Assignment -2 in LaTeX

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11.16.3.12: One urn contains two black balls (labelled B1 and B2) and one white ball. A second urn contains one black ball and two white balls (labelled W1 and W2). Suppose the following experiment is performed. One of the two urns is chosen at random. Next a ball is randomly chosen from the urn. Then a second ball is chosen at random from the same urn without replacing the first ball.

(a) What is the probability that two black balls are chosen?
(b) What is the probability that two balls of opposite colour are chosen?

Solution:

Let X be a Bernoulli random variable

$$X = \begin{cases} 0, & \text{Urn 1} \\ 1, & \text{Urn 2} \end{cases} \tag{1}$$

Since both events are equally likely

$$\Pr(X = 0) = \Pr(X = 1) \tag{2}$$
$$= \frac{1}{2} \tag{3}$$

Let Y_i be a random variable to denote the turn

$$Y_i = \begin{cases} 0, & \text{Black ball} \\ 1, & \text{White ball} \end{cases} \tag{4}$$

Y_1 denotes the first ball and Y_2 denotes the second ball.

Y_1	Y_2	**Description**
0	0	Both Black
1	1	Both White
0	1	Black,White
1	0	White,Black

TABLE 1: Random variables for each ball

1)

$$E = (Y_1 + Y_2)' \tag{5}$$
$$= Y_1' Y_2' \tag{6}$$

Required Probability:

$$\Pr(Y_1' Y_2') = \Pr(Y_1' Y_2' | X') \Pr(X') \tag{7}$$
$$= \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2}$$
$$= 1/6 \tag{8}$$

Therefore,

$$\Pr(E) = \frac{1}{6} \tag{9}$$

2)

$$E = Y_1 Y_2' + Y_1' Y_2 \tag{10}$$

Required Probability :

$$\Pr(Y_1 Y_2' + Y_1' Y_2) = \Pr(Y_1 Y_2') + \Pr(Y_1' Y_2) \tag{11}$$
$$= \left(\frac{1}{3} \times \frac{1}{2} + \frac{2}{3} \times \frac{1}{2} \times \frac{1}{2} \right) \times 2 \tag{12}$$
$$= \frac{2}{3} \tag{13}$$

Therefore,

$$\Pr(E) = \frac{2}{3} \tag{14}$$