Bisection Method and Algorithm for Solving The Electrical Circuits

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Abstract—- **Iteration is the process to solve a problem or defining a set of processes to called repeated with different values. The method mentioned in this survey article, we will find the roots of equations which is described. This method is called bisection. The use of this method is implemented on a electrical circuit element. The solution of the problem is only finding the real roots of the equation. In different types of applications, sometimes the real roots can not be find.In this situation, the complex roots of the equation is determined. On the other hand, the finding of the complex roots is needed to make numeric analysis. The numeric analysis is except for this article. Defined by the flow chart of the method can be present different approach for this method with using Fortran,C, Matlab programming language**

Keywords—: Iteration,Bisection Method, Fortran, C, MatLab

I. INTRODUCTION

In mechanical, electrical, construction as well as during the implementation of the different engineering disciplines which are created for the solution of the problem of mathematical modeling some of the systems are nonlinear equations, or equations [1]. Non-linear or non-linear equations are including two or more higher order polynomials equations or exponential, logarithmic, trigonometric called mathematical equations that contain terms such as non-linear. Graphical representation of the non-linear equations, linear expressions can be found. For example;

$$x^4+3x^3-2x^2+5cosx=0 \qquad (1,1)$$

or

$$x+2cotx=4e^{-x} \qquad (1,2)$$

These statements are only unknowns mathematical expressions. Nonlinear expressions,

$$f(x)=0 \qquad (1,3)$$

is defined like this. Similarly, more than one variable in the expressions,

$$f(x_1,x_2,x_{3...}) = 0 \qquad (1,4)$$

is defined as. In general mathematical equation sollutions, when n number of equations are need n number of variables. If we solve and determine n number of non linear equations, we have to use n number of variables like general mathematical equations. There is a basically the situation allocated for solving systems of linear and nonlinear systems. Meanwhile, there is only one solution for the linear mathematical equations; multiple roots of nonlinear equations, or complex roots can be obtained at the end of operations. Therefore, different numerical analysis techniques can be used to solve non-linear systems [2].

Defined as a closed loop and expressed by one variable of

(1.3) equation is defined as a reference for using bisection method. A type of iteration method which is bisection is an instrument for the determination of the roots involves the application of the system for a given range of values, are estimated [3]. Also outside the scope of this article and used as a method of repetition, are Newton, Secant, Regula-Falsa and Fixed Points (Fixed Point) studies can be used different methods of analysis [4].

II. BISECTION METHOD

In the iteration methods, bisection is used basically. The method helps to achieve the exact solution when all the parameters are correctly reconstructed. But the biggest drawback of this method is slow convergence. First step of the bisection method for solving any equations is determining a range of [a,b] is including equation's root[5]. This range is not selected by assigning random values. This range value is found with drawing a graph or mathematical algorithms to calculate of the function at the regular intervals. The steps are repeated again divided into two parts of the root (Figure 1.). The important thing in here is one root in the range. If more than one root in this range, the process can not be predicted which converge to the root.
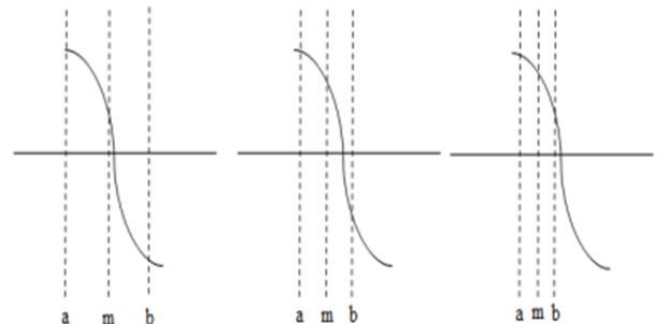
Figure 1.1. Root is in the between of a-m (the graphic on the left), Root is in the between of m-b (the graphic in the middle), Root is in the between of a-m (the graphic on the right) [Reproduced from R. Tapramaz]

III. ALGORITHMIC APPROACH FOR THE METHOD

- In the range of [a,b], for one variable and continuous function f(x), if f(a).f(b)<0, determined function in the range has a root. Because f(a) and f(b) have opposite signs.

- If f(a).f(b)=0; f(a)=0 or f(b)=0. Then, the root of equations is a or b.

- If f(a).f(b)>0; there is not ant root in the range of [a,b]. Because, f(a) and f(b) has same signs

- For reached a root in the range of [a,b], mid-point of the range is reached. This value is m=(a+b)/2. If f(a).f(m)<0; the root of the function is in the range of [a,m]. If f(a).f(m)>0, the root of the function is in the range of [m,b]. If f(a).f(m)=0; the root is 'm'

- If the root is in the range of [a,m], 'b=m' is defined. Otherwise, expressed by 'a=m'. Then, turning process is repeated with the newly defined range

- The interval will be cut into half. The repeating process will be continue until the value of the minimum specific $\in$.The value of $\in$ is expressed by $|(b-a)/2| \leqslant \in$. $\in$ is generally 10-8 or selected convergence of the solution or sensebility in the solution. So the solution has 0,00000001 sensitivity value or means to be more sensitive [6,7].

Being expected to the finding roots of equations,the procedure can not determine initially how to define a mathematical. But, logarithmic, trigonometric or exponential equations such as the overall estimate can be made. And used during the determination of the roots by means known in open methods, the roots need not be determined in a range that. Although the use of these methods are very efficient and sometimes beyond the conclusion. Open methods are, Basic Iteration, Newton Raphson and Secant Methods. Open Methods can not produce a solution for the problem cases, Graphics, Bisection and Regula-Falsi methods may be preferred [8,9].

After determining the ranges of bisection method is basically the root of the equation by reducing the root of the approach based on the principle that certain intervals range. One of the points specified in negative equity, while the other leads to positive equity. Changes the signs with using in the function. The main point of the solution, marking the midpoint of the independent variables that produce different values, by finding the value function, the result is going to aim for a new set. Process until you reach the desired precision in the range bisection process is continued [10,11]. And, this approach can be seen in Figure 1.2.

IV. AN APPLICATION FOR BISECTION METHOD

The method may be use in many areas which are mechanical , electrical and other enineering disciplines. If some types of equipments are re-examined from electrical engineers, thermistor is seen that the characteristic nonlinear equations. In this paper, we say about an example from Autar Kaw about thermistor [12].

Thermistors are temperature-measuring equipments. Thermistor material shows a change in electrical resistance with a change in temperature. By measuring the resistance of the thermistor, one can then establish the temperature. For a 10K3A Betatherm thermistor,
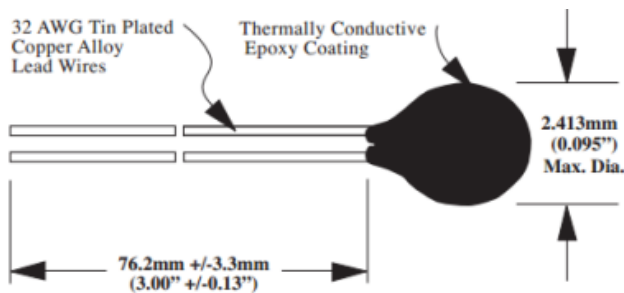

Figure 4.1. A Typical Thermistor [13]

the relationship between the resistance R of the thermistor and the temperature is given by

$$\frac{1}{T} = a + b\ln(R) + c\ln^3(R) \tag{4,1}$$

$$y = \frac{a - \frac{1}{T}}{c} \tag{4,2}$$

$$\frac{1}{T} = 1.129241 \times 10^{-3} + 2.341077 \times 10^{-4}\ln(R) + 8.775468 \times 10^{-8}\{\ln(R)\}^3 \tag{4,3}$$

where T is in Kelvin and R is in ohms.

A thermistor temperature range is no more than $\pm 0.010C$ is acceptable. To find the range of the resistance that is within this admissible limit at 190C , we need to solve,

$$\frac{1}{19.01+273.15} = 1.129241 \times 10^{-3} + 2.341077 \times 10^{-4}\ln(R) + 8.775468 \times 10^{-8}\{\ln(R)\}^3$$

And

$$\frac{1}{18.99+273.15} = 1.129241 \times 10^{-3} + 2.341077 \times 10^{-4}\ln(R) + 8.775468 \times 10^{-8}\{\ln(R)\}^3$$

We will find the roots of equations with using bisection method to determine the resistance R at 18.99 . For this solving, three iterations are conducted to estimate the root of the above equation. Absolute relative approximate error is found at the end of each iteration and the number of significant digits at least correct at the end of each iteration.

Solving,

$$\frac{1}{18.99+273.15} = 1.129241 \times 10^{-3} + 2.341077 \times 10^{-4}\ln(R) + 8.775468 \times 10^{-8}\{\ln(R)\}^3$$

At this point,

$$y = \frac{1.129241 \times 10^{-3} - \frac{1}{18.99+273.15}}{8.775468 \times 10^{-8}}$$

we get,

$$f(R) = 2.341077 \times 10^{-4}\ln(R) + 8.775468 \times 10^{-8}\{\ln(R)\}^3 - 2.293775 \times 10^{-3}$$

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Assume that,

$R_l = 11000 \quad R_u = 14000$

Check if the function changes sign between R_l and R_u

$$f(R_l) = f(11000)$$
$$= 2.341077 \times 10^{-4}\ln(11000) + 8.775468 \times 10^{-8}\ln^3(11000) - 2.293775 \times 10^{-3}$$
$$= -4.4536 \times 10^{-5}$$

$$f(R_u) = f(14000)$$
$$= 2.341077 \times 10^{-4}\ln(14000) + 8.775468 \times 10^{-8}\ln^3(14000) - 2.293775 \times 10^{-3}$$
$$= 1.7563 \times 10^{-5}$$

Hence,

$$f(R_l)\, f(R_u) = (-4.4536 \times 10^{-5}) \times (1.7563 \times 10^{-5}) < 0$$

So there is at least one root between R_l and R_u that is, between 11000 and 14000.

Iteration 1,

$$R_m = \frac{R_l + R_u}{2} \qquad (4,4)$$

$$= \frac{11000 + 14000}{2}$$

$$= 12500 \text{ ohm}$$

$$f(R_m) = f(12500)$$
$$= 2.341077 \times 10^{-4}\ln(12500) + 8.775468 \times 10^{-8}\ln^3(12500) - 2.293775 \times 10^{-3}$$
$$= -1.1655 \times 10^{-6}$$

Hence,

$$f(R_l)\, f(R_m) = (-4.4536 \times 10^{-5}) \times (-1.1655 \times 10^{-5}) > 0$$

The root is bracketed between Rm and Ru , that is, between 12500 and 14000. So, the lower and upper limits of the new bracket are

$R_l = 12500 \quad \text{and} \quad R_u = 14000$

the absolute relative approximate error | | can not be computed as we do not have a previous approximation.

Ieration 2,

$$R_m = \frac{R_l + R_u}{2}$$

$$= \frac{12500 + 14000}{2}$$

$$= 13250 \text{ ohm}$$

$$f(R_m) = f(13250)$$
$$= 2.341077 \times 10^{-4}\ln(13250) + 8.775468 \times 10^{-8}\ln^3(13250) - 2.293775 \times 10^{-3}$$
$$= 3.3599 \times 10^{-6}$$

$$f(R_l)\, f(R_m) = (-1.1655 \times 10^{-5}) \times (3.3599 \times 10^{-6}) < 0$$

Hence, the root is bracketed between Rl and Rm , that is, between 12500 and 13250. So the lower and upper limits of the new bracket are

$R_l = 12500 \quad R_u = 13250$

The absolute relative approximate error | | at the end of Iteration 2 is

$$|\epsilon| = \left| \frac{mnew - mold}{mnew} \right| \times 100 \qquad (4,5)$$

$$= \left| \frac{13250 - 12500}{13250} \right| \times 100$$

$$= 5.6604\%$$

None of the significant digits are at least correct in the estimated root

$R_m = 13250$

as the absolute relative approximate error is greater than 5%.

Iteration 3,

$$R_m = \frac{R_l + R_u}{2}$$

$$= \frac{12500 + 13250}{2}$$

$$= 12875 \text{ ohm}$$

$$f(R_m) = f(12875)$$
$$= 2.341077 \times 10^{-4}\ln(12875) + 8.775468 \times 10^{-8}\ln^3(12875) - 2.293775 \times 10^{-3}$$
$$= -4.0403 \times 10^{-6}$$

$$f(R_l)\, f(R_m) = (-1.1655 \times 10^{-5}) \times (-4.0403 \times 10^{-6}) > 0$$

Hence, the root is bracketed between Rm and Ru , that is, between 12875 and 13250. So, the lower and upper limits of the new bracket are

$R_l = 12875 \quad \text{and} \quad R_u = 13250$

The absolute relative approximate error | | at the end of Iteration 3 is,

$$|\epsilon| = \left| \frac{mnew - mold}{mnew} \right| \times 100$$

$$= \left| \frac{12875 - 13250}{12875} \right| \times 100$$

$$= 2.9126\%$$

One of the significant digits is at least correct in the estimated root of the equation as the absolute relative approximate error is less than 5%.

Seven more iterations were conducted and these iterations are shown in the Table 1.

Table 1. Root of as a function of the number ofiterations for bisection method. [Reproduced from A. Kaw]

Iteration	R_ℓ	R_u	R_m	$|\epsilon_a|\%$	$f(R_m)$
1	11000	14000	12500	----------	1.1655×10^{-5}
2	12500	14000	13250	5.6604	3.3599×10^{-6}
3	12500	13250	12875	2.9126	-4.0403×10^{-6}
4	12875	13250	13063	1.4354	-3.1417×10^{-7}
5	13063	13250	13156	0.71259	1.5293×10^{-6}
6	13063	13156	13109	0.35757	6.0917×10^{-7}
7	13063	13109	13086	0.17910	1.4791×10^{-7}
8	13063	13086	13074	0.089633	-8.3022×10^{-8}
9	13074	13086	13080	0.044796	3.2470×10^{-8}
10	13074	13080	13077	0.022403	-2.5270×10^{-8}

At the end of the 10th iteration,

$$|\epsilon_a| = 0.022403\%$$

Hence the number of significant digits at least correct is given by the largest value of for which

$$|\epsilon_a| \leq 0.5 \times 10^{2-m}$$
$$0.022403 \leq 0.5 \times 10^{2-m}$$
$$0.044806 \leq 10^{2-m}$$
$$\log(0.044806) \leq 2-m$$
$$m \leq 2 - \log(0.044806) = 3.3487$$

So,

m=3

The number of significant digisat least correct in the estimated root 13077 is 3.

V. CONCLUSION

The initial value is incorrect repetitions faulty condition is selected, so the root will be reflected in the process of obtaining. Function which will be in the process of settlement of the range set for the number of repetitions is not connected, i.e the sensitivity of the value depends on the sensitivity of the equation. Convergence of bisection method is slower than other methods, and the number of repetitions are more than the other methods. The biggest drawback is that the number of repetitions compared to other methods. Be determined for the maximum number of repetitions by the person who analyzes the problem [7,14].

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Figure 1.2. The Flow Chart of the Algorithm