Name _____

Final Exam
Intertemporal Choice
Fall, 2016

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Part I. Short Discussion Questions.

'Real' and 'Financial' Explanations of Asset Price Movements

A large literature at the intersection of finance and macroeconomics aims to explain movements of stock prices. Roughly speaking, much of that literature attempts to identify movements that can be explained by (a) 'real' and observable factors like news about a firm's profits ('cash flow') and dividends, versus (b) other unobservable factors that are often called 'discount rate' shocks. (The literature typically assumes that the 'discount rate' or, synonymously, the 'stochastic discount factor,' is impossible to observe *except* through its consequences for asset prices.)

1. Interpret the foregoing statement using the Lucas (1978) model. In that framework, what would you identify as 'real' and what would you identify as 'financial' factors?

2. Explain why it is problematic, for the status of finance as a science, to attribute all movements in asset prices that are not explainable as reflecting observable 'real' factors to movements in a variable which is in principle unobservable ('the stochastic discount rate').

3. Using the insights from the prior questions, explain why Robert Shiller's work in attempting to find credible ways of surveying market participants to measure their beliefs and preferences and attitudes would, if successful, be useful.

Dynamic Inefficiency and Japan's Woes.

Define dynamic inefficiency and explain why a plausible way of thinking about Japan's economic problems after 1990 is to argue that Japan's economy has been dynamically inefficient. Explain why the fact that Japan's capital/output ratio is not much higher than the U.S. capital/output ratio presents a challenge to the dynamic inefficiency interpretation of Japan's problems, and discuss how the question of dynamic inefficiency is connected not just to the level of aggregate capital but also to the efficiency with which financial markets allocate capital to productive uses.

Part II. Longer Analytical Questions

Brock and Mirman (1972) Multiplied

A large body of recent research has provided support for the old Keynesian idea that, at least under certain conditions,[1] the amount of aggregate output depends partly on the amount of 'aggregate demand.' Under such conditions, any shock that results in a change in a component of 'aggregate demand' will have a 'multiplier' effect (that is, the resulting change in output will be greater than the change in the component of aggregate demand).

Krueger, Mitman, and Perri (2016) propose that the role of a 'consumption multiplier' can be approximately captured by augmentation of the usual productivity term in the production function with a component that is increasing in the level of consumption:

$$\mathrm{F}_t(K_t, L_t) = C_t^{\omega} Z_t K_t^{\alpha} L_t^{1-\alpha} \tag{1}$$

where $0 \leq \omega < 1$ determines the size of the multiplier, and $A_t = C_t^{\omega} Z_t$ would be measured as the level of 'productivity' using conventional approaches. (We might call call Z 'primitive' or 'structural' productivity, as it is what would be manifested if not for the distorting effects of consumption's multiplier).

This question explores the implications of this production function in a growth model in which consumers are not aware of the existence of the multiplier; instead they interpret any changes in output that are attributable to the multiplier as reflecting the usual mysterious shocks to aggregate productivity that drive many other business cycle models. We assume that the 'multiplier effect' takes one time period to manifest itself (most attempts to provide deeper foundations for multipliers would suggest even longer lags).

We examine these ideas in a `BrockMirman` model. Thus, our consumers perceive their problem to be

$$\max \sum_{n=0}^{\infty} \beta^n \log C_{t+n} \tag{2}$$

s.t.

$$Y_t = A_t K_t^{\alpha} \tag{3}$$

$$K_{t+1} = Y_t - C_t \tag{4}$$

with Bellman equation

$$\mathrm{V}_t(K_t) = \max_{C_t} \log C_t + \beta \mathrm{V}_{t+1}(A_t K_t^{\alpha} - C_t). \tag{5}$$

However, in truth the level of income is given by

$$Y_t = C_{t-1}^{\omega} Z_t K_t^{\alpha} \tag{6}$$

(questions begin on the next page)

[1] Most notably, when interest rates are stuck at the zero lower bound

1. Show that the consumption function will be

$$C_t = \overbrace{(1-\alpha\beta)}^{\equiv\kappa} Y_t \tag{7}$$

For the rest of the question, suppose that consumers believe that the log of productivity $a \equiv \log A$ follows a random walk:

$$a_{t+1} = a_t + \phi_{t+1} \tag{8}$$

and suppose that consumers observe the current level of output and derive their believed log level of 'productivity' as $a_t = y_t - \alpha k_t = z_t + \omega c_{t-1}$. Finally, assume that z does indeed follow a random walk:

$$z_{t+1} = z_t + \zeta_{t+1}. \tag{9}$$

2. Show that if throughout an indefinitely long prior history the log-level of z has been $z = 0$, there will be 'pseudo-steady-state' values of the logs of the model's main variables, $\{\hat{c}, \hat{y}, \hat{k}, \hat{y}\}$ to which the economy will converge. (Note: Do *not* try actually to do the tedious algebra required to find the solutions; instead, you should give a mathematical argument for why it is likely that such a solution will exist, at least for some parameter values).

3. Assume that in period 0, the economy was in the pseudo-steady-state associated with a perpetual past history in which $z_{-n} = 0 \; \forall \; \geq 0$, ($\{c_0, y_0, k_0, a_0\} = \{\hat{c}, \hat{y}, \hat{k}, \hat{a}\}$). In period 1, a shock of size $\zeta_1 > 0$ occurs (and no other shocks occur thereafter: $\zeta_t = 0 \; \forall \; t \neq 1$). Show that the pattern of income growth is

$$\begin{aligned} \Delta y_1 &= \zeta_1 \\ \Delta y_2 &= \zeta_1(\alpha + \omega) \\ \Delta y_3 &= \zeta_1(\alpha + \omega)^2 \\ \vdots &= \vdots \\ \Delta y_n &= \zeta_1(\alpha + \omega)^n \end{aligned}$$

Hint: you will want to use these steady-state facts:

$$\hat{z} = 0 \tag{10}$$

$$\hat{a} = \hat{z} + \omega\hat{c} = \omega\hat{c} \tag{11}$$

and the dynamic equations:

$$z_t = z_{t-1} + \zeta_t \tag{12}$$

$$a_t = z_t + \omega c_{t-1} \tag{13}$$

$$k_t = \log \alpha\beta + y_{t-1} \tag{14}$$

$$y_t = a_t + \alpha k_t \tag{15}$$

$$c_t = \log(1 - \alpha\beta) + y_t. \tag{16}$$

Further hint: Begin by calculating how the shock ζ_1 changes the period 1 values of a, k, y, and c from their steady state values (for example, $a_1 = \zeta_1 + \omega\hat{c}$), and then use the dynamic equations to move forward.

4. Draw diagrams showing the dynamics of a, k, and y in the experiment described above. On the same diagrams, show the dynamics that consumers *expect* these variables will follow. Using the comparison of the expected and the actual dynamics, explain the sense in which this model can be said to exhibit a 'consumption multiplier.'

5. Most of the deeper mechanisms by which multipliers are hypothesized to work (e.g., having workers put in extra hours when demand surges, or stickiness of prices) imply that the stimulative effect on productivity eventually dies out. In the framework outlined above, this might be captured by supposing that the multiplier depends not upon the level of C but rather on its ratio to some new 'steady-state' value. That is, the 'productivity' term in the production function is $A_t(C_{t-1}/C^{\mathrm{SS}})^{\omega}$ where C^{SS} catches up slowly but inexorably to actual consumption (for example, if $C_t^{\mathrm{SS}} = \lambda C_{t-1}^{\mathrm{SS}} + (1-\lambda)C_{t-1}$ for some value of λ not much less than 1) so that eventually (in the absence of further shocks) the new 'pseudo-steady-state' would be reached at a point where $C_t/C^{SS} = 1$. Draw the diagrams you would expect this economy to exhibit in response to the same experiment performed earlier.

Infrastructure and Interest Rates.

Starting near the beginning of the Great Recession (henceforth, 'GR'), economists including Larry Summers, Paul Krugman, Brad deLong, and even Martin Feldstein argued that an appropriate response to a crisis that combined low aggregate demand and historically low interest rates would be a large increase in infrastructure investment.

From 2009 to 2015, Republican party dogma was that an increase in infrastructure investment was not desirable. But, in the 2016 presidential election campaign, Donald Trump criticized Hillary Clinton's proposals to boost infrastructure spending by saying that she was not calling for *enough of an increase* in infrastructure investment.

This question asks you to think about what a neoclassical model might say about infrastructure investment, and then to critique those insights.

Suppose that the private (that is, non-infrastructure) capital stock k is exogenously fixed at k_0 and does not depreciate but cannot be augmented by extra saving (there is an 'endowment' of capital).

Using n to designate the quantity of i*n*frastructure capital (which, like private capital, does not depreciate), suppose we can capture the long-run effect of n in the production function by:

$$\mathrm{f}(k, n) \quad = \quad k^{\alpha} n^{\mu} \tag{17}$$

so that (under the maintained assumption $0 < \mu < 1$), a country with more n gets higher productivity from its private capital k. (For the remainder of the question, assume the population is constant at 1).

1. Suppose that, leading up to the GR, the government had chosen a level of infrastructure capital $n = n_0$, paid for by borrowing on international capital markets, and had imposed lump sum taxes on households sufficient to pay the interest on the debt incurred thereby, $\tau_0 = \mathsf{r}_0 n_0$, so that after-tax national income was

$$y_0 = \mathrm{f}(k_0, n_0) - \tau_0. \tag{18}$$

Calculate the level of the infrastructure capital stock $n_{0,*}$ that a government that wanted to maximize after-tax income would have chosen for this economy. *Explain intuitively* the reasons for the effects of the various parameters. Defining $\mathrm{f}_{0,*}$ as the production function that applies when n has been chosen optimally, and $\mathrm{f}^n_{0,*}$ as the marginal product of an additional unit of n, you should obtain the result that:

$$\mathrm{f}^n_{0,*} \equiv \mathrm{f}^n(k_0, n_{0,*}) \quad = \quad \mathsf{r}_0 \tag{19}$$

(The next page is left blank to provide room for your answer)

Answer below

2. Continue to assume that up to the GR, the government had chosen n optimally so that $y_{0,*} = \mathrm{f}_{0,*} - \mathsf{r}_0 n_{0,*} = k_0^\alpha n_{0,*}^\mu - \mathsf{r}_0 n_{0,*}$. Calculate explicit formulae for the level of after-tax income under the following alternative government policies, assuming that when the GR hits, interest rates drop to $\mathsf{r}_1 = \mathsf{r}_0(1-\epsilon)$ for some 'small' $\epsilon > 0$ (and are expected to stay at r_1 forever):

   a) Cut taxes to the level sufficient to pay the new, lower, interest rate costs associated with maintaining the original n_0

   b) Leave taxes at the old level of $\tau_{0,*} = \mathsf{r}_0 n_{0,*}$ and change n to the amount that can be paid for with those tax revenues. (Hints: (1) Begin by showing that your earlier FOC for $n_{0,*}$ implies that $\mu \mathrm{f}_{0,*} = \mathsf{r}_0 n_0$; (2) you will need to pay attention to the concavity of the production function in n either by using a 2nd order Taylor expansion or just by staring at the equation and seeing the relevant intuition)

   c) Change the stock of infrastructure to the level optimal for the new level of interest rates, while changing the tax rate accordingly

3. Can the three policies above be ranked from best to worst? If so, what is the ranking? If not, explain why and what additional information you would need to be able to rank them?

4. The preceding analysis assumed that the private capital stock k was fixed at k_0. We now want to consider how things might change if k is chosen optimally. (For the following analysis, suppose that k_0 was in fact the optimal level of the private capital stock at an interest rate of r_0 and with infrastructure of n_0).

   a) Suppose the government pursues option (a) above, keeping $n = n_0$. Using a ϱ model of investment, show the paths of investment and private capital k that you would expect to arise from optimizing firms after the drop in the world interest rate to r_1 (assuming that this is the only thing that has changed for the firms; that is, they are not expecting a fall in sales or other consequences of the GR)

b) Suppose now that when the GR hits, not only do interest rates drop to $\mathsf{r}_1 < \mathsf{r}_0$ but firms' expectations about their future productivity also deteriorate, by an amount that is just enough to keep their investment spending the same as it was before, i_0 (assuming that aggregate infrastructure remained at its pre-GR level n_0). That is, if firms expected the goverment to pursue the household tax-cut option in the prior question, firms would not change their behavior. Discuss the consequences for firm investment spending if, instead, the government pursues each of the other two options in the prior question. Can the size of any effect on firm investment spending be ranked between these two alternatives? Taking this into account, comment on any new insights about the relative desirability of the alternatives.

References

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