Матанализ. Семинар.

Nikita Muzykantov

26 октября 2017 г.

$$\lim_{x\to 0}\frac{a^x-1}{x}=\lim_{\log_a(1+y)\to 0}\frac{y}{log_a(1+y)}=\lim_{y\to 0}\frac{y\ln a}{\ln(y+1)}=\ln a$$

$$\lim_{x\to 0}\frac{a^x-b^x}{x}=\lim_{x\to 0}\left(\frac{a^x-1}{x}-\frac{b^x-1}{x}\right)=\ln a-\ln b=\ln\frac{a}{b}$$

$$\lim_{x\to 1}\frac{x^x-1}{x-1}=\lim_{x\to 1}\left(\frac{e^{x\ln x}}{x-1}\times\frac{x\ln x}{x\ln x}\right)=\lim_{x\to 1}\left(\frac{e^{x\ln x}}{x\ln x}\times\frac{x\ln x}{x-1}\right)$$

Замена: $x=1+y$

$$\lim_{x\to 1}\frac{\sin\pi x^\alpha}{\sin\pi x^\beta}=\lim_{x\to 1}\frac{-\sin\pi(x^\alpha-1)}{-\sin\pi(x^\beta-1)}=\lim_{x\to 1}\frac{x^\alpha-1}{x^\beta-1}=$$

$$=\lim_{x\to 1}\frac{(x-1)(x^\alpha-1)+(x^\alpha-2)+\cdots+1}{(x-1)(x^\beta-1)+(x^\beta-2)+\cdots+1}=\frac{\alpha}{\beta}$$

1 Задачи на выделение главного члена

1.Дано: $x\to 1; x^3-3x+2; c(x-1)^n$. Пусть $x=1+y$

$$\lim_{x\to 1}\frac{x^3-3x+2}{(x-1)^n}=\lim_{y\to 0}\frac{1+3y+3y^2+y^3-3y-3+2}{y^n}=lim_{y\to 0}\frac{y^3+3y^2}{y^n}=$$

$$lim_{y\to 0}\frac{y+3}{y^{n-2}}=3$$

$$x^3-3x+2=3(x-1)^2+O((x-1)^3)$$

2. Дано: $x \to 1$; $\sqrt[3]{1-\sqrt{x}}; c(x-1)^n$.

$$\lim_{x\to 1}\frac{\sqrt[3]{1-\sqrt{x}}}{(x-1)^n}=\lim_{y\to 0}\sqrt[3]{\frac{(1-sqrt(x))(1+sqrt(x))}{(x-1)^{3n}(1+sqrt(x))}}=lim_{y\to 0}\frac{y^3+3y^2}{y^n}=$$

$$=lim_{x\to 1}\frac{-1}{\sqrt[3]{}}$$

3.Дано: $x \to 1; \ln x; c(x-1)^n$.

$$\lim_{x\to 1}\frac{\ln x}{(x-1)^n}=\lim_{y\to 0}\frac{\ln(1+y)}{y^n}=c=1$$

4. Дано: $x \to 1; e^x-e; c(x-1)^n$.

$$\lim_{x\to 1}\frac{e^x-e}{(x-1)^n}=\lim_{y\to 0}\frac{e^{1+y}-e}{y^n}=\lim_{y\to 0}e\frac{e^y-1}{y^n}$$

Ответ: $e^x-e=e(x-1)+o(x-1)$

5.

$$\lim_{n\to\infty}\left(\frac{6n}{3n^2-1}\sin(n^2)+\frac{sqrt(n)}{sqrt(n+1)+sqrt(n)}\right)=$$

$$=\lim_{n\to\infty}\dots$$

6.

$$\lim_{x\to 0}\left(\frac{1+x\times 2^x}{1+x\times 3^x}\right)^{\frac{1}{x^2}}=$$

$$=\lim_{x\to 0}\left(1+\frac{x(2^x-3^x)}{1+x\times 3^x}\right)^{\frac{1}{x^2}}=\lim_{x\to 0}\left(1+\frac{x(2^x-3^x)(1+\ln 2-1-x\ln 3)}{(1+x\times 3^x)(1+\ln 2-1-x\ln 3)}\right)^{\frac{1}{x^2}}=$$

$$=\lim_{x\to 0}\left(1+\frac{x^2(2^x-3^x)\ln\frac{2}{3}}{(x\ln\frac{2}{3}(1+x\times 3^x))}\right)^{\frac{1}{x^2}\times\frac{\ln\frac{2}{3}}{\ln\frac{2}{3}}}=e^{\ln\frac{2}{3}}=\frac{2}{3}$$

Т.к. $\frac{a^x-1}{x}\to_{x\to 0}\ln a$

7.

$$\lim_{x\to\frac{\pi}{4}}\frac{\sqrt[3]{\operatorname{tg} x}-1}{\log_{\frac{\pi}{4}}-1}=$$

$$=\lim_{y\to 0}\ln\frac{\pi}{4}\frac{\frac{4y}{\pi}}{\ln(\frac{4y}{\pi}+1)\times\frac{4y}{\pi}}\times\frac{\left(\sqrt[3]{\frac{2\sin y}{\cos y-\sin x}}-1\right)}{den}$$

8. $\lim f(x)$

$x\to +-0$