Name _____

Final Exam

Intertemporal Choice

Fall, 2018

You are expected to answer all parts of all questions. If you cannot solve part of a question, *do not give up*. The exam is written so that you should be able to answer later parts even if you are stumped by earlier parts.

Write all answers on the exam itself; if you run out of room, use the back of the previous page.

Short Questions.

1. **Asymmetric Information and Investment.** The handout on imperfect capital markets and investment indicated that investment would occur only if the condition

$$\gamma > 1 + r + A(c, r, W, \gamma) \tag{1}$$

held true. Explain what each of the arguments of the A function are and briefly describe how and why an increase in each parameter would affect the amount of investment, holding all the other parameters constant. (Be sure to compare the effects with the effects, if any, that would occur in the standard 'perfect capital markets' model).

2. **Capital Market Imperfections and Slow Recoveries.** Reinhart and Rogoff (2009) present evidence that the recovery from recessions caused by financial crises tends to be slower than recoveries from other kinds of recessions. Use the model of capital market imperfections presented in class to discuss some reasons why this might be so (under the assumption that 'other kinds of recessions' mostly are caused by temporary factors that do not impede the efficient functioning of capital markets).

Medium-Length Questions

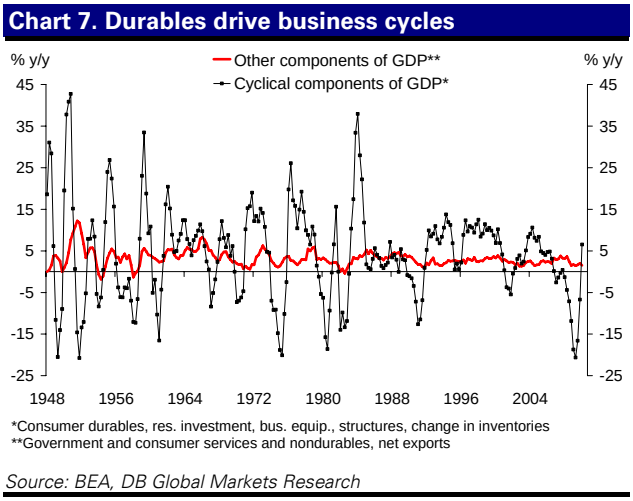
1. **Lucas (1978) Prices for Risky and Safe Firms.** Consider an island economy with two kinds of food production.

   - Pick berries from bushes that grow in a particular spot on the island. The production of berries is not very much affected by the weather, so the variation in berry production is not very great.
   - Obtain coconuts from a set of (identical) coconut trees. Coconut production varies much more from year to year because sometimes a monsoon will come and blow away many of the coconuts.

   a) Suppose the residents of the island participate in a perfectly efficient financial market in which shares to the production of the berry bushes and coconut trees are traded. What would the Lucas asset pricing model imply about the variation in the price of berry bushes compared to the variation in the price of coconut trees? Why?

b) Suppose global warming intensifies the size of weather fluctuations from year to year, making the amount of coconut production more variable but leaving the variability of berry production unchanged. What would the model imply about the effects of global warming on the average relative prices for trees versus berries?

Figure 1 Spending on Durables and Nondurables



c) Figure 1 shows the business-cycle movements of spending on durable goods and nondurable goods in the U.S.

   i. Interpret this figure in light of the Mankiw (1982) model of durables; specifically, explain why that model implies that expenditures on durable goods should be much more volatile than expenditures on nondurables.

   ii. Explain how this figure relates to the earlier discussion of the Lucas asset pricing model. In particular, explain what that model suggests about the average price-to-dividend ratio of firms that produce durable goods compared to that of firms that produce nondurable goods.

2. **Capital Dynamics in Growth Models**

a) `BrockMirman` shows that the Brock and Mirman (1972) model yields the following equation for capital in logs,

$$k_{t+1} = \log \alpha\beta + \alpha k_t + a_t \tag{2}$$

where a_t is the productivity shock in the production function (and recall that the model implies that $K_{t+1} = \alpha\beta Y_t$).

Show that this implies that the variance of k is

$$\text{var}(k) = \frac{\sigma_a^2}{1-\alpha^2} \tag{3}$$

Hints: $\text{var}(k_{t+1}) = \text{var}(k_t)$ and for some constant a and some stochastic variable b, $\text{var}(ab) = a^2 \,\text{var}(b)$.

b) Now we want to consider a case where the level of productivity a is serially correlated:

$$a_t = \zeta a_{t-1} + \nu_t \tag{4}$$

for some shock ν with variance σ_ν^2 and a serial correlation coefficient $0 < \zeta < 1$. Calculate the variance of k under this new assumption.

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c) Similarly, since log output is simply $y = a + \alpha k$, the dynamics of output can be obtained from

$$\begin{aligned} y_{t+1} &= a_{t+1} + \alpha k_{t+1} && (5)\\ &= \alpha(\log K_{t+1}) + a_{t+1} && (6)\\ &= \alpha(\log \alpha\beta y_t) + a_{t+1} && (7)\\ &= \alpha(\underbrace{y_t + \log \alpha\beta}_{\text{log saving}}) + a_{t+1} && (8) \end{aligned}$$

it is possible to show that in a more general version of the model in which utility is not logarithmic, an equation of the form

$$y_{t+1} \approx \bar{y} + \omega_1(y_t - \bar{y}) + a_{t+1} \quad (9)$$

does a good job in describing the dynamics of income near the target level $\bar{y}$, and note that ω_1 is kind of like a marginal 'consequence' of saving (a combination of how much you save and how much it influences income in the next period). This reduces to the Brock-Mirman model for $\omega_1 = \alpha$. Derive an expression for the variance of the capital stock if this were a 'structural' equation, and make a remark about what this says about the breadth of the insight that comes from the prior analysis. .

d) Unlike (8), equation (9) is not 'structural': That is, while it may do a good job in describing the dynamics of the model near $\bar{y}$, if a parameter of the model changes (say, relative risk aversion), the ω coefficient will change.

Consider, for example, an increase in the coefficient of relative risk aversion, starting from the benchmark Brock-Mirman value of $\rho = 1$. Explain how you would expect the coefficient ω_1 to change, and what the limits of the extent of the change are.

Hint: ρ^{-1} is the intertemporal elasticity of substitution; a consumer who is very reluctant to substitute consumption over time will set consumption C_t near the $\mathbb{E}[y_{t+1}] = y_t$ locus in the phase diagram. (For purposes of answering this question you can assume that the variance of a is arbitrarily small).

e) Explain intuitively why the determinants of the long-run variance of y are the (log) marginal propensity to save and the serial correlation of productivity growth, and explain why the MPS is related to α.

3. **An Increase in Interest Rates in the ϱ Model.** Consider a perfectly competitive industry with costs of adjustment to capital that was in equilibrium with interest rate $\underline{\mathsf{r}}$ leading up to period t, at which date the interest rate increases permanently to $\bar{\mathsf{r}} > \underline{\mathsf{r}}$.

   a) Draw a diagram that shows how the $\Delta\varrho_{t+1} = 0$, $\Delta k_{t+1} = 0$ loci and the saddle path change in response to the increase in the interest rate. Is the new equilibrium level of the capital stock $\check{k}_{\text{post}}$ higher or lower than before? Why?

b) Draw diagrams showing the time paths of share prices, investment, and the capital stock following the increase in the interest rate. Explain the time pattern of share prices over time; in particular, explain the relationship between any depicted movements in share prices, and the proposition that stock prices follow a random walk.

c) Suppose that, in response to the increase in interest rates, the government wanted to pursue a tax policy that would prevent any changes in the level of the capital stock. Describe the two options the government has, and explain how the two policies would differ or be similar with respect to their implications for dynamics of φ and λ.

4. **Infrastructure and Growth.**

Standard growth models ignore the role of a country's public 'infrastructure' (roads, bridges, sewers, other public facilities) in determining a country's level of income per capita. Yet looking across countries, it seems clear that countries with well-designed and well-maintained infrastructure are more prosperous than countries with crumbling, deficient, or nonexistent infrastructure.

Suppose we can capture the long-run effect of government infrastructure expenditures e in the per-capita production function:

$$f(k, e) \quad = \quad k^{\alpha} e^{\eta} \tag{10}$$

where a country with more infrastructure spending has a higher value of e. (Assume the population and the level of productivity are normalized at 1, and $\eta < 1$).

Suppose government infrastructure expenditures translate one-for-one into productive efficiency e, and assume that the government must satisfy a balanced budget criterion by the use of lump-sum taxes of amount τ:

$$e \quad = \quad \tau. \tag{11}$$

For simplicity, suppose that the capital stock is exogenously fixed at $k = \bar{k}$ and does not depreciate but cannot be augmented by extra saving (there is an endowment of capital).

a) Calculate the level of taxes that maximizes per-capita after-tax income $f(k, e) - \tau$ and explain intuitively the reasons for the effects that the parameters have on the optimal choice of government expenditures.

b) Now suppose this economy suffers from corruption. Specifically, some of the tax revenues that are raised do not get spent on efficient government expenditures but instead are wasted. Again using e for the amount of efficient expenditures, and again imposing the balanced budget constraint, the new level of after-tax income is

$$f(\bar{k}, e) - \underbrace{\tau}_{=e\chi} \tag{12}$$

where $\chi > 1$ measures the degree of corruption. Thus, taxes paid τ exceed expenditures e (the extra taxes represent waste and corruption). Now calculate the level of e that maximizes after-tax per capita output. Is it higher or lower than in the honest economy (where $\chi = 1$)? Why? Is there a cost to the economy beyond the fact that the tax burden is higher by amount χ? Why?

c) Hall and Jones (1999) find that, looking across countries in the world, only a very small proportion of the differences in output per capita are explained by differences in private capital, natural resources, or other private factors of production. Discuss how this finding might be related to the modeling choice above to assume a fixed level of capital $\bar{k}$. Speculate on whether permitting private capital accumulation would be likely to reinforce, undermine, or leave unchanged the results from the baseline model. If the theory sketched above about the effectiveness of infrastructure investment were true, discuss what effect it would have had for Hall and Jones to include some measure of public capital (as well as private capital) in their model.

References

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