

EXAMPLE 6 | A Homogeneous System

Use Gauss-Jordan elimination to solve the homogeneous linear system

$$\begin{aligned}x_1 + 3x_2 - 2x_3 &+ 2x_5 = 0 \\2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 &= 0 \\5x_3 + 10x_4 &+ 15x_6 = 0 \\2x_1 + 6x_2 &+ 8x_4 + 4x_5 + 18x_6 = 0\end{aligned}\tag{4}$$

Solution. Observe that this system is the same as that in Example 5 except for the constants on the right side, which in this case are all zero. The augmented matrix for this system is

$$\left[\begin{array}{ccccccc} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & 0 \\ 0 & 0 & 5 & 10 & 0 & 15 & 0 \\ 2 & 6 & 0 & 8 & 4 & 18 & 0 \end{array} \right] \tag{5}$$

which is the same as that in Example 5 except for the entries in the last column, which are all zeros in this case. Thus, the reduced row echelon form of this matrix will be the same as that of the augmented matrix in Example 5, except for the last column. However, a moment's reflection will make it evident that a column of zeros is not changed by an elementary row operation, so the reduced row echelon form of (5) is

$$\left[\begin{array}{ccccccc} 1 & 3 & 0 & 4 & 2 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \tag{6}$$

The corresponding system of equations is

$$\begin{aligned}x_1 + 3x_2 + 4x_4 + 2x_5 &= 0 \\x_3 + 2x_4 &= 0 \\x_6 &= 0\end{aligned}$$

Solving for the leading variables, we obtain

$$\begin{aligned}x_1 &= -3x_2 - 4x_4 - 2x_5 \\x_3 &= -2x_4 \\x_6 &= 0\end{aligned}\tag{7}$$

If we now assign the free variables x_2, x_4 , and x_5 arbitrary values r, s , and t , respectively, then we can express the solution set parametrically as

$$x_1 = -3r - 4s - 2t, \quad x_2 = r, \quad x_3 = -2s, \quad x_4 = s, \quad x_5 = t, \quad x_6 = 0$$

Note that the trivial solution results when $r = s = t = 0$.