Recollection of Discrete Mathematics Test

6/27/2023

1 True or False

1. 1+1=2 only if $\emptyset \in \emptyset$.
2. The Hasse diagram of $\{1, 2, 3, 4, 5, 7, 8, 9 \mid \mid\}$is a 4-ary tree.
3. A complete graph can be transformed into a disconnected graph by removing a single vertex.
4. At least two people share the same number of hair in Hangzhou.
5. The relation on set $\emptyset$ is an equivalent relation.
6. Minimal spanning tree can be constructed by chooseing the n-1 edge with the least weight.
7. If $f(1) = 1, f(n) = 2f\left(\lfloor \frac{n}{2} \rfloor\right)$, then $f(n) \leq n$.
8. n and k are positive integers, $\lceil \frac{n}{k} \rceil = \lfloor \frac{n-1}{k} \rfloor + 1$.
9. $D_n = (n-1)(D_{n-1} - D_{n-2})$ is a linear homogeneous constant coefficient recurrence equation.
10. Oh, I forgot it. (lol

2 Multiple Choice

One option for each question.

1. Choose the false one.

   A.$\forall x(P(x) \bigwedge Q(x)) \equiv \forall x P(x) \bigwedge \forall x Q(x)$
   B.$A- > \forall x P(x) \equiv \forall x(A- > P(x))$
   C.$\exists x(P(x) \bigwedge Q(x)) \equiv \exists x P(x) \bigwedge \exists x Q(x)$
   D.forgotten

2. Choose the equivalent relation: x and y are English words.
   A.x and y share at least one letter.
   B.x and y have the same starting letter.
   C.
   D.

3. $0\lambda \in S, \lambda 1 \in S$ count the number of strings of length 5. (Need to be clearfied by others!)

4. Something about recursion, easy.

5. Counting order pair $\{(A, B)|A \subseteq B\}$ on set S.

6. Find the shortest path (using Dijkstra).

7. (Several questions are lost.)

3 Short Answer

1. extend(1,2),(3,3),(4,1) into equivalent relation.

2. $7^{222} \equiv x(mod 11)$. x is ____.

3. Put 4 candies into a 4*4 grid,no two in the same row or column, count the number of ways.

4. $29x \equiv 1(mod 37)$, find x, the smallest positive integer satisfies the equation.

5. Postorder traversal.
   (It jsut looks like $*15 - 2/xy \rightarrow +36$ or something.)

6. (Several questions are lost.)

4 Long Answer

1. 3-ary string with no consecutive 0s.
   (The string only contains 0,1,2)
   (Actually this question comes from our Book Page 537,exercise 13,version 8)

   (a) Recurrence relation and initial condition.

   (b) Solve using generating function.

   (c) Solve using characteristic equation.

2. Proof an equivalence relation.
   (The general train of thought to solve this problem is: When P is reflexive, we can conclude that Q is reflexive. The same as symmatric and transitive.)

3. Huffman Tree and calculate the average length.
   (The question will give you the information on the frequency of occurence of about 7 letters ABCDEFG. For example: A:0.03 B:0.10 C:0.06 etc. After obtaining the Huffman tree, calculate the average number of bits. Almost the same as the book page 806,question 23,version 8.)

4. Graph problem

   (a) Count the paths.

   (b) Connectivity.

   (c) Hamiltonian or not.

   (d) Chromatic number using backtracking.

   (e) BFS tree.