Recovering Dynamics of Latent Variables Using Recurrence Quantification Analysis

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1 INTRODUCTION

Intensive longitudinal methods are useful for capturing and analyzing dynamic, real-time variations in individuals' behaviors and experiences over a period of time (**?**). They come with a unique set of methodological challenges[1]. Psychological variables are generally latent: they are not directly observable and our knowledge of their mechanisms is incomplete at best (**?**). Therefore, data from latent constructs is usually less precise, as we have to rely on subjective assessment to estimate them (**?**). Whereas between-person methods rely on averaging out the effects of time and within-person variation to deal with the complications this causes, (quantitative) dynamical within-person methods rely on that variation to make inferences about the underlying trajectory: how the variable fluctuates over time (**? ? ?**). We aim to see whether Recurrence Quantification Analysis (RQA) can be used to recover aspects of the true trajectory from this less precise data (**?**).

To introduce our topic, we make a number of assumptions about the nature of psychological constructs that are studied using intensive longitudinal methods. It is important to make these assumptions explicit to increase the ability to reject their tenets if they turn out not to hold (**?**). We will use these assumptions to introduce the topic and embed the study in the literature. We note that these assumptions are very close to 'common sense' beliefs within the quantitative dynamical within-person research community. To our understanding, however, these assumptions are not made explicit all that often. We invite the reader to evaluate them critically, and even think of arguments or experiments to disprove them. To help this process along, we present some questions that can function as a starting point to think about the validity of these assumptions.

The first assumption is our working definition of psychological constructs. We take the perspective that these are latent variables that attempt to measure the phenomena of interest in psychology (**?**). They are not directly observable and our knowledge of these phenomena is incomplete (**? ?**). As such, one does not know the true value of these constructs, both because of the incapacity to measure them directly and the 'error' that comes with measuring them. For example, take the construct of happiness. To measure

[1] Note that we avoid the terms 'idiographic' and 'nomothetic' here. They seem to bring more confusion than clarity. For reasons why this is the case, see Lamiell's work (**?**)

this construct, we could ask someone how happy they are. There is no external 'measuring tape' to judge whether their account of their happiness is equivalent to their 'true' state of happiness.

Questions for assumption one:

- Can you quantify happiness?
- What about community spirit?
- Can you be happy without knowing it?
- Can you make errors when judging your own feelings?
- If so, how do we define feelings if we cannot judge them ourselves?

Secondly, we posit that there is an underlying continuous real-valued trajectory of the features that constructs aim to measure (**?**). This follows from the fact that if someone has a 'true score', this score needs to be definable even when it cannot be measured, i.e., if somebody is asked to rate themselves, there is a true score that is being approximated using the latent construct. We also assume that that true score changes smoothly over time. It can drop or increase very quickly, but not instantaneously. Further, intensive longitudinal measures are time-dependent. These measures are shaped by different forces and the previous trajectory of the construct (**?**). They are 'complex' measures, meaning that they come to be through the interdependencies of the numerous non-trivially interacting forces that influence the system (**?**).

Questions for assumption two:

- How depressed are you when you are asleep?
- How agreeable are you when you are very focused on something?

The last assumption is that ordinal likert-type scales are approximations of this underlying continuous measure (**?**). Thus, we assume that there is information loss that comes with these ordinal representations of continuous variables (**?**). We see this as 'error', because we postulate it to be a deviation from its actual, continuous state. For our purposes, we discard other types of error.

Questions for assumption three:

- Did you ever feel there were not enough answering options to answer a likert-type questionnaire fully? (Did you ever feel a 3.5 out of five about your shopping experience?)
- Would you have trouble answering a question with too many (ordinal) answer options?

A technique that stands out for its broad applicability is RQA.This method identifies recurrent patterns of time series data (**?**). It results in different indicators for the stability, predictability, and dynamic behavior inherent in these systems. Our research question follows naturally from these assumptions and uses recurrence methods to try to find a solution: if one were to make an explicit theoretical prediction for trajectory, it would be difficult to validate it, as we would have to test our predictions about this continuous trajectory using ordinal measurements of low granularity (?). We do not have sufficient information to consider the trajectory immediately, but there needs to be an intermediary step where we reconstruct relevant aspects of the trajectory from these ordinal measurements. Our research goal is to find out whether RQA is suitable to fulfill this role.

We start by using computational methods to simulate each of the three assumptions described above. We then try to infer characteristics of the trajectory from the degraded data using RQA.We first simulate a trajectory through dynamical computational modelling (? ?), and then break it down by binning the data and removing time points. We use a toy model that simulates symptomatology by Gauld & Depannemaecker to generate the trajectories (?). Symptomatology is a subset of latent variable constructs, which makes it well-suited for our purposes. It allows for the specification of trajectories with a wide range of behaviour.

The overarching goal is to develop methods to recover aspects of a trajectory empirically using intensive longitudinal studies based on infrequent, low-resolution measurements. While full recovery of trajectory is impossible, it may be possible to recover some relevant aspects of the system under study. The research question is 'Given that a psychological construct has a real-valued continuous trajectory, can we recover elements of it using RQA from limited sampling occurrences on an ordinal scale?'. The major elements to be examined include the stability of several recurrence indicators under degradation, the implications for measurement and analysis of time series of latent variable constructs, and the weaknesses and oversights that we found when we tried to simulate a theoretical trajectory and degrade it.

A strength of this project is that it explicates normally tacit assumptions, and uses these assumptions to model the entire process: we model the underlying trajectory, the latent variable that estimates this trajectory, and we make an explicit prediction for its relationship if these assumptions are met. We use computational methods to generate the data because it is impossible to answer this question in the same way empirically as the real underlying trajectory is unknown. There is, however, an important trade-off being made: because we simulate our data, many of the complicating aspects that would come up during empirical studies are overlooked. We do not use real data, and that means that the inferences are only correct when our assumptions are correct. Serious objections to these assumptions can be found in the discussion section. Another weakness is that the performance of recurrence methods can be sensitive to the parameter settings of the computational model. In the current project, we treat only four different trajectories.

2 MATERIALS AND METHODS

2.1 Stage 1: Data generation

In the first stage, we used a toy model to simulate the data based on the *3 + 1 Dimensions Model* introduced by (?). This model captures clinical observations found in psychiatric symptomatology by modeling internal factors (y), environmental noise (z), temporal specificities (f), and symptomatology (x) using coupled differential equations. x is the basis of the time series. The original aim of this toy model is to simulate the trajectory of symptomatology of psychiatric symptoms over time. It is suitable for our project because the model creates realistic looking trajectories for psychological phenomena. The model uses four coupled differential equations to model the effect of time on symptom intensity. Symptom intensity is a subset of latent constructs, and we see its behaviour over time as similar to other latent constructs. It is of note that many different systems could have let to similar results to the ones outputted here. The data generating process is of secondary importance: it should result in somewhat plausible trajectories. We chose this model over more traditional choices, such as the Lorenz attractor, because we prioritized its flexibility in capturing realistic trajectories based on latent variable constructs in the social sciences.

2.1.1 Symptom intensity

$$\tau_x \frac{dx}{dt} = \frac{S_{\max}}{1 + \exp(\frac{Rs - y}{\lambda_s})} - x \tag{1}$$

2.1.2 Modelling of internal elements

$$\tau_y \frac{dy}{dt} = \frac{P}{1 + \exp(\frac{R_b - y}{\lambda_b})} + L - xy - z \tag{2}$$

2.1.3 Modelling of perceived environment

$$\tau_z \frac{dz}{dt} = S(ax + \beta y)\,\zeta(t) - z \tag{3}$$

2.1.4 Temporal specificities

$$\tau_f \frac{df}{dt} = y - \lambda_f f \tag{4}$$

2.1.4.1 Parameter definitions

Parameter definitions and parameter settings are shortly mentioned here. For a more in-depth treatment, see Gauld and Depannemaecker (**?**). α & β are the weight of the effect of variables x and y on environmental perception. $\tau_{x,y,z,f}$ are the different time scales the equations operate on. $S_{\max}$ is the maximum level of the symptoms. $R_{s,b}$ is the sensitivity to triggering the system. $\lambda_{s,b}$ are the slopes of the internal and symptom curves. P is the maximal rate of internal elements of the systems. S is the overall sensitivity to the environment. L is the level of predisposing factors. λ_f is the scaling factor of the slow evolution of fluctuations affecting L. $\zeta(t)$ is a point in the normal distribution where $\sigma = 0.5$. It is calculated at each $0.01t$, and is clamped between -1 and 1.

2.1.4.2 Parameter settings

There are four initial parameter settings that we have taken from the same source (**?**). They represent four different disorders. Their initial conditions are given at page **??**. Each time series is representative of a different kind of chaotic behaviour. The time series of the 'healthy'-trajectory moves randomly around 0.1. The time series of the 'schizophrenia' time series moves close to 8, before dropping for intervals. Both 'bereavement' and 'bipolar' oscillate quickly in symptom strength, covering the full total range. For visualizations, see page **??**.

2.1.5 Solvers

We used the Tsitouras 5/4 Runge-Kutta method as the solver for the differential equations, as implemented in the DifferentialEquations.jl package (**?**). We used standard settings for all of the parameters, aside from a higher number of maximum iterations ($1e^7$). The baseline is calculated for $0.01t$, where t represents one day in the model.

2.2 Stage 2: Binning data and removing time points

Afterwards, we systematically reduced the quality of the data. We binned the range of the width of the data into n intervals of equal length, where n stands for the number of bins. Moreover, we removed time points from the data by keeping the first and every k^{th} observation of the simulated data. We systematically decreased the number of bins and the number of time points, and re-analyze the data. k at 1 is set at the baseline. This implies no reduction. The other k-values include 2, 4, and 8. For binning, n = 100 is set at the baseline, and is equivalent to a visual analog scale. Other n-values include 20, 7, 6, 5, 4, 3, and 2. These were chosen to reflect different types of measuring instruments, such as several types of likert and forced-choice scales.

2.3 Stage 3: Data analysis

We judged the sensitivity of the data by deriving the recurrence indicators introduced before for each time series in each state of degradation. We calculated the deviation of each of these values from the baseline, which are the recurrence values derived for the intact dataset. We mapped the changes as the deviation for these indicators between the baseline and a set of degraded data, adjusting indicators based on line length by multiplying them by the reduction factor.

2.3.1 Recurrence Quantification Analysis

RQA is a method that is based on the identification of recurrent points in a time series. A point recurs if it is within the recurrence threshold of another point in time (**?**). Indicators can then be derived from this matrix. The development of these indicators has seen considerable development (**?**). We chose to focus on the core set of indicators, as described by Marwan and Webber (**?**). The recurrence threshold was set at the size of the bins of the degraded data set. E.g., if the range of the trajectory was 0 to 2, and the number of bins is 7 (data is degraded so that it is similar to likert-scale data), then the recurrence threshold would have been set at $\frac{2-0}{7} = \frac{2}{7}$. A visualization of these recurrences for the four trajectories can be found on page **??**.

2.3.2 Recurrence Indicators

The *recurrence rate* is the proportion of points in the phase space that reoccur at later times (**?**). Higher recurrence rates indicate that an underlying function is more periodic. *Determinism* is the share of recurrent points that are part of diagonal lines, which indicate that the structure might be deterministic. It should be noted that it is a necessary condition, not sufficient by itself, to indicate determinism (**?**). *Average and maximum length of diagonal structures* are also given. A longer average length means more predictable dynamics. A longer maximum indicates the longest segment. *Entropy of diagonal structures* concerns the Shannon entropy of diagonal line lengths (**?**). It is an indicator of the amount of randomness, or information, in the data. *Trapping time* is the average length of vertical lines in the plot. It is a measure of how long a system stays in a particular state. *Most probable recurrence time*, similarly, is the mode of the length of the vertical lines in the plot.

2.4 Software

We used the Julia language, and in particular the ‘DynamicalSystems.jl’, ‘RecurrenceAnalysis.jl’, and ‘Statistics.jl’ packages to implement the toy model and run the recurrence analyses (**? ? ?**). Analyses were run on a personal computer. Full information about dependencies and version numbers can be found in a machine-readable format in the Manifest.toml file in the Github-repository. Instructions for running the analysis through a sandboxed project environment identical to our system can be found on the main page of this repository.

CONFLICT OF INTEREST STATEMENT

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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DATA AVAILABILITY STATEMENT

The code, additional material, and generated data for this study can be found on GitHub.

FIGURES

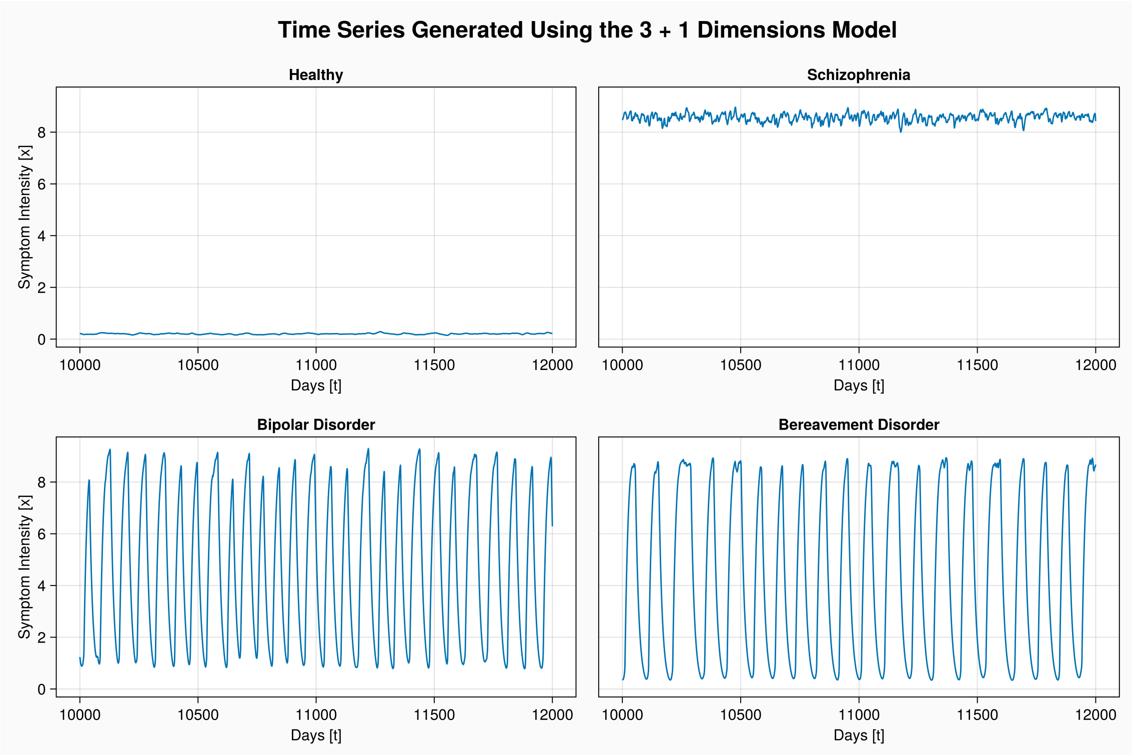

Figure 1. A section of the time series created using the coupled differential equations and parameter settings specified in section 2.1.1. This is the intact data, before degradation takes place.

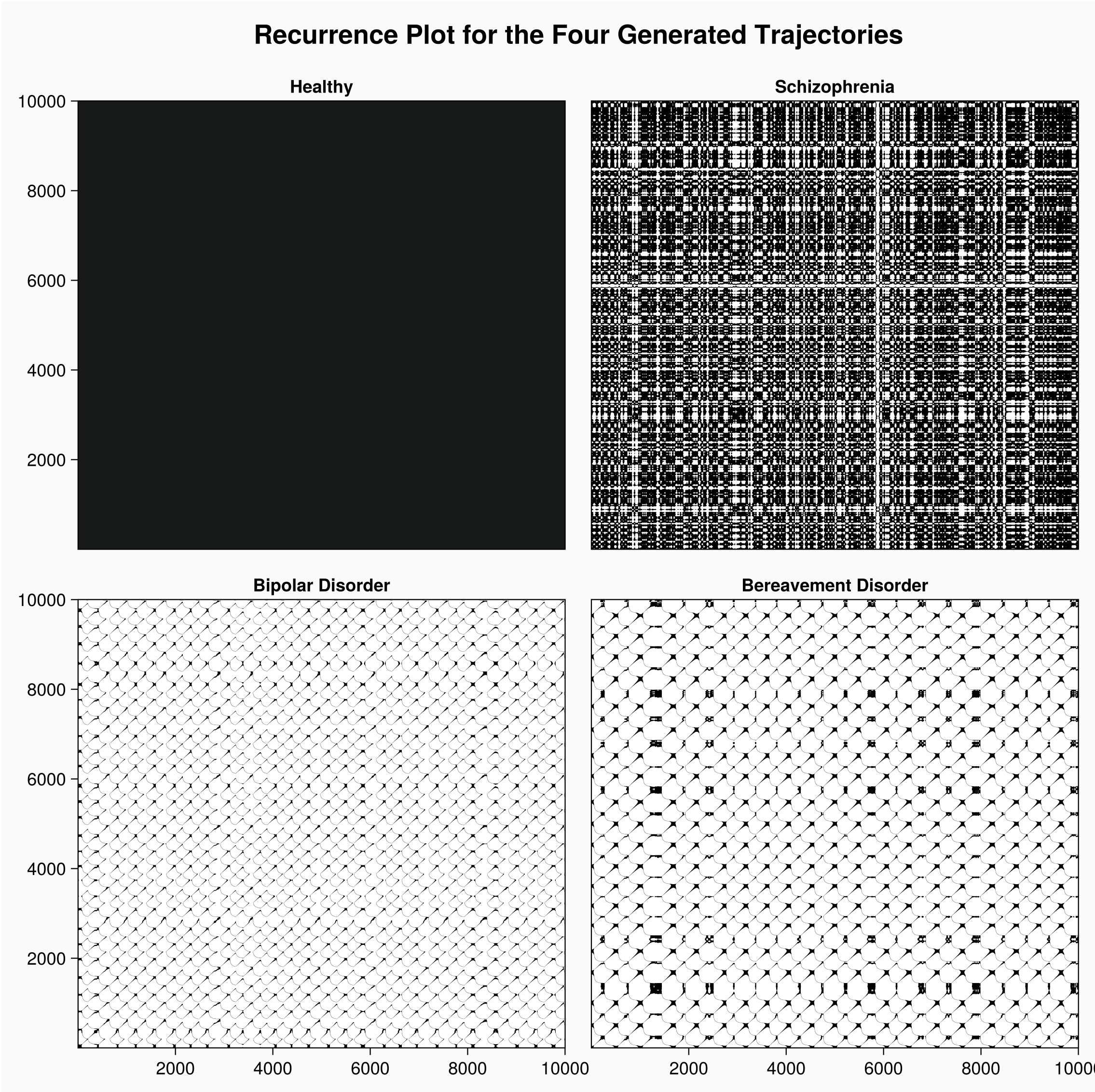

Figure 2. Recurrence plot for the four time series generated using the coupled differential equations and parameter settings specified in section 2.1.1. A point recurs when it is within the recurrence threshold of another point. Recurrent points are black, non-recurrent points are white. The axes represent time points, each location on the matrix represents a combination of time points. The recurrence threshold is set at 0.2 for illustration purpose. Note that the plot for the 'healthy' trajectory is completely black: this is because every point in the plot falls within the recurrence threshold. Also note the black 'boxes' where the bottom two trajectories are stagnant.

TABLES

Parameter	S_{max}	R_s	λ_s	τ_x	P	R_b	λ_b	L	τ_y	S	α	β	τ_z	λ_d	τ_f
Healthy	10	1	0.1	14	10	1.04	0.05	0.2	14	4	0.5	0.5	1	1	720
Schizophrenia	10	1	0.1	14	10	0.904	0.05	0.2	14	4	0.5	0.5	1	1	720
Bipolar	10	1	0.1	14	10	1.04	0.05	1.01	14	10	0.5	0.5	1	1	720
Bereavement	10	1	0.1	14	10	1	0.05	0.6	14	4.5	0.5	0.5	1	1	720

Table 1. The parameter settings used as initial parameter settings for the coupled differential equations specified in paragraph 2.1.1