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Discrete Structures
Assignment01

Answer#1:

a. Proposition : True
b. Proposition : False
c. Proposition: True
d. Proposition: False
e. Not a proposition
f. Not a proposition

Answer #2:

a. let,
p : B has more RAM than A. → true
q : B has more RAM than C → true

⇒ $p \wedge q$
⇒ True Ans.

p	q	p^q
T	T	T
T	F	F
F	T	F
F	F	F

b. let,
p : C has more ROM than B. → false
q : C has higher resolution than B. → true

⇒ $p \vee q$
⇒ True Ans.

p	q	pvq
T	T	T
T	F	T
F	T	T
F	F	F

c. let,
p : B has more RAM than A. → true
q : B has more ROM than A. → true
r : B has higher resolution than A. → false

⇒ $p \wedge q \wedge r$
⇒ False Ans.

p	q	r	p^q^r
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

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d. let,

p: B has more RAM than C. → true
q: B has more ROM than C. → true
r: B has higher resolution than C. → false

⇒ $(p \wedge q) \rightarrow r$

⇒ False

p	q	r	$(p \wedge q)$	$(p \wedge q \rightarrow r)$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	T
T	F	F	F	T
F	T	T	F	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

e. let,

p: A has more RAM than B. → false
q: B has more RAM than A. → true

⇒ $p \leftrightarrow q$

⇒ False

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Answer #3:

a. let,

p: Quixote had more revenue than Acme. → (False)
q: Quixote had more revenue than Nadir Software. → (True)

⇒ $p \wedge q$

⇒ False

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

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b. let,

p: Nadir software had lowest net profit. → True
q: Acme had larget revenue. → True

⇒ $p \wedge q$

⇒ True

c.

let,

p: Acme had more net profit than Nadir. → (True)

q: Acme had more net profit than Quixote. → (False)

r: Quixote had more net profit than Acme. → (True)

s: Quixote had more net profit than Nadir. → (True)

$\Rightarrow (p \wedge q) \vee (r \wedge s)$

$\Rightarrow F \vee T$

$\Rightarrow$ True

d. let,

p: Quixote had smallest net profit. → False

q: Acme had largest revenue. → True

$\Rightarrow p \rightarrow q$

$\Rightarrow$ True

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

e. let,

p: Nadir had smallest net profit. → True

q: Acme had largest revenue. → True

$\Rightarrow p \leftrightarrow q$

$\Rightarrow$ True

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Answer # 4:

a. $p \rightarrow q$: If you have the flu, then you will miss the final examination. Ans.

b. $\neg q \leftrightarrow r$: You will not miss the final examination if and only if you pass the course. Ans.

c. $q \rightarrow \neg r$: If you miss the final examination, then you will not pass the course. Ans.

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d. $p \vee q \vee r$:

You have the flu or you miss the final examination or you pass the course. Ans.

e. $(p \to \neg r) \vee (q \to \neg r)$:

If you have the flue, then you will not pass the course or if you miss the final examination, then you will not pass the course. Ans.

f. $(p \wedge q) \vee (\neg q \wedge r)$:

You have the flu and you will miss the final examination or you will not miss the final examination and you will pass the course. Ans.

Answer#5:

a. $r \wedge \neg q$

b. $p \wedge q \wedge r$

c. $r \to p$

d. $(p \wedge \neg q) \wedge r$

e. $(p \wedge q) \to r$

f. $r \leftrightarrow (q \vee p)$.

Answer #6:

a. If I will remember to send you the address, then you send me an email message.

b. If you were born in the United States, then you are a citizen of this country.

c. If you keep your textbook, then it will be a useful reference in your future courses.

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d. If the Red Wing's goalie plays well, then they will win the Stanley Cup.

e. If you got the job, then you had the best credentials.

f. If there is a storm, then the beach erodes.

g. If you want to log on to the server, then you must have a valid password.

h. If you do not begin your climb too late, then you will reach the summit.

Answer#7

a. $p \to q$ can be written as:

- p is sufficient for q: Tomorrow being sunny is sufficient for me to go for a walk in the woods.
- a necessary condition for q is p: Tomorrow being sunny is a necessary condition for me to go for a walk in the woods.
- p implies q: Tomorrow being sunny implies I will go for a walk in the woods.
- q unless $\neg p$: I will go for a walk in the woods unless it is not sunny tomorrow.
- q provided that p: I will go for a walk in the woods provided that it is sunny tomorrow.
- a sufficient condition for q is p: A sufficient condition for me going for a walk in the woods is tomorrow being sunny.

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b. $p \to q$:

converse: $q \to p$

If I go for a walk in the woods, then it will be sunny tomorrow.

inverse: $\neg p \to \neg q$

If it is not sunny tomorrow, then I will not go for a walk in the woods.

contrapositive: $\neg q \to \neg p$

If I do not go for a walk in the woods, then it is not sunny tomorrow.

c. $p \to q$ (using part B)

- inverse of inverse:

Ans. If it is sunny tomorrow, then I will go for a walk in the woods.

- inverse of converse:

Ans. If I do not go for a walk in the woods, then it will not be sunny tomorrow.

- inverse of contrapositive:

Ans. If I go for a walk in the woods, then it will be sunny tomorrow.

Answer # 8

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a. $\underbrace{\text{Jan is rich}}_{p}$ and $\underbrace{\text{happy}}_{q}$.

$\wedge$

$\Rightarrow \neg(p \wedge q) = \neg p \vee \neg q$

Ans. Jan is not rich or Jan is not happy.

b. $\underbrace{\text{Carlos will bicycle}}_{p}$ or $\underbrace{\text{run tomorrow}}_{q}$.

$\vee$

$\Rightarrow \neg(p \vee q) = \neg p \wedge \neg q$

Ans. Carlos will not bicycle and will not run tomorrow.

c. $\underbrace{\text{The fan is slow}}_{p}$ or $\underbrace{\text{it is very hot}}_{q}$.

$\vee$

$\Rightarrow \neg(p \vee q) = \neg p \wedge \neg q$

Ans. The fan is not slow and it is not very hot.

d. $\underbrace{\text{Akram is unfit}}_{p}$ and $\underbrace{\text{Saleem is injured}}_{q}$.

$\wedge$

$\Rightarrow \neg(p \wedge q) = \neg p \vee \neg q$

Ans. Akram is not unfit or Saleem is not injured.

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Answer # 9:

a. Exclusive sense, because the person will chose one alternabive at a time and not both; they can either stay at home or go out to watch a movie but not both.

b. Inclusive sense, because it is possible that a person could fail to make the payment as well as it could be a incorrect amount at the same time.

c. Inclusive sense, because it is inevitable that the person cannot go to the trip if both events happen.

d. Inclusive sense, because the person will still be denied of service if he doesn't wear shoes and shirt simultaneously or either one of them.

Answer # 10:

a. Prove: $(p \wedge (\neg(\neg p \vee q))) \vee (p \wedge q) \equiv p$

$\Rightarrow (p \wedge (\neg\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$ $\quad \because$ Demorgan's law

$\Rightarrow (p \wedge (p \wedge \neg q)) \vee (p \wedge q) \equiv p$ $\quad \because \neg\neg p = p$ [Double Negation]

$\Rightarrow ((p \wedge p) \wedge \neg q) \vee (p \wedge q) \equiv p$ $\quad \because$ Associative law

$\Rightarrow (p \wedge \neg q) \vee (p \wedge q) \equiv p$ $\quad \because$ Idempotent law

$\Rightarrow p \wedge (\neg q \vee q) \equiv p$ $\quad \because$ Distributive law

$\Rightarrow p \wedge (T) \equiv p$ $\quad \because$ Negation law $(\neg q \vee q \equiv T)$

$\Rightarrow p \equiv p$ $\quad \because$ Identity law

Hence Proved,

L.H.S $\equiv$ R.H.S

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6. Prove: $\neg(p \leftrightarrow q) \equiv (p \leftrightarrow \neg q)$

Taking R.H.S:

$\Rightarrow (p \leftrightarrow \neg q)$

$\Rightarrow (p \rightarrow \neg q) \wedge (\neg q \rightarrow p)$ ∴ By definition of Bi-implication

$\Rightarrow (\neg p \vee \neg q) \wedge (q \vee p)$ ∴ Implication law

$\Rightarrow \neg(\neg((\neg p \vee \neg q) \wedge (q \vee p)))$ ∴ Double negation: $\neg\neg r \equiv r$

$\Rightarrow \neg(\neg(\neg p \vee \neg q) \vee \neg(q \vee p))$ ∴ Demorgan law

$\Rightarrow \neg((p \wedge q) \vee (\neg q \wedge \neg p))$ ∴ Demorgan law

$\Rightarrow \neg((p \vee \neg q) \wedge (p \vee \neg p) \wedge (q \vee \neg q) \wedge (q \vee \neg p))$ ∴ Distributive law

$\Rightarrow \neg((p \vee \neg q) \wedge (T) \wedge (T) \wedge (q \vee \neg p))$ ∴ Identity law

$\Rightarrow \neg((p \vee \neg q) \wedge (q \vee \neg p))$

$\Rightarrow \neg((\neg q \vee p) \wedge (\neg p \vee q))$ ∴ Commutative law

$\Rightarrow \neg((q \rightarrow p) \wedge (p \rightarrow q))$ ∴ Implication law

$\Rightarrow \neg(q \leftrightarrow p)$ ∴ Definition of Bi-implication

$\Rightarrow \neg(p \leftrightarrow q) \equiv L.H.S$ ∴ Commutative law

Hence proved,

$L.H.S \equiv R.H.S$

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c. $\neg p \leftrightarrow q \equiv p \leftrightarrow \neg q$

L.H.S:

$\Rightarrow \neg p \leftrightarrow q$

$\Rightarrow (\neg p \rightarrow q) \wedge (q \rightarrow \neg p)$ $\therefore$ Definition of Bi-implication

$\Rightarrow (\neg\neg p \vee q) \wedge (\neg q \vee \neg p)$ $\therefore$ Implication law

$\Rightarrow (p \vee q) \wedge (\neg q \vee \neg p)$ $\therefore$ Double negation

$\Rightarrow (q \vee p) \wedge (\neg p \vee \neg q)$ $\therefore$ Commutative law

$\Rightarrow (\neg q \rightarrow p) \wedge (\neg\neg p \rightarrow \neg q)$ $\therefore$ Implication law

$\Rightarrow (\neg q \rightarrow p) \wedge (p \rightarrow \neg q)$ $\therefore$ Double negation

$\Rightarrow (\neg q \leftrightarrow p)$ $\therefore$ Definition of Bi-implication

$\Rightarrow p \leftrightarrow \neg q \equiv$ R.H.S $\therefore$ Commutative law

Hence proved,
L.H.S $\equiv$ R.H.S

d. $(p \wedge q) \rightarrow (p \rightarrow q) \equiv T$

L.H.S:

$\Rightarrow (p \wedge q) \rightarrow (p \rightarrow q)$

$\Rightarrow \neg(p \wedge q) \vee (p \rightarrow q)$ $\therefore$ Implication law

$\Rightarrow (\neg p \vee \neg q) \vee (p \rightarrow q)$ $\therefore$ Demorgan law

$\Rightarrow (\neg p \vee \neg q) \vee (\neg p \vee q)$ $\therefore$ Implication law

$\Rightarrow (\neg p \vee \neg p) \vee (\neg q \vee q)$ $\therefore$ Associative law

$\Rightarrow \neg p \vee (\neg q \vee q)$ $\therefore$ Idempotent law

$\Rightarrow \neg p \vee (T)$ $\therefore$ Negation law: $\neg q \vee q \equiv T$

$\Rightarrow T \equiv$ R.H.S $\therefore$ Universal Bound law

Hence proved,
L.H.S $\equiv$ R.H.S

e. Prove: $\neg(p \vee \neg(p \wedge q)) \equiv F$ 23k-2001 BCS-3J

L.H.S:

$\Rightarrow \neg(p \vee \neg(p \wedge q))$

$\Rightarrow \neg p \wedge \neg\neg(p \wedge q)$ ∴ Demorgan law

$\Rightarrow \neg p \wedge (p \wedge q)$ ∴ Double negation: $\neg\neg r = r$

$\Rightarrow (\neg p \wedge p) \wedge q$ ∴ Associative law

$\Rightarrow F \wedge q$ ∴ Negation law: $\neg p \wedge p \equiv F$

$\Rightarrow F \equiv F$ ∴ Universal bound law

Hence proved,

L.H.S ≡ R.H.S

Answer#11: Equivalence by Truth Table

a. $(p \to r) \wedge (q \to r)$ & $(p \vee q) \to r$

p	q	r	$p \to r$	$q \to r$	$p \vee q$	$(p \to r) \wedge (q \to r)$	$(p \vee q) \to r$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	T	T	T	T	T
T	F	F	F	T	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	T	T	F	T	T
F	F	F	T	T	F	T	T

=

Ans. Equivalent,

Hence proved

b. $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$

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p	q	r	$q \vee r$	$p \to q$	$p \to r$	$(p \to q) \vee (p \to r)$	$p \to (q \vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

=

Ans. Equivalent,
Hence Proved.

c. $(p \to q) \to (r \to s)$ & $(p \to r) \to (q \to s)$.

p	q	r	s	$p \to q$	$p \to r$	$r \to s$	$q \to s$	$(p \to q) \to (r \to s)$	$(p \to r) \to (q \to s)$
T	T	T	T	T	T	T	T	T	T
T	T	T	F	T	T	F	F	F	F
T	T	F	T	T	F	T	T	T	T
T	T	F	F	T	F	T	F	T	T
T	F	T	T	F	T	T	T	T	T
T	F	T	F	F	T	F	T	T	T
T	F	F	T	F	F	T	T	T	T
T	F	F	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T	T	T
F	T	T	F	T	T	F	F	F	F
F	T	F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	F	T	F
F	F	T	T	T	T	T	T	T	T
F	F	T	F	T	T	F	T	F	T
F	F	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T	T

$\neq$

Ans. Not equivalent, Hence Proved

Answer#12:

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a. $P(4,5)$

Ans. False — when $m:4$, $n:5$ — $\frac{5}{4} \neq$ any integer

b. $P(2,4)$

Ans. True — when $m:2$, $n:4$ — $\frac{4}{2} = 2$ (integer)

c. $\forall m \forall n (P(m,n))$

Ans. False — not every integer divides every other integer
e.g: $P(4,5) \Rightarrow$ False

d. $\exists m \forall n P(m,n)$

Ans. True — some value m exists that divides every n
e.g: $P(1,n) \Rightarrow$ True since 1 can divide any integer

e. $\exists n \forall m P(m,n)$

Ans. False — some value n is not divisible by all m
e.g: $P(m,5)$ only when m also 5 but not for all m.

f. $\forall n P(1,n)$

Ans. True — 1 can divide any integer n.

Answer#13:

a. $\exists x (x^2=2)$

Ans. True — $x = \{\sqrt{2}, -\sqrt{2}\}$ satisfy $x^2 = 2$

b. $\exists x (x^2 = -1)$

Ans. False — For any real number $x^2 \geq 0$ always, so, no value x can satisfy $x^2 = -1$.

c. $\forall x (x^2+2 \geq 1)$

Ans. True — since $x^2 \geq 0$ always, then $x^2 + \geq 2$ always so, for all x, $x^2+2 \geq 1$ can be satisfied.

d. $\exists x (x^2 = x)$

Ans. True — $x = \{0, 1\}$ satisfy $x^2 = x$

Answer #14:

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a. Everybody can fool Bob.

Ans. $\forall x\, F(x, Bob)$

b. Alice can fool everybody.

Ans. $\forall y\, F(Alice, y)$

c. Everybody can fool somebody.

Ans. $\forall x\, \exists y\, F(x,y)$

d. There is no one who can fool everybody.

Ans. $\neg \exists x\, \forall y\, F(x,y)$

e. Everyone can be fooled by somebody.

Ans. $\forall y\, \exists x\, F(x,y)$

Answer #15:

a. There is a student at your school who can speak Russian and who knows C++

Ans. $\exists x\, (P(x) \wedge Q(x))$

b. There is a student at your school who can speak Russian but doesn't know C++.

Ans. $\exists x\, (P(x) \wedge \neg Q(x))$

c. Every student at your school either speaks Russian or knows C++.

Ans. $\forall x\, (P(x) \vee Q(x))$

d. No student at your school can speak Russian or C++.

Ans. $\neg \exists x (P(x) \vee Q(x))$ representation #1

$\forall x (\neg P(x) \wedge \neg Q(x))$ representation #2

(Demorgans law)

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Answer#16:

a. $\exists x \exists y\, Q(x,y)$: There is a student in your class who has sent an email message to some student in your class

b. $\exists x \forall y\, Q(x,y)$: There is a student in your class who has sent an email message to every student in your class.

c. $\forall x \exists y\, Q(x,y)$: Every student in your class has sent an email message to atleast one student in your class.

d. $\exists y \forall x\, Q(x,y)$: There is a student in your class who has been sent a message by every student in your class.

e. $\forall y \exists x\, Q(x,y)$: Every student in your class has been sent a message from atleast one student of your class.

f. $\forall x \forall y\, Q(x,y)$: Every student in your class has sent an email message to every student in your class.

Answer#17:

a. $\exists x \exists y\, P(x,y)$: Atleast one student has taken a computer science class

b. $\exists x \forall y\, P(x,y)$: Atleast one student has taken all computer science classes

c. $\forall x \exists y\, P(x,y)$: All students have taken atleast one computer science class.

d. $\exists y \forall x\, P(x,y)$: Atleast one computer science class has been taken by all students.

e. $\forall y \exists x\, P(x,y)$: All computer science classes have been taken by atleast one student.

f. $\forall x \forall y\, P(x,y)$: All students have taken all computer science classes.

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Answer#18:

a. $\dfrac{p}{\therefore p \vee q}$ Ans: Addition

b. $\dfrac{p \wedge q}{\therefore p}$ Ans: Simplification

c. $p \to q$
$\underline{p}$
$\therefore q$
Ans: Modus ponen

d. $p \to q$
$\underline{\neg q}$
$\therefore \neg p$
Ans: Modus Tollen

e. $p \to q$
$\underline{q \to r}$
$\therefore p \to r$
Ans: Hypothetical Syllogism

Answer # 19:

a. let,
p: Today is tuesday q: I have a test in Mathematics
r: I have a test in Economics. s: Economics professor is sick.

$p \to (q \vee r)$
$s \to \neg r$
$\underline{p \wedge s}$
$\therefore q$

i)	$p \wedge s$	(premise)
ii)	p	∵ Simplification law on (i)
iii)	$p \to (q \vee r)$	(premise)
iv)	$q \vee r$	∵ Modus ponen
v)	s	∵ Simplification law on (i)
vi)	$s \to \neg r$	(premise)
vii)	$\neg r$	∵ Modus ponen
viii)	$r \vee q$	∵ Commutative law on (iv)
ix)	$\therefore q$	∵ Elimination

$\neg r \wedge (r \vee q)$
$(\neg r \wedge r) \vee q$
$F \vee q$
q

Ans. The conclusion matches, hence the argument is valid

Conclusion: I have a test in mathematics.

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b. let,

p: Ali is a lawyer. q: Ali is ambitious.

r: Ali is an early riser. s: Ali does not like chocolates.

$p \to q$
$r \to s$
$q \to r$

$\therefore p \to s$

i) $p \to q$ (premise)

ii) $q \to r$ (premise)

---

iii) $p \to r$ $\because$ Hypothetical syllogism

iv) $r \to s$ (premise)

---

v) $\therefore p \to s$ $\because$ Hypothetical syllogism

Ans. The conclusion matches, hence the argument is valid.

Conclusion: If Ali is a lawyer, then he does not like chocolates.

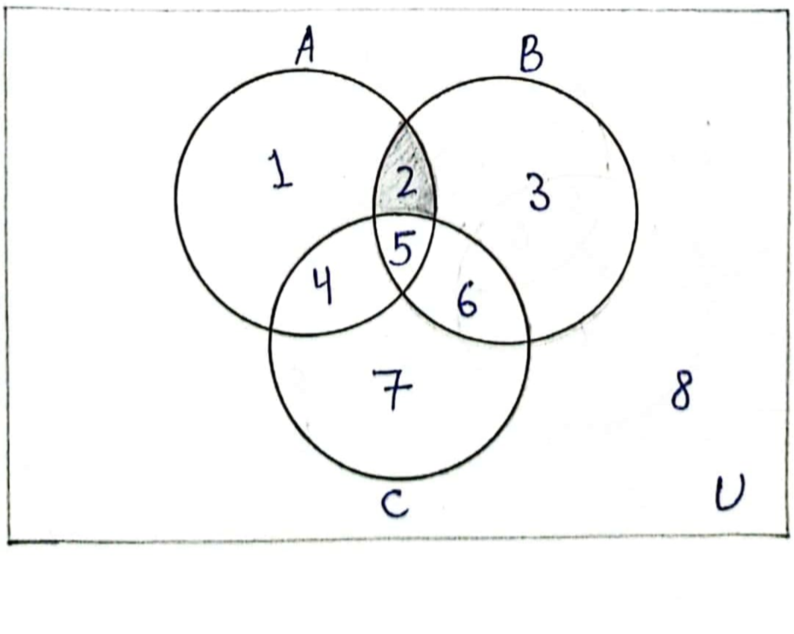
Answer#20:

a. $(A \cap B) \cap \overline{C}$:

$A \cap B = \{1,2,4,5\} \cap \{2,3,5,6\} = \{2,5\}$

$\overline{C} = U - C = \{1,2,3,8\}$

$(A \cap B) \cap \overline{C} = \{2,5\} \cap \{1,2,3,8\} = \{2\}$ Ans.



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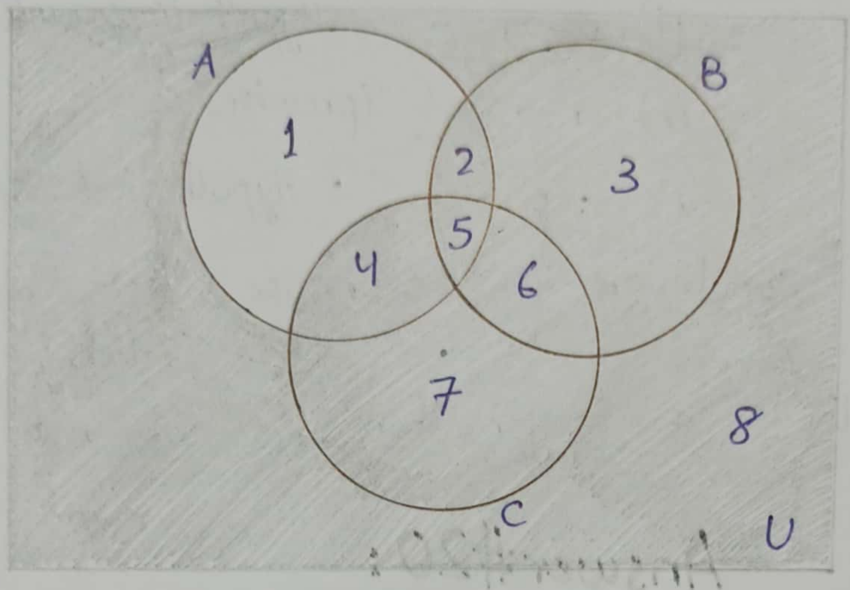
b. $\bar{A} \cup (B \cup C)$:

$\bar{A} = U - C = \{3, 6, 7, 8\}$

$B \cup C = \{2,3,5,6\} \cup \{4,5,6,7\} = \{2,3,4,5,6,7\}$

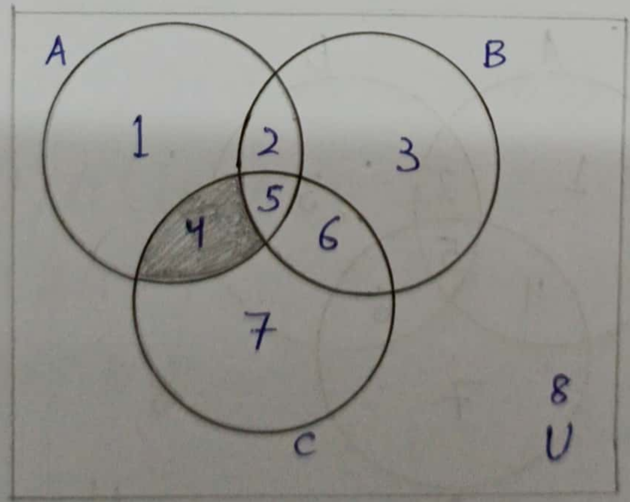
$\bar{A} \cup (B \cup C) = \{3,6,7,8\} \cup \{2,3,4,5,6,7\} = \{2,3,4,5,6,7,8\}$ Ans.



c. $(A - B) \cap C$:

$A - B = \{1,2,4,5\} - \{2,3,5,6\} = \{1, 4\}$

$(A-B) \cap C = \{1, 4\} \cap \{4,5,6,7\} = \{4\}$ Ans.



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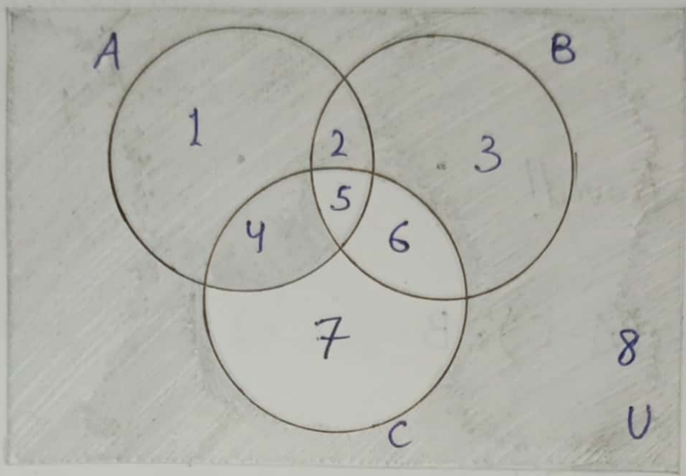
d. $(A \cap \overline{B}) \cup \overline{C}$:

$\overline{B} = U - B = \{1, 4, 7, 8\}$

$\overline{C} = U - C = \{1, 2, 3, 8\}$

$A \cap \overline{B} = \{1, 2, 4, 5\} \cap \{1, 4, 7, 8\} = \{1, 4\}$

$(A \cap \overline{B}) \cup \overline{C} = \{1, 4\} \cup \{1, 2, 3, 8\} = \{1, 2, 3, 4, 8\}$



Answer#21:

a. Prove: $(A - (A \cap B)) \cap (B - (A \cap B)) = \emptyset$

Taking L.H.S:

$\Rightarrow (A - (A \cap B)) \cap (B - (A \cap B))$

$\Rightarrow (A \cap (A \cap B)') \cap (B \cap (A \cap B)')$ ∵ Set difference law

$\Rightarrow (A \cap (A' \cup B')) \cap (B \cap (A' \cup B'))$ ∵ Demorgan's Law

$\Rightarrow ((A \cap A') \cup (A \cap B')) \cap ((B \cap A') \cup (B \cap B'))$ ∵ Distributive law

$\Rightarrow (\emptyset \cup (A \cap B')) \cap ((B \cap A') \cup \emptyset)$ ∵ Complement law

$\Rightarrow (A \cap B') \cap (B \cap A')$ ∵ Identity law

$\Rightarrow (A \cap A') \cap (B \cap B')$ ∵ Associative law

$\Rightarrow \emptyset \cap \emptyset$ ∵ Complement law

$\Rightarrow \emptyset = R.H.S$

Hence Proved!

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b. $(A-B)\cup(A\cap B) = A$

Taking L.H.S:

$\Rightarrow (A-B)\cup(A\cap B)$

$\Rightarrow (A\cap B')\cup(A\cap B)$ $\quad \because$ Set Difference Law

$\Rightarrow A\cap(B'\cup B)$ $\quad \because$ Distributive law

$\Rightarrow A\cap(U)$ $\quad \because$ Complement law

$\Rightarrow A = R.H.S$ $\quad \because$ Identity law

Hence Proved!

c. $(A-B)-C = (A-C)-B$

Taking L.H.S:

$\Rightarrow (A-B)-C$

$\Rightarrow (A\cap B')-C$ $\quad \because$ Set difference law

$\Rightarrow (A\cap B')\cap C'$ $\quad \because$ Set difference law

$\Rightarrow (A\cap C')\cap B'$ $\quad \because$ Associative law

$\Rightarrow (A-C)\cap B'$ $\quad \because$ Set difference Law (reversed)

$\Rightarrow (A-C)-B = R.H.S$ $\quad \because$ Set difference Law (reversed)

Hence proved!

d. $\overline{(\bar{B}\cup(\bar{B}-A))} = B$

Taking L.H.S:

$\Rightarrow (B'\cup(B'-A))'$

$\Rightarrow (B'\cup(B'\cap A'))'$ $\quad \because$ Set Difference Law

$\Rightarrow \bar{\bar{B}}\cap(B'\cap A')'$ $\quad \because$ Demorgan's law

$\Rightarrow B\cap(\bar{\bar{B}}\cup\bar{\bar{A}})$ $\quad \because$ Demorgan's law

$\Rightarrow B\cap(B\cup A)$ $\quad \because$ Double complement

$\Rightarrow B = R.H.S$ $\quad \because$ Absorption law: $[x\cap(x\cup y) = x]$

Hence Proved!

Answer#22:

23K-2001
BCS-3J

a. Total apples : $100 \rightarrow n(A)$

apples with worms : $20 \rightarrow n(W)$

apples with bruises : $15 \rightarrow n(B)$

apples with worms & bruises : $n(W \cap B) \rightarrow 10$

apples that can be sold : ?

$\Rightarrow$ apples with worms or bruise : $\rightarrow n(W \cup B) = n(W) + n(B) - n(W \cap B)$

$$n(W \cup B) = 20 + 15 - 10$$

$$n(W \cup B) = 25$$

$\Rightarrow$ apples that can be sold : $\rightarrow n(S) = n(A) - n(W \cup B)$

$$n(S) = 100 - 25$$

$$n(S) = 75 \quad \text{Ans.}$$

b. Total students : $1000 \rightarrow n(S)$

students like CS : $350 \rightarrow n(CS)$

students like SE : $450 \rightarrow n(SE)$

students like both : $100 \rightarrow n(CS \cap SE)$

students who like either : ? $\rightarrow n(CS \cup SE)$

students who like neither : ? $\rightarrow n(S) - n(CS \cup SE)$

$\Rightarrow n(CS \cup SE) = n(CS) + n(SE) - n(CS \cap SE)$

$n(CS \cup SE) = 350 + 450 - 100$

$n(CS \cup SE) = 700$, students like either subject Ans.

$\Rightarrow n(S) - n(CS \cup SE) = 1000 - 700$

$n(S) - n(CS \cup SE) = 300$, students like neither subject Ans.

23k-2001
BCS-3J

c. like mixed berry : 78 → $n(M)$
like Irish icecream : 32 → $n(I)$
like tiramisu : 57 → $n(T)$
like both mixed berry and Irish icecream : 13 → $n(M \cap I)$
like both Irish icecream and tiramisu : 21 → $n(I \cap T)$
like both tiramisu and mixed berry : 16 → $n(M \cap T)$
like all three : 5 → $n(M \cap I \cap T)$
like none : 14 → $n(X)$
total surveyed : ?

⇒ like either : ? $n(M \cup I \cup T)$

$$n(M \cup I \cup T) = n(M) + n(I) + n(T) - n(M \cap I) - n(I \cap T) - n(M \cap T) + n(M \cap I \cap T)$$

$$n(M \cup I \cup T) = 78 + 32 + 57 - 13 - 21 - 16 + 5$$
$$n(M \cup I \cup T) = 122$$

⇒ total $= n(M \cup I \cup T) + n(X)$
total $= 122 + 14$
total $= 136$, students were surveyed altogether. Ans.

d. Prove: $A \times (B \cap C) = (A \times B) \cap (A \times C)$, Taking L.H.S:

⇒ $\{x, y : (x \in A) \wedge (y \in B \cap C)\}$ ∵ Definition of cross-product
⇒ $\{x, y : (x \in A) \wedge (y \in B) \wedge (y \in C)\}$ ∵ Definition of intersection
⇒ $\{x, y : (x \in A) \wedge (x \in A) \wedge (y \in B) \wedge (y \in C)\}$ ∵ Idempotent law
⇒ $\{x, y : ((x \in A) \wedge (y \in B)) \wedge ((x \in A) \wedge (y \in C))\}$ ∵ Commutative law
⇒ $\{x, y : (x \in A) \wedge (y \in B)\} \cap \{x, y : (x \in A) \wedge (y \in C)\}$ ∵ Definition of intersection
⇒ $(A \times B) \cap (A \times C) = R.H.S$ ∵ Definition of cross-product

Hence proved.

Answer #23:

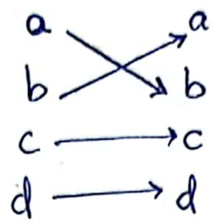
23K-2001
BCS-3J

a. $f(a)=b,\ f(b)=a,\ f(c)=c,\ f(d)=d$

i) i. Domain: $D=\{a,b,c,d\}$

ii. Codomain: $C=\{a,b,c,d\}$

iii. Range: $R=\{a,b,c,d\}$



ii) The function is bijective.

iii) Inverse:

$\Rightarrow f^{-1}(b)=a$

$\Rightarrow f^{-1}(a)=b$

$\Rightarrow f^{-1}(c)=c$

$\Rightarrow f^{-1}(d)=d$

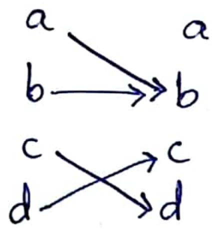
---

b. $f(a)=b,\ f(b)=b,\ f(c)=d,\ f(d)=c$

i) i. Domain: $D=\{a,b,c,d\}$

ii. Co-domain: $C=\{a,b,c,d\}$

iii. Range: $R=\{b,c,d\}$



ii) The function is neither of the three.

iii) Inverse: Doesn't exist.

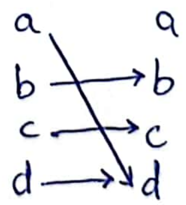
---

c. $f(a)=d,\ f(b)=b,\ f(c)=c,\ f(d)=d$

i) i. Domain: $D=\{a,b,c,d\}$

ii. Codomain: $C=\{a,b,c,d\}$

iii. Range: $R=\{b,c,d\}$



ii) The function is neither of the three.

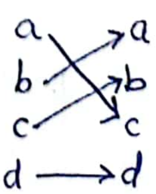
iii) Inverse: Doesn't exist.

---

d. $f(a)=c,\ f(b)=a,\ f(c)=b,\ f(d)=d$

i) Domain: $D=\{a,b,c,d\}$, Codomain: $C=\{a,b,c,d\}$

Range: $R=\{a,b,c,d\}$



ii) The function is Bijective.

iii) Inverse:

$f^{-1}(c)=a,\ f^{-1}(a)=b,\ f^{-1}(b)=c,\ f^{-1}(d)=d$

23K-2001
BCS-3J

Answer # 24:

a. $f(x) = \left\lfloor \frac{x^2}{3} \right\rfloor$, find $f(S)$

i) $S = \{-2, -1, 0, 1, 2, 3\}$

$\Rightarrow f(-2) = \left\lfloor \frac{-2^2}{3} \right\rfloor = 1$ $\quad \Rightarrow f(-1) = \left\lfloor \frac{-1^2}{3} \right\rfloor = 0$ $\quad \Rightarrow f(0) = \left\lfloor \frac{0^2}{3} \right\rfloor = 0$

$\Rightarrow f(1) = \left\lfloor \frac{1^2}{3} \right\rfloor = 0$ $\quad \Rightarrow f(2) = \left\lfloor \frac{2^2}{3} \right\rfloor = 1$ $\quad \Rightarrow f(3) = \left\lfloor \frac{3^2}{3} \right\rfloor = 3$

Ans. $f(S) = \{1, 0, 0, 0, 1, 3\}$

---

ii) $S = \{0, 1, 2, 3, 4, 5\}$

$\Rightarrow f(0) = \left\lfloor \frac{0^2}{3} \right\rfloor = 0$ $\quad \Rightarrow f(1) = \left\lfloor \frac{1^2}{3} \right\rfloor = 0$, $f(2) = \left\lfloor \frac{2^2}{3} \right\rfloor = 1$

$\Rightarrow f(3) = \left\lfloor \frac{3^2}{3} \right\rfloor = 3$ $\quad \Rightarrow f(4) = \left\lfloor \frac{4^2}{3} \right\rfloor = 5$, $f(5) = \left\lfloor \frac{5^2}{3} \right\rfloor = 8$

Ans. $f(S) = \{0, 0, 1, 3, 5, 8\}$

---

iii) $S = \{1, 5, 7, 11\}$

$\Rightarrow f(1) = \left\lfloor \frac{1^2}{3} \right\rfloor = 0$, $\Rightarrow f(5) = \left\lfloor \frac{5^2}{3} \right\rfloor = 8$, $f(7) = \left\lfloor \frac{7^2}{3} \right\rfloor = 16$, $f(11) = \left\lfloor \frac{11^2}{3} \right\rfloor = 40$

Ans. $f(S) = \{0, 8, 16, 40\}$

---

iv) $S = \{2, 6, 10, 14\}$

$\Rightarrow f(2) = \left\lfloor \frac{2^2}{3} \right\rfloor = 1$, $\quad \Rightarrow f(6) = \left\lfloor \frac{6^2}{3} \right\rfloor = 12$, $\Rightarrow f(10) = \left\lfloor \frac{10^2}{3} \right\rfloor = 33$

$\Rightarrow f(14) = \left\lfloor \frac{14^2}{3} \right\rfloor = 65$

Ans. $f(S) = \{1, 12, 33, 65\}$

b. i) $\left\lceil \frac{3}{4} \right\rceil = 1$ ii) $\left\lfloor \frac{7}{8} \right\rfloor = 0$ iii) $\left\lceil -\frac{3}{4} \right\rceil = 0$

iv) $\left\lfloor -\frac{7}{8} \right\rfloor = -1$ v) $\lceil 3 \rceil = 3$ vi) $\lfloor -1 \rfloor = -1$

vii) $\left\lfloor \frac{1}{2} + \left\lceil \frac{3}{2} \right\rceil \right\rfloor = \left\lfloor \frac{1}{2} + 2 \right\rfloor = 2$

viii) $\left\lfloor \frac{1}{2} \cdot \left\lfloor \frac{5}{2} \right\rfloor \right\rfloor = \left\lfloor \frac{1}{2} \cdot 2 \right\rfloor = 1$

c. Prove or disproof:

$\lfloor -x \rfloor = -\lceil x \rceil$ & $\lceil -x \rceil = -\lfloor x \rfloor$; $x \in R$

i) For $x = \frac{7}{8}$

$\left\lfloor -\left(\frac{7}{8}\right) \right\rfloor = -1$

$-\left\lceil \frac{7}{8} \right\rceil = -1$

$-1 = -1$
Hence, it is true.

ii) For $x = \frac{7}{8}$

$\left\lceil -\left(\frac{7}{8}\right) \right\rceil = 0$

$-\left\lfloor \frac{7}{8} \right\rfloor = 0$

$0 = 0$
Hence, it is true

Ans. Both statements are True.

23k-2001
BCS-3J

Answer #25:

23k-2001
BCS-3J

$f(a) = 2a+3$, $g(a) = 3a+2$

a) $f \circ g$: ?

$f \circ g = f(g(a))$

$f \circ g = 2(3a+2)+3$

$f \circ g = 6a+4+3$

$f \circ g = 6a+7$ Ans.

$g \circ f$: ?

$g \circ f = g(f(a))$

$g \circ f = 3(2a+3)+2$

$g \circ f = 6a+9+2$

$g \circ f = 6a+11$ Ans.

b) $f(a) = 2a+3$ (non-zero slope)

Ans. linear and Injective

$g(a) = 3a+2$ (non-zero slope)

Ans. linear and Injective

c) $f(a) = 2a+3$:

Injective → due to non-zero slope

not surjective → cannot map to even integers

Ans. Not Invertible ; since it is not bijective.

$g(a) = 3a+2$:

Injective → due to non-zero slope

not surjective → cannot map to odd integers sometimes

Ans. Not Invertible; since it is not bijective.