PowerToUkraine

Implementing the Decentralization of Power Grid as Linear Optimization Problem

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***Abstract*—In Russia's war of aggression against Ukraine it is actively targeting the power grid in an effort to demoralize its victims. The standard tree shaped network topology is highly vulnerable to attacks on power plants or high voltage power line cuts. We seek to employ micro grids and rooftop solar panels as a means to produce electricity locally, making bombings of power plants and lines less effective. We look at suburban neighbourhoods and use a linear optimisation task to find optimal locations for a given amount of solar panels while retaining the networks current topology.**

I. Introduction

A. Motivation

Ukraine, like many other countries, has a tree shaped power grid. These were effective in an era of big centralised power plants that produce electricity for large swathes of land e.g. the Ukrainian nuclear power plant "Zaporizhzhia". However in an era where there are more and more small producers and a growing number of "Prosumers" [11] these grids are harder and harder to manage [2]. They also make a country more susceptible to attacks on its grid, as it turns out, only few targets have to be struck to ensure a large scale black-out. Furthermore these big power plants are very hard to repair and the damage is long-lasting. Figure 1 shows Ukraine at night on Nov. 24. 2022, and parts of the adjoining countries. The stark difference in illumination shows the devastating effect of Russias attacks. Our proposal to reduce the bombings effect is to decentralise the power grid by installing photovoltaic (PV) cells on civilians houses and producing the electricity as close as possible to where it is consumed. We concentrate on rooftop solar panels, as they are cheap, flexible and sustainable while being widely available due to existing infrastructure. Although only using solar panels is an impracticable solution to electricity needs in general, they might provide a stopgap during war times. We assumed the solar panels to be given to Ukraine by other countries, as countries try to bolster their local PV production they would give panels produced in their country.

Further our paper concentrates on the low voltage grid, as high voltage grids will be less important in a localized grid. Medium voltage grids on the other hand tend to be rather meshed when compared to low voltage ones [1] resulting in a grid sturdier against attacks. Interestingly enough Denys Shmyhal Ukraines Prime Minister (as of Feb. 23. 2023) announced that Ukraine had similar plans saying: "Russian attacks are pushing us towards a fundamental reform — building a decentralized energy system," [4] and "It will make it less vulnerable to enemy attacks. We are talking about building mini-power plants and small generation facilities, integrated into the existing power grid." [4].

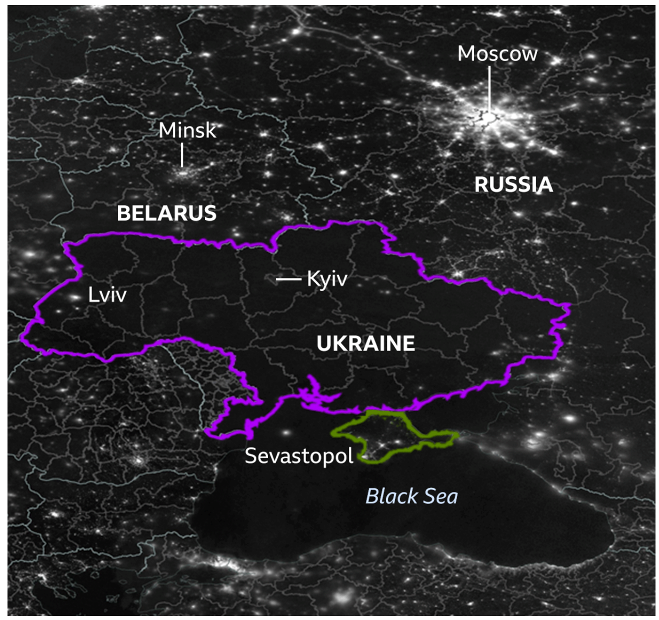

Fig. 1. Satellite image of Ukraine at night on Nov. 24. 2022 [3]

II. Mathematical Model

Our system models a neighborhood or small city. The neighborhood consists of different building types, which include apartment buildings with multiple households sharing one power connector and detached single family houses. Furthermore there are commercial buildings, including bakeries, small grocery shops, barbers etc.

A. Model definition

For simplicity's sake, we will group them under the generic term house. In addition the neighborhood has a slack node, representing the connection to the existing middle voltage grid and can produce or absorb an unlimited amount of power if necessary. Not all houses are connected to the slack node directly but they can still receive power indirectly via connection to other houses.

The houses are numbered from 1 to N while the slack node is denoted by s and has index $N+1$.

Our aim is to find an optimal distribution of a given amount

of solar panels on these houses. As power consumption changes during the course of a day or season our model can adopt different time points creating a build-out that performs well all year over.
We use a discrete set of times, snapshots of the power consumption, which we will denote as $T = \{t_1, t_2, ..., t_k\}$. The area of solar panels built on house i is called a_i, which should be minimized to use the panels efficiently. We modeled our system such that solar panels are given in square meters and not as individual panels of fixed size. This keeps our feasible set continuous and in the real world it would be possible to round the size to the next solar panel. If that would not fit onto the roof in our data in reality it will just slightly hang off the roof.
If solar panels are not needed in a village they should be used in another one, otherwise the power has to flow over a long range again.

One house will be connected to other houses by power lines. The power (in Watt) transported by the power line from House i to House j at a given time t is denoted as $p_{i,j}(t)$. This implies that $p_{i,i}$ is the amount of power house i produces, which solely depends on the area of solar panels built on its roof. These are all decision variables of our problem. Note that the direction of the current is not given by the signum but by the indices.
For example $p_{1,2}(t)$ is the amount of power transported to house 1 from house 2 and $p_{2,1}(t)$ is the amount of power transported to house 2 from house 1. The length between houses i and j is denoted with $l_{i,j}$. Each power line has a given rating determining the maximal current flow possible. It is denoted by $r_{i,j}$. The ratings $r_{i,j}$, the lengths between the houses $l_{i,j}$ and the amount of power sent between houses $p_{i,j}(t)$ can be interpreted as matrices $L, R, P(t) \in \mathbb{R}^{(N+1)\times(N+1)}$. The roof sizes a_i of the houses meanwhile is a vector $a \in \mathbb{R}^N$, the slack node does not have a roof even though we model it similarly to a house.

Our model works with very simplified physics, it does not account for many problems that occur in a real world setting e.g. friction, loss of power, voltage drops, instabilities caused by many small providers etc.
Further it does not distinguish between multiple flats in one building and assumed they share one connection.
The area PVs we can build on one house is treated as continuous, additional infrastructure is also not taken into account. The power generated by the PVs only depends on the sun and the efficiency of the PVs formalized by the function $sun(t), t \in T$. We assume the same type of PVs on every house, since sending different ones to one small village would not make sense from a logistical perspective.

B. Constraints

First we want to give an overview of the constraints, we formulated:

1. Most importantly, the energy consumption $c_i(t)$ of house i is to be met for every time t. To calculate the power that a house has available, the outgoing power $p_{j,i}(t)$ is subtracted from the incoming $p_{i,j}(t)$ produced power $p_{i,i}(t)$:

$$c_i(t) = \sum_{j=1}^{N+1} (p_{i,j}(t) - p_{j,i}(t)) + p_{i,i}(t), \forall i \in \{1, ..., N\}$$

2. Incoming power minus outgoing power of the slack node must equal the amount of power offered or absorbed by the slack node s:

$$p_{N+1,N+1}(t) = \sum_{j=1}^{N} p_{j,N+1}(t) - p_{N+1,j}(t)$$

3. The amount of power house i produces depends on the area of PVs built on its roof and a function $sun(t)$ which represents the amount of sun energy the panel can convert at given time t:

$$p_{i,i}(t) = a_i \cdot sun(t), \forall i \in \{1, \ldots, N\}$$

4. The rating of the power line $r_{i,j}$ limits the power flow on the line $p_{i,j}(t)$ and can not be negative:

$$0 \leq p_{i,j}(t) \leq r_{i,j}, \forall i, j \in \{1, \ldots, N+1\} \text{ and } i \neq j$$

5. Each individual house i has a maximum roof area on which we can build PVs a_i^{max}:

$$0 \leq a_i \leq a_i^{max}, \forall i \in \{1, \ldots, N\}$$

6. We do not have an unlimited amount of solar panels, we can build on all houses A_{tot}:

$$\sum_{i=1}^{N} a_i \leq A_{tot}$$

All of the constraints have to be met $\forall t \in T$

C. Cost Function

The cost function comprises of three terms controlling the outcome. We will use different colors to visualize the aforementioned:

$$\sum_{t\in T}\Big(\sum_{i=j=1}^{N} {\color{blue} l_{i,j} \cdot p_{i,j}(t)} \Big) \;\; {\color{red} + \;\; M \cdot p_{N+1,N+1}(t)} \Big) + \sum_{i=1}^{N} {\color{green} \varepsilon \cdot a_i}$$

The underlying idea of the blue term in our cost function is decentralization of the power grid. We multiply the length of the power lines, with the power that is sent over those lines and sum these for all times t of our discrete set of times T. This punishes sending large amounts of power over long distances,

which should be avoided. Meanwhile providing a neighbour with self produced energy should be promoted. This makes sure that we try to find a solution that is as decentralized as possible.
The red term deals with the slack node s. In order to prevent the usage of outside power, we punish it with a very large term M. In an optimal setting, the houses would sustain themselves, without the need of a connection to the outside grid. This is called the "island mode" of the decentralized grid [11]. That might of course not always be possible. At night for example the houses still have power consumption, but can not produce any power with their PVs. The green term makes sure we do not build more solar panels than are necessary by introducing a rather small scalar ε. In a real life setting more PVs would not be build if a neighbourhood is already able to cover its power consumption by itself. Instead these PVs would be distributed to other neighbourhoods.

D. Optimization Problem

Note that we did not formulate our problem in the notation of the lecture. We decided against it, as it hinders intuitive interpretation, although it might be preferable in abstraction or for proofs. Here is the whole problem with the cost function and all its constraints:

$$\min_{P(t),a} \sum_{t\in T} (\sum_{i=j=1}^{N} l_{ij} \cdot p_{ij}(t)) + M \cdot p_{N+1N+1}(t)) + \sum_{i=1}^{N} \varepsilon \cdot a_i$$

- $c_i(t) = \sum_{j=1}^{N+1} (p_{ij}(t) - p_{ji}(t)) + p_{ii}(t) \forall i \in \{1, \dots, N\}$
- $p_{N+1N+1}(t) = \sum_{j=1}^{N} (p_{jN+1}(t) - p_{N+1j}(t)$
- $p_{ii}(t) = a_i \cdot sun(t) \qquad \forall i \in \{1, \dots, N\}$
- $0 \leq p_{ij}(t) \leq r_{ij} \qquad \forall i, j \in \{1, \dots, N+1\}$ and $i \neq j$
- $a_i \leq a_i^{max} \qquad \forall i \in \{1, \dots, N\}$
- $\sum_{i=1}^{N} a_i \leq A_{tot}$

All constraints have to be met $\forall t \in T$.

Both the cost function and the constraints are linear, which is fortunate as very good solvers exist for linear problems (LPs). In our implementation we chose the solver "IPOPT" [10] and "Casadi" [9] as the framework. We chose "IPOPT" for its efficiency and robustness, but also for its ability to exploit sparsity in problems. The robustness came in handy while designing the model, as it allows to change the problem without also changing the solver. The latter was important in earlier iterations of our model, in which the $P_{i,j}$ were often zero. Because $P(t)$ and a have values in $\mathbb{R}$, the resulting feasible set is convex, as we only have linear constraints. The optimization problem has $(N+1)^2 \cdot |T| + N$ decision variables in total, where $|T|$ is the number of snapshots. Further there are $|T| \cdot ((N+1)^2 + 2N + 1) + N + 1$ constraints in our optimization task.

III. Data

A. Simulation of Power Grids

On any given day a buildings energy consumption varies, depending on different factors like season, temperature, weather, as well as the things people inside are doing (working, cooking, watching tv..).
Obviously energy providers cannot know peoples daily rhythms, but they are in possession of various statistical tools to estimate demand. One such tool is the standard load profile", the expected normalized live power consumption for a type of residential or commercial building [5].

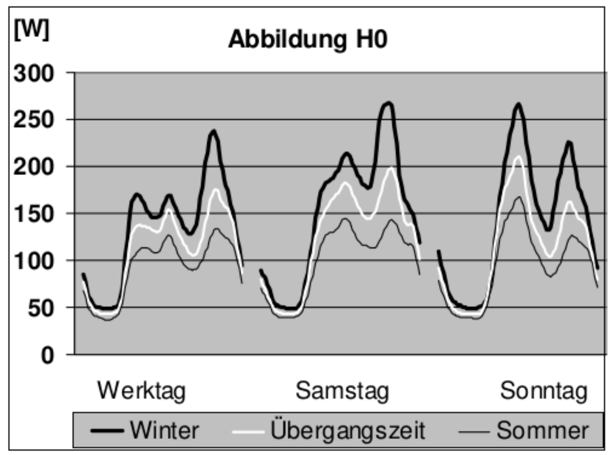

Fig. 2. Standard Load Profile H0 for Households

For residential buildings providers use just one load profile, as different preferences average out quickly, given the sheer number of private homes. However for commercially used buildings there is quite a lot of variation, depending on the industry. While a store with fixed opening hours has a comparatively consistent consumption throughout the day, a bakery consumes the most electricity when preparing goods in the early morning and a farm with cows needs little energy during the day, but a lot for milking and cooling down the produce directly afterwards at dusk and dawn.
The load profiles provided by the German Association of Energy and Water Industries [5] divides one day into 96 15-minute intervals in which they assume a constant energy demand for every recipient. As mentioned above the provided data is normalized, meaning it is scaled to a yearly total of 1000kWh. So when using a standard load profile to predict demand the actual consumption needs to be accounted for by using an appropriate factor; e.g. 4.8 in case of a yearly total of 4800kWh.

IV. Results

We will present two different examples in the following section. The first example consists of a few nodes at one point in time only. It is meant to illustrate how houses are modelled, panels allocated and to what degree changing parameters influence the results. Details on power consumption, restraints and constructed PV are given. The second example covers more houses and several sample times showing how to go about modelling larger townships. However, providing all the details of this second implementation would go beyond the scope of this report.

A. *Small Scale Example*

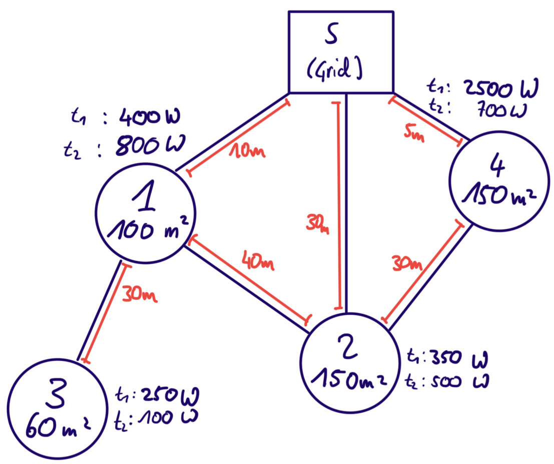

Fig. 3. Setting: Four Houses with slack note

Figure 3 shows the four houses with different energy needs and available roof size, one of which has no direct connection to the slack node s. The lines represent the power grid with the corresponding lengths noted in orange. In this setting of two snapshots with constant PV output of 180 $\frac{\mathrm{W}}{\mathrm{m}^2}$ we decided on a total of 18m^2 of available PVs; a little less than necessary in order to utilize the main grid. Note that in this particular example we set the rating values of the power lines quite high, making the process more transparent.

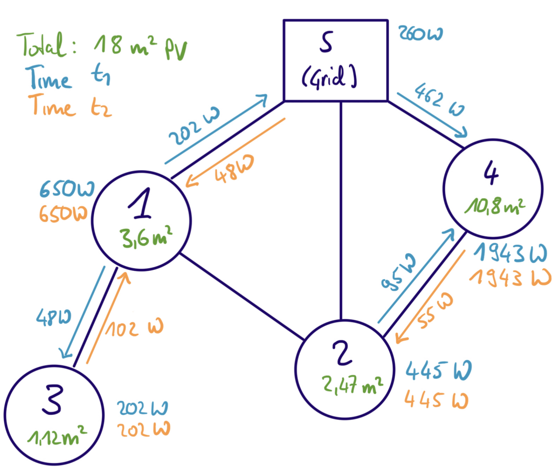

Fig. 4. Results at 180$\frac{\mathrm{W}}{\mathrm{m2}}$ (Four Houses with Slack Node)

The results of our optimization are shown in 4. Some details stand out. Firstly, a large portion of the PVs are built on house *4*. This makes sense, since it is the biggest consumer. Secondly, the power line between house *1* and *2* is avoided, as is is longer than the alternative. Finally, for time t_1 the system has to get some power from the grid to match the demand, but send most of it to the largest consumer. And because the power consumption of the houses is lower at time t_2 the slack node provides no extra electricity. In general these results are in line with our preliminary considerations.

B. *A Larger Model*

The village modelled in our second example consists of twenty houses and one slack node s as shown in Figure 3 below. The power lines are represented by the connecting lines with corresponding lengths again in orange. Electricity demand and available roof area have been omitted this time in order not to overload the graphics. We decided to concentrate on two of the most common scenarios: village centre with some small businesses and residential area with mainly one- or two-family homes. To make the most of the solar panels we decided on a sunny summer working day as setting.

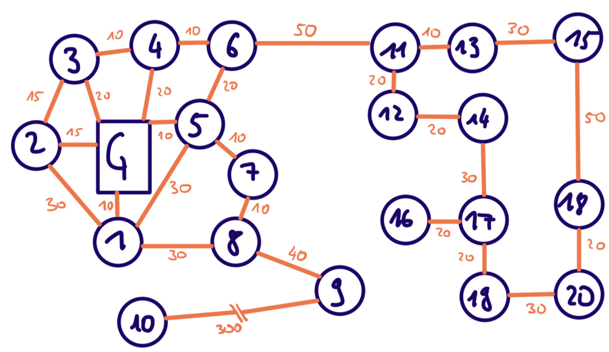

Fig. 5. Schematics of a Village

1) Times: Four sample times have been chosen to show how the simulation holds up to several very different and somewhat realistic scenarios.

We decided on nighttime between 3 to 4am as the first time point. Usually, the lowest demand is recorded in the middle of the night when people are sleeping and businesses are closed, providing a baseline with very low demand. The major drawback to this: we cannot expect any output from the solar panels, so everything needs to be provided by the slack node.

The second sample between 9 to 10am represents the beginning of the day for most people. Demand in many buildings has been going up for some time, but has not peaked yet. The farm as the only exception to that, here electricity consumption is at its highest. In return we can expect a decent amount of solar power and hope to balance some if not all of the higher demand.

In the afternoon around 3 to 4pm consumption has overcome its peak for most of the small businesses, while power usage in private homes is relatively low, with people still working or spending their time outside. Solar output meanwhile is at

it absolutely highest and should meet the needs.
The last sampling time is set in the evening. Businesses have closed already or are about to, but people are at home cooking, watching tv etc therefore creating a higher demand than before. On the other hand, the sun is about to set and while we can still expect some output from PVs, it is not going to be very much.

2) Village Center: Typically, village centers consists of larger apartment buildings mixed with some businesses. Our scenario aims at a scaled down but realistic layout for a small village, i.e. no mega malls or similar, which still utilizes different load profiles. Altogether the layout includes four apartment buildings, a bakery (*6*), a small supermarket (*7*), a clothing store (*2*), a hairdresser (*3*) and further out one villa (*9*) and a farm (*10*). The yearly consumption of the businesses is based on average values per square meters according to industry provided by the Austrian Energy Agency [8].

3) Residential Area: Suburban residential area with mainly one- or two-family homes of different sizes. These private homes all share the same load profile, only the overall electricity consumption varies. The main influences on energy usage are area of living, number of people in the household and type of house, since a single family home typically needs more energy per square meter than an apartment in a larger building. The values are variations of the average electricity usage per year based on the denominators (see [7]) mentioned above as provided by the German Federal Department for Statistics [6].

4) Result: For the larger model our system works well and optimizes as intended. Just like in the small scale example when given the option the solver builds enough panels and tries to meet the demand of large consumers by sending surplus power generated with PVs instead of drawing from the existing grid. When provided with less PVs than needed the buildings around the slack note are left undeveloped. In the residential area because of the long distance to the grid connector enough panels are built to meet the demand. The outliers (*9* and *10*) also try to stay self-sufficient. As much as possible is again being built the roof of the major consumers, e.g. the supermarket. In the edge cases of no panels or no sun output everything works as expected with the generator providing all the power. Interestingly enough, if the rating of a power line is low the solver builds less PVs even if the capacity of the roofs is not exhausted, as the surplus energy can't be used anywhere else.

V. Discussion

Our model seems to offer a reliable method to allocate solar panels in small neighbourhoods and should also work for larger examples. We only considered the low voltage grid not the itegration into the existing middle or high voltage grid in detail, problems may arise here. Important physical realities like voltage drops and frequency of the whole grid were disregarded, limiting the applicability in a real-world scenario. It would be interesting to see how well our system performs for multiple different villages and large cities with many districts. We did not model other sustainable solutions such as wind turbines, as they are less realistic in Ukraine's scenario. However our current model could be easily adapted to use these or other sources of electricity, e.g. diesel generators, by adding further nodes with output capacity and no consumption. So far we looked at few discrete time samples only, but could extend the model by a continuous function. Furthermore instead of only considering an unclouded summer day, it could be expanded to a realistic model of sunlight over the whole year. We tested varying types of buildings each with a different load profile, but for larger cities even more and varied types could be considered. Finally the ε in our cost function prevents some edge cases, but does not hinder the model from building more solar panels on the roofs than are actually needed by the village. This is because it will always be better for the cost function if the slack node absorbs electricity produced in the village, a necessity as we want to be as autarkic as possible. However this results in a negative cost function in our model. This problem is solvable, but it requires more work to do it in an effective manner and the resulting problem would likely not be linear anymore. One could of course also always choose to just use the effective but less scenic route and calculate the necessary amount of solar panels beforehand and then taking the minimum of the available solar panels and the necessary solar panels to optimize over.
All in all a solid foundation that could be developed further but not yet used for practical applications.

Acknowledgements

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References

[1] Giuliano Andrea Pagani and Marco Aiello, "Towards Decentralized Trading: A Topological Investigation of the Dutch Medium and Low Voltage Grids" arXiv preprint arXiv:1101.1118, pp. 45–46, 14 Feb 2011.

[2] Riaz, Shariq, et al. "Generic demand model considering the impact of prosumers for future grid scenario analysis." IEEE Transactions on Smart Grid 10.1(2017): 819-829.

[3] Ukraine at night. Visited on 02.10.2023 https://commons.wikimedia.org/wiki/File:Ukraine_at_night_from_space_(November_24,_2022).jpg

[4] Denys Shmyhal. Visited on Feb 23. 2023 https://english.nv.ua/nation/ukraine-s-energy-system-will-\focus-on-renewables-news-50295235.html

[5] C. Fünfgeld and R. Tiedemann, "Anwendung der Repräsentativen VDEW-Lastenprofile" Cottbus: VDEW-Frankfurt, 31. Jan 2000.

[6] Statistisches Bundesamt. Visited on Feb 6 2023 https://www.destatis.de/DE/Themen/Gesellschaft-Umwelt/Umwelt/UGR/private-haushalte/Tabellen/stromverbrauch-haushalte.html

[7] WEMAG "Durchschnittlicher Stromverbrauch im 1, 2, 3 und 4 Personen Haushalt" 15 Dec 2022. Visited on Feb 5. 2023. https://www.wemag.com/energiesparberatung/stromverbrauch-single-2-3-4-personen-haushalt

[8] D. Mandl and F. Kapusta, "Auswertung der Ergebnisse der KMU-Scheck-Beratungen" Energieinstitut der Wirtschaft GmbH, p.16, 30 Sep 2011.

[9] J.A.E. Andersson, J. Gillis, G. Horn, J. B. Rawlings, and M. Diehl. CasADi – A software framework for nonlinear optimization and optimal control. Mathematical Programming Computation, 11(1):1–36, 2019

[10] A. Wachter and L.T. Biegler. On the implementation of an interior-point filter line search algorithm for large-scale nonlinear programming. Mathematical Programming, 106(1):25–

[11] How Microgrids Work U.S. Department of Energy, June 2014 https://www.energy.gov/articles/how-microgrids-work