Hand-in sheet 1

- This hand-in sheet **will** be graded. It will count for 20% of the total grade.
- Please hand-in your answers before **Noon Friday 16 May**.
- Answers may be handed in physically to or digitally through e-mail to: maarten.meent@nbi.ku.dk.
- You are allowed (and encouraged) to work in groups to discuss the exercises. However, everyone **must** write and hand-in their own answers (Show your work!).
- You are also allowed to use computer algebra software (e.g. Mathematica or SageMath) to aid with your computations. However, if you do, please include your notebooks with your answer.
- The **bonus** questions are included because they complete the "story" of the questions. They are however likely too much work.

1 Marginally bound geodesics

We defined geodesics with $\mathcal{E}^2 > \mu$ as **unbound** and geodesic with $\mu > \mathcal{E}^2$ as **bound**. The geodesics on the boundary between these two cases with $\mathcal{E}^2 = \mu$ are referred to as **marginally bound**.
a. Argue that all marginally bound geodesics must be timelike.
b. Show that for marginally bound geodesics the radial potential P_r is a third order polynomial in r.
c. How many zeroes does P_r have for marginally bound orbits? How many of those zeroes can be real?
d. For marginally bound orbits, how many of P_r's zeroes can lie outside the event horizon $r = r_+$?
e. Based on the possible configurations of zeroes of P_r, describe the possible solutions for marginally bound geodesics that have at least some part of their solution outside r_+.
f. Show that any marginally bound circular orbit (outside the horizon) *must* be unstable.
g. Show that for any *equatorial* marginally bound circular orbit at radius r_o we have $\mathcal{L}^2 = 4\mu M r_o$.
h. **(bonus)** Show that for any equatorial marginally bound circular orbit $r_o = 2M(1 + \sqrt{1 - a/M}) - a$.

(more on next page)

2 Kerry Schwarzschild

a. Expand the Kerr metric around $a = 0$ to linear order in a. I.e. find $h^{\delta a}_{\mu\nu}$ in

$$g^{\text{Kerr}}_{\mu\nu} = g^{\text{Schw.}}_{\mu\nu} + a h^{\delta a}_{\mu\nu} + \mathcal{O}(a^2).$$

b. Show that $h^{\delta a}_{\mu\nu}$ satisfies the Lorenz gauge condition.
c. Expand the Schwarzschild metric around $M = M_0$ to first order in $\delta M = M - M_0$. I.e. find $h^{\delta M}_{\mu\nu}$ in

$$g[M]^{\text{Schw.}}_{\mu\nu} = g[M_0]^{\text{Schw.}}_{\mu\nu} + \delta M h^{\delta M}_{\mu\nu} + \mathcal{O}(\delta M^2).$$

d. Does $h^{\delta M}_{\mu\nu}$ satisfy the Lorenz gauge condition?
e. **(bonus)** Find the gauge transformation ξ_μ that brings $h^{\delta M}_{\mu\nu}$ to the Lorenz gauge.