

Dynamic Inefficiency and the Capital Share Coefficient in an OLG Model.

Consider a [Diamond \(1965\)](#) OLG economy like the one in the handout [OLGModel](#) and the notebook [DiamondOLG](#), assuming logarithmic utility and a Cobb-Douglas aggregate production function,

$$Y = F(K, PL) \quad (1)$$

where P is a measure of labor productivity that grows according to

$$P_{\tau+1} = GP_{\tau}. \quad (2)$$

Population growth is zero ($\Xi = 1$; for convenience normalize the population at $L_{\tau} = 1 \forall \tau$), and until date t productivity growth has occurred at the rate $g > 0$ (equivalently, $1 + g = G \geq 1$) forever. Under these assumptions, it can be shown that the dynamic process for aggregate $k \equiv K/PL$ is

$$k_{\tau+1} = \left(\frac{(1 - \alpha)\beta}{G_{\tau+1}(1 + \beta)} \right) k_{\tau}^{\alpha} \quad (3)$$

1. Derive the steady-state level of k_{τ} that the economy will have achieved by date t if the rate of productivity growth has always been $G_{\tau} = G \forall \tau$.

Now suppose that, after an eternity of remaining in the steady state, all of a sudden at the beginning of period t everybody learns that henceforth and forever more, the exponent on capital in the production function will change to $\hat{\alpha} > \alpha$.

2. Define the new steady-state as \bar{k} . Will this be larger or smaller than the original steady state \bar{k} ? *Explain your answer.*
3. Next, use a diagram to show how the $k_{\tau+1}(k_{\tau})$ curve changes when the new α takes effect, and show the dynamic adjustment process for the capital stock toward its new steady-state, assuming that the economy was at its original steady state leading up to period t .
4. Define an index of aggregate consumption per efficiency unit of labor in period τ as $\chi_{\tau} = c_{1,\tau} + c_{2,\tau}/G$, and derive a formula for the sustainable level of χ associated with a given level of k .
5. Derive the conditions under which a marginal increase in α will result in an increase in the steady-state level of χ , and explain in words why this result holds.

References

DIAMOND, PETER A. (1965): "National Debt in a Neoclassical Growth Model," *American Economic Review*, 55, 1126–1150, <http://www.jstor.org/stable/1809231>.