

Assignment -2 in L^AT_EX

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11.16.3.12: One urn contains two black balls (labelled B1 and B2) and one white ball. A second urn contains one black ball and two white balls (labelled W1 and W2). Suppose the following experiment is performed. One of the two urns is chosen at random. Next a ball is randomly chosen from the urn. Then a second ball is chosen at random from the same urn without replacing the first ball.

- What is the probability that two black balls are chosen?
- What is the probability that two balls of opposite colour are chosen?

Solution:

Let X be a Bernoulli random variable

$$X = \begin{cases} 0, & \text{Urn 1} \\ 1, & \text{Urn 2} \end{cases} \quad (1)$$

Since both events are equally likely

$$\Pr(X = 0) = \Pr(X = 1) \quad (2)$$

$$= \frac{1}{2} \quad (3)$$

Let Y_i be a random variable to denote the turn

$$Y_i = \begin{cases} 0, & \text{Black ball} \\ 1, & \text{White ball} \end{cases} \quad (4)$$

Y_1 denotes the first ball and Y_2 denotes the second ball.

Y_1	Y_2	Description
0	0	Both Black
1	1	Both White
0	1	Black, White
1	0	White, Black

TABLE 1

Required Probability:

$$\Pr(Y'_1 Y'_2) \quad (7)$$

$$= \Pr(Y'_1 Y'_2 | X') \Pr(X')$$

$$= \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2}$$

$$= 1/6 \quad (8)$$

Therefore,

$$\Pr(E) = \frac{1}{6} \quad (9)$$

2)

$$E = Y_1 \oplus Y_2 \quad (10)$$

$$= Y_1 Y'_2 + Y'_1 Y_2 \quad (11)$$

Required Probability :

$$\Pr(Y_1 Y'_2 + Y'_1 Y_2) \quad (12)$$

$$= \Pr(Y_1 Y'_2) + \Pr(Y'_1 Y_2) \quad (13)$$

$$= \left(\frac{1}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2} \right) \times 2 \quad (14)$$

$$= \frac{2}{3} \quad (15)$$

Therefore,

$$\Pr(E) = \frac{2}{3} \quad (16)$$

1)

$$E = (Y_1 + Y_2)' \quad (5)$$

$$= Y'_1 Y'_2 \quad (6)$$