

1. (a) $O(1)$
 (b) $\Theta(n + m)$
 (c) $O(n^2 * \log n)$
 (d) $O(n)$
 (e) $O(n + N)$
 (f) $\Omega(d)$
 (g) $O(n)$
 (h) $O(n^2)$
 (i) $O((n + m) * \log(\log n))$

2.

Proof : Since $u.d < v.d$, it means that vertex u is discovered before vertex v . Also, there is at least one path from u to v . Therefore, things must be that in DFS, vertex u is discovered and then from the edges connected to u , vertex v is discovered. Therefore, v is a descendant of u in corresponding DFS forest.

3. (a) False

- (b) $(n^2 - m)$
- (c) the number of different value of characters in this digit
- (d) False
- (e) False
False
- (f) three, which is 10, 24 and 03.
- (g) True.
- (h) Z_k is an LCS of X and Y_{n-1} .

4.

The error of the algorithms is at the second line. Instead, we should choose the min-weight edge which connect from the vertex in the MST to the vertex not in the MST.

For example, in undirected graph

The vertices are A B C D E and the edges and weight are $(A,B) = 1$, $(A,D) = 2$, $(B,D) = 4$, $(B, C) = 4$, $(B, E) = 3$, $(C, E) = 5$, $(C,D) = 2$, $(D, E) = 4$.

For the algorithms, first pick A and add $(A,B) = 1$; then pick B add $(B,E) = 3$; then pick E and add $(D,E) = 4$; finally pick D and add $(D,C) = 2$. The total weight is 10.

However, there is another tree, which is $(A,B) = 1$, $(A,D) = 2$, $(D,C) = 2$, $(B,E) = 3$. The total weight is 8.

Therefore, the algorithms is wrong.

5.

Solution(A, S)

```
{
    boolean res[][] = new boolean[A.length][S+1];
    boolean memo[][] = new boolean[A.length][S+1];
    for i = 0 to A.length
        for j = 0 to S
            res[i][j] = false;
            memo[i][j] = false;
    return findSet(A, S, A.length, res, memo);
}
```

```

findSet(A, S, end, res, memo)
{
    if(end == 0)
    {
        res[end][S] = (A[end] == S);
        memo[end][S] = true;
    }

    if memo[end][S] = true;
        return res[end][S];

    res[end][S] = findSet(A, S, end - 1, res, memo) || findSet(A, S - A[end], end - 1, res,
memo);
    memo[end][S] = true;
    return res[end][S];
}

```

6.

RABIN - KARP($T[0 \dots n - 1]$, $P[0 \dots m - 1]$, d)

```

{
    Let q to be an prime number.
    int n = T.length
    int m = P.length
    int h =  $d^{(m - 1)} \bmod q$ 
    int t = 0;
    int p = 0;
    for i = 0 to m
        t = (d * p + T[i]) mod q;
        p = (d * t + P[i] mod q);
    for i = 0 to n - m
        if(p == t)
            int k = 0;
            while(k < m)
                if(P[k] != T[i + k])
                    break;

            if(k == m)
                print i;

        if i < n - m
            t = ((d * (t - T[i]) * h) + T[i + m]) mod q;
    }
}

```

7.

// Assume that the vertices are labeled as the number of sequence of 0, 1, 2, 3 ... n - 1

// G.AdjList[0] means the head node of list of vertices connected to vertex 0.

// G.AdjList[0].next means the next node of the list.

// G.AdjList[0].val means the label value of the node

vertexDegrees(G.AdjList, m, n)

```

{
    int outDegree[] = new int[n];
    int inDegree[] = new int[n];
    for i = 0 to n - 1

```

```

        outDegree[i] = 0;
        inDegree[i] = 0;
    for i = 0 to n - 1
        node p = G.AdjList[i];
        while(p != NIL)
            outDegree[i] = outDegree[i] + 1;
            inDegree[p.val] = inDegree[p.val] + 1;
            p = p.next;
    for i = 0 to n - 1
        System.out.print(outDegree[i]);
        System.out.print(" ");
        System.out.print(inDegree[i]);
        System.out.println(" ");
    }
}
running time : initialization:  $\Theta(n)$  since we iterate all the vertices
                calculation :  $\Theta(m)$  since we iterate all the edges of the graph.
                total running time :  $\Theta(n + m)$ 

```

8.

I will use counterexample to prove that the greedy method is wrong.

counterexample : Assume that there are three items total which are labeled as 1, 2, 3.

and the most W pounds is 100. The value of 1 is 15 and its weight is 51; the value of 2 and 3 is 10 and its weight is 50;

For greedy strategy, thief will choose the maximum ratio. $15/51 = 0.29$, $10/50 = 0.2$. So the thief will choose the item 1 and he can only can choose item one since $51 + 50 > 100$. Therefore he take 15 value away.

However, the optimal solution is to take item 2 and item 3. The weight is 100 which can be carried and the value of the two items is $10 + 10 = 20$ which is bigger than 15.

Therefore, the greedy strategy in the question is not the optimal solution.