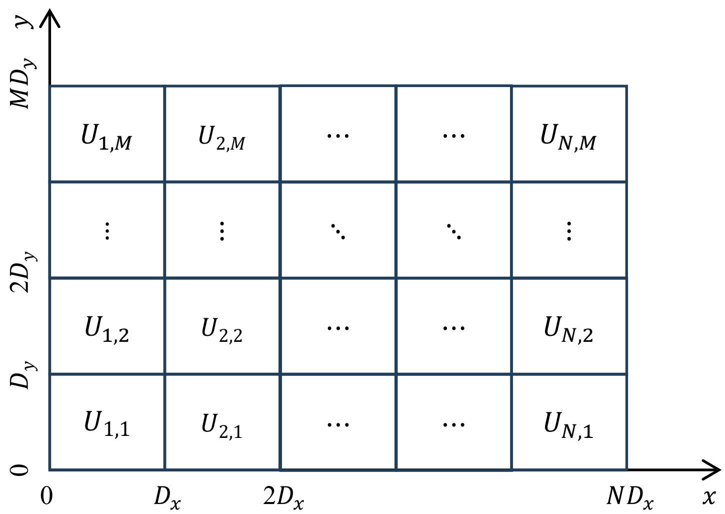
Definition of centroid

For a 2D rectangle $C = [0, L_x] \times [0, L_y] \in \mathbb{R}^2$ where the mass density function is $f(x, y)$ for $(x, y) \in C$. The centroid of C is defined as the coordinate (x_c, y_c) where

$$x_c = \frac{\int_0^{L_y} \int_0^{L_x} x f(x,y)\,dxdy}{\int_0^{L_y} \int_0^{L_x} f(x,y)\,dxdy}, \qquad y_c = \frac{\int_0^{L_y} \int_0^{L_x} y f(x,y)\,dxdy}{\int_0^{L_y} \int_0^{L_x} f(x,y)\,dxdy}$$

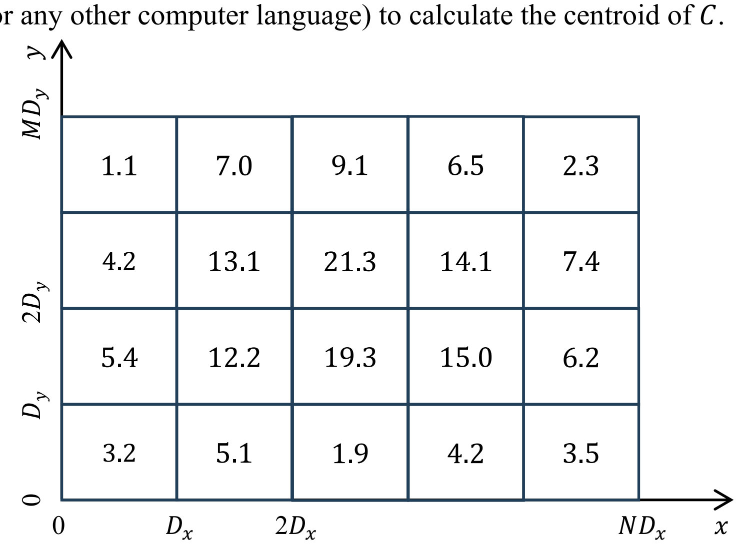
Problem 1

As shown in the figure, suppose C is partitioned into smaller rectangles (called pixels) $C_{n,m} = [(n-1)D_x, nD_x] \times [(m-1)D_y, mD_y]$ for $n = 1, 2, \ldots, N$ and $m = 1, 2, \ldots, M$. That is, there are a total of $N \times M$ pixels and they have the same size of $D_x \times D_y$. The mass over each pixel is uniformly distributed, that is, the density function $f(x, y)$ on each pixel is a constant but unknown. However, the total mass on the (n, m)th pixel is known and is equal to $U_{n,m}$ for $n = 1, 2, \ldots, N$ and $m = 1, 2, \ldots, M$. From the above definition of centroid, derive the centroid formula of C in terms of $U_{n,m}$, D_x, D_y, N and M.



Problem 2

As shown in the figure below, suppose $D_x = 100$, $D_y = 120$, $N = 5$, $M = 4$, and the mass of each pixel $U_{n,m}$ is given in the figure below. Based on your formula in Problem 1, make your program on MATLAB (or any other computer language) to calculate the centroid of C.



Submit your solution to Problem 1 and programing codes with the calculated centroid in Problem 2.