

Assignment -5 in L^AT_EX

Muzaan Mohammed Faizel A P
EE22BTECH11036

GATE 2023 BM QN.12 For a Binomial random variable X , $E(X)$ and $\text{Var}(X)$ are the expectation and variance, respectively. Which one of the following statements CANNOT be true?

S.No	$E(X)$	$\text{Var}(X)$
1	20	16
2	6	5.4
3	10	15
4	64	12.8

TABLE 1

Solution:

$$X \sim \text{Bin}(n, p)$$

We know ,

$$\begin{aligned} E(X) &= np & (1) \\ \text{Var}(X) &= np(1-p) & (2) \\ 0 &\leq p \leq 1 & (3) \\ \Rightarrow -1 &\leq -p \leq 0 & (4) \\ \Rightarrow 0 &\leq 1-p \leq 1 & (5) \\ \Rightarrow np(1-p) &\leq np & (6) \end{aligned}$$

Therefore,

$$\text{Var}(X) \leq E(X) \quad (7)$$

From the four options, the statement that cannot be true is option (3)

Simulation steps

Step 1: Generate a Range of Probabilities

The program generates a range of probabilities (p) in increments of 0.0098, ranging from 0.01 to 0.99. This range is used to create different binomial distributions for subsequent calculations.

Step 2: Generating binomial r.v from Uniform distribution

$U \sim \text{Uniform}(0, 1)$.

Defining a random variable X as:

$$X = \begin{cases} 1 & \text{if } U > p \\ 0 & \text{otherwise} \end{cases}$$

This becomes a Bernoulli r.v. The count variable evaluates the Binomial r.v by the summation of Bernoulli r.v

$$\text{count} = \sum_{i=1}^n X$$

Step 3: Calculate Variances for Each Probability

For each probability (p) in the generated range, the program generates a binomial random variable using the given mean and the inverse of the probability (mean/p) from uniform distribution.

Step 4: Find Maximum and Minimum Variances

After calculating variances for each probability, the program identifies the maximum and minimum variances in the generated set.

Step 5: User Input

The program prompts the user to input a variance value for validation.

Step 6: Check Validity

The user-input variance is compared against the computed maximum and minimum variances. If the input variance falls within this range (inclusive of the minimum and exclusive of the maximum), the program outputs "Valid." Otherwise, it outputs "Invalid."

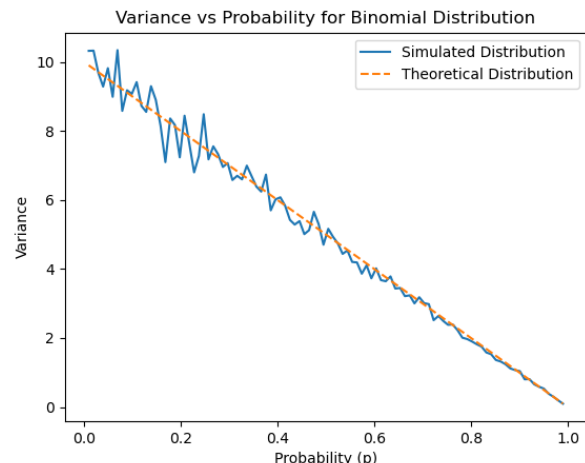


Fig. 0: Variance for mean=10