

Assignment -1 in L^AT_EX

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Question 1.4.1 Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$$

The equation of the perpendicular bisector of BC is

$$\left(\mathbf{x} - \frac{\mathbf{B} + \mathbf{C}}{2} \right) (\mathbf{B} - \mathbf{C}) = 0 \quad (1)$$

To find the equations of perpendicular bisectors of BC, AB, CA .

Solution:

- 1) BC : Given equation for the perpendicular bisector of BC :

$$\left(\mathbf{x} - \frac{\mathbf{B} + \mathbf{C}}{2} \right) (\mathbf{B} - \mathbf{C}) = 0 \quad (2)$$

On substituting the values,

$$\frac{\mathbf{B} + \mathbf{C}}{2} = \begin{pmatrix} -\frac{7}{2} \\ \frac{1}{2} \end{pmatrix} \quad (3)$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} -1 \\ 11 \end{pmatrix} \quad (4)$$

Solving using matrix multiplication:

$$(\mathbf{B} - \mathbf{C})^\top \left(\mathbf{x} - \frac{\mathbf{B} + \mathbf{C}}{2} \right) = 0 \quad (5)$$

$$\Rightarrow (\mathbf{B} - \mathbf{C})^\top \mathbf{x} = (\mathbf{B} - \mathbf{C})^\top \left(\frac{\mathbf{B} + \mathbf{C}}{2} \right) \quad (6)$$

$$(\mathbf{B} - \mathbf{C})^\top = (-1 \ 11) \quad (7)$$

$$(\mathbf{B} - \mathbf{C})^\top \left(\frac{\mathbf{B} + \mathbf{C}}{2} \right) \quad (8)$$

$$= (-1 \ 11) \begin{pmatrix} -\frac{7}{2} \\ \frac{1}{2} \end{pmatrix} = 9 \quad (9)$$

Therefore, perpendicular bisector of BC is

$$(-1 \ 11) \mathbf{x} = 9 \quad (10)$$

- 2) AB : Similar to the equation for the perpendicular bisector of BC ,

$$\frac{\mathbf{A} + \mathbf{B}}{2} = \begin{pmatrix} -\frac{3}{2} \\ \frac{5}{2} \end{pmatrix} \quad (11)$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 5 \\ -7 \end{pmatrix} \quad (12)$$

And,

$$(\mathbf{A} - \mathbf{B})^\top = (5 \ -7) \quad (13)$$

Now,

$$(\mathbf{A} - \mathbf{B})^\top \left(\frac{\mathbf{A} + \mathbf{B}}{2} \right) \quad (14)$$

$$= (5 \ -7) \begin{pmatrix} -\frac{3}{2} \\ \frac{5}{2} \end{pmatrix} = -25 \quad (15)$$

Therefore, perpendicular bisector of AB is

$$(5 \ -7) \mathbf{x} = -25 \quad (16)$$

- 3) CA :

$$\frac{\mathbf{C} + \mathbf{A}}{2} = \begin{pmatrix} -1 \\ -3 \end{pmatrix} \quad (17)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} \quad (18)$$

And,

$$(\mathbf{C} - \mathbf{A})^\top = (-4 \ -4) \quad (19)$$

Now,

$$(\mathbf{C} - \mathbf{A})^\top \left(\frac{\mathbf{C} + \mathbf{A}}{2} \right) \quad (20)$$

$$= (-4 \ -4) \begin{pmatrix} -1 \\ -3 \end{pmatrix} = 16 \quad (21)$$

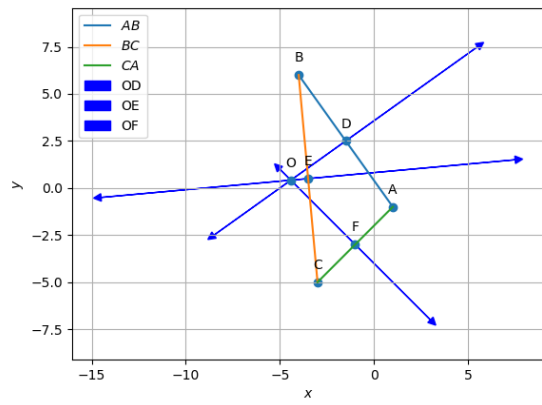


Fig. 3: Plot of the perpendicular bisectors

Therefore, perpendicular bisector of CA is

$$\begin{pmatrix} -4 & -4 \end{pmatrix} \mathbf{x} = 16 \quad (22)$$