Discrete Assignment #02

23K-2001 BCS-3J

Question #01

(i)

Ans. → Graph has undirected edges.
→ Graph has multiple edges.
→ Graph has no loops.
Hence, it is an undirected multigraph.

(ii)

Ans. → Graph has undirected edges.
→ Graph has no multiple edges.
→ Graph has no loops.
Hence, it is an undirected simple graph.

(iii)

Ans. → It has undirected edges.
→ It has multiples edges.
→ It has 3 loops.
→ Hence, it is an undirected 'Pseudo Graph'.

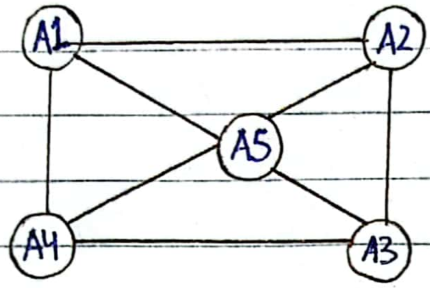
(iv)

Ans. → Graph has directed edges.
→ Graph has multiple edges.
→ Graph has 2 loops.
Hence, it is a directed multi-graph.

Question#02

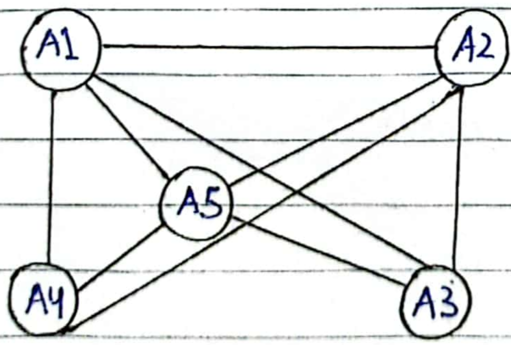
(i)

$A1 = \{0, 2, 4, 6, 8\}$, $A2 = \{0, 1, 2, 3, 4\}$
$A3 = \{1, 3, 5, 7, 9\}$, $A4 = \{5, 6, 7, 8, 9\}$, $A5 = \{0, 1, 8, 9\}$



(ii)

$A1 = \{\dots, -4, -3, -2, -1, 0\}$, $A2 = \{\dots, -2, -1, 0, 1, 2 \dots\}$
$A3 = \{\dots, -6, -4, -2, 0, 2, 4, 6\}$, $A4 = \{\dots, -5, -3, -1, 1, 3, 5 \dots\}$
$A5 = \{\dots, -6, -3, 0, 3, 6, \dots\}$



Question #03

a.

i. Vertices: 5

Edges: 13

Degree of Vertices:

$d(a): 6$

$d(b): 6$

$d(c): 6$

$d(d): 5$

$d(e): 3$

Neighbourhood Vertice:

$N(a) = \{a, b, e\}$

$N(b) = \{a, c, d, e\}$

$N(c) = \{b, c, d\}$

$N(d) = \{b, c, e\}$

$N(e) = \{a, b, d\}$

ii. Vertices: 9

Edges: 12

Degree of Vertices:

$d(a) = 3$ $\quad d(f) = 0$

$d(b) = 2$ $\quad d(g) = 4$

$d(c) = 4$ $\quad d(h) = 2$

$d(d) = 0$ $\quad d(i) = 3$

$d(e) = 6$

Neighbourhood Vertice:

$N(a) = \{c, e, i\}$

$N(b) = \{e, h\}$

$N(c) = \{a, e, g, i\}$

$N(d) = \emptyset$, (no vertices)

$N(e) = \{a, b, c, g\}$

$N(f) = \emptyset$, (no vertices)

$N(g) = \{c, e\}$

$N(h) = \{b, i\}$, $N(i) = \{a, c, h\}$

b.

i. In-degree of vertices:

$d^-(a) = 6$, $d^-(b) = 1$, $d^-(c) = 2$

$d^-(d) = 4$, $d^-(e) = 0$

Out-degree of vertices:

$d^+(a) = 1$, $d^+(b) = 5$, $d^+(c) = 5$

$d^+(d) = 2$, $d^+(e) = 0$

ii. In-degree of vertices:

$d^-(a) = 2$, $d^-(b) = 3$, $d^-(c) = 2$

$d^-(d) = 1$

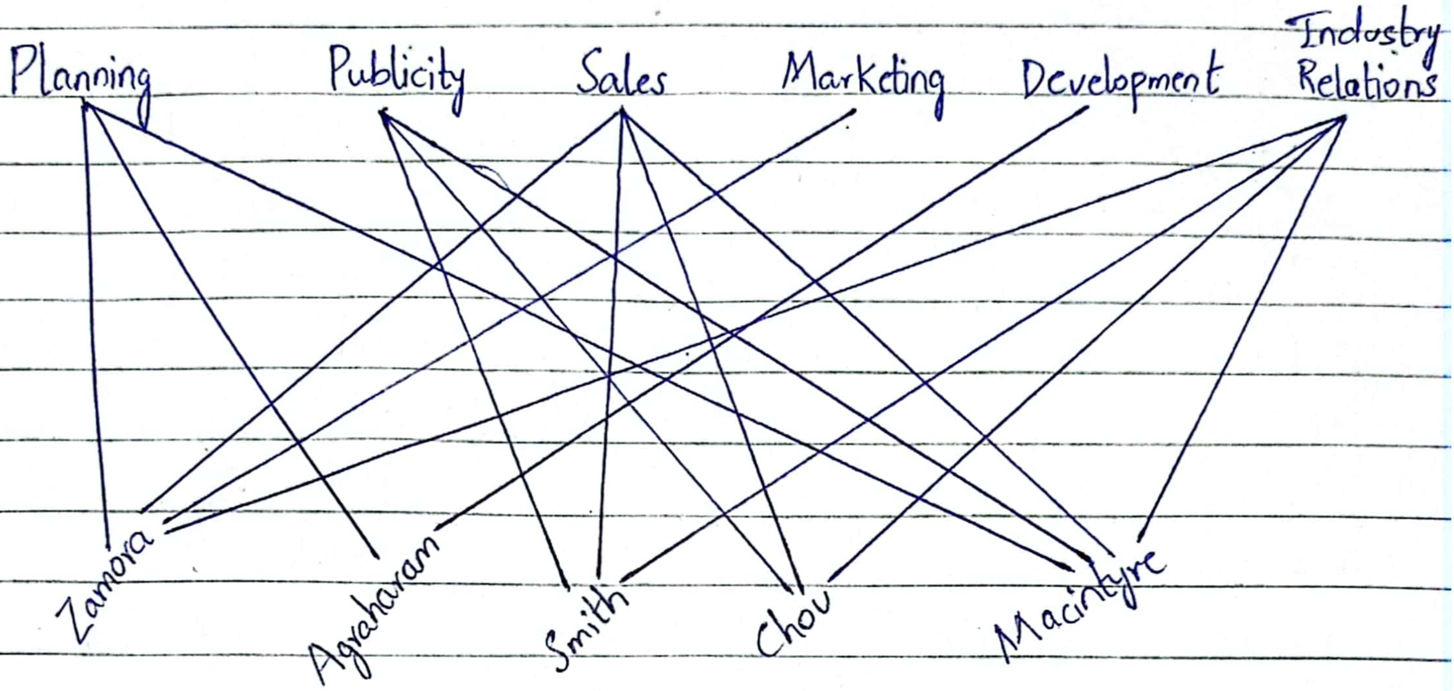
Out-degree of vertices:

$d^+(a) = 2$, $d^+(b) = 4$, $d^+(c) = 1$

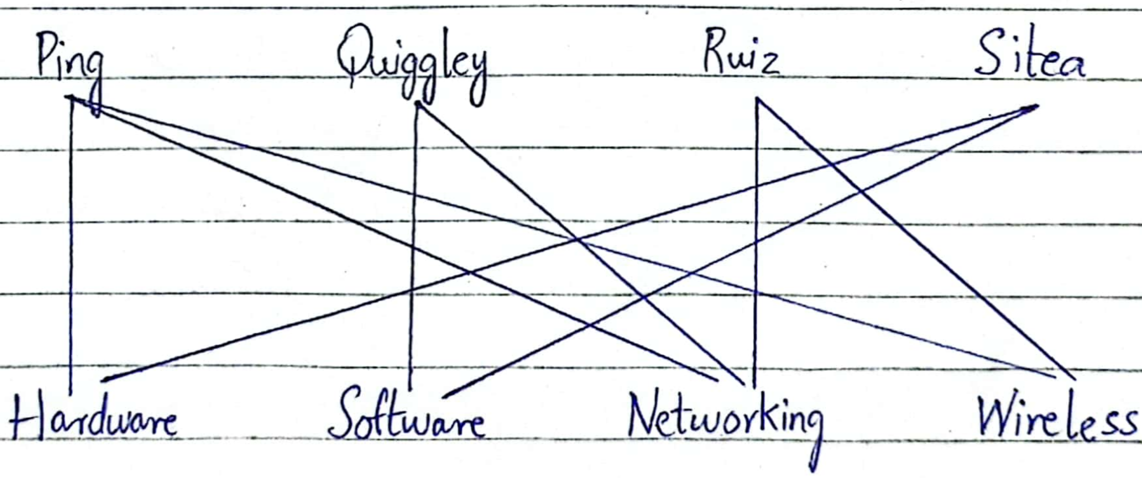
$d^+(d) = 1$

Question #04

a. Bipartite Graph:



b. Bipartite Graph:



Question #05

i. Not a bipartite. ⇒ (as 'c' is adjacent to 'a' & 'd')
ii. Bipartite graph. ⇒ $A(v_1, v_3, v_5)$ & $B(v_2, v_4, v_6)$
iii. Not a bipartite. ⇒ (as 'v_2' & 'v_4' are adjacent)
iv. Not a bipartite. ⇒ (as 'b' is adjacent to 'd' & 'e')

Question #06

(i)

four vertices. $d(a) = 1$, $d(b) = 1$, $d(c) = 2$, $d(d) = 3$

Sol:

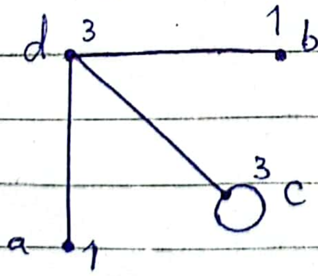
Total degree of graph: $1 + 1 + 2 + 3 = 7$ (odd)

According to Handshaking Theorem, total degree of graph should be 'even'.

Hence graph not possible.

(ii)

four vertices. $d(a) = 1$, $d(b) = 1$, $d(c) = 3$, $d(d) = 3$



(iii)

Ans. A simple graph with 4 vertices $d(a) = 1$, $d(b) = 1$, $d(C) = 3$, $d(d) = 3$ does not exist.

Question # 07

(a)

By using Handshaking Theorem:

$$15 \times 3 = 45 \quad \text{(odd)}$$
$$\Rightarrow 45 \neq 2e \qquad \text{No!}$$

Since, total degree of the graph is odd, it is not possible to make the graph such that each of the 15 vertices have degree = 3.

(b)

By using Handshaking Theorem:

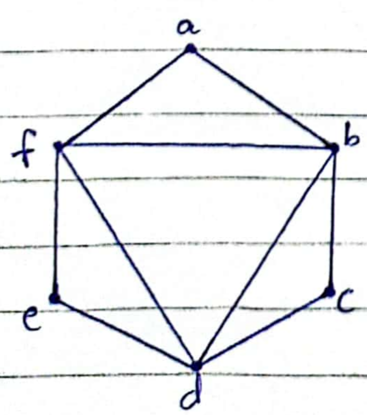
$$4 \times 3 = 12$$
$$\Rightarrow 12 = 2e \qquad \text{Yes!}$$

Since, total degree is even, it is possible to make the graph such that 4 vertices have degree 3.

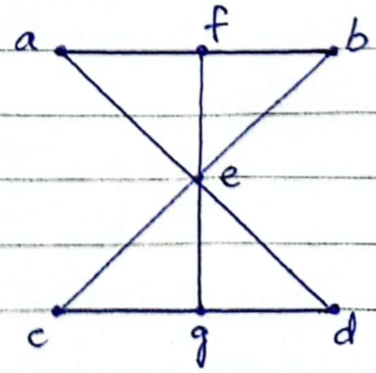
Question # 08

a.

i.



ii.



b.

Regular graph

degree = 4

total edges = 10

Using Handshaking Theorem:

$$\Rightarrow 2e = (\text{vertices} \times \text{degree})$$
$$2(10) = n \times 4$$
$$20 = 4n$$
$$n = 5$$

Ans: 5 vertices.

Question#09

i. vertices → 5
edges → 7
degree sequence → 4, 3, 3, 2, 2

Conditions are satisfied

Hence they are isomorphic.

$g : V(G) \rightarrow V(G') \Rightarrow$

$g(v_1) = w_2$, $g(v_2) = w_3$, $g(v_3) = w_1$
$g(v_4) = w_5$, $g(v_5) = w_4$

ii. vertices → 6
edges → 7
degree sequence → 3, 3, 3, 2, 2, 1

Conditions are satisfied

Hence they are isomorphic

$g : V(G) \rightarrow V(G') \Rightarrow$

$g(v_1) = u_5$, $g(v_2) = u_2$, $g(v_3) = u_4$, $g(v_4) = u_3$
$g(v_5) = u_1$, $g(v_6) = u_6$

iii. vertices → 7
edges → 9
degree sequence → 4, 3, 3, 2, 2, 2, 2

Conditions are satisfied

Hence, they are are isomorphic.

$g : V(G) \to V(G') \Rightarrow$

$g(v_1) = u_5$, $g(v_2) = u_4$, $g(v_3) = u_3$
$g(v_4) = u_2$, $g(v_5) = u_7$, $g(v_6) = u_1$
$g(v_7) = u_6$

iv. vertices → 5
edges → 7
degree sequence → different
in G' $d(v_1) = 4$ whereas
no such vertex in G for $d = 4$
Hence, they are NOT isomorphic.

N	D(b)	D(c)	D(d)	D(e)	D(f)	D(g)	D(h)	D(i)	D(j)	D(k)	D(l)	D(m)	D(n)	D(o)	D(p)	D(q)	D(r)	D(s)	D(t)	D(z)
a	2,a	4,a	**1,a**																	
ad	**2,a**	4,a			6,d	5,d														
adb		4,a		**3,b**	6,d	5,d														
adbe		**4,a**			6,d	5,d	6,e													
adbec					6,c	**5,d**	6,e													
adbeg					**6,c**		6,e			7,g										
adbegf							**6,e**	8,f	10,f	7,g										
adbegfh								8,f	10,f	**7,g**	7,h			14,h						
adbegfhk								8,f	10,f		**7,h**		11,k	14,k			9,k			
adbegfhkl								**8,f**	10,f			10,l	11,k	13,l			9,k			
adbegfhkli									10,f			10,l	11,k	13,l			**9,k**			
adbegfhklir									**10,f**			10,l	10,r	13,l		17,r			14,r	
adbegfhklirj												**10,l**	10,r	13,l		17,r			14,r	
adbegfhklirjm													**10,r**	13,l	12,m	17,r			14,r	
adbegfhklirjmn														13,l	**12,m**	12,n			14,r	
adbegfhklirjmnp														13,l		**12,n**		14,p	13,p	
adbegfhklirjmnpq														**13,l**				14,p	13,p	
adbegfhklirjmnpqo																		14,p	**13,p**	
adbegfhklirjmnpqot																		**14,p**		21,t
adbegfhklirjmnpqots																				**16,s**
adbegfhklirjmnpqotsz																				

Question#10:

(i)

(ii)

N	D(b)	D(c)	D(d)	D(e)	D(f)	D(g)	D(z)
a	4, a	3, a	∞	∞	∞	∞	∞
ac			6, c	9, c	∞	∞	∞
acb			6, c	9, c	∞	∞	∞
acbd				7, d	11, d	∞	∞
acbde					11, d	12, e	∞
acbdef						12, e	18, f
acbdefg							16, g
acbdefgz	4, a	3, a	6, c	7, d	11, d	12, e	16, g

Question#11:

(i)

Sol: ABCDA = 125 , ABDCA = 140 , ACBDA = 155

Route: $A \rightarrow B \rightarrow C \rightarrow D \rightarrow A = 125$ (Hamilton Circuits)

Hence, 125 is the minimum distance coverable by route: ABCDA. Ans.

(ii)

Sol: ABCDA = 97 , ABDCA = 108 , ACBDA = 141

Route: $A \rightarrow B \rightarrow C \rightarrow D \rightarrow A = 97$ (Hamilton Circuits)

Hence, 97 is the minimum distance coverable by route: ABCDA. Ans.

Question#12:

a.

Sol:

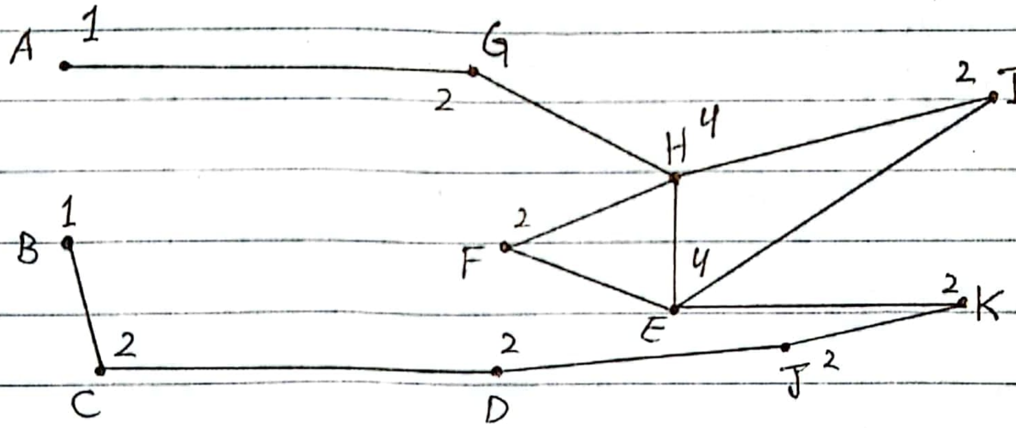
It is possible by the path:

$A \to H \to G \to B \to C \to D \to G \to F \to E$ Ans.

b.

Sol:

We can represent the floor as:



$d(A) = d(B) = 1 \Rightarrow$ odd

$d(C) = d(D) = d(E) = d(F) = d(G) = d(H) = d(I) = d(J) = d(K)$

$\Rightarrow$ even

Hence,

it is an Euler Path:

$A \to G \to H \to F \to E \to I \to H \to E \to K \to J \to D \to C \to B$

Ans.

Question#13:

(i)

Hamilton Circuit: $V_0, V_1, V_2, V_6, V_5, V_4, V_7, V_3, V_0$

Hamilton Path: $V_0, V_1, V_2, V_6, V_5, V_4, V_7, V_3$ Ans.

(ii)

Hamilton Circuit: DOES NOT EXIST.

(as a closed loop cannot be formed that returns to the starting vertex, without repetition)

Hamilton Path: b, c, f, g, h, e, a, d Ans.

(iii)

Hamilton Circuit: d, c, b, a, g, f, e, d Ans.

Hamilton Path: d, c, b, a, g, f, e

Question#14:

a. (i)

Sol: Euler circuit exists. (even degrees)

$\Rightarrow V_1, V_2, V_5, V_4, V_5, V_2, V_3, V_4, V_1$ Ans.

a. (ii)

Sol: Euler circuit Does Not Exist.

V_1, V_5, V_7, V_8 & V_9 have odd degrees. Ans.

For Euler circuits,

all vertices should have even degrees.

b. (i)

Euler Path does not exist. Ans.

As more than 2 vertices have odd degree.

b. (ii)

Euler Path exists.

As exactly 2 vertices have odd degree.

$\Rightarrow$ $U, V_1, V_0, V_7, U, V_2, V_3, V_4, V_2, V_6, V_5, W, V_6, V_4, W$

Ans.

Question# 15:

a.

(i)

Incidence matrix:

$$\begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{array} \begin{array}{c} \begin{array}{ccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

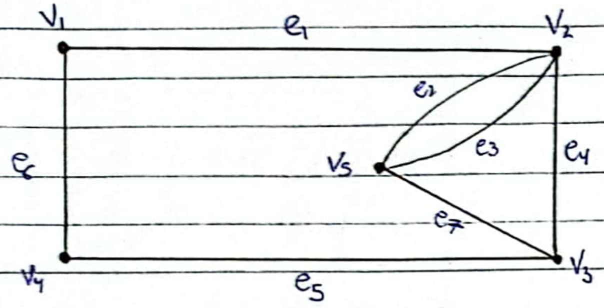
(ii)

Incidence matrix:

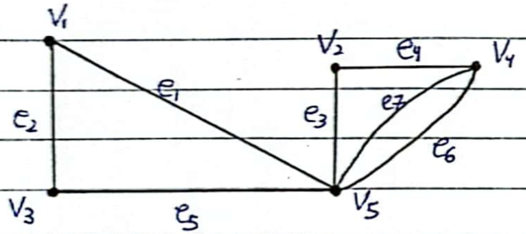
$$\begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{array} \begin{array}{c} \begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

(b)

(i) Graph:



(ii) Graph:



Question#16:

i. List:

Initial vertex	Terminal Vertex
a	a,b,c,d
b	d
c	a, b
d	b,c,d

Adjacency Matrix:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

Ans

ii. List:

Initial Vertex	Terminal Vertex
a	b, d
b	a, c, d, e
c	b, c
d	a, e
e	c, e

Adjacency Matrix:

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Ans.

iii. list:

Initial Vertex	Terminal Vertex
a	b, d
b	a, c
c	b, d
d	a, c

Adjacency Matrix:

$$\begin{bmatrix} 0 & 3 & 0 & 1 \\ 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 3 \\ 1 & 0 & 3 & 0 \end{bmatrix}$$

Ans.

iv. list:

Initial Vertex	Terminal Vertex
a	a, c, d
b	b, c, d
c	a, b, c
d	a, c, d

Adjacency Matrix:

$$\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

Ans.

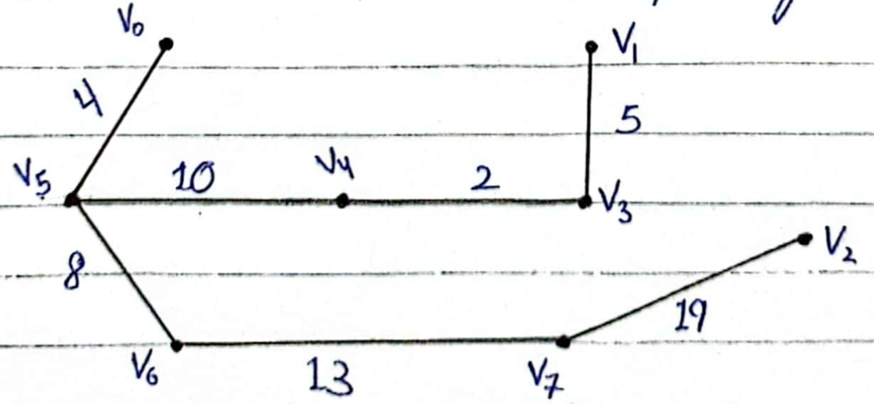
Question #17:

i) 3
ii) 0
iii) 5
iv) 'u' & 'v' are children of n.
v) d
vi) 'k' & 'i' are siblings of j.
vii) m, s, t, x, y are descendants of f.
viii) a, b, e, k, c, f, m, t, d, h, i, n, o, v
ix) v, n, h, d, a are ancestors of z.
x) j, l, q, r, s, x, y, g, p, u, w, z

Question #18:

(i)

$(V_0, V_5) = 4$ $(V_1, V_3) = 5$
$(V_3, V_4) = 2$ $(V_4, V_5) = 10$
$(V_2, V_7) = 19$ $(V_6, V_7) = 13$
$(V_5, V_6) = 8$ $\Rightarrow$ Minimum Spanning Tree Cost : 61



Ans.

(ii)

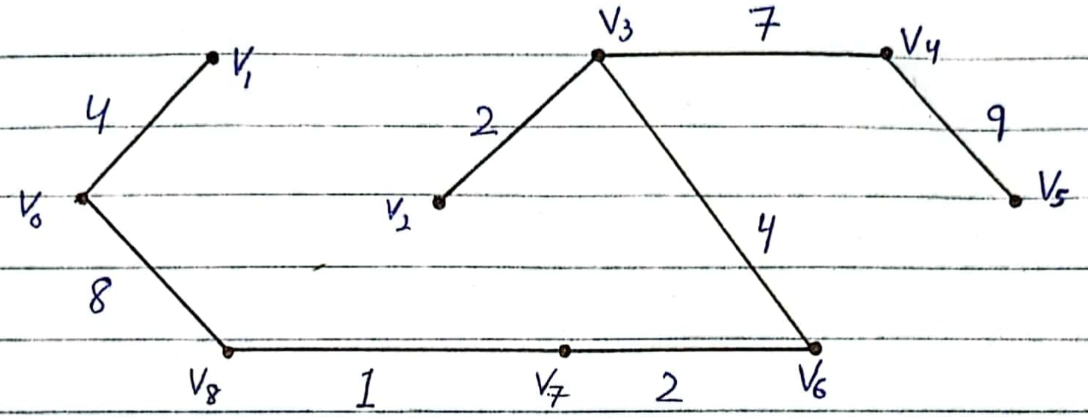
$(V_0, V_1) = 4$ $\quad$ $(V_3, V_6) = 4$

$(V_6, V_7) = 2$ $\quad$ $(V_4, V_5) = 9$

$(V_3, V_4) = 7$ $\quad$ $(V_7, V_8) = 1$

$(V_0, V_8) = 8$ $\quad$ $(V_2, V_3) = 2$

$\Rightarrow$ Minimum Spanning Tree cost : 37



Ans.

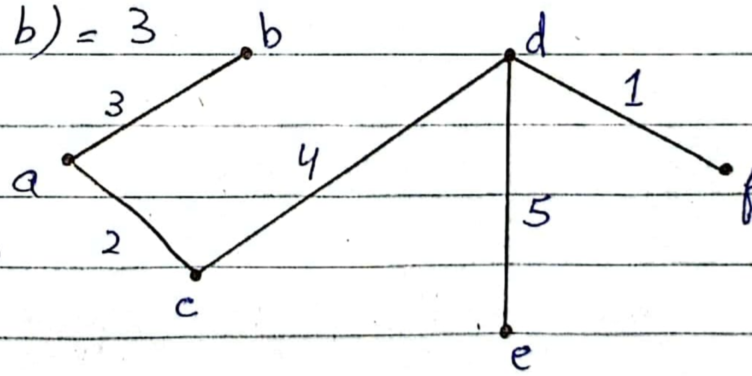
Question#19:

(i)

Sol:

$(d,f) = 1$ $\quad$ $(a,c) = 2$ $\quad$ $(a,b) = 3$

$(c,d) = 4$ $\quad$ $(d,e) = 5$



$\Rightarrow$ Minimum Spanning Tree Cost : 15

Ans.

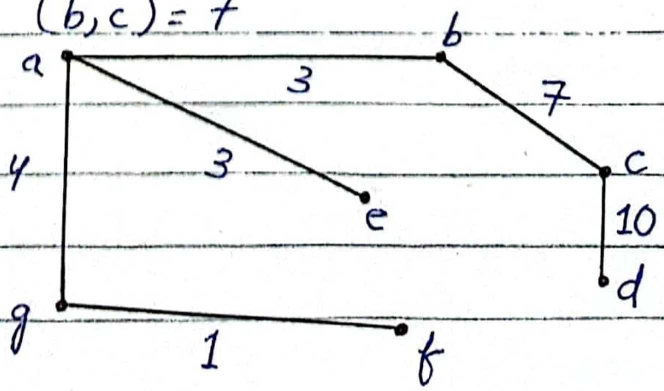
(ii)

Sol:

$(g,f) = 1$ $\quad$ $(c,d) = 10$ $\quad$ $(a,g) = 4$

$(a,b) = 3$ $\quad$ $(a,e) = 3$ $\quad$ $(b,c) = 7$

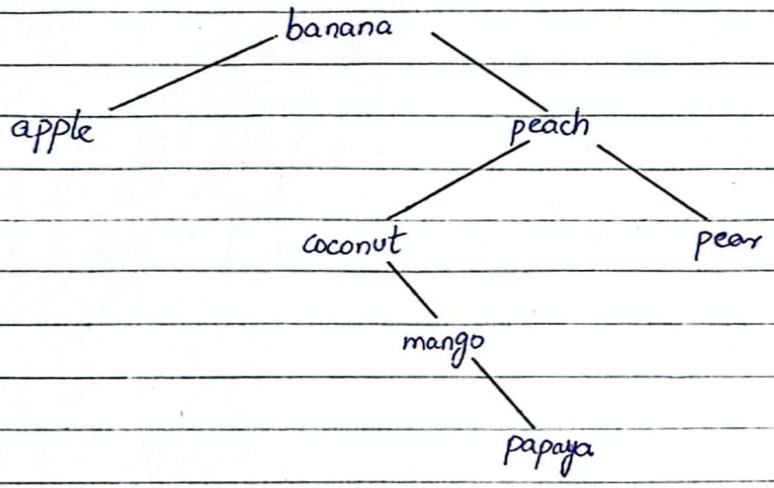
$\Rightarrow$ Minimum Spanning Tree Cost : 28.



Question #20

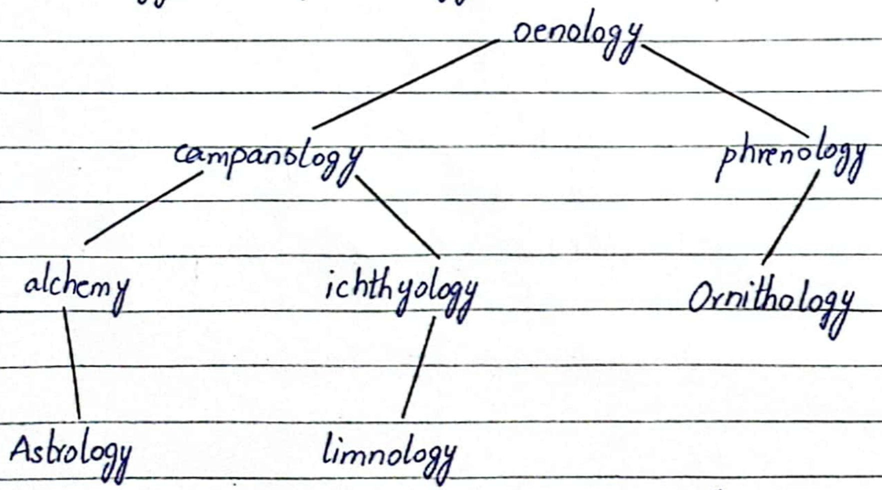
a. (Binary Search Trees)

i. banana, peach, apple, pear, coconut, mango, papaya



Ans.

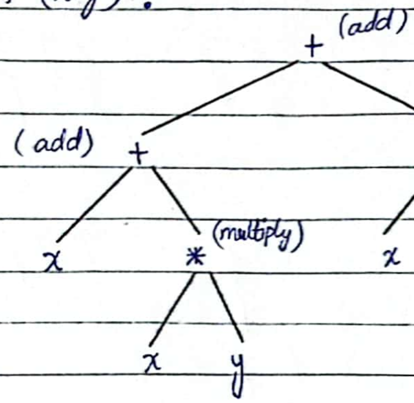
ii. oenology, phrenology, campanology, ornithology, ichthyology, limology, alchemy, astrology



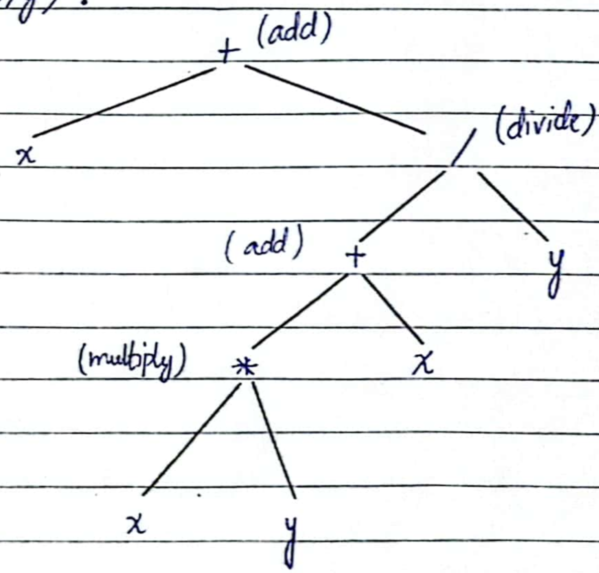
Ans.

b.

i. $(x+xy)+(x/y)$:



ii. $x+((xy+x)/y)$:



Question#21:

i. Sol:

Preorder ⇒ a b e k l m f g n r s c d h o l j p q

Inorder ⇒ k e l m b f r n s g a c o h d i p j q

Post order ⇒ k l m e f r s n g b c o h l p q j d a

ii. Sol:

Preorder ⇒ a b d e i j m n o c f g h k l p

Inorder ⇒ d b i e m j n o a f c g k h p l

Postorder ⇒ d i m n o j e b f g k p l h c a

Question#22:

a. Tree with (n-1)edges produce n-vertices

⇒ vertices = 10000

edges = 10000 - 1 = 9999 Ans.

b. Full-binary tree has two edges per internal vertex.

⇒ internal vertices = 1000

edges = 1000 × 2 = 2000 Ans.

c. Full m-ary tree with I (internal) vertices has ($n = mI + 1$) vertices

⇒ $m = 5$, $I = 100$

$n = 5 \times 100 + 1 = 501$ vertices Ans.

Question#23

a.

i. $(x + xy) + (x/y)$:

Sol:

Prefix: + + x * x y / x y

Postfix: x x y * + x y / +

ii. $x + ((xy + x)/y)$:

Sol:

Prefix: + x / + * x y x y

Postfix: x x y * x + y / +

b.

i. $+ - \uparrow 3 2 \uparrow 2 3 / 6 - 4 2$

Ans : 4

ii. $4 8 + 6 5 - * 3 2 - 2 2 + * /$

Ans : 3

Question#24:

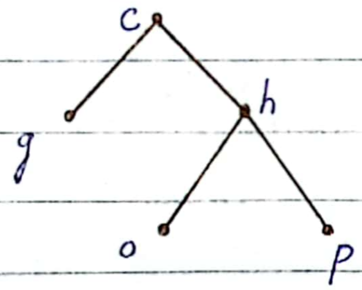
i. Ans. It is not a full m-ary tree because some of the internal vertices have 2 children while others have 3 children.

ii. Ans. It is not a balanced m-ary tree because it has leaves at level 2, 3, 4, 5.

iii.

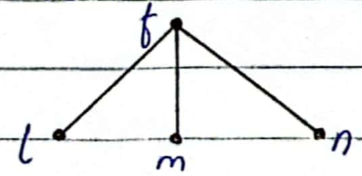
a.

Sub-tree rooted at 'c'



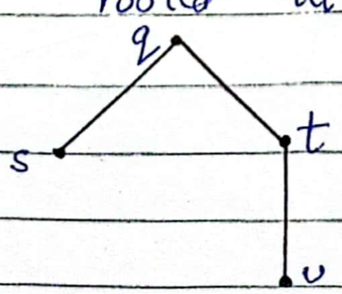
b.

Sub-tree rooted at 'f'



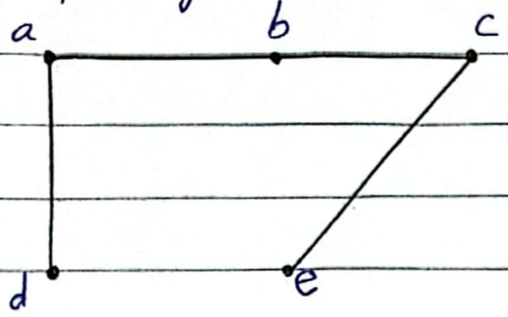
c.

Sub-tree rooted at 'q'



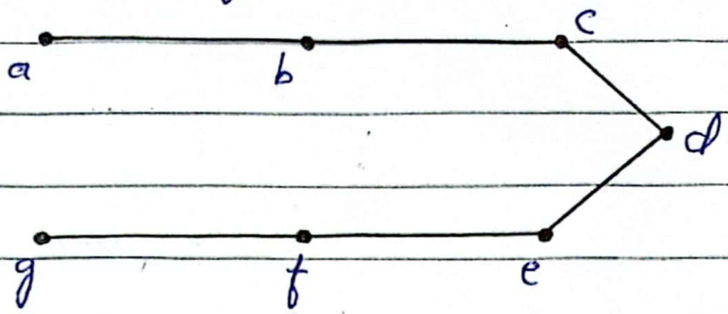
Question#25:

i. Spanning Tree:



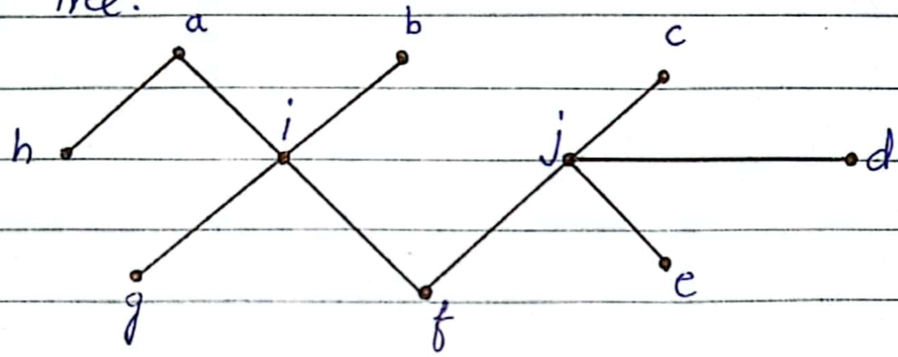
Removed: (b,d),(b,e)

ii. Spanning Tree:



Removed: (a,d),(d,g),(c,e),(b,e),(b,f),(b,g)

iii. Spanning Tree:

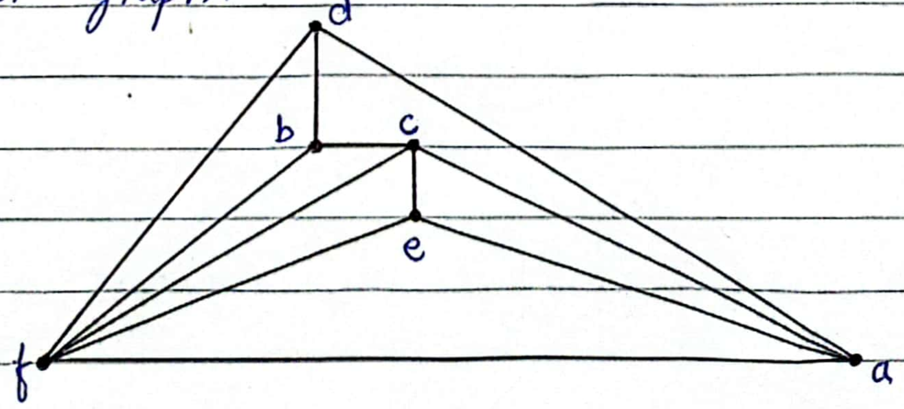


Removed:
(a,b),(b,c),(c,d)
(d,e),(e,f),(f,g)
(g,h),(h,i),(i,j)

Question #26:

a. The graph is a $K_{3,3}$ graph which is a known non-planar graph, which cannot be drawn without edge crossing. Ans. Non-planar

b. The graph can be arranged to represent a planar graph.



Question#27:

a. R is reflexive as R contains: $(a,a), (b,b), (c,c), (d,d)$
b. R is NOT symmetric as R contains: (a,c) but not $(c,a) \in R$
c. R is NOT antisymmetric as both $(b,c) \in R$ & $(c,b) \in R$, but $b = c$
d. R is NOT transitive as both $(a,c) \in R$ & $(c,b) \in R$, but not $(a,b) \in R$
e. R is NOT irreflexive as R contains: $(a,a), (b,b), (c,c), (d,d)$
f. R is NOT asymmetric because R is not antisymmetric.

Question#28:

$A = \{0,1,2,3,4\}$, $B = \{0,1,2,3\}$ where $(a,b) \in R$

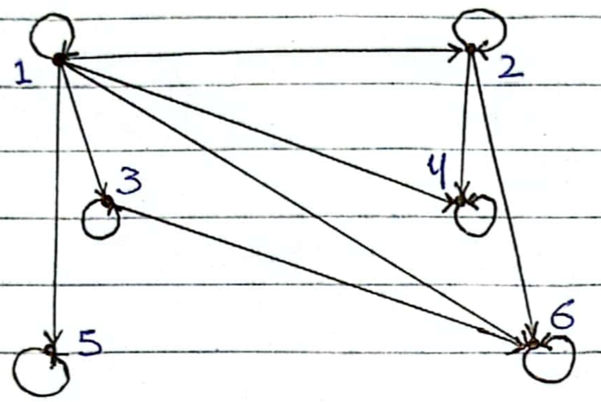
a. $a = b$:
Ans: $\{(0,0), (1,1), (2,2), (3,3)\}$
b. $a + b = 4$:
Ans. $\{(1,3), (2,2), (3,1), (4,0)\}$
c. $a > b$:
Ans. $\{(1,0), (2,0), (3,0), (4,0), (2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\}$
d. $a \mid b$:
Ans. $\{(1,0), (2,0), (3,0), (4,0), (1,1), (1,2), (2,2), (1,3), (3,3)\}$
e. $\gcd(a,b) = 1$:
Ans. $\{(1,0), (0,1), (1,1), (1,2), (1,3), (2,1), (3,1), (4,1), (2,3), (3,2), (4,3)\}$
f. $\text{lcm}(a,b) = 2$:
Ans. $\{(1,2), (2,1), (2,2)\}$

Question#29:

Sol:

$R = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,2),(2,4),(2,6)$
$(3,3),(3,6),(4,4),(5,5),(6,6)\}$

Directed Graph:



Matrix form:

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 01 \end{bmatrix}$$

Ans.

Question#30:

$\{1,2,3,4\}$

a. $\{(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)\}$

Sol:

i. NOT reflexive: does not contain $(1,1)$ & $(4,4)$

ii. NOT symmetric: R contains $(2,4)$ but not $(4,2) \in R$

iii. R is NOT antisymmetric: contains $(2,3)$ & $(3,2)$ but $2 \neq 3$

iv. R is Transitive: for any 3 numbers a, b, c:
if $(a,b),(b,c) \in R$ then $(a,c) \in R$.

b. $\{(1,1),(1,2),(2,1),(2,2),(3,3),(4,4)\}$

Sol:

i. R is reflexive: contains $(1,1),(2,2),(3,3),(4,4)$

ii. R is symmetric: as (a,b) & $(b,a) \in R \Rightarrow (1,2),(2,1)$

iii. NOT antisymmetric: contains $(1,2),(2,1)$ but $1 \neq 2$

iv. R is Transitive: for any 3 numbers a, b, c
if $(a,b),(b,c) \in R$ then $(a,c) \in R$.

c. $\{(2,4),(4,2)\}$

Sol:

i. NOT reflexive: does not contain $(1,1),(2,2),(3,3),(4,4)$

ii. R is symmetric: contains $(2,4)$ & $(4,2) \in R$

iii. NOT antisymmetric: contains $(2,4),(4,2)$ but $2 \neq 4$

iv. NOT transitive: $(2,4),(4,2) \in R$ but not $(2,2) \in R$

d. $\{(1,2),(2,3),(3,4)\}$

Sol:

i. NOT reflexive: does not contain $(1,1),(2,2),(3,3),(4,4)$

ii. NOT symmetric: $(1,2) \in R$ but not $(2,1) \in R$

iii. R is antisymmetric: contains (a,b) but not $(b,a) \in R$

iv. NOT transitive: $(1,2),(2,3) \in R$ but not $(1,3) \in R$

e. $\{(1,1),(2,2),(3,3),(4,4)\}$

Sol:

i. R is reflexive: contains $(1,1),(2,2),(3,3),(4,4)$

ii. R is symmetric: contains (a,b) & $(b,a) \in R$

iii. R is antisymmetric: contains (a,b) & $(b,a) \in R$, then $a=b$

iv. R is transitive: for any 3 numbers a,b,c
if $(a,b),(b,c) \in R$ then $(a,c) \in R$

f. $\{(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)\}$

Sol:

i. NOT reflexive: does not contain $(1,1),(2,2),(3,3),(4,4)$

ii. NOT symmetric: contains $(1,4) \in R$ but not $(4,1) \in R$

iii. NOT antisymmetric: contains $(1,3)$ & $(3,1) \in R$ but $1 \neq 3$

iv. NOT transitive: Contains $(1,3)$ & $(3,1) \in R$ but not $(1,1) \in R$

Question#31:

a. a is taller than b.

i. NOT reflexive: the person cannot be taller than themself.

ii. NOT symmetric: if A is taller than B, then B cannot be taller than A.

iii. Relation is antisymmetric: $(a,b) \in R$ and $(b,a) \in R$ cannot occur simultaneously. (i.e one person is alway taller than the other but not the other way around).

iv. Relation is transitive: if A is taller than B, and B is taller than C, then A is also taller than C.

b. a & b were born on the same day.

i. Relation is reflexive: the person is born the same day as themself.

ii. Relation is symmetric: if A & B are born on the same day, then B is also born on the same day as A.

iii. NOT antisymmetric: if A & B are born the same day as B & A, it is not necessary that A & B are the same person.

iv. Relation is transitive: if A & B are born on the same day, and B & C are born on the same day, then A & C are also born on the same day.

c. a has the same first name as b.

i. Relation is symmetric: if A has the same name as B, then B also has the same first name as A.

ii. Relation is reflexive: A person have the same name as themself.

iii. NOT antisymmetric: if A has the same first name as B, and also B has same first name as A, it's not necessary that A & B are the same person.

iv. Relation is transitive: if A has same first name as B, and B has same first name as C, then A also has same first name as C.

d. a & b have a common grandparent.

i. Relation is reflexive: A person has the same grandparents as themself.

ii. Relation is symmetric: if A & B have a common grandparent, then B & A also have a common grandparent.

iii. NOT antisymmetric: if A & B have a common grandparent, and B & A have a common grandparent, then it is not necessary that A & B are the same people/person.

iv. NOT transitive: if A & B have a common grandparent & B & C have a common grandparent, then it is not necessary that A & C have a common grandparent.

Question # 32:

a. both symmetric & antisymmetric:

Ans: $\{(1,1),(2,2),(3,3)\}$

b. neither symmetric nor antisymmetric:

Ans: $\{(1,2),(2,1),(3,4)\}$

Question # 33:

$A = \{1,2,3\}$

$R_1 = \{(2,1),(3,1),(3,2)\}$

$R_2 = \{(1,1),(2,1),(2,2),(3,1),(3,2),(3,3)\}$

$R_3 = \{(1,2),(1,3),(2,3)\}$

$R_4 = \{(1,1),(2,2),(3,3),(1,2),(1,3),(2,3)\}$

$R_5 = \{(1,1),(2,2),(3,3)\}$

$R_6 = \{(1,2),(1,3),(2,1),(2,3),(3,1),(3,2)\}$

a. $R_2 \cup R_4 = \{(1,1),(2,2),(3,3),(2,1),(3,1),(3,2),(1,2),(1,3),(2,3)\}$

b. $R_3 \cup R_6 = \{(1,2),(1,3),(2,1),(2,3),(3,1),(3,2)\}$

c. $R_3 \cap R_6 = \{(1,2),(1,3),(2,3)\}$

d. $R_4 \cap R_6 = \{(1,2),(1,3),(2,3)\}$

e. $R_3 - R_6 = \emptyset$

f. $R_6 - R_3 = \{(2,1),(3,1),(3,2)\}$

g. $R_2 \oplus R_6 = \{(1,1),(2,2),(3,3),(1,2),(1,3),(2,3)\}$

h. $R_3 \oplus R_5 = \{(1,1),(2,2),(3,3),(1,2),(1,3),(2,3)\}$

i. $R_2 \circ R_1 = \{(2,1),(3,1),(3,2)\}$

j. $R_6 \circ R_6 = \{(1,1),(2,2),(3,3),(2,1),(3,1),(3,2),(1,2),(1,3),(2,3)\}$

Question#34:

a.

$\{1, 2, 3\}$

i. $\{(1,1), (1,2), (1,3)\}$

Sol:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

ii. $\{(1,2), (2,1), (2,2), (3,3)\}$

Sol:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

iii. $\{(1,1), (1,2), (1,3), (2,2), (2,3), (3,3)\}$

Sol:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

iv. $\{(1,3), (3,1)\}$

Sol:

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

b.

i. $R = \{(1,1), (1,3), (2,2), (3,1), (3,3)\}$

ii. $R = \{(1,2), (2,2), (3,2)\}$

iii. $R = \{(1,1), (1,2), (1,3), (2,1), (2,3), (3,1), (3,2), (3,3)\}$

Question #35:

a. Reflexivity: As $I(A) = I(B)$, it follows ARA for all strings A.

Symmetry: Suppose ARB, since $I(A) = I(B)$, $I(B) = I(A)$ also holds & BRA.

Transitivity: Suppose that ARB & BRC, since $I(A) = I(B)$ & $I(B) = I(C)$, $I(A) = I(C)$ also holds & ARC.

b. Reflexivity: $a \equiv a \pmod{m}$ since $a - a = 0$ is divisible by m as $0 = 0.m$

Symmetry: suppose $a \equiv b \pmod{m}$, then $a-b$ is divisible by m, so $a-b = km$, where 'K' is an integer. It follows $b-a = (-k)m$, so $b \equiv a \pmod{m}$

Transitivity: Suppose, $a \equiv b \pmod{m}$ & $b \equiv c \pmod{m}$, then m divides both $(a-b)$ & $(b-c)$, hence: $a-b = km$ & $b-c = lm$. By adding equations $\Rightarrow$ $a-c = (k+l)m$, therefore: $a \equiv c \pmod{m}$

Question#36: a.

i. $2^n - 1$: 1, 3, 7, 15, 31

Ans:

ii. $10 - (3/2)n$:

Ans: $(17/2), 7, (11/2), 4, (5/2)$

iii. $((-1)^n/n^2)$:

Ans. $-1, \frac{1}{4}, \frac{-1}{9}, \frac{1}{16}, \frac{-1}{25}$

iv. $((3n+4)/(2n-1))$:

Ans. $7, \frac{10}{3}, \frac{13}{5}, \frac{16}{7}, \frac{19}{9}$

b.

i. Arithmetic sequence

$d = -7$

$T_n = a + (n-1)d$

$T_{11} = -15 + (11-1)(-7) = -85$

Ans.

ii. Arithmetic sequence

$d = 3b$

$T_n = a + (n-1)d$

$T_{15} = (a - 42b) + (15-1)(3b) = a$

Ans.

iii. Geometric sequence

$r = 3/4$

$T_n = ar^{n-1}$

$T_{17} = 4(3/4)^{17-1} = (3^{16}/4^{15})$

Ans.

iv. Geometric sequence

$r = 1/2$

$T_n = ar^{n-1}$

$T_9 = 32(1/2)^{9-1} = (1/8)$

Ans.

Question#37:

a. GP

i. $T_3 = 10$, $T_5 = (2\ 1/2)$

$T_3 = ar^2 = 10$, $T_5 = ar^4 = 5/2$.

Divide both equations $\Rightarrow r^2 = \frac{5}{2} \times \frac{1}{10} \Rightarrow r = \pm\frac{1}{2}$

$\Rightarrow a(1/2)^2 = 10$

$a = 40$

Hence, the required G.P is:

$40, 20, 10, 5, \frac{5}{2}, \ldots$ or $40, -20, 10, -5, \frac{5}{2}, \ldots$

Ans.

ii. $T_5 = 8$ & $T_8 = -64/27$

$T_5 = ar^4 = 8$, $T_8 = ar^7 = -64/27$

After dividing we get, $\Rightarrow r^3 = \frac{-64}{27} \times \frac{1}{8}$, $\Rightarrow r = \frac{-2}{3}$

$\Rightarrow a\left(\frac{-2}{3}\right)^4 = 8$

$a = 81/2$

Hence, the required G.P is:

$\frac{81}{2}, -27, 18, -12, 8, \ldots$

Ans.

b. AP

i. $T_4 = 7$ & $T_{16} = 31$

$T_4 = a + 3d = 7$, $T_{16} = a + 15d = 31$

After subtracting we get, $\Rightarrow 12d = 24$, $\Rightarrow d = 2$

$\Rightarrow a + 3(2) = 7$

$a = 1$

Hence, the required A.P is:

$1, 3, 5, 7, 9, 11, \ldots$ Ans.

ii. $T_5 = 86$ & $T_{10} = 146$

$T_5 = a + 4d = 86$, $T_{10} = a + 9d = 146$

After subtracting we get, $\Rightarrow 5d = 60$, $d = 12$

$\Rightarrow a + 4(12) = 86$

$a = 38$

Hence, the required AP is:

$38, 50, 62, 74, 86, \ldots$ Ans.

Question#38:

a. $d = 7 \rightarrow$ AP: $259, 266, 273, 280, \ldots, 784$

Since, $T_n = a + (n-1)d$

$784 = 259 + (n-1)(7)$

$n = 76$ numbers ans.

$\because S_n = (n/2)[2a + (n-1)d]$

$S_{76} = \frac{76}{2}[2(259) + (76-1)(7)]$

$S_{76} = 39{,}634$ Ans.

b. $T_1 = 1/n$, $T_L = \dfrac{n^2 - n + 1}{n}$

Since, $S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$

$T_n = a + (n-1)d$

$\dfrac{n^2 - n + 1}{n} = \dfrac{1}{n} + (n-1)d$

$\Rightarrow d = 1$

Hence,

$S_n = \dfrac{n}{2}\left[\dfrac{2}{n} + (n-1)(1)\right]$

$\Rightarrow S_n = \dfrac{2 + n^2 - n}{2}$ Ans.

Question #39:

a. $\sum_{j=1}^{100} (1/j)$ Ans.

b. i. $\sum_{k=4}^{8} (-1)^k$

Ans: $(-1)^4 + (-1)^5 + (-1)^6 + (-1)^7 + (-1)^8$

$= 1 - 1 + 1 - 1 + 1$

$= 1$ Ans.

ii. $\sum_{j=1}^{5} (j)^2$

Ans: $(1)^2 + (2)^2 + (3)^2 + (4)^2 + (5)^2$

$= 1 + 4 + 9 + 16 + 25$

$= 55$ Ans.

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Question # 40:

a. $a_n = -2a_{n-1}$, $a_0 = -1$

Sol:

$$a_0 = -1$$

$$a_1 = -2a_0 = -2(-1) = 2$$

$$a_2 = -2a_1 = -2(2) = -4$$

$$a_3 = -2a_2 = -2(-4) = 8$$

$$a_4 = -2a_3 = -2(8) = -16$$

$$a_5 = -2a_4 = -2(-16) = 32$$

first six terms : $-1, 2, -4, 8, -16, 32$ Ans.

b. $a_n = a_{n-1} - a_{n-2}$, $a_0 = 2$, $a_1 = -1$

Sol:

$$a_0 = 2$$

$$a_1 = -1$$

$$a_2 = a_1 - a_0 = -1 - 2 = -3$$

$$a_3 = a_2 - a_1 = -3 - (-1) = -2$$

$$a_4 = a_3 - a_2 = -2 - (-3) = 1$$

$$a_5 = a_4 - a_3 = 1 - (-2) = 3$$

first six terms : $2, -1, -3, -2, 1, 3$ Ans.

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c. $a_n = 3a^2_{n-1}$, $a_0 = 1$

Sol:

$a_0 = 1$

$a_1 = 3a_0^2 = 3(1)^2 = 3$

$a_2 = 3a_1^2 = 3(3)^2 = 27$

$a_3 = 3a_2^2 = 3(27)^2 = 2187$

$a_4 = 3a_3^2 = 3(2187)^2 = 14348907$

$a_5 = 3a_4^2 = 3(14348907)^2 = 6.17673 \times 10^{14}$

first six terms: $1, 3, 27, 2187, 14348907, 6.17673 \times 10^{14}$ Ans.

d. $a_n = n a_{n-1} + a^2_{n-2}$, $a_0 = -1$, $a_1 = 0$

Sol:

$a_0 = -1$

$a_1 = 0$

$a_2 = 2a_1 + a_0^2 = 2(0) + (-1)^2 = 1$

$a_3 = 3a_2 + a_1^2 = 3(1) + (0)^2 = 3$

$a_4 = 4a_3 + a_2^2 = 4(3) + (1)^2 = 13$

$a_5 = 5a_4 + a_3^2 = 5(13) + (3)^2 = 74$

first six terms: $-1, 0, 1, 3, 13, 74$ Ans.