Name ____________________________

Midterm Exam

Intertemporal Choice

Fall, 2015

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Part I

Consumption, Saving, and the Great Recession.

In the U.S. during the Great Recession, household consumption expenditures fell even more than household income, leading to a sharp rise in the saving rate. This question explores potential explanations within the context of the partial equilibrium/small-open-economy model described in PerfForesightCRRA, which provides an approximate formula for consumption that can be rewritten as:

$$\boldsymbol{c}_t \quad \approx \quad \mathbf{p}_t \left(1 - \text{þ}_\gamma/(\mathsf{r}-\gamma)\right) - \text{þ}_{\mathsf{r}} b_t$$

where noncapital income is $\boldsymbol{p}_t$, and

$$\text{þ}_\gamma \quad \equiv \quad \rho^{-1}(\mathsf{r}-\vartheta) - \gamma$$

is the 'growth patience' rate while the 'return patience' rate is

$$\text{þ}_{\mathsf{r}} \quad \equiv \quad \rho^{-1}(\mathsf{r}-\vartheta) - \mathsf{r}.$$

1. Briefly:
   a) Explain in words why we need to impose $\text{þ}_{\mathsf{r}} < 0$
   b) Explain in words why we need to impose $\gamma < \mathsf{r}$
   c) Explain in words why we might want to impose $\text{þ}_\gamma < 0$

2. Capital income is the interest rate multiplied by the amount of financial assets, so the saving rate (income minus consumption, divided by income) is approximately:

$$s_t \quad \approx \quad \left(\frac{\mathsf{r} b_t + \boldsymbol{p}_t - \boldsymbol{c}_t}{\mathsf{r} b_t + \boldsymbol{p}_t} \right). \tag{1}$$

Assuming various quantities are 'small' in convenient ways,[1] for people whose financial wealth ratio $b_t \equiv b_t / \boldsymbol{p}_t$ is small (that is, for most people), show that the saving rate will be approximately:

$$s_t \quad = \quad \left(\frac{\text{þ}_{\gamma}}{\mathsf{r} - \gamma} \right) + \text{þ}_{\mathsf{r}} b_t. \tag{2}$$

Interpret (2) *in words*, explaining why each component has the effect that it does.

[1]Warning: Some of the required assumptions will be bold ones that are not easy to justify.

3. PerfForesightCRRA shows that b_t gets very large, the saving rate approaches

$$s = \left(\frac{\rho^{-1}(\mathsf{r}-\vartheta)}{\mathsf{r}}\right). \tag{3}$$

a) Explain (in words as well as math) why the saving rate for very (financially) rich people does not depend on þ_γ or on b.

b) Show that, for these people, the response of the saving rate to the interest rate is

$$\left(\frac{ds}{d\mathsf{r}}\right) = \rho^{-1}\vartheta\mathsf{r}^{-2}, \tag{4}$$

and, for plausible parameter values, describe what this implies about the likely nature *and magnitude* of the response of the saving rate to changes in the interest rate.

4. Good economists do not want to explain the increase in the saving rate during the Great Recession by saying something silly like 'suddenly everyone just become more patient' or 'everyone's ρ parameters suddenly changed.' What do these formulas suggest are the only three other potential explanations? Explain *in words as well as math* how the changes in each of these *ceteris paribus* would affect saving *rates* of the very (financially) rich (with b approaching ∞) and of the non-rich (with reasonably 'small' values of $\mathsf{r}b_t$). Explain what kind of microeconomic data you would ideally like to have in order to test each of these ideas.

5. In practice, the explanations available from the formulas above are not very satisfying. In particular, interest rates seem to have moved in the wrong direction for the theories, and movements in the other explanators do not seem very persuasive. Of course, the perfect foresight, perfect-capital-markets model of `PerfForesightCRRA` leaves out two categories of explanation: Uncertainty, and changes in capital market conditions (e.g., credit availability).

    a) Explain intuitively, and using the consumption Euler equation, how an increase in the degree of uncertainty about future consumption could increase the saving rate.

b) Explain intuitively why restrictions on people's ability to borrow might change the implications of the theories for either or both categories of people discussed above.

c) Discuss what kinds of data you might want to obtain in order to test the proposition that either greater uncertainty or capital market imperfections could explain the change in saving behavior.

Part II

Dynamic Inefficiency and the Capital Share Coefficient in an OLG Model.

Consider a Diamond (1965) OLG economy like the one in the handout `OLGModel` and the notebook DiamondOLG, assuming logarithmic utility and a Cobb-Douglas aggregate production function,

$$Y = \mathrm{F}(K, PL) \tag{5}$$

where P is a measure of labor productivity that grows according to

$$P_{\tau+1} \;=\; \mathsf{G}P_{\tau}. \tag{6}$$

Population growth is zero ($\Xi = 1$; for convenience normalize the population at $L_{\tau} = 1\,\forall\,\tau$), and until date t productivity growth has occurred at the rate $\mathsf{g} > 0$ (equivalently, $1 + \mathsf{g} = \mathsf{G} \geq 1$) forever. Under these assumptions, it can be shown that the dynamic process for aggregate $k \equiv K/PL$ is

$$k_{\tau+1} \;=\; \left(\frac{(1-\alpha)\beta}{\mathsf{G}_{\tau+1}(1+\beta)}\right) k_{\tau}^{\alpha} \tag{7}$$

1. Derive the steady-state level of k_{τ} that the economy will have achieved by date t if the rate of productivity growth has always been $\mathsf{G}_{\tau} = \mathsf{G}\,\forall\,\tau$.

Now suppose that, after an eternity of remaining in the steady state, all of a sudden at the beginning of period t everybody learns that henceforth and forever more, the exponent on capital in the production function will change to $\hat{\alpha} > \alpha$.

2. Define the new steady-state as $\bar{\hat{k}}$. Will this be larger or smaller than the original steady state $\bar{k}$? *Explain your answer.*

3. Next, use a diagram to show how the $k_{\tau+1}(k_\tau)$ curve changes when the new α takes effect, and show the dynamic adjustment process for the capital stock toward its new steady-state, assuming that the economy was at its original steady state leading up to period t.

4. Define an index of aggregate consumption per efficiency unit of labor in period τ as $\chi_\tau = c_{1,\tau} + c_{2,\tau}/\mathsf{G}$, and derive a formula for the sustainable level of χ associated with a given level of k.

5. Derive the conditions under which a marginal increase in α will result in an increase in the steady-state level of χ, and explain in words why this result holds.

References

Diamond, Peter A. (1965): "National Debt in a Neoclassical Growth Model," *American Economic Review*, 55, 1126–1150, http://www.jstor.org/stable/1809231.