

EE23010 Assignment

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Question 11.16.3.12: Check whether the following probabilities $\Pr(A)$ and $\Pr(B)$ are consistently defined

$$1) \Pr(A) = 0.5, \Pr(B) = 0.7, \Pr(AB) = 0.6$$

$$2) \Pr(A) = 0.5, \Pr(B) = 0.4, \Pr(A + B) = 0.8$$

Solution: The given probabilities are consistently defined if it satisfies the following properties:

$$\Pr(AB) \leq \Pr(A), \Pr(B) \quad (1)$$

$$0 \leq \Pr(A), \Pr(B), \Pr(AB), \Pr(A + B) \leq 1 \quad (2)$$

$$\Pr(A + B) = \Pr(A) + \Pr(B) - \Pr(AB) \quad (3)$$

Proof of (1):

$$\Pr(A) = \Pr(A(B + B')) \quad (4)$$

$$\implies \Pr(A) = \Pr(AB) + \Pr(AB') \quad (5)$$

$$\text{Therefore, } \Pr(AB) \leq \Pr(A) \quad (6)$$

$$\text{On Interchanging A and B, we get :} \quad (7)$$

$$\Pr(AB) \leq \Pr(B) \quad (8)$$

1) Given:

$$\Pr(A) = 0.5, \quad (9)$$

$$\Pr(B) = 0.7, \quad (10)$$

$$\Pr(AB) = 0.6 \quad (11)$$

Since

$$\Pr(AB) \geq \Pr(A) \quad (12)$$

$\Pr(A)$ and $\Pr(B)$ are not consistently defined.

2) Given:

$$\Pr(A) = 0.5, \quad (13)$$

$$\Pr(B) = 0.4, \quad (14)$$

$$\Pr(A + B) = 0.8 \quad (15)$$

From (3),

$$\Pr(AB) = 0.5 + 0.4 - 0.8 \quad (16)$$

$$= 0.1 \quad (17)$$

This also satisfies (2) and (1)

Therefore $\Pr(A)$ and $\Pr(B)$ are consistently defined.