Assignment -1 in LaTeX

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Question 1.4.1 Consider a triangle with vertices

$$\mathbf{A} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$$

The equation of the perpendicular bisector of BC is

$$\left(\mathbf{x} - \frac{\mathbf{B}+\mathbf{C}}{2}\right)(\mathbf{B}-\mathbf{C}) = 0 \tag{1}$$

To find the equations of perpendicular bisectors of AB,BC,CA.

Solution:

1) BC: Given equation for the perpendicular bisector of BC:

$$\left(\mathbf{x} - \frac{\mathbf{B}+\mathbf{C}}{2}\right)(\mathbf{B}-\mathbf{C}) = 0 \tag{2}$$

On substituting the values,

$$\frac{\mathbf{B}+\mathbf{C}}{\mathbf{2}} = \begin{pmatrix} -\frac{7}{2} \\ \frac{1}{2} \end{pmatrix} \tag{3}$$

$$\mathbf{B}-\mathbf{C} = \begin{pmatrix} -1 \\ 11 \end{pmatrix} \tag{4}$$

Solving using matrix multiplication

$$(\mathbf{A}-\mathbf{B})^\top \left(\frac{\mathbf{A}+\mathbf{B}}{2}\right) = \begin{pmatrix} 7 & -2 \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & 5 \end{pmatrix} = -\frac{27}{2} \tag{5}$$

The required equation is:

$$(\mathbf{B}-\mathbf{C})^\top \left(\mathbf{x} - \frac{\mathbf{B}+\mathbf{C}}{2}\right) = 0 \tag{6}$$

$$\implies (\mathbf{B}-\mathbf{C})^\top \mathbf{x} = (\mathbf{B}-\mathbf{C})^\top \left(\frac{\mathbf{B}+\mathbf{C}}{2}\right) \tag{7}$$

$$(\mathbf{B}-\mathbf{C})^\top = \begin{pmatrix} -1 & 11 \end{pmatrix}$$

$$(\mathbf{B}-\mathbf{C})^\top \left(\frac{\mathbf{B}+\mathbf{C}}{2}\right) = \begin{pmatrix} -1 & 11 \end{pmatrix}\begin{pmatrix} -\frac{7}{2} \\ \frac{1}{2} \end{pmatrix} = 9 \tag{8}$$

Therefore,from (5)
Perpendicular bisector of BC is

$$\begin{pmatrix} -1 & 11 \end{pmatrix}\mathbf{x} = 9$$

2) AB: Similar to the equation for the perpendicular bisector of BC,

$$\frac{\mathbf{A}+\mathbf{B}}{\mathbf{2}} = \begin{pmatrix} -\frac{1}{2} \\ 5 \end{pmatrix} \tag{9}$$

$$\mathbf{A}-\mathbf{B} = \begin{pmatrix} 7 \\ -2 \end{pmatrix} \tag{10}$$

And,

$$(\mathbf{A}-\mathbf{B})^\top = \begin{pmatrix} 7 & -2 \end{pmatrix} \tag{11}$$

$$(\mathbf{A}-\mathbf{B})^\top \left(\frac{\mathbf{A}+\mathbf{B}}{2}\right) = \begin{pmatrix} 7 & -2 \end{pmatrix}\begin{pmatrix} -\frac{1}{2} \\ 5 \end{pmatrix} = \frac{27}{2} \tag{12}$$

Therefore,perpendicular bisector of AB is

$$\begin{pmatrix} 7 & -2 \end{pmatrix}\mathbf{x} = -\frac{27}{2}$$

3) CA:

$$\frac{\mathbf{C}+\mathbf{A}}{\mathbf{2}} = \begin{pmatrix} 0 \\ -\frac{1}{2} \end{pmatrix} \tag{13}$$

$$\mathbf{C}-\mathbf{A} = \begin{pmatrix} -6 \\ -9 \end{pmatrix} \tag{14}$$

And,

$$(\mathbf{C}-\mathbf{A})^\top = \begin{pmatrix} -6 & -9 \end{pmatrix} \tag{15}$$

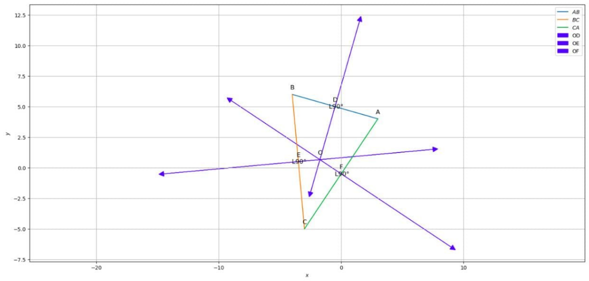

Fig. 3: Plot of the perpendicular bisectors

$$(\mathbf{C}-\mathbf{A})^{\top}\left(\frac{\mathbf{C}+\mathbf{A}}{2}\right) = \begin{pmatrix}-6 & -9\end{pmatrix}\begin{pmatrix}0\\ -\frac{1}{2}\end{pmatrix} \tag{16}$$
$$= \frac{9}{2}$$

Therefore,perpendicular bisector of CA is $\begin{pmatrix}-6 & -9\end{pmatrix}\mathbf{x} = \frac{9}{2}$