Name _____

Final Exam
Intertemporal Choice
Fall, 2015

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Part I. Long Question.

Theory and Evidence on the Effects of Credit Expansion.

Aydin (2015) reports the results of an experiment in which consumers with credit cards received an exogenous increase in their credit limit. This question asks you to think about what results you would expect from such an experiment if all the consumers behaved according to a modified version of the `TractableBufferStock` model.

The modification permits the model to analyze a change in credit availability. In order to explicitly incorporate a borrowing constraint that can be relaxed, the model must be changed in one respect: We will assume that unemployed consumers, rather than receiving income of zero, instead receive an 'unemployment insurance' (UI) benefit whose value in the first period ('period 1') of unemployment (the 'replacement rate') is μ times the value of their income in the last period of employment ('period 0'); the benefit thereafter grows by the same factor Γ as the aggregate wage.[1] To simplify the analysis, suppose that the government finances the UI program by some windfall source of revenue (say, an oil discovery) and thus does no increase in taxes on employed consumers is required to finance the new program.

1. Explain the 'growth impatience condition' and show that, if it holds for the unemployed consumer, the PDV of UI benefits that an employed consumer knows he will receive if he becomes unemployed next period, relative to his current income, is:

$$\underline{h} \;=\; \left(\frac{\mu}{1-\Gamma/\mathsf{R}}\right)/\mathsf{R} \tag{1}$$

   where Γ is the growth factor for aggregate wages, and R is the interest factor.

[1] This follows the approach elaborated in Carroll, Slacalek, and Sommer (2019).

2. Explain why, in a model with no exogenously imposed credit constraints, the creation of the UI program (an increase from $\mu = 0$ to $0 < \mu < 1$) implies that the consumer will be willing to borrow, but will never choose to allow his end-of-period cash-on-hand to fall to $-\underline{h}$ or below. Explain why the creation of the UI system can be interpreted as relaxing a 'natural borrowing constraint.'

3. Assume that before the creation of the UI system at date 1, the employed consumer was at the target level of market resources, $m_0 = \check{m}_0$. (For convenience, henceforth please assume that the growth factor for aggregate wages is $\Gamma = 1$.)

   a) Show how the consumption function diagram changes for the employed consumer, and show a series of dots that trace out the immediate and eventual effects on consumption and market resources.

b) Show that, if the MPC at the target level of wealth, $\check{\kappa}_0$, is much greater than the interest rate (as is true for plausible calibrations of the model for impatient consumers), the infinite-horizon 'propensity to consume' out of the 'windfall' represented by the introduction of the UI program is (very) approximately $-\mathsf{r}\underline{h}$. That is, show that, if $\check{c}_0$ was the target level of consumption before the expansion and $\check{c}_\infty$ is the new target toward which consumption tends, then $\check{c}_\infty - \check{c}_0 \approx -\mathsf{r}\underline{h}$. Explain why eventually consumption is *lower* as a result of the relaxation of the constraint.

c) Draw diagrams showing the time series paths for c and m, and indicate the eventual new 'steady-state' levels of c and m. How does concavity of the consumption function modify the results from what they would be if the consumption function were linear?

d) Consider an increase in credit availability that occurs at date 0. Out of such an increase, define the 'marginal propensity to *have consumed*' (MPTHC) by the date $n > 0$ as the amount by which market wealth m has declined divided by the amount by which credit was originally increased. That is, for a consumer whose wealth at date 0 was at its target value, $m_0 = \check{m}$, the marginal propensity to *have consumed* by date n would be $(m_0 - m_n)/\underline{h}$. Carroll (2001) shows that, in a model with a more realistic income process (transitory and permanent shocks calibrated to the PSID), a relaxation of an exogenous liquidity constraint results in a change in the target level of m that is about the same size as the change in the availability of credit. In the simplified model considered here, the results in Carroll (2001) would correspond to the proposition that $\lim_{n\uparrow\infty} m_0 - m_n \approx \underline{h}$. Under this assumption, draw a graph that shows the evolution of the MPTHC from date 0 toward date ∞.

4. Give some intuition for why the response to a relaxation of an 'artificial' borrowing constraint (such as a limitation imposed by lenders that requires the debt-to-income ratio to be smaller than a certain amount) is likely to be similar to the relaxation of the 'natural' borrowing constraint described above. (You can take this to be true; your answer will be judged by the quality of your intuitive explanation for why).

5. Aydin (2015) defines the *marginal propensity to consume out of liquidity* as 'the dollar response of debt to a $1 change in borrowing capacity.' Given your answer to the question above about the MPTHC, critique this definition, particularly with respect to what it has to say about the time horizon.

6. Given his definition of the MPCL, Aydin (2015) makes the following statements about what his data show. For each statement, discuss the claimed result by deciding whether it is either (a) consistent with the results above from the tractable model; (b) inconsistent with the results above; or (c) ambigious in whether it is consistent or inconsistent with the model.

    a) 'MPCL is significantly larger than zero for three quarters of the population, most of whom are not immediately constrained.'

b) ‘In the short-run, MPCL exhibits a clear heterogeneity in cash-on-hand, exclusively in line with a concave consumption function.’

c) 'In the long-run, MPCL is mean-reverting: an increase in credit capacity does not permanently shift leverage, but individuals accumulate debt only to deaccumulate it after 6-18 months.'[2]

[2]It must be noted that Aydin's evidence for this proposition seems weak. The figure cited as proof does not appear to me to exhibit any robust evidence of a tendency to decumulate. But, your task is to evaluate whether the proposition as stated is consistent with the model.

The final part of this question invites you to contemplate consequences of the paper's findings for the analysis of the Great Recession. Assume for the purposes of the discussion that the paper's results suggest that there are two types of consumers: Higly impatient ones, with a low target buffer stock of assets; and mildly impatient ones whose buffer stock is very large. Mian and Sufi in a series of papers, and especially Mian, Rao, and Sufi (2013), have suggested that the credit boom that led up to 2007 largely reflected an increase in credit to the 'impatient' consumers who had small buffer stocks of assets, and that the post-2007 contraction in credit was concentrated on the same set of consumers. Recent work by Adelino, Schoar, and Severino (2015) challenges this proposition, arguing instead that much of the credit boom was directed toward consumers with substantial assets and income.

7. Draw the consumption function diagrams for highly impatient and for mildly impatient consumers, and use those diagrams to explain what the model says about why it might matter, for the consumption response, whether the credit boom was directed toward patient consumers (with initially large amounts of assets) or toward impatient consumers (with initially small amounts of assets).

8. Some work has argued that another factor was involved in the consumption boom and subsequent bust: Uncertainty. Specifically, in the years leading up to 2007, macroeconomists were writing about the 'great moderation' in macroeconomic performance, the unemployment rate was stable at near-record low levels for many years, and direct measures of consumer uncertainty registered at low levels. After 2007, uncertainty by various measures spiked to near record high levels. Suppose that the degree of uncertainty faced by everyone is the same. Does the model imply that one group or the other (the 'highly impatient' or the 'mildly impatient') might react more to the increase in uncertainty? Sketch some implications for the research agenda.

Part II. Discussion Question.

Open Source Software and Knowledge Capital. "Open Source" software is created largely by computer programmers who are not directly paid for their contributions. Some of the most central components of the infrastructure of the internet are open-source (for example, the Apache web server software that powers perhaps half of the world's webservers; the FireFox browser; the LaTeX typographical system; and much more).

Explaining the production of valuable services by volunteer labor is a challenge for neoclassical economics (to say the least!). This question asks you to make a first stab at the job, using the Lucas (1988) growth model with an aggregate production function

$$Y = AK^{\alpha}(\ell h L)^{1-\alpha} \tag{2}$$

Recall that in that model, "human capital" h accumulates according to

$$\dot{h}/h \;\; = \;\; \phi(1-\ell) \tag{3}$$

where ℓ is the proportion of time that people spend working.

1. Write a paragraph or two about reasons for and against a reinterpretation of Lucas's model in which what he calls "human capital" is thought of as "knowledge capital" produced by people in their free-time contributions to open source knowledge projects. Comment, in particular, about what you need to assume about the parameter that captures the degree of externalities in aggregate h in order for this interpretation to have any force.

2. Suppose a government concludes that the value of open source software in boosting aggregate productivity is very large. Under the appropriate assumptions (that is, the assumptions that make the Lucas model sketched above appropriate for addressing the question), suppose the government wants to evaluate a subsidy that will increase the leisure time of computer programmers (in the belief that in their leisure time they will create more open-source software).

   - Assume that only computer programmers contribute to knowledge capital in the relevant sense (everybody else just contributes raw labor). Also, programmers are "born that way" (they can't help becoming programmers, and nobody else can become one) so that their proportion in the population remains constant at Π. Show that under these assumptions the effective aggregate production function becomes

     $$y = \hat{A}k^{\alpha}(\ell h)^{1-\alpha} \tag{4}$$

     where h is the knowldege capital per capita of the computer programmers and ℓ is the proportion of their hours they spend working.

   - Explain why the optimal policy for the government is likely to involve subsidizing the programmers *not* to work. Discuss how would the appropriate subsidy depends (qualitatively) on the different parameters in the model (including the Cobb-Douglas parameter for capital's share in output).

- Now suppose that labor market imperfections mean that the company that a programmer works for is able to temporarily capture a substantial portion of the "knowledge" benefits of the programmer's "free time" programming activities, but the amount that the company captures depends on the number of hours that the programmer spends working at the company (the ℓ). Will the company agree with the government about the optimal size of the subsidy? Explain why or why not, and discuss what determines whether the company is likely to benefit (on net) from the externalities mentioned above.

- Assuming that parameter values are such that the subsidy to leisure of programmers has a positive net value, would the model suggest that such a subsidy would have a positive effect on the level of output, on the growth rate of output, or both?

- How might such a subsidy compare, in its effects, to a more traditional policy like a permanent investment tax credit?

- Explain why the government's optimization problem becomes much harder if some people can pretend to be computer programmers even though their only real talent is in English Literature and they hate programming and in their free time they read Shakespeare instead of coding the latest update to Firefox.

Part III. An Entrepreneurial Country.

The handout `EntrepreneurPF` combines results from `PerfForesightCRRA` and `qModel` to produce a model of the behavior of an entrepreneur who owns a business whose investments incur costs of adjustment. You should be able to skim the handout to get the gist of how the model works (since it is basically a mashup of two frameworks that you are already familiar with). Use the model in the handout to answer the following questions, under the following assumptions:

- We are now reinterpreting the model to describe a small open economy run by a benevolent social planner, rather than a firm run by an entrepreneur
- Leading up to period t, the economy was in equilibrium with a stable and positive amount of monetary resources $\bar{m}$ and a constant (equilibrium) capital stock $\bar{k}$.
- For all three of the experiments below, show the dynamics of consumption, investment, national net worth, and the aggregate capital stock following the shock at period t, and explain why the results look the way they do.

1. Suppose the country experiences a war that suddenly destroys 10 percent of its physical physical capital stock, so that $k_t = 0.9\bar{k}$.

2. Suppose that year t was 2008 and the country had accumulated a substantial amount of monetary assets so that $a_{t-1} >> 0$, but those assets happened to be deposited in Lehmann Brothers right at the time it collapsed. The value of those assets therefore falls to zero.

3. Suppose now that what happens at date t is that another country starts producing the same goods that this country has produced before, and the consequence is that this country's production function for the firm suddenly, permanently, and unexpectedly drops; specifically, leading up to period t the firm was in steady state, but in periods $t+1$ and beyond the production function will be $\mathrm{f}_{\geq}(k) = \Psi \mathrm{f}_{<}(k)$ for some $\Psi < 1$ where $\mathrm{f}_{<}$ and $\mathrm{f}_{\geq}$ indicate the production functions before and after the increase in productivity.

Consider a firm characterized by the following:

k_t	-	Firm's capital stock at the beginning of period t
$\mathrm{f}(k)$	-	The firm's total output depends only on k
i_t	-	Investment in period t
$\mathrm{j}(i,k)$	-	AdJustment costs associated with investment i given capital k
$\xi_t = i_t + \mathrm{j}_t$	-	eXpenditures (purchases plus adjustment costs) on investment
$\beta = 1/\mathsf{R}$	-	Discount factor for future profits (inverse of interest factor)

Suppose that the firm's goal is to pick the sequence i_t that solves:

$$\mathrm{e}(k_t) = \max_{\{\mathrm{i}\}_t^\infty} \sum_{n=0}^{\infty} \beta^n \left(f_{t+n} - i_{t+n} - \mathrm{j}_{t+n}\right) \tag{5}$$

subject to the transition equation for capital,

$$k_{t+1} = (k_t + i_t)\urcorner \tag{6}$$

where $\urcorner = (1-\delta)$ is the amount of capital left after one period of depreciation at rate δ.[3] e_t is the value of the profit-maximizing firm: If capital markets are efficient this is the equity value that the firm would command if somebody wanted to buy it.

The firm's Bellman equation can be written:

[3]There are some small differences between the formulation of the model here and in `qModel`. Here, investment costs are paid at the time of investment and the depreciation factor applies to $(k_t + i_t)$ rather than just k_t. These changes simplify the computational solution without changing any key results.

Define j_t^i as the derivative of adjustment costs with respect to the level of investment.

The first order condition for optimal investment implies:

Now suppose that a steady state exists in which the capital stock is at its optimal level and is not adjusting, so costs of adjustment are zero: $\mathrm{j}_t = \mathrm{j}_{t+1} = j_t^i = j_{t+1}^i = j_t^k = j_{t+1}^k = 0$.

Another way to analyze this problem is in terms of the marginal value of capital, $\lambda_t \equiv \mathrm{e}_t^k(k_t)$.

We now wish to modify the problem in two ways. First, we have been assuming that the firm has only physical capital, and no financial assets. Second, we have been assuming that the manager running the firm only cares about the PDV of profits; suppose instead we want to assume that the firm is a small business run by an entrepreneur who must live off the dividends of the firm, and thus they are maximizing the discounted sum of utility from dividends $\mathrm{u}(c_t)$ rather than just the level of discounted profits. (Note that we designate dividends by c_t; dividends were not explicitly chosen in the ϱ-model version of the problem, because the Modigliani-Miller theorem says that the firm's value is unaffected by its dividend policy).

We call the maximizer running this firm the 'entrepreneur.' The entrepreneur's level of monetary assets m_t evolves according to

$$m_{t+1} = f_{t+1} + (m_t - i_t - \mathrm{j}_t - c_t)\,\mathsf{R}. \tag{7}$$

That is, next period the firm's money is next period's profits plus the return factor on the money at the beginning of this period, minus this period's investment and associated adjustment costs, minus dividends paid out (which, having been paid out, are no longer part of the firm's money).

The entrepreneur's Bellman equation can now be written

$$\mathrm{v}_t(k_t, m_t) = \max_{\{i_t, c_t\}} \quad \mathrm{u}(c_t) + \beta \mathrm{v}_{t+1}(k_{t+1}, m_{t+1})$$

Assume that f and j do not depend directly on m_t. That is, their partial derivatives with respect to m_t are zero.

Then we will have

Now note that the value function can be rewritten as

$$\mathrm{v}_t(k_t, m_t) = \max_{\{i_t, m_{t+1}\}} \quad \mathrm{u}((f_{t+1} - m_{t+1})/\mathsf{R} + m_t - i_t - \mathrm{j}_t) + \beta \mathrm{v}_{t+1}(k_{t+1}, m_{t+1})$$

For the version in (8) the FOC with respect to i_t is

$$\mathrm{u}'(c_t)((1+\mathrm{j}_t^i) - f_{t+1}^k \daleth/\mathsf{R}) = \daleth\beta v_{t+1}^k \tag{8}$$

Now we can use the envelope theorem with respect to k_t to show that

$$v_t^k = \mathrm{u}'(c_t)(f_{t+1}^k \daleth/\mathsf{R} - \mathrm{j}_t^k) + \beta\daleth v_{t+1}^k \tag{9}$$

Now we can combine (8) and (9) to derive the Euler equation for investment

$$(1 + \mathrm{j}_t^i) = \beth \beta \left[\mathrm{f}^k(k_{t+1}) + (1 + \mathrm{j}_{t+1}^i - \mathrm{j}_{t+1}^k) \right]. \tag{10}$$

Now consider a firm of this kind that happens to have arrived in period t with positive monetary assets $m_t > 0$ and with capital equal to the steady-state target value $k_t = \check{k}$.

Suppose that a thief steals all the firm's monetary assets.

Now consider another kind of shock: The firm's main building gets hit by a meteor, destroying some of the firm's capital stock.

References

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