

Assignment -1 in L^AT_EX

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EE22BTECH11036

Question 1.4.1 Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$$

The equation of the perpendicular bisector of BC is

$$\left(\mathbf{x} - \frac{\mathbf{B} + \mathbf{C}}{2} \right) (\mathbf{B} - \mathbf{C}) = 0 \quad (1)$$

To find the equations of perpendicular bisectors of AB, BC, CA .

Solution:

1) BC :

Given equation for the perpendicular bisector of BC :

$$\left(\mathbf{x} - \frac{\mathbf{B} + \mathbf{C}}{2} \right) (\mathbf{B} - \mathbf{C}) = 0$$

On substituting the values,

$$\frac{\mathbf{B} + \mathbf{C}}{2} = \begin{pmatrix} -\frac{7}{2} \\ \frac{1}{2} \end{pmatrix} \quad (2)$$

$$\mathbf{B} - \mathbf{C} = \begin{pmatrix} -1 \\ 11 \end{pmatrix} \quad (3)$$

Solving using matrix multiplication

The required equation is:

$$(\mathbf{B} - \mathbf{C})^\top \left(\mathbf{x} - \frac{\mathbf{B} + \mathbf{C}}{2} \right) = 0 \quad (4)$$

$$\Rightarrow (\mathbf{B} - \mathbf{C})^\top \mathbf{x} = (\mathbf{B} - \mathbf{C})^\top \left(\frac{\mathbf{B} + \mathbf{C}}{2} \right) \quad (5)$$

$$(\mathbf{B} - \mathbf{C})^\top = (-1 \quad 11)$$

$$(\mathbf{B} - \mathbf{C})^\top \left(\frac{\mathbf{B} + \mathbf{C}}{2} \right) = (-1 \quad 11) \begin{pmatrix} -\frac{7}{2} \\ \frac{1}{2} \end{pmatrix} = 9 \quad (6)$$

Therefore, from (5)

Perpendicular bisector of BC is

$$(-1 \quad 11) \mathbf{x} = 9$$

2) AB :

Similar to the equation for the perpendicular bisector of BC ,

$$\frac{\mathbf{A} + \mathbf{B}}{2} = \begin{pmatrix} -\frac{1}{2} \\ 5 \end{pmatrix} \quad (7)$$

$$\mathbf{A} - \mathbf{B} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \quad (8)$$

And,

$$(\mathbf{A} - \mathbf{B})^\top = (7 \quad -2) \quad (9)$$

$$(\mathbf{A} - \mathbf{B})^\top \left(\frac{\mathbf{A} + \mathbf{B}}{2} \right) = (7 \quad -2) \begin{pmatrix} -\frac{1}{2} \\ 5 \end{pmatrix} = -13.50 \quad (10)$$

Therefore, perpendicular bisector of AB is

$$(7 \quad -2) \mathbf{x} = -13.50$$

3) CA :

$$\frac{\mathbf{C} + \mathbf{A}}{2} = \begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix} \quad (11)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} -6 \\ -9 \end{pmatrix} \quad (12)$$

And,

$$(\mathbf{C} - \mathbf{A})^\top = (-6 \quad -9) \quad (13)$$

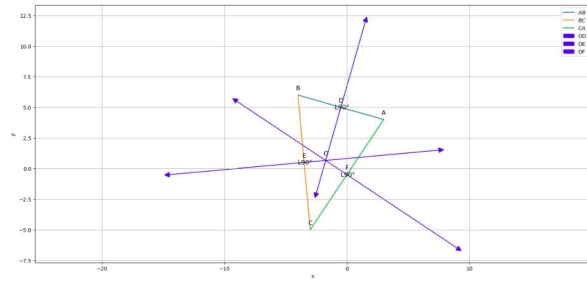


Fig. 3: Plot of the perpendicular bisectors

$$(\mathbf{C} - \mathbf{A})^\top \left(\frac{\mathbf{C} + \mathbf{A}}{2} \right) = \begin{pmatrix} -6 & -9 \end{pmatrix} \begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix} = 4.50 \quad (14)$$

Therefore, perpendicular bisector of CA is
 $\begin{pmatrix} -6 & -9 \end{pmatrix} \mathbf{x} = 4.50$