Saving, Interest Rates, and Growth. PerfForesightCRRA shows that in a perfect foresight infinite horizon CRRA-utility consumption model, the formula for consumption

$$c_t \quad = \quad (h_t \boldsymbol{p}_t + b_t)\kappa \tag{1}$$

implies that

$$\mathbf{c}_t \quad \approx \quad \overbrace{\mathbf{p}_t \left(1 - \mathfrak{b}_\gamma/(\mathsf{r}-\gamma)\right)}^{\text{from human wealth}} + \overbrace{(\mathsf{r} - \rho^{-1}(\mathsf{r}-\vartheta))\mathbf{b}_t}^{\text{from nonhuman wealth}} \tag{2}$$

where

$$\begin{aligned} \gamma &= \text{Growth rate of permanent noncapital income} \\ \mathfrak{b}_\gamma &= \rho^{-1}(\mathsf{r}-\vartheta) - \gamma \\ &\equiv \text{'growth impatience rate'} \end{aligned} \tag{3}$$

ExplainRIC

1. Explain, in words, why the 'return impatience condition' $\mathfrak{b}_\mathsf{r} < 0$ needs to be imposed in order for the model to have a sensible solution. Explain, in words, why the 'finite human wealth' condition, $\gamma < \mathsf{r}$ needs to be imposed. Explain, in words, what imposing the 'growth impatience condition' $\mathfrak{b}_\gamma < 0$ accomplishes and why it might be desirable to impose that condition in an infinite-horizon model. (Reminder: $\mathfrak{b}_\mathsf{r} \equiv \rho^{-1}(\mathsf{r}-\vartheta) - \mathsf{r}$).

   Answer:

   These points are all explained in PerfForesightCRRA.

DeriveSavRate

2. Use (2) and the fact that the level of saving $\mathbf{s}$ can be defined as total income minus total consumption:

$$\mathbf{s}_t \quad = \quad \mathsf{r}a_{t-1} + \mathbf{p}_t - \mathbf{c}_t \tag{4}$$

   to show that the ratio of saving to permanent labor income $\boldsymbol{p}_t$ is approximately

$$s_t \quad \approx \quad \mathfrak{b}_\gamma/(\mathsf{r}-\gamma) + \rho^{-1}(\mathsf{r}-\vartheta)a_{t-1} \tag{5}$$

   and that the ratio of saving to total income (the 'saving rate') is

$$\varsigma_t \quad = \quad \left(\frac{\mathfrak{b}_\gamma/(\mathsf{r}-\gamma) + \rho^{-1}(\mathsf{r}-\vartheta)a_{t-1}}{1 + \mathsf{r}a_{t-1}}\right). \tag{6}$$

   Answer:

   These derivations are in PerfForesightCRRA.

SavSens

3. Use the tools available at Econ-ARK/HARK to plot the relationship between the saving rate and assets under various combinations of parameter values, with the objective of exploring the sensitivity of saving to the model's calibration

4. Opinions about the long-term growth rate of income, γ, are widely divergent today. Some scholars (e.g., Robert Gordon (2012)) believe that productivity growth in the U.S. is likely to be slow, perhaps 1 percent, over the next 50 years, while others

(e.g., Erik Brynjolfsson and McAfee (2014)) are considerably more optimistic, projecting little if any slowdown from the growth rate of 2.5 percent a year that has characterized the postwar period. Furthermore, beliefs about long-run growth have become considerably more pessimistic in the period since the Great Recession. GrowthCastsDoubt

a) Explain why these facts cast doubt upon this model as a useful or reliable guide to understanding actual saving choices. Relate this point to the argument of Summers (1981) about the magnitude of the human wealth effect in perfect foresight models.

Answer:

Basically, the model says that the saving rate is insanely sensitive to the exact values of the various parameters and the relationship among them. If the model is taken (quantitatively) literally, it implies that even modest revisions in beliefs about long-run growth should result in massive shifts in saving rates. Or, minor differences across people in their beliefs about long-run growth should result in huge differences in their saving behavior. This is an illustration of the point made in class: Economists have gradually moved from merely asking their models to provide some *qualitative* guidance about what *kinds* of effects to expect to see, to expecting the models to have some *quantitative* plausibility. The perfect foresight model fails dismally on the quantitative side, not because it predicts the wrong quantity but because it is so sensitive to tiny calibration differences that it can predict almost anything for some easy-to-make-up combination of reasonable parameter values.

WhyHigher

b) Explain why the saving rate is higher in a `TractableBufferStock` model than in the perfect foresight model

Answer:

The saving rate is higher in the model with risk because now the consumers have a *precautionary motive*. The impetus to save is not merely to smooth consumption across (an infinite number of) periods, but now also to provide a bulwark against catastrophe which takes the form of a transition into the unemployed state.

PFvsBS

c) Using the same Jupyter notebook as before, compare Compare the degree of sensitivity of the saving rate to the growth rate of income in this model compared to the perfect foresight model. Give the intuition for any differences.

Answer:

The saving rate is relatively less responsive to the growth rate in the face of uncertainty than it is with perfect foresight. The intuition is that now changes in the growth rate have an additional effect on precaution. In the perfect foresight model, an increase in income growth results in less saving because the consumer no longer has

to save as much to transfer the same amount of resources into the future – those resources are already there due to the up-tick in growth. However, in the buffer stock model this also means that the unemployed state is even scarier from the perspective of the employed – it is now a further fall, and this tends to increase the precautionary motive. The presence of this countervailing force will tend to dampen the response of saving to changes in the growth rate.

References

Brynjolfsson, Erik, and Andrew McAfee (2014): *The second machine age: work, progress, and prosperity in a time of brilliant technologies*. WW Norton & Company.

Gordon, Robert J. (2012): "Is U.S. Economic Growth Over? Faltering Innovation Confronts the Six Headwinds," Working Paper 18315, National Bureau of Economic Research.

Summers, Lawrence H. (1981): "Capital Taxation and Accumulation in a Life Cycle Growth Model," *American Economic Review*, 71(4), 533–544, http://www.jstor.org/stable/1806179.