Orbits in Barred Galaxies:

Knowledge of the stellar orbits in a barred galaxy can help us to understand the dynamical properties of bars, their extension, and pattern velocity. It is also fundamental to understanding resonances. Bars are three-dimensional components, and they can be modeled as triaxial ellipsoids. Although the disks of spiral galaxies are very thin components, with axis ratios around 10, bars can be thicker because of vertical resonances, and reveal box or peanut shapes when seen edge-on. In a first approach, I will describe the stellar orbits as if they were confined in the galaxy plane. This much simpler approach gives already most of the characteristics of bar dynamics. the characteristics of orbits in an axisymmetric potential $\Phi(r)$ in the plane $z = 0$. A circular orbit has an angular velocity $\omega^2 = 1 / r\, d\phi / dr$. In linearizing the potential in the neighborhood of a circular orbit, the motion of any particle can be expressed in first order by an epicyclic oscillation, of frequency κ,

$$\kappa^2 = d^2\Phi / dr^2 + 3\omega^2 = r\, d\omega^2 / dr + 4\omega^2$$

The bar creates a bisymmetric gravitational potential, which rotates in the galaxy with the pattern speed ω_b. To be left with a potential independent of time, where the energy of particles is conserved, we must consider the orbits in the rotating frame. The equivalent potential in this frame is then:

$$\Phi_{eq} = \Phi(r, \theta) - 1/2\, \omega_b^2 r^2$$

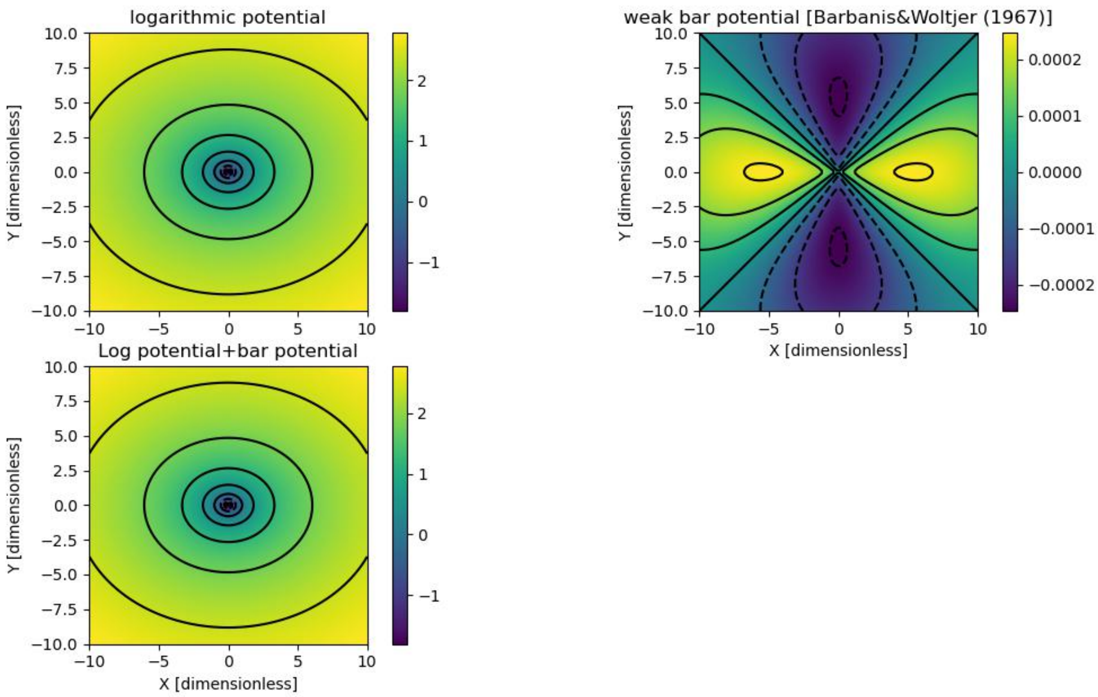
in cylindrical coordinates (r, θ, z), for $z = 0$. Here I also add the bar potential from the paper of Barbaris-Woltjer (1967) i.e.,

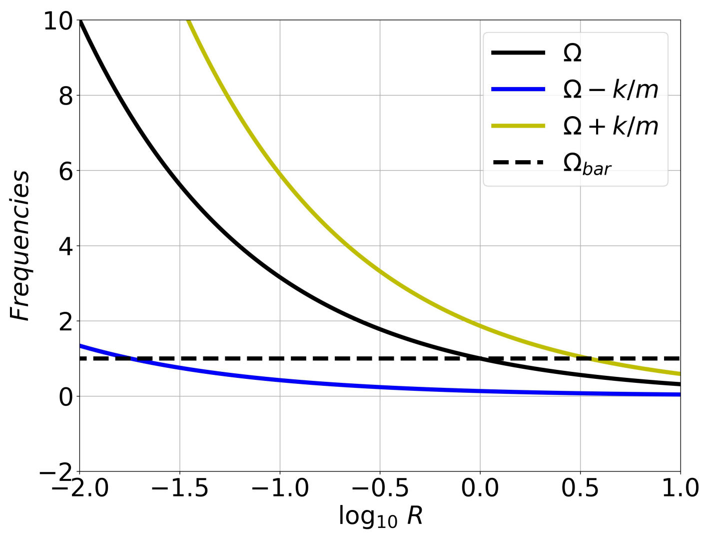
$$\Phi_{\text{Bar pot}} = \xi\, r^{1/2}(16-r) \cos 2\theta$$

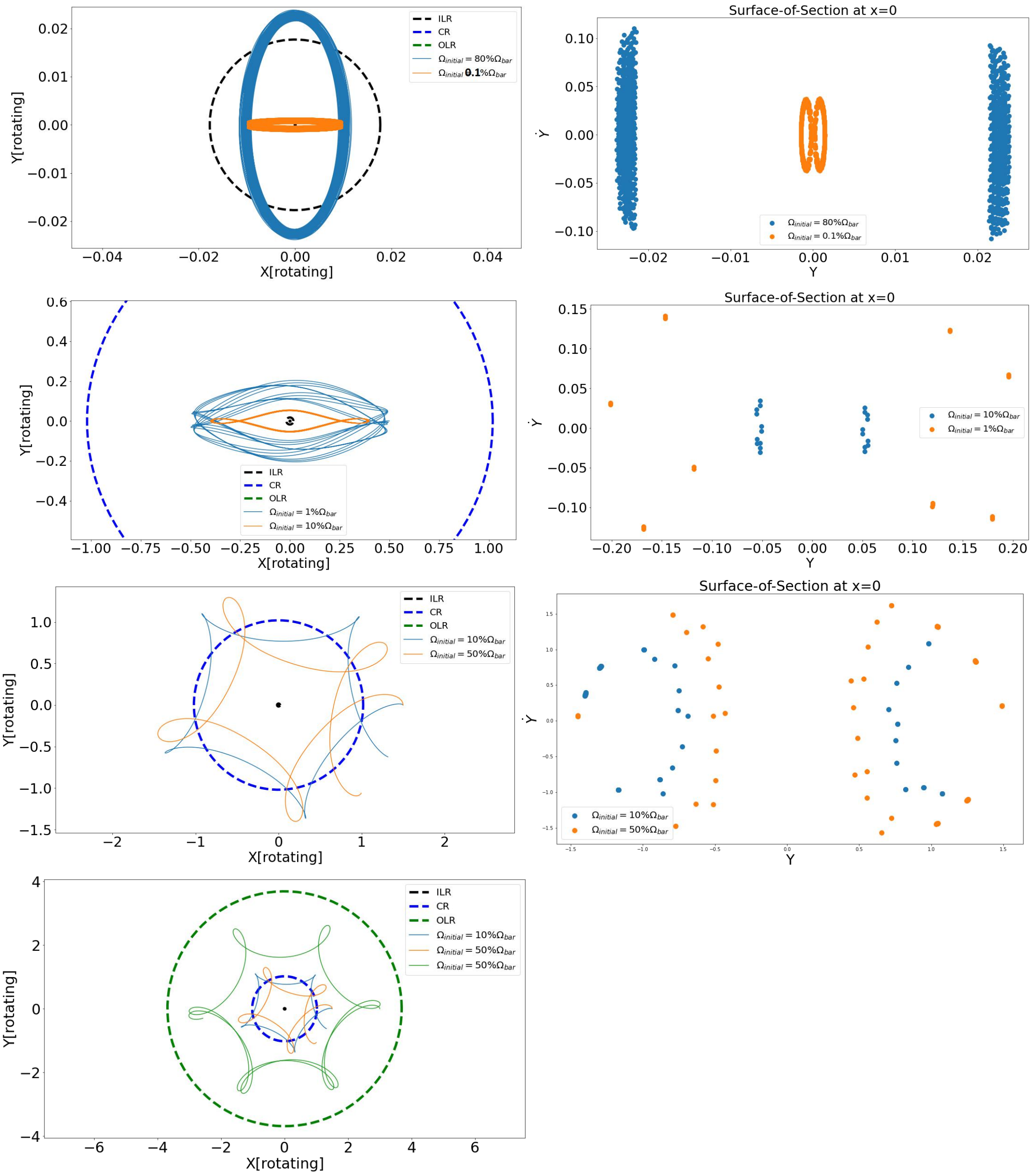
So the final potential is the sum of the logarithmic potential, bar potential and centrifugal potential. It is illustrated in below potential map. Bar potential is very weak that has negligible effect on the log potential.

In the rotating frame, the effective angular velocity of a particle is $\omega' = \omega - \omega_b$. There exists then regions in the galaxy where $\omega' = \kappa / m$, i.e. where the epicyclic orbits close themselves after m lobes. The corresponding stars are aligned with the perturbation and closely follow it; they interact with it always with the same sign, and resonate with it. These zones are the Lindblad resonances, sketched with dashed circles in Figures below. And because the bar is a bisymmetric perturbation, the most important resonances are those for $m = 2$. An obvious resonance is corotation, for which $\omega = \omega_b$ ($\omega' = 0$).

The periodic orbits are numerous are: the x_1 family is the main family supporting the bar. Orbits are elongated parallel to the bar, within corotation. They can look like simple ellipses, and with energy increasing, they can form a cusp, and even two loops at the extremities. the x_2 family exists only between the two inner Lindblad resonances (ILR), when they exist. They are more round, and elongated perpendicular to the bar. There are also the x_4 retrograde orbits, and the x_3 unstable orbits, but with less impact for galaxies. When getting towards corotation (CR) all higher order resonances $\omega' = \kappa / m$ with $m \geq 3$ are encountered. Spanning this region, on the inner or outer side of CR, the periodic families 6 / 1 and 4 / 1 are displayed.


logarithmic potential
weak bar potential [Barbanis&Woltjer (1967)]
Log potential+bar potential
Y [dimensionless]
X [dimensionless]


Ω
$\Omega - k/m$
$\Omega + k/m$
Ω_{bar}
Frequencies
$\log_{10} R$



ILR
CR
OLR
$\Omega_{initial} = 80\%\Omega_{bar}$
$\Omega_{initial}$ 0.1$\%\Omega_{bar}$
X[rotating]
Y[rotating]
Surface-of-Section at x=0
$\Omega_{initial} = 80\%\Omega_{bar}$
$\Omega_{initial} = 0.1\%\Omega_{bar}$
Y
$\dot{Y}$
ILR
CR
OLR
$\Omega_{initial} = 1\%\Omega_{bar}$
$\Omega_{initial} = 10\%\Omega_{bar}$
X[rotating]
Y[rotating]
Surface-of-Section at x=0
$\Omega_{initial} = 10\%\Omega_{bar}$
$\Omega_{initial} = 1\%\Omega_{bar}$
Y
$\dot{Y}$
ILR
CR
OLR
$\Omega_{initial} = 10\%\Omega_{bar}$
$\Omega_{initial} = 50\%\Omega_{bar}$
X[rotating]
Y[rotating]
Surface-of-Section at x=0
$\Omega_{initial} = 10\%\Omega_{bar}$
$\Omega_{initial} = 50\%\Omega_{bar}$
Y
$\dot{Y}$
ILR
CR
OLR
$\Omega_{initial} = 10\%\Omega_{bar}$
$\Omega_{initial} = 50\%\Omega_{bar}$
$\Omega_{initial} = 50\%\Omega_{bar}$
X[rotating]
Y[rotating]