Name ____________________

Midterm Exam

Intertemporal Choice

Fall, 2019

Answers

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up.* The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Part I

Buffer Stock Savers with Sticky Expectations. This question asks you to combine insights from the tractable buffer stock model and the sticky expectations model.

StickyExpectationsC examined C dynamics in an economy populated by consumers who, if they had perfect information, would be **?** random walk consumers:

$$c_t = o_t \kappa, \tag{1}$$

but instead of rational they have 'sticky' expectations, so they consume $\hat{o}_{i,t}\kappa$ where $\hat{o}_{i,t}$ is consumer i's *perceived* value of o_t. If the probability with which each consumer updates information about the macroeconomy is Π, and the only kinds of shocks in this economy are transitory, aggregate C dynamics will be characterized by

$$\Delta C_{t+1} = (1-\Pi)\mathsf{R}\Delta C_t + \underbrace{\Pi\hat{\Theta}_{t+1}\kappa}_{\equiv \xi_{t+1}} \tag{2}$$

where $\hat{\Theta}_{t+1} = \Theta_{t+1} - 1$ is the deviation of the aggregate transitory shock in period $t+1$ from its mean value. (Assume $\mathbb{E}_t[\Theta_{t+n}] = 1 \ \forall \ n > 0$ where $\mathbb{E}_t$ is the rational expectation).

1. Consider an economy populated by Hall consumers which had experienced no transitory or permanent shocks for a long time leading up to period t and then experienced a one-time positive shock to transitory income so that $\hat{\Theta}_t > 0$. Draw a diagram showing the dynamics of aggregate consumption leading up to t and for several periods thereafter.

   Answer:

   Consumers who did not notice the shock (proportion $(1-\Pi)$ will not change their consumption. Consumers who did notice the shock will change their consumption by $\kappa\hat{\Theta}$. So the total change in aggregate consumption will be

   $$\begin{aligned} \Delta C_{t+1} &= (1-\Pi) \times 0 + \Pi \times \kappa\hat{\Theta}_t & (3) \\ &= \Pi\kappa\hat{\Theta}_t & (4) \end{aligned}$$

   Thereafter the dynamics of consumption increase as per the figure in StickyExpectationsC.

2. Suppose an econometrician studying this economy can directly observe the magnitude of the transitory and permanent shocks it experiences in real time. Show that this econometrician's prediction for the change in income in period t will be $\mathbb{E}_t[\Delta Y_{t+1}] = -\hat{\Theta}_t$.

   Answer:

   The econometrician expects tomorrow's transitory shock to be $\Theta_{t+1} = 1$. Since today's shock is $1 + \hat{\Theta}_t$, the expected change in income is $\mathbb{E}_t[\Delta Y_{t+1}] = \Theta_{t+1} - \Theta_t = -\hat{\Theta}_t$.

3. Suppose this econometrician were to estimate a Campbell-Mankiw model of aggregate C growth under the assumption that households' expectations of income growth match the rational expectation:

$$\Delta C_{t+1} \;=\; \alpha_0 + \alpha_1 \mathbb{E}_t[\Delta Y_{t+1}] \tag{5}$$

What would the econometrician obtain for the coefficient α_1? Explain why, and discuss what the econometrician would have to say about whether the economy exhibits 'excess sensitivity' and/or 'excess smoothness' relative to the random walk benchmark.

Answer:

$$\begin{aligned} \Delta C_t &= \Pi\kappa\hat{\Theta}_t && (6)\\ \Delta C_{t+1} &= (1-\Pi)\mathsf{R}\Delta C_t + \Pi \overbrace{\hat{\Theta}_{t+1}}^{=0 \text{ by assumption}} \kappa && (7)\\ &= (1-\Pi)\mathsf{R}\Pi\kappa\hat{\Theta}_t && (8)\\ &= (1-\Pi)\mathsf{R}\Pi\kappa(-\mathbb{E}_t[\Delta Y_{t+1}]) && (9) \end{aligned}$$

so a regression of the form (5) should yield coefficients of $\alpha_0 = 0$ and $\alpha_1 = -(1-\Pi)\mathsf{R}\Pi\kappa$. The coefficient is negative because the predicted change in income for period $t+1$ is negative while the change in C will be positive.

4. Now consider a different economy. This one is populated by buffer stock consumers, and it very occasionally exhibits a change in its growth rate. The econometrician observes this economy over a long time span, in the middle of which the economy exhibited a change in the growth rate of income. (For example, in the U.S. income grew faster in the period 1947-1973 than it has grown in the period 1973 to 2011). Suppose that in the earlier half of the sample consumers did not forsee that the growth rate might change; they believed it was permanently at some level Γ^A, and suppose further that when the growth rate changes to $\Gamma^B < \Gamma^A$ partway through the sample, consumers see this immediately and perceive that this change will be permanent (the econometrician can see this too). Draw diagrams showing the patterns of income growth and C growth over the entire sample, and explain approximately what coefficient α_1 the econometrician could expect to obtain in a regression like the one above if the sample size is long.

Answer:

C growth is constant at Γ^A leading up to the date at which income growth changes. In that period there is a sudden drop in the level of C (so a negative spike in the growth rate of C). Thereafter C growth will converge toward the new growth rate Γ^B from above.

5. Now suppose that the buffer stock consumers who populate this economy have sticky expectations rather than rational expectations, and the economy is subject

to transitory aggregate shocks like in the sticky expectations economy as well as permanent shocks to growth as above. Explain how the econometrician's estimate of α_1 is likely to depend on the sample size and the precision with which it is possible to measure permanent components versus transitory components of income growth.

Answer:

> If the sample is long and the magnitude of transitory shocks is small, the econometrician is likely to estimate $\alpha_1 \approx 1$ because most of the time the growth rate of consumption will be close to the growth rate of income. If the sample is short, or the transitory shocks are large relative to the permanent ones, the econometrician is likely to estimate a coefficient on α_1 closer to the one obtained in the Hall-sticky-expectations economy, because the bulk of the high frequency variation will resemble that of the regular sticky expectations model. (Basically, the effects of the permanent change in growth will be dominated by 'sticky expectations' dynamics that resemble those in the economy populated by Hall consumers.)

Part II

Precautionary Saving and Convex Marginal Utility.

Consider a consumer whose last period of life is T and who is trying to decide how much to save in period $T-1$. Suppose the interest factor and the time preference factor are $\mathsf{R} = \beta = 1$ and so consumer's dynamic budget constraint is

$$c_T = a_{T-1} + y_T. \tag{10}$$

and define an end-of-period value function as

$$\mathfrak{v}'_{T-1}(a_{T-1}) = \mathbb{E}_{T-1}[\mathrm{u}'(c_T)] \tag{11}$$

Assuming CRRA utility in periods T and $T-1$, draw a diagram that shows:

1. Marginal end-of-period value as a function of a_{T-1} if income is perfectly certain at $y_T = 0$
2. Marginal end-of-period value as a function of a_{T-1} if income is

$$y_T = \begin{cases} \epsilon & \text{with probability } 0.5 \\ -\epsilon & \text{with probability } 0.5 \end{cases} \tag{12}$$

3. Draw $\mathrm{u}'(m_{T-1} - a_{T-1})$ and explain why a_{T-1} increases as a result of either an increase in risk aversion or an increase in the size of uncertainty ϵ.

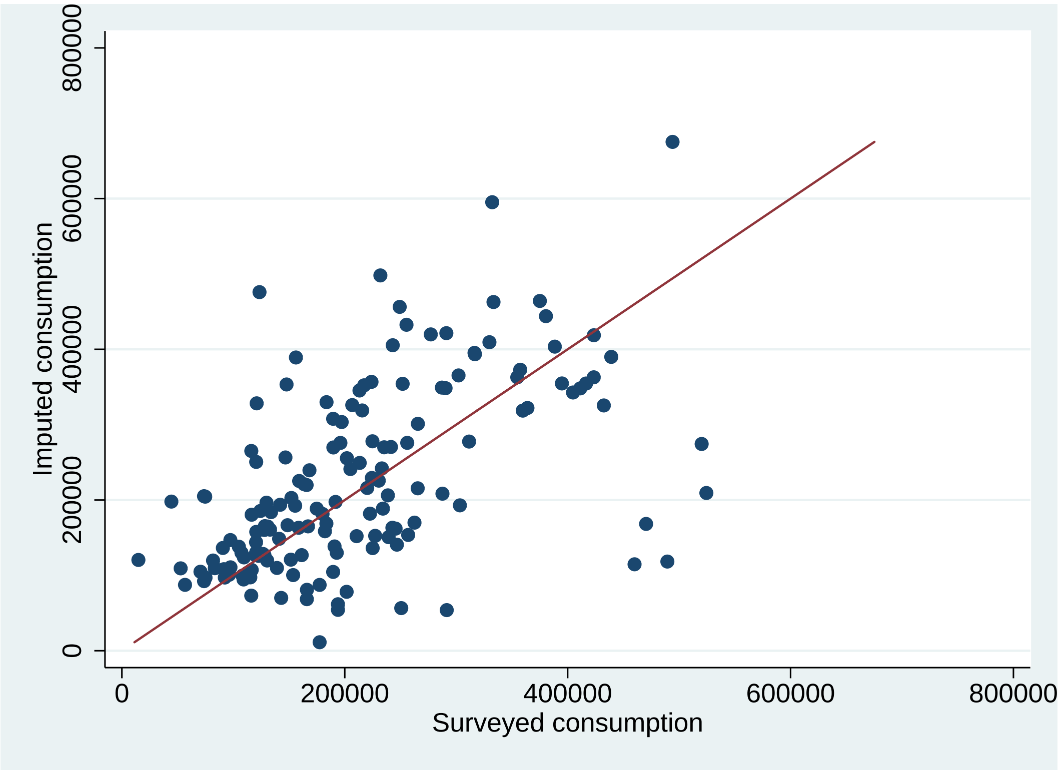
Answer:

This question essentially repeats the analysis presented in **?** discussed in class.

?'s Permanent Income Hypothesis With Noisy c**.** Answer the following questions under the assumption that Friedman's Permanent Income Hypothesis $c_i = \boldsymbol{p}_i$ is true. **?** present household spending data from two different sources ("survey" – designated below by an "s" superscript) and "imputed" (designated below by an "m" superscript) which can be assumed to have independent measurement errors $\epsilon_i^s \perp \epsilon_i^m$:

$$c_i^s = c_i + \epsilon_i^s \tag{13}$$

$$c_i^m = c_i + \epsilon_i^m \tag{14}$$



?

1. Suppose that the errors from the imputed measure of spending were $\epsilon_i^m = 0 \; \forall \; i$. That is, the imputation manages to capture the true level of spending perfectly. On the other hand, the errors from the survey have variance σ_s^2. Explain what coefficients you would expect to get from the following two regressions:

$$c_i^s = \alpha_0 + \alpha_1 c_i^m + u_i \quad (15)$$

$$c_i^m = \gamma_0 + \gamma_1 c_i^s + v_i \quad (16)$$

if $\sigma_c^2 = \sigma_{\epsilon^s}^2$.

Answer:

If $c_i^m = c_i$ and $c_i^s = c_i + \epsilon_i^s$ then the "true" version of (15) is

$$c_i^s = 0 + 1 \times c_i^m + u_i \quad (17)$$

where u_i is an iid variable. So the estimated coefficient should be $\alpha_1 = 1$.

Now rewrite (16) as

$$c_i^m = \gamma_0 + \gamma_1 (c_i + \epsilon_i^s) + v_i \quad (18)$$

The Friedman errors-in-variables formula tells us that if we estimate (16) we will obtain

$$\gamma_1 = \left(\frac{\sigma_c^2}{\sigma_c^2 + \sigma_{\epsilon^s}^2} \right) \quad (19)$$

but if $\sigma_c^2 = \sigma_{\epsilon^s}^2$ this reduces to $\gamma_1 = 1/2$.

2. The authors compare their results to those from a similar Danish study that also has data from similar "survey" and "imputed" sources. They find that when they run a regression like (16) their γ_1 coefficient is lower than the corresponding coefficient estimated in Danish data. What hypothesis about the relative variances of ϵ_i^s might explain these results? What other hypothesis might explain these results even if the variances of the ϵ_i^s variables are the same across Denmark and Sweden?

   Answer:

   A natural hypothesis is that the Swedish survey data have larger errors than the Danish ones. This would bias downward the estimate of γ_1 from the Swedish versus the Danish data. The authors present some evidence that the Swedish survey data are surprisingly inaccurate (though they do not have similar metrics for the Danish survey data so whether the Swedish data are actually *worse* than the Danish data cannot be answered using their results.)

   Another natural hypothesis is that maybe the variance in c is different in Denmark and Sweden. If, for example, the Danes have much larger differences in permanent income than do the Swedes (perhaps the welfare state is more generous in Sweden, pushing everyone's permanent income closer together), then even for the same size of survey measurement error, the relative size of that survey error compared to the differences in permanent income would be larger in Sweden.

3. Separately, the authors present persuasive evidence that suggests that their estimate of c_i^m is substantially better than the estimate of c_i^m from the Danish data (that is, their value of $\sigma^2_{\epsilon^m}$ is smaller than the $\sigma^2_{\epsilon^m}$ in the Danish data). If this is true, what differences might you expect in the coefficient on α_1 if the authors were to estimate that equation on their data?

   Answer:

   The same formula used before yields

   $$\alpha_1 = \left(\frac{\sigma_c^2}{\sigma_c^2 + \sigma^2_{\epsilon^m}} \right) \tag{20}$$

   and since σ_c^2 is by assumption the same in the two datasets, the implication of lower measurement error in the imputed data is that α_1 should be larger.