A Two-Asset Savings Model with an Income-Contribution Scheme REMARK

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Abstract

This paper contains the highlights from the REMARK file in Code>Python folder.

Keywords Lifecycle, Portfolio Choice, Social Security, Replication

GitHub: `http://github.com/econ-ark/REMARK/REMARKS/CGMPortfolio`
(In GitHub repo, see `/Code` *for tools for solving and simulating the model)*

CLICK HERE for an interactive Jupyter Notebook that uses the Econ-ARK/HARK toolkit to produce our figures (warning: it may take several minutes to launch). Information about citing the toolkit can be found at Acknowleding Econ-ARK.

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All numerical results herein were produced using the Econ-ARK/HARK toolkit; for further reference options see Acknowleding Econ-ARK. Thanks to Chris Carroll and Sylvain Catherine for comments and guidance.

1 Introduction

This paper develops a two-asset consumption-savings model and serves as the documentation of an open-source implementation of its solution and simulation methods in the HARK toolkit (Christopher D. Carroll, Alexander M. Kaufman, Jacqueline L. Kazil, Nathan M. Palmer, and Matthew N. White, 2018). The model represents an agent who can save using two different assets—one risky and the other risk-free—to insure against fluctuations in his income, but faces frictions to transferring funds between assets. The flexibility of its implementation and its inclusion in the HARK toolkit allows users to adapt the model to realistic life-cycle calibrations, and also to embedded it in heterogeneous-agents macroeconomic models.

Macro applications The last decade of research in macroeconomics has experienced and increased interest in how households allocate their savings to assets with different risk exposures and liquidity. Heterogeneous asset allocations have been shown to be determinant to the effects of fiscal and monetary policy (Kaplan and Violante, 2014; Luetticke, 2021).

Micro applications

2 The model

I now discuss the main components of the model informally, and leave its full recursive mathematical representation for Section 3.

2.0.1 Time, mortality, and utility

Time advances in discrete steps that I will index with t. The model can be used in both infinite and finite-horizon versions.

Agents face an exogenous risk of death δ_t each period, which becomes certain at the maximum age of the finite-horizon version. There are no intentional bequests; agents will consume all of their resources if they reach the last period, but they can leave accidental bequests upon premature death.

In each period, agents derive utility from consumption only. Their utility function follows a constant relative risk aversion specification. Formally, for a level of consumption C, the agent derives instant utility

$$u(C) = \frac{C^{1-\rho}}{1-\rho}. \tag{1}$$

2.0.2 Income process

Agents supply labor inelastically. Their labor earnings $Y_{i,t}$ are the product of a permanent component $P_{i,t}$ and a transitory stochastic component $\theta_{i,t}$ as in Carroll (1997), where i indexes different agents. Formally,

$$\ln Y_{i,t} = \ln P_{i,t} + \ln \theta_{i,t}$$
$$\ln P_{i,t} = \ln P_{i,t-1} + \ln \Gamma_{i,t} + \ln \psi_{i,t}$$

where $\Gamma_{i,t}$ is a deterministic growth factor that can capture life-cycle patterns in earnings, and $\ln \psi_{i,t} \sim \mathcal{N}(-\sigma^2_{\psi,t}/2, \sigma_{\psi,t})$ is a multiplicative shock to permanent income[1].

The transitory component $\theta_{i,t}$ is a mixture that models unemployment and other temporal fluctuations in income as

$$\ln \theta_{i,t} = \begin{cases} \ln \mathcal{U}, & \text{With probability } \mho \\ \ln \tilde{\theta}_{i,t} \sim \mathcal{N}(-\sigma^2_{\theta,t}/2, \sigma_{\theta,t}), & \text{With probability } 1 - \mho, \end{cases}$$

with $\mho$ representing the probability of unemployment and $\mathcal{U}$ the replacement factor of unemployment benefits.

This specification of the income process is parsimonious and flexible enough to accommodate life-cycle patterns in income growth and volatility, transitory unemployment and exogenous retirement. Introduced by Carroll (1997), this income specification is common in studies of life-cycle wealth accumulation and portfolio choice; see e.g., Cagetti (2003); Cocco, Gomes, and Maenhout (2005); Fagereng, Gottlieb, and Guiso (2017). The specification has also been used in studies of income volatility, which have yielded calibrations of its stochastic shocks' distributions (see e.g., Carroll, 1992; Carroll and Samwick, 1997; Sabelhaus and Song, 2010)

2.0.3 Financial assets and frictions

Agents smooth their consumption by saving and have two assets available for this purpose. The first is a risk-free liquid account with constant per-period return factor R. The second has a stochastic return factor $\tilde{R}$ that is log-normally distributed and independent across time. Various interpretations such as stocks, a retirement fund, or entrepreneurial capital could be given to the risky asset. Importantly, consumption must be paid for using funds from the risk-free account. The levels of risk-free and risky assets owned by the agent will both be state variables, denoted with $M_{i,t}$ and $N_{i,t}$ respectively.

Portfolio rebalancing takes place by moving funds between the risk-free and risky accounts. These flows are one of the agents' control variables and are denoted as $D_{i,t}$, with $D_{i,t} > 0$ representing a movement of funds from the risk-free to the risky account. Withdrawals from the risky account are subject to a constant-rate tax τ which can represent, for instance, capital-gains realization taxes or early retirement-fund withdrawal penalties. In sum, denoting post-rebalancing asset levels with $\tilde{\cdot}$,

$$\tilde{M}_{i,t} = M_{i,t} - D_{i,t}(1 - 1_{[D_{i,t} \leq 0]}\tau)$$
$$\tilde{N}_{i,t} = N_{i,t} + D_{i,t}.$$

At any given period, an agent might not be able to rebalance his portfolio. This ability

[1]The mean of the shock is set so that $E[\psi_{i,t}] = 1$.

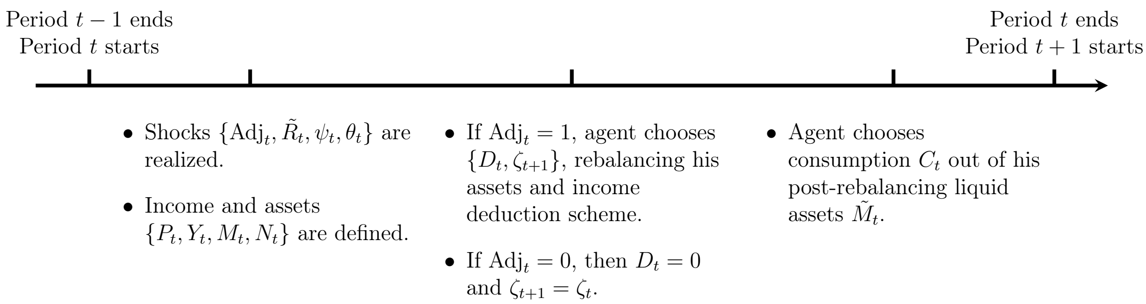

Figure 1 Summary of the Model's Timing

is governed by an exogenous stochastic shock that is realized at the start of the period

$$\text{Adj}_t \sim \text{Bernoulli}(p_t),$$

with $\text{Adj}_t = 1$ meaning that the agent can rebalance and $\cancel{\text{Adj}}_t = 1$ ($\text{Adj}_t = 0$) forcing him to set $D_{i,t} = 0$. This friction is a parsimonious way to capture the fact that portfolio rebalancing is costly and households do it sporadically. Recent studies have advocated for (Giglio, Maggiori, Stroebel, and Utkus, 2021) and used (Luetticke, 2021) this kind of rebalancing friction.

To partially evade the possibility of being unable to rebalance their accounts, agents can use an income deduction scheme. By default, labor income ($Y_{i,t}$) is deposited to the risk-free liquid account at the start of every period. However, agents can pre-commit to have a fraction $\zeta_t \in [0, 1]$ of their income diverted to their risky account instead. This fraction can be tweaked by the agent whenever $\text{Adj}_t = 1$; otherwise it stays at its previous value, $\zeta_{t+1} = \zeta_t$.

2.0.4 Timing

Figure 1 summarizes the timing of stochastic shocks and optimizing decisions that occur within a period of the life cycle model.

3 Recursive representation of the model

Individual subscripts are dropped for simplicity. The value function for an agent who is not allowed to rebalance his portfolio at time t is

$$\begin{aligned} V_t^{\cancel{\text{Adj}}}(M_t, N_t, P_t, \zeta_t) = \max_{C_t} u(C_t) + p_{t+1}\beta\delta_{t+1}E_t\left[V_{t+1}^{\text{Adj}}\left(M_{t+1}, N_{t+1}, P_{t+1}\right)\right] + \\ (1-p_{t+1})\beta\delta_{t+1}E_t\left[V_{t+1}^{\cancel{\text{Adj}}}\left(M_{t+1}, N_{t+1}, P_{t+1}, \zeta_{t+1}\right)\right] \end{aligned}$$

$$\begin{aligned} &\text{Subject to:} \\ 0 \leq& C_t \leq M_t \\ A_t =& M_t - C_t \\ M_{t+1} =& RA_t + (1-\zeta_{t+1})Y_{t+1} \\ N_{t+1} =& \tilde{R}_{t+1}N_t + \zeta_{t+1}Y_{t+1} \\ P_{t+1} =& \Gamma_{t+1}\psi_{t+1}P_t \\ Y_{t+1} =& \theta_{t+1}P_{t+1} \\ \zeta_{t+1} =& \zeta_t \end{aligned}$$

and that of agent who is allowed to rebalance is

$$\begin{aligned} V_t^{\text{Adj}}(M_t, N_t, P_t) = \max_{C_t, D_t, \zeta_{t+1}} u(C_t) + p_{t+1}\beta\delta_{t+1}E_t\left[V_{t+1}^{\text{Adj}}\left(M_{t+1}, N_{t+1}, P_{t+1}\right)\right] + \\ (1-p_{t+1})\beta\delta_{t+1}E_t\left[V_{t+1}^{\cancel{\text{Adj}}}\left(M_{t+1}, N_{t+1}, P_{t+1}, \zeta_{t+1}\right)\right] \end{aligned}$$

$$\text{Subject to:}$$

$$-N_t \leq D_t \leq M_t, \quad \zeta_{t+1} \in [0,1], \quad 0 \leq C_t \leq \tilde{M}_t$$

$$\begin{aligned} \tilde{M}_t =& M_t - D_t\left(1 - 1_{[D_t \leq 0]}\tau\right) \\ \tilde{N}_t =& N_t + D_t \\ A_t =& \tilde{M}_t - C_t \\ M_{t+1} =& RA_t + (1-\zeta_{t+1})Y_{t+1} \\ N_{t+1} =& \tilde{R}_{t+1}\tilde{N}_t + \zeta_{t+1}Y_{t+1} \\ P_{t+1} =& \Gamma_{t+1}\psi_{t+1}P_t \\ Y_{t+1} =& \theta_{t+1}P_{t+1} \end{aligned}$$

The problem can be normalized by permanent income, following Carroll (2020). Using lower case variables to denote their upper-case counterparts normalized by permanent income ($x_t \equiv X_t/P_t$) and defining $\tilde{\Gamma}_t = \Gamma_t \psi_t$, we can write normalized problems

$$
\begin{aligned}
v_t^{\cancel{\text{Adj}}}(m_t, n_t, \zeta_t) = \max_{c_t} u(c_t) + p_{t+1}\beta\delta_{t+1}E_t\left[\tilde{\Gamma}_{t+1}^{1-\rho} v_{t+1}^{\text{Adj}}(m_{t+1}, n_{t+1})\right] + \\
(1 - p_{t+1})\beta\delta_{t+1}E_t\left[\tilde{\Gamma}_{t+1}^{1-\rho} v_{t+1}^{\cancel{\text{Adj}}}(m_{t+1}, n_{t+1}, \zeta_{t+1})\right] \\
\text{Subject to:} \\
0 \leq & c_t \leq m_t \\
a_t = & m_t - c_t \\
m_{t+1} = & \frac{R}{\tilde{\Gamma}_{t+1}} a_t + (1 - \zeta_{t+1})\theta_{t+1} \\
n_{t+1} = & \frac{\tilde{R}_{t+1}}{\tilde{\Gamma}_{t+1}} n_t + \zeta_{t+1}\theta_{t+1} \\
\zeta_{t+1} = & \zeta_t
\end{aligned}
\tag{2}
$$

and

$$
\begin{aligned}
v_t^{\text{Adj}}(m_t, n_t) = \max_{c_t, d_t, \zeta_{t+1}} u(c_t) + p_{t+1}\beta\delta_{t+1}E_t\left[\tilde{\Gamma}_{t+1}^{1-\rho} v_{t+1}^{\text{Adj}}(m_{t+1}, n_{t+1})\right] + \\
(1 - p_{t+1})\beta\delta_{t+1}E_t\left[\tilde{\Gamma}_{t+1}^{1-\rho} v_{t+1}^{\cancel{\text{Adj}}}(m_{t+1}, n_{t+1}, \zeta_{t+1})\right] \\
\text{Subject to:} \\
-n_t \leq d_t \leq m_t, \quad \zeta_{t+1} \in & [0, 1], \quad 0 \leq c_t \leq \tilde{m}_t \\
\\
\tilde{m}_t = & m_t - d_t\left(1 - 1_{[d_t \leq 0]}\tau\right) \\
\tilde{n}_t = & n_t + d_t \\
a_t = & \tilde{m}_t - c_t \\
m_{t+1} = & \frac{R}{\tilde{\Gamma}_{t+1}} a_t + (1 - \zeta_{t+1})\theta_{t+1} \\
n_{t+1} = & \frac{\tilde{R}_{t+1}}{\tilde{\Gamma}_{t+1}} \tilde{n}_t + \zeta_{t+1}\theta_{t+1}
\end{aligned}
\tag{3}
$$

It can be shown that

$$
V_t^{\text{Adj}}(M_t, N_t, P_t) = P_t^{1-\rho} v_t^{\text{Adj}}(m_t, n_t), \quad V_t^{\cancel{\text{Adj}}}(M_t, N_t, P_t, \zeta_t) = P_t^{1-\rho} v_t^{\cancel{\text{Adj}}}(m_t, n_t, \zeta_t),
$$

and that the policy functions of both problems are related through

$$
\begin{aligned}
C_t^{\cancel{\text{Adj}}}(M_t, N_t, P_t, \zeta_t) = P_t c_t^{\cancel{\text{Adj}}}(m_t, n_t, \zeta_t), \quad C_t^{\text{Adj}}(M_t, N_t, P_t) = P_t c_t^{\text{Adj}}(m_t, n_t), \\
D_t(M_t, N_t, P_t) = P_t d_t(m_t, n_t), \quad \zeta_{t+1}(M_t, N_t) = \zeta_{t+1}(m_t, n_t).
\end{aligned}
$$

Therefore, the model's implementation solves the problem in normalized form, and rescales the relevant states and choices using permanent income when simulating.

3.1 Partition into stages

An additional insight that facilitates solving the model is that the three decisions that an agent might take in a period (rebalancing his assets, choosing his income deduction fraction and consuming) can be seen as happening sequentially. This is convenient because:

- The sub-problems are easier to solve than the multi-choice full problem.
- Since the non-adjusting agent only chooses his consumption and we must solve his problem for every combination of $(m_t, n_t, \zeta_t = \zeta_{t+1})$, we can re-utilize his solution by expressing the adjusting agent's problem as

$$v_t^{\text{Adj}}(m_t, n_t) = \max_{\tilde{m}_t, \tilde{n}_t, \zeta_{t+1}} v_t^{\cancel{\text{Adj}}}(\tilde{m}_t, \tilde{n}_t, \zeta_{t+1}).$$

  This insight is similar to the "nested" reformulation suggested by Druedahl (2020).

To start re-expressing the problem, I take the order of the ageent's decisions to be: rebalance assets, define income contribution share and finally consume. I denote the stages at which these decisions are taken with `Reb`, `Sha`, and `Cns` respectively. I will use $v^{\texttt{Reb}}(\cdot)$, $v^{\texttt{Sha}}(\cdot)$ and $v^{\texttt{Cns}}(\cdot)$ to represent *stage value functions*.

I now present each stage in detail, working backwards in time.

3.1.1 Consumption stage, `Cns`

An agent who takes his assets and income contribution share as given is one who can not adjust them and can only choose his consumption. This corresponds to the problem of the non-adjusting agent (Equation 2). The important facts to realize at this stage are that

$$v_t^{\text{Adj}}(m_t, n_t) = v_t^{\texttt{Reb}}(m_t, n_t), \quad v_t^{\cancel{\text{Adj}}}(m_t, n_t, \zeta_t) = v_t^{\texttt{Cns}}(m_t, n_t, \zeta_t),$$

with the first fact being true because we have assumed that asset rebalancing is the first decision that an adjusting agent takes, and he assumes that his subsequent decisions will be optimal. Therefore, the consumption stage problem is

$$\begin{aligned} v_t^{\texttt{Cns}}(\tilde{m}_t, \tilde{n}_t, \zeta_{t+1}) = \max_{c_t} u(c_t) + p_{t+1}\beta\delta_{t+1}E_t\left[\tilde{\Gamma}_{t+1}^{1-\rho} v_{t+1}^{\texttt{Reb}}\left(m_{t+1}, n_{t+1}\right)\right] + \\ (1-p_{t+1})\beta\delta_{t+1}E_t\left[\tilde{\Gamma}_{t+1}^{1-\rho} v_{t+1}^{\texttt{Cns}}\left(m_{t+1}, n_{t+1}, \zeta_{t+1}\right)\right] \end{aligned}$$

Subject to:

$$\begin{aligned} 0 \leq & c_t \leq \tilde{m}_t \\ a_t = & \tilde{m}_t - c_t \\ m_{t+1} = & \frac{R}{\tilde{\Gamma}_{t+1}} a_t + (1-\zeta_{t+1})\theta_{t+1} \\ n_{t+1} = & \frac{\tilde{R}_{t+1}}{\tilde{\Gamma}_{t+1}} \tilde{n}_t + \zeta_{t+1}\theta_{t+1} \end{aligned}$$

3.1.2 Income contribution share stage, `Sha`

An agent with the option to set his income contribution share will do so taking his asset allocation as given and assuming that he will optimally pick his consumption in the next stage. His problem is

$$v_t^{\texttt{Sha}}(\tilde{m}_t, \tilde{n}_t) = \max_{\zeta_{t+1}\in[0,1]} v_t^{\texttt{Cns}}(\tilde{m}_t, \tilde{n}_t, \zeta_{t+1})$$

3.1.3 Rebalancing stage, `Reb`

The first decision that an agent takes, if allowed, is how to reallocate his assets. At this stage, his value function is

$$\begin{aligned} v_t^{\texttt{Reb}}(m_t, n_t) =& \max_{d_t} v_t^{\texttt{Sha}}(\tilde{m}_t, \tilde{n}_t) \\ \text{Subject to:}& \\ -n_t \leq& d_t \leq m_t \\ \tilde{m}_t =& m_t - d_t\left(1 - 1_{[d_t\leq 0]}\tau\right) \\ \tilde{n}_t =& n_t + d_t \end{aligned} \tag{4}$$

The solution to this stage problem will be the policy function $d_t(\cdot,\cdot)$ that gives the optimal flow from risk-free to risky assets, which can be negative. However, it is convenient to define a normalized policy function $\underline{d}_t$ as

$$\underline{d}_t(m, n) = \begin{cases} d_t(m,n)/m, & \text{if } d_t(m,n) \geq 0 \\ d_t(m,n)/n, & \text{if } d_t(m,n) < 0 \end{cases}$$

so that $-1 \leq \underline{d}(m,n) \leq 1$ for all (m,n).

4 Examples of Solutions and Simulations

This section examines various instances of the model under different specifications for lives' lengths and financial frictions. The main purpose of these exercises is to illustrate the model's capabilities and the implications of financial frictions for asset allocations.

4.1 Infinite-horizon

For a first exercise, consider the infinite-horizon version of the model. In this version, parameters are constant and the optimization problem that the agent solves every period is—up to the values of state variables—the same. I set the parameters to the values reported in Table 1.

The infinite-horizon solution is obtained by value-function iteration backwards in time. I define trivial starting value functions $v_T^{\text{Adj}}(\cdot)$ and $v_T^{\cancel{\text{Adj}}}(\cdot)$ and use Equations 3 and 2 to find $v_{T-1}^{\text{Adj}}(\cdot)$ and $v_{T-1}^{\cancel{\text{Adj}}}(\cdot)$. I repeat this process until the value functions obtained in successive iterations are similar enough.

Table 1 Infinite Horizon Example Calibration

Name in HARK	Mathematical Symbol	Value
`CRRA`	ρ	5.0
`Rfree`	R	1.03
`DiscFac`	β	0.9
`LivPrb`	δ	0.98
`PermGroFac`	Γ	1.01
`PermShkStd`	σ_ψ	0.1
`TranShkStd`	σ_θ	0.1
`UnempPrb`	$\mho$	0.05
`IncUnemp`	$\mathcal{U}$	0.3
`RiskyAvg`	$E[\tilde{R}]$	1.08
`RiskyStd`	$\sqrt{V[\tilde{R}]}$	0.18
`AdjustPrb`	p	Varying
`tau`	τ	Varying

To illustrate the effect of the different financial frictions in the model, I present solutions for three different parametrizations.

- **Base**: a version without financial frictions. The probability of being able to rebalance is $p = 1$ and the risky withdrawal is $\tau = 0$.
- **Tax**: a version with a risky withdrawal tax of 10% ($\tau = 0.1$). The probability of being able to rebalance is $p = 1$.
- **Calvo**: a version with a stochastic inability to rebalance that occurs with a 75% chance ($p = 0.25$). The risky withdrawal is $\tau = 0$.

The first stage of an agent's period (if he is allowed to rebalance) is to solve the asset-rebalancing problem (Equation 4). The solution to this problem is the rebalancing fraction function $\underline{d}(m, n)$, which I present in Figure 5 for the different parametrizations and various combinations of (m, n). The figure shows how individual with low risk-free resources withdraw funds from their risky accounts in order to finance their consumption. The version of the model with the withdrawal tax has regions where $\underline{d} = 0$ as a result of the tax's asymmetry. As illustrated by the figure, the stochastic inability to rebalance pushes agents to keep less funds in the risky asset, withdrawing them at higher rates when they get the chance.

The second stage in a period, which an agent also participates in only if he $\text{Adj}_t = 1$, consists of choosing the income contribution share ζ. Figure 3 presents the policy function $\zeta(\tilde{m}, \tilde{n})$ for various $(\tilde{m}, \tilde{n})$ combinations. The income deduction scheme becomes irrelevant in the "Base" and "Tax" versions of the model, since agents are always able to rebalance their assets ($p = 1$) and thus pre-committing funds yields no gains. The

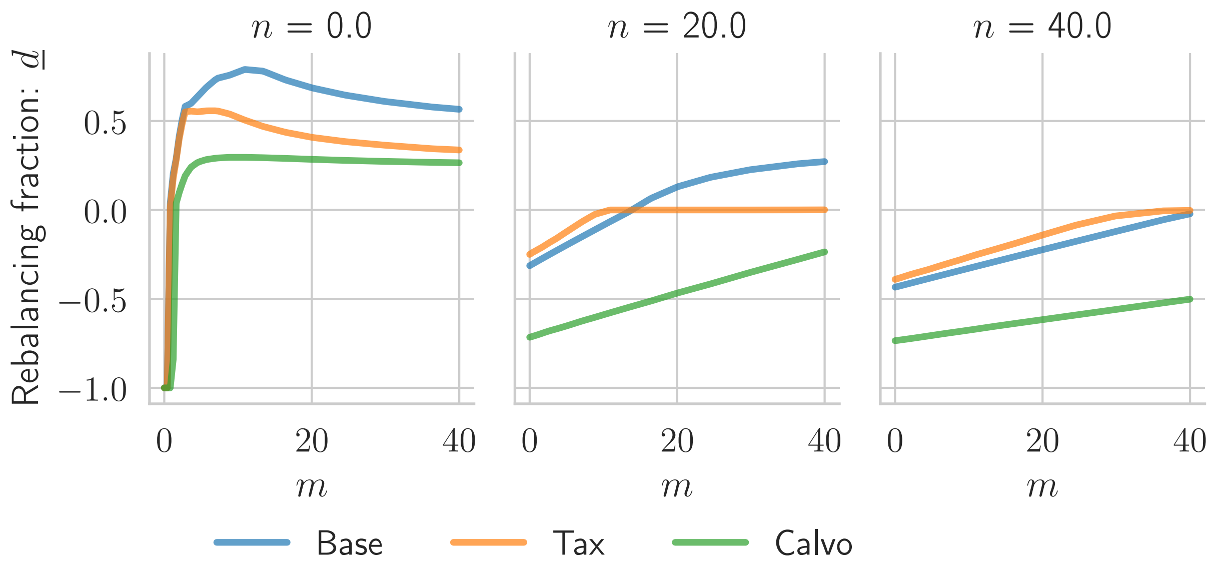

Figure 2 Optimal rebalancing fraction $\underline{d}$ in the infinite horizon model.

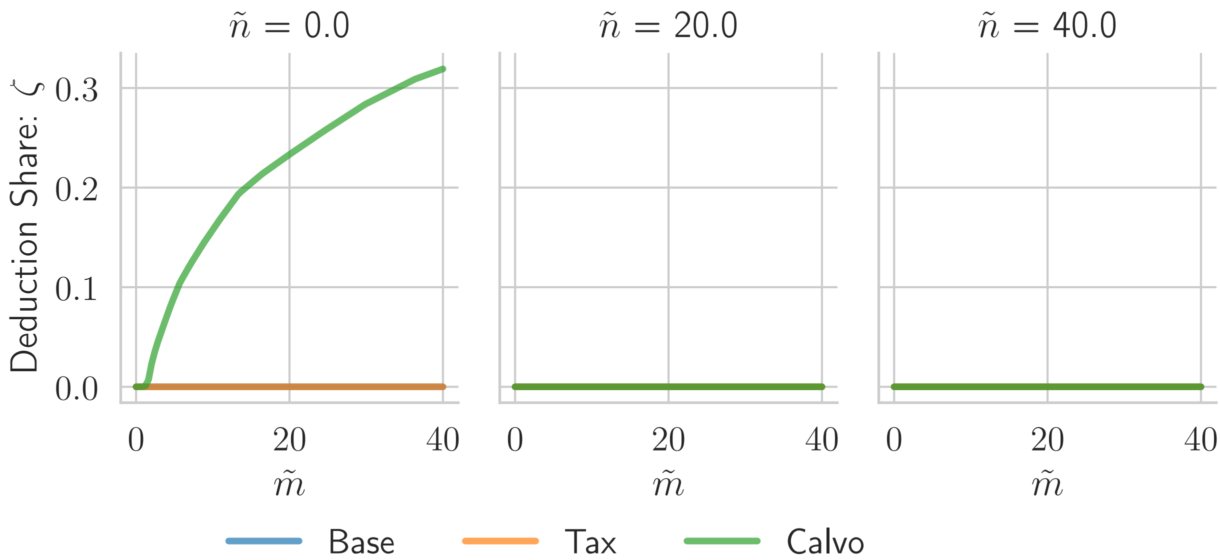

Figure 3 Convergence of the Consumption Rules

figure shows that when $p < 1$ agents will make use of the system, especially if the agent has low risky asset balances.

The final stage in the agent's problem, or the only one if $\text{Adj}_t = 0$, is to choose how much of his risk-free resources to consume. Figure 4 presents the consumption functions for different combinations of post-rebalancing assets $(\tilde{m}, \tilde{n})$ and income contribution fractions ζ. Consumption is increasing in both assets as would be expected, and financial frictions reduce the level of consumption at any given state. A first characteristic to note is that the contribution share has no effect on the consumption functions of the "Base" model and little effect in that of the "Tax" model. Since these agents know they will be able to rebalance their assets, the only relevance of where their income is initially deposited comes from potentially paying the rebalancing tax.

4.2 Life-Cycle finite horizon

5 Conclusion

6 Puzzles and Questions

7 Robustness Analyses

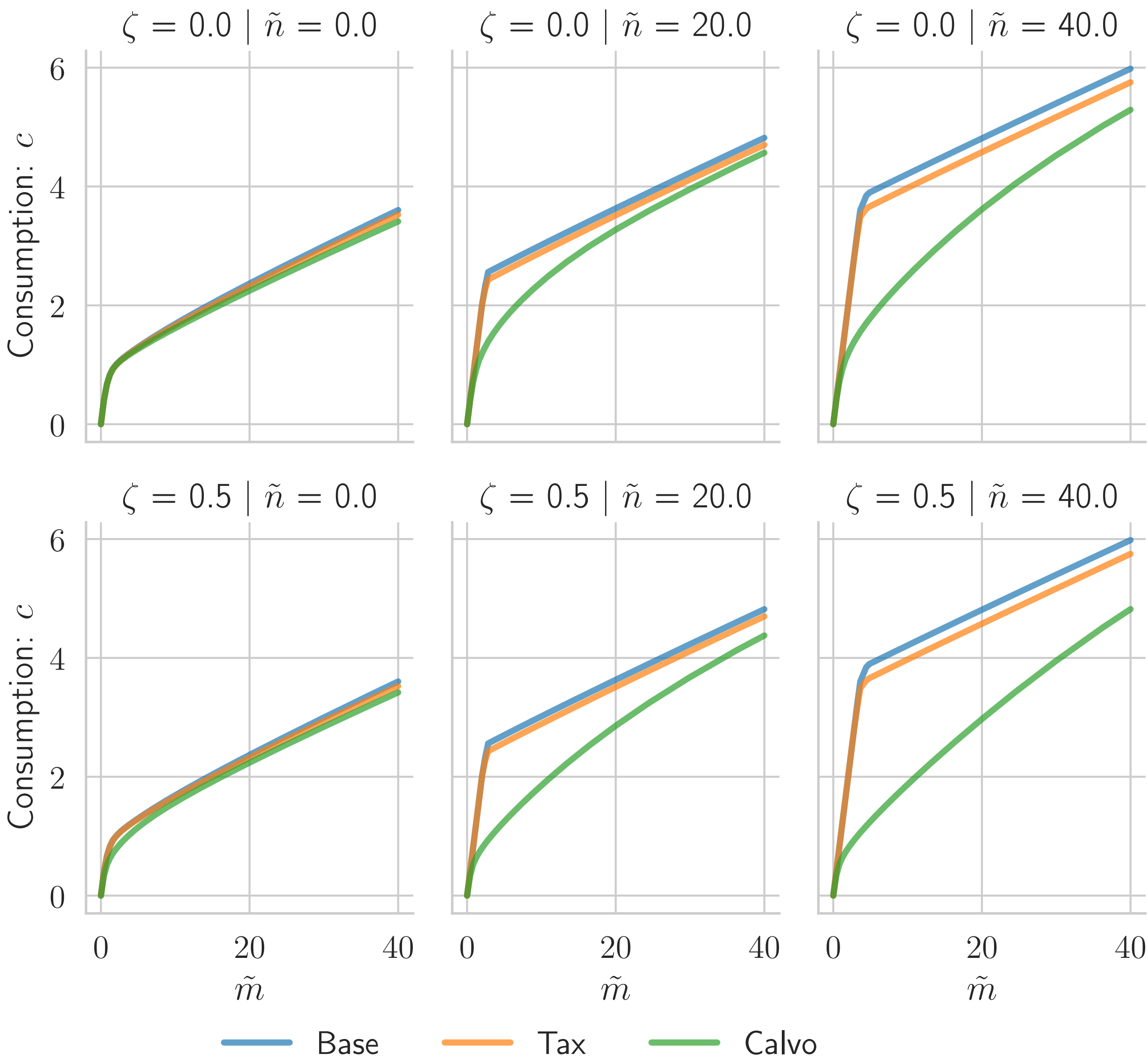

Figure 4 Convergence of the Consumption Rules

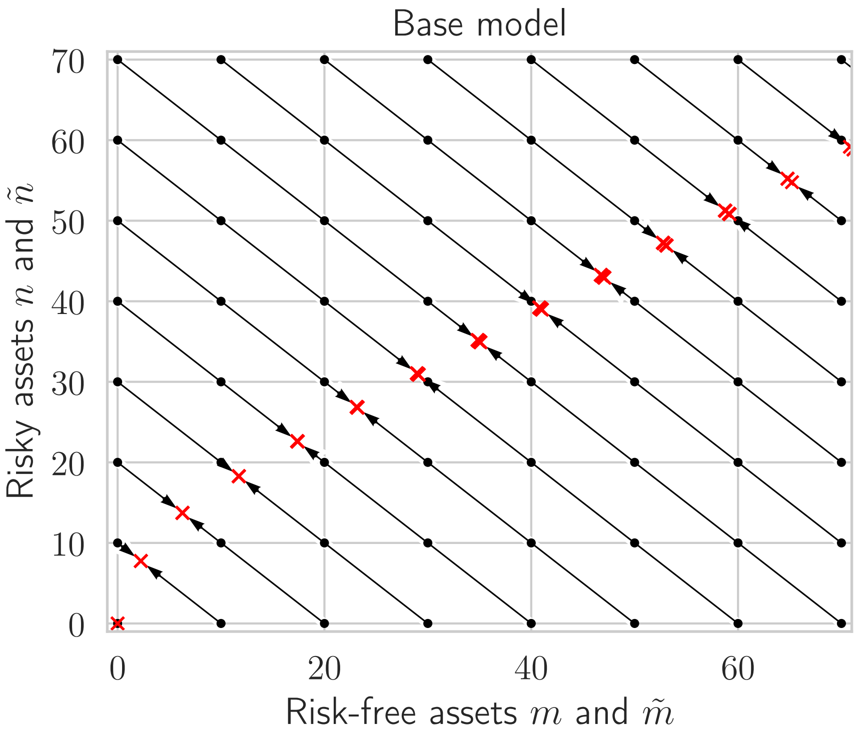

Figure 5 Convergence of the Consumption Rules

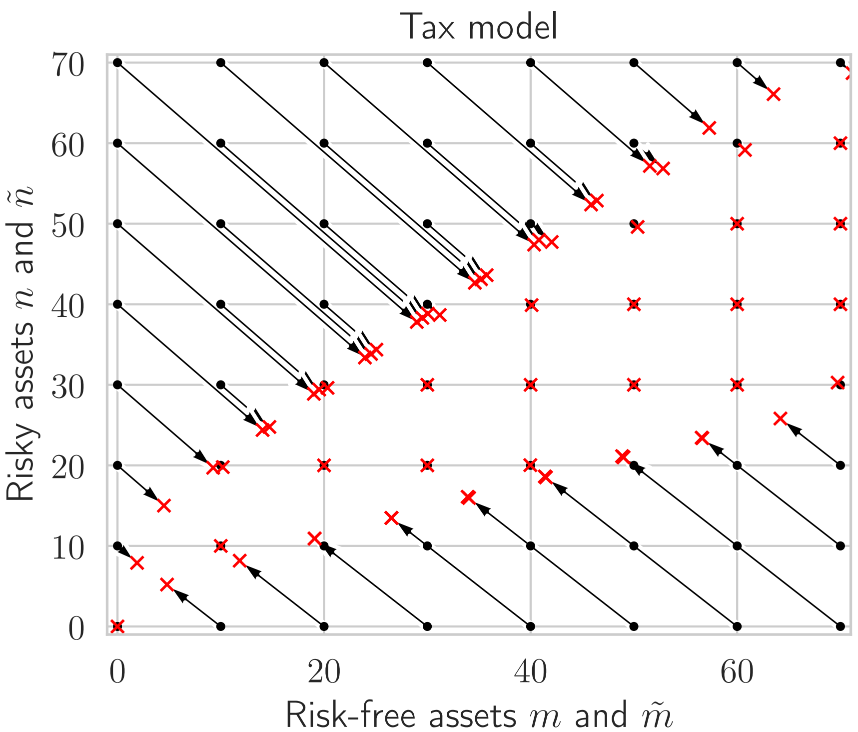

Figure 6 Convergence of the Consumption Rules

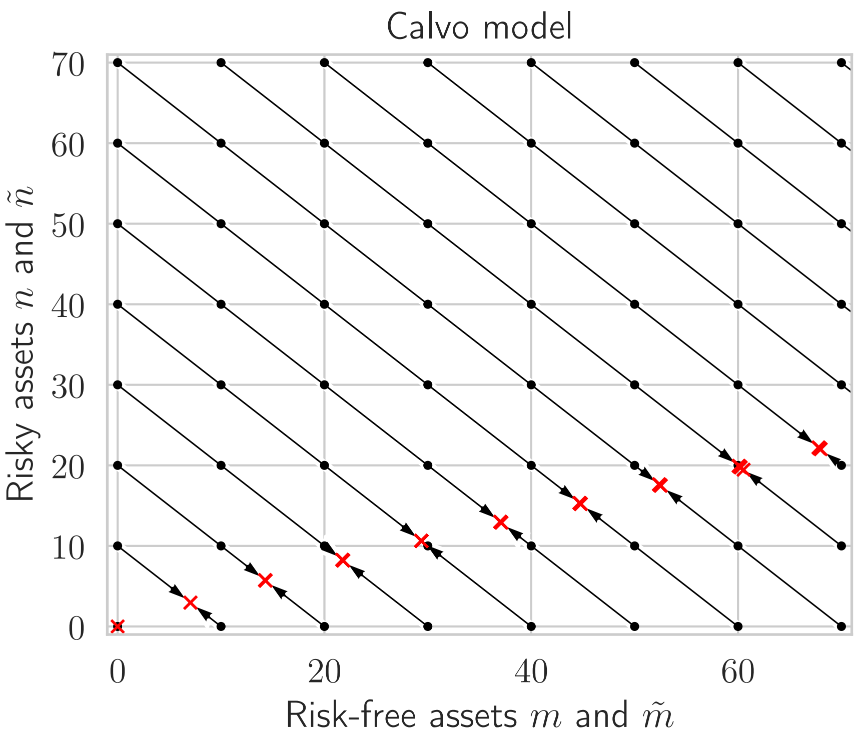

Figure 7 Convergence of the Consumption Rules

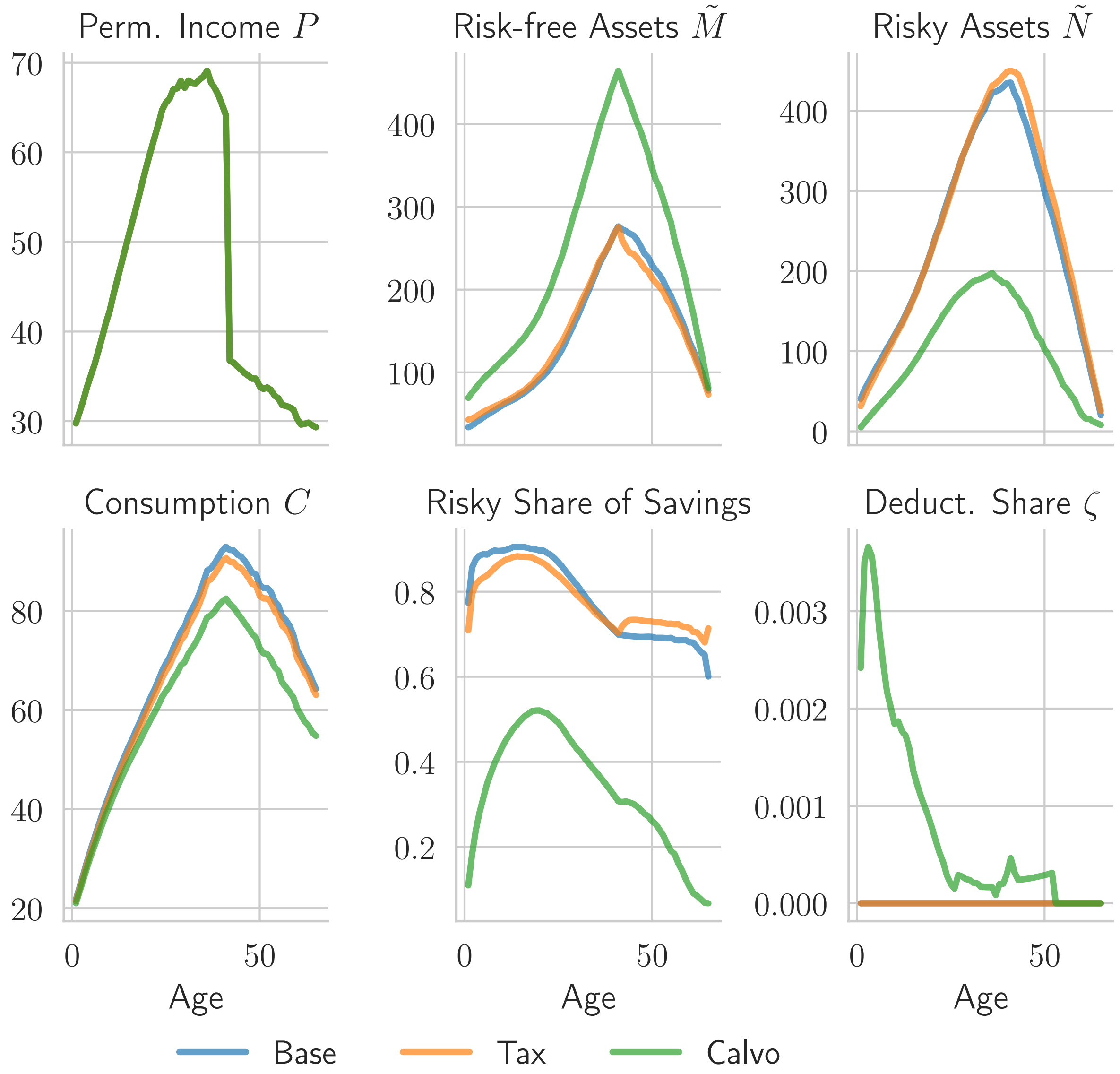

Figure 8 Convergence of the Consumption Rules

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