1 First order conditions and value-function derivatives

The computational solution of the model uses the stage-problems' first-order conditions extensively. This appendix writes the first-order conditions and the value-function derivatives that appear in them explicitly.

1.0.1 *Consumption stage,* `Cns`

The first order condition for an interior solution ($c < \tilde{m}$) of the consumption stage problem (Equation **??**) is

$$
\begin{aligned}
u'(c_t) = p_{t+1}\beta R\delta_{t+1}E_t\left[\tilde{\Gamma}_{t+1}^{-\rho}\frac{\partial v_{t+1}^{\texttt{Reb}}}{\partial \tilde{m}_{t+1}}\right] + \\
(1-p_{t+1})\beta R\delta_{t+1}E_t\left[\tilde{\Gamma}_{t+1}^{-\rho}\frac{\partial v_{t+1}^{\texttt{Cns}}}{\partial \tilde{m}_{t+1}}\right]
\end{aligned}
$$

The derivatives of the stage value function are

$$
\frac{\partial v_t^{\texttt{Cns}}\left(m_t, n_t, \zeta_{t+1}\right)}{\partial \tilde{m}_t} = u'(c_t) \tag{1}
$$

$$
\begin{aligned}
\frac{\partial v_t^{\texttt{Cns}}\left(m_t, n_t, \zeta_{t+1}\right)}{\partial \tilde{n}_t} = p_{t+1}\beta\delta_{t+1}E_t\left[\tilde{R}_{t+1}\tilde{\Gamma}_{t+1}^{-\rho}\frac{\partial v_{t+1}^{\texttt{Reb}}}{\partial \tilde{m}_{t+1}}\right] + \\
(1-p_{t+1})\beta\delta_{t+1}E_t\left[\tilde{R}_{t+1}\tilde{\Gamma}_{t+1}^{-\rho}\frac{\partial v_{t+1}^{\texttt{Cns}}}{\partial \tilde{m}_{t+1}}\right]
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
\frac{\partial v_{t+1}^{\texttt{Cns}}\left(m_t, n_t, \zeta_{t+1}\right)}{\partial \zeta_{t+1}} = p_{t+1}\beta\delta_{t+1}E_t\left[\tilde{\Gamma}_{t+1}^{1-\rho}\theta_{t+1}\left(\frac{\partial v_{t+1}^{\texttt{Reb}}}{\partial \tilde{n}_{t+1}} - \frac{\partial v_{t+1}^{\texttt{Reb}}}{\partial \tilde{m}_{t+1}}\right)\right] + \\
(1-p_{t+1})\beta\delta_{t+1}E_t\left[\tilde{\Gamma}_{t+1}^{1-\rho}\theta_{t+1}\left(\frac{\partial v_{t+1}^{\texttt{Cns}}}{\partial \tilde{n}_{t+1}} - \frac{\partial v_{t+1}^{\texttt{Cns}}}{\partial \tilde{m}_{t+1}}\right) + \tilde{\Gamma}_{t+1}^{1-\rho}\frac{\partial v_{t+1}^{\texttt{Cns}}}{\partial \zeta_{t+2}}\right]
\end{aligned} \tag{3}
$$

1.0.2 *Income contribution share stage,* `Sha`

The first order condition for an interior solution ($\zeta \in (0,1)$) of the income contribution share stage (Equation **??**) is

$$
0 = \frac{\partial v_t^{\texttt{Cns}}}{\partial \zeta_{t+1}}, \tag{4}
$$

and the derivatives of the stage value function are

$$
\frac{\partial v_t^{\texttt{Sha}}(\tilde{m}_t, \tilde{n}_t)}{\partial \tilde{m}_t} = \frac{\partial v_t^{\texttt{Cns}}}{\partial \tilde{m}_t}, \qquad \frac{\partial v_t^{\texttt{Sha}}(\tilde{m}_t, \tilde{n}_t)}{\partial \tilde{n}_t} = \frac{\partial v_t^{\texttt{Cns}}}{\partial \tilde{n}_t}
$$

1.0.3 Rebalancing stage, `Reb`

The first order condition for a solution of the type $d \in [(-n, 0) \cup (0, m)]$the rebalancing stage problem (Equation **??**) is

$$\left(1 - 1_{[d_t \leq 0]}\tau\right) \frac{\partial v_t^{\texttt{Sha}}}{\partial \tilde{m}} = \frac{\partial v_t^{\texttt{Sha}}}{\partial \tilde{n}}, \tag{5}$$

and a necessary condition for a solution of the type $d = 0$ is

$$(1 - \tau) \frac{\partial v_t^{\texttt{Sha}}}{\partial \tilde{m}} \leq \frac{\partial v_t^{\texttt{Sha}}}{\partial \tilde{n}} \leq \frac{\partial v_t^{\texttt{Sha}}}{\partial \tilde{m}} \tag{6}$$

The derivatives of the stage value function are

$$\frac{\partial v_t^{\texttt{Reb}}(m_t, n_t)}{\partial m_t} = \max\left\{\frac{\partial v_t^{\texttt{Sha}}}{\partial \tilde{m}}, \frac{\partial v_t^{\texttt{Sha}}}{\partial \tilde{n}}\right\} \tag{7}$$

$$\frac{\partial v_t^{\texttt{Reb}}(m_t, n_t)}{\partial n_t} = \max\left\{(1 - \tau)\frac{\partial v_t^{\texttt{Sha}}}{\partial \tilde{m}}, \frac{\partial v_t^{\texttt{Sha}}}{\partial \tilde{n}}\right\} \tag{8}$$