Gravitational Potential of Disks and their Orbits:

In my earlier attempt, I have seen some spherical mass distribution cases, some are a) Keplerian Potential, b) Spherical Harmonic Potential, c) Isochrone Potential (modification of keplerian), in which the orbits are rather simple with the conservation in angular momentum that makes the orbit mostly closed. In this attempt, we will see different potentials which are mostly used to defined the galaxies.

1) Razor-thin Disk: The Kuzmin Model

A simple flattened axisymmetric potential is given as,

$$\phi(R, z) = -\frac{GM}{\sqrt{R^2 + (|z| + a)^2}}$$

With a > 0 is the parameter that defined the altitude along z-component. This potential resembles the keplerian potential (point mass) located at (R,z) for any z≠0. MilkyWay follows the same behavior with the ratio of scale length to scale height is (R/z ~ 10). Figure 1 is the Razor-thin potential on x,y,z planes and the orbit based on arbitrary values. Since I gave positive values to potential, therefore larger values represents the stronger potential region. Here we can noticed from z-component, that point-mass moved toward (a).

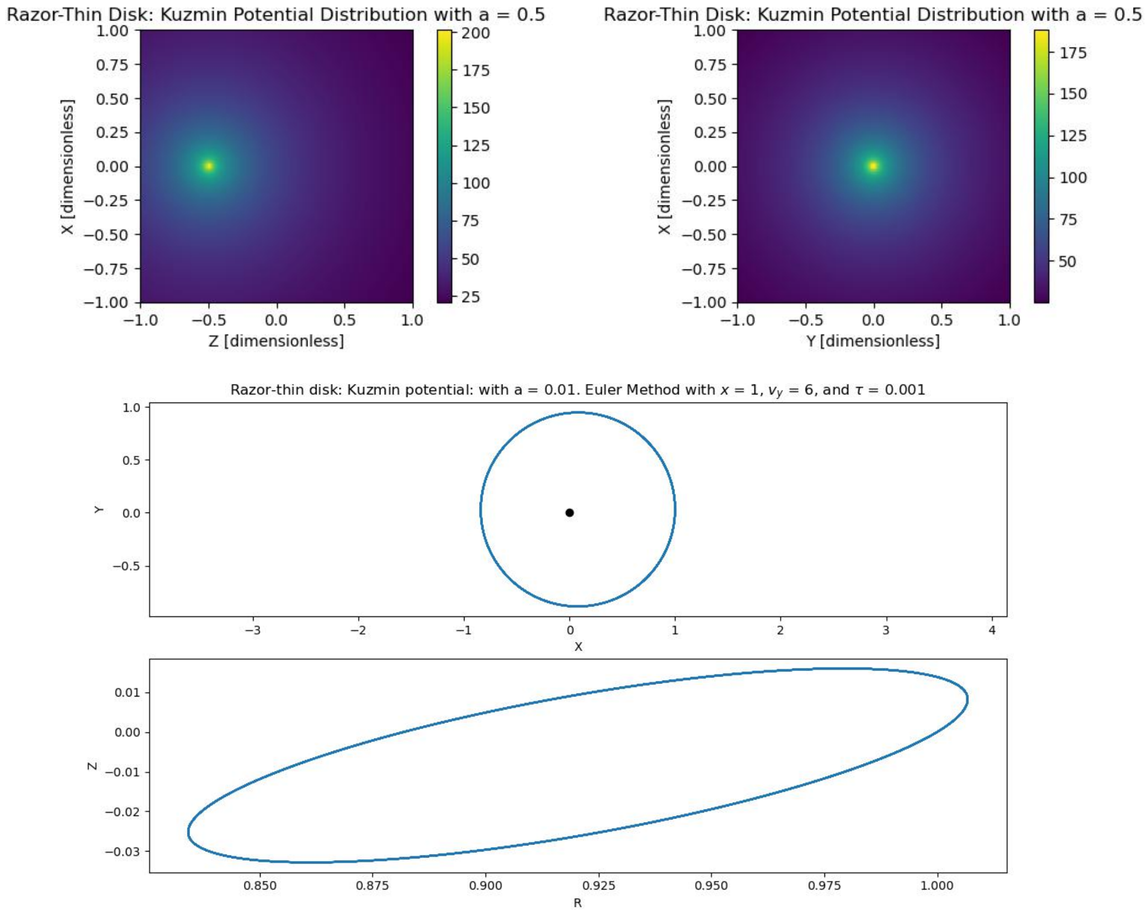


Figure 1: Potential and Orbit of Razor Thin Disk Model.

2) Thickened Disk: Miyamoto Nagai Model:

Thick disk can be obtained from the thin-disk model just by replacing $|z| \rightarrow \sqrt{z^2 + b^2}$, b is a new parameter that upgrade potential as,

$$\phi(R, z) = -\frac{GM}{\sqrt{R^2 + (\sqrt{z^2 + b^2} + a)^2}}$$

Given potential includes the Plummer model if a = 0 or the Kuzmin model for b = 0. Thus it describe the wide range of shapes in which the ratio b/a shows the flattening of mass distribution. b/a >> 1 corresponds to the spherical distribution while b/a << 1 shows the significant flattened distribution.

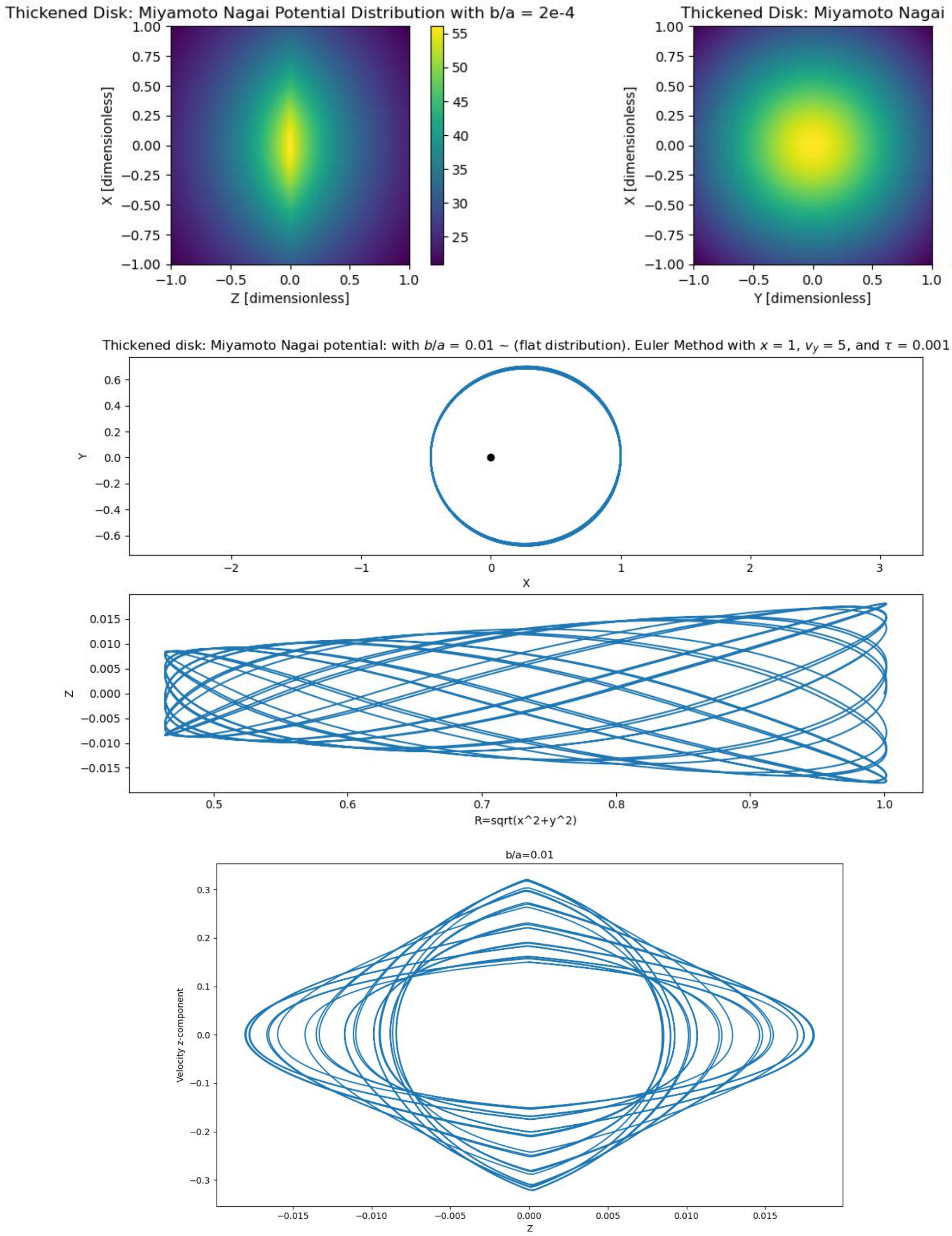


Figure 2: Thick-Disk: Miyamoto Nagai Potential

The limitation of Thick-Model comes at large radii that is quite different than observed galactic disk. Since the density distribution is the exponential relation with radii (ρ α exp(-R)), however for Thick-model the density decreases as a power-law (ρ α $1/R^3$).

3) Logarithmic Potential:

The idea of logarithmic potential is to include the constant circular velocity parameter to the potential that is responsible for the flat rotation curve that many galaxies are observed to have.

$$\phi(R,z) = \frac{v_c^2}{2} \ln\left(R^2 + \frac{z^2}{q^2}\right)$$

Here the parameter q defined how flattening the potential is, q varies between 0.1-0.9 in which smaller means more flattening and vice verse for large q. The problem arises with the density distribution at R~0, where the density is ‘dimpled’ at the poles, if $q^2 < 1/2$ the density becomes negative that is non-physical. Nevertheless, this model is still useful for the large radii where the ‘dimpling effect’ is unnecessary.

For disk potential, rotation curve can be computed in the midplane (z=0) by balancing the centripetal acceleration with the radial force as,

$$a_R(R) = v_c^2/R = -F_R(R, z=0) = \frac{\partial \phi(R, z=0)}{\partial R}$$

Using this relation, we can estimate different rotation curves by varying potential accordingly. For logarithmic potential, rotation speed is independent of radii and remain constant.

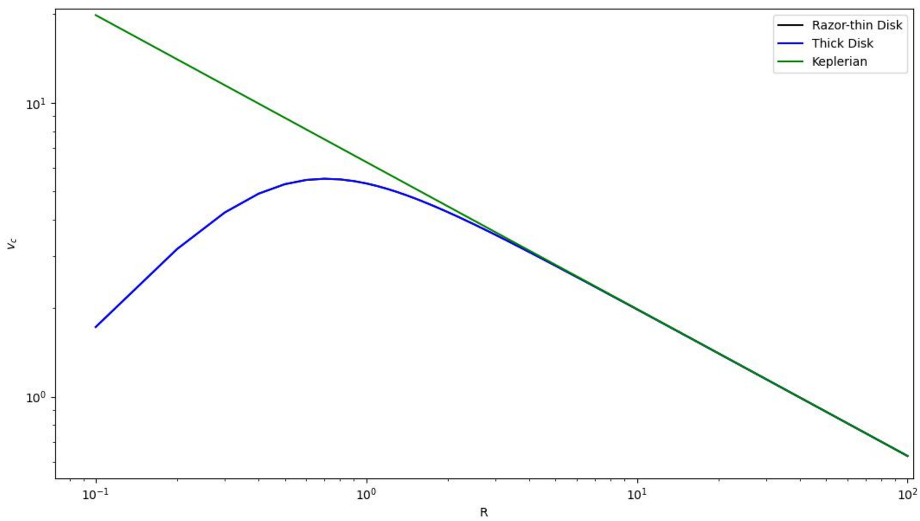
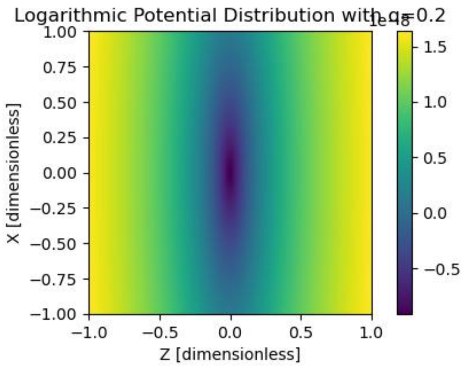
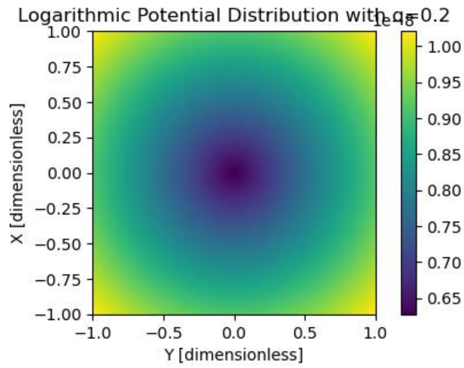
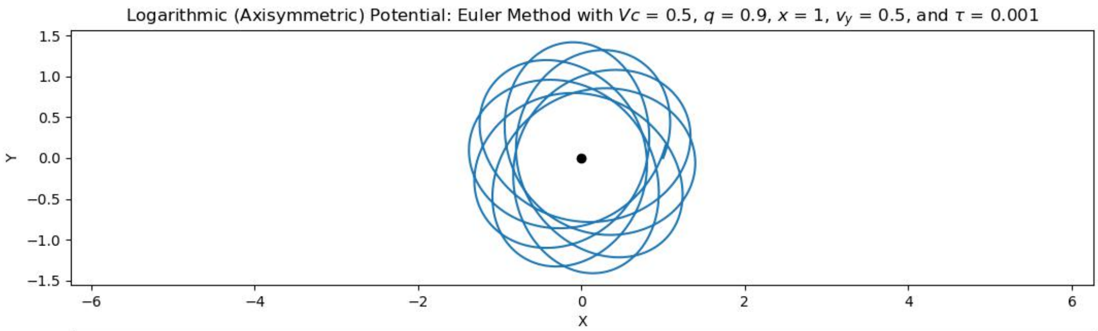


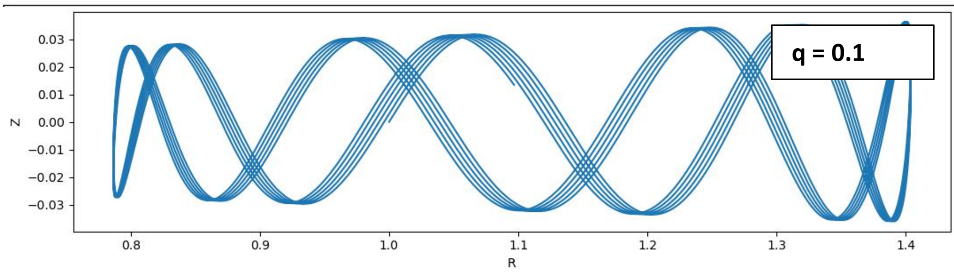
Figure 3: Rotation curve of Keplerian, Razor thin and thick disk models

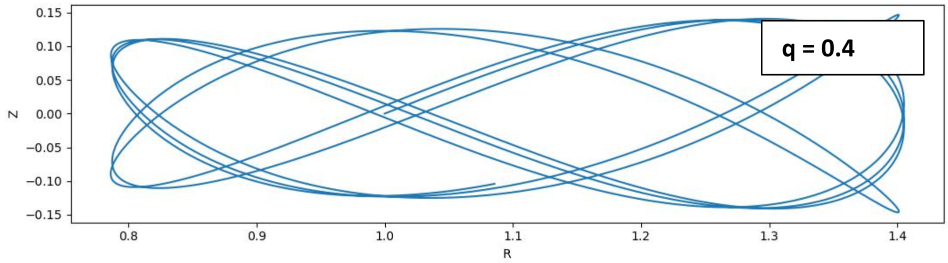
Query 1: All the potentials and their resulted orbits are very sensitive to their inputs, especially the initial velocities. So, how do we know which orbits are correct?

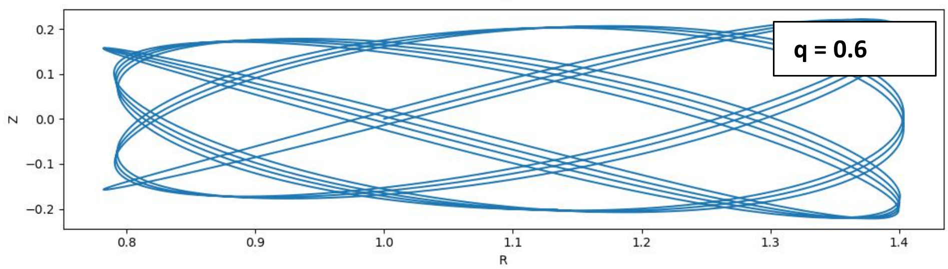

Logarithmic Potential Distribution with q=0.2
X [dimensionless]
Z [dimensionless]

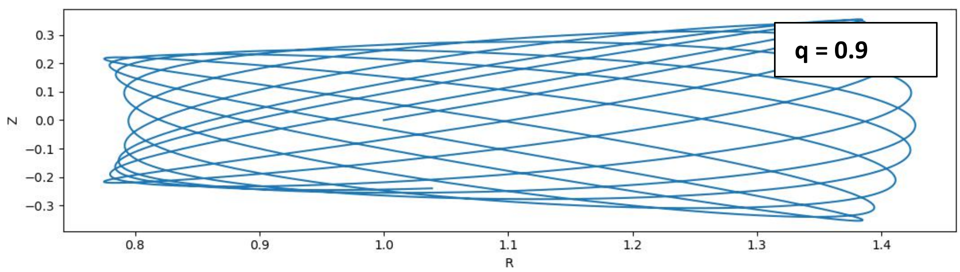

Logarithmic Potential Distribution with q=0.2
X [dimensionless]
Y [dimensionless]


Logarithmic (Axisymmetric) Potential: Euler Method with Vc = 0.5, q = 0.9, x = 1, v_y = 0.5, and τ = 0.001
Y
X


q = 0.1
Z
R


q = 0.4
Z
R


q = 0.6
Z
R


q = 0.9
Z
R