Name _____

Final Exam

Intertemporal Choice

Fall, 2019

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Short Questions.

1. **Predicting Microeconomic Behavior of Buffer Stock Consumers**

   Consider an economy populated by buffer stock savers subject only to transitory shocks to income. This economy has reached the equilibrium 'ergodic' distribution of wealth in date t; at the end of t you obtain a dataset containing several years' worth of recent data on a representative sample of the population. Answer the following questions about how *and why* the behavior of selected subsamples of this population can be predicted to be different from the population as a whole, in three dimensions: (1) Their expected labor income growth; (2) Their expected consumption growth; (3) Their expected wealth level growth

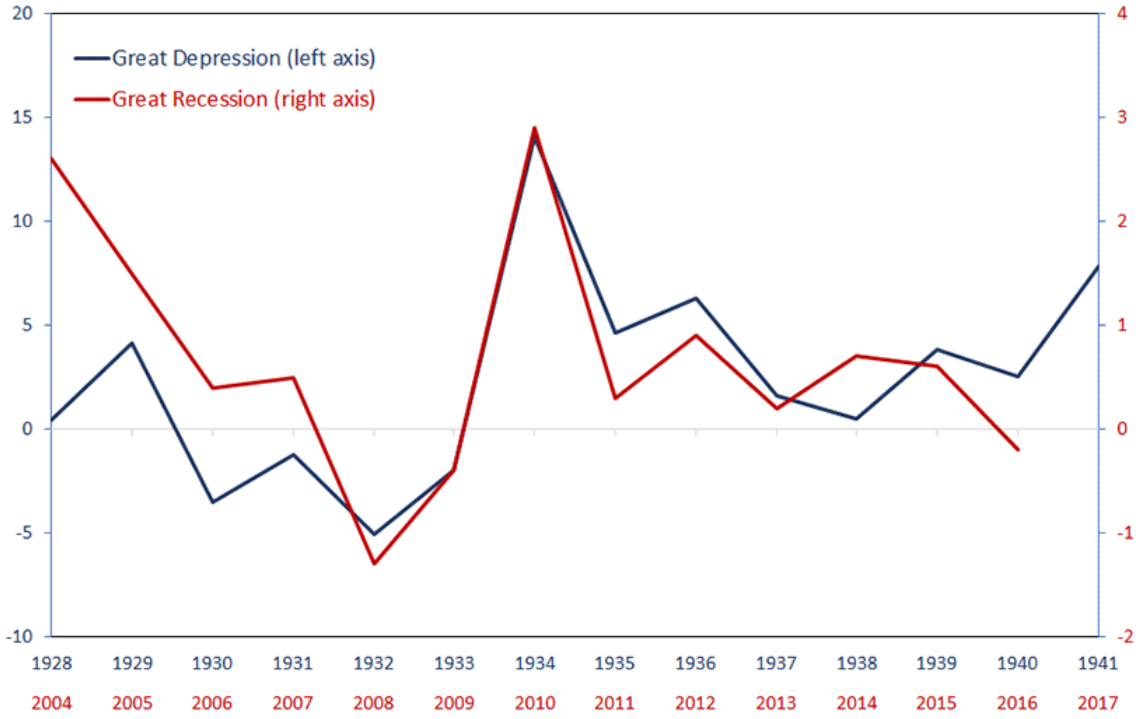
   For example, if your subsample were completely randomly selected you would say: "I expect their income growth, consumption growth, and wealth growth to be identical to that of the population, *because they were randomly selected.*"

   a) Consumers who in year t experienced a substantial positive shock to their income

   b) Consumers who in year t are poorer than average

   c) Consumers whose saving rate has been unusually high for several years in a row before t.

Figure 1 Total factor productivity growth in the Great Depression and the Great Recession (percent)



2. **Effects of a Positive Productivity Shock in an RBC Model.**

   The rate of productivity growth in the United States during the two-year period in 2009-10 was much higher than historical norms, though most forecasters by late 2010 were expecting that the rate of productivity growth would subside (as it did). Describe the effects that such a 'positive productivity shock' should have had on the total amount of labor hours during this period in a Real Business Cycle model of the type advocated by Prescott (1986). Does this match well with the actual experience of 2008-2010? Can you think of another explanation for why productivity growth might have appeared strong, especially in 2009?

Medium-Length Questions

1. **Saving and Growth and Accounting.** Consider a RamseyCassKoopmans model with labor-augmenting technological progress at rate g and suppose there is no population growth ($\xi = 0$ in the handout in the link above). Under standard assumptions (Cobb-Douglas production with capital share α; CRRA utility with risk aversion γ), such a model can be normalized by the level of labor productivity, and we can solve a normalized version of the problem:

$$\max \int_0^\infty \left(\frac{c_t^{1-\gamma}}{1-\gamma} \right) e^{-\hat{\vartheta} t} dt \tag{1}$$

$$\text{s.t.}$$
$$\dot{k}_t = y_t - c_t - (\delta + g) k_t$$
$$y_t = k_t^\alpha$$

where $\hat{\vartheta}$ is a 'growth-adjusted' time preference rate $\hat{\vartheta} = (\vartheta - (1-\gamma)g)$ where ϑ is the 'pure' time preference rate.

If for simplicity we further assume further assume that depreciation is zero ($\delta = 0$), the usual national accouting definition of the gross and the net national saving rates are the same:

$$\begin{aligned} s &= \frac{y - c}{y} \\ &= 1 - c/y \end{aligned}$$

Barro (2019) argues that this definition is inappropriate, because it does not account for the fact that in a growing economy, maintaining the value of k requires the saving rate to be adjusted for growth. This problem explores that proposition, and other relationships between saving and growth in and out of equilibrium.

a) Using the fact from RamseyCassKoopmans that the consumption Euler equation is $\dot{c}/c = \gamma^{-1}(\mathrm{f}'(k) - (\hat{\vartheta} + g))$, show that the steady-state value of k is

$$\check{k} = \left(\frac{\alpha}{\gamma g + \vartheta} \right)^{\frac{1}{1-\alpha}} \tag{2}$$

b) Use this and the definition of the saving rate (2) to show that the optimal steady-state saving rate is

$$\check{s} = g\check{k}^{1-\alpha}, \tag{3}$$

and determine whether the steady-state saving rate increases or decreases when the growth rate increases.

c) In economic terms, explain the various considerations at work, and which might be likely to be strongest.

d) Using a phase diagram, analyze the effects of an unexpected permanent increase in g. Next, make a graph showing the path of the aggregate saving rate over time after the economy switches into the fast-growth regime. *Explain the reason both graphs look the way they do in intuitive terms.* Discuss what determines whether the saving rate rises or falls in the instant when consumers learn that growth has increased.

2. **Dynamics of Investment in Response to a Temporary $\tau \uparrow$ in the φ Model.**

   Answer the following questions using an Abel (1981)-Hayashi (1982) φ model of investment.

   You are expected to answer the questions not just quantitatively (e.g., with figures or numbers) but also conceptually. That is, you must explain, in intuitive terms, *why* the variables do what they do.

   a) Leading up to date t, the economy is in steady state. At date t, the government unexpectedly introduces a permanent increase in the corporate tax rate, $\tau \uparrow$. Show the effects on a phase diagram and show dynamics of investment, capital, share prices, and φ following the tax change. In particular explain what, if anything, happens to λ, the share price of the firm, when the τ is implemented.

b) Leading up to date t, the economy is in steady state. At date t, the government unexpectedly introduces a *temporary* increase in the corporate tax rate, $\tau \uparrow$. The high τ will last for two years, and then the τ will revert back to its normal level. Show the effects on a phase diagram and show dynamics of investment, capital, share prices, and ϱ, and the capital stock under two scenarios: (1) costs of adjustment for the capital stock, ω, are high; (2) costs of adjustment are low. EXPLAIN your results.

c) Leading up to date t, the economy is in steady state, and a τ of 20 percent has existed since the beginning of time. At date t, the government unexpectedly *announces* that in three years (that is, in year $t+3$), there will be a *permanent* decrease in the corporate tax rate, $\tau \downarrow \bar{\tau}$. Show and explain the effects on a phase diagram and show dynamics of investment, capital, share prices, and ϱ, and the capital stock under two scenarios: (1) costs of adjustment for the capital stock, ω, are high; (2) costs of adjustment are low. EXPLAIN your results.

d) Do your results have any implications for whether, when lawmakers introduce a bill to increase the corporate tax rate, they will want to make it 'retroactive' (that is, if the τ change ever passes, it would apply to profits made during the period between the introduction of the bill and its passage into law). Is this the same as or different from the implications for the ITC?

e) How would your results to the previous question change if corporations had some accounting tricks they could use, at a smoothly convex increasing cost, to shift "reported" profits between calendar years? (By smoothly conves increasing costs, we mean costs like those for investment in the q model; that is, the marginal cost to move a negligible amount ϵ of reported taxes between t and $t+1$, is tiny (proportional to ϵ^2). (You do not need to make diagrams or do quantitative analysis for this question; intuition is enough).

Long Question

1. **Labor Income and Capital Income Uncertainty Over the Business Cycle.**

   Consider an employed consumer who ordinarily behaves according to the `TractableBufferStock` model discussed in class, in which there is a probability $\mathcal{U} > 0$ of becoming permanently unemployed. At the end of period t, after the consumption decision has been made, this consumer is offered a one-time-only, never-to-be-repeated opportunity (an "MIT shock") to invest in a risky asset. The returns on the asset will be revealed at the beginning of period $t+1$, and thereafter the consumer will behave again according to the optimal solution to the standard tractable buffer stock model.

   The risky asset's return factor $\mathbf{R}_{t+1}$ will take one of two values:

   $$\mathbf{R}_{t+1} = \begin{cases} \underline{\mathbf{R}} = 0 & \text{with probability } \wp > 0 \text{ (a stock 'crash')} \\ \bar{\mathbf{R}} = \left(\frac{\mathsf{R}\Phi}{1-\wp}\right) & \text{with probability } \not{\wp} \equiv (1-\wp) \end{cases} \tag{4}$$

   where $\Phi > 1$ is the 'risky return premium' that compensates the consumer for the fact that the asset is riskier than the safe asset which earns R. (Assume that the unemployment shock and the stock crash events are statistically independent.)

   Call the share that the consumer invests in the risky asset ς_t. Thus, if $b_{t+1} = \mathsf{R}a_t$ is the bank balnces the consumer would have with zero investment in the risky asset, the consumer arrives in period $t+1$ with bank balances of

   $$\tilde{b}_{t+1} \quad = \quad b_{t+1}\,(1 + \varsigma_t \phi_{t+1}) \tag{5}$$

   where $\phi_{t+1} \equiv (\mathbf{R}_{t+1}/\mathsf{R} - 1)$ is the 'excess return' on the risky asset yielding two possible realizations of $\tilde{b}_{t+1}$:

   $$\tilde{b}_{t+1} = \begin{cases} \bar{b}_{t+1} \equiv b_{t+1}(1 + \varsigma_t \bar{\phi}) & \text{with probability } \not{\wp} = (1-\wp) \\ \underline{b}_{t+1} \equiv b_{t+1}(1 - \varsigma_t\ \ ) & \text{with probability } \wp \end{cases} \tag{6}$$

   a) Explain why the consumer's portfolio choice problem is solved by

   $$\max_{\varsigma_t} \wp \left(\mathrm{v}^u(\underline{b}_{t+1})\mathcal{U} + \mathrm{v}^e(\underline{b}_{t+1}+1)\not{\mathcal{U}}\right) + \not{\wp}\left(\mathrm{v}^u(\bar{b}_{t+1})\mathcal{U} + \mathrm{v}^e(\bar{b}_{t+1}+1)\not{\mathcal{U}}\right) \tag{7}$$

b) Prove that the first order condition for the optimal portfolio share can be rewritten as below, and explain this condition.

$$\mathbb{E}_t[\phi_{t+1}\mathrm{u}'(\mathrm{c}(\tilde{m}_{t+1}))] \quad = \quad 0. \tag{8}$$

c) Explain why (8) implies that this consumer will never choose to invest $\varsigma_t = 1$ in the risky asset.

d) Now define $m_{t+1} = b_{t+1}+1$ as the value that m_{t+1} would take in the absence of the opportunity to invest in the risky asset, so that $\tilde{m}_{t+1} = m_{t+1} + b_{t+1}\phi_{t+1}\varsigma_t$ and explain how the consumption function can be approximated by

$$\mathrm{c}(\tilde{m}_{t+1}) \quad \approx \quad \mathrm{c}(m_{t+1}) + \mathrm{c}'(m_{t+1})b_{t+1}\phi_{t+1}\varsigma_t. \tag{9}$$

e) Use the approximation (9) to explain why the consumer will not choose $\varsigma_t = 0$ by showing that at $\varsigma_t = 0$ a small increase in the risky share of the portfolio would increase expected utility. Explain the intuition behind this result.

f) Using insights implicit in the questions above (even if you were not able to do all of the derivations), answer the following questions:

   i. Explain why a model like this implies that, holding the expected excess return ϕ constant, higher perceived unemployment risk causes consumers to be less willing to invest in the risky asset.

   ii. Explain why a model like this implies that, holding the expected excess return ϕ constant, a consumer who perceives that the riskiness of investing has gone up will want to invest less in the risky asset

   iii. Explain why the 'holding the expected return ϕ constant' is not a very sensible thing to do. What are the implications of the experiments above for asset prices if ϕ is determined in general equilibrium?

g) Now consider informally some potential business cycle implications of these interactions. Suppose that there is an exogenous increase in perceived riskiness of financial investments (perhaps caused by the collapse of a financial institution like Lehmann Brothers). Capture this by an increase in the assumed value of $\wp$.

i. What implications does this perceived increase in $\wp$ have for the levels of asset prices and for consumption?

ii. Suppose that these consumers live in a 'New Keynesian' economy in which the amount of 'aggregate demand' (part of which is consumption demand) affects firms' hiring decisions in the next period. (Lower aggregate demand means that firms need less labor so $\mho$ turns out to be higher.) Suppose further that consumers update their expectations of $\mho$ based on experience: $E_t[\mho_{t+1}] = \mho_t$. Discuss the potential interesting dynamics, when, after they see the period $t+1$ consequences of the aggregate demand shock caused by the 'Lehmann shock,' they update their expectations of $\mho$.

iii. Suppose a fiscal policymaker understands how all of this works. What options might the policymaker have to respond to the circumstances? Discuss when a fiscal response might be welfare improving.

iv. Consider an alternative model in which all consumers behave according to the Merton-Samuelson model examined in Portfolio-CRRA. The handout C-With-Optimal-Portfolio shows that if such consumers have logarithmic utility, an increase in riskiness of returns has no effect on consumption. How would your responses to the question above change in this model?

v. Consider another alternative in which, in equilbrium, half of aggregate consumption is done by Merton-Samuelson consumers, and half by 'rule-of-thumb' consumers who simply spend all of their income in every period. How would your answers to the questions above change?

References

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Barro, Robert J. (2019): "Double-Counting of Investment," *Manuscript, Harvard University*, Presentation at Johns Hopkins University macroeconomics seminar.

Hayashi, Fumio (1982): "Tobin's Marginal Q and Average Q: A Neoclassical Interpretation," *Econometrica*, 50(1), 213–224, Available at http://ideas.repec.org/p/nwu/cmsems/457.html.

Prescott, Edward C. (1986): "Theory Ahead of Business Cycle Measurement," *Carnegie-Rochester Conference Series on Public Policy*, 25, 11–44, http://ideas.repec.org/p/fip/fedmsr/102.html.