



UNIVERSIDAD NACIONAL AUTÓNOMA DE MÉXICO

FACULTAD DE CIENCIAS

Tarea 2

Análisis de algoritmos

Rivera Morales David
320176876

Gallegos Cortes José Antonio
320316566

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Solución de los 8 pares de funciones

1. Par 1:

$$f(n) = \log(n^2), \quad g(n) = \log(n^2 + 1).$$

Análisis: Para n grande,

$$n^2 + 1 \sim n^2 \implies \log(n^2 + 1) \sim \log(n^2) = 2 \log(n).$$

Por lo tanto,

$$\frac{\log(n^2)}{\log(n^2 + 1)} \xrightarrow{n \rightarrow \infty} \text{constante positiva},$$

lo que nos permite concluir que:

$$f(n) \in \Theta(g(n)).$$

2. Par 2:

$$f(n) = \log(n), \quad g(n) = \log(n^2).$$

Análisis:

$$\log(n^2) = 2 \log(n) \implies \frac{f(n)}{g(n)} = \frac{\log(n)}{2 \log(n)} = \frac{1}{2}.$$

Por ello,

$$f(n) \in \Theta(g(n)).$$

3. Par 3:

$$f(n) = (n^3)^2 = n^6, \quad g(n) = n^3.$$

$$\frac{f(n)}{g(n)} = \frac{n^6}{n^3} = n^3.$$

Análisis:

$$\frac{n^6}{n^3} = n^3 \xrightarrow{n \rightarrow \infty} \infty.$$

Por lo tanto,

$$f(n) \in \omega(g(n)).$$

4. Par 4:

$$f(n) = n \log(n^2), \quad g(n) = n \log(n).$$

$$f(n) = n \cdot 2 \log(n) = 2n \log(n).$$

$$\frac{f(n)}{g(n)} = \frac{2n \log(n)}{n \log(n)} = 2.$$

Análisis:

$$f(n) \in \Theta(g(n)).$$

5. Par 5:

$$f(n) = \log(\log(n)), \quad g(n) = \sqrt{\log(n)}.$$

$$\frac{f(n)}{g(n)} = \frac{\log(\log(n))}{(\log(n))^{1/2}}.$$

Análisis:

$$\lim_{n \rightarrow \infty} \frac{\log(\log(n))}{(\log(n))^{1/2}} = 0.$$

$$f(n) \in o(g(n)).$$

6. Par 6:

$$f(n) = n^{\log(n)}, \quad g(n) = n \log(n) - n.$$

$$g(n) = n(\log(n) - 1).$$

$$n^{\log(n)} = \exp(\log(n) \cdot \log(n)) = \exp((\log(n))^2).$$

$$\frac{f(n)}{g(n)} \approx \frac{\exp((\log(n))^2)}{n \log(n)}.$$

$$\log\left(\frac{f(n)}{g(n)}\right) \approx (\log(n))^2 - \log(n \log(n)).$$

Análisis:

$$\lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \infty.$$

Por lo tanto,

$$f(n) \in \omega(g(n)).$$

7. Par 7:

$$f(n) = \log(n^{n \log n}), \quad g(n) = \sqrt{n \log(n)}.$$

$$\log(n^{n \log n}) = n \log n \cdot \log n = n (\log n)^2.$$

$$g(n) = \sqrt{n \log(n)} = (n \log(n))^{1/2} = n^{1/2} (\log(n))^{1/2}.$$

$$\frac{f(n)}{g(n)} = \frac{n (\log(n))^2}{n^{1/2} (\log(n))^{1/2}} = n^{1/2} (\log(n))^{3/2}.$$

Análisis:

$$\lim_{n \rightarrow \infty} n^{1/2} (\log(n))^{3/2} = \infty.$$

Así,

$$f(n) \in \omega(g(n)).$$

8. **Par 8:**

$$f(n) = 2^{n^2+1}, \quad g(n) = 4^n.$$

$$4^n = (2^2)^n = 2^{2n}.$$

$$\frac{f(n)}{g(n)} = \frac{2^{n^2+1}}{2^{2n}} = 2^{n^2+1-2n}.$$

$$n^2 + 1 - 2n = n^2 - 2n + 1 = (n - 1)^2.$$

Análisis:

$$f(n) \in \omega(g(n)).$$