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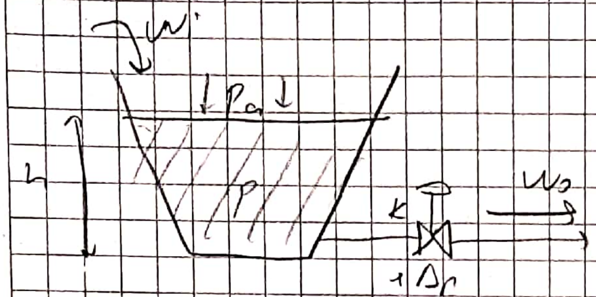
EE-407
HW-2

Question-1



$w_o = k\sqrt{\Delta p}$ k: constant
where
$\Delta p = p - p_a$

$p = p_a + \rho g h$
where ρ: liquid density

a) We assume the rate of change of cross section will be wrt $h(t)$. $(A(h))$

$$w_o = k\sqrt{p - p_a} = k\sqrt{p_a + \rho g h - p_a} = k\sqrt{\rho g h}$$

By our assumption;

$$\dot{V} = A(h) \cdot \dot{h}$$ (Rate of change of volume = Cross section when $h = h(t)$, rate of change of h)

Also,

$$\dot{V} = w_i - w_o = w_i - k\sqrt{\rho g h}$$ (Accumulation (conservation) law)

$$\dot{V} = A(h) \cdot \dot{h} = w_i - k\sqrt{\rho g h}$$

$$\boxed{\dot{h} = \frac{w_i}{A(h)} - \frac{k\sqrt{\rho g h}}{A(h)}}$$

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b) $\Delta p = \rho g h$

$$h' = \frac{\Delta p'}{\rho g} \qquad h = \frac{\Delta p}{\rho g}$$

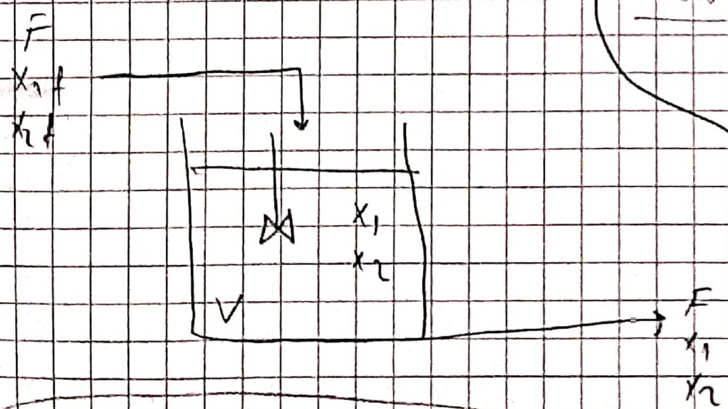
$$\boxed{\Delta p' = \frac{\rho \cdot g \cdot w_i(t)}{A(h)} - \frac{\rho g \cdot k \cdot \sqrt{\Delta p'}}{A(h)}}$$

c) output constant $\Rightarrow$ $h' = 0$, $\boxed{h = r}$ given

$$0 = \frac{w_i(t)}{\cancel{A(t)}} - \frac{k\sqrt{\rho g h}}{\cancel{A(t)}}$$

$$\boxed{w_{i\,\text{steady state}} = u_{ss} = k\sqrt{\rho g r}}$$

Question 2



Generation rate

$r_1 = \mu X_1$

Specific growth coefficient of X_2

concentration of (mass/volume) substrate

Assumptions = no biomass feed stream $X_{1f} = 0$, $d = F/V \rightarrow$ constant

flow rate

dilution rate

$$Y = \frac{r_1}{r_2} \Rightarrow \text{constant}$$

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a) $\dot{X}_1 = \mu X_1 - X_1 \cdot \frac{F}{V}^{d} + X_{1f} \cdot \frac{F}{V}^{d} = (\mu - d) X_1$

$\dot{X}_2 = -\frac{\mu}{Y} X_1 - X_2 \cdot \frac{F}{V}_d + X_{2f} \cdot \frac{F}{V}^{d} = -\frac{\mu}{Y} X_1 - d X_2 + d X_{2f}$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} \mu - d & 0 \\ -\frac{\mu}{Y} & -d \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ d \end{bmatrix} \cdot X_{2f}$$

b) when $\mu = \frac{\mu_m \cdot x_2}{(k_m + x_2)}$ / $\mu_m, k_m > 0$, $d < \mu_m$

(1) $\dot{x}_1 = 0 = \Big(\underbrace{\frac{\mu_m \cdot x_{2eq}}{k_m + x_{2eq}}}_{\mu} - d \Big) x_{1eq}$

(2) $\dot{x}_2 = 0 = -\frac{1}{Y} \underbrace{\left(\frac{\mu_m \cdot x_{2eq}}{k_m + x_{2eq}} \right)}_{\mu} \cdot x_{1eq} - d x_{2eq} + d x_{2f}$

From (1)

$d \cdot k_m + d x_{2eq} = \mu_m \cdot x_{2eq}$

$$\boxed{x_{2eq} = \frac{d \cdot k_m}{(\mu_m - d)}}$$

Continues →

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From (2)

$$-d\,X_{2eq} - \frac{1}{Y}\left(\frac{\mu m \cdot X_{2eq}}{Km + X_{2eq}}\right) \cdot X_{1eq} + d \cdot X_{2f} = 0$$

$$X_{1eq} = \frac{Y \cdot d\,(X_{2f} - X_{2eq})\,(Km + X_{2eq})}{\mu m \cdot X_{2eq}}$$

where $X_{2eq} = \dfrac{d \cdot Km}{(\mu m - d)}$ (Found before)

$$\boxed{X_{1eq} = Y\left(X_{2f} - \frac{d \cdot Km}{(\mu m - d)}\right)}$$

Or as trivial solution

$X_{1eq} = 0$

$X_{2eq} = X_{2f}$

Continue

c) Write in the form of

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 - x_{1eq} \\ x_2 - x_{2eq} \end{bmatrix}$$

Now, find 1, 2, 3, 4

$$\dot{x}_1 = 0 + \underbrace{\left(\frac{\mu_m x_{2eq}}{K_m + x_{2eq}} - d \right)}_{1} (x_1 - x_{1eq})$$

$$+ \underbrace{x_1 \mu_m \left(\frac{K_m}{(K_m + x_{2eq})^2} \right)}_{2} (x_2 - x_{2eq})$$

Find 1

$$\frac{\mu_m \left(\frac{d K_m}{\mu_m - d} \right)}{K_m + \frac{d K_m}{\mu_m - d}} - d = \boxed{0} \quad 1$$

Find 2

$$\boxed{\frac{x_1 \cdot \mu_m \cdot K_m}{\left(K_m + \frac{d K_m}{\mu_m - d} \right)^2}} \quad 2$$

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To find 3, 4 use $\dot{x}_2$

$$\dot{x}_2 = \underbrace{0 + \left(\frac{1}{y} \cdot \frac{\mu_m x_{2eq}}{(k_m + x_{2eq})} - d\right)}_{3}(x_1 - x_{1eq}) + \underbrace{\left(-d - \left(\frac{1}{y} x_{1eq}\, \mu_m \cdot \frac{k_m}{(k_m + x_{2eq})^2}\right)\right)}_{4}(x_2 - x_{2eq})$$

Find 3

$$3 = \frac{-d}{y}$$

Find 4

$$-d - \left(x_{2} + \frac{d k_m}{\mu_m - d}\right) \frac{\mu_m \cdot k_m}{\left(k_m + \frac{d k_m}{\mu_m - d}\right)^2}$$

So finally we now all unknowns

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 - x_{1eq} \\ x_2 - x_{2eq} \end{bmatrix}$$

d) $\mu = \dfrac{\mu_m x_2}{(k_m + x_2 + k_1 x_2^2)}$.

① $\dot{x_1} = 0 = -d x_{1eq} + \left(\dfrac{\mu_m x_{2eq}}{(k_m + x_{2eq} + k_1 x_{2eq}^2}\right) \cdot x_{1eq}$

② $\dot{x_2} = 0 = d x_{2f} - d x_{2eq} - \dfrac{1}{Y} \cdot \dfrac{\mu_m x_{2eq}}{(k_m + x_{2eq} + k_1 x_{2eq}^2)} \cdot x_{1eq}$

From ①

$$x_{2eq} = \frac{\frac{\mu_m - d}{d} \pm \sqrt{\left(\frac{d - \mu_m}{d}\right)^2 - 4 k_1 k_m}}{2 k_1}$$

$$\boxed{x_{2eq} = \frac{(\mu_m - d) \pm \sqrt{d^2 + \mu_m^2 + 2 d \mu_m - 4 d^2 k_1 k_m}}{2 d k_1}}$$

From ②

$$x_1^{eq} = \frac{Y(x_{2f} - x_{2eq})(k_m + x_{2eq} + k_1 x_{2eq}^2)}{\mu_m x_{2eq}}$$

put x_{2eq}

$$\boxed{x_1^{eq} = Y\left(x_{2f} - \frac{\mu_m - d \pm \sqrt{d^2 + \mu_m^2 + 2 d \mu_m - 4 d^2 k_1 k_m}}{2 d k_1}\right)}$$

As trivial, $\boxed{x_{1eq} = 0}$ $\boxed{x_{2eq} = x_{2f}}$

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