bmatrix} 3 & 5 & -2 & 6 \\ 1 & 2 & -1 & 1 \\ 2 & 4 & 1 & 5 \\ 3 & +7 & 5 & 3 \end{bmatrix}$$

By Cofactor Expansion

Swap R_1 & R_2

$$= -1 \begin{bmatrix} 1 & 2 & -1 & 1 \\ 3 & 5 & -2 & 6 \\ 2 & 4 & 1 & 5 \\ 3 & +7 & 5 & 3 \end{bmatrix}$$

$-3R_1 + R_2$

$$= (-1) \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & -1 & 1 & 3 \\ 2 & 4 & 1 & 5 \\ 3 & +7 & 5 & 3 \end{bmatrix}$$

$-2R_1 + R_3$

$$= (-1) \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & -1 & 1 & 3 \\ 0 & 0 & 3 & 3 \\ 3 & +7 & 5 & 3 \end{bmatrix}$$

$-3R_1 + R_4$

$$= (-1) \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & -1 & 1 & 3 \\ 0 & 0 & 3 & 3 \\ 0 & \cancel{-3} & 8 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 3 \\ 9 & 3 \end{bmatrix}$$

$=$

$9 - 27$

$= -18$ Ans.

$$= (-1)\begin{bmatrix} -1 & 1 & 3 \\ 0 & 3 & 3 \\ \cancel{-13}\,1 & 8 & 0 \end{bmatrix}$$

$$= -1(-1)\begin{bmatrix} 1 & -1 & -3 \\ 0 & 3 & 3 \\ \overset{1}{\cancel{-13}} & 8 & 0 \end{bmatrix}$$

$-R_1 + R_3$

$$= +1\begin{bmatrix} 1 & -1 & -3 \\ 0 & 3 & 3 \\ 0 & 9 & 3 \end{bmatrix}$$

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Ex 2.1

Determinant

Minor Entry $a_{ij} = M_{ij}$

Cofactor entry $a_{ij} = C_{ij} = (-1)^{i+j} \cdot M_{ij}$

Q1) Find the value C_{11} & C_{32}

$$A = \begin{bmatrix} 3 & 1 & -4 \\ 2 & 5 & 6 \\ 1 & 4 & 8 \end{bmatrix}$$

i) $C_{11} = (-1)^{1+1} M_{11}$

$$A = \begin{bmatrix} 3 & 1 & -4 \\ 2 & 5 & 6 \\ 1 & 4 & 8 \end{bmatrix}$$

$$M_{11} = \begin{bmatrix} 5 & 6 \\ 4 & 8 \end{bmatrix}$$

$= 40 - 24$

$= 16$

$C_{11} = (-1)^2 \cdot 16$

$C_{11} = 16$

ii) $C_{32} = (-1)^{3+2} M_{32}$

$$A = \begin{bmatrix} 3 & 1 & -4 \\ 2 & 5 & 6 \\ 1 & 4 & 8 \end{bmatrix}$$

$$M_{32} = \begin{bmatrix} 3 & -4 \\ 2 & 6 \end{bmatrix}$$

$= 18 - (-8)$

$= \cancel{14}\ 26$

$C_{32} = (-1)^5 \cdot 26$

$C_{32} = -26$

Q2) Find all the minors & cofactors of matrix A.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 3 & 6 \\ 0 & 1 & 4 \end{bmatrix} \qquad C_{ij} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$M_{11} =$ ~~$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 3 & 6 \\ 0 & 1 & 4 \end{bmatrix}$~~

$M_{11} = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 3 & 6 \\ 0 & 1 & 4 \end{bmatrix}$

$M_{11} = \begin{bmatrix} 3 & 6 \\ 1 & 4 \end{bmatrix}$

$M_{11} = 12 - 6$

$M_{11} = 6$

$C_{11} = (-1)^2 \cdot 6$

$C_{11} = 6$

$M_{12} = \begin{bmatrix} 3 & 6 \\ 0 & 4 \end{bmatrix}$

$= 12 - 0$

$= \cancel{6}12$

$C_{12} = (-1)^3 \cdot 6$

$C_{12} = -\cancel{6}12$

$M_{13} = \begin{bmatrix} 3 & 3 \\ 0 & 1 \end{bmatrix}$

$= 3 - 0$

$C_{13} = (-1)^4 \cdot 3$

$C_{13} = 3$

$M_{21} = \begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}$

$= 4 - 2$

$= 2$

$C_{21} = -2.$

$M_{22} = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$

$= 4 - 0$

$C_{22} = 4$

$M_{23} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$= 1 - 0$

$C_{23} = -1$

$M_{31} = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

$= 6 - 6$

$= 0$

$C_{31} = 0$

$M_{32} = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

$= 6 - 6$

$C_{32} = 0$

$M_{33} = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix}$

$= 0$

$C_{33} = 0$

$$C_{ij} = \begin{bmatrix} 6 & -12 & 3 \\ -2 & 4 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Q3) Find the Inverse

$$A = \begin{bmatrix} -5 & 7 \\ -7 & -2 \end{bmatrix}$$

$$\boxed{A^{-1} = \frac{Adj(A)}{|A|}}, \quad \boxed{Adj(A) = \begin{bmatrix} \nwarrow & \pm \\ \pm & \searrow \end{bmatrix}}$$

$$Adj(A) = \begin{bmatrix} -2 & -7 \\ 7 & -5 \end{bmatrix}$$

$$|A| = 10 + 49 = 59$$

$$A^{-1} = \begin{bmatrix} -2/59 & -7/59 \\ 7/59 & -5/59 \end{bmatrix}$$

Q4) Find the det(A) by Arrow technique

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -4 & 5 & 6 \\ 7 & -8 & 9 \end{bmatrix}$$

$$det(A) = \left[\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 2 \\ -4 & 5 & 6 & -4 & 5 \\ 7 & -8 & 9 & 7 & -8 \end{array}\right]$$

$$= [45 + 84 + 96] - [105 - 48 - 72]$$

$$= 240$$

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ii) $\begin{bmatrix} -2 & 7 & 6 \\ 5 & 1 & -2 \\ 3 & 8 & 4 \end{bmatrix}$

$$\det(A) = \left[\begin{array}{ccc|cc} -2 & 7 & 6 & -2 & 7 \\ 5 & 1 & -2 & 5 & 1 \\ 3 & 8 & 4 & 3 & 8 \end{array}\right]$$

$$= [-2(1)(4) - 52 + 240] - [18 + 32 + 140]$$
$$= 0$$

Q18) Find all values of λ for which $\det(A) = 0$

$$\begin{bmatrix} \lambda-4 & 4 & 0 \\ -1 & \lambda & 0 \\ 0 & 0 & \lambda-5 \end{bmatrix}$$

By Arrow Technique

$$\left[\begin{array}{ccc|cc} \lambda-4 & 4 & 0 & \lambda-4 & 4 \\ -1 & \lambda & 0 & -1 & \lambda \\ 0 & 0 & \lambda-5 & 0 & 0 \end{array}\right]$$

$$= [(\lambda-4)(\lambda)(\lambda-5) + 0 + 0] - [0 + 0 + (\lambda-5)(-1)(4)]$$
$$= \cancel{\lambda^2 - 4\lambda} \; \cancel{[-1} \quad \lambda-5(\lambda^2 - 4\lambda + 4)$$

$\boxed{\lambda = 5}$ $\boxed{\lambda = 2}$ $\boxed{\lambda = 2}$ Ans ~~1, 3, 2~~

$$= (\lambda-4)[\lambda(\lambda-5) - 0] - 4[(-1)(\lambda-5) - 0] + 0$$
$$= \lambda-5[\lambda(\lambda-4)] + 4(\lambda-5) = 0$$

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[Chapter 4] Vector Space

V is non complete set of object for which two operation are defined add & multiplication.

1- If u & v are object in V, then u+v is in V (closed)

2- $u + v = v + u$ (commotative)

3- $u + (v + w) = (u + v) + w$ (Associative)

4- $u + (-u) = 0$ (zero vector)

5- $0 + u = u + 0 = u$

6- If k is any scalar u is any object in V, then ku is in V

7- $k(u+v) = ku + kv$

8- $(k+m)u = ku + mu$ ∴ k & m are scalar

9- $k(mu) = (km)u$

10- $1 \cdot u = u$

Q) The set of all real numbers with standard operations of Addition & multiplication.

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(Q2) The set of all pairs of real no. of the form $(n,0)$ with the standard operation on R^2

u, v

$u = (u_1, u_2)$

$v = (v_1, v_2)$

$V = R^2$

1- $(x,0) \rightarrow (y,0) = (x+y, 0)$ is in $V = R^2$, x, y are Real

2- $(x,0) + (y,0) = (y,0) + (x,0) = (y+x, 0)$

3- $(x,0) + [(y,0) + (z,0)] = (x,0) + (y+z,0) = (x+y+z,0)$
$= (x+y, 0) + (z,0) = [(x,0) + (y,0)] + (z,0)$, x,y,z are re

4- $(x,0) + (-x,0) = (0,0)$

5- $(x,0) + (0,0) = (0,0) + (x,0) = (x,0)$

6- $k(x,0) = (kx, 0)$

7- $k[(x,0) + (y,0)] = k(x+y,0) = (kx+ky,0) = k(x,0) + k(y,0)$, k, y are real

8- $(k+m)(x,0) = ((k+m)x, 0) = (kx+mx, 0) = k(x,0) + m(x,0)$

9. $km(x,0) = k(mx,0) = (kmx,0) = km(x,0)$

10 - $1(x,0) = (x,0)$, x is real no.

Consistent → Linear

IN Consilent → Not Linear

Q5 The set of all pairs of real no. of the form (x, y) where $x \geq 0$ with the standard operation on R^2

Property 4 and 6-a are not following property as k can be any number (+ve & -ve) & $x \geq 0$.

Ex 4.3

Linear Combination (L.C).

$$\cancel{K_1}\ K_1V_1 + K_2V_2 + \cdots K_nV_n = w$$

Q1) Which of following L.C of $u = (0,-2,2)$ & $v = (1,3,-1)$

a) $(2,2,2)$, w

b) $(0,4,5)$

c) $(0,0,0)$

$$K_1(0,-2,2) + \cancel{K_2(0,-2,2)})\ K_2(1,3,-1) = (2,2,2)$$

Augmented Matrix

$0K_1 + K_2 = 2$

$-2K_1 + 3K_2 = 2$

$2K_1 - K_2 = 2$

Reduced Echelon Form

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$K_1 = 2$, $K_2 = 2$

One Sol , System is Consistent

Hence u & v are L.C of $(2,2,2)$

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Q2) Which of the L.C

$$A = \begin{bmatrix} 4 & 0 \\ -2 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix}$$

(a) $\begin{bmatrix} 6 & -8 \\ -1 & -8 \end{bmatrix}$

$$K_1 A + K_2 B + K_3 C = \omega$$

$$K_1 \begin{bmatrix} 4 & 0 \\ -2 & -2 \end{bmatrix} + K_2 \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} + K_3 \begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 6 & -8 \\ -1 & -8 \end{bmatrix}$$

Augmented Matrix

$$\begin{bmatrix} 4 & 1 & 0 & 6 \\ 0 & -1 & 2 & -8 \\ -2 & 2 & 1 & -1 \\ -2 & 3 & 4 & -8 \end{bmatrix}$$

Reduced Echelon Form

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$K_1 = 1$

$K_2 = 2$

$K_3 = -3$

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Q3) In Each part the vector as a L.C

$p_1 = 2 + x + 4x^2$
$p_2 = 1 - x + 3x^2$
$p_3 = 3 + 2x + 5x^2$

a) $-9 - 7x - 15x^2$ $\quad w$

L.C

$K_1 P_1 + K_2 P_2 + K_3 P_3 = w$

$K_1(2 + x + 4x^2) + K_2(1 - x + 3x^2) + K_3(3 + 2x + 5x^2) = -9 - 7x - 15x^2$

Augmented Matrix

$$\begin{bmatrix} 2 & 1 & 3 & -9 \\ 1 & -1 & 2 & -7 \\ 4 & 3 & 5 & -15 \end{bmatrix}$$

$K_1 = -2$
$K_2 = 1$
$K_3 = -2$

Q4) In Each part, determine whether the vector span R^3

$V_1 = (2, 2, 2)$, $V_2 (0, 0, 3)$, $V_3 = (0, 1, 1)$

L.C

$K_1 V_1 + K_2 V_2 + K_3 V_3 = w$

$K_1 (2, 2, 2) + K_2 (0, 0, 3) + K_3 (0, 1, 1) = w (w_1, w_2, w_3)$

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Agmented Matrix

$$\begin{bmatrix} 2 & 0 & 0 & \omega_1 \\ 2 & 0 & 1 & \omega_2 \\ 2 & 3 & 1 & \omega_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 0 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

$\det(A) = -6$ Spam

~~if (det~~ if $\det(A) = 0$, Not Spam

otherwise , Spam

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Ex4.4

Linearly Independence

$k_1v_1 + k_2v_2 + \dots k_nv_n = 0$

If $k_1 = k_2 \dots = k_n = 0$: The system is L.independent
otherwise ; The system is L.dependent

Q1) Determine whether the vectors are L.Indep or L.dep

$(3,8,7,-3)$, $(1,5,3,-1)$, $(2,-1,2,6)$, $(4,2,6,4)$

By L.indep

$k_1(3,8,7,-3) + k_2(1,5,3,-1) + k_3(2,-1,2,6) + k_4(4,2,6,4) = (0,0,0,0)$

Augmented Matrix $\rightarrow$ $\left[\begin{array}{ccccc} 3 & 1 & 2 & 4 & 0 \\ 8 & 5 & -1 & 2 & 0 \\ 7 & 3 & 2 & 6 & 0 \\ -3 & -1 & 6 & 4 & 0 \end{array}\right]$

Reduced Echelon Form
$\downarrow$

$$\left[\begin{array}{ccccc} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Let $k_4 = t$

$k_1 = -t$

$k_2 = t$

$k_3 = -t$

The system is Consistent

$k_1 = k_2 = k_3 = k_4 \neq 0$

The system is L.dep

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Q2) L. Ind or L. dep

$2 - x + 4x^2$, $3 + 6x + 2x^2$, $2 + 10x - 4x^2$

By L. Indep

$K_1 p_1 + K_2 p_2 + K_3 p_3 = 0$

$K_1(2 - x + 4x^2) + K_2(3 + 6x + 2x^2) + K_3(2 + 10x - 4x^2) = 0$

Augmented Matrix

$$\begin{bmatrix} 2 & 3 & 2 & 0 \\ -1 & 6 & 10 & 0 \\ 4 & 2 & -4 & 0 \end{bmatrix}$$

Reduced Echelon form

$$\begin{bmatrix} \quad \end{bmatrix}$$

$K_1 = 0$, $K_2 = 0$, $K_3 = 0$

Hence it is Lin. Indep

Q3) $K = ?$

The matrix is Lin. Indep $\rightarrow \det(A) = 0$

$$\begin{bmatrix} 1 & 0 \\ 1 & K \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ K & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$$

By L. Indep

$$K_1 \begin{bmatrix} 1 & 0 \\ 1 & K \end{bmatrix} + K_2 \begin{bmatrix} -1 & 0 \\ K & 1 \end{bmatrix} + K_3 \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Aug Matrix

$$\left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & k & 1 & 0 \\ k & 1 & 3 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & -1 & 2 & 0 \\ 1 & k & 1 & 0 \\ k & 1 & 3 & 0 \end{array}\right]$$

Convert Augmented Matrix to A Matrix

$$\begin{bmatrix} 1 & -1 & 2 \\ 1 & k & 1 \\ k & 1 & 3 \end{bmatrix}$$

$$\det(A) = 3k - 1 + 3 - k + 2(1 - k^2)$$
$$= -2k^2 + 2k + 4$$
$$= -2(k^2 - k \cancel{*} - 2)$$
$$= -2(k^2 - 2k + k - 2)$$
$$= -2(k(k-2) + 1(k-2))$$
$$= -2(k+1)(k-2)$$

$\det(A) = 0$

$0 = (k+1)(k-2)$

$\{ k = -1, 2 \}$

Q4) In Each part, Let $T_A : R^2 - R^2$ be multiplication by A, and Let $u = (1,2)$, ~~K~~ $u_2 = (-1, 1)$. Determine whether the set $\{ T_A(u_1), T_A(u_2) \}$ are L. Indep.

a) $A = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$

$$T_A(u_1) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$T_A(u_2) = \begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

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By L. Indep

$K_1 T_A(u_1) + K_2 T_A(u_2) = 0$

$K_1(-1, 4) + K_2(-2, 2) = 0$

Aug Matrix

$$\begin{bmatrix} -1 & -2 & 0 \\ 4 & 2 & 0 \end{bmatrix}$$

Reduced Echelon Form

$K_1 = 0$, $K_2 = 0$

The System is L.Indep

$\{ T_A(u_1), T_A(u_2) \}$

Wronskian Determinant

$$w(x) = \begin{bmatrix} f_1 & f_2 & \cdots & f_x \\ f_1' & f_2' & \cdots & f_x' \\ \cdots & \cdots & \cdots & \vdots \\ f_1^{(x)} & f_2^{(x)} & \cdots & f_x^{(x)} \end{bmatrix}$$

If $w(x) \neq 0$; L.Indep

otherwise ; L.dep

Q5) Find the system whether L.indep or L.dep using wronskian

$f_1(x) = x$, $f_2(x) = \cos x$

By Wronskian Det

$$w(x) = \begin{bmatrix} f_1 & f_2 \\ f_1' & f_2' \end{bmatrix}$$

$$= \begin{bmatrix} x & \cos x \\ 1 & -\sin x \end{bmatrix}$$

$$= -x\sin x - \cos x$$

$\neq 0$

Hence $f_1(x)$ & $f_2(x)$ are L.Indep.

Q2) $f_1(x) = e^x$

$f_2(x) = xe^x$

$f_3(x) = x^2e^x$

$f_2' = x(e^x) + 1(e^x) = e^x(x+1)$

$f_2'' = e^x(x+1) + e^x(1)$

$f_3' = x^2(e^x) + e^x(2x) = e^x(x^2+2x)$

$f_3'' = (e^x)(x^2+2x) + e^x(2x+2)$

By W.d

$$w(x) = \begin{bmatrix} f_1 & f_2 & f_3 \\ f_1' & f_2' & f_3' \\ f_1'' & f_2'' & f_3'' \end{bmatrix}$$

$$= \begin{bmatrix} e^x & xe^x & x^2e^x \\ e^x & e^x(x+1) & e^x(x^2+2x) \\ e^x & e^x(x+2) & e^x(x^2+4x+2) \end{bmatrix}$$

Det $\neq 0$

L. Ind


eclipse
Ex-4.5
Basis:
If S = {v1, v2, ... vn} is set of vectors in a finite-dimensional
Vector space V, then S is called a basis for V if:
(a) S spans V
(b) S is linearly indep.
det (A) ≠ 0 ; It is basis
otherwise; It is not basis
Q.1 Show that following polynomial form a basis for P2
x²+1 , x²−1 , 2x−1
c1P1 + c2P2 + c3P3 = 0 ; by linearly Indep.
c1P1 + c2P2 + c3P3 = w ; by linear Combinati
c1 (x²+1) + c2 (x² −
1
0
1

$$c_1(x^2+1)+c_2(x^2-1)+c_3(2x-1)=0$$

$$\begin{aligned} 1 \quad -1 \quad -1 &= 0 \\ 0 \quad 0 \quad 2 &= 0 \\ 1 \quad 1 \quad 0 &= 0 \end{aligned}$$

$$A=\begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & 2 \\ 1 & 1 & 0 \end{bmatrix}$$

$\det(A)=$

$$c_1(x^2+1)+c_2(x^2-1)+c_3(2x-1)=(w_1, w_2, w_3)$$

$$\begin{aligned} 1 \quad -1 \quad -1 &= w_1 \\ 0 \quad 0 \quad 2 &= w_2 \\ 1 \quad 1 \quad 0 &= w_3 \end{aligned}$$

Ex4.5

Q2) First show that $S = \{A_1, A_2, A_3, A_4\}$ as a bases for M_2 the express A as a linear combination of the vector is S, & then find the coordinate vectors of A relation to S

$$A_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$K_1 A_1 + K_2 A_2 + K_3 A_3 + K_4 A_4 = 0$; Linear Indep

$K_1 A_1 + K_2 A_2 + K_3 A_3 + K_4 A_4 = W$; Linear Combination

$$K_1 \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} + K_2 \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} + K_3 \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} + K_4 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

'' '' '' '' $= \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 0 & 0 & a \\ 1 & 1 & 0 & 0 & b \\ 1 & 1 & 1 & 0 & c \\ 1 & 1 & 1 & 1 & d \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \overset{\text{det}}{1} \neq 0$$

This is Basis

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By Linear Combination

$$\left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right]$$

$K_1 = 1$
$K_2 = -1$
$K_3 = 1$
$K_4 = -1$

This is Linear Comb

$(K_1, K_2, K_3, K_4) = (1, -1, 1, -1)$
is Coordinate Vectors

Q3). In Each part
$T_A : R^3 - R^3$ be multiply by A & let $u = \{1, -2, -1\}$.
Find coordinate vector of $T_A(u)$ relative to the basis
$S = \{(1,1,0), (0,1,1) (1,1,1)\}$ for R^3

a) $A = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & -1 & 2 \end{bmatrix}$

$$T_A(u) = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 1 & 1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 0 \end{bmatrix}$$

By Linear Comb

$$K_1 V_1 + K_2 V_2 + K_3 V_3 = w$$

$$K_1 (1,1,0) + K_2 (0,1,1) + K_3 (1,1,1) = (4,-2,0)$$

$$\left[\begin{array}{cccc} 1 & 0 & 1 & 4 \\ 1 & 1 & 1 & -2 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

Augmented Matrix

$$\begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

By Reduced Echelon form

$K_1 = -2$, $K_2 = -6$, $K_3 = 6$

Coordinate Vector $(K_1, K_2, K_3) = (-2, -6, 6)$

Q4) The first four Hermite polynomial are $1, 2t, -2+4t^2, -12t+18t^3$

a) Show that first four H.P form a basis for P_3.

b) Find the coordinate vector of polynomial $P(t) = -1-4t+8t^2+8t^3$ relative to.

By Linearly Indep] ①

$K_1P_1 + K_2P_2 + K_3P_3 + K_4P_4 = 0$

By Linear Combination] ②

$K_1P_1 + K_2P_2 + K_3P_3 + K_4P_4 = \omega$

$K_1(1) + K_2(2t) + K_3(-2+4t^2) + K_4(-12t+18t^3)$

from 1

$$\begin{bmatrix} 1 & 0 & -2 & 0 & 0 \\ 0 & 2 & 0 & -12 & 0 \\ 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 18 & 0 \end{bmatrix}$$

from 2

$$\begin{bmatrix} 1 & 0 & -2 & 0 & \omega_1 \\ 0 & 2 & 0 & -12 & \omega_2 \\ 0 & 0 & 4 & 0 & \omega_3 \\ 0 & 0 & 0 & 18 & \omega_4 \end{bmatrix}$$

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from

$$A = \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 2 & 0 & -12 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 18 \end{bmatrix}$$

$\det(A) = \overset{144}{\cancel{64}} \neq 0$ This is Basis

For Coordinate Vectors

$$\begin{bmatrix} 1 & 0 & -2 & 0 & -1 \\ 0 & 2 & 0 & -12 & -4 \\ 0 & 0 & 4 & 0 & 8 \\ 0 & 0 & 0 & 18 & 8 \end{bmatrix}$$

$K_1 =$
$K_2 =$
$K_3 =$
$K_4 =$

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Ex 4.6 [Dimensions]

$Dim(R^n) = n$; The standard basis has (n) vectors

$Dim(P_n) = n+1$ " $(n+1)$

$Dim(M_{mn}) = mn$ " (mn)

$Dim[span\{v_1, v_2 \; --- \; v_r\}] = r$

Q1) Find a basis for the sol space of system & find the dimensions of that space.

$$x_1 - 4x_2 + 3x_3 - x_4 = 0$$
$$2x_1 - 8x_2 + 6x_3 - 2x_4 = 0$$

$$\begin{bmatrix} 1 & -4 & 3 & -1 & 0 \\ 2 & -8 & 6 & -2 & 0 \end{bmatrix}$$

$-2R_1 + R_2$

$$\begin{bmatrix} 1 & -4 & 3 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Infinitely many sol

Parametric Eq

$$x_1 - 4x_2 + 3x_3 - x_4 = 0 \qquad \boxed{x_2 = a}, \; \boxed{x_3 = b}, \; \boxed{x_4 = c}$$
$$x_1 - 4a_2 + 3b - c = 0$$
$$x_1 = 4a - 3b + c$$

Basis

$$(x_1, x_2, x_3, x_4) = (4a - 3b + c,\ a,\ b,\ c)$$
$$= a(4, 1, 0, 0) + b(-3, 0, 1, 0)$$
$$+ c(1, 0, 0, 1)$$

$v_1 = (4, 1, 0, 0)$

$v_2 = (-3, 0, 1, 0)$

$v_3 = (1, 0, 0, 1)$

Dimension $= 3$

Q2) a) The plane $3x - 2y + 5z = 0$

b) The plane $x - y = 0$

c) The Line $x = 2t,\ y = -t,\ z = 4t$

a) $3x - 2y + 5z = 0$

Let $y = t$, $z = s$

$3x - 2t + 5s = 0$

$x = \frac{2t - 5s}{3}$

Basis

$(x, y, z) = \left(\frac{2t-5s}{3}, t, s\right)$

$= t\left(\frac{2}{3}, 1, 0\right) + s\left(-\frac{5}{3}, 0, 1\right)$

$v_1 = (2/3, 1, 0)$

$v_2 = (-5/3, 0, 1)$

Dimensions : 2

The plane $x - y = 0$

Let $y = t$

$x - t = 0$

$x = t$

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Basis

$(x, y) = (t, t)$

$t(1, 1)$

$v_1 = (1, 1)$

Dimension $= 1$

c) ~~Basis~~ $x = 2t$, $y = -t$, $z = 4t$

$(x, y, z) = (2t, -t, 4t)$

$t \cdot (2, -1, 4)$

$v_1 = (2, -1, 4)$

Dimensions $= 1$

Q3) Find the basis for the subspace of R^3 that is spanned by vector

$v_1 = (1, 0, 0)$

$v_2 = (1, 0, 1)$

$v_3 = (2, 0, 1)$

$v_4 = (0, 0, -1)$

By L.Indep (which spanned)

$k_1 v_1 + k_2 v_2 + k_3 v_3 + k_4 v_4 = 0$

$k_1 (1, 0, 0) + k_2 (1, 0, 1) + k_3 (2, 0, 1) + k_4 (0, 0, -1) = 0$

$$\begin{bmatrix} 1 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$-R_1 + R_2$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$k_1 + k_3 + k_4 = 0$ (1)

$k_2 + k_3 - k_4 = 0$ (2)

Taking (1)

$k_1 + k_3 + k_4 = 0$

Let $k_3 = s$, $k_4 = t$

$k_1 + s + t = 0$

$k_1 = -s - t$

$(k_1, k_2, k_3, k_4) = (-s-t, 0, s, t)$

$s(-1, 0, 1, 0) + t(-1, 0, 0, 1)$

$v_1 = (-1, 0, 1, 0)$

$v_2 = (-1, 0, 0, 1)$

Dimensions = 2

eq (2)

$k_2 + k_3 - k_4 = 0$

Let $k_3 = s$, $k_4 = t$

$k_2 + s - t = 0$

$k_2 = -s + t$

$(k_1, k_2, k_3, k_4) = (0, -s+t, s, t)$

$s(0, -1, 1, 0) + t(0, 1, 0, 1)$

Dimensions : 2

$v_1 = (0, -1, 1, 0)$

$v_2 = (0, 1, 0, 1)$

Ex 4.8

Row, Col & Null Space

Q1) Express the product A_n as a Linear Combination of the col vectors of A.

$$\begin{bmatrix} 4 & 0 & -1 \\ 3 & 6 & 2 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} -2 \\ 3 \\ 5 \end{bmatrix}$$

$$= -2 \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} 0 \\ 6 \\ -1 \end{bmatrix} + 5 \begin{bmatrix} -1 \\ 2 \\ 4 \end{bmatrix}$$

Q2) Determine whether b is in the col space of A, & if so express b as a linear combination of the col vectors of A.

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 9 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}$$

Augmented Matrix

$$\begin{bmatrix} 1 & -1 & 1 & 5 \\ 9 & 3 & 1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

$-9R_1 + R_2$

$$\begin{bmatrix} 1 & -1 & 1 & 5 \\ 0 & 12 & -8 & -44 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

$-R_1+R_3$ & $R_2/12$

$$\begin{bmatrix} 1 & -1 & 1 & 5 \\ 0 & 1 & -8/12 & -44/12 \\ 0 & 2 & 0 & -6 \end{bmatrix}$$

$-2R_2+R_3$

$$\begin{bmatrix} 1 & -1 & 1 & 5 \\ 0 & 1 & -2/3 & -11/3 \\ 0 & 0 & 4/3 & -4/3 \end{bmatrix}$$

$\vdots$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$x_1 = 1$
$x_2 = -3$
$x_3 = 1$

Linear Combination

$$= \begin{bmatrix} 1 \\ 9 \\ 1 \end{bmatrix} - 3 \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Q3) Suppose that $x_1 = 3, x_2 = 0, x_3 = -1, x_4 = 5$, is a Sol of a Non Homogeneous Linear System $Ax=b$ & that the sol set the Homogeneous system $Ax=0$ is given functions.

$\cancel{x_1 = 5r}$ $x_1 = 5r - 2s$, $x_2 = s$, $x_3 = s+t$, $x_4 = t$

a) Find a vector $Ax = 0$

b) Find a vector $Ax = b$

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a) $Ax=0$

$(x_1, x_2, x_3, x_4) = (5r-2s,\ s,\ s+t,\ t)$
$= r(5,0,0,0) + s(-2,1,1,0) + t(0,0,1,1)$

$v_1 = (5,0,0,0)$
$v_2 = (-2,1,1,0)$
$v_3 = (0,0,1,1)$

b) $Ax=b$

$(x_1, x_2, x_3, x_4) = (3,0,-1,5) + (5r-2s, s, s+t, t)$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -1 \\ 6 \end{bmatrix} + r\begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$$

Ans

Q4) Find the vector of the general sol of the Linear System $Ax=b$, and then use that ~~matrix~~ result to find the vector form of general sol of $Ax=0$

$x_1 + x_2 + 2x_3 = 5$
$x_1 + x_3 = -2$
$2x_1 + x_2 + 3x_3 = 3$

$$\begin{bmatrix} 1 & 1 & 2 & 5 \\ 1 & 0 & 1 & -2 \\ 2 & 1 & 3 & 3 \end{bmatrix}$$

$R_1 - R_2$ $\quad 2R_1 - R_3$

$$\begin{bmatrix} 1 & 1 & 2 & 5 \\ 0 & 1 & 1 & 7 \\ 0 & 1 & 1 & 7 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 2 & 5 \\ 0 & 1 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 - R_3$

$-R_2 + R_1$

$$\begin{bmatrix} 1 & 0 & 1 & -2 \\ 0 & 1 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Parametric Eq

$x_1 = -2 - t \quad , x_2 = 7 - t \quad , x_3 = t$

$(x_1, x_2, x_3) = (-2-t, 7-t, t)$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 7 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$ Non Homo. $Ax = b$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = t \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$ Homogeneous $Ax = 0$

Q5) Find Bases for the null space & row space of A.

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$$

$Ax = 0$

$$\begin{bmatrix} 1 & -1 & 3 & 0 \\ 5 & -4 & -4 & 0 \\ 7 & -6 & 2 & 0 \end{bmatrix}$$

$-5R_1 + R_2 \rightarrow -7R_1 + R_3$

$$\begin{bmatrix} 1 & -1 & 3 & 0 \\ 0 & 1 & -19 & 0 \\ 0 & 1 & -19 & 0 \end{bmatrix}$$

$-R_2 + R_3 \rightarrow R_2 + R_1$

$$\begin{bmatrix} 1 & 0 & -16 & 0 \\ 0 & 1 & -19 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 - 16x_3 = 0$

$x_2 - 19x_3 = 0$

Let $x_3 = t$

$x_1 = 16t$

$x_2 = 19t$

$(x_1, x_2, x_3) = (16t, 19t, t)$

$= t(16, 19, 1)$ 1D = vec. space

Null Space $= \left\{ t \begin{bmatrix} 16 \\ 19 \\ 1 \end{bmatrix} \right.$

Row Space:

$r_1 = [1 \quad 0 \quad -16]$

$r_2 = [0 \quad 1 \quad -19]$

Q 6) Find basis for row space & col space

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$ (Already Reduced)

Row Space

$r_1 =$ ~~(1, 0, 2~~ $(1 \quad 0 \quad 2)$

$r_2 =$ $(0 \quad 0 \quad 1)$

Col space

$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$

Ex4.9

Rank, Nullity

Dimension Theorem for Matrix.

If A is a matrix with n col, then ~~Rank (A)~~ -

Rank (A) + Nullity (A) = n

Find the rank & nullity of the matrix A by reducing it to row echelon form.

$$A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & -2 & 2 \\ 3 & 6 & -3 & 3 \\ 4 & 8 & -4 & 4 \end{bmatrix}$$

Reduced Echelon Form

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Non Zero Leading Element

Rank (A) = 1

n = 4

Rank (A) + Nullity = n

1 + Nullity = 4

Nullity = 3

Q) The matrix R is reduced row echelon form of matrix A.

a) Find Rank & Nullity of A.

b) Confirm rank & Nullity by formula

c) Find the no. of leading variables & No. of parameter's in the general Sol $Ax = 0$, without Solving.

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$$A = \begin{bmatrix} 2 & -1 & -3 \\ -1 & 2 & -3 \\ 1 & 1 & 4 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

a) Rank (A) $= 3$

Nullity $= 3 - 3 = 0$

b) Rank(A) + Nullity (A) $= n$

$3 + 0 = 3$

$3 = 3$

c) Leading variable $= 3$

Parameter $= 0$

b)

$$A = \begin{bmatrix} 2 & -1 & -3 \\ -1 & 2 & -3 \\ 1 & 1 & -6 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

a) Rank (A) $= 2$

Nullity(A) $= 1$

)

; Prove

)

Leading var $= 2$

Parameter $= 0$

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Q3) Find the Largest possible value for the rank of A and the Smallest possible value for Nullity of A.

a) 4×4
b) 3×5
c) 5×3

a) Largest Rank(A) = 4
Nullity = 0

b)

L Rank (A) = 3
S Nullity = 2

c)

1		
	1	
		1

L Rank(A) = 3
S Nullity = col − Rank
= 3 − 3
= 0

Q 4) Verify that Rank(A) = Rank(A^T)

$$A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ -3 & 1 & 5 & 2 \\ -2 & 3 & 9 & 2 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & -3 & -2 \\ 2 & 1 & 3 \\ 4 & 5 & 9 \\ 0 & 2 & 2 \end{bmatrix}$$

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$3R_1+R_2$ & $R_2/7$

$$\begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 1 & 17/7 & 2/7 \\ -2 & 3 & 9 & 2 \end{bmatrix}$$

$2R_1+R_3$

$$\begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 1 & 17/7 & 2/7 \\ 0 & 7 & 17 & 2 \end{bmatrix}$$

$-7R_2+R_3$

$$\begin{bmatrix} 1 & 2 & 4 & 0 \\ 0 & 1 & 17/7 & 2/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank = 2

Nullity = 4 − 2
= 2

$$A^T = \begin{bmatrix} 1 & -3 & -2 \\ 2 & 1 & 3 \\ 4 & 5 & 9 \\ 0 & 2 & 2 \end{bmatrix}$$

$-2R_1+R_2$ & $R_2/7$ & $-4R_1+R_3$ & R

$$\begin{bmatrix} 1 & -3 & -2 \\ 0 & 1 & 1 \\ 0 & 17 & 17 \\ 0 & 2 & 2 \end{bmatrix}$$

$17R_2-R_3$

Swap

$$\begin{bmatrix} 1 & -3 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 2 & 2 \end{bmatrix}$$

Swap R_3 & R_4

$$\begin{bmatrix} 1 & -3 & -2 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

~~$\text{Rank}(A^T) = 3$~~

$2R_2 - R_3$

$$\begin{bmatrix} 1 & -3 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\text{Rank}(A^T) = 2$

$\text{Nullity} = 3 - 2$

$= \underline{1}$

Hence $\text{Rank}(A) = \text{Rank}(A^T)$

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Eigen Values & Eigen vectors

$Ax = \lambda x$

Characteristic Eq

$\det(\lambda I - A) = 0$ λ is eigen values.

Q1) Confirm by multiplication that x is an Eigen vector of A, & find the corresponding Eigen values.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$Ax = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$= -1 \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$\boxed{\lambda = -1}$$

Q2)

$$A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

Q3) Find the characteristic Eq, the eigenvalue, and bases for Eigen space of the matrix

$$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

λ is an Eigen Value

I is an Identity Matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

The characteristic Eq $\det(\lambda I - A) = 0$

$$\det\left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} \lambda-1 & 0-4 \\ 0-2 & \lambda-3 \end{bmatrix}\right) = 0$$

$(\lambda-1)(\lambda-3) - (-2)(-4) = 0$

$\cancel{(\lambda-1)(\lambda-3)}$ $\cancel{-8}$ $\lambda^2 - 3\lambda - \lambda + 3 - 8 = 0$

λ $\quad \lambda = +5 \quad , \quad \lambda = -1$

$\lambda = -1$

$(-1-1)(-1-3) - 8 = 0$

$8 - 8 = 0$

$0 = 0$

$\lambda = -1$

$$\begin{bmatrix} \lambda-1 & -4 \\ -2 & \lambda-3 \end{bmatrix}$$

$$\begin{bmatrix} -1-1 & -4 \\ -2 & -1-3 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -4 \\ -2 & -4 \end{bmatrix}$$

Reduced Echelon Form

$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

$(-1\lambda - A) = 0$

$x_1 = -2t$

$x_2 = 2t$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2t \\ \cancel{2}t \end{bmatrix}$$

$$= t\begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ is an Eigen Vector with $\lambda = -1$

$\lambda = 5$

$$\begin{bmatrix} 5-1 & -4 \\ -2 & 5-3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -4 \\ -2 & 2 \end{bmatrix}$$

By Reduced Echelon Form

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

$x_1 = t$

$x_2 = t$

$(5I - A) = 0$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} t \\ t \end{bmatrix}$$

$\lambda = +ve$ Unstable
$\lambda = -ve$ Stable
$\lambda = 0$ Critical Point

$= t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is Eigen Vector with $\lambda = 5$.

Q4) $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$ $\quad \det(\lambda I - A) = 0$

$$\det\left(\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}\right)$$

$$\det\begin{pmatrix} \lambda-4 & 0 & -1 \\ 2 & \lambda-1 & 0 \\ 2 & 0 & \lambda-1 \end{pmatrix} = 0$$

$\lambda-4\,((\lambda-1)\cdot(\lambda-1)) - 0) - 0 - 1\,(0 - 2(\lambda-1))$

$(\lambda-4)(\lambda^2-2\lambda+1) + 2\lambda - 2 = 0$

$\lambda^3 - 2\lambda^2 + \lambda - 4\lambda^2 + 8\lambda - 4 + 2\lambda - 2 = 0$

$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$

$\lambda = 1,\ \lambda = 2,\ \lambda = 3$

$\lambda = 1$

$\begin{bmatrix} \cancel{1-4}\,-3 & 0 & -1 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix}$

Reduced Echelon Form
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

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$x_1 = 0$

$x_3 = 0$

$x_2 = t$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ t \\ 0 \end{bmatrix} = t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ is Eigen Vector of $\lambda = 1$

$\underline{\lambda = 2}$

$\begin{bmatrix} -1/2 \\ 1 \\ 1 \end{bmatrix}$ = by = $\begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$ is Eigen Vector of $\lambda = 2$

$\underline{\lambda = 3}$

$\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$ is Eigen Vector of $\lambda = 3$

Q) Find the characteristic Eq of matrix $\begin{bmatrix} 3 & 0 & 0 \\ -2 & 7 & 0 \\ 4 & 8 & 1 \end{bmatrix}$

$\det(\lambda I - A) = 0$

$$\det\left(\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ -2 & 7 & 0 \\ 4 & 8 & 1 \end{bmatrix}\right)$$

$$\det\begin{pmatrix} \lambda - 3 & 0 & 0 \\ 2 & \lambda - 7 & 0 \\ -4 & -8 & \lambda - 1 \end{pmatrix}$$

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$(\lambda-3)[(\lambda-1)(\lambda-7)]=0$

$\lambda=3 \quad \lambda=1 \quad \lambda=7$

Q Find the Eigen value & a basis for each Eigen space of the linear operator defined by the stated formula.

$$T(x,y,z) = (2x-y-z,\ x-z,\ -x+y+2z)$$

$$T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2x-y-z \\ x-z \\ -x+y+2z \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ -1 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

St. Matrix

$\det(\lambda I - A) = 0$

$$\det\left(\lambda\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ -1 & 1 & 2 \end{bmatrix}\right)$$

$$\det\begin{pmatrix} \lambda-2 & 1 & 1 \\ -1 & \lambda & 1 \\ 1 & -1 & \lambda-2 \end{pmatrix} = 0$$

$$[\lambda-2(\lambda(\lambda-2)+1) - 1[-\lambda+2-1] + 1[+1-\lambda] = 0]$$

$$\lambda^3-2\lambda^2+\lambda-2\lambda^2+4\lambda-2+\lambda-1+1-\lambda=0$$

$$\lambda^3-4\lambda^2+5\lambda-2=0 \qquad \boxed{\lambda=1,\quad \lambda=2}$$

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$\lambda \neq 1$

$$\begin{bmatrix} \lambda-2 & 1 & 1 \\ -1 & \lambda & 1 \\ 1 & -1 & \lambda-2 \end{bmatrix}$$

$\lambda = 1$

$$\begin{bmatrix} -1 & 1 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$$

R_1 & R_3

$$\begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

$R_1 + R_2$ & $R_1 + R_3$

$$\begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_2 = t$ $\quad x_3 = h$

$x_1 - t - h = 0$

$x_1 = t + h$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} t+h \\ t \\ h \end{bmatrix}$$

$$t \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, h \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Eigen value

$\lambda = 2$

$$\begin{bmatrix} 0 & 1 & 1 \\ -1 & 2 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\begin{array}{c} \begin{matrix} x & y & z \end{matrix} \\ \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \end{array}$$

$x - y = 0$

$y + z = 0$

$z = s$ Parametric.

$y = -s$

$x = -s$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow s \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

Eigen Val

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Q) $T(x,y) = (x+4y, 2x+3y)$

Ex 1.8 Transformation

$$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$\det(\lambda I - A)$

$$\det\left[\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}\right]$$

$$\det \begin{bmatrix} \lambda - 1 & -4 \\ -2 & \lambda - 3 \end{bmatrix} = 0 \quad (1)$$

$$(\lambda-1)(\lambda-3) + 4(2) = 0$$

$\lambda = 5 \qquad \lambda = -1$

$\boxed{\lambda = 5}$

Substitute in eq (1)

$$\begin{bmatrix} 4 & -4 \\ -2 & 2 \end{bmatrix}$$

$$\begin{array}{cc} x_1 & x_2 \end{array}$$
$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$ Row Echelon

$\boxed{x_2 = t}$ Parametric

$x_1 - x_2 = 0$

$x_1 = x_2$

$\boxed{x_1 = t}$

$$t \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\lambda = -1$

$$\begin{bmatrix} -2 & -4 \\ -2 & -4 \end{bmatrix}$$

$$\begin{array}{cc} x_1 & x_2 \end{array}$$
$$\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

$\boxed{x_2 = t}$

$x_1 + 2x_2 = 0$

$x_1 + 2t = 0$

$\boxed{x_1 = -2t}$

$$= t \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

Ex 5.2

Diagonalization

Similar Matrix

1) $|A| = |B|$
2) $P^{-1} . A . P$

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Q2) Find the matrix P that diagonalizes A, & check your work by Computing $P^{-1}.A.P$

$$\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$$

$6R_1 - R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$-R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\cancel{\lambda_1 = 0}$

$\lambda_2 \neq 0$

$$\det(\lambda I - A) = 0$$

$$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}\right) = 0$$

$$\det\begin{pmatrix} \lambda - 1 & -4 \\ -2 & \lambda - 3 \end{pmatrix} = 0$$

$$(\lambda-1)(\lambda-3) + 8 = 0$$

$$\lambda^2 - 3\lambda - \lambda + 3 + 8 = 0$$

$$\lambda^2 - 4\lambda + 11 = 0$$

$$\lambda = 2 + \sqrt{7}\, i \qquad \lambda = 2 - \sqrt{7}\, i$$

Hence Order would be changed $\rightarrow \det(A - \lambda I) = 0$

$$\det\left(\begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right) = 0$$

$$\det\begin{pmatrix} 1-\lambda & 0 \\ 6 & -1-\lambda \end{pmatrix} = 0$$

$\cancel{(1-\lambda)(3-\lambda) - 8 = 0}$

$\cancel{3 - \lambda - 3\lambda + \lambda^2 - 8 = 0}$

$\cancel{\lambda^2 - 4\lambda - 5 = 0}$

$$(1-\lambda)(-1-\lambda) - 0 = 0$$

$$-1 - \lambda + \lambda + \lambda^2 = 0$$

$$\lambda^2 - 1 = 0$$

$$\lambda = 1 \qquad \lambda = -1$$

$$\begin{bmatrix} 1-\lambda & 0 \\ 6 & -1-\lambda \end{bmatrix}$$

$\underline{\lambda = 1}$

$$\begin{bmatrix} 0 & 0 \\ 6 & -2 \end{bmatrix}$$

$x_2 = t$

$6x_1 - 2t = 0$

$x_1 = \frac{t}{3}$

$t \begin{bmatrix} 1/3 \\ 1 \end{bmatrix} \times 3$

$p_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$\lambda = -1$

$$\begin{bmatrix} 2 & 0 \\ 6 & 0 \end{bmatrix}$$

$3R_1 - R_2$

$$\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$x_2 = t$

$2x_1 = 0$

$x_1 = 0$

$t \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$p_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$P = [P_1 \quad P_2]$

$= \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

$P^{-1} = \frac{\text{Adj } P}{|P|}$

$= \dfrac{\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix}}{1}$

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$$P^{-1} \cdot A \cdot P = \begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

Q3)

$$A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$$

a) Find the Eigen Value of A.

b) For Each Eigen Value λ, Find the rank of matrix $\lambda I - A$

c) Is A is diagonalisable ? Justify your Conclusion.

$$\det(\lambda I - A) = 0$$

$$\det\left(\lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}\right)$$

$$\begin{bmatrix} \lambda - 4 & 0 & -1 \\ -2 & \lambda - 3 & -2 \\ -1 & 0 & \lambda - 4 \end{bmatrix}$$

$$(\lambda-4)\left[(\lambda-4)(\lambda-3) - 0\right] - 1\left[0 - (-1)(\lambda-3)\right]$$

$$\cancel{(\lambda-4)}(\lambda-4)\left[\lambda^2 - 7\lambda + 12\right] - \lambda + 3$$

$$\lambda^3 - 7\lambda^2 + 12\lambda - 4\lambda^2 + 28\lambda - 48 - \lambda + 3 = 0$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$\boxed{\lambda = 5}$ $\boxed{\lambda = 3}$ $\boxed{\lambda = 3}$

$$\begin{bmatrix} \lambda-4 & 0 & -1 \\ -2 & \lambda-3 & -2 \\ -1 & 0 & \lambda-4 \end{bmatrix}$$

$\lambda = 3$

$$\begin{bmatrix} -1 & 0 & -1 \\ -2 & 0 & -2 \\ -1 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_3 = t \qquad x_2 = s$

$x_1 + x_3 = 0$

$x_1 = -t$

$$t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\lambda = 5$

$$\begin{bmatrix} 1 & 0 & -1 \\ -2 & 2 & -2 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$[\ x_3 = t$

$\left[\begin{array}{l} x_2 - 2x_3 = 0 \\ x_2 = 2t \end{array}\right.$

$\left[\begin{array}{l} x_1 - x_3 = 0 \\ x_1 = t \end{array}\right.$

$$t \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

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Ex 5.2
Diagonalization.

Q1) The characteristic Eq of matrix A is given. Find the size of the matrix & dimension.

a) $(\lambda-1)(\lambda+3)(\lambda-5)=0$; 3×3 , Dim = 1

b) $\lambda^2(\lambda-1)(\lambda-2)^2=0$ 6×6 , Dim = 1

Q2) Uses the Method to compute the matrix A^{10}.

$$A = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$$

$P^{-1}AP = D$

$A^{10} = PD^{10}P^{-1}$

$\det(\lambda I - A) = 0$

$$\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}\right) = 0$$

$$\det\begin{pmatrix} \lambda & -3 \\ -2 & \lambda+1 \end{pmatrix} = 0$$

$(\lambda+1)(\lambda) - 6 = 0$

$\lambda^2 + \lambda - 6 = 0$

$\lambda = -3$, $\lambda = 2$

$$\begin{bmatrix} \lambda & -3 \\ -2 & \lambda+1 \end{bmatrix}$$

$\lambda = 2$

$$\begin{bmatrix} 2 & -3 \\ -2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 \\ 0 & 0 \end{bmatrix}$$

$x_2 = t$

$2x_1 - 3x_2 = 0$

$2x_1 = 3t$

$x_1 = \frac{3t}{2}$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t \begin{bmatrix} 3/2 \\ 1 \end{bmatrix} \times 2$$

$P_1 = [P_1 \quad P_2]$

$$P = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$$

$P^{-1} = \frac{\text{Adj } P}{|P|}$

$$= \frac{\begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}}{5}$$

$$= \begin{bmatrix} 1/5 & 1/5 \\ -2/5 & 3/5 \end{bmatrix}$$

$\lambda = -3$

$$\begin{bmatrix} -3 & -3 \\ -2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} \cancel{-3}\,1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

$x_2 = t$

$x_1 + x_2 = 0$

$x_1 = -t$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = t \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$$D = P^{-1} \cdot A \cdot P$$

$$= \begin{bmatrix} 1/5 & 1/5 \\ -2/5 & 3/5 \end{bmatrix} \cdot \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 0 \\ 0 & -3 \end{bmatrix}$$

$$D^{10} = \begin{bmatrix} 2^{10} & 0 \\ 0 & (-3)^{10} \end{bmatrix}$$

$$A^{10} = P \cdot D^{10} \cdot P^{-1}$$

$$= \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2^{10} & 0 \\ 0 & (-3)^{10} \end{bmatrix} \cdot \begin{bmatrix} 1/5 & 1/5 \\ -2/5 & 3/5 \end{bmatrix}$$

$$= \begin{bmatrix} 24{,}234 & -34815 \\ -23210 & 35839 \end{bmatrix}$$

Q3) $A = \begin{bmatrix} -1 & 7 & -1 \\ 0 & 1 & 0 \\ 0 & 15 & -2 \end{bmatrix}$, $P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 5 \end{bmatrix}$

Confirm that P diagonalizes A, & then compute each of the following powers of A.

a) A^{1000}

b) A^{2301}

$$P^{-1} = \cancel{\text{Adj } P} = \begin{bmatrix} 1 & -5 & 1 \\ 1 & 4 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$D = P^{-1} . A . P$$

$$= \begin{bmatrix} 1 & -5 & 1 \\ 1 & 4 & -1 \\ 0 & 1 & 0 \end{bmatrix} . \begin{bmatrix} -1 & 7 & -1 \\ 0 & 1 & 0 \\ 0 & 15 & -2 \end{bmatrix} . \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D^{1000} = \begin{bmatrix} (-2)^{1000} & 0 & 0 \\ 0 & (-1)^{1000} & 0 \\ 0 & 0 & (1)^{1000} \end{bmatrix}$$

$$A^{1000} = P D^{1000} P^{-1}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 5 \end{bmatrix} . \begin{bmatrix} (-2)^{1000} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} 1 & -5 & 1 \\ 1 & 4 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$=$$

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Let $A = \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$, $P = \begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

Confirm that P diagonalizes A, & then compute each of the following power of A.

a) A^{1000}

b) A^{2301}.

a) $A^{1000} = P D^{1000} P^{-1}$

$D = P^{-1} A P$

$$D = \overset{P^{-1}}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}} \overset{A}{\begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}} \overset{P}{\begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}$$

$$D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$D^{1000} = \begin{bmatrix} (-1)^{1000} & 0 & 0 \\ 0 & (-1)^{1000} & 0 \\ 0 & 0 & (1)^{1000} \end{bmatrix}$$

$$A^{1000} = \overset{P}{\begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}} \overset{D^{1000}}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \overset{P^{-1}}{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For $A^{2301} = \underbrace{\begin{bmatrix} 1 & -4 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{D^{2301}} \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & 4 \end{bmatrix}}_{P^{-1}}$

$$A^{2301} = \begin{bmatrix} 1 & -2 & 8 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$