Solución de los 8 pares de funciones con explicaciones detalladas

1. **Par 1:**
$$f(n) = \log(n^2), \quad g(n) = \log\bigl(n^2 + 1\bigr).$$
Análisis: Para n grande,
$$n^2 + 1 \sim n^2 \quad \Longrightarrow \quad \log\bigl(n^2+1\bigr) \sim \log(n^2) = 2\log(n).$$
Por lo tanto,
$$\frac{\log(n^2)}{\log\bigl(n^2+1\bigr)} \xrightarrow[n\to\infty]{} \text{constante positiva,}$$
lo que nos permite concluir que:
$$f(n) \in \Theta\bigl(g(n)\bigr).$$

2. **Par 2:**
$$f(n) = \log(n), \quad g(n) = \log(n^2).$$
Análisis:
$$\log(n^2) = 2\log(n) \quad \Longrightarrow \quad \frac{f(n)}{g(n)} = \frac{\log(n)}{2\log(n)} = \frac{1}{2}.$$
Por ello,
$$f(n) \in \Theta\bigl(g(n)\bigr).$$

3. **Par 3:**
$$f(n) = (n^3)^2 = n^6, \quad g(n) = n^3.$$
$$\frac{f(n)}{g(n)} = \frac{n^6}{n^3} = n^3.$$
Análisis:
$$\frac{n^6}{n^3} = n^3 \xrightarrow[n\to\infty]{} \infty.$$
Por lo tanto,
$$f(n) \in \omega\bigl(g(n)\bigr).$$

4. **Par 4:**

$$f(n) = n\ \log(n^2), \quad g(n) = n\ \log(n).$$
$$f(n) = n \cdot 2\log(n) = 2n\log(n).$$
$$\frac{f(n)}{g(n)} = \frac{2n\log(n)}{n\log(n)} = 2.$$

Análisis:

$$f(n) \in \Theta\big(g(n)\big).$$

5. **Par 5:**

$$f(n) = \log\big(\log(n)\big), \quad g(n) = \sqrt{\log(n)}.$$
$$\frac{f(n)}{g(n)} = \frac{\log(\log(n))}{(\log(n))^{1/2}}.$$

Análisis:

$$\lim_{n\to\infty} \frac{\log(\log(n))}{(\log(n))^{1/2}} = 0.$$
$$f(n) \in o\big(g(n)\big).$$

6. **Par 6:**

$$f(n) = n^{\log(n)}, \quad g(n) = n\ \log(n) - n.$$
$$g(n) = n\big(\log(n) - 1\big).$$
$$n^{\log(n)} = \exp\big(\log(n)\cdot\log(n)\big) = \exp\big((\log(n))^2\big).$$
$$\frac{f(n)}{g(n)} \approx \frac{\exp\big((\log(n))^2\big)}{n\ \log(n)}.$$
$$\log\bigg(\frac{f(n)}{g(n)}\bigg) \approx (\log(n))^2 - \log\big(n\ \log(n)\big).$$

Análisis:

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty.$$

Por lo tanto,

$$f(n) \in \omega\big(g(n)\big).$$

7. **Par 7:**

$$f(n) = \log\big(n^{\,n\log n}\big), \quad g(n) = \sqrt{n\ \log(n)}.$$

$$\log\big(n^{\,n\log n}\big) = n\log n \cdot \log n = n\,(\log n)^2.$$

$$g(n) = \sqrt{n\ \log(n)} = (n\ \log(n))^{1/2} = n^{1/2}\,(\log(n))^{1/2}.$$

$$\frac{f(n)}{g(n)} = \frac{n\,(\log(n))^2}{n^{1/2}\,(\log(n))^{1/2}} = n^{1/2}\,(\log(n))^{3/2}.$$

Análisis:

$$\lim_{n\to\infty} n^{1/2}\,(\log(n))^{3/2} = \infty.$$

Así,

$$f(n) \in \omega\big(g(n)\big).$$

8. **Par 8:**

$$f(n) = 2^{\,n^2+1}, \quad g(n) = 4^n.$$

$$4^n = (2^2)^n = 2^{2n}.$$

$$\frac{f(n)}{g(n)} = \frac{2^{n^2+1}}{2^{2n}} = 2^{\,n^2+1-2n}.$$

$$n^2 + 1 - 2n = n^2 - 2n + 1 = (n-1)^2.$$

Análisis:

$$f(n) \in \omega\big(g(n)\big).$$