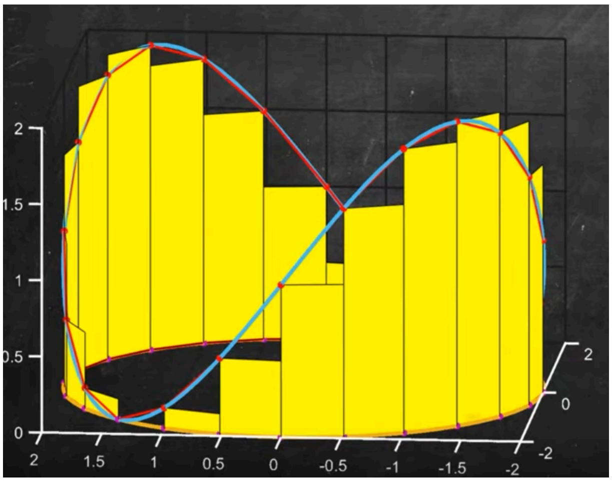
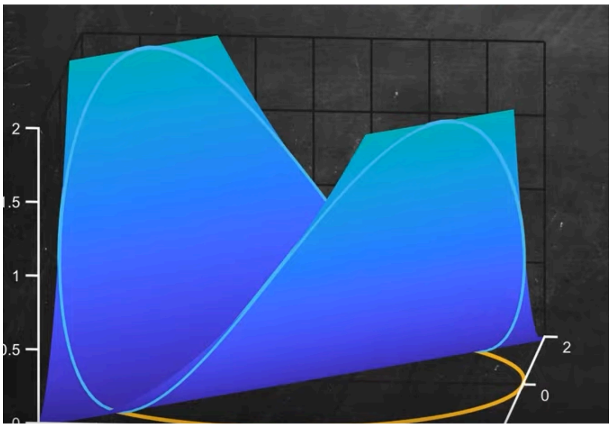
Line integrals

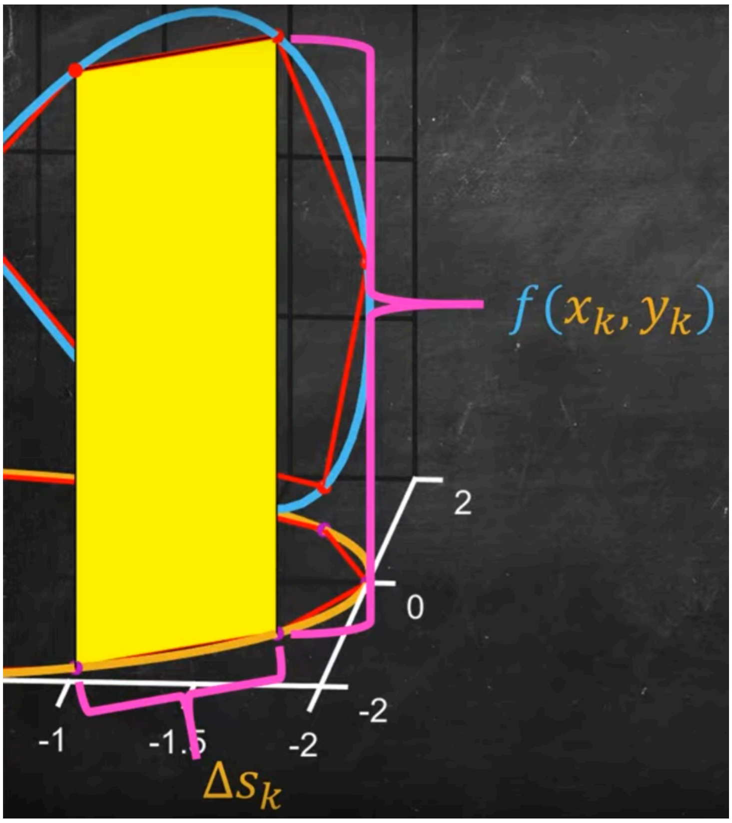
On veut intégrer $f(x, y) = z$ (en bleu) selon le cercle, que l'on paramétrise comme $\vec{r}(t) = g(t)\vec{i} + h(t)\vec{j}$.



On peut d'abord réécrire notre fonction comme $f(t) = f(g(t), h(t))$. Pourquoi ? Parce que les seuls points qui nous intéressent sont ceux selon $g(h), h(t)$!



Notre fonction aurait pu être comme ça, mais on veut juste être sur les points du cercle.



Ici on veut l'aire donc

$$A_k = f(x_k, y_k)\Delta s_k$$

$$A_k = f(x_k, y_k)\sqrt{(\Delta x_k)^2 + (\Delta y_k)^2}$$

$$\Rightarrow dA = f(g(t), h(t))\sqrt{g'(t)^2 + h'(t)^2}dt$$

Theorem (Gauss / Green)

Let $\Omega \subseteq \mathbb{R}^2$ be as in the main auxiliary theorem

Let $\vec{F} : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a diff'able vector field. Then

$$\iint_{\Omega} \operatorname{div} \vec{F}(x_1, x_2)\, dx_1 dx_2 = \int_{\partial\Omega} \vec{F} \cdot \vec{n}\, dl$$

Gauss theorem / divergence theorem

$$\iint_{\Omega} \operatorname{curl} \vec{F}(x_1, x_2)\, dx_1 dx_2 = \int_{\partial\Omega} \vec{F} \cdot \vec{\tau}\, dl$$

Green theorem

ccw rotation by $90°$: $(-y, x)$

cw rotation by $90°$: $(y, -x)$

Outlook

$\boxed{2D}$ $\iint \operatorname{div} \vec{F} = \oint \vec{F} \cdot \vec{n}$

area integral — line integral

$\iint \operatorname{curl} \vec{F} = \oint \vec{F} \vec{T}$

area integral — line integral

$\boxed{3D}$ $\iiint \operatorname{div} \vec{F} = \oiint \vec{F} \cdot \vec{n}$

volume integral — surface integral

$\oiint \operatorname{curl} \vec{F} = \oint \vec{F} \vec{T}$

surface integral — line integral

Similar in higher dimensions:
hypervolume vs. hypersurface

Higher dimensions:
so much more difficult

Trouver un potentiel

- compute $\int_0^x F(t)dt$
- compute $\int_x^y F(t)dt$
- sum them