

Name \_\_\_\_\_

Final Exam  
Intertemporal Choice

Fall, 2019  
Answers

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

## Short Questions.

### 1. Predicting Microeconomic Behavior of Buffer Stock Consumers

Consider an economy populated by buffer stock savers subject only to transitory shocks to income. This economy has reached the equilibrium ‘ergodic’ distribution of wealth in date  $t$ ; at the end of  $t$  you obtain a dataset containing several years’ worth of recent data on a representative sample of the population. Answer the following questions about how *and why* the behavior of selected subsamples of this population can be predicted to be different from the population as a whole, in three dimensions: (1) Their expected labor income growth; (2) Their expected consumption growth; (3) Their expected wealth level growth

For example, if your subsample were completely randomly selected you would say: “I expect their income growth, consumption growth, and wealth growth to be identical to that of the population, *because they were randomly selected.*”

- a) Consumers who in year  $t$  experienced a substantial positive shock to their income

*Answer:*

The positive income shock this year will make these consumers wealthier than average, so in year  $t$ , income, wealth, and consumption will all be above average. However, the model says that all three of these variables can be expected to return to their population averages.

- b) Consumers who in year  $t$  are poorer than average

*Answer:*

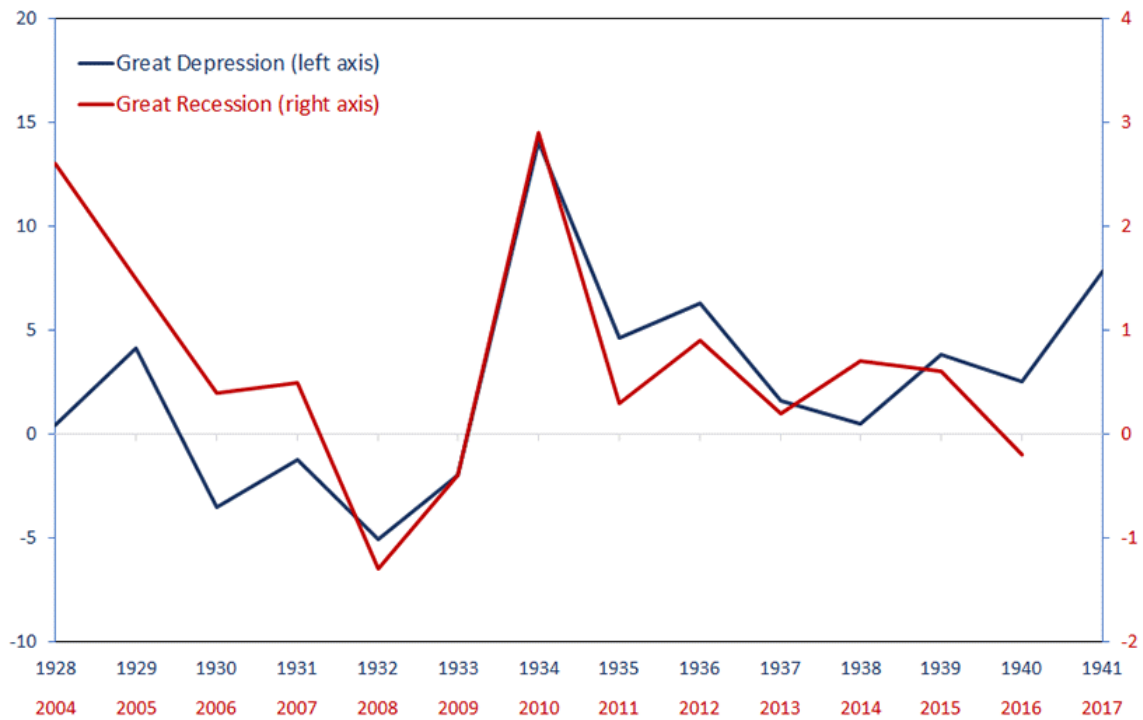
The only reason for them to be poorer is that they must have experienced a net negative set of transitory shocks relatively recently. So they are basically the opposite of group 1 in all respects

- c) Consumers whose saving rate has been unusually high for several years in a row before  $t$ .

*Answer:*

There are two reasons you might have a high saving rate in a particular year: Your income could be temporarily higher than average this year (like group 1). Or you could have experienced a set of negative shocks that made you poorer than other people. However, only the latter group of people would have a *sustained* higher-than-average saving rate, as they attempt to build their assets back up toward their targets. So, on average, this is likely to be a group of people whose consumption and wealth will be rising over time. But since they were selected on the basis of saving before period  $t$ , their average

**Figure 1 Total factor productivity growth in the Great Depression and the Great Recession (percent)**



income in  $t$  and expected income in future periods is the same as for everyone else. So their income growth is also expected to be the same as average.

## 2. Effects of a Positive Productivity Shock in an RBC Model.

The rate of productivity growth in the United States during the two-year period in 2009-10 was much higher than historical norms, though most forecasters by late 2010 were expecting that the rate of productivity growth would subside (as it did). Describe the effects that such a 'positive productivity shock' should have had on the total amount of labor hours during this period in a Real Business Cycle model of the type advocated by ?. Does this match well with the actual experience of 2008-2010? Can you think of another explanation for why productivity growth might have appeared strong, especially in 2009?

*Answer:*

In the Prescott model, a transitory positive productivity shock causes a boost to wages which induces people to work harder in order to take advantage of the temporarily high wage rate. Thus, the Prescott model would have implied robust real wage growth accompanied by a boom in labor hours during the 2009-2010 period. To put it mildly, this is not what happened.

Solow argued for many years that the Solow residual (the measure of “total factor productivity growth” depicted in the figure) was not a plausible indicator of the rate of technological progress over short periods of time like business cycles, because aggregate demand movements cause much more rapid movements in output than in employment. Basically, demand fell off a cliff, and while firms were firing people at a prodigious pace, it still took quite a while to adjust the labor force downward in face of lower demand. So, income fell more than employment (and the capital stock of course did not change much), leading almost mechanically to a decline in measured productivity. The point is that when the economy is not operating near its efficiency frontier, TFP is not a very sensible measure of “technology.”

## Medium-Length Questions

1. **Saving and Growth and Accounting.** Consider a **RamseyCassKoopmans** model with labor-augmenting technological progress at rate  $g$  and suppose there is no population growth ( $\xi = 0$  in the handout in the link above). Under standard assumptions (Cobb-Douglas production with capital share  $\alpha$ ; CRRA utility with risk aversion  $\gamma$ ), such a model can be normalized by the level of labor productivity, and we can solve a normalized version of the problem:

$$\max \int_0^\infty \left( \frac{c_t^{1-\gamma}}{1-\gamma} \right) e^{-\hat{\vartheta}t} dt \quad (1)$$

s.t.

$$\dot{k}_t = y_t - c_t - (\delta + g)k_t$$

$$y_t = k_t^\alpha$$

where  $\hat{\vartheta}$  is a ‘growth-adjusted’ time preference rate  $\hat{\vartheta} = (\vartheta - (1 - \gamma)g)$  where  $\vartheta$  is the ‘pure’ time preference rate.

If for simplicity we further assume further assume that depreciation is zero ( $\delta = 0$ ), the usual national accounting definition of the gross and the net national saving rates are the same:

$$\begin{aligned} s &= \frac{y - c}{y} \\ &= 1 - c/y \end{aligned}$$

? argues that this definition is inappropriate, because it does not account for the fact that in a growing economy, maintaining the value of  $k$  requires the saving rate to be adjusted for growth. This problem explores that proposition, and other relationships between saving and growth in and out of equilibrium.

- a) Using the fact from **RamseyCassKoopmans** that the consumption Euler equation is  $\dot{c}/c = \gamma^{-1}(f'(k) - (\hat{\vartheta} + g))$ , show that the steady-state value of  $k$  is

$$\check{k} = \left( \frac{\alpha}{\gamma g + \hat{\vartheta}} \right)^{\frac{1}{1-\alpha}} \quad (2)$$

*Answer:*

$$0 = \dot{c}/c = \gamma^{-1}(f'(\check{k}) - (\hat{\vartheta} + g))$$

$$\alpha \check{k}^{\alpha-1} = \hat{\vartheta} + g$$

$$\check{k}^{\alpha-1} = \left( \frac{\hat{\vartheta} + g}{\alpha} \right)$$

$$\begin{aligned}
\check{k}^{1-\alpha} &= \left( \frac{\alpha}{\hat{\vartheta} + g} \right) \\
\check{k} &= \left( \frac{\alpha}{\hat{\vartheta} + g} \right)^{1/(1-\alpha)} \\
&= \left( \frac{\alpha}{\vartheta + g(\gamma - 1) + g} \right)^{1/(1-\alpha)} \\
&= \left( \frac{\alpha}{\vartheta + g\gamma} \right)^{1/(1-\alpha)}
\end{aligned}$$

- b) Use this and the definition of the saving rate (2) to show that the optimal steady-state saving rate is

$$\check{s} = g\check{k}^{1-\alpha}, \quad (3)$$

and determine whether the steady-state saving rate increases or decreases when the growth rate increases.

*Answer:*

At steady state,  $\dot{k} = 0$ . Thus,

$$\begin{aligned}
0 = \dot{k} &= \check{y} - \check{c} - g\check{k} \\
\check{s} &= \frac{\check{y} - \check{c}}{\check{y}} \\
&= \frac{g\check{k}}{\check{k}^\alpha} \\
&= g\check{k}^{1-\alpha}.
\end{aligned}$$

Substituting for  $\check{k}$  from (2), the steady state saving rate can be written

$$\check{s} = g \left( \frac{\alpha}{\gamma g + \vartheta} \right) = \frac{\alpha}{\gamma + \vartheta/g}.$$

Thus, the optimal saving rate depends on the time preference rate. While ? and ? had qualms about assuming  $\vartheta > 0$ , the mathematical necessity of the assumption in infinite horizon modeling has made it conventional. With  $\vartheta > 0$ ,  $g \uparrow$  would lead to  $\check{s} \downarrow$ : The optimal steady-state saving rate increases when the growth rate increases.

- c) In economic terms, explain the various considerations at work, and which might be likely to be strongest.

*Answer:*

In the partial equilibrium context, we learned in class that if the current level of permanent labor income is  $y$  and the expected growth

rate is  $g$ , human wealth is

$$h = \left( \frac{y}{r - g} \right) \quad (4)$$

and thus even small changes in  $g$  can have large effects on human wealth; a small increase in growth, say, will have a large positive effect on  $h$  and therefore a large positive effect on  $c$  and a large negative effect on  $s$ .

In that partial equilibrium discussion, though, the interest rate  $r$  was taken as exogenous. Here, the equilibrium interest rate is endogenous:

$$\begin{aligned} \check{r} &= \alpha \check{k}^{\alpha-1} \\ &= \alpha \left( \frac{\gamma g + \vartheta}{\alpha} \right) \\ &= \gamma g + \vartheta. \end{aligned}$$

Thus, according to the model, the equilibrium interest rate rises with the rate of productivity growth. This is because an increase in growth will result in lower capital per unit of effective labor. Under the usual assumption that  $\gamma > 1$  the increase in the interest rate exceeds the increase in the growth rate. While the pure “growth rate” contribution to the human wealth effect ( $g$  in (4) boosts human wealth) is negative, the “interest rate” effect on human wealth is of the opposite sign and larger.

(The size of the interest rate effect on saving can be interpreted as the net of the effects of growth on income and on consumption; the consumption effects, as usual, can be interpreted as income, substitution, and human wealth effects, but with the additional complication that the equilibrium level of nonhuman wealth is also affected by the change in  $g$ ).

This result was not emphasized in class because empirical evidence does not support the model’s implication that interest rates are higher in fast-growing countries. This may be because international capital markets are sufficiently open that no country can have much effect on them, or it may be for other reasons. But whatever the reason, the empirical fact that interest rates do not seem to be higher for fast-growing countries is a good reason to focus on the partial equilibrium result.

- d) Using a phase diagram, analyze the effects of an unexpected permanent increase in  $g$ . Next, make a graph showing the path of the aggregate saving rate over time after the economy switches into the fast-growth regime. *Explain the reason both graphs look the way they do in intuitive terms.* Discuss what

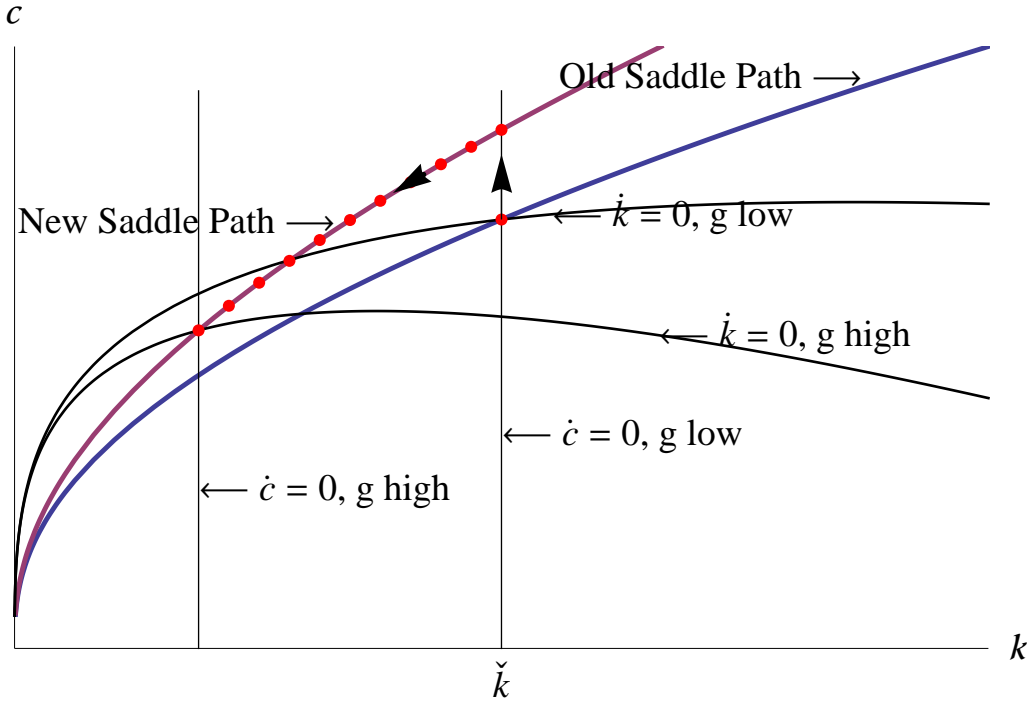


Figure 1

determines whether the saving rate rises or falls in the instant when consumers learn that growth has increased.

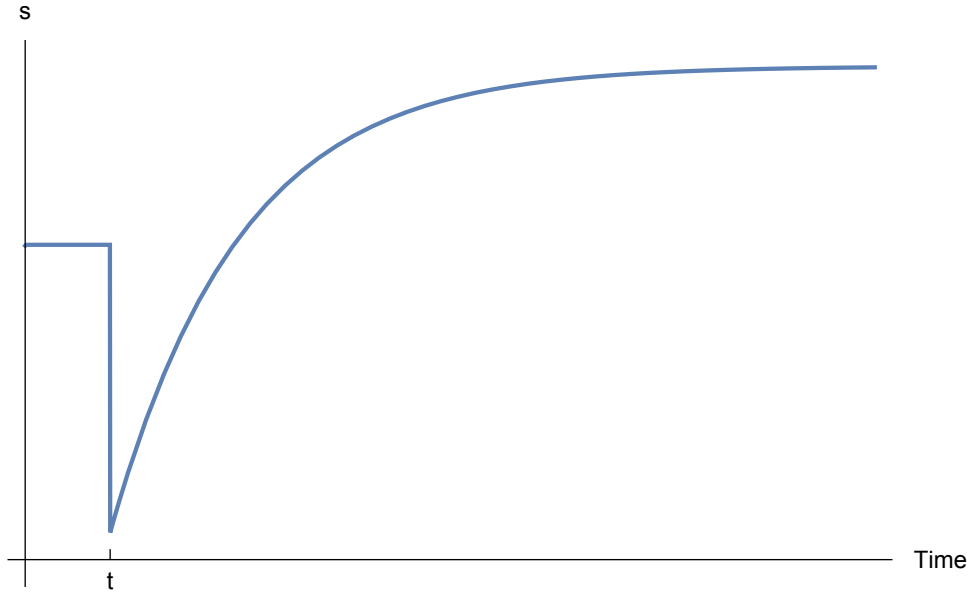
*Answer:*

The effects of  $g$  on the  $\dot{c} = 0$  locus are qualitatively the same as an increase in  $\vartheta$ : the locus moves left because the steady-state level of  $k$  is lower. The reason that  $\vartheta \uparrow$  and  $g \uparrow$  have similar effects is that increasing either of them makes consumers want to spend more now; higher  $\vartheta$  directly makes them more impatient, and higher  $g$  means that they will be richer in the future than they are now, and because they are forward-looking this extra future income means they can ‘afford’ to spend more now (this is the human wealth effect of the faster growth).

$g$  also affects the  $\dot{k} = 0$  locus. This is because  $k$  is capital per efficiency unit of labor, and if efficiency units of labor start growing faster, then capital must also grow faster in order to maintain capital-per-efficiency-unit constant.

The key insight for understanding the behavior of the saving rate is that the increase in  $g$  also implies that the rate of return on capital increases, because any specific amount of physical capital will be more productive than previously expected because it will be





**Figure 2**

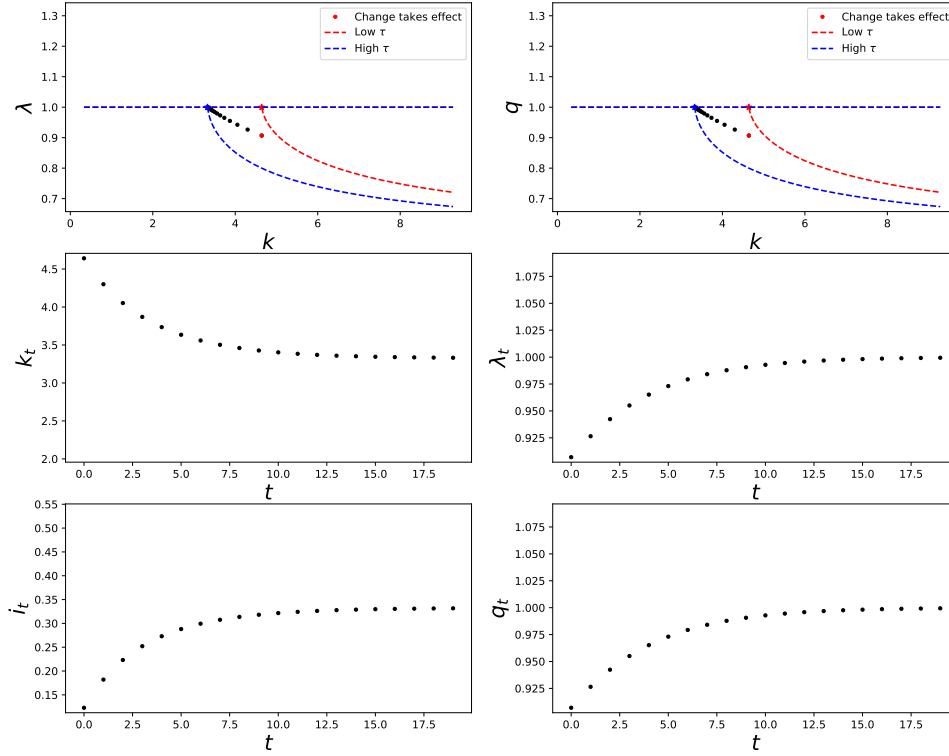
combined with more effective labor in the future. The higher interest rate, in turn, eventually gives the representative agent an incentive to save more than at the old interest rates.

But immediately after the growth increase, the value of  $k$  is the same as it was before, and therefore interest rates are the same. As a result, the initial effect on the saving rate is negative. This is because, at the same interest rate and the same  $k$ , both substitution and income effects are zero. The only one of the three usual effects that is operative is the human wealth effect: The consumer anticipates being richer in the future, and that justifies more spending now.

Several things are unambiguous: (1) since the new equilibrium  $k$  is lower, the new equilibrium interest rate is higher than before the increase in  $g$ ; (2) the final steady-state saving rate is higher than the saving rate experienced immediately after the increase in  $g$ , because with faster growth more saving is needed to maintain the capital/income ratio constant (this is partly, but not fully, offset by the fact that the steady-state capital/income ratio is lower); (3) the variation in saving rates over time is less for  $\gamma$  large than for  $\gamma$  small, because for  $\gamma$  large consumers are less willing to substitute consumption over time.

## 2. Dynamics of Investment in Response to a Temporary $\tau \uparrow$ in the $\varphi$ Model.

Answer the following questions using an  $\varphi$ - $\tau$  model of investment.



**Figure 3** Instant  $\tau$  increase

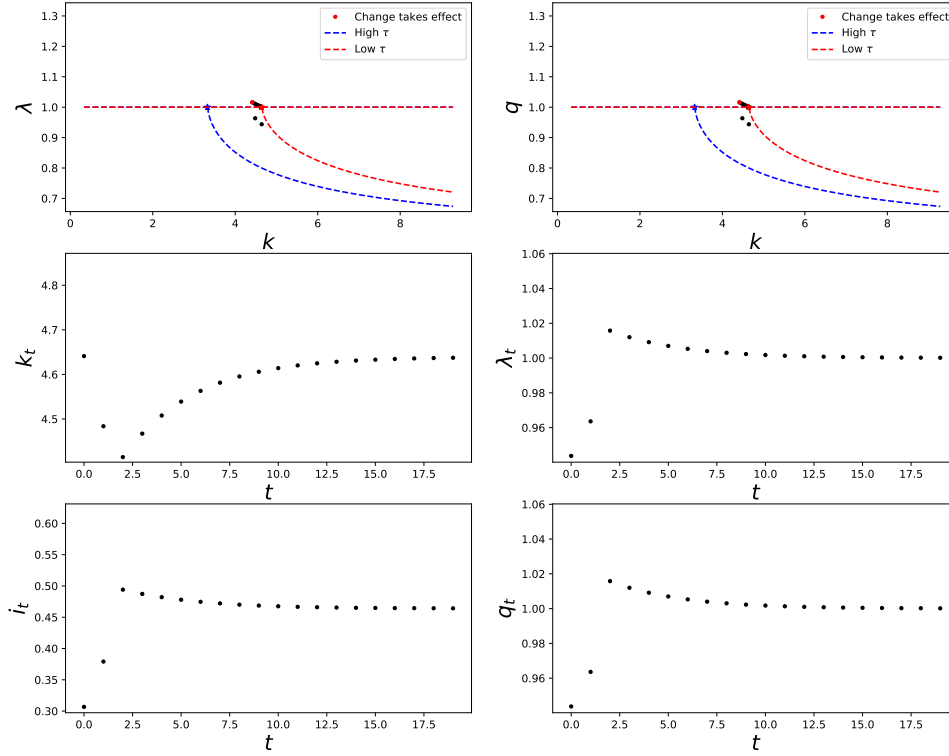
You are expected to answer the questions not just quantitatively (e.g., with figures or numbers) but also conceptually. That is, you must explain, in intuitive terms, *why* the variables do what they do.

- a) Leading up to date  $t$ , the economy is in steady state. At date  $t$ , the government unexpectedly introduces a permanent increase in the corporate tax rate,  $\tau \uparrow$ . Show the effects on a phase diagram and show dynamics of investment, capital, share prices, and  $\varphi$  following the tax change. In particular explain what, if anything, happens to  $\lambda$ , the share price of the firm, when the  $\tau$  is implemented.

*Answer:*

The effects on the phase diagram and optimal dynamics are presented in Figure 3. The figure shows that  $\lambda$  instantly drops when the change is announced.

- b) Leading up to date  $t$ , the economy is in steady state. At date  $t$ , the government unexpectedly introduces a *temporary* increase in the corporate tax



**Figure 4** Temporary  $\tau$  increase

rate,  $\tau \uparrow$ . The high  $\tau$  will last for two years, and then the  $\tau$  will revert back to its normal level. Show the effects on a phase diagram and show dynamics of investment, capital, share prices, and  $q$ , and the capital stock under two scenarios: (1) costs of adjustment for the capital stock,  $\omega$ , are high; (2) costs of adjustment are low. EXPLAIN your results.

*Answer:*

The effects on the phase diagram and optimal dynamics are presented in Figure 4.

The way to think about problems like this is, first, to figure out what the long-run phase diagrams will look like, then to figure out what the phase diagram will look like in the short run. A key point to realize is that the  $\Delta k$  phase diagram at any moment of time is a purely mechanical budget equation, so it is usually fairly easy to figure out what happens to the  $\Delta k$  locus. A second point is that the tax terms that enter the equations are always the value of those

taxes at the current date; any effects of future taxes have to come through their effects on  $\lambda$  or  $\varphi$ .

Upon introduction of the temporary increase in  $\tau$  at date  $t$ ,  $\lambda$  jumps down by exactly the amount such that if it evolves according to the new equations of motion it will arrive back at the original saddle path at date  $t + 2$ , and thereafter,  $\lambda$  will move on the original saddle path to the old equilibrium level. The point here is that there can be no anticipated jump in  $\lambda$ . When costs of adjustment for the capital stock are low, the initial jump of  $\lambda$  is big; when costs of adjustment for the capital stock are high, the initial jump of  $\lambda$  is small.

$\lambda$  jumps down because the higher corporate tax makes the rate of profit per unit of capital (marginal or average) smaller.

Dynamics of  $\varphi$ : since there is no change in the outside price of capital,  $\varphi$  dynamics mimic those of  $\lambda$ .

Dynamics of  $i$ :  $i$  closely follows the dynamics of  $\varphi$  since  $i = (\iota(\varphi) + \delta)k = (\frac{\varphi-1}{\omega} + \delta)k$ . At date  $t$ ,  $i$  jumps down. Between date  $t$  and date  $t + 2$ ,  $i$  increases gradually. After  $t + 2$ ,  $i$  gradually goes back to the initial level.

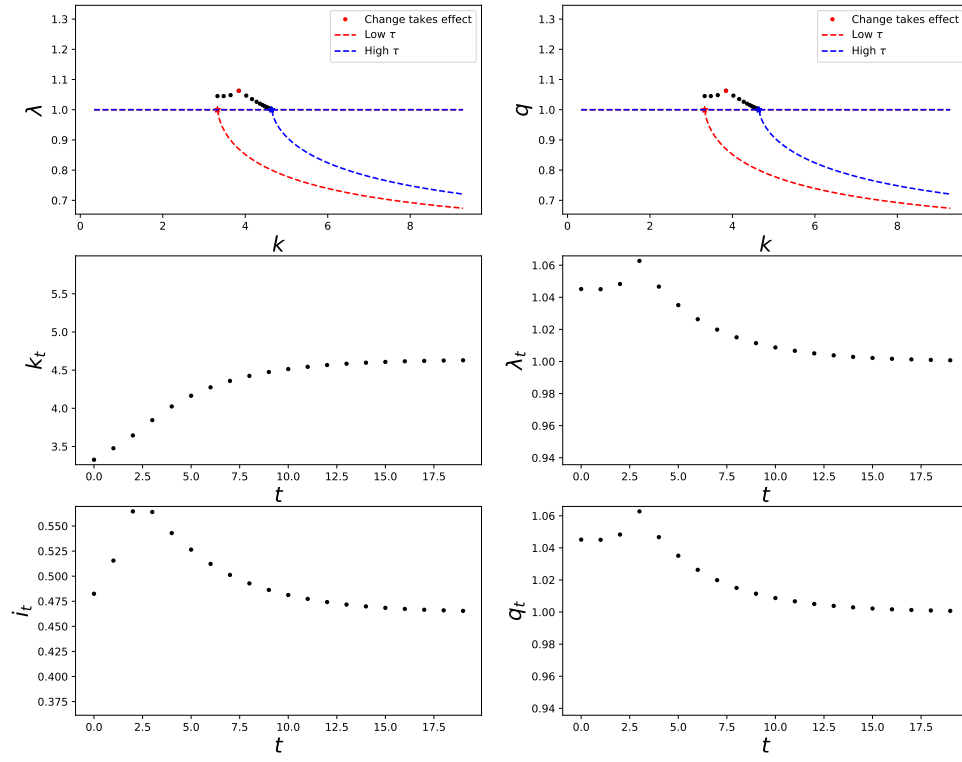
Dynamics of  $k$ :  $k$  decreases between date  $t$  and date  $t + 2$  and then increases to its original level.

- c) Leading up to date  $t$ , the economy is in steady state, and a  $\tau$  of 20 percent has existed since the beginning of time. At date  $t$ , the government unexpectedly *announces* that in three years (that is, in year  $t + 3$ ), there will be a *permanent* decrease in the corporate tax rate,  $\tau \downarrow \bar{\tau}$ . Show and explain the effects on a phase diagram and show dynamics of investment, capital, share prices, and  $\varphi$ , and the capital stock under two scenarios: (1) costs of adjustment for the capital stock,  $\omega$ , are high; (2) costs of adjustment are low. EXPLAIN your results.

*Answer:*

The effects on the phase diagram and optimal dynamics are presented in Figure 5.

Upon announcement of the permanent decrease in  $\tau$  in three years at date  $t$ ,  $\lambda$  jumps up and then gravitates to the saddle path dictated by the new  $\tau$ . At date  $t + 3$ ,  $\lambda$  will be on the new saddle path when the permanent  $\tau$  increase is introduced. Afterwards,  $\lambda$  will move on the new saddle path to the new equilibrium level. The point here is again that there can be no anticipated jump in  $\lambda$ . When costs of adjustment for the capital stock are low, the initial jump of  $\lambda$  is big; when costs of adjustment for the capital stock are high, the initial jump of  $\lambda$  is small.



**Figure 5** Future  $\tau$  increase

Dynamics of  $\varphi$ : since there is no change in the outside price of capital,  $\varphi$  follows the dynamics of  $\lambda$ .

Dynamics of  $i$ :  $i$  closely follows the dynamics of  $\varphi$  since  $i = (\iota(\varphi) + \delta)k = (\frac{\varphi-1}{\omega} + \delta)k$ .

Dynamics of  $k$ :  $k$  increases even between  $t$  and date  $t + 3$  in order to have capital already installed when the change takes effect. This is because of convexity in the adjustment costs: the firm prefers to slowly build up capital in advance rather than making large investments after the change takes effect. After the change,  $k$  continues increasing smoothly.

The point here is that if firms know that capital will be much more productive three years in the future, they will have a strong incentive to anticipate purchases of capital even before the change.

- d) Do your results have any implications for whether, when lawmakers introduce a bill to increase the corporate tax rate, they will want to make it ‘retroactive’ (that is, if the  $\tau$  change ever passes, it would apply to profits made during the period between the introduction of the bill and its passage into law). Is this the same as or different from the implications for the ITC?

*Answer:*

Even a member of Congress is smart enough to know that if a cut in the  $\tau$  is passed that will not take effect until some future date, firms will increase investment even before the tax change comes into effect so, they can take advantage of lower future taxes. Since Members of Congress generally want investment to be high, they would be willing to pass a future tax cut even if it is not retroactive.

This implication is different from the ITC because it does not alter the outside price of capital. Therefore, the incentive to postpone future investment until the change takes effect is not present.

- e) How would your results to the previous question change if corporations had some accounting tricks they could use, at a smoothly convex increasing cost, to shift “reported” profits between calendar years? (By smoothly convex increasing costs, we mean costs like those for investment in the  $q$  model; that is, the marginal cost to move a negligible amount  $\epsilon$  of reported taxes between  $t$  and  $t + 1$ , is tiny (proportional to  $\epsilon^2$ ). (You do not need to make diagrams or do quantitative analysis for this question; intuition is enough).

*Answer:*

If tax-shifting costs were zero, corporations would report all of their profits as occurring in periods where the tax rate is lowest. If costs of tax shifting are very large, they will do little of it.

# Long Question

## 1. Labor Income and Capital Income Uncertainty Over the Business Cycle.

Consider an employed consumer who ordinarily behaves according to the **TractableBufferStock** model discussed in class, in which there is a probability  $\mathfrak{U} > 0$  of becoming permanently unemployed. At the end of period  $t$ , after the consumption decision has been made, this consumer is offered a one-time-only, never-to-be-repeated opportunity (an “MIT shock”) to invest in a risky asset. The returns on the asset will be revealed at the beginning of period  $t + 1$ , and thereafter the consumer will behave again according to the optimal solution to the standard tractable buffer stock model.

The risky asset’s return factor  $\mathbf{R}_{t+1}$  will take one of two values:

$$\mathbf{R}_{t+1} = \begin{cases} \underline{\mathbf{R}} = 0 & \text{with probability } \wp > 0 \text{ (a stock ‘crash’)} \\ \bar{\mathbf{R}} = \left( \frac{\mathbf{R}\Phi}{1-\wp} \right) & \text{with probability } \wp \equiv (1 - \wp) \end{cases} \quad (5)$$

where  $\Phi > 1$  is the ‘risky return premium’ that compensates the consumer for the fact that the asset is riskier than the safe asset which earns  $\mathbf{R}$ . (Assume that the unemployment shock and the stock crash events are statistically independent.)

Call the share that the consumer invests in the risky asset  $\varsigma_t$ . Thus, if  $b_{t+1} = \mathbf{R}a_t$  is the bank balances the consumer would have with zero investment in the risky asset, the consumer arrives in period  $t + 1$  with bank balances of

$$\tilde{b}_{t+1} = b_{t+1} (1 + \varsigma_t \phi_{t+1}) \quad (6)$$

where  $\phi_{t+1} \equiv (\mathbf{R}_{t+1}/\mathbf{R} - 1)$  is the ‘excess return’ on the risky asset yielding two possible realizations of  $\tilde{b}_{t+1}$ :

$$\tilde{b}_{t+1} = \begin{cases} \bar{b}_{t+1} \equiv b_{t+1}(1 + \varsigma_t \bar{\phi}) & \text{with probability } \wp = (1 - \wp) \\ \underline{b}_{t+1} \equiv b_{t+1}(1 - \varsigma_t) & \text{with probability } \wp \end{cases} \quad (7)$$

a) Explain why the consumer’s portfolio choice problem is solved by

$$\max_{\varsigma_t} \wp (v^u(\underline{b}_{t+1})\mathfrak{U} + v^e(\underline{b}_{t+1} + 1)\mathfrak{J}) + \wp (v^u(\bar{b}_{t+1})\mathfrak{U} + v^e(\bar{b}_{t+1} + 1)\mathfrak{J}) \quad (8)$$

*Answer:*

The consumer’s goal is to maximize expected value,

$$\max_{\varsigma_t} \mathbb{E}_t[v(\tilde{m}_{t+1})] \quad (9)$$

and (8) simply writes out explicitly the elements of that expected value, equal to the probability-weighted outcomes that the consumer will experience in each of the four possible conditions: Risky asset bust, unemployed; risky asset bust, employed; risky asset boom, unemployed; risky asset boom, employed.

- b) Prove that the first order condition for the optimal portfolio share can be rewritten as below, and explain this condition.

$$\mathbb{E}_t[\phi_{t+1}u'(c(\tilde{m}_{t+1}))] = 0. \quad (10)$$

*Answer:*

$$\max_{\varsigma_t} \mathbb{E}_t[v(a_t(R + (\mathbf{R}_{t+1} - R)\varsigma_t) + 1)] \quad (11)$$

but because the portfolio choice is assumed to have no effect on probability of becoming unemployed, this is equivalent to

$$\mathbb{E}_t[v'(\tilde{m}_{t+1})(\mathbf{R}_{t+1} - R)a_t] = 0 \quad (12)$$

but since  $a_t$  is predetermined at the time that  $\varsigma_t$  is chosen, and since the Envelope theorem tells us that  $u'(c_{t+1}) = v'(\tilde{m}_{t+1})$ , (12) is equivalent to (10) (dividing by  $R$ ).

The ‘explain’ part is as follows. Defining  $\bar{c} = \mathbb{E}_t[\tilde{c}_{t+1}]$  so that  $\tilde{c}_{t+1} = \bar{c} + \zeta_{t+1}$  for some  $\zeta_{t+1}$ , if the expression  $\mathbb{E}_t[\phi_{t+1}u'(\bar{c} + \zeta_{t+1})]$  is zero while  $u'$  is always positive, it must be that the higher-order (‘risk’) terms end up being negative (to REALLY see why this is, notice that the  $\zeta_{t+1}$  will itself be lower when  $\phi_{t+1}$  is higher). So, on the one hand, positive expected mean return times positive expected marginal utility makes the whole expression want to be positive. But the negative covariance term (higher return, lower  $u'$ ) means that the risk terms contribute a negative component to the expectation. The FOC says the sum must be zero, which is saying that the advantages from holding the risky asset (from its higher return) must be weighed against the disadvantages from its riskiness (the negative covariance with  $u'$ ).

- c) Explain why (10) implies that this consumer will never choose to invest  $\varsigma_t = 1$  in the risky asset.

*Answer:*

If the consumer were to invest all his wealth in the risky asset and it returned  $\mathbf{R}_{t+1} = 0$ , and the consumer also became unemployed in that same period, his  $\tilde{m}_{t+1}$  would be zero forcing consumption to be zero and so his marginal utility would be infinity and so the expectation in (10) could not be satisfied for any  $\wp$  and  $\mathfrak{U}$  both  $> 0$ .

- d) Now define  $m_{t+1} = b_{t+1} + 1$  as the value that  $m_{t+1}$  would take in the absence of the opportunity to invest in the risky asset, so that  $\tilde{m}_{t+1} = m_{t+1} + b_{t+1}\phi_{t+1}\varsigma_t$  and explain how the consumption function can be approximated by

$$c(\tilde{m}_{t+1}) \approx c(m_{t+1}) + c'(m_{t+1})b_{t+1}\phi_{t+1}\varsigma_t. \quad (13)$$



*Answer:*

(13) is a first order Taylor expansion of  $c(\tilde{m}_{t+1})$  around  $\varsigma_t = 0$  using the definition of (6).

- e) Use the approximation (13) to explain why the consumer will not choose  $\varsigma_t = 0$  by showing that at  $\varsigma_t = 0$  a small increase in the risky share of the portfolio would increase expected utility. Explain the intuition behind this result.

*Answer:*

$$\mathbb{E}_t [\phi_{t+1} (u'(c(m_{t+1})) + u''(c(m_{t+1}))c'(m_{t+1})b_{t+1}\phi_{t+1}\varsigma_t)] \approx 0 \quad (14)$$

Following up on the hint, if none of the portfolio is invested in the risky asset, then the covariance between the risky asset's returns and consumption must be zero, because the second term in the equation above is zero. But then for  $a_t > 0$ , since  $\mathbb{E}_t[\phi_{t+1}] > 0$  and  $\mathbb{E}_t[u'(c(m_{t+1}))] > 0$  and at  $\varsigma = 0$  the covariance is zero, it must be the case that at  $\varsigma$  the expectation on the LHS of is a strictly positive number.

This is a general implication of the C-CAPM model: any asset whose covariance with consumption is zero, and which earns a positive expected return premium, must be marginally preferable to the riskless asset. So the consumer will want to increase his share in the risky asset from zero to some positive amount.

- f) Using insights implicit in the questions above (even if you were not able to do all of the derivations), answer the following questions:
- i. Explain why a model like this implies that, holding the expected excess return  $\phi$  constant, higher perceived unemployment risk causes consumers to be less willing to invest in the risky asset.
  - ii. Explain why a model like this implies that, holding the expected excess return  $\phi$  constant, a consumer who perceives that the riskiness of investing has gone up will want to invest less in the risky asset
  - iii. Explain why the 'holding the expected return  $\phi$  constant' is not a very sensible thing to do. What are the implications of the experiments above for asset prices if  $\phi$  is determined in general equilibrium?

*Answer:*

- g) Now consider informally some potential business cycle implications of these interactions. Suppose that there is an exogenous increase in perceived riskiness of financial investments (perhaps caused by the collapse of a financial

institution like Lehmann Brothers). Capture this by an increase in the assumed value of  $\wp$ .

- i. What implications does this perceived increase in  $\wp$  have for the levels of asset prices and for consumption?

*Answer:*

Asset prices and consumption will both fall

- ii. Suppose that these consumers live in a ‘New Keynesian’ economy in which the amount of ‘aggregate demand’ (part of which is consumption demand) affects firms’ hiring decisions in the next period. (Lower aggregate demand means that firms need less labor so  $\mathcal{U}$  turns out to be higher.) Suppose further that consumers update their expectations of  $\mathcal{U}$  based on experience:  $E_t[\mathcal{U}_{t+1}] = \mathcal{U}_t$ . Discuss the potential interesting dynamics, when, after they see the period  $t+1$  consequences of the aggregate demand shock caused by the ‘Lehmann shock,’ they update their expectations of  $\mathcal{U}$ .

*Answer:*

The idea is simple: When people are surprised to see the unemployment rate go up, they project that the future unemployment rate will also be higher than they expected. Greater uncertainty induces greater prudence (a stronger precautionary saving motive) so they cut back on spending. That cutback on spending leads to a further negative shock to aggregate demand, and so on.

- iii. Suppose a fiscal policymaker understands how all of this works. What options might the policymaker have to respond to the circumstances? Discuss when a fiscal response might be welfare improving.

*Answer:*

A fiscal stimulus package that puts money in people’s pockets could counteract the negative impulse to aggregate demand coming from the precautionary motive, by giving money to people with a high enough marginal propensity to consume.

- iv. Consider an alternative model in which all consumers behave according to the Merton-Samuelson model examined in **Portfolio-CRRA**. The handout **C-With-Optimal-Portfolio** shows that if such consumers have logarithmic utility, an increase in riskiness of returns has no effect on consumption. How would your responses to the question above change in this model?

*Answer:*

If there is no effect on consumption there is no multiplier/accelerator effect of the increase in uncertainty.

- v. Consider another alternative in which, in equilibrium, half of aggregate consumption is done by Merton-Samuelson consumers, and half by ‘rule-of-thumb’ consumers who simply spend all of their income in every period. How would your answers to the questions above change?

*Answer:*

Neither Merton-Samuelson consumers with log utility, nor  $C = Y$  consumers, change their consumption. If neither subgroup responds, then the aggregate economy does not respond. So in this special case, there is no effect.