Name _____

Final Exam

Intertemporal Choice

Fall, 2018

Answers

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Short Questions.

1. **Asymmetric Information and Investment.** The handout on imperfect capital markets and investment indicated that investment would occur only if the condition

$$\gamma > 1 + r + A(c, r, W, \gamma) \tag{1}$$

held true. Explain what each of the arguments of the A function are and briefly describe how and why an increase in each parameter would affect the amount of investment, holding all the other parameters constant. (Be sure to compare the effects with the effects, if any, that would occur in the standard 'perfect capital markets' model).

Answer:

- c is the cost of verification. With higher costs of verification, the cutoff value at which projects are undertaken is higher, which means fewer projects will be carried out, so investment will fall. In the perfect capital markets model, c does not exist so increases in c cannot be considered/have no effect.
- r is the interest rate. Higher interest rates interact with c to reduce investment by more than it would be reduced in the standard model.
- W is the entrepreneur's wealth. Higher entrepreneurial wealth increases the creditworthiness of entrepreneurs and increases the amount of investment
- γ is the quality of entrepreneurs. Increasing this increases the amount of investment because projects are more likely to be successful and therefore loans are more likely to be paid off

2. **Capital Market Imperfections and Slow Recoveries.** Reinhart and Rogoff (2009) present evidence that the recovery from recessions caused by financial crises tends to be slower than recoveries from other kinds of recessions. Use the model of capital market imperfections presented in class to discuss some reasons why this might be so (under the assumption that 'other kinds of recessions' mostly are caused by temporary factors that do not impede the efficient functioning of capital markets).

Answer:

There is no single 'correct' answer to this question; good answers would be those that explained why the kinds of factors captured in that model (like a breakdown in the ability of financial intermediaries to assess the likelihood of default) might be more long-lasting than factors involved in usual business cycle fluctuations

Good answers will also be couched in terms of the A in the Romer model which measures the degree of capital market imperfection, and relate the discussion to determinants of A.

Medium-Length Questions

1. **Lucas (1978) Prices for Risky and Safe Firms.** Consider an island economy with two kinds of food production.

   - Pick berries from bushes that grow in a particular spot on the island. The production of berries is not very much affected by the weather, so the variation in berry production is not very great.
   - Obtain coconuts from a set of (identical) coconut trees. Coconut production varies much more from year to year because sometimes a monsoon will come and blow away many of the coconuts.

   a) Suppose the residents of the island participate in a perfectly efficient financial market in which shares to the production of the berry bushes and coconut trees are traded. What would the Lucas asset pricing model imply about the variation in the price of berry bushes compared to the variation in the price of coconut trees? Why?

   Answer:

   The price of the berry bushes is less volatile because the dividends are less volatile.

   b) Suppose global warming intensifies the size of weather fluctuations from year to year, making the amount of coconut production more variable but leaving the variability of berry production unchanged. What would the model imply about the effects of global warming on the average relative prices for trees versus berries?

   Answer:

   The relative price of the threes will go down, because the increased variability makes them less desirable.

   c) Figure 1 shows the business-cycle movements of spending on durable goods and nondurable goods in the U.S.

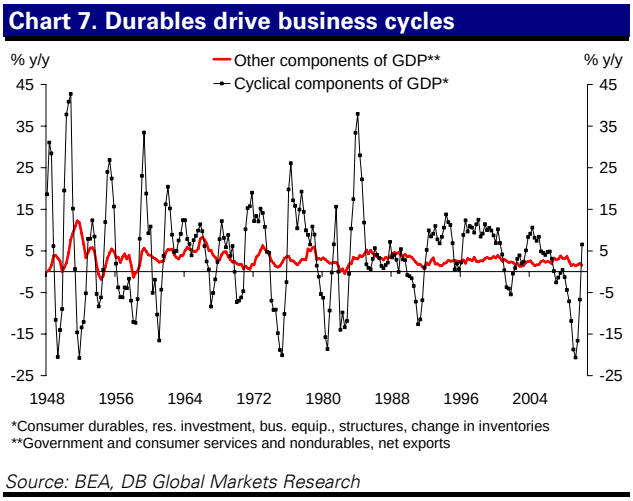
      i. Interpret this figure in light of the Mankiw (1982) model of durables; specifically, explain why that model implies that expenditures on durable goods should be much more volatile than expenditures on nondurables.

      ii. Explain how this figure relates to the earlier discussion of the Lucas asset pricing model. In particular, explain what that model suggests about the average price-to-dividend ratio of firms that produce durable goods compared to that of firms that produce nondurable goods.

   Answer:

      i. `Durables` explains this point

Figure 1 Spending on Durables and Nondurables



ii. Assuming that the fluctuations in spending translate into fluctuations in profits, ownership of shares of firms that produce durable goods is like ownership of a tree that produces coconuts: The profits are very volatile, compared to profits from owning shares of a firm that produces nondurable goods (resp., ownership of berry bush shares). The P/D ratio should be lower for firms that have risker returns, because people don't like to hold those shares because of their riskiness.

2. Capital Dynamics in Growth Models

a) `BrockMirman` shows that the Brock and Mirman (1972) model yields the following equation for capital in logs,

$$k_{t+1} = \log \alpha\beta + \alpha k_t + a_t \tag{2}$$

where a_t is the productivity shock in the production function (and recall that the model implies that $K_{t+1} = \alpha\beta Y_t$).

Show that this implies that the variance of k is

$$\text{var}(k) = \frac{\sigma_a^2}{1-\alpha^2} \tag{3}$$

Hints: $\text{var}(k_{t+1}) = \text{var}(k_t)$ and for some constant a and some stochastic variable b, $\text{var}(ab) = a^2\,\text{var}(b)$.

Answer:

$$k_{t+1} = \log \alpha\beta + \alpha k_t + a_t \tag{4}$$
$$\text{var}(k) = \alpha^2\,\text{var}(k) + \text{var}(a) \tag{5}$$

$$\begin{aligned} \text{var}(k)(1-\alpha^2) &= \sigma_a^2 && (6) \\ \text{var}(k) &= \frac{\sigma_a^2}{1-\alpha^2} && (7) \end{aligned}$$

b) Now we want to consider a case where the level of productivity a is serially correlated:

$$a_t = \zeta a_{t-1} + \nu_t \tag{8}$$

for some shock ν with variance σ_ν^2 and a serial correlation coefficient $0 < \zeta < 1$. Calculate the variance of k under this new assumption.

Answer:

$$\begin{aligned} a_t &= \zeta a_{t-1} + \nu_t && (9) \\ \text{var}(a) &= \frac{\text{var}(\nu)}{1-\zeta^2} && (10) \end{aligned}$$

so

$$\text{var}(k) = \frac{\sigma_\nu^2}{(1-\alpha^2)(1-\zeta^2)} \tag{11}$$

c) Similarly, since log output is simply $y = a + \alpha k$, the dynamics of output can be obtained from

$$\begin{aligned} y_{t+1} &= a_{t+1} + \alpha k_{t+1} && (12) \\ &= \alpha(\log K_{t+1}) + a_{t+1} && (13) \\ &= \alpha(\log \alpha\beta y_t) + a_{t+1} && (14) \\ &= \alpha(\underbrace{y_t + \log \alpha\beta}_{\text{log saving}}) + a_{t+1} && (15) \end{aligned}$$

it is possible to show that in a more general version of the model in which utility is not logarithmic, an equation of the form

$$y_{t+1} \approx \bar{y} + \omega_1(y_t - \bar{y}) + a_{t+1} \tag{16}$$

does a good job in describing the dynamics of income near the target level $\bar{y}$, and note that ω_1 is kind of like a marginal 'consequence' of saving (a combination of how much you save and how much it influences income in the next period). This reduces to the Brock-Mirman model for $\omega_1 = \alpha$. Derive an expression for the variance of the capital stock if this were a 'structural' equation, and make a remark about what this says about the breadth of the insight that comes from the prior analysis.

Answer:

$$\text{var}(y) = \left(\frac{\sigma_a^2}{1-\omega_1^2}\right) \tag{17}$$

The 'remark' should be that this result tells us that model dynamics will be driven by the magnitude of the marginal consequense of saving and of the shocks, and the consequence of saving in turn is related to α because it determines how much a unit of saving in period t translates into income in period $t+1$.

d) Unlike (15), equation (16) is not 'structural': That is, while it may do a good job in describing the dynamics of the model near $\bar{y}$, if a parameter of the model changes (say, relative risk aversion), the ω coefficient will change.

Consider, for example, an increase in the coefficient of relative risk aversion, starting from the benchmark Brock-Mirman value of $\rho = 1$. Explain how you would expect the coefficient ω_1 to change, and what the limits of the extent of the change are.

Hint: ρ^{-1} is the intertemporal elasticity of substitution; a consumer who is very reluctant to substitute consumption over time will set consumption C_t near the $\mathbb{E}[y_{t+1}] = y_t$ locus in the phase diagram. (For purposes of answering this question you can assume that the variance of a is arbitrarily small).

Answer:

ρ^{-1} measures the intertemporal elasticity of substitution, so an increase in ρ implies a reduction in the intertemporal elasticity. A low IES means a value such that

$$1 \quad = \quad \beta\, \mathbb{E}_t[\mathbf{R}_{t+1}(C_t/C_{t+1})^{\rho}] \tag{18}$$

but near the steady state where $\beta\, \mathbb{E}_t[\mathbf{R}_{t+1}] = 1$ this means that C_{t+1} needs to be very close to C_t. The means by which a consumer could achieve $C_{t+1} = C_t$ would be to consume exactly the 'sustainable' amount such that $\mathbb{E}[Y_{t+1}] = Y_t$. This means that as ρ gets larger, the consumption function must lie closer to the sustainable consumption locus than in the logarithmic utility case. Since the slope of the $\mathbb{E}[Y_{t+1}] = Y_t$ locus is shallower than the $C = (1 - \alpha\beta)Y$ line that characterizes the Brock-Mirman consumption function, we know that near the steady state the increase in ρ must have increased the level of the consumption function but reduced the marginal propensity to consume, so the marginal propensity to save must be higher (that is, $\omega_1 < \alpha$).

e) Explain intuitively why the determinants of the long-run variance of y are the (log) marginal propensity to save and the serial correlation of productivity growth, and explain why the MPS is related to α.

Answer:

When there is a shock to productivity/income, the dynamics depend on two things:

i. How persistent is the shock? Its persistence translates directly into persistence in income

ii. How much extra saving (and thus extra future capital, and thus extra future capital income) does the shock induce?

- This is determined immediately by the effect of current saving on next period's capital
- But the saving rate is influenced by α because with a higher α the saving is more effective in producing future income and therefore more attractive

3. **An Increase in Interest Rates in the q Model.** Consider a perfectly competitive industry with costs of adjustment to capital that was in equilibrium with interest rate $\underline{r}$ leading up to period t, at which date the interest rate increases permanently to $\bar{r} > \underline{r}$.

   a) Draw a diagram that shows how the $\Delta q_{t+1} = 0$, $\Delta k_{t+1} = 0$ loci and the saddle path change in response to the increase in the interest rate. Is the new equilibrium level of the capital stock $\check{k}_{\text{post}}$ higher or lower than before? Why?

   Answer:

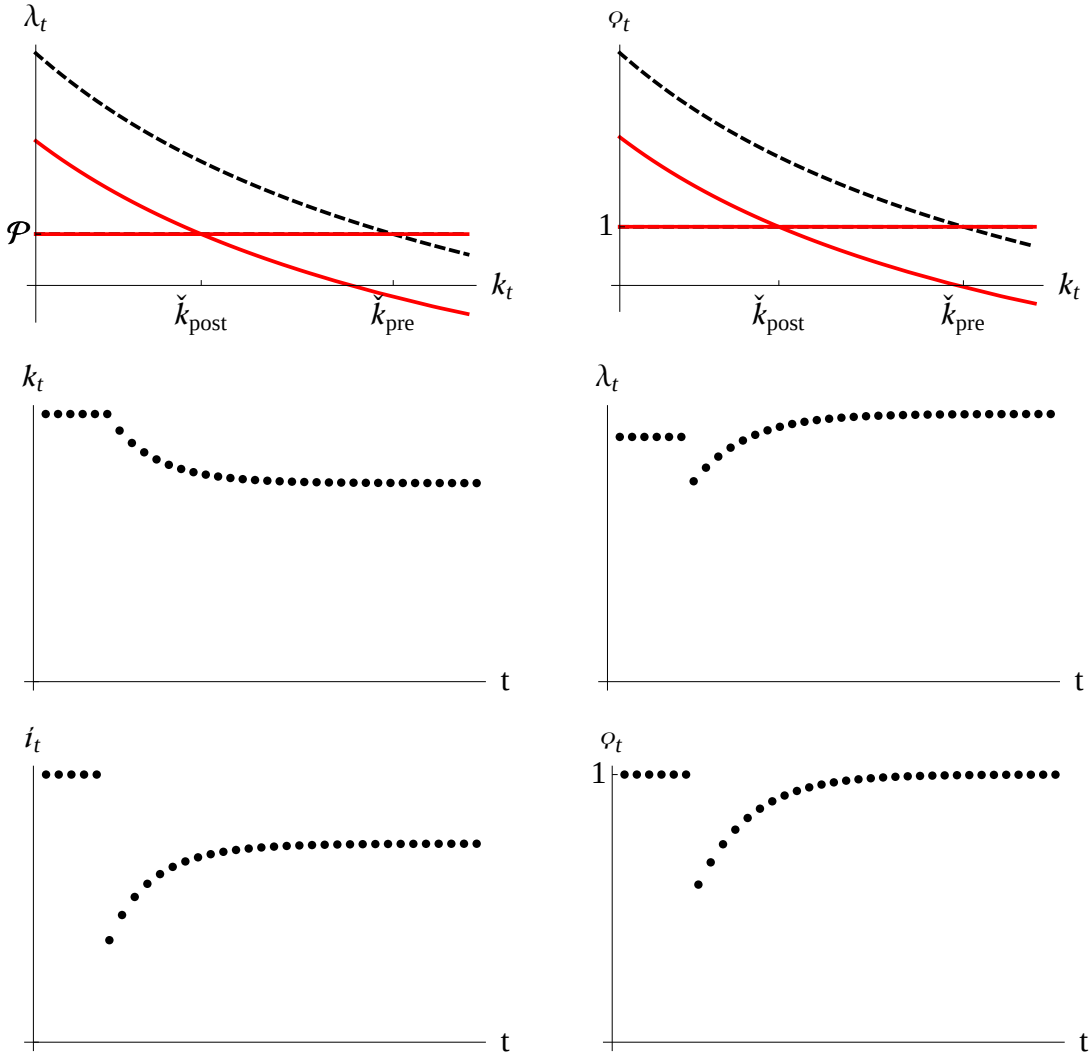


Figure 2 Increase in R: phase diagrams with saddle paths (dashed-black and continuous-red lines respectively pre and post the R change) and impulse response functions

Steady-state capital is lower (because with a higher interest rate, capital is more expensive).

b) Draw diagrams showing the time paths of share prices, investment, and the capital stock following the increase in the interest rate. Explain the time pattern of share prices over time; in particular, explain the relationship between any depicted movements in share prices, and the proposition that stock prices follow a random walk.

Answer:

The figures are plotted above. When interest rates rise, share prices fall; you can think of this as reflecting the fact that future earnings are being discounted at a higher interest rate (so it is kind of like a human wealth effect in the consumption model). Gradually over time share prices recover to the point where the value of a unit of capital inside the firm is equal to the original (unchanged) price of purchasing a unit of capital. Investment drops very sharply, then gradually recovers as the firm gradually accomplishes its goal of achieving a lower target capital.

The proposition that share prices follow a random walk has little relevance except to the inital movement in λ that happens when interest rates change. That jump in share prices was assumed to reflect the fact that the change in interest rates was unanticipated, and so a resulting jump can be interpreted as a shock to expectations. The gradual movements in share prices thereafter are consistent with the need for capital gains or losses on share prices to counterbalance the dynamics of the cash flow of the firm's profits.

c) Suppose that, in response to the increase in interest rates, the government wanted to pursue a tax policy that would prevent any changes in the level of the capital stock. Describe the two options the government has, and explain how the two policies would differ or be similar with respect to their implications for dynamics of ϱ and λ.

Answer:

The government could either cut the corporate tax rate, or increase the investment tax credit. See `qModel` for detailed discussion of the consequences. Briefly, the key equilibrium equations that tell us what must happen are

$$(\mathsf{r}+\delta)\lambda_t \approx \not{\pi}\mathrm{f}^k(k_t) \tag{19}$$

and

$$\varrho = \lambda/\not{\zeta}. \tag{20}$$

If the government wants to keep capital constant without changing the ITC, the equilibrium fact that $\varrho = 1$ tells us that the equilibrium value of λ must be unchanged. In this case (19) tells us that it will need to boost the portion of corporate earnings that are untaxed, $\not{\pi}$,

by an amount that exactly offsets on the LHS of the equation from a higher value of r.

On the other hand, if it wants to leave the corporate tax rate unchanged, then we know that in equilibrium $\lambda = \not{\zeta}$, so we can solve (19) for the value of

$$(\mathsf{r} + \delta)\not{\zeta} \approx \not{\pi}\mathrm{f}^k(\check{k}) \tag{21}$$

that counterbalances the higher r while keeping $\not{\pi}\mathrm{f}^k(\check{k})$ constant.

4. **Infrastructure and Growth.**

Standard growth models ignore the role of a country's public 'infrastructure' (roads, bridges, sewers, other public facilities) in determining a country's level of income per capita. Yet looking across countries, it seems clear that countries with well-designed and well-maintained infrastructure are more prosperous than countries with crumbling, deficient, or nonexistent infrastructure.

Suppose we can capture the long-run effect of government infrastructure expenditures e in the per-capita production function:

$$f(k, e) = k^\alpha e^\eta \tag{22}$$

where a country with more infrastructure spending has a higher value of e. (Assume the population and the level of productivity are normalized at 1, and $\eta < 1$).

Suppose government infrastructure expenditures translate one-for-one into productive efficiency e, and assume that the government must satisfy a balanced budget criterion by the use of lump-sum taxes of amount τ:

$$e = \tau. \tag{23}$$

For simplicity, suppose that the capital stock is exogenously fixed at $k = \bar{k}$ and does not depreciate but cannot be augmented by extra saving (there is an endowment of capital).

a) Calculate the level of taxes that maximizes per-capita after-tax income $f(k, e) - \tau$ and explain intuitively the reasons for the effects that the parameters have on the optimal choice of government expenditures.

Answer:

$$\max_e \bar{k}^\alpha e^\eta - e \tag{24}$$

has FOC with respect to e of

$$\bar{k}^\alpha \eta e^{\eta-1} = 1 \tag{25}$$

$$e = \left(\bar{k}^\alpha \eta\right)^{1/(1-\eta)} \tag{26}$$

which says that expenditures/taxes will be higher when 1) the capital stock is higher (because there is more productivity to "enhance" by

government expenditures; 2) when the coefficient on capital is higher (α is larger), for the same reasons; 3) η is larger, because the larger is η the smaller is the rate at which government efficiency improvements have diminishing marginal productivity effects.

b) Now suppose this economy suffers from corruption. Specifically, some of the tax revenues that are raised do not get spent on efficient government expenditures but instead are wasted. Again using e for the amount of efficient expenditures, and again imposing the balanced budget constraint, the new level of after-tax income is

$$f(\bar{k}, e) - \underbrace{\tau}_{=e\chi} \tag{27}$$

where $\chi > 1$ measures the degree of corruption. Thus, taxes paid τ exceed expenditures e (the extra taxes represent waste and corruption). Now calculate the level of e that maximizes after-tax per capita output. Is it higher or lower than in the honest economy (where $\chi = 1$)? Why? Is there a cost to the economy beyond the fact that the tax burden is higher by amount χ? Why?

Answer:

The FOC are

$$\bar{k}^{\alpha}\eta e^{\eta-1} = \chi \tag{28}$$

$$e = \left(\frac{\bar{k}^{\alpha}\eta}{\chi}\right)^{1/(1-\eta)} \tag{29}$$

and since $\chi > 1$ this is clearly a smaller number than the e that was optimal for the honest economy. Notice that after-tax income is lower for *two* reasons: 1) with a lower e the economy produces less output; 2) with a higher χ the effective tax rate is higher. So pretax income is less while taxes are higher.

c) Hall and Jones (1999) find that, looking across countries in the world, only a very small proportion of the differences in output per capita are explained by differences in private capital, natural resources, or other private factors of production. Discuss how this finding might be related to the modeling choice above to assume a fixed level of capital $\bar{k}$. Speculate on whether permitting private capital accumulation would be likely to reinforce, undermine, or leave unchanged the results from the baseline model. If the theory sketched above about the effectiveness of infrastructure investment were true, discuss what effect it would have had for Hall and Jones to include some measure of public capital (as well as private capital) in their model.

Answer:

The Hall and Jones (1999) finding suggests that private capital accumulation is not one of the main influences that make some countries

rich and others poor, so an intensive and complex study of optimal intertemporal allocation decisions may not yield much fundmanetal insight about the process of economic growth.

Permitting private capital accumulation would likely reinforce but not change the logic outlined above; in the more efficient economy, the incentives to save (returns on capital) would be higher, and therefore it is likely that there would be more saving.

If the theory were true, we should expect to see measures of infrastructure per capita having a high degree of explanatory power for measures of income per capita.

References

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