6.4书后习题答案

$$1.\frac{\partial F}{\partial x}(x,y,z)+\frac{\partial F}{\partial y}(x,y,z)+\frac{\partial F}{\partial z}(x,y,z)\left(\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}\right)=0;$$

$$\Rightarrow\frac{\partial z}{\partial x}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}}\qquad;\frac{\partial z}{\partial y}=-\frac{\frac{\partial F}{\partial z}}{\frac{\partial F}{\partial y}}$$

2.解

(1)原式对x求导

$$y\cdot\mathrm{cosxy}+x\frac{\mathrm{dx}}{\mathrm{dy}}\mathrm{cosxy}-\left(y+x\frac{\mathrm{dx}}{\mathrm{dy}}\right)e^{\mathrm{xy}}-2\mathrm{xy}-x^2\frac{\mathrm{dx}}{\mathrm{dy}}=0;$$

$$\Rightarrow y\,\mathrm{cosxy}-y\cdot e^{\mathrm{xy}}-2\mathrm{xy}=(-x\cos xy+x\cdot e^{xy}+x^2)\frac{\mathrm{dy}}{\mathrm{dx}}$$

$$\Rightarrow\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{y\,\mathrm{cosy}-y\cdot e^{\mathrm{xy}}-2\mathrm{xy}}{-\mathrm{xcos}\,xy+x\cdot e^{\mathrm{xy}}+x^2}$$

(2)原式对x求导

$$\frac{1}{\sqrt{x^2+y^2}}\cdot\frac{1}{2}\cdot\frac{1}{\sqrt{x^2+y^2}}(2x+2y\cdot y')=\frac{1}{1+\frac{y^2}{x^2}}\cdot\frac{y'x-y}{x^2};$$

$$\Rightarrow\frac{\mathrm{dy}}{\mathrm{dx}}=y'=\frac{x+y}{x-y}$$

$$\Rightarrow\frac{d^2y}{\mathrm{dx}^2}=\frac{d}{\mathrm{dx}}\left(\frac{x+y}{x-y}\right)=\frac{(1+y')(x-y)-(1-y')(x+y)}{(x-y)^2}=\frac{2(x^2+y^2)}{(x-y)^3}$$

(3)两边对x求导

$$x^y\left(y^x\ln^x+\frac{y}{x}\right)=y^x\left(\ln^y+\frac{x}{y}\cdot y'\right)$$

$$\Rightarrow\left(x^y\ln^x-y^x\cdot\frac{x}{y}\right)y'=y^x\ln^y-\frac{y}{x}\cdot x^y$$

$$\Rightarrow\frac{\mathrm{dy}}{\mathrm{dx}}=y'=\frac{\mathrm{xy}\ln^y-y^2}{\mathrm{xy}\ln^x-x^2}=\frac{y^2(\ln^x-1)}{x^2(\ln^y-1)};$$

$$\Rightarrow\frac{d^2y}{\mathrm{dx}^2}=\frac{d}{\mathrm{dx}}\left(\frac{y^2(\ln^x-1)}{x^2(\ln^y-1)}\right)=\frac{[2\mathrm{yy}'(\ln^x-1)]x^2(\ln^y-1)-\left[2x(\ln^y-1)-\frac{1}{y}y'.x^2\right]y^2(\ln^x-1)}{[x^2(\ln^y-1)]^2}$$

$$\Rightarrow\frac{d^2y}{\mathrm{dx}^2}=\frac{y^2[x(\ln^y-1)^2+2(x-y)(\ln^x-1)(\ln^y-1)-y(\ln^x-1)^2]}{x^4(\ln^y-1)^3}$$

(4)

$$e^z\frac{\partial z}{\partial x}-2\frac{\partial z}{\partial x}-y\cdot e^{-\mathrm{xy}}=0$$

$$\Rightarrow\frac{\partial z}{\partial x}=-\frac{ye^{-\mathrm{xy}}}{2-e^z}$$

同理：$\frac{\partial z}{\partial y} = -\frac{x e^{-xy}}{2-e^z}$

$\because -\left(x + \frac{\partial x}{\partial y}\cdot y\right) e^{-\mathrm{xy}} = 0 \quad \Rightarrow \frac{\partial x}{\partial y} = -\frac{x}{y}$

$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(-\frac{y e^{-\mathrm{xy}}}{2-e^z}\right) = \frac{(2-e^z)^2 y^2 e^{-\mathrm{xy}} + e^z y^2 e^{-2\mathrm{xy}}}{(2-e^z)^3}$

(5)

$\frac{\partial z}{\partial x} e^z - x y \frac{\partial z}{\partial x} - y z = 0 \quad \Rightarrow \frac{\partial z}{\partial x} = \frac{\mathrm{yz}}{(e^z - x y)}$

同理：$\frac{\partial z}{\partial y} = \frac{x z}{(\mathrm{e}^z - \mathrm{xy})}$;

$\because -\left(x + \frac{\partial x}{\partial y} y\right) z = 0 \quad \Rightarrow \frac{\partial x}{\partial y} = -\frac{x}{y}$

$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\mathrm{yz}}{e^z - x y}\right) = \frac{y \frac{\partial z}{\partial x}(e^z - x y) - y z\left(\frac{\partial z}{\partial x} e^z - y\right)}{(e^z - x y)^2} = \frac{-z(z^2 - 2z + 2)}{x^2 (z-1)^3}$

(6)

$\frac{z - x\frac{\partial z}{\partial x}}{z^2} = \frac{y}{z}\cdot\frac{1}{y}\cdot\frac{\partial z}{\partial x} \quad \Rightarrow z - x\cdot\frac{\partial z}{\partial z} = z\cdot\frac{\partial z}{\partial x} \qquad \Rightarrow \frac{\partial z}{\partial x} = \frac{z}{x+z}$

$-\frac{x}{z^2}\cdot\frac{\mathrm{dz}}{\mathrm{dy}} = \frac{y}{z}\cdot\frac{-z + y\frac{\partial z}{\partial y}}{y^2}$

$\Rightarrow \frac{\partial z}{\partial y} = \frac{z^2}{y(x+z)}$

$\frac{1}{z}\frac{\partial x}{\partial y} = \frac{y}{z}\cdot - \frac{z}{y^2} = -\frac{1}{y}$

$\Rightarrow \frac{\partial x}{\partial y} = -\frac{z}{y}$

$\frac{\partial^2 z}{\partial x^2} = \frac{\frac{\partial z}{\mathrm{dx}}(x+z) - \left(1 + \frac{\partial z}{\partial x}\right)\cdot z}{(x+z)^2} = -\frac{z^2}{(x+z)^3}$

(7)

对x求偏导

$F_1' + F_2' + F_3'\left(1 + \frac{\partial z}{\partial x}\right) = 0$

$\Rightarrow \frac{\partial z}{\partial x} = -\frac{F_1' + F_2' + F_3'}{F_3'}$

对y求偏导

$\frac{\partial z}{\partial y} = -\frac{F_2' + F_3'}{F_3'}$

(8)

$\left(z+x\frac{\partial z}{\partial x}\right)F_1'+y\frac{\partial z}{\partial x}F_2'=0$

$\Rightarrow\frac{\partial z}{\partial x}=\frac{-zF_2'}{xF_1'+\mathrm{yF_2'}}$

$x\frac{\partial z}{\partial y}F_1'+\left(z+y\frac{\partial z}{\partial y}\right)F_2'=0$

$\Rightarrow\frac{\partial z}{\partial y}=\frac{-zF_1'}{xF_1'+yF_2'}$

3.证明

$\frac{\partial z}{\partial x}=-\frac{F_x'}{F_z'}$ $\frac{\partial x}{\partial y}=-\frac{F_y'}{F_x'}$ $\frac{\partial y}{\partial z}=-\frac{F_z'}{F_y'}$

$\Rightarrow\frac{\partial x}{\partial y}\cdot\frac{\partial y}{\partial z}\cdot\frac{\partial z}{\partial x}=-1$

4.

(1)

$-2\,\mathrm{cosx}\,\mathrm{sinx}+2\mathrm{cosz}\,(-\mathrm{sinz})\,\frac{\partial z}{\partial x}=0$

$\Rightarrow\frac{\partial z}{\partial x}=-\frac{\sin 2x}{\sin 2z}$

同理：$\frac{\partial z}{\partial y}=-\frac{\sin 2y}{\sin 2z}$

$\Rightarrow\mathrm{dz}=-\frac{\sin 2x}{\sin 2z}\mathrm{dx}-\frac{\sin 2y}{\sin 2y}\mathrm{dy}$

(2)

$y\left(z+x\frac{\partial z}{\partial x}\right)=1+\frac{\partial z}{\partial x}$

$\Rightarrow\frac{\partial z}{\partial x}=\frac{1-yz}{yx-1}$

同理$\frac{\partial z}{\partial y}=\frac{1-xz}{xy-1}$

$\Rightarrow\mathrm{dz}=\frac{1-yz}{xy-1}\mathrm{dx}+\frac{1-\mathrm{xz}}{xy-1}\mathrm{dy}$

(3)

$3u^2\frac{\partial u}{\partial x}-3u^2-6\,(x+y)\,u\frac{\partial u}{\partial x}=0$

$\Rightarrow\frac{\partial u}{\partial x}=\frac{u^2}{u^2-2u\,(x+y)}=\frac{u}{u-2\,(x+y)}$

同理：$\frac{\partial u}{\partial z}=-\frac{u}{u-2\,(x+y)}$

$3u^2\frac{\partial u}{\partial z}-6u\,(x+y)\,\frac{\partial u}{\partial z}+3z^2=0$

$\Rightarrow\frac{\partial u}{\partial z}=-\frac{z^2}{u^2-2u(x+y)}$

$\Rightarrow\mathrm{du}=\frac{u}{u-2(x+y)}(\mathrm{dx}+\mathrm{dy})-\frac{z^2}{u^2-2u(x+y)}\mathrm{dz}$

(4)

$F_1'-\frac{\partial z}{\partial x}F_2'+\left(\frac{\partial z}{\partial x}-1\right)F_3'=0$

$\Rightarrow\frac{\partial z}{\partial x}=\frac{F_3'-F_1'}{F_3'-F_2'}$

同理：$\frac{\partial z}{\partial y}=\frac{F_1'-F_2'}{F_3'-F_2'}$

$\Rightarrow\mathrm{dz}=\frac{F_3'-F_1'}{F_3'-F_2'}\mathrm{dx}+\frac{F_1'-F_2'}{F_3'-F_2'}\mathrm{dy}$

5.证明

$y+x\frac{\mathrm{dy}}{\mathrm{dx}}=k\left(1-\frac{\mathrm{dy}}{\mathrm{dx}}\right)$

$\Rightarrow\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{k-y}{x+k}$

$\Rightarrow\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{k-y}{x+k}=\frac{\frac{1+xy}{(x-y)}-y}{x+\frac{1+xy}{(x-y)}}=\frac{y^2+1}{x^2+1}$

$\Rightarrow\frac{\mathrm{dx}}{1+x^2}=\frac{\mathrm{dy}}{1+y^2}$

6.证明

$\begin{cases}2\cos(x+2y-3z)\left(1-3\frac{\partial z}{\partial x}\right)=1-3\frac{\partial z}{\partial x}\\2\cos(x+2y-3z)\left(2-3\frac{\partial z}{\partial y}\right)=2-3\frac{\partial z}{\partial y}\end{cases}$

$\therefore\cos(x+2y-3z)=1$

或$1-3\frac{\partial z}{\partial x}=0$且$2-3\frac{\partial z}{\partial y}=0$

若$\cos(x+2y-3z)=1$

$\Rightarrow x+2y-3z=2n\pi,n$为整数

$\Rightarrow\frac{\partial z}{\partial x}=\frac{1}{3},\frac{\partial z}{\partial y}=\frac{2}{3}$

$\Rightarrow\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=1$

若$1-3\frac{\partial z}{\partial x}=0$且$2-3\frac{\partial z}{\partial y}=0$

$\Rightarrow\frac{\partial z}{\partial x}=\frac{1}{3},\frac{\partial z}{\partial y}=\frac{2}{3}$

$\Rightarrow \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$

证毕

7.证明

$\varphi_1'\left(c - a\frac{\partial z}{\partial x}\right) + \varphi_2'\left(-b\frac{\partial z}{\partial x}\right) = 0 \qquad \Rightarrow \frac{\partial z}{\partial x} = \frac{c\varphi_1'}{a\varphi_1' + b\varphi_2'}$

$\varphi_1'\left(-a\frac{\partial z}{\partial y}\right) + \varphi_2'\left(c - b\frac{\partial z}{\partial y}\right) = 0 \qquad \Rightarrow \frac{\partial z}{\partial y} = \frac{c\varphi_2'}{a\varphi_1' + b\varphi_2'}$

$\Rightarrow a\frac{\partial z}{\partial x} + b\frac{\partial z}{\partial y} = \frac{ac\varphi_1'}{a\varphi_1' + b\varphi_2'} + \frac{bc\varphi_2'}{a\varphi_1' + b\varphi_2'} = c$

8.解

$2x - y - x\frac{\mathrm{dy}}{\mathrm{dx}} + 2y\frac{\mathrm{dy}}{\mathrm{dx}} = 0$

$\frac{\mathrm{dy}}{\mathrm{dx}} = \frac{y - 2x}{2y - x}$

$\frac{\mathrm{dz}}{\mathrm{dx}} = 2x + 2y\cdot\frac{\mathrm{dy}}{\mathrm{dx}} = 2x + 2y\cdot\frac{y - 2x}{2y - x} = \frac{2(y^2 - x^2)}{2y - z}$

$\frac{d^2z}{\mathrm{dx}^2} = \frac{2[(2\mathrm{yy}' - 2x)(2y - x) - (2y' - 1)(y^2 - x^2)]}{(2y - x)^3} = \frac{4x - 2y}{x - 2y} + \frac{6x}{(x - 2y)^3}$

9.解

$u = \varphi(u) + \int_y^x p(t)\,\mathrm{dt}$

$\Rightarrow \frac{\partial u}{\partial x} = \varphi'(u)\frac{\partial u}{\partial x} + p(x) \qquad ; \frac{\partial u}{\partial y} = \varphi'(u)\frac{\partial u}{\partial y} - p(y)$

$\Rightarrow \frac{\partial u}{\partial x} = \frac{p(x)}{1 - \varphi'(x)} \qquad ; \frac{\partial u}{\partial y} = \frac{-p(y)}{1 - \varphi'(u)}$

$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x} = f'(u)\cdot\frac{p(x)}{1 - \varphi'(u)}$

$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y} = f'(u)\cdot\frac{-p(y)}{1 - \varphi'(u)}$

$\Rightarrow p(y)\frac{\partial z}{\partial x} + p(x)\frac{\partial z}{\partial y} = 0$

10.解

(1)

$x^2 + (-x - z)^2 + z^2 = 2x^2 + 2zx + 2z^2 = 1$

$\Rightarrow x^2 + xz + z^2 = \frac{1}{2}$

$$\Rightarrow 2x\cdot\frac{\mathrm{dx}}{\mathrm{dz}}+x+z\cdot\frac{\mathrm{dx}}{\mathrm{dz}}+2z=0$$

$$\Rightarrow\frac{\mathrm{dx}}{\mathrm{dz}}=\frac{y-z}{x-y}$$

同理$\frac{\mathrm{dy}}{\mathrm{dz}}=\frac{z-x}{x-y}$

(2)

$$\begin{cases} F_1'+F_2'\frac{\mathrm{dy}}{\mathrm{dx}}+F_3'\frac{\mathrm{dz}}{\mathrm{dx}}=0 \\ G_1'+G_2'\frac{\mathrm{dy}}{\mathrm{dx}}+G_3'\frac{\mathrm{dz}}{\mathrm{dx}}=0 \end{cases}$$

$$\Rightarrow\begin{cases} \frac{\mathrm{dy}}{\mathrm{dx}}=\frac{G_1'F_3'-G_3'F_1'}{F_2'G_3'-G_2'F_3'}=\frac{\partial(F,G)}{\partial(y,z)}\Big/\frac{\partial(F,G)}{\partial(y,z)} \\ \frac{\mathrm{dz}}{\mathrm{dx}}=\frac{G_1'F_2'-G_2'F_1'}{F_3'G_2'-G_3'F_2'}=\frac{\partial(F,G)}{\partial(x,y)}\Big/\frac{\partial(F,G)}{\partial(y,z)} \end{cases}$$

11.

(1)

$$u^2+(-u-x-y)^2+x^2+y^2=1$$

$$\Rightarrow u^2+ux+uy+xy+x^2+y^2=\frac{1}{2}$$

$$\Rightarrow 2u\frac{\partial u}{\partial x}+u+x\cdot\frac{\partial u}{\partial x}+y\cdot\frac{\partial u}{\partial y}+y+2x=0$$

$$\Rightarrow\frac{\partial u}{\partial x}=-\frac{u+y+2x}{x+y+2u}=\frac{v-x}{u-v}$$

同理

$$\frac{\partial u}{\partial y}=-\frac{u+x+2y}{x+y+2u}=\frac{v-y}{u-v}$$

$$\frac{\partial v}{\partial x}=-\frac{v+y+2x}{x+y+2v}=\frac{x-u}{u-v}$$

$$\frac{\partial v}{\partial y}=-\frac{v+x+2y}{x+y+2v}=\frac{y-u}{u-v}$$

(2)

$$\begin{cases} \frac{\partial u}{\partial x}=f_1'\cdot\left(u+x\cdot\frac{\partial u}{\partial x}\right)+f_2'\cdot\frac{\partial v}{\partial x} \\ \frac{\partial v}{\partial x}=g_1'\cdot\left(\frac{\partial u}{\partial x}-1\right)+g_2'\left(2vy\cdot\frac{\partial v}{\partial x}\right) \end{cases}$$

$$\Rightarrow\begin{cases} \frac{\partial u}{\partial x}=\frac{uf_1'(1-2vyg_2')-g_1'f_2'}{(1-\mathrm{xf}_1')(1-2vyg_2')-g_1'f_2'} \\ \frac{\partial v}{\partial x}=\frac{g_1'(uf_1'+xf_1'-1)}{(1-\mathrm{xf}_1')(1-2vyg_2')-g_1'f_2'} \end{cases}$$

$$\begin{cases} \frac{\partial u}{\partial y}=xf_1'\cdot\frac{\partial u}{\partial y}+f_2'\left(1+\frac{\partial v}{\partial y}\right) \\ \frac{\partial v}{\partial y}=g_1'\cdot\frac{\partial u}{\partial y}+g_2'\left(v^2+2vy\frac{\partial v}{\partial y}\right) \end{cases}$$

$$\Rightarrow\begin{cases}\frac{\partial u}{\partial y}=\frac{f_2'(1-2vyg_2')+f_2'g_2'v^2}{(1-xf_1')(1-2vyg_2')-g_1'f_2'}\\\frac{\partial v}{\partial y}=\frac{g_1'f_2'+(1-xf_1')g_2'v^2}{(1-xf_1')(1-2vyg_2')-g_1'f_2'}\end{cases}$$

12.

(1)

$$\begin{cases}1=f_1'\cdot\frac{\partial u}{\partial x}+f_2'\cdot\frac{\partial v}{\partial x}\\0=g_1'\cdot\frac{\partial u}{\partial x}+g_2'\cdot\frac{\partial v}{\partial x}\end{cases}$$

$$\Rightarrow\begin{cases}\frac{\partial u}{\partial x}=\frac{g_2'}{f_1'g_2'-g_1'f_2'}\\\frac{\partial v}{\partial x}=\frac{g_1'}{g_1'f_2'-g_2'f_1'}\end{cases}$$

$$\begin{cases}0=f_1'\frac{\partial u}{\partial y}+f_2'\frac{\partial v}{\mathrm{d}y}\\1=g_1'\frac{\partial u}{\partial y}+g_2'\frac{\partial v}{\partial y}\end{cases}$$

$$\Rightarrow\begin{cases}\frac{\partial u}{\partial y}=\frac{f_2'}{g_1'f_2'-g_2'f_1'}\\\frac{\partial v}{\partial y}=\frac{f_1'}{f_1'g_2'-f_2'g_1'}\end{cases}$$

(2)

$$\begin{cases}1=\frac{\partial u}{\partial x}\cdot e^u+\frac{\mathrm{d}u}{\partial x}\sin v+u\,\frac{\partial v}{\partial x}\cos v\\0=\frac{\partial u}{\partial x}\cdot e^u-\frac{\partial u}{\partial x}\cos v+u\,\frac{\partial u}{\partial x}\sin v\end{cases}$$

$$\Rightarrow\begin{cases}\frac{\partial u}{\partial x}=\frac{\sin v}{e^u(\sin v-\cos v)+1}\\\frac{\partial v}{\partial x}=\frac{\cos v-e^u}{u\,e^u(\sin v-\cos v)+u}\end{cases}$$

$$\begin{cases}0=\frac{\partial u}{\partial y}\cdot e^u+\frac{\partial u}{\partial y}\sin v+u\frac{\partial v}{\partial y}\cos v\\1=\frac{\partial u}{\partial y}\cdot e^u-\frac{\partial u}{\partial y}\cos v+u\frac{\partial v}{\partial y}\sin v\end{cases}$$

$$\Rightarrow\begin{cases}\frac{\partial u}{\partial y}=\frac{-\cos v}{e^u(\sin v-\cos v)+1}\\\frac{\partial v}{\partial y}=\frac{e^u+\sin v}{u\,e^u(\sin v-\cos v)+u}\end{cases}$$

13.解

$$\frac{\mathrm{du}}{\mathrm{dx}}=f_1'+f_2'\cdot\frac{\mathrm{dy}}{\mathrm{dx}}+f_3'\cdot\frac{\partial z}{\partial x}$$

$$\frac{\mathrm{dy}}{\mathrm{dx}}=\cos x;$$

$$\frac{\partial\varphi}{\partial x}=\varphi_1'\cdot 2x+\varphi_2'\cdot e^y\cos x+\varphi_3'\cdot\frac{\partial z}{\partial x}=0$$

$$\Rightarrow\frac{\partial z}{\partial x}=-\frac{2x\varphi_1'+\varphi_2'\cdot e^y\cos x}{\varphi_3'}$$

$$\Rightarrow \frac{\mathrm{du}}{\mathrm{dx}} = f_1' + f_2' \cdot \cos x - f_3' \frac{\varphi_1' \cdot 2x + \varphi_2' \cdot e^{\sin x} \mathrm{cosx}}{\varphi_3'}$$

14.解

$$\begin{cases} \frac{\mathrm{dz}}{\mathrm{dx}} = f(x+y) + x f'(x+y)\left(1 + \frac{\mathrm{dy}}{\mathrm{dx}}\right) \\ F_1' + F_2' \cdot \frac{\mathrm{dy}}{\mathrm{dx}} + F_3' \cdot \frac{\mathrm{dz}}{\mathrm{dx}} = 0 \end{cases}$$

$$\Rightarrow \begin{cases} \frac{\mathrm{dz}}{\mathrm{dx}} = \frac{F_2' f(x+y) + x F_2' f'(x+y) - F_1' x f'(x+y)}{F_2' + x F_3' f'(x+y)} \\ \frac{\mathrm{dy}}{\mathrm{dz}} = -\frac{F_3' F_2' f(x+y) + F_2' F_3' x f'(x+y) + F_1' F_2'}{F_2'(F_2' + x F_3' f'(x+y))} \end{cases}$$

15.解

$$\begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{pmatrix} = -\begin{pmatrix} \frac{\partial F}{\partial u} & \frac{\partial F}{\partial v} \\ \frac{\partial G}{\partial u} & \frac{\partial G}{\partial v} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \\ \frac{\partial G}{\partial x} & \frac{\partial g}{\partial y} \end{pmatrix} = -\begin{pmatrix} F_3' & F_4' \\ G_3' & G_4' \end{pmatrix}^{-1} \cdot \begin{pmatrix} F_1' & F_2' \\ G_1' & G_2' \end{pmatrix} = \frac{-1}{F_3' G_4' - F_4' G_3'} \begin{pmatrix} G_4' & -F_4' \\ -G_3' & F_3' \end{pmatrix} \cdot \begin{pmatrix} F_1' & F_2' \\ G_1' & G_2' \end{pmatrix}$$

$$= \frac{-1}{F_3' G_4' - F_4' G_3'} \begin{pmatrix} G_4' F_1' - F_4' G_1' & G_4' F_2' - F_4' G_2' \\ F_3' G_1' - G_3' F_1' & F_3' G_2' - G_3' F_2' \end{pmatrix}$$

$$\Rightarrow \mathrm{du} = \frac{-1}{F_3' G_4' - F_4' G_3'} [(G_4' F_1' - F_4' G_1')\,\mathrm{dx} + (G_4' F_2' - F_4' G_2')\,\mathrm{dy}];$$

$$\mathrm{dv} = \frac{-1}{F_3' G_4' - F_4' G_3'} [(F_3' G_1' - G_3' F_1')\,\mathrm{dx} + (F_3' G_2' - G_3' F_2')\,\mathrm{dy}]$$

16.解

设$F = f(x, y, z, t) - u \quad G = g(y, z, t) \qquad , H = h(z, t)$

$$\begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial z}{\partial x} & \frac{\partial z}{\partial y} \\ \frac{\partial t}{\partial x} & \frac{\partial t}{\partial y} \end{pmatrix} = -\begin{pmatrix} \frac{\partial F}{\partial u} & \frac{\partial F}{\partial z} & \frac{\partial F}{\partial t} \\ \frac{\partial G}{\partial u} & \frac{\partial G}{\partial z} & \frac{\partial G}{\partial t} \\ \frac{\partial H}{\partial u} & \frac{\partial H}{\partial z} & \frac{\partial H}{\partial t} \end{pmatrix}^{-1} \cdot \begin{pmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} \\ \frac{\partial G}{\partial x} & \frac{\partial G}{\partial y} \\ \frac{\partial H}{\partial x} & \frac{\partial H}{\partial y} \end{pmatrix} = -\begin{pmatrix} -1 & f_3' & f_4' \\ 0 & g_2' & g_3' \\ 0 & h_1' & h_2' \end{pmatrix}^{-1} \cdot \begin{pmatrix} f_1' & f_2' \\ 0 & g_1' \\ 0 & 0 \end{pmatrix}$$

$$= -\frac{-1}{g_2' h_2' - g_3' h_1'} \begin{pmatrix} g_2' h_2' - g_3' h_1' & -f_3' h_2' + h_1' f_4' & f_3' g_3' - f_4' g_2' \\ 0 & -h_2' & g_3' \\ 0 & h_1' & -g_2' \end{pmatrix} \cdot \begin{pmatrix} f_1' & f_2' \\ 0 & g_1' \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \frac{\partial u}{\partial x} = f_1' \qquad ; \frac{\partial u}{\partial y} = f_2' + \frac{1}{g_2' h_2' - g_3' h_1'} . (-f_3' h_2' + h_1' f_4')\, g_1'$$

17.解

设$F = \cos(2x + 2y + 2z) + y - 1 \quad ; G = \sin(12x + 6y - 6z) - 6z$

$$\begin{pmatrix}\frac{\mathrm{dy}}{\mathrm{dx}}\\\frac{\mathrm{dz}}{\mathrm{dx}}\end{pmatrix}=-\begin{pmatrix}\frac{\partial F}{\partial y}&\frac{\partial F}{\partial z}\\\frac{\partial G}{\partial y}&\frac{\partial G}{\partial z}\end{pmatrix}^{-1}\cdot\begin{pmatrix}\frac{\partial F}{\partial x}\\\frac{\partial G}{\partial x}\end{pmatrix}=$$

$$-\begin{pmatrix}-2\sin(2x+2y+2z)+1&-2\sin(2x+2y+2z)\\6\cos(12x+6y-6z)&-6\cos(12x+6y-6z)-6\end{pmatrix}^{-1}\cdot\begin{pmatrix}-2\sin(2x+2y+2z)\\12\cos(12x+6y-6z)\end{pmatrix}$$

$$=-\frac{\begin{pmatrix}-6\cos(12x+6y-6z)-6&2\sin(2x+2y+2z)\\-6\cos(12x+6y-6z)&-2\sin(2x+2y+2z)+1\end{pmatrix}\cdot\begin{pmatrix}-2\sin(2x+2y+2z)\\12\cos(12x+6y-6z)\end{pmatrix}}{24\sin(2x+2y+2z)\cos(12x+6y-6z)+12\sin(2x+2y+2z)-6\cos(12x+6y-6z)-6}$$

$$=-\frac{\begin{pmatrix}24\sin(2x+2y+2z)+24\cos(12x+6y-6z)\sin(2x+2y+2z)\\12\cos(12x+6y-6z)\end{pmatrix}}{24\sin(2x+2y+2z)\cos(12x+6y-6z)+12\sin(2x+2y+2z)-6\cos(12x+6y-6z)-6}$$

$$\Rightarrow\frac{\mathrm{dy}}{\mathrm{dx}}=-\frac{24\sin(2x+2y+2z)+24\cos(12x+6y-6z)\sin(2x+2y+2z)}{24\sin(2x+2y+2z)\cos(12x+6y-6z)+12\sin(2x+2y+2z)-6\cos(12x+6y-6z)-6}$$

$$=-\frac{4\sin(2x+2y+2z)+4\cos(12x+6y-6z)\sin(2x+2y+2z)}{4\sin(2x+2y+2z)\cos(12x+6y-6z)+2\sin(2x+2y+2z)-\cos(12x+6y-6z)-1}$$

$$\frac{\mathrm{dz}}{\mathrm{dx}}=-\frac{12\cos(12x+6y-6z)}{24\sin(2x+2y+2z)\cos(12x+6y-6z)+12\sin(2x+2y+2z)-6\cos(12x+6y-6z)-6}$$

$$=-\frac{2\cos(12x+6y-6z)}{4\sin(2x+2y+2z)\cos(12x+6y-6z)+2\sin(2x+2y+2z)-\cos(12x+6y-6z)-1}$$

$\Rightarrow$当$(x,y,z)=(0,0,0)$时$y'(0)=0;z'(0)=1$

令$A=(4+8\cos(12x+6y-6z))\cos(2x+2y+2z)(1+y'+z')$

令$A'=24\sin(2x+2y+2z)\sin(12x+6y-6z)(2+y'-z')$

令$B=4\sin(2x+2y+2z)\cos(12x+6y-6z)+2\sin(2x+2y+2z)-\cos(12x+6y-6z)-1$

令$C=2\sin(2x+2y+2z)+4\cos(12x+6y-6z)\sin(2x+2y+2z)$

令$D=(2-24\sin(12x+6y-6z)(2+y'-z'))\sin(2x+2y+2z)$

令$E=8\cos(12x+6y-6z)\cos(2x+2y+2z)(1+y'+z')+6\sin(12x+6y-6z)(2+y'-z')$

令$F=-12\sin(12x+6y-6z)(2x+y'-z')$

$$\Rightarrow\frac{d^2y}{\mathrm{dx}^2}=-\frac{(A+A')\cdot B-C(D+E)}{B^2}\quad;\frac{d^2z}{\mathrm{dx}^2}=-\frac{F\cdot B-2\cos(12x+6y-6z)(D+E)}{B^2}$$

$$\Rightarrow\frac{d^2y}{\mathrm{dx}^2}\Big|_{x=0}=16\quad;\frac{d^2z}{\mathrm{dx}^2}\Big|_{x=0}=8$$

（完结）

撒花！！！