EE 407 Homework-2

1. A hydraulic system
2. A biochemical reactor
3. An Ecological System

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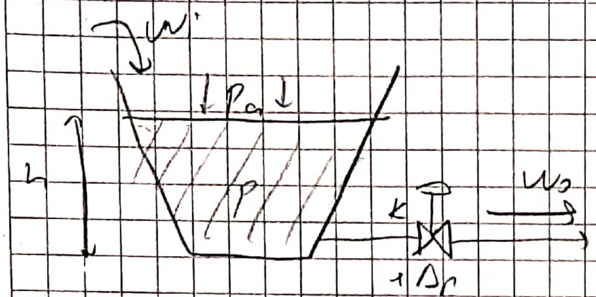
EE-407
HW-2

Question-1



$w_o = k\sqrt{\Delta p}$ k: constant

where

$\Delta p = p - p_a$

$p = p_a + \rho g h$

where ρ: liquid density

a) We assume the rate of change of cross section will be wrt $h(t)$. $(A(h))$

$$w_o = k\sqrt{p-p_a} = k\sqrt{p_a+\rho g h-p_a} = k\sqrt{\rho g h}$$

By our assumption;

$$\dot V = A(h)\cdot \dot h$$ (Rate of change of volume = Cross section when $h=h(t)$ · rate of change of h)

Also,

$$\dot V = w_i - w_o = w_i - k\sqrt{\rho g h}$$ (Accumulation (conservation) law)

$$\dot V = A(h)\cdot \dot h = w_i - k\sqrt{\rho g h}$$

$$\boxed{\dot h = \frac{w_i}{A(h)} - \frac{k\sqrt{\rho g h}}{A(h)}}$$

b) $\Delta p = \rho g h$

$h' = \dfrac{\Delta p'}{\rho g}$ $\quad h = \dfrac{\Delta p}{\rho g}$

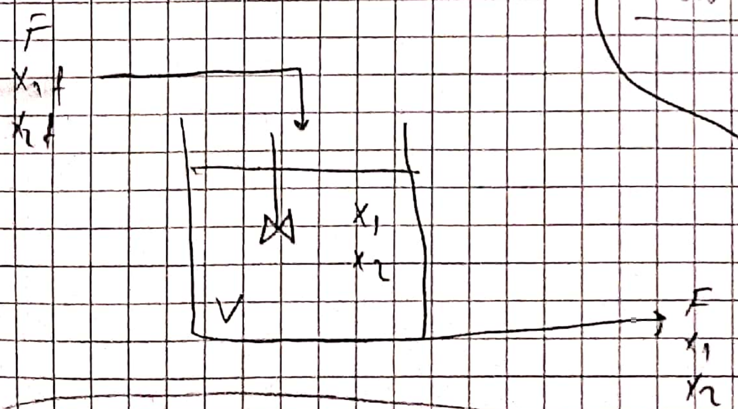
$$\boxed{\Delta p' = \frac{\rho \cdot g \cdot w_i(t)}{A(h)} - \frac{\rho g \cdot k \cdot \sqrt{\Delta p'}}{A(h)}}$$

c) output constant $\Rightarrow$ $h' = 0$, $\boxed{h = r}$ given

$$0 = \frac{w_i(t)}{\cancel{A(t)}} - \frac{k\sqrt{\rho g h}}{\cancel{A(h)}}$$

$$\boxed{W_{i\,\text{steady state}} = u_{ss} = k\sqrt{\rho g r}}$$

Question 2



Generation rate

$r_1 = \mu x_1$

μ: Specific growth coefficient of x_1

x_1: concentration of (mass/volume) substrate

Assumptions = no biomass feed stream $x_{1f} = 0$, $d = F/V \rightarrow$ constant

F: flow rate

d: dilution rate

$Y = \dfrac{r_1}{r_2} \Rightarrow$ constant

a) $\dot{x}_1 = \mu x_1 - x_1 \cdot \frac{F}{V} + x_{1f} \cdot \frac{F}{V} = (\mu - d) x_1$

d $x_{1f} = 0$

$\dot{x}_2 = -\frac{\mu x_1}{Y} - x_2 \cdot \frac{F}{V} + x_{2f} \cdot \frac{F}{V} = -\frac{\mu}{Y} x_1 - d x_2 + d x_{2f}$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} \mu - d & 0 \\ -\frac{\mu}{Y} & -d \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ d \end{bmatrix} \cdot x_{2f}$$

b) when $\mu = \frac{\mu_m \cdot x_2}{(k_m + x_2)}$ / $\mu_m, k_m > 0$

$d < \mu_m$

(1) $\dot{x}_1 = 0 = \left(\underbrace{\frac{\mu_m \cdot x_{2eq}}{k_m + x_{2eq}}}_{\mu} - d \right) x_{1eq}$

(2) $\dot{x}_2 = 0 = -\frac{1}{Y} \underbrace{\left(\frac{\mu_m \cdot x_{2eq}}{k_m + x_{2eq}} \right)}_{\mu} \cdot x_{1eq} - d x_{2eq} + d x_{2f}$

From (1)

$d \cdot k_m + d x_{2eq} = \mu_m \cdot x_{2eq}$

$$\boxed{x_{2eq} = \frac{d \cdot k_m}{(\mu_m - d)}}$$

Continued

From (2)

$$-d\,x_{2eq} - \frac{1}{Y}\left(\frac{\mu_m . x_{2eq}}{K_m + x_{2eq}}\right) . x_{1eq} + d . x_{2f} = 0$$

$$x_{1eq} = \frac{Y . d\,(x_{2f} - x_{2eq})\,(K_m + x_{2eq})}{\mu_m . x_{2eq}}$$

where $x_{2eq} = \dfrac{d . K_m}{(\mu_m - d)}$ (Found before)

$$\boxed{x_{1eq} = Y\left(x_{2f} - \frac{d . K_m}{(\mu_m - d)}\right)}$$

Or as trivial solution

$x_{1eq} = 0$

$x_{2eq} = x_{2f}$

Continued

c) Write in the form of

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} ① & ② \\ ③ & ④ \end{bmatrix} \begin{bmatrix} x_1 - x_{1eq} \\ x_2 - x_{2eq} \end{bmatrix}$$

Now, find ①, ②, ③, ④

$$\dot{x}_1 = 0 + \underbrace{\left(\frac{\mu_m x_{2eq}}{K_m + x_{2eq}} - d \right)}_{①} (x_1 - x_{1eq})$$

$$+ \underbrace{x_1 \mu_m \left(\frac{K_m}{(K_m + x_{2eq})^2} \right)}_{②} (x_2 - x_{2eq})$$

Find ①

$$\frac{\mu_m \left(\frac{d K_m}{\mu_m - d} \right)}{K_m + \frac{d K_m}{\mu_m - d}} - d = \boxed{0} \quad ①$$

d

Find ②

$$\boxed{\frac{x_1 \cdot \mu_m \cdot K_m}{\left(K_m + \frac{d K_m}{\mu_m - d} \right)^2}} \quad ②$$

To find 3, 4 use $\dot{X}_2$

$$\dot{X}_2 = 0 + \underbrace{\left(\frac{1}{Y}\cdot\frac{\mu_m X_{2eq}}{(K_m + X_{2eq})}\right)}_{3}(X_1 - X_{1eq}) +$$

$$\underbrace{\left(-d - \left(\frac{1}{Y} X_{1eq}\, \mu_m \cdot \frac{K_m}{(K_m + X_{2eq})^2}\right)\right)}_{4}(X_2 - X_{2eq})$$

Find 3

$$3 = \frac{-d}{Y}$$

Find 4

$$-d - \left(X_{2f} - \frac{d K_m}{\mu_m - d}\right)\frac{\mu_m \cdot K_m}{\left(K_m + \frac{d K_m}{\mu_m - d}\right)^2}$$

So finally we now all unknowns

M

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} X_1 - X_{1eq} \\ X_2 - X_{2eq} \end{bmatrix}$$

d) $\mu = \dfrac{\mu_m x_2}{(k_m + x_2 + k_1 x_2^2)}$

① $\dot{x}_1 = 0 = -d\, x_{1eq} + \left(\dfrac{\mu_m x_{2eq}}{(k_m + x_{2eq} + k_1 x_{2eq}^2)}\right) \cdot x_{1eq}$

② $\dot{x}_2 = 0 = d\, x_{2f} - d\, x_{2eq} - \dfrac{1}{Y} \dfrac{\mu_m x_{2eq}}{(k_m + x_{2eq} + k_1 x_{2eq}^2)} \cdot x_{1eq}$

From ①

$$x_{2eq} = \frac{\dfrac{\mu_m - d}{d} \pm \sqrt{\left(\dfrac{d - \mu_m}{d}\right)^2 - 4 k_1 k_m}}{2 k_1}$$

$$\boxed{x_{2eq} = \frac{(\mu_m - d) \pm \sqrt{d^2 + \mu_m^2 + 2 d \mu_m - 4 d^2 k_1 k_m}}{2 d k_1}}$$

From ②

$$x_1^{eq} = \frac{Y (x_{2f} - x_{2eq})(k_m + x_{2eq} + k_1 x_{2eq}^2)}{\mu_m x_{2eq}}$$

put x_{2eq}

$$\boxed{x_1^{eq} = Y \left(x_{2f} - \frac{\mu_m - d \pm \sqrt{d^2 + \mu_m^2 + 2 d \mu_m - 4 d^2 k_1 k_m}}{2 d k_1} \right)}$$

As trivial, $\boxed{x_{1eq} = 0}$ $\boxed{x_{2eq} = x_{2f}}$

3. An Ecological System

a)

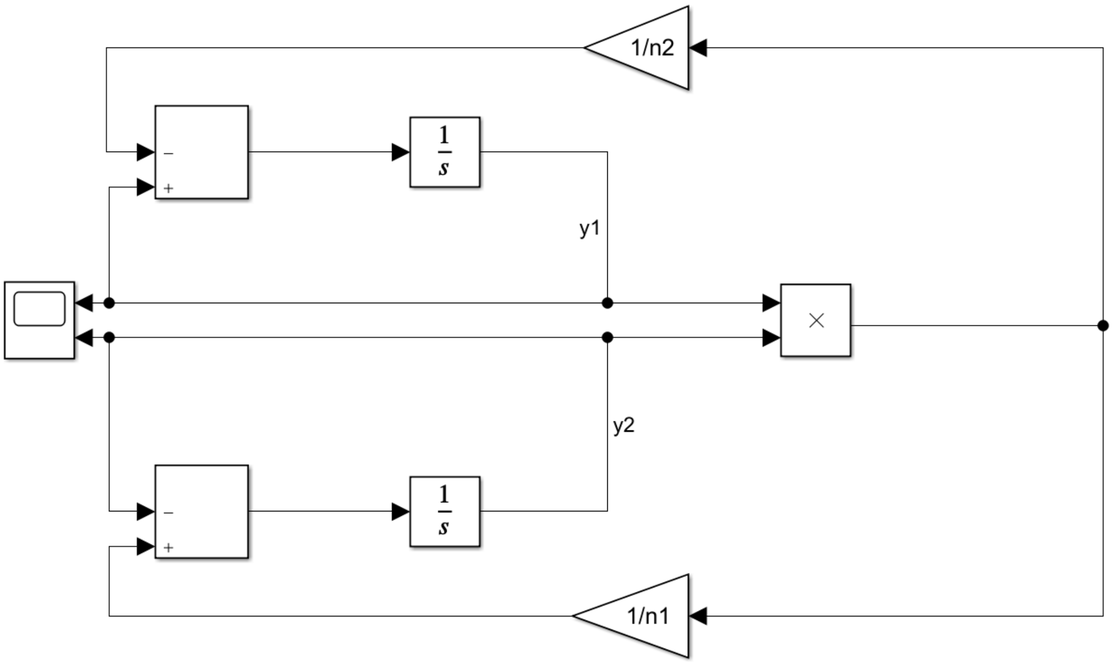


Figure.1: Prey-Predator Simulink Model

b)

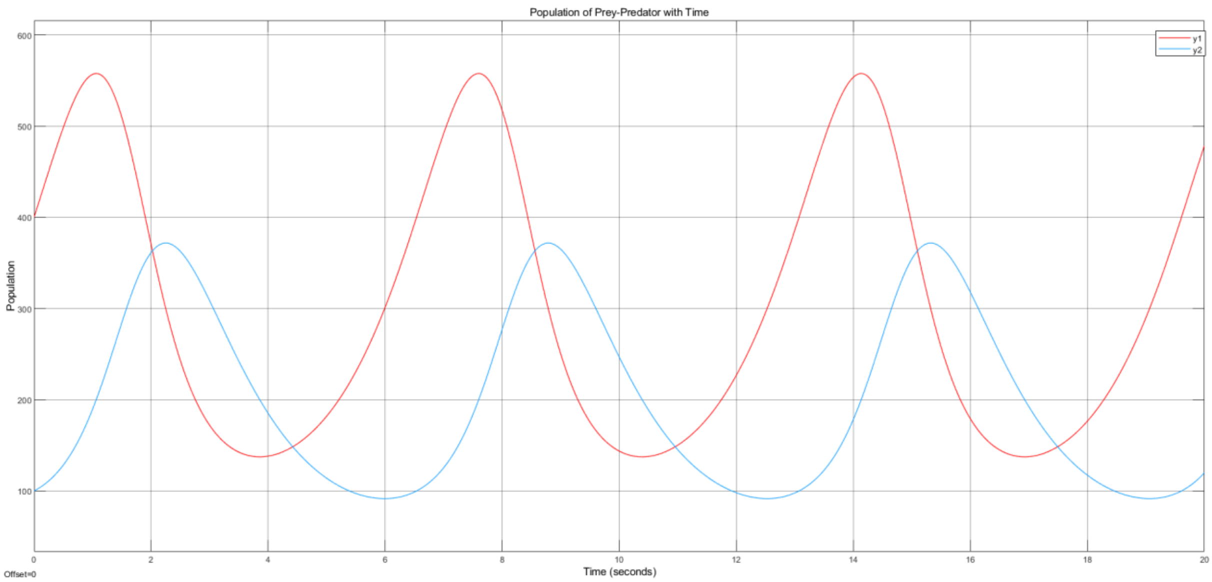


Figure.2: Prey-Predator Plot for y1,i=400, y2,i=100, n1=300, n2=200

c) Periodicity is 6.5s and both populations are increasing. They are living in harmony. Lions and plants are an example of this situation. They have similar properties.

e) Fort he euilibrium point, derivatives of them should be zero. So, y1,i=n1=300, y2,i=n2=200. They reach quilibrium and doesn't change. Oscillations stop and perioda are nearly zero and they have constant population.

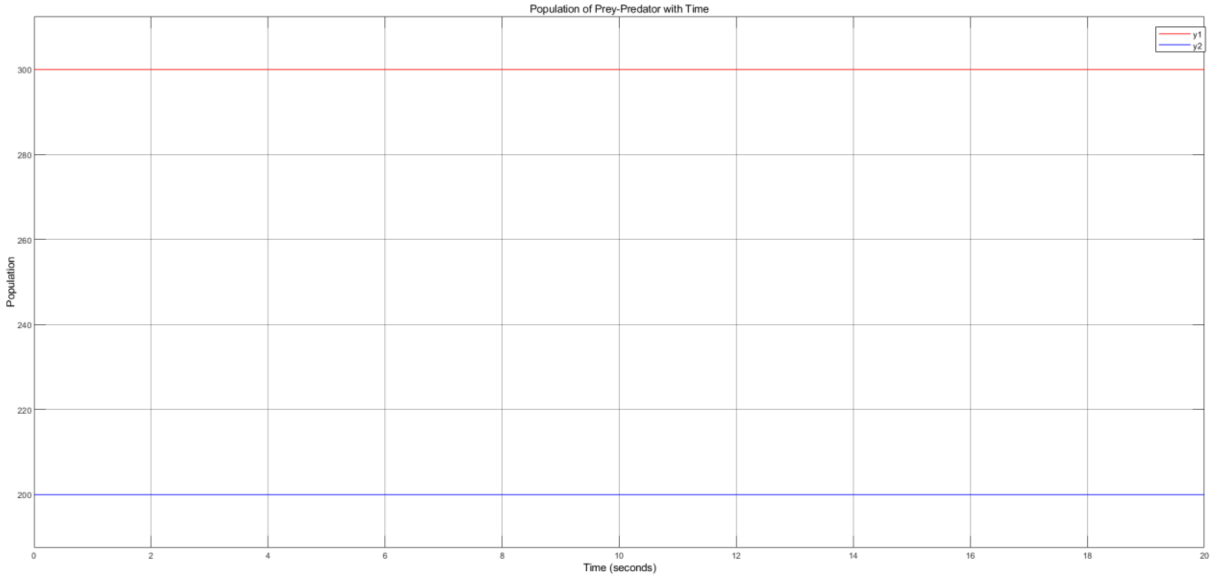


Figure.3: Prey-Predator Plot for y1,i=n1=300, y2,i=n2=200

f)

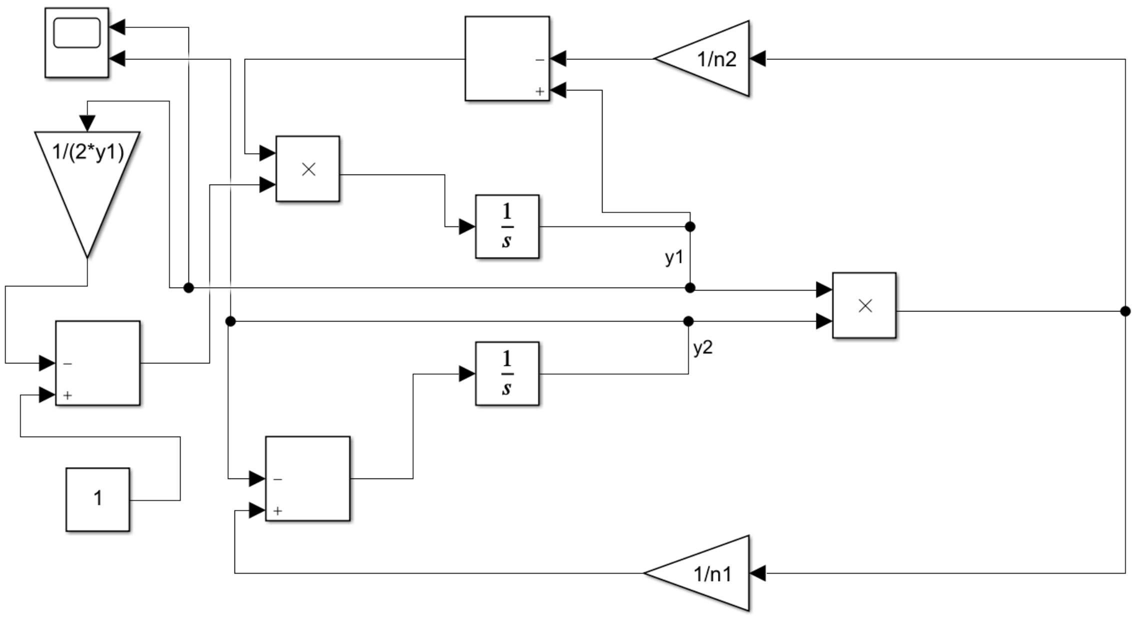


Figure.4: Prey-Predator Simulink Model for part(f)

g)

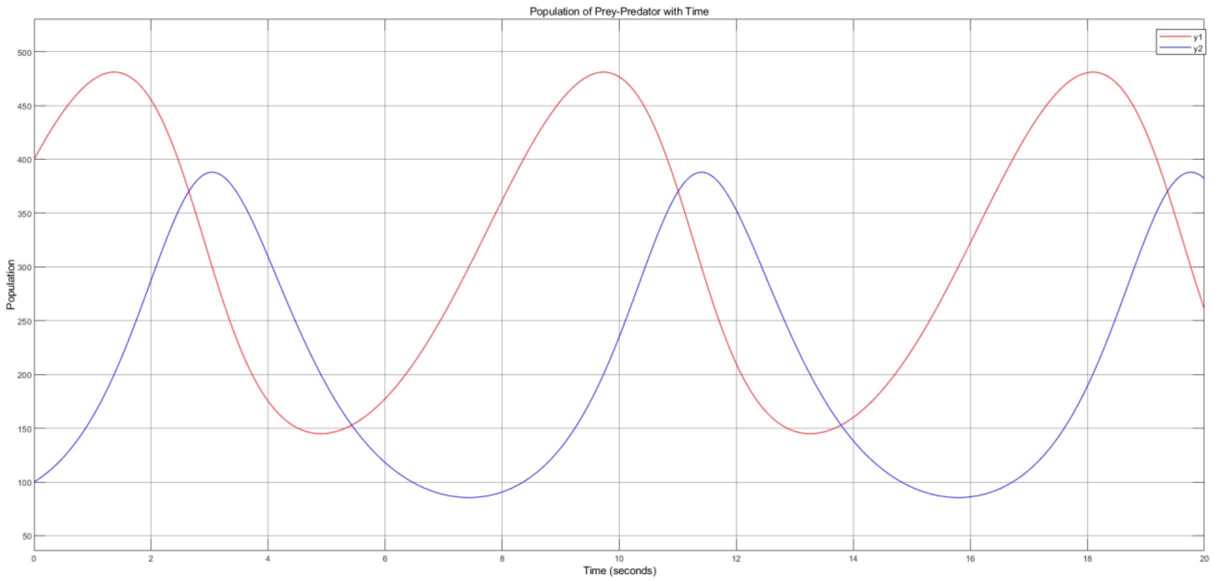


Figure.4: Prey-Predator Plot for part(g)

Oscillations happen and peak of prey decreased, mean is same. Predator is changed very little. Period are increased as nearly 8.5 since derivative of prey decreased.