UNIVERSITÁ BOCCONI
SPECIALIZED MASTER IN QUANTITATIVE FINANCE AND RISK MANAGEMENT

Exotic Derivatives Assignment

CAPPED BONUS CERTIFICATE
S&P MIDCAP 400 INDEX

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1 Introduction

This report presents a comprehensive quantitative and qualitative analysis of an investment certificate issued by Vontobel on April 5,024. The structured financial product presented by Vontobel is linked to the performance of the S&P MidCap 400 Index and was issued at 100% of its nominal value of USD 1,000.00. Denominated in USD, the product includes key dates such as the initial fixing on April 5, 2024, the final fixing on October 6, 2025, and the repayment date on October 14, 2025.

2 Product Description

The product's return is tied to the S&P MidCap 400 Index. At inception, the index had a spot reference price of USD 2,989.16. The strike price was set at USD 3,198.40, representing 107% of the spot reference price, while a cap level was defined at USD 3,422.59, corresponding to 114.50% of the spot. Additionally, a barrier level was established at USD 2,391.33, equivalent to 80% of the spot price.

Barrier monitoring is continuous over the period from April 5, 2024, to October 6, 2025. A barrier event occurs if, at any point during this time frame, the index value touches or falls below the barrier level. The redemption mechanism is contingent on whether such an event takes place:

- **No Barrier Event:** If the underlying index does not breach the barrier during the observation period, the redemption amount is based on the final fixing level of the index. This value is adjusted to ensure it is not lower than the strike price and does not exceed the cap. The resulting amount is then multiplied by the number of underlyings (0.33454).
- **Barrier Event:** If the barrier is breached at any point during the observation period, the payout corresponds directly to the final fixing level of the index, subject to the cap. This amount is also multiplied by the number of underlyings.

The redemption formula defined by the issuer is:

$$\text{Redemption} = R \cdot \min\left(\text{Cap},\ SF + I \cdot \max(0,\ X - SF)\right) \tag{1}$$

where:

- SF = Final fixing level of the underlying index
- X = Strike price
- R = Number of underlyings (0.33454)
- $I = \begin{cases} 1, & \text{if the barrier is never breached during the observation period} \\ 0, & \text{if the barrier is breached at any point} \end{cases}$

Payoff Structure and Analytical Framework

The payoff structure depends on whether the barrier level is breached during the observation period. Therefore, based on the possible outcomes, two distinct cases can be identified, as detailed in the tables below:

Final Index Level (SF)	**Barrier Not Breached ($I = 1$)**
Barrier $\leq SF \leq$ Cap	$R \cdot (SF + \max(0,\ X - SF))$
$SF >$ Cap	$R \cdot$ Cap

Table 1: Payoff when no barrier event occurs

where cap and barrier correspond to the predefined levels introduced at the beginning of this report.

Final Index Level (SF)	**Barrier Breached ($I = 0$)**
$SF <$ Barrier	$R \cdot SF$
Barrier $\leq SF \leq$ Cap	$R \cdot SF$
$SF >$ Cap	$R \cdot$ Cap

Table 2: Payoff when a barrier event occurs

By conditioning and combining the 2 possible events outlined in Table 1 and 2, we can write the general payoff formula as follows:

$$\begin{aligned} \text{Payoff} &= R \cdot \min\left(\text{Cap},\ I \cdot (SF + \max(0,\ X - SF)) + (1 - I) \cdot SF\right) & (2) \\ &= R \cdot \min\left(\text{Cap},\ SF + I \cdot \max(0,\ X - SF)\right) & (3) \end{aligned}$$

After expanding and simplifying the terms in the general payoff formula that we construct by combining the two possible events (barrier breached or not breached), we observe that it matches the issuer's defined redemption formula for the certificate. The payoff of the product can be visualized through the following payoff graph, shown in Figure 1.

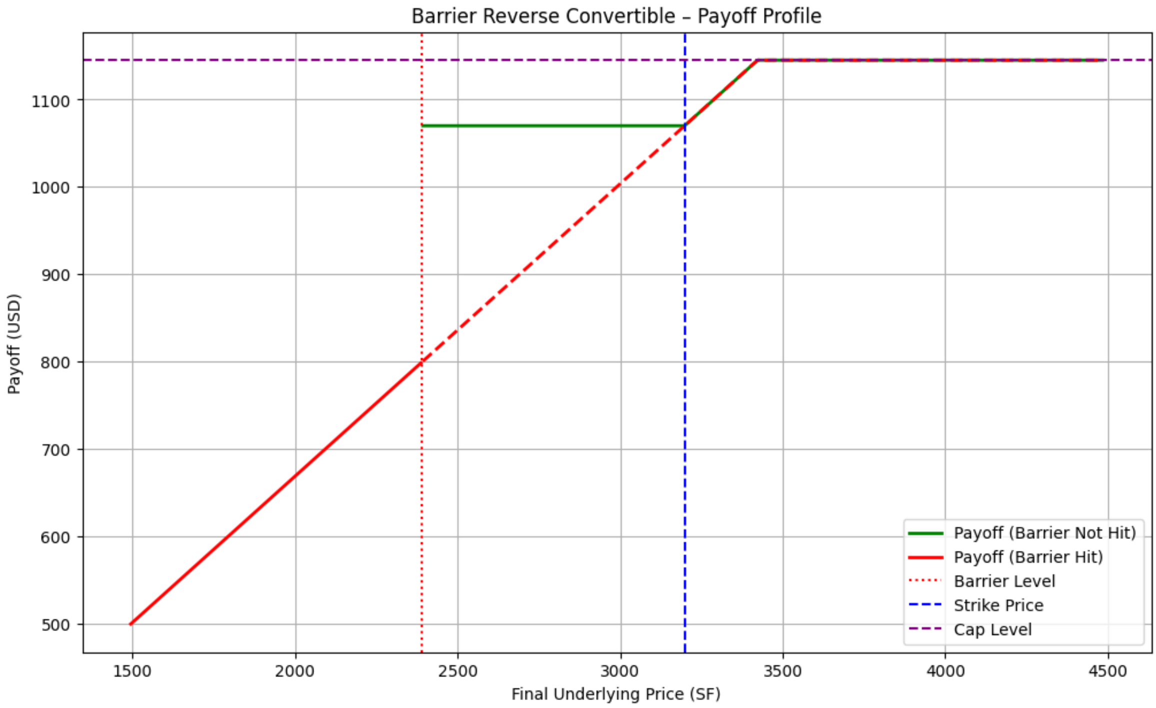

Figure 1: Payoff profile of the investment certificate

3 Replicating Strategy

To determine the price of the certificate, we begin by constructing a replicating portfolio that mimics the payoff function of the product outlined in Figuer 1. The goal is to break down the payoff structure into simpler components that can be directly replicated with financial instruments. To identify the components, we simplified and rearranged the terms of the payoff function, arriving at the following formula:

$$\begin{aligned} \text{Payoff} &= R \cdot \min\left(\text{Cap},\ I \cdot (SF + \max(0,\ K - SF)) + (1 - I) \cdot SF\right) & (4) \\ &= R \cdot \text{Cap} + I \cdot R \cdot \max(0, K - SF) - R \cdot \max(0, SF - \text{Cap}) & (5) \end{aligned}$$

We can now break the payoff into the following three distinct components:

- $R{\cdot}\text{Cap}$: This term can be replicate by the payoff from a zero-coupon bond (ZCB) that guarantees a fixed return of Cap, which reflects the capped value of the product.
- $I \cdot R{\cdot}\max(0, SF - X)$: This term can be replicated by entering a long position in a down-and-out put option, where the payoff is activated when the underlying asset price breaches the Barrier.
- $-I \cdot R{\cdot}\max(0, K - SF)$: This term can be represented by entering a short position in a standard put option with strike $K = CapLevel$.

Thus, the replicating strategy involves combining the following components:

- A zero-coupon bond (ZCB) with a face value equal to the Cap,
- A long position in a down-and-out put option with a barrier at H = 80% of the spot price ,
- A short position in a standard put option with a strike price $K = CapLevel$.

This combination of instruments exactly replicates the payoff profile of the certificate outlined in Figure 1, allowing us to price the certificate using the Black-Scholes Model and Monte Carlo Simulation.

4 Main Risk Factors from the Issuer's Perspective

- **Market Risk**: The product's payoff depends on the performance of the S&P Midcap 400 Index. Poor performance can lead to higher payouts if the barrier is not hit and the final price exceeds the strike price. High volatility increases the likelihood of the barrier being breached, which could raise the issuer's liability.
- **Barrier Risk**: The barrier level, if breached, changes the payoff structure, increasing the issuer's payout obligation.
- **Interest Rate Risk**: Changes in interest rates impact the present value of future obligations, potentially affecting the issuer's liability.
- **Liquidity Risk**: The issuer must maintain sufficient liquidity to meet payment obligations, including worst-case scenario payouts. Additionally, possible lack of liquidity means that issuers may struggle to exit positions or sell the products without incurring financial losses.
- **Credit Risk**: This involves the risk that a counterpart, such as a guarantor or other involved entities, fails to meet their financial obligations. For issuers, the possibility of default by these parties poses a threat to the stability and reliability of the product.
- **Regulatory and Compliance Risk**: Non-compliance with financial regulations could result in fines, legal actions, or reputational damage.

5 Pricing Model & Data Calibration

Calibration Procedure

The historical data used for calibration was collected from Yahoo Finance using the `yfinance` Python package. A three-year period of daily closing prices was chosen to ensure a statistically significant sample without being too outdated.

Logarithmic returns were computed to estimate the drift (μ) and volatility (σ) under the assumption of a Geometric Brownian Motion (GBM) process. This approach is commonly adopted when option-implied volatilities are unavailable.

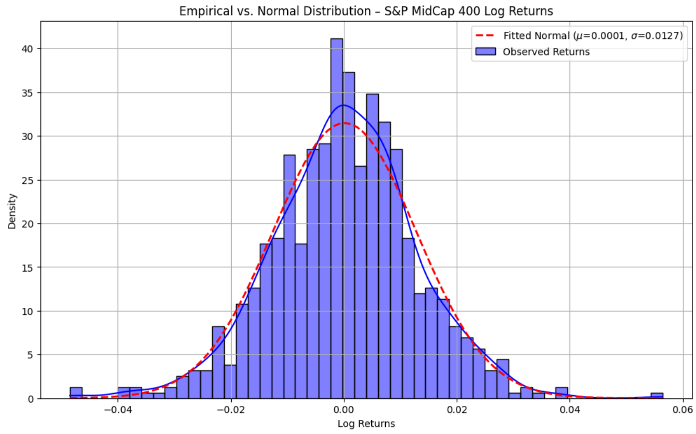

Figure 2: Historical Distribution S&P MidCap 400 Index

Fit Quality

A normal distribution was fitted to the empirical log returns to assess how well the GBM model captures the observed dynamics. While the fit was generally adequate, potential underestimation of tail risk suggests the GBM model might be extended using stochastic volatility (Heston) or jump-diffusion models for more accuracy in extreme scenarios.

Parameters	Value	Type	Motivation
S_0	2989.16	Market data	Most recent closing price of S&P MidCap 400 (from Yahoo Finance)
μ	0.0354	Historical drift	Estimated from log returns of 3-year daily close prices
σ	0.2013	Historical volatility	Standard deviation of log returns from the same period
r	0.045	Assumed constant	Typical actual long-term annualized risk-free rate
T	1.5	Model input	Time to maturity (in years)
Strike	3198.40	Market data	Strike of the Investment Certificate
Ratio	0.33454	Product-specific	Defined by product specification
$n_{\mathbf{paths}}$	10,000	Simulation parameters	Sufficient for Monte Carlo convergence
$n_{\mathbf{steps}}$	378	Simulation parameters	Approx. number of trading days in 1.5 years

Table 3: Summary of Input Parameters, Sources, and Motivation

Monte Carlo Simulation for the Underlying:

Given that the payoff of the structured product depends on the final value of the S&P MidCap 400 Index, we began our analysis by examining the historical returns of the index over the past three years, deliberately excluding earlier periods to avoid the volatility distortions caused by the pandemic. By fitting a normal distribution and applying the Kolmogorov-Smirnov test for normality, we found that the historical returns closely follow a normal distribution. Based on this, we employed a Monte Carlo simulation using a geometric Brownian motion model, with the drift parameter μ set to the mean and the volatility σ set to the standard deviation of the log returns. This approach allowed us to simulate a wide range of possible future price paths and estimate the expected payoff under various market conditions.

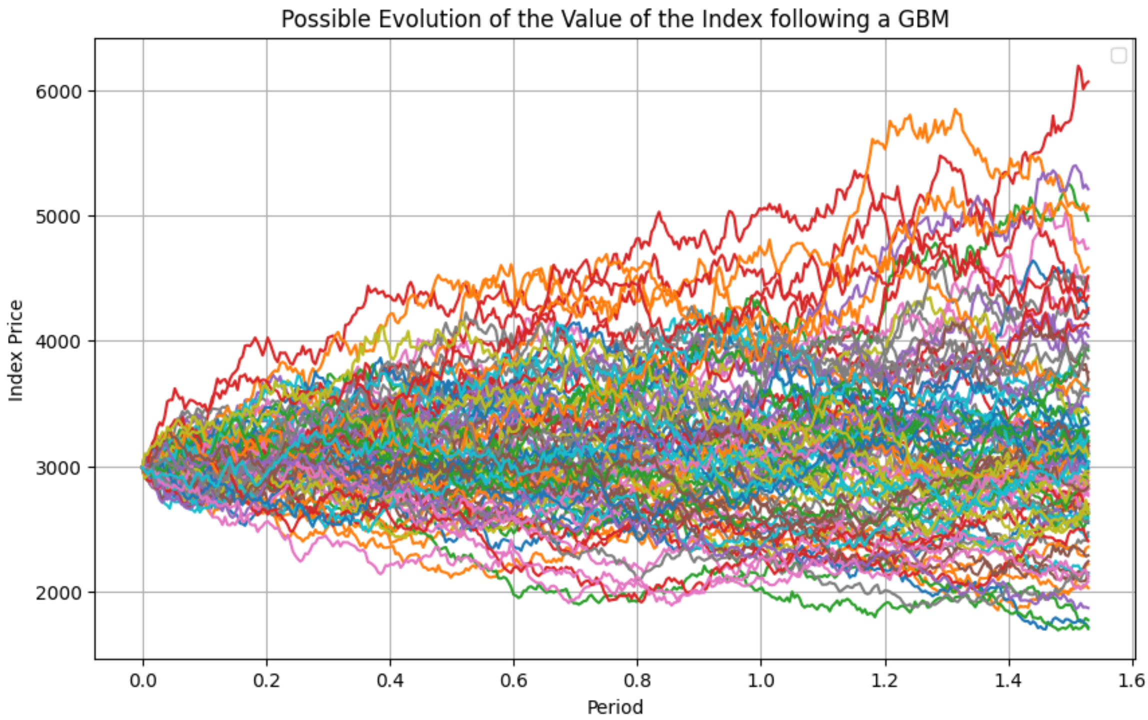

Figure 3: S&P MidCap 400 Monte Carlo Simulation

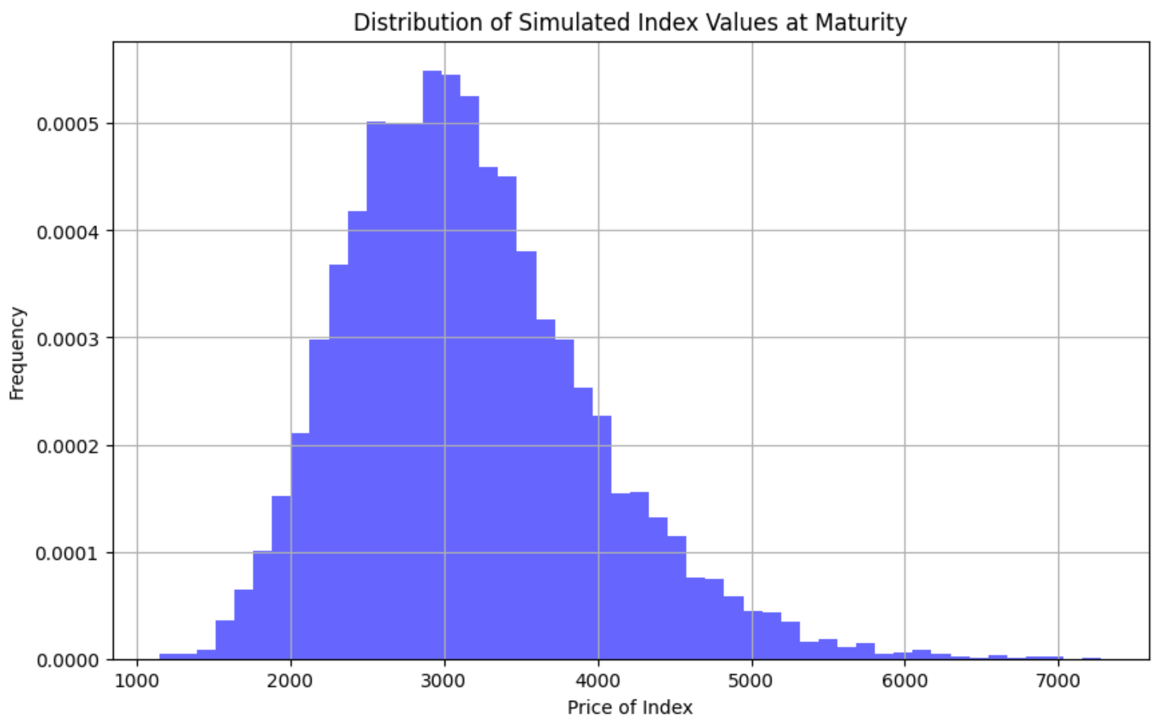

Figure 4: Distribution of S&P MidCap 400 Simulated Index at Maturity

Monte Carlo Simulation for pricing the Down-and-Out Put:

Monte Carlo simulations are ideal for pricing path-dependent options, as they allow us to model a wide range of possible price trajectories and determine whether the barrier is breached along each path.

To price the down-and-out put option, we implemented a Monte Carlo simulation using the volatility parameter σ estimated from historical log returns. A barrier was set at 80% of the current spot price, where if the simulated path reaches or falls below this barrier at any point before maturity, the option becomes worthless, resulting in a zero payoff. Otherwise, the payoff is given by $\max(K - S_T, 0)$, where K is the strike price and S_T is the simulated terminal price of the index. The option value was then estimated by taking the average of all non-zero payoffs and discounting them to present value.

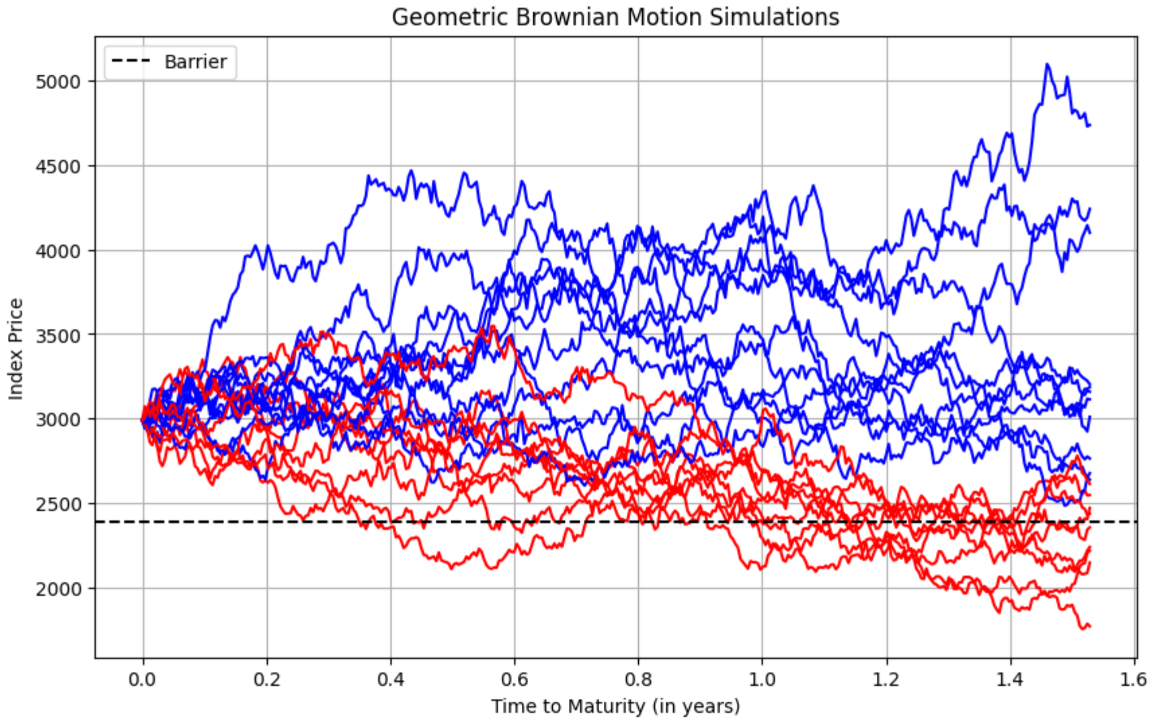

Figure 5: Down & Out Put Monte Carlo Simulation

Pricing the Short Put using the lack-Scholes model:

To price the short put option in the replicating startegy, we used the the widely accepted Black-Scholes model. Since this part of the product is not affected by the path of the underlying price and has no barrier.

The price of the put option was calculated using the following formula:

$$\text{Put Price} = Ke^{-rT}N(-d_2) - S_0N(-d_1) \tag{6}$$

where d_1 and d_2 are given by:

$$d_1 = \frac{\ln(S_0/K) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T} \tag{7}$$

Normal Discounting for the Zero Coupon Bond (ZCB):

The Zero-Coupon Bond represents a guaranteed payment at maturity. Its value today is found by discounting the final amount.

The price of the zero-coupon bond was calculated using the following formula:

$$\text{ZCB Price} = \text{Final Value} \times e^{-rT} \tag{8}$$

6 Certification Price

Certificate

Given that we now have the price of the three components of the replicating strategy, the price of the certificate can be derived as:

$$\text{Certificate Price} = \text{ZCB Price} + \text{Option Price} - \text{Put Price} \tag{9}$$

Results

The estimated prices for each component and the final certificate price are as follows:

Component	**Price**
Put Option	140.5146 USD
Down-and-Out Put Option	25.7464 USD
Zero-Coupon Bond	1068.8771 USD
Certificate	954.11 USD

Table 4: Estimated Prices of Components

The replicating strategy provides a comprehensive approach to pricing a financial certificate by combining different financial instruments. The estimated price of the certificate, based on the given assumptions and calculations, is approximately 954.11 USD.

7 Greeks

For our sensitivity analysis, we calculated the Greeks using the central difference method, applying a 1% relative shift to each relevant input variable. For Theta, we used a backward difference approach, since the option ceases to exist after maturity, making forward shifts in time inappropriate.

Delta (Δ)

Measures the sensitivity of the option price to changes in the underlying index price S.

$$\Delta = \frac{V(S + \Delta S, \sigma, r, T) - V(S - \Delta S, \sigma, r, T)}{2 \cdot h} \tag{10}$$

Value: $\Delta = 0.1714$

A Delta of \$0.1714 means the option's price rises by about \$0.17 for every \$1 increase in the underlying asset's price. This aligns with expectations, as a higher index level brings the payoff closer to the cap and further from the barrier.

Gamma (Γ)

Indicates the rate of change of Delta with respect to S.

$$\Gamma = \frac{V(S + \Delta S, \sigma, r, T) - 2V(S, \sigma, r, T) + V(S - \Delta S, \sigma, r, T)}{h^2} \tag{11}$$

Value: $\Gamma = -0.00134$

A Gamma of -0.0013 indicates that Delta decreases slightly as the underlying price increases. This suggests the position becomes less sensitive to price changes at higher levels, which appears consistent with the restriction imposed by the cap.

Vega ($\mathcal{V}$)

Measures sensitivity to changes in implied volatility σ.

$$\mathcal{V} = \frac{V(S, \sigma + \Delta\sigma, r, T) - V(S, \sigma - \Delta\sigma, r, T)}{2 \cdot h} \tag{12}$$

Value: $\mathcal{V} = -0.0539$

A negative Vega of -0.0539 implies that the option's price falls by about \$0.054 for every 1% increase in volatility. This reflects the fact that higher volatility increases the likelihood of breaching the barrier, reducing the value of the down-and-out put.

Theta (Θ)

Represents time decay (sensitivity to the passage of time T).

$$\Theta = \frac{V(S, \sigma, r, T) - V(S, \sigma, r, T - h)}{h} \tag{13}$$

Value: $\Theta = -0.0148$

A negative Theta of -0.0148 means the option loses about \$0.015 in value each day as time passes, assuming all else remains constant. This time decay is expected, as the probability of the barrier being hit reduces over time.

Rho (ρ)

Shows sensitivity to changes in the risk-free interest rate r.

$$\rho = \frac{V(S, \sigma, r + \Delta r, T) - V(S, \sigma, r - \Delta r, T)}{2 \cdot h} \tag{14}$$

Value: $\rho = 0.0088$

A Rho of 0.0088 indicates that the option's value increases by around \$0.0088 for every 1 percentage point rise in interest rates. This is mainly driven by the zero-coupon bond component, which becomes more valuable as rates increase.

8 Main risk factors for all option components

Option Component	**Main Risk Factors**
Zero-Coupon Bond (ZCB)	Interest Rate Risk, Credit Risk, Liquidity Risk
Put Option	Volatility Risk, Market Risk, Liquidity Risk, Credit Risk
Down-and-Out Put Option	Barrier Risk, Volatility Risk, Market Risk, Liquidity Risk, Credit Risk

Table 5: Main Risk Factors for Each Option Component

Zero-Coupon Bond (ZCB) A Zero-Coupon Bond is particularly sensitive to **interest rate risk**, as its value is the present value of a single future payment; fluctuations in interest rates directly impact its discounted value. Additionally, ZCBs carry **credit risk**, since the bondholder is exposed to the possibility that the issuer may default on the payment at maturity. **Liquidity risk** also arises if the market for such bonds is thin, potentially making it difficult to sell the bond quickly without significantly affecting its price.

Put Option The value of a Put Option is heavily influenced by **volatility risk**, as increased volatility in the underlying asset typically raises the premium of the option. It also bears **market risk**, given that broad movements in the market or in the underlying asset's price can alter its intrinsic and time value. **Liquidity risk** is present if the option cannot be traded swiftly at a fair price. Finally, when the option is not exchange-traded, **credit risk** may be significant due to the possibility that the counterparty fails to fulfill their obligations.

Down-and-Out Put Option A Down-and-Out Put Option introduces unique **barrier risk**, since the option becomes worthless if the underlying asset's price touches a predefined barrier level. Like standard options, it is also exposed to **volatility risk**, as changes in the underlying's volatility affect the likelihood of hitting the barrier and the option's overall price. **Market risk** continues to play a role through general asset price movements. **Liquidity risk** may occur due to less active markets for exotic options. Finally, **credit risk** arises in over-the-counter (OTC) contracts if the counterparty defaults.

9 Worst of a basket of 3 stocks

When transitioning from a single asset product to one based on the worst-performing asset in a basket of three stocks, several additional risk factors emerge. Key considerations mainly include correlation risk, where increased correlation among stocks can amplify losses. Concentration risk also increases, as the basket's performance can be heavily impacted if one stock underperforms. Liquidity risk varies across the individual stocks, potentially resulting in higher transaction costs or challenges in exiting positions. Volatility risk is more pronounced in the worst-performing stock, which can cause larger fluctuations in the overall basket's value. Credit risk may be affected if the issuer faces financial difficulties, impacting product performance. Market risk also broadens, as economic downturns or sector-specific events affecting any of the basket's components can drive the worst-of outcome. Lastly, operational and regulatory risks increase due to the added complexity in managing multiple assets and complying with evolving legal frameworks. These risks require careful consideration to ensure the product's structure aligns with investor objectives and risk tolerance.

10 Conclusion

Overall, based on the Monte Carlo simulation using the calibrated parameters, the estimated value of the structured investment certificate is very close to the actual market price of 960, indicating that our model provides a reliable and reasonably accurate approximation. This product may be considered suitable for short-term investment, especially under the assumption that the S&P MidCap 400 index does not experience a severe drop (greater than 80%) during the investment horizon. Such an extreme decline is historically rare and statistically unlikely under normal market conditions.Therefore, for investors with a moderate risk appetite and a short investment horizon, this certificate offers a compelling risk-reward profile, particularly given the embedded yield enhancement and conditional protection features.