

# MPTorch-FPGA: a Custom Mixed-Precision Framework for FPGA-based DNN Training

Sami Ben Ali  
Inria, Univ. Rennes  
sami.ben-ali@inria.fr

Silviu-Ioan Filip  
Inria, Univ. Rennes  
silviu.filip@inria.fr

Olivier Sentieys  
Inria, Univ. Rennes  
olivier.sentieys@inria.fr

Guy Lemieux  
University of British Columbia  
lemieux@ece.ubc.ca

**Abstract**—Training Deep Neural Networks (DNNs) is computationally demanding, leading to a growing interest in reduced precision formats to enhance hardware efficiency. Several frameworks explore custom number formats with parameterizable precision through software emulation on CPUs or GPUs. However, they lack comprehensive support for different rounding modes and struggle to accurately evaluate the impact of custom precision for FPGA-based targets. This paper introduces MPTorch-FPGA, an extension of the MPTorch framework for performing custom, multi-precision inference and training computations in CPU, GPU, and FPGA environments in PyTorch. MPTorch-FPGA can generate a model-specific accelerator for DNN training, with customizable sizes and arithmetic implementations, providing bit-level accuracy with respect to emulated low precision DNN training on GPUs or CPUs. An offline matching algorithm selects one of several pre-generated (static) FPGA configurations using a custom performance model to estimate latency. To showcase the versatility of MPTorch-FPGA, we present a series of training benchmarks using diverse DNN models, exploring a range of number format configurations and rounding modes. We report both accuracy and hardware performance metrics, verifying the precision of our performance model by comparing estimated and measured latencies across multiple benchmarks. These results highlight the flexibility and practical value of our framework.

## I. INTRODUCTION

Deep Neural Networks (DNNs) have revolutionized AI, enabling significant advancements but at the cost of substantial computational demands. As models scale in size and complexity, developing efficient training methods is crucial. Mixed-precision training emerges as a promising solution to boost performance without compromising accuracy. However, most frameworks exploring custom mixed precisions for DNN training rely on software emulation, with limited direct evaluation on hardware platforms like FPGAs.

This paper introduces MPTorch-FPGA, illustrated in Figure 1, an extension of the MPTorch framework [1], that integrates custom mixed-precision arithmetic for DNN training in CPU-GPU-FPGA environments. Our framework is designed for researchers and developers who seek to explore and optimize hardware designs with minimal accuracy loss. By combining software emulation and hardware execution within a unified framework, MPTorch-FPGA offers flexibility and precision, allowing for detailed exploration of custom number formats, rounding modes, and arithmetic configurations. The main contributions are:

- **Unified Emulation and Hardware Framework :** MPTorch-FPGA integrates bit-accurate emulation and

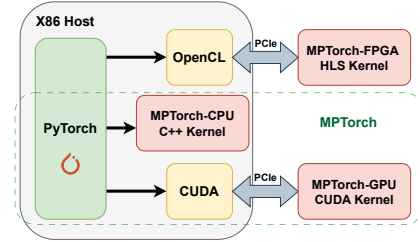


Fig. 1. MPTorch framework overview. It includes CPU, GPU, and the new FPGA component. The MPTorch-FPGA component is the focus of this paper.

hardware exploration, seamlessly combining software and FPGA components. This enables rapid and precise exploration of custom number formats and arithmetic modes. We also provide detailed and fine-grained performance and accuracy benchmarks for various Floating-Point (FP) and Fixed-Point (FXP) configurations across different stages of DNN training.

- **Model-Specific Accelerator Optimization:** Our framework offers a complete flow for selecting an optimized accelerator configuration tailored to each model, optimizing FPGA usage based on the training workload.
- **Efficient Hardware Design:** MPTorch-FPGA uses a single bitstream for each training task, eliminating the need for bitstream switching, and leverages High-Bandwidth Memory (HBM) to efficiently manage multiple cores, addressing FPGA memory constraints in the context of DNN training.

The manuscript is organized as follows. In Sec. II, we discuss previous work, while in Sec. III we discuss the concept and ideas behind MPTorch and MPTorch-FPGA. Sec. IV delves into the implementation aspects of MPTorch-FPGA. Results are presented in Sec. V, followed by a conclusion. The MPTorch-FPGA framework will be released as open-source within the same repository as MPTorch.

## II. BACKGROUND AND RELATED WORK

### A. Custom precision for DNN training

Custom arithmetic precision in DNN training seeks to reduce memory consumption while improving computational efficiency. In this section, we review prominent techniques for custom low-precision training.

General Matrix Multiplications (GEMM), the most computationally expensive operations in DNN training, have been a

primary focus of precision reduction research [2]–[10]. Many studies explore the use of 8-bit Floating-Point (FP8) formats for GEMM [2]–[6], [10], though accumulations are often still performed with higher 16- or 32-bit precision. To reduce accumulator overhead, other works [6], [10] perform accumulations using lower-precision adders (e.g., FP12). Additionally, [3], [5], [6] use loss scaling and different FP8 formats for the FWD and BWD passes to minimize accuracy loss and memory access overhead.

Beyond FP8, BFloat16 [8], [11] and block floating-point formats [9] have been explored. Some approaches also investigate integer arithmetic [9], [12]–[16]. Stochastic rounding has also been proposed to mitigate rounding errors in low-precision training, in particular due to stagnation [10], [17].

### B. Emulation frameworks

Several tools (see Table I) have been proposed in recent years to assess the impact of various hardware and arithmetic choices on DNN training accuracy.

In terms of FPGA-oriented work, Langroudi et al. [18] introduced Cheetah, a co-design framework for DNN inference and training that emulates FP and posit formats using CPU cores and compiles a softcore of Multiply And Accumulate (MAC) operators on FPGA to evaluate hardware characteristics. Tatsumi et al. [6] introduce Archimedes-MPO, a C++-based mixed-precision inference and training framework for FPGAs. Models compiled with Archimedes-MPO use custom templated data types for each low-level operation. These types support up to 32-bit FP or FXP with custom policies including exponent/mantissa/integer/fractional word lengths, optional subnormals, and whether the multiplier output in a MAC is fused or rounded. The FPGA component accelerates GEMM using one single SA synthesized using a single MAC design that supports a fixed mixture of data types and policies.

Table I compares existing frameworks and their features. While frameworks like [6], [18] allow synthesizing custom FPGA operators, their lack of integrated support complicates performance benchmarking across arithmetic configurations. In contrast, MPTorch-FPGA provides a built-in, model-specific FPGA implementation with customizable operators, facilitating easier performance evaluation and supporting various rounding options, including Stochastic Rounding (SR) [19] and Round to Odd (RO) [20].

### C. FPGA DNN training accelerators

While simulation frameworks explore precision impact on accuracy, other research focuses on hardware architectures for custom precision DNN training on FPGAs. Vink et al. [26] introduced the Barista toolchain to simplify DNN accelerator deployment on FPGAs. Integrated with the Caffe framework [27], it allows users to define networks and includes an FPGA-based accelerator with an OpenCL runtime for CNN training. Convolutions use a compile-time sized 2D systolic array, and the authors optimize performance by evaluating various tile sizes to identify the best configuration for specific CNN models. Luo et al. [28] introduced DARK FPGA, a training

framework for FPGA accelerators that employs batch-level parallelism and a hardware/software co-design approach for mixed precision training. DARK FPGA enhances computation by determining optimal tile sizes and accelerator parameters.

Previous works [26], [28], [29] provide user-friendly FPGA acceleration frameworks connected to x86 hosts, facilitating quick testing and integration of their designs. However, they are often difficult to adapt for custom arithmetic configurations and do not investigate the impact of these configurations on overall performance and accuracy. In contrast, our work supports easy evaluation of various arithmetic configurations through a versatile GEMM accelerator.

## III. THE MPTORCH FRAMEWORK AND ITS FPGA EXTENSION

The goal of MPTorch is to offer a comprehensive resource for researchers investigating DNN acceleration at the arithmetic level, with a strong focus on mixed-precision DNN training. MPTorch includes GPU and CPU components. This paper focuses on its new FPGA component.

As illustrated in Figure 1, MPTorch is built on top of PyTorch. The CPU and GPU components are designed to offer bit-accurate emulation of arithmetic operations. For the CPU, custom precision operators and quantization functions are implemented in C++, while CUDA is leveraged for GPU implementations. These implementations are exposed to Python via PyBind and seamlessly integrated as PyTorch extensions, ensuring compatibility within the PyTorch ecosystem.

MPTorch-FPGA extends MPTorch with support for FPGA accelerator implementations. FPGA logic is designed using C++ High-Level Synthesis (HLS), controlled through an OpenCL-based Python interface. A detailed discussion of the FPGA accelerator architecture is presented in Section IV.

Figure 2 illustrates the mixed-precision training process in MPTorch-FPGA for a single training iteration. During both forward and backward passes, inputs are quantized to the desired precision before undergoing matrix multiplication. Our framework emphasizes the impact of custom precisions on GEMM operations<sup>1</sup>.

MPTorch-FPGA enables FPGA-based GEMM computation or emulation on GPU/CPU. Results are cast back to full precision. Additionally, the framework supports custom precision simulation for weight updates, where weights are quantized, updated in custom precision, and stored in full precision.

Our framework integrates seamlessly with the PyTorch workflow. Custom-precision operations are easily defined within layer declarations, as illustrated in Figure 3. The parameters for these layers mirror those of standard PyTorch layers, and an extra parameter group to specify the arithmetic configuration for forward and backward passes and quantization formats for executing the layers’ GEMM operations.

MPTorch-FPGA provides fine-grained control over arithmetic configurations, allowing independent customization of

<sup>1</sup>Convolution operations are transformed into GEMM computations using the im2col and col2im transformations, performed on the CPU host.

TABLE I  
COMPARISON OF DIFFERENT DNN TRAINING SIMULATION FRAMEWORKS. MPTORCH-FPGA IS THE ONLY ONE OFFERING MODEL-SPECIFIC ACCELERATOR SUPPORT

Work	AdaPT [21]	ApproxTrain [22]	Cheetah [18]	GoldenEye [23]	QPytorch [24]	FASE [25]	Archimedes-MPO [6]	MPTorch-FPGA Ours
Framework	PyTorch	TensorFlow	TensorFlow	PyTorch	PyTorch	PyTorch, Caffe	TinyDNN	PyTorch
GPU acceleration	✗	✓	✗	✓	✓	✗	✓	✓
Built-in FPGA support	✗	✗	✗	✗	✗	✗	✓	✓
Transformer support	✓	✓	✗	✓	✗	✓	✗	✓
FMA support	✗	✗	✗	✗	✗	✓	✓	✓
Operator emulation	✓	✓	✓	✓	✗	✓	✓	✓
Formats	FXP	FP	Posit, FP	FXP, FP, BFP	FXP, FP, BFP	FP	FXP, FP	FXP, FP
Rounding	-	RZ	RN	RN, RZ	RN, RZ, SR	RN	RN	RN, RZ, SR, RO

formats for multiplication and accumulation, precision settings, and rounding modes. It supports both fixed-point, floating-point, and blocked FP arithmetic, offering a variety of rounding modes, including Stochastic Rounding (SR), Round to Nearest Even (RN), Round to Odd (RO), Round to Zero (RZ) and No Rounding (NR), in which the result is calculated exactly.

During emulation (on CPU or GPU), computations are performed using FP32 operators. For both multiplication and addition, pre-quantized inputs are operated on using standard FP32 hardware. The result can be rounded by truncating the least significant bits (RZ mode) or by adding and truncating those bits for RN, RO, and SR. Alternatively, the FP32 result can emulate fused multiply-add (FMA) behavior for low precisions.

Emulating custom precision operators introduces significant latency overhead. While this overhead is smaller with GPU implementations, training tasks on CPU can be notably slow. Full precision operators ensure bit-accurate emulation for low-precision formats, though issues like double rounding may arise for higher-precision formats. On the other hand, FPGA implementation provides exact computation using the specified formats and allows for a thorough evaluation of the performance characteristics of custom arithmetic configurations.

#### IV. FPGA IMPLEMENTATION

Our FPGA design, which allows for fast and easy benchmarking, is a GEMM accelerator composed of several Systolic Arrays (SAs), also referred to as cores in this paper. Specifically, we use the one-dimensional systolic array introduced by de Fine Licht et al. [30], as shown in Figure 4. This systolic array is composed of  $N$  Processing Elements (PEs), each containing  $M$  MAC units. The GEMM computation is performed as inputs are streamed across the PEs. We have significantly modified the original design to accommodate custom arithmetic units and rounding modes. Additionally, our implementation synthesizes multiple SAs within the same chip, as illustrated in Figure 5. This approach addresses the inefficiencies of large tile sizes in conventional SAs, where the tile size is equal to the total number of MAC units  $N \times M$ . Large tile sizes often result in low utilization for most DNNs, as the input shapes are usually a fraction of the tile size.

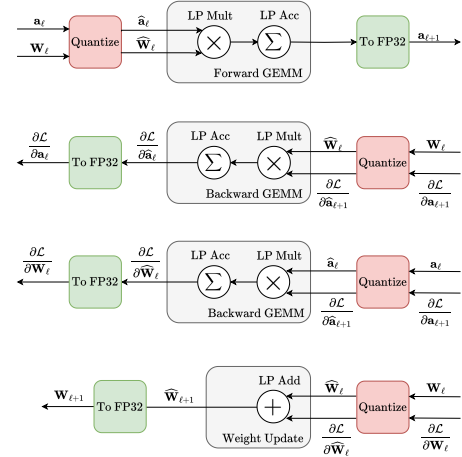


Fig. 2. Low precision computation flow through a linear layer during one iteration of training. The  $\mathbf{W}$  variables are the weights, activations are denoted with  $\mathbf{a}$ , and the loss function with  $\mathcal{L}$ .

```
import mptorch.quant as qpt

formats = qpt.QAffineFormats(fwd_mac, fwd_rnd, bwd_mac,
                             bwd_rnd, weight_quant,
                             input_quant, grad_quant,
                             bias_quant)

qlinear_layer = qpt.QLinear(in_features, out_features,
                             formats, device)
```

Fig. 3. MPTorch layer declaration. To enable the FPGA extension, the user should designate an 'fpga' value to the device parameter.

Moreover, large SAs complicate routing and reduce the design frequency. To mitigate these issues, we deploy multiple smaller SAs, each utilizing separate HBM ports for parallel memory transfer and computation.

#### A. Performance Model

A training iteration at the layer level consists of a series of GEMM operations. Each GEMM operation input needs to be padded to match the tile size of the selected configuration. By knowing the input shapes and the tiling parameters of the accelerator, we can estimate the latency of each operation. The total training iteration latency is the sum of the latencies of all consecutive GEMM operations.

The input matrices  $A \in \mathbb{R}^{n \times k}$  and  $B \in \mathbb{R}^{k \times m}$  undergo three padding stages. The first stage ensures that computation can be evenly partitioned among cores by padding either  $A$  or  $B$ . The second and third stages pad the partitioned inputs for each core. The first two stages are executed on the host CPU before transferring the matrices to the FPGA's HBM memory, while the third stage occurs on the FPGA fabric during data loading.

- 1) **First Stage:** The input dimensions  $n$  or  $m$  (depending on the partitioned input) are padded to match the number of cores  $C$ , producing  $C$  sub-matrices with dimensions  $(n_{\text{core}}, k)$  and  $(k, m_{\text{core}})$ . Assuming  $A$  is partitioned, each core receives sub-matrix  $A_c$ , where  $c \in \{1, \dots, C\}$  : with dimensions  $(n_{\text{core}}, k)$  and the full matrix  $B$  with dimensions  $(k, m)$ .
- 2) **Second Stage:** For each core inputs  $A_c$  and  $B$ , the  $m$  and  $k$  dimensions are padded to align with the memory tile size,  $T_{\text{mem}}$ , defined by the capacity of one HBM port width (512 bits). For instance, for an 8-bit value, the memory pack size is  $512/8 = 64$ . After this stage, the padded inputs for each core become: submatrices  $A_c \in \mathbb{R}^{n_{\text{core}} \times k_{\text{mem}}}$  and matrix  $B \in \mathbb{R}^{k_{\text{mem}} \times m_{\text{mem}}}$ .
- 3) **Third Stage:** Padding is further applied to core inputs  $A_c$  and  $B$  padded in the second stage. Each matrix  $B$  (with dimensions  $(k_{\text{mem}}, m_{\text{mem}})$ ) is padded further in the  $m_{\text{mem}}$  dimension to size  $m_{\text{comp}}$ , which is divisible by the compute tile size  $T_{\text{MAC}} = N \times M$ , corresponding to the total number of MAC units per core. Each sub-matrix  $A_c$  (with dimensions  $(n_{\text{core}}, k_{\text{mem}})$ ) is padded further in the  $n_{\text{core}}$  dimension to size  $n_{\text{comp}}$ , which is divisible by the compute tile size  $T_{\text{PE}} = N$ , corresponding to the total number of MAC units per PE. After this stage, the final padded inputs for each core are:  $A_c \in \mathbb{R}^{n_{\text{comp}} \times k_{\text{mem}}}$  and  $B \in \mathbb{R}^{k_{\text{mem}} \times m_{\text{comp}}}$ .

The performance of the GEMM operation within each core is calculated as  $L_{\text{Core}} = L_{\text{MAC}} + L_{\text{write}}$ , where the latency of MAC computations is defined as

$$L_{\text{MAC}} = \frac{n_{\text{comp}} \cdot m_{\text{comp}} \cdot k_{\text{mem}}}{N \cdot M \cdot F},$$

with  $F$  being the operating frequency. Since data reads from off-chip memory to each GEMM core occur in parallel with MAC computations, only the sequential operations for writing to off-chip memory are considered.  $L_{\text{write}}$  is given by:

$$L_{\text{write}} = \frac{n_{\text{comp}} \cdot m_{\text{comp}}}{T_{\text{out}} \cdot F},$$

where  $T_{\text{out}} = M$  is the tile size for off-chip data writes.

The memory transfer latency between the host CPU and FPGA off-chip memory is constrained by the PCIe bandwidth:

$$L_{\text{data}} = \frac{S_{\text{data}}}{B_{\text{PCIe}}},$$

where  $S_{\text{data}}$  is the total data size computed as

$$S_{\text{data}} = \underbrace{C \cdot n_{\text{core}} \cdot k_{\text{mem}}}_{\text{First input matrix}} + \underbrace{k_{\text{mem}} \cdot m_{\text{mem}}}_{\text{Second input matrix}} + \underbrace{C \cdot n_{\text{core}} \cdot m_{\text{mem}}}_{\text{Output matrix}}.$$

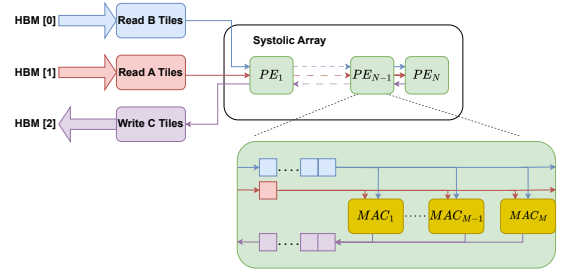


Fig. 4. Systolic array architecture of a GEMM core.

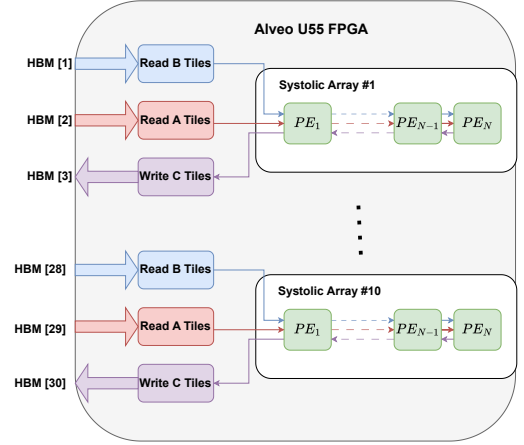


Fig. 5. Accelerator multicore architecture.

Since all cores are of the same size and operate in parallel, the total latency of the accelerator is:

$$L_{\text{total}} = L_{\text{Core}} + L_{\text{data}}.$$

### B. Accelerator Configuration

To maximize the accelerator's performance, we select the configuration that delivers the best results for a given DNN training workload. We rely on the performance model from Sec. IV-A to accurately estimate the latency of a training iteration for a specific model. We then apply an optimized mapping strategy that ensures the best performance for the DNN model and the accelerator configuration.

Through a unified two-level approach, we minimize latency incurred by padding overhead. The first step involves determining whether to feed the inputs in their original or transposed forms. Transposing allows switching input dimensions, potentially reducing padding overhead. However, this decision is coupled with the second step: optimizing input partitioning.

For both transposed and non-transposed inputs, we evaluate which input to partition across cores to minimize execution latency. Using a brute-force approach, we calculate the latency for every combination of input format and partitioning strategy. The objective is to identify the configuration—original or transposed format, combined with the optimal partitioning—that achieves the lowest overall latency.



TABLE II  
TEST ACCURACY COMPARISON ACROSS DIFFERENT MULTIPLIER AND  
ACCUMULATOR CONFIGURATIONS FOR VARIOUS CNNs.

Multiplier	Accumulator	LeNet5 †	ResNet20 ‡	VGG16 ‡	ResNet50 Δ
E5M2-NR	E6M5-RZ	97.10	10.00	10.00	10.00
	E6M5-RO	98.00	10.00	10.00	10.00
	E6M5-RN	98.61	10.00	10.00	10.00
	E6M5-SR	99.00	90.55	88.99	80.88
	E5M10-RN	99.05	91.24	89.81	82.97
E8M23-RN	E8M23-RN	99.18	91.91	90.67	82.92
FXP4, 4-RN		99.06	10.00	10.00	10.00
FXP4, 4-SR		99.14	10.00	10.00	10.00
FXP4, 4-RZ	FXP8, 8	98.85	10.00	10.00	10.00
FXP4, 4-RO		10.00	10.00	10.00	10.00

Datasets: †MNIST, ‡CIFAR10, ΔImagewoof

## V. EVALUATION

In this section, we illustrate the effectiveness of our framework by exploring how different arithmetic configurations influence model training through bit-accurate emulation. Additionally, we extend this analysis by designing and implementing an FPGA accelerator based on one of the evaluated arithmetic formats. To accomplish this, we apply the methodology from Sec. IV to determine an optimized configuration for the accelerator. Finally, we compare the actual performance of the FPGA accelerator with the predictions made by our model across various DNN training benchmarks, validating the robustness of our approach.

### A. Training Setup

We conducted our experiments using various convolutional models, including LeNet5, ResNet20, VGG16, ResNet50, and a Transformer model. The details of the datasets and training configurations for each model are outlined below. In all experiments, we employed adaptive loss scaling [7] with an initial scaling factor of 256 to mitigate accuracy loss during mixed precision training.

1) *CNN Experiments*: For LeNet5, we performed training on the MNIST dataset for 10 epochs, using a batch size of 64 and a learning rate of 0.1.

In the case of ResNet20 and VGG16, we followed the original training configurations [31], [32]. Both models were trained on the CIFAR10 dataset. For ResNet20, the initial learning rate was set to 0.1, and weight decay was 0.0001, while VGG16 used an initial learning rate of 0.01 and a weight decay of 0.0005. Both models were trained for 200 epochs with a batch size of 128. We used stochastic gradient descent (SGD) with a momentum coefficient of 0.9 throughout.

For ResNet50, we selected the more challenging Imagewoof dataset, a subset of ImageNet that focuses on 10 dog classes out of the 1,000 total classes in the full ImageNet dataset. The model was trained with a batch size of 16 and an initial learning rate of 0.01, highlighting the increased computational demand posed by the larger dataset and more complex task.

2) *Transformer Experiments*: We also conducted training on the Shakespeare dataset, which contains text sequences drawn from Shakespeare’s works. We used the Nano-GPT generative model [33] comprising 6 layers, 6 attention heads,

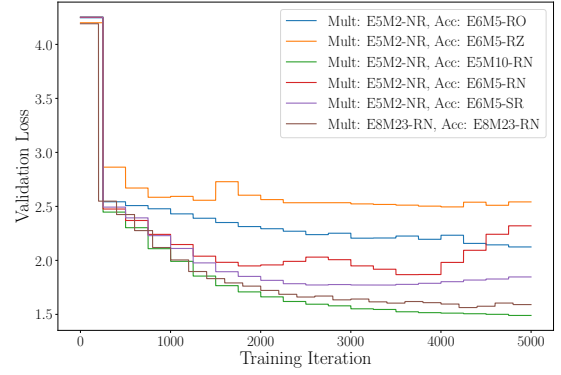


Fig. 6. Nano-GPT validation loss for different arithmetic configurations.

a 384 embedding size, and 256 for the block size. Training was carried out for 5,000 iterations using a learning rate of  $10^{-4}$  and the Adam optimizer.

### B. Training Results

This section demonstrates the effectiveness of our framework in evaluating the impact of custom precision configurations on DNN model training. Table II presents the emulated test accuracy for various CNN models and arithmetic configurations, while Figure 6 shows the validation loss for the Transformer benchmark. Floating-point formats are denoted as  $EeMm$ , where  $e$  represents the exponent size and  $m$  denotes the mantissa size. For fixed-point formats, the  $FXPi.f$  notation is used, with  $i$  indicating the size of the signed integer part and  $f$  representing the fractional part.

1) *Floating Point Formats*: We trained models using FMA operators with FP8 multipliers and FP12 or FP16 adders. The FP12 format, previously studied in [6], [10], was further evaluated with different rounding modes. Our results confirm that SR consistently outperforms other rounding modes at the same precision. Although RO and RZ performed well on the LeNet5 benchmark, they failed to converge on other tasks. In [10], the authors demonstrated that increasing the number of random bits can match FP16RN accuracy using FP12SR with 13 random bits. In our experiments, using 10 bits caused slight degradation compared to FP16RN but still provided a notable accuracy advantage over other FP12 configurations.

2) *Fixed-Point Formats*: We also experimented with FXP8.4 multipliers and FXP16.8 adders. All FXP configurations failed to converge, except for the Lenet5 experiments, where all rounding modes except RO resulted in near FP32 baseline accuracy.

### C. Accelerator Performance

Our framework facilitates exploration of performance characteristics for different arithmetic formats at the accelerator level. The template-based code allows for easy modification of the accelerator to accommodate custom arithmetic formats or operators. In our experiments, we focus on the FP8 multiplier, FP12SR accumulator FMA configuration, which shows minimal accuracy degradation across all benchmarks. Using the performance model and mapping strategy from Sec. IV, we

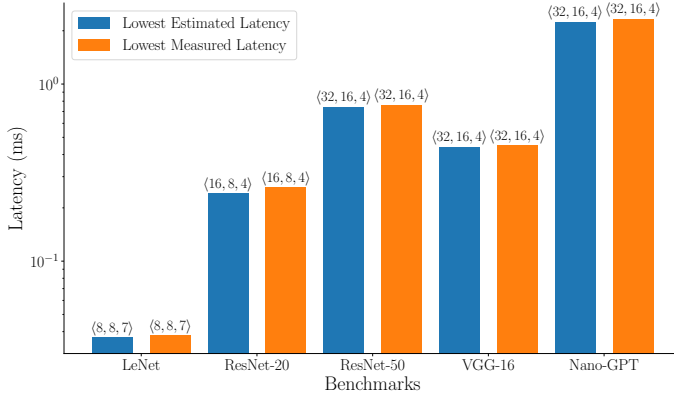


Fig. 7. Lowest Estimated vs Measured Latency and  $\langle N, M, C \rangle$  configurations, for different training benchmarks.

identify the optimal accelerator configuration for this format. We validate the model’s accuracy by comparing its predictions with actual hardware performance.

We use AMD Vitis HLS 2023.1 to synthesize various accelerator configurations, targeting the U55 Alveo FPGA. However, our framework is adaptable to other Alveo and datacenter boards.

In our experiments, we explore the design space by varying the accelerator parameters  $N$ ,  $M$ , and  $C$ , subject to specific constraints:  $N$  and  $M$  must be powers of two,  $M$  must be divisible by  $N$ , and  $C$  is capped at 10 due to the U55’s 32 memory ports (with 3 ports per core). We synthesize each configuration at the maximum core count and highest achievable frequency. As detailed in Table III, we present the synthesis results in terms of overall resource utilization on the FPGA, specifically reporting the consumption of Look-Up Tables (LUTs), BRAMs (block RAMs), and Digital Signal Processing blocks (DSPs). The arithmetic operators are implemented using LUTs, while the DSP usage is due to address generation logic within the SAs. As the size of the systolic arrays increases, the number of cores that fit on-chip decreases. The largest systolic array we can accommodate has  $N = 64, M = 32$  with  $C = 1$ .

Using fewer cores can sometimes improve performance due to higher operating frequencies. Table IV presents the estimated training latencies for different models using systolic arrays of size  $N = M = 8$ , synthesized with varying core counts. The optimal core count is the one resulting in minimum latency.

To identify the optimal accelerator configuration for a training task, we first determine the optimal core count for each SA size. This process is repeated for varying SA sizes to ultimately find the  $\langle N, M, C \rangle$  combination that minimizes the estimated training latency.

To confirm the accuracy of our performance model, we compare the optimal configurations calculated through the model with those measured on the hardware. Figure 7 shows the latencies of optimal configurations for various training benchmarks. The model successfully identifies all optimal configurations, though measured latencies are slightly higher,

TABLE III  
POSSIBLE ACCELERATOR CONFIGURATIONS ( $N = \#PEs$ ,  $M = \#MACs$  PER PE,  $C = \#CORES$  OF SIZE  $N \times M$ , SHOWING MAXIMAL  $C$  AND  $F$ , WITH CHIP RESOURCE UTILIZATION).

$N$	$M$	$C$	$F$ (MHz)	LUT (%)	BRAM(%)	DSP(%)
1	1	10	320.9	14.12	13.78	8.56
2	1	10	320.1	14.80	13.80	7.98
2	2	10	320.1	15.10	14.44	8.05
4	2	10	311.0	18.06	15.99	9.76
4	4	10	328.4	21.30	18.20	9.80
8	4	10	197.7	28.20	17.09	11.53
8	8	10	196.2	37.51	21.50	11.53
16	8	10	180.0	61.60	30.3	11.6
16	16	7	160.0	62.73	33.57	7.45
32	16	4	198.4	73.26	33.26	5.72
32	32	2	197.3	62.19	71.48	2.77
64	32	1	150.0	52.57	71.64	1.93

TABLE IV  
ESTIMATED TRAINING LATENCY PER ITERATION.

$C$	$F$ (MHz)	Estimated Training Latency (s), $N \times M = 8 \times 8$				
		CNN				Transformer
		LeNet5 †	VGG16 ‡	ResNet20 ‡	ResNet50 Δ	Nano-GPT ◊
1	378.3	0.0081	5.42	1.12	8.35	25.17
2	330.9	0.0055	3.12	0.66	4.82	14.54
3	298.0	0.0047	2.33	0.51	3.61	11.00
4	298.0	0.0041	1.75	0.40	2.72	8.23
5	299.8	0.0038	1.41	0.34	2.21	6.67
6	270.6	0.0039	1.30	0.33	2.05	6.22
7	274.7	<b>0.0037</b>	1.12	<b>0.29</b>	1.75	5.45
8	203.1	0.0045	1.30	0.35	2.02	6.14
9	203.1	0.0044	1.17	0.33	1.84	5.76
10	196.2	0.0043	<b>1.10</b>	0.32	<b>1.73</b>	<b>5.35</b>

Datasets: †MNIST, ‡CIFAR10, ΔImagewoof, ◊Shakespeare

due to the PCIe bandwidth being capped at 80% of its maximum capacity.

## VI. CONCLUSION

In this paper, we introduce MPTorch-FPGA, a versatile framework that bridges the gap between software emulation and hardware execution for DNN training. By enabling seamless exploration of arithmetic configurations in FPGA-based systems, it offers researchers a powerful tool to optimize model-specific accelerators and explore precision trade-offs with minimal overhead. We report both accuracy and hardware performance metrics, confirming the precision of our performance model by comparing estimated latency with measured results across various benchmarks. These findings emphasize the flexibility and practical utility of MPTorch-FPGA, demonstrating its effectiveness as a framework for optimizing DNN training across diverse hardware setups. Future extensions of this work could involve finer-grained model matching, such as applying different arithmetic configurations to the forward and backward passes. Additionally, support for more advanced DNN architectures and larger datasets could further expand the applicability of the framework.

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