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A practical solution approach for the green vehicle routing problem

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ABSTRACT

Green vehicle routing (GVRP) is an active research area that is concerned with the development and analysis of distribution activities with eco-friendly vehicles. We propose a practical solution approach for the GVRP that uses a mixed-integer linear formulation and a reduction procedure. The newly formulation offers two significant advantages: compactness and flexibility. We provide empirical evidence that the formulation and the reduction procedure enable to derive optimal solutions for medium-sized instances using a general-purpose solver. We show that the proposed exact approach consistently outperforms a state-of-the-art branch-and-cut algorithm and constitutes an appealing and practical alternative for optimally solving GVRPs.

1. Introduction

In response to an ever-increasing concern about climate change, *green logistics* has emerged as an active discipline seeking to design and operate logistics systems that achieve high-standard of energy efficiency while aggressively reducing carbon footprint. The development of environmentally friendly logistics has been prompted by the high impact of transportation activities on air pollution, and consequently on global warming. Also, there is a growing social awareness about environmental issues, new regulations, and public incentives that aim at promoting R&D activities and investments in eco-friendly transportation and distribution models (Sinha and Labi, 2007). In this regard, the last decade witnessed the emergence of an active new field of transportation research that has focused on the investigation of distribution problems arising in environment-friendly settings. This new area is generically referred to as *green vehicle routing*. Basically, all these novel distribution models are rooted in the standard Vehicle Routing Problem (VRP). This cornerstone problem has been introduced, about sixty years ago, by Dantzig and Ramser (1959) and is concerned with designing optimal routes for a set of vehicles from a central depot to a set of customers. For an overview of properties, solution approaches, and variants of the VRP, we refer to the books by Golden et al. (2008) and by Toth and Vigo (2014), and to the paper by Laporte (2009). Compared to the conventional powered vehicles, the alternative energy-powered vehicles, that are modeled in green vehicle routing problems, have a limited driving range (Juan et al., 2014) and they need to be frequently recharged, therefore, technical and operating conditions influence their effectiveness (Margaritis et al., 2016). In this context, effectively planning the visits to charging stations, and establishing the number, location and capacity of these stations are new issues that need to be addressed (Juan et al., 2016) (see e.g. Wang and Lin, 2009, 2013).

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An overview of recent research in green vehicle routing have been presented by Bektaş et al. (2016), where emissions and energy consumption models are extensively discussed together with an overview of pollution routing problems (Bektaş and Laporte, 2011; Suzuki, 2016). In Pelletier et al. (2016), the authors present a review of transportation issues of electric vehicles, providing information on the technological and marketing background, presenting a survey of the state-of-the-art research and proposing perspectives for future researches.

The VRP with the possibility to contemplate a recharge of energy at customer locations (RVRP) with time-windows has been originally introduced by Conrad and Figliozzi (2011).

In Erdoğan and Miller-Hooks (2012), the authors consider the so-called *Green Vehicle Routing Problem* (GVRP) in which vehicles can be refueled at alternative fueling stations at most once during their routes. The authors propose a mixed-integer linear programming (MILP) formulation for minimizing the traveled distance of a limited fleet of biodiesel-powered uncapacitated vehicles having a limited driving range and restricted tour duration. The refueling time is assumed constant and the vehicles are fully recharged. They propose also two heuristics, the Modified Clarke and Wright Saving heuristic and the Density-Based Clustering Algorithm followed by a post-optimization improvement. Their work has been extended by Schneider et al. (2014) who introduced the Electric Vehicle Routing Problem with Time Windows and Recharge Stations (E-VRPTW). The E-VRPTW aims at minimizing the number of capacitated vehicles and the total traveled distance, while considering that the charging time is dependent on the battery charge of the vehicles on arrival at the charging stations. They also assumed that the number of recharging stops per route is unlimited, and that the customers can be visited only during fixed time windows. The authors propose a hybrid Variable Neighborhood Search with a Tabu search heuristic that significantly improves the results in Erdoğan and Miller-Hooks (2012) if applied to the GVRP. In Montoya et al. (2016), the authors propose a two phase-heuristic for the GVRP. In the first phase, a randomized route-first cluster-second heuristic is used for collecting a pool of routes, and in the second phase the routes are assembled by solving a set partitioning formulation. Many heuristic algorithms have been developed in order to identify feasible solutions for the GVRP, and to the best of our knowledge, the first attempt to solve this problem to optimality in a reasonable computation time for medium size instances has been proposed by Koç and Karaoglan (2016). The authors present a mathematical formulation that uses a reduced number of variables and constraints and propose a set of valid inequalities. A branch-and-cut algorithm is implemented in combination with a simulated annealing heuristic that improves the initial solution and the upper bounds.

In addition to the GVRP, many authors considered different green vehicle routing variants. In this regard, green inventory routing problems with heterogeneous fleets were addressed by many authors (see, e.g., Ene et al. (2016), Xiao and Konak (2016), Cheng et al. (2017)). Also, Yin and Chuang (2016) developed an Artificial Bee Colony algorithm for green vehicle routing with cross-docking, and Norouzi et al. (2017) and Hosseini-Nasab and Lotfalian (2017) described mathematical models for minimizing travel times and total fuel consumption. Schiffer and Walther (2017) presented a location routing problem model of electric vehicles that combines simultaneously routing and siting decisions. Different charging options, time-windows and capacity constraints are considered. Moreover, the impact of different objective functions is analyzed. Wang et al. (2016) proposed mixed-integer programming models for finding an optimal deployment of recharging stations and an optimal recharging schedule for a fleet of electric vehicles. Felipe et al. (2014) extended the GVRP to the green vehicle routing problem with multiple technologies and partial recharges (GVRP-MTPR), in which the charging stations have different recharge technologies and the vehicles can be partially recharged. The authors present a MILP formulation where the objective function is to minimize the total charging costs and the total traveled distance. They also proposed a local search and a simulated annealing-based heuristic. In Keskin and Çatay (2016), the authors present an adaptive large neighborhood search algorithm for the electric vehicle routing problem with time-windows and partial recharging (EVRP-TWPR) in which the full recharge restriction is relaxed and partial recharging is allowed. Furthermore, Roberti and Wen (2016) investigated the Electric Traveling Salesman Problem with Time Windows. They proposed a MILP model and a fast heuristic algorithm.

In this paper, we address the exact solution of the GVRP. This problem requires designing a set of routes for a homogeneous fleet of environment-friendly vehicles (such as plug-in hybrid electric, electric and alternative-fuel powered vehicles). Each route should start from and end at a single depot, while respecting time duration limits and contemplating the possibility of intermediate stops at recharging stations. The objective is to minimize the total travelling distance (or cost). Toward this end, as a main contribution, we propose a novel MILP formulation for solving the GVRP together with a preprocessing reduction procedure. The newly derived formulation offers two significant advantages. First, it includes a polynomial number of variables and constraints and can thereby be directly solved using a commercial solver. Second, it is flexible and can accommodate many variants of vehicle routing problems. In sharp contrast to previously proposed formulations, we provide empirical evidence that attests to the efficacy of the proposed formulation for solving medium-sized instances. Actually, we found that directly solving the formulation with a general commercial solver (CPLEX) consistently outperforms a state-of-the-art branch-and-cut algorithm. Hence, the proposed exact approach constitutes an appealing, simple, and yet effective alternative for optimally solving medium-sized GVRP instances without resorting to sophisticated integer programming approaches that require significant time, efforts and expertise. We present reduction procedures for early fixing binary variables with the effect of reducing the problem size and thereby decreasing the computation time. We test the formulation on benchmark instances proposed by Erdoğan and Miller-Hooks (2012) and we compare our results with the branch-and-cut algorithm presented in Koç and Karaoglan (2016).

The remainder of the paper is organized as follows. In Section 2, we propose a formal description of the GVRP, and we report two state-of-the-art formulations. In Section 3, we present a nonlinear compact formulation for the GVRP and we linearize it using the Reformulation-Linearization Technique (RLT). Section 4 describes a preprocessing reduction procedure that allows to fix some variables. Computational results are reported and analyzed in Section 5. Finally, in Section 6, we provide some concluding remarks and outline directions for future research.

2. State-of-the-art formulations

Let C denote the set of the customer nodes, R the set of the charging station nodes, and 0 the index of the depot node. The GVRP can be defined on a complete directed graph $G = (V, A)$ where $V \equiv \{0\} \cup C \cup R$ is the set of nodes, and A is the set of the arcs.

A fleet of homogeneous vehicles is available at the central depot. Each vehicle has a energy consumption rate r (kW h per mile), a maximum storage of energy denoted by E_{max} (kW h) and a minimum reserve of energy denoted by E_{min} (kW h). Moreover, each vehicle is supposed to have a constant average speed sp (miles per hour).

With each arc $(i,j) \in A$ are associated a nonnegative distance d_{ij} as well as a nonnegative travel time $t_{ij} = d_{ij}/sp$. We assume that the triangular inequality holds for the distances: $d_{ij} \leqslant d_{ik} + d_{kj}$ for all $i,j,k \in V$ with $i \neq j \neq k$.

If arc (i,j) is traversed by a vehicle, then its level of energy decreases by an amount $e_{ij} \equiv rd_{ij}$. The depot is considered as a charging station, therefore a copy of the depot $\bar{0}$ is also an element of R. Each time a vehicle visits a charging station its level of energy assumes value E_{max}. All the charging stations have unlimited capacities.

The total duration of each tour is limited by a maximum allowed time limit denoted by T_{max}. A service time s_i is associated with each node of the graph: if $i \in C$, then s_i is the service time at customer i, whereas if $i \in R$, then s_i represents the recharging time. The service time is constant at each customer node, and also the charging time is constant at each charging station.

The problem is to design a set of vehicle routes covering the minimum total distance while satisfying the following restrictions:

- Each tour starts at the depot where the vehicles have a fully level of energy, E_{max};
- Each tour end at the depot within the time limit T_{max};
- Each customer is visited exactly once by one single vehicle;
- Each charging station can be visited more than once by the vehicles (subtours are allowed);
- Between two consecutive customers, it is allowed at most one visit to a charging station;
- Each vehicle is used for at most one route;
- Upon the arrival at each node, the energy level of each vehicle should be at least E_{min};
- The total number of routes should not exceed m.

In the sequel, we shall assume that, for each node i, the service time is included in the travel time of the arcs outgoing from i. That is $t_{ij} = s_i + d_{ij}/sp$, for all $i,j \in V$, with $i \neq j$.

Given that the distances satisfy the triangular inequality, then also the travel times are such that $t_{ij} \leqslant t_{ik} + t_{kj}$, for all $i,j,k \in V$, with $i \neq j \neq k$ as well as the energy consumption, that is: $e_{ij} \leqslant e_{ik} + e_{kj}$, for all $i,j,k \in V$, with $i \neq j \neq k$.

To alleviate the notation, we assume that each time two nodes i and j are selected then they are different.

2.1. Formulation F_{EMH}

In Erdoğan and Miller-Hooks (2012), the authors propose a formulation denoted by F_{EMH} that has been later improved by Schneider et al. (2014) and that we describe in the following.

First, to allow multiple visits to the charging stations, the set R is augmented by dummy copies of the charging stations (one copy for each potential visit). Let R' be the set of all the possible charging nodes. Let $V' \equiv \{0\} \cup C \cup R'$, A' be the set of all the arcs (i,j) with $i,j \in V'$ and $G' = (V', A')$ the resulting graph.

For each arc $(i,j) \in A'$, let x_{ij} be a binary variable that takes value 1 if arc (i,j) is traversed by a vehicle and 0 otherwise. Let f_i denote an energy level variable: for each customer node $i \in C, f_i$ is the energy level upon the arrival at node i, whereas f_i is reset to E_{max} whenever $i \in R'$ is a charging station node. Finally, for each node $i \in V'$, let τ_i be a time variable that indicates the arrival time of a vehicle at node i, with $\tau_0 = 0$ at the depot node.

With the introduction of the minimum level of energy E_{min} at each node (that was originally set to zero in Erdoğan and Miller-Hooks (2012)), formulation F_{EMH} reads as follows:

$$F_{EMH} : \text{Minimize} \sum_{i,j \in V'} d_{ij} x_{ij} \tag{1}$$

subject to :

$$\sum_{j \in V'} x_{ij} = 1, \qquad \forall i \in C, \tag{2}$$

$$\sum_{j \in V'} x_{ij} \leqslant 1, \qquad \forall i \in R', \tag{3}$$

$$\sum_{i\in V'} x_{ij} - \sum_{i\in V'} x_{ji} = 0, \qquad \forall j \in V', \tag{4}$$

$$\sum_{j\in V'\setminus\{0\}} x_{0j} \leqslant m, \tag{5}$$

$$\sum_{j\in V'\setminus\{0\}} x_{j0} \leqslant m, \tag{6}$$

$$\tau_i + t_{ij}x_{ij} - T_{max}(1 - x_{ij}) \leqslant \tau_j, \qquad \forall i \in V', j \in V' \setminus \{0\}, \tag{7}$$

$$t_{0j} \leqslant \tau_j \leqslant T_{max} - t_{j0}, \qquad \forall j \in V' \setminus \{0\}, \tag{8}$$

$$0 \leqslant \tau_0 \leqslant T_{max}, \tag{9}$$

$$f_j \leqslant f_i - e_{ij}x_{ij} + E_{\max}(1 - x_{ij}), \qquad \forall i \in V', j \in C, \tag{10}$$

$$f_j = E_{\max}, \qquad \forall j \in R' \cup \{0\}, \tag{11}$$

$$f_j \geqslant e_{ji}x_{ji} + E_{min}, \qquad \forall i \in R' \cup \{0\}, j \in C, \tag{12}$$

$$x_{ij} \in \{0, 1\}, \qquad \forall i, j \in V', \tag{13}$$

where the objective function (1) minimizes the travel distance of the vehicles. Constraint (2) enforces the existence of exactly one arc outgoing from each customer node. Constraint (3) enables the possibility that a charging station is not visited at all, or it has at most one outgoing arc. Constraint (4) is a flow conservation constraint that states that the sum of the outgoing arcs is equal to the sum of the incoming arcs in each node of the graph. Constraints (5) and (6) limit the number of tours that start and end at the depot node to the number of vehicles m. Constraint (7) is a Miller-Tucker-Zemlin (MTZ) constraint (Miller et al., 1960; Desrochers and Laporte, 1991; Kara et al., 2004), that in combination with (8) and (9), determines the arrival time at each node of the graph and ensures that each vehicle returns to the depot within the time limit T_{max}. Constraint (10) is also an MTZ constraint that determines the energy level of the vehicles during their routes. By Constraint (11), the energy level is set to E_{max} every time the vehicles visit a charging station or the depot, whereas (12) ensures that at each customer node there is enough energy to reach an adjacent charging station or the depot. Finally, Constraint (13) enforces the integrality of the x-variables.

2.2. Formulation F_{KK}

The introduction of dummy copies of the charging stations is surely a shortcoming of formulation F_{EMH}. Indeed, it consistently increases the size of the graph and consequently the number of binary variables of the formulation.

In order to avoid this drawback, Koç and Karaoglan (2016) proposed a different formulation by introducing additional binary variables that model the existence of 3-node paths that start from a customer or from the depot, visit a charging station and end at a customer node or at the depot.

Consequently, in addition to the already defined binary variables x_{ij}, with $i, j \in C \cup \{0\}$, additional binary variable z_{irj} takes value 1 if a vehicle travels from node i to charging station r and immediately after to node j, and 0 otherwise, with $i, j \in C \cup \{0\}$ and $r \in R$. We set $z_{0r0} = 0$ for all $r \in R$ and $z_{0\bar{0}j} = z_{j\bar{0}0} = 0$ for all $j \in C$.

Furthermore, time variables τ_i and the energy level variables $f_i$are defined only for customer nodes $i \in C$.

For simplifying the notation, let $\widehat{d}_{irj} = d_{ir} + d_{rj}$ and $\widehat{t}_{irj} = t_{ir} + t_{ri}$, with $i, j \in C \cup \{0\}$ and $r \in R$ be the distance and the total duration (including the service times and charging times) of a path from i to j through r, respectively.

The MILP formulation proposed by Koç and Karaoglan is the following:

$$F_{KK} : \text{Minimize} \sum_{i,j\in C\cup\{0\}} d_{ij}x_{ij} + \sum_{i,j\in C\cup\{0\}} \sum_{r\in R} \widehat{d}_{irj}z_{irj} \tag{14}$$

subject to :

$$\sum_{i\in C\cup\{0\}} \left(x_{ij} + \sum_{r\in R} z_{irj} \right) = 1, \qquad \forall j \in C, \tag{15}$$

$$\sum_{i\in C\cup\{0\}} \left((x_{ij} - x_{ji}) + \sum_{r\in R} (z_{irj} - z_{jri}) \right) = 0, \qquad \forall j \in C \cup \{0\}, \tag{16}$$

$$\sum_{j\in C} \left(x_{0j} + \sum_{r\in R} z_{0rj} \right) \leqslant m, \tag{17}$$

$$\tau_i - \tau_j + (M_{irj} - \hat{t}_{irj})x_{ij} + (M_{irj} - \hat{t}_{irj} - t_{ij} - t_{ji})x_{ji} + (M_{irj} - t_{ij})z_{irj} + (M_{irj} - t_{ij} - \hat{t}_{irj} - \hat{t}_{jri})z_{jri} \\ \leqslant M_{irj} - t_{ij} - \hat{t}_{irj}, \qquad \forall i, j \in C, r \in R, \tag{18}$$

$$\tau_j \geqslant t_{0j} + \sum_{r\in R} \widehat{t}_{0rj} z_{0rj}, \qquad \forall j \in C, \tag{19}$$

$$\tau_j \leqslant T_{max} - (T_{max} - t_{0j})x_{0j} - \sum_{r\in R}(T_{max} - \widehat{t}_{0rj})z_{0rj}, \qquad \forall j \in C, \tag{20}$$

$$\tau_j \leqslant T_{max} - t_{j0}x_{j0} - \sum_{r\in R} \widehat{t}_{jr0} z_{jr0} \qquad \forall j \in C, \tag{21}$$

$$f_j - f_i + \bar{M}_{ij}x_{ij} + (\bar{M}_{ij} - e_{ij} - e_{ji})x_{ji} \leqslant \bar{M}_{ij} - e_{ij}, \qquad \forall i,j \in C, \tag{22}$$

$$f_j \leqslant E_{max} - e_{0j}x_{0j} - \sum_{i\in C\cup\{0\}} \sum_{r\in R} e_{rj} z_{irj}, \qquad \forall j \in C, \tag{23}$$

$$f_j \geqslant E_{\min} + e_{j0}x_{j0} + \sum_{i\in C\cup\{0\}} \sum_{r\in R} e_{jr} z_{jri}, \qquad \forall j \in C, \tag{24}$$

$$x_{ij} \in \{0,1\}, \qquad \forall i,j \in C \cup \{0\}, \tag{25}$$

$$z_{irj} \in \{0,1\}, \qquad \forall i,j \in C \cup \{0\}, r \in R, \tag{26}$$

where $M_{irj} = T_{max} + t_{ij} + \widehat{t}_{irj} - t_{i0} - t_{0j}$ for all $i,j \in C \cup \{0\}$ and for all $r \in R$ and $\bar{M}_{ij} = E_{max} - e_{ij} - \min_{r\in R} e_{rj} - \min_{r\in R} e_{ir}$ for all $i,j \in C \cup \{0\}$.

The objective function (14) minimizes the sum of the costs of the arcs connecting directly depot and customers and the costs of the 3-node paths. Constraint (15) ensures that each customer is visited exactly once by a vehicle, Constraint (16) is a flow conservation constraint, and (17) ensures that at most m vehicle routes are included in the solution. Constraints (18)–(21) guarantee the respect of the time limit duration, whereas Constraints (22)–(24) ensure the respect of the consumption requirements. Finally, (25) and (26) are the binary restrictions on the x- and z-variables, respectively.

Remark 2.1. In the original model of Koç and Karaoglan (2016), the fleet of vehicles is unlimited and, thus, constraint (17) is relaxed. However Koç and Karaoglan bound from below the number of routes of the vehicles.

3. A novel compact linearized formulation F_{RLT}

In this section, we propose a new compact formulation based on Reformulation-Linearization Technique (RLT) of Sherali and Adams (1994, 1990). First, we start by proposing a valid nonlinear formulation for the GVRP, and then we show how to derive an equivalent MILP formulation.

3.1. Nonlinear compact formulation F_{NL}

We define binary y-variables as follows

$$y_{ij} = x_{ij} + \sum_{r\in R} z_{ikj} \qquad \forall i \in C \cup \{0\}, j \in C.$$

We observe that, because of (15) if $x_{ij} = 1$, then $\sum_{r\in R} z_{ikj} = 0$ and if there exists $\bar{r} \in R$ such that $z_{i\bar{r}j} = 1$, then $x_{ij} = \sum_{r\in R\setminus\{\bar{r}\}} z_{irj} = 0$. Hence, if $y_{ij} = 1$, then either node i is directly connected to node j or there is a 3-node path that connects i to j through a charging station r. Clearly, in the former case, we have $\tau_j = \tau_i + t_{ij}$, and in the latter case, there exists a charging station $r \in R$ such that $\tau_j = \tau_i + \widehat{t}_{irj}$. In other words, if $y_{ij} = 1$, then

$$\tau_j = \tau_i + t_{ij}x_{ij} + \sum_{r\in R} \widehat{t}_{irj} z_{irj}, \qquad \forall i \in C \cup \{0\}, j \in C.$$

Therefore, the time duration constraints can be formulated in a nonlinear form as follows:

$$\tau_j y_{ij} = \tau_i y_{ij} + t_{ij}x_{ij}y_{ij} + \sum_{r\in R} \widehat{t}_{irj} z_{irj} y_{ij}, \quad \forall i \in C \cup \{0\}, j \in C. \tag{27}$$

By observing that $x_{ij}y_{ij} = x_{ij}$, and that $z_{irj}y_{ij} = z_{irj}$, we derive that

$$\tau_j y_{ij} = \tau_i y_{ij} + t_{ij}x_{ij} + \sum_{r\in R} \widehat{t}_{irj} z_{irj}, \qquad \forall i \in C \cup \{0\}, j \in C. \tag{28}$$

Moreover, given that the travel times satisfy the triangular inequality, then the arrival time at customer node j belongs to the interval:

$$t_{0j} \leqslant \tau_j \leqslant T_{\max} - t_{j0}, \qquad \forall j \in C. \tag{29}$$

Consequently, it holds that:

$$t_{0j}y_{ij} \leqslant \tau_j y_{ij} \leqslant (T_{\max} - t_{j0})y_{ij}, \qquad \forall i \in C \cup \{0\}, j \in C, \tag{30}$$

and

$$t_{0j}y_{ji} \leqslant \tau_j y_{ji} \leqslant (T_{\max} - t_{j0})y_{ji}, \qquad \forall i \in C \cup \{0\}, j \in C. \tag{31}$$

On the other hand, considering the energy level constraints, we see that the following propositions hold:

(a) $x_{ij} = 1 \Rightarrow f_j = f_i - e_{ij}$, for every $i,j \in C$;
(b) $x_{0j} = 1 \Rightarrow f_j = E_{max} - e_{0j}$, for every $j \in C$, and
$\sum_{i \in C \cup \{0\}} z_{irj} = 1 \Rightarrow f_j = E_{max} - e_{rj}$, for every $j \in C$ and $r \in R$;
(c) $x_{j0} = 1 \Rightarrow f_j \geqslant E_{min} + e_{j0}$ for every $j \in C$, and
$\sum_{i \in C \cup \{0\}} z_{jri} = 1 \Rightarrow f_j \geqslant E_{min} + e_{jr}$, for every $j \in C$ and $r \in R$.

As a consequence from (a), we derive that

$$f_j x_{ij} = (f_i - e_{ij})x_{ij}, \qquad \forall i,j \in C. \tag{32}$$

From (b), we get

$$f_j x_{0j} = (E_{max} - e_{0j})x_{0j}, \qquad \forall j \in C, \tag{33}$$

and

$$f_j \sum_{i \in C \cup \{0\}} z_{irj} = (E_{max} - e_{rj}) \sum_{i \in C \cup \{0\}} z_{irj}, \qquad \forall j \in C, r \in R. \tag{34}$$

Finally, from (c), we obtain that

$$f_j x_{j0} \geqslant (E_{min} + e_{j0})x_{j0}, \qquad \forall j \in C, \tag{35}$$

and

$$f_j \sum_{i \in C \cup \{0\}} z_{jri} \geqslant (E_{min} + e_{jr}) \sum_{i \in C \cup \{0\}} z_{jri}, \qquad \forall j \in C, r \in R. \tag{36}$$

Using these relationships, we propose the following quadratic formulation for the GVRP

$$F_{NL}: \text{Minimize} \sum_{i,j \in C \cup \{0\}} d_{ij}x_{ij} + \sum_{i,j \in C \cup \{0\}} \sum_{r \in R} \widehat{d}_{irj} z_{irj}$$

subject to : (15)–(17), (25), (26), and

$$y_{ij} = x_{ij} + \sum_{r \in R} z_{ikj}, \qquad \forall i \in C \cup \{0\}, j \in C, \tag{37}$$

$$\tau_j y_{ij} = \tau_i y_{ij} + t_{ij}x_{ij} + \sum_{r \in R} \widehat{t}_{irj} z_{irj}, \qquad \forall i \in C \cup \{0\}, j \in C, \tag{38}$$

$$t_{0j}y_{ij} \leqslant \tau_j y_{ij} \leqslant (T_{\max} - t_{j0})y_{ij}, \qquad \forall i \in C \cup \{0\}, j \in C, \tag{39}$$

$$t_{0j}y_{ji} \leqslant \tau_j y_{ji} \leqslant (T_{\max} - t_{j0})y_{ji}, \qquad \forall i \in C \cup \{0\}, j \in C, \tag{40}$$

$$\tau_0 = 0, \tag{41}$$

$$f_j x_{ij} = (f_i - e_{ij})x_{ij}, \qquad \forall i,j \in C, \tag{42}$$

$$f_j x_{0j} = (E_{max} - e_{0j})x_{0j}, \qquad \forall j \in C, \tag{43}$$

$$f_j \sum_{i \in C \cup \{0\}} z_{irj} = (E_{max} - e_{rj}) \sum_{i \in C \cup \{0\}} z_{irj}, \qquad \forall j \in C, r \in R, \tag{44}$$

$$f_j x_{j0} \geqslant (E_{min} + e_{j0})x_{j0}, \qquad \forall j \in C, \tag{45}$$

$$f_j \sum_{i \in C \cup \{0\}} z_{jri} \geqslant (E_{min} + e_{jr}) \sum_{i \in C \cup \{0\}} z_{jri}, \qquad \forall j \in C, r \in R. \tag{46}$$

Clearly, the exact solution of the *nonlinear* mixed-integer formulation F_{NL} is a daunting task. In the sequel, and for the sake of computational convenience, we shall use F_{NL} to derive an equivalent linearized formulation.

3.2. Linearization of the time duration constraints (38)–(40)

The aim of this section is to linearize the time duration constraints (38)–(40) through the Reformulation Linearization Technique.

First, we observe that combining (15) with (16) we get:

$$\sum_{j\in C\cup\{0\}}(x_{ij}+\sum_{k\in R}z_{ikj})=1,\qquad \forall i\in C. \tag{47}$$

We introduce two additional variables $\lambda$and μthat are defined as follows:

$$\lambda_{ij}=\tau_j y_{ij}\qquad \forall i,j\in C\cup\{0\},$$

and

$$\mu_{ij}=\tau_i y_{ij}\qquad \forall i,j\in C\cup\{0\}.$$

Hence, the quadratic constraint (38) becomes

$$\lambda_{ij}=\mu_{ij}+t_{ij}x_{ij}+\sum_{r\in R}\hat{t}_{irj}z_{irj},\qquad \forall i\in C\cup\{0\},j\in C. \tag{48}$$

Moreover, since $\tau_0=0$, then

$$\mu_{0j}=0\qquad \forall j\in C, \tag{49}$$

and

$$\lambda_{j0}=0\qquad \forall j\in C. \tag{50}$$

Multiplying (15) and (47) by τ_j, we get:

$$\left[\sum_{i\in C\cup\{0\}}(x_{ij}+\sum_{r\in R}z_{irj})=1\right]\cdot\tau_j,\qquad \forall j\in C, \tag{51}$$

$$\left[\sum_{i\in C\cup\{0\}}(x_{ji}+\sum_{r\in R}z_{jri})=1\right]\cdot\tau_j,\qquad \forall j\in C. \tag{52}$$

Using equality (37) and linearizing, we derive the following identities:

$$\sum_{i\in C\cup\{0\}}\lambda_{ij}=\tau_j,\qquad \forall j\in C, \tag{53}$$

$$\sum_{i\in C\cup\{0\}}\mu_{ji}=\tau_j,\qquad \forall j\in C. \tag{54}$$

Consequently, the τ-variables can be eliminated from the formulation.

By substituting (48) into (53), we obtain that

$$\tau_j=\sum_{i\in C\cup\{0\}}\mu_{ij}+\sum_{i\in C\cup\{0\}}t_{ij}x_{ij}+\sum_{i\in C\cup\{0\}}\sum_{r\in R}\hat{t}_{irj}z_{irj},\qquad \forall j\in C. \tag{55}$$

In view of (54), we derive the linear constraint

$$\sum_{i\in C\cup\{0\}}\mu_{ji}=\sum_{i\in C\cup\{0\}}\mu_{ij}+\sum_{i\in C\cup\{0\}}t_{ij}x_{ij}+\sum_{i\in C\cup\{0\}}\sum_{r\in R}\hat{t}_{irj}z_{irj},\qquad \forall j\in C. \tag{56}$$

Linearizing (39) and (40), we derive

$$t_{0j}\left(x_{ij}+\sum_{r\in R}z_{irj}\right)\leqslant\lambda_{ij}\leqslant(T_{\max}-t_{j0})(x_{ij}+\sum_{r\in R}z_{irj}),\quad \forall i\in C\cup\{0\},j\in C, \tag{57}$$

and

$$t_{0i}\left(x_{ij}+\sum_{r\in R}z_{irj}\right)\leqslant\mu_{ij}\leqslant(T_{\max}-t_{i0})(x_{ij}+\sum_{r\in R}z_{irj}),\quad \forall i\in C,j\in C\cup\{0\}, \tag{58}$$

respectively.

Furthermore, substituting (48) in (57), we retrieve that

$$(t_{0j} - t_{ij})x_{ij} + \sum_{r\in R}(t_{0j} - \hat{t}_{irj})z_{irj} \leqslant \mu_{ij} \leqslant (T_{\max} - t_{j0} - t_{ij})x_{ij} + \sum_{r\in R}(T_{\max} - t_{j0} - \hat{t}_{irj})z_{irj}, \quad \forall i,j, \in C. \tag{59}$$

Consequently, we derive the lower bounding constraint

$$\max(t_{0j} - t_{ij}, t_{0i})x_{ij} + \sum_{r\in R}\max(t_{0j} - \hat{t}_{irj}, t_{0i})z_{irj} \leqslant \mu_{ij} \quad \forall i,j, \in C, \tag{60}$$

and the upper bounding constraint

$$\mu_{ij} \leqslant \min(T_{max} - t_{j0} - t_{ij}, T_{\max} - t_{i0})x_{ij} + \sum_{r\in R}\min(T_{max} - t_{j0} - \hat{t}_{irj}, T_{\max} - t_{i0})z_{irj}, \quad \forall i,j, \in C. \tag{61}$$

Moreover, setting $j = 0$ in (58), we have that

$$t_{0i}\left(x_{i0} + \sum_{r\in R}z_{ir0}\right) \leqslant \mu_{i0} \leqslant (T_{\max} - t_{i0})\left(x_{i0} + \sum_{r\in R}z_{ir0}\right), \quad \forall i \in C. \tag{62}$$

In addition to the linearized time constraints, because of (21), it holds that

$$\tau_i \leqslant T_{\max} - t_{i0}x_{i0} - \sum_{r\in R}\hat{t}_{ir0}z_{ir0}, \qquad \forall i \in C. \tag{63}$$

This last inequality can be multiplied by y_{i0} and, then linearized as follows:

$$\mu_{i0} \leqslant (T_{\max} - t_{i0})x_{i0} + \sum_{r\in R}(T_{\max} - \hat{t}_{ir0})z_{ir0}, \qquad \forall i \in C. \tag{64}$$

Therefore, combining (62) with (64) together with $\hat{t}_{ir0} \geqslant t_{i0}$, it holds that

$$t_{0i}\left(x_{i0} + \sum_{r\in R}z_{ir0}\right) \leqslant \mu_{i0} \leqslant (T_{\max} - t_{i0})x_{i0} + \sum_{r\in R}(T_{max} - \hat{t}_{ir0})z_{ir0}, \quad \forall i \in C. \tag{65}$$

In summary, the linear time duration constraints are (49), (56), (60), (61) and (65).

3.3. Linearization of the energy level constraints (42)–(46)

The energy level constraints can also be linearized using RLT. Toward this end, we define three sets of variables α, β and γ as follows:

$$\alpha_{ij} = f_j x_{ij}, \qquad \forall i \in C \cup \{0\}, j \in C,$$

$$\beta_{ij} = f_i x_{ij}, \qquad \forall i \in C, j \in C \cup \{0\},$$

and

$$\gamma_{jr} = \begin{cases} f_j \sum_{i\in C\cup\{0\}} z_{jri}, & \forall j \in C, r \in R, \\ f_j x_{j0}, & \forall j \in C, r = 0. \end{cases}$$

Using these definitions, the linearization of constraints (42) and (43) are:

$$\alpha_{ij} = \beta_{ij} - e_{ij}x_{ij}, \qquad \forall i,j \in C, \tag{66}$$

and

$$\alpha_{0j} = (E_{\max} - e_{0j})x_{0j}, \qquad \forall j \in C, \tag{67}$$

respectively.

Multiplying (15) by f_j and using (67) and (44), we get:

$$f_j = \left[\sum_{i\in C\cup\{0\}}(x_{ij} + \sum_{r\in R}z_{irj})\right] \cdot f_j = \sum_{i\in C}\alpha_{ij} + (E_{\max} - e_{0j})x_{0j} + \sum_{i\in C\cup\{0\}}\sum_{r\in R}(E_{max} - e_{rj})z_{irj}, \qquad \forall j \in C. \tag{68}$$

Also, multiplying (47) by f_j, we derive

$$f_j = \left[\sum_{i\in C\cup\{0\}}(x_{ji} + \sum_{r\in R}z_{jri})\right] \cdot f_j = \sum_{i\in C}\beta_{ji} + f_j x_{j0} + \sum_{r\in R}\gamma_{jr} = \sum_{i\in C}\beta_{ji} + \sum_{r\in R\cup\{0\}}\gamma_{jr} \qquad \forall j \in C. \tag{69}$$

Equating (68) and (69), and using (66) we obtain that

$$\sum_{i\in C}\alpha_{ij} - \sum_{i\in C}\alpha_{ji} = \sum_{r\in R\cup\{0\}}\gamma_{jr} + \sum_{i\in C}e_{ji}x_{ji} - \sum_{i\in C\cup\{0\}}\sum_{r\in R}(E_{max} - e_{rj})z_{irj} - (E_{max} - e_{0j})x_{0j}, \quad \forall j \in C. \tag{70}$$

The linearized form of constraints (45) and (46) are:

$$\gamma_{j0} \geqslant (E_{\min} + e_{j0})x_{j0}, \qquad \forall j \in C, \tag{71}$$

and

$$\gamma_{jr} \geqslant \sum_{i\in C\cup\{0\}}(E_{min} + e_{jr})z_{jri}, \qquad \forall j \in C, r \in R, \tag{72}$$

respectively.

We observe that at any customer node j, the energy level f_j should be enough to reach at least its nearest charging station or the depot and it should be at most equal to the remaining energy after covering the distance from the nearest charging station to j, that is, we have

$$E_{min} + \min_{r\in R\cup\{0\}} e_{jr} \leqslant f_j \leqslant E_{max} - \min_{r\in R\cup\{0\}} e_{rj}. \tag{73}$$

Therefore, it holds that

$$(E_{min} + \min_{r\in R\cup\{0\}} e_{jr})x_{ij} \leqslant f_j x_{ij} \leqslant (E_{max} - \min_{r\in R\cup\{0\}} e_{rj})x_{ij}, \qquad \forall i,j \in C, \tag{74}$$

$$(E_{min} + \min_{r\in R\cup\{0\}} e_{jr})x_{ji} \leqslant f_j x_{ji} \leqslant (E_{max} - \min_{r\in R\cup\{0\}} e_{rj})x_{ji}, \quad \forall i \in C\cup\{0\}, j \in C, \tag{75}$$

and,

$$(E_{min} + \min_{r\in R\cup\{0\}} e_{jr})\sum_{i\in C\cup\{0\}} z_{jri} \leqslant f_j \sum_{i\in C\cup\{0\}} z_{jri} \leqslant (E_{max} - \min_{r\in R\cup\{0\}} e_{rj})\sum_{i\in C\cup\{0\}} z_{jri}, \quad \forall j \in C, r \in R. \tag{76}$$

The linearization of (74) and (75) are:

$$(E_{min} + \min_{r\in R\cup\{0\}} e_{jr})x_{ij} \leqslant \alpha_{ij} \leqslant (E_{max} - \min_{r\in R\cup\{0\}} e_{rj})x_{ij}, \quad \forall i,j \in C, \tag{77}$$

and

$$(E_{min} + \min_{r\in R\cup\{0\}} e_{jr})x_{ji} \leqslant \beta_{ji} \leqslant (E_{max} - \min_{r\in R\cup\{0\}} e_{rj})x_{ji}, \quad \forall i,j \in C, \tag{78}$$

and, when i coincides with the depot in (75), we have that

$$(E_{min} + \min_{r\in R\cup\{0\}} e_{jr})x_{j0} \leqslant \gamma_{j0} \leqslant (E_{max} - \min_{r\in R\cup\{0\}} e_{rj})x_{j0}, \qquad \forall j \in C. \tag{79}$$

Using the inequalities (77), (78) and (66), we derive

$$\max(E_{min} + \min_{r\in R\cup\{0\}} e_{jr}, E_{min} + \min_{r\in R\cup\{0\}} e_{ir} - e_{ij})x_{ij} \leqslant \alpha_{ij}, \qquad \forall i,j \in C, \tag{80}$$

and

$$\alpha_{ij} \leqslant \min(E_{max} - \min_{r\in R\cup\{0\}} e_{ri} - e_{ij}, E_{max} - \min_{r\in R\cup\{0\}} e_{rj})x_{ij}, \qquad \forall i,j \in C. \tag{81}$$

Furthermore, (79) in combination with (71) leads to the following inequalities:

$$(E_{min} + e_{j0})x_{j0} \leqslant \gamma_{j0} \leqslant (E_{max} - \min_{r\in R\cup\{0\}} e_{rj})x_{j0}, \qquad \forall j \in C. \tag{82}$$

Finally, linearizing (76) we get:

$$(E_{min} + \min_{r\in R\cup\{0\}} e_{jr})\sum_{i\in C\cup\{0\}} z_{jri} \leqslant \gamma_{jr} \leqslant (E_{max} - \min_{r\in R\cup\{0\}} e_{rj})\sum_{i\in C\cup\{0\}} z_{jri}, \qquad \forall j \in C, r \in R, \tag{83}$$

Combining this latter with (72) yields

$$(E_{min} + e_{jr})\sum_{i\in C\cup\{0\}} z_{jri} \leqslant \gamma_{jr} \leqslant (E_{max} - \min_{r\in R\cup\{0\}} e_{rj})\sum_{i\in C\cup\{0\}} z_{jri}, \qquad \forall j \in C, r \in R. \tag{84}$$

In summary, the linearized energy level constraints are (70), (80), (81), (82) and (84).

3.4. Formulation F_{RLT}

The linear compact formulation that we propose is thus:

$$F_{RLT1}: \text{ Minimize} \sum_{i,j\in C\cup\{0\}} d_{ij}x_{ij} + \sum_{i,j\in C\cup\{0\}}\sum_{r\in R}\widehat{d}_{irj}z_{irj}$$

$$\text{subject to}: (15),\ (17),\ (25),\ (26),\ (47), \text{ and}$$

$$\sum_{i\in C\cup\{0\}}\mu_{ji} = \sum_{i\in C\cup\{0\}}\mu_{ij} + \sum_{i\in C\cup\{0\}} t_{ij}x_{ij} + \sum_{i\in C\cup\{0\}}\sum_{r\in R}\widehat{t}_{irj}z_{irj}, \qquad \forall j\in C, \tag{85}$$

$$t_{0j}(x_{j0} + \sum_{r\in R} z_{jr0}) \leqslant \mu_{j0} \leqslant (T_{\max} - t_{j0})x_{j0} + \sum_{r\in R}(T_{\max} - \widehat{t}_{jr0})z_{jr0}, \quad \forall j\in C, \tag{86}$$

$$\mu_{0j} = 0, \quad \forall j\in C \tag{87}$$

$$\max(t_{0j} - t_{ij}, t_{0i})x_{ij} + \sum_{r\in R}\max(t_{0j} - \widehat{t}_{irj}, t_{0i})z_{irj} \leqslant \mu_{ij} \quad \forall i,j\in C, \tag{88}$$

$$\mu_{ij} \leqslant \min(T_{max} - t_{j0} - t_{ij}, T_{\max} - t_{i0})x_{ij} + \sum_{r\in R}\min(T_{max} - t_{j0} - \widehat{t}_{irj}, T_{\max} - t_{i0})z_{irj}, \quad \forall i,j\in C. \tag{89}$$

$$\sum_{i\in C}\alpha_{ij} - \sum_{i\in C}\alpha_{ji} = \sum_{r\in R\cup\{0\}}\gamma_{jr} + \sum_{i\in C} e_{ji}x_{ji} - \sum_{i\in C\cup\{0\}}\sum_{r\in R}(E_{max} - e_{rj})z_{irj} - (E_{max} - e_{0j})x_{0j}, \quad \forall j\in C, \tag{90}$$

$$\max(E_{min} + \min_{r\in R\cup\{0\}} e_{jr}, E_{min} + \min_{r\in R\cup\{0\}} e_{ir} - e_{ij})x_{ij} \leqslant \alpha_{ij}, \ \forall i,j\in C, \tag{91}$$

$$\alpha_{ij} \leqslant \min(E_{max} - \min_{r\in R\cup\{0\}} e_{ri} - e_{ij}, E_{max} - \min_{r\in R\cup\{0\}} e_{rj})x_{ij}, \qquad \forall i,j\in C, \tag{92}$$

$$(E_{min} + e_{j0})x_{j0} \leqslant \gamma_{j0} \leqslant (E_{max} - \min_{r\in R\cup\{0\}} e_{rj})x_{j0}, \qquad \forall j\in C, \tag{93}$$

$$(E_{min} + e_{jr})\sum_{i\in C\cup\{0\}} z_{jri} \leqslant \gamma_{jr} \leqslant (E_{max} - \min_{r\in R\cup\{0\}} e_{rj})\sum_{i\in C\cup\{0\}} z_{jri}, \qquad \forall j\in C, r\in R. \tag{94}$$

This formulation has the same number of integer variables as formulation F_{KK}, $\max(O(|C^2|), O(|C|\,|R|))$ continuous variables, and $\max(O(|C|^2), O(|C|\,|R|))$ constraints.

With the aim of tightening the linear programming (LP) relaxation of F_{RLT1}, we propose the following valid inequality:

$$x_{ij} + x_{ji} + \sum_{r\in R}(z_{irj} + z_{jri}) \leqslant 1, \qquad \forall i,j\in C, \tag{95}$$

which is a 2-customer subtour elimination constraint: if i and j are two consecutive customers in a tour of a vehicle, then either i precedes j possibly through a charging station r or *vice versa*.

We denote by F_{RLT2} the formulation that is obtained by appending the valid inequality (95) to F_{RLT1}.

4. Preprocessing conditions

In this section, we describe sufficient conditions that (possibly) allow to fix some binary variables.

In what follows, we shall assume that $e_{ij} \leqslant E_{max} - E_{min}$ and that $t_{ij} \leqslant T_{max}$ for all the nodes i and j of the graph.

Before fixing some variables, the conditions that ensure the feasibility of the instances (possibly eliminating unreachable customers), and that check whether it is possible to eliminate useless charging stations are the following:

C1: Let $v \in C\cup R$. If $t_{0v} + t_{v0} > T_{max}$, then node v is eliminated. This is an immediate consequence of the fact that the triangular inequality holds. Indeed, $t_{0v} + t_{v0}$ represents the minimum traveling time for a route that visits node v.

C2: Let $r \in R$. If the minimum traveling time for connecting the depot, a customer node and the charging station r is greater than T_{max} , that is, if $\min_{i\in C}(t_{0i} + t_{ir}) + t_{r0} > T_{max}$, then the charging station r can be eliminated because it will never be included in a feasible route.

C3: Let $i \in C$. If the path from the nearest charging station (or depot) to customer i and back to the charging station (or depot) exceeds the maximum acceptable level of energy, that is, if $\min_{r\in R} e_{ri} + \min_{l\in R} e_{il} > E_{max} - E_{min}$, then customer i is eliminated.

Else, if for every $r \in R\cup\{0\}$ it holds that: ($e_{ri} + e_{i0} \leqslant E_{max} - E_{min}$ and $t_{0r} + t_{ri} + t_{i0} > T_{max}$) or (denoting by $\overline{R}$ the subset of $R\cup\{0\}$ such that $e_{ri} + e_{il} \leqslant E_{max} - E_{min}$, for all $l \in \overline{R}$ it holds that $t_{0r} + t_{ri} + \min_{l\in\overline{R}}(t_{il} + t_{l0}) > T_{max}$) then customer i is eliminated.

In the first case, despite the possibility of having enough energy for the paths from r to i and back to the depot, the traveling time for the fastest route $0 \to r \to i \to 0$ exceeds the time duration limit. In the second case, the duration of all the routes that visit i and that are feasible (from the energy level point of view) exceeds the time limit.

4.1. Fixing variables

The x-variables can be fixed to zero if the following conditions are satisfied.
Let $i, j \in C$.

C4: If $e_{0i} + \min_{r \in R \cup \{0\}} e_{ir} > E_{max} - E_{min}$, then $x_{0i} = 0$.
That is, if a vehicle that travels directly from the depot to customer i has not enough energy level to reach the nearest charging station, then it has to visit a charging station in the route from the depot to i and, therefore, it is not possible to directly connect the depot to i.

C5: If $\min_{r \in R \cup \{0\}} e_{ri} + e_{i0} > E_{max} - E_{min}$, then $x_{i0} = 0$.
This condition is analogous to C4 applied to a route that ends at the depot immediately after visiting customer i.

C6: If $t_{0i} + t_{ij} + t_{j0} > T_{max}$ or $\min_{l \in R \cup \{0\}} e_{li} + e_{ij} + \min_{r \in R \cup \{0\}} e_{jr} > E_{max} - E_{min}$, then $x_{ij} = 0$.
Indeed, the arc (i, j) can be eliminated if the minimum duration of a tour that directly connects i to j exceeds the time limit or if the energy level of a path from the nearest charging station to i and, then, to j is not enough to reach the nearest charging station of node j.

The next condition allows to possibly fix to zero some of the z-variables.
Let $i, j \in C \cup \{0\}$, let $r \in R$.

C7: If $(t_{0i} + t_{ir} + t_{rj} + t_{j0} > T_{\max})$ or $(\min_{l \in R \cup \{0\}} e_{li} + e_{ir} > E_{\max} - E_{min})$ or $(e_{rj} + \min_{l \in R \cup \{0\}} e_{jl} > E_{\max} - E_{min})$, then $z_{irj} = 0$.

That is, the path from i to the charging station r and then to j is infeasible if the duration of the route $0 \to i \to r \to j \to 0$ exceeds the time duration limit, or the maximum energy level at i is not enough to reach the charging station r, or the energy level at j (after visiting r) is not sufficient to reach the nearest charging station.

We denote by F_{RLT3} and F_{RLT4} the formulations that are obtained from F_{RLT1} and F_{RLT2}, respectively, after applying the reduction tests C4-C7 and fixing some x- and z-variables.

Finally, we denote by F_{RLT5} the formulation that is derived from F_{RLT4} after relaxing the constraint on the maximum number of routes (17).

5. Computational results

The proposed formulations were coded in C and directly solved by using the default settings of IBM ILOG CPLEX 12.6.1. All the tests were run on an Intel Xeon PC with 64 GB and 3.30 GHz.

The objective of the computational experiments is twofold. First, to empirically investigate the respective performance of formulations F_{EMH} and F_{KK} against F_{RLT1} and F_{RLT2}. Second, to compare the efficacy of the branch-and-cut proposed by Koç and Karaoglan in Koç and Karaoglan (2016) against the enhanced formulations F_{RLT3}, F_{RLT4} and F_{RLT5}.

Toward this end, we used the same benchmark instances that were generated by Erdoğan and Miller-Hooks (2012). Nodes are located in a grid of 330 by 300 miles and the depot is located near the center of the grid. In these instances, the number of customers $|C|$ is set to 20 and the number of charging stations $|R|$ varies from 2 to 10. Four different scenarios were considered. In Scenarios 1 and 2, there are three charging stations and the customers are uniformly located and clustered, respectively. In Scenarios 3 and 4, the test instances are selected from Scenarios 1 and 2, however in Scenario 3 three additional charging stations are randomly located in the grid, whereas in Scenario 4 the number of charging stations is increased from 2 to 10 by increments of two. These last two sets explore the impact of the spatial configuration and the density of the charging stations, respectively.

The original test instances may be infeasible due to the charging restrictions and maximum duration of the routes. In a preliminary phase, infeasible nodes are dropped from the instances. Hence, the actual number of customers is reported in Table 1 together with the value of m that in Erdoğan and Miller-Hooks (2012) has been identified from heuristic solutions. We highlight that all the test instances satisfy the following additional assumption that has been made in Erdoğan and Miller-Hooks (2012): there exists a feasible solution where each vehicle visits at most one charging station during its tour.

In the experiments, the maximum energy E_{max} is equivalent to the energy that is available for biodiesel-powered vehicles with fuel tank capacity of 60 gallons, the energy consumption rate equivalent to the fuel consumption rate of 0.2 gallons and the minimum energy level E_{min} is set to zero. The value sp is assumed to be 40 miles per hour (mph).

We highlight that, according to the settings in Koç and Karaoglan (2016), T_{max} is set to 10 h and 45 min (instead of 11 h as in Erdoğan and Miller-Hooks (2012)). The waiting time is equal to 30 min at a customer node and to 15 min at a charging station.

5.1. Empirical analysis of the performance of the formulations

In the following tables, we denote by T_{LP} the CPU time for solving the LP relaxation of the formulations and by T_{OPT} the CPU time for finding optimal solutions. The CPU time limit has been set to 3600 s.

Table 1
Comparison of formulations on the test instances of Scenarios 1 and 2.

				F_{EMH}	F_{KK}		F_{RLT1}			F_{RLT2}		
Instance	$\|C\|$	$\|R\|$	m	UB	T_{opt}	UB	T_{LP}	T_{opt}	UB	T_{LP}	T_{opt}	UB
20c3sU1	20	3	6	1797.51	>3600	1797.49	0.11	432.57	**1797.49**	0.13	233.46	**1797.49**
20c3sU2	20	3	6	1574.82	>3600	1574.78	0.13	277.69	**1574.78**	0.17	78.50	**1574.78**
20c3sU3	20	3	7	1765.9	>3600	1704.48	0.14	63.03	**1704.48**	0.11	33.17	**1704.48**
20c3sU4	20	3	5	1482.00	>3600	1482.00	0.13	520.64	**1482.00**	0.13	170.37	**1482.00**
20c3sU5	20	3	6	1689.35	>3600	1689.37	0.13	92.13	**1689.37**	0.14	124.08	**1689.37**
20c3sU6	20	3	6	1643.05	>3600	1618.65	0.16	167.40	**1618.65**	0.16	148.42	**1618.65**
20c3sU7	20	3	6	1715.13	>3600	1713.79	0.11	2349.85	**1713.66**	0.12	>3600	1713.66
20c3sU8	20	3	6	1709.43	>3600	1706.50	0.11	39.69	**1706.50**	0.16	118.86	**1706.50**
20c3sU9	20	3	6	1708.84	>3600	1708.82	0.12	1380.20	**1708.82**	0.16	>3600	1708.82
20c3sU10	20	3	5	1261.15	923.10	**1181.31**	0.11	4.88	**1181.31**	0.13	4.03	**1181.31**
Average					3332.31		0.12	532.81		0.14	811.09	
20c3sC1	20	3	5	1235.21	>3600	1179.16	0.14	45.73	**1173.57**	0.13	23.84	**1173.57**
20c3sC2	19	3	5	1539.94	>3600	1539.97	0.11	29.53	**1539.97**	0.11	38.87	**1539.97**
20c3sC3	12	3	4	985.41	95.07	**880.20**	0.03	2.06	**880.20**	0.02	1.75	**880.20**
20c3sC4	18	3	5	1080.16	>3600	1059.35	0.08	536.01	**1059.35**	0.08	238.28	**1059.35**
20c3sC5	19	3	7	2190.68	>3600	2156.01	0.11	>3600	2156.01	0.14	2012.25	**2156.01**
20c3sC6	17	3	9	2785.86	>3600	2758.17	0.08	19.66	**2758.17**	0.09	9.56	**2758.17**
20c3sC7	6	2	5	1393.98	0.66	**1393.99**	0.02	0.48	**1393.99**	0.00	0.13	**1393.99**
20c3sC8	18	3	10	3319.71	1155.00	**3139.72**	0.06	12.52	**3139.72**	0.09	3.70	**3139.72**
20c3sC9	19	3	6	1799.95	>3600	1799.94	0.09	47.45	**1799.94**	0.09	16.36	**1799.94**
20c3sC10	15	3	8	2583.42	>3600	2583.42	0.05	26.01	**2583.42**	0.06	9.33	**2583.42**
Average					2645.07		0.08	431.94		0.08	235.41	

Table 2
Comparison of formulations on the test instances of Scenarios 3 and 4.

				F_{EMH}	F_{KK}		F_{RLT1}			F_{RLT2}		
Instance	$\|C\|$	$\|R\|$	m	UB	T_{opt}	UB	T_{LP}	T_{opt}	UB	T_{LP}	T_{opt}	UB
S1_2i6s	20	6	6	1578.15	>3600	1578.12	0.14	712.03	**1578.12**	0.41	1009.61	**1578.12**
S1_4i6s	20	6	5	1438.89	>3600	1397.27	0.14	66.85	**1397.27**	0.17	32.65	**1397.27**
S1_6i6s	20	6	6	1571.28	>3600	1560.49	0.12	94.35	**1560.49**	0.17	145.33	**1560.49**
S1_8i6s	20	6	6	1692.34	>3600	1692.32	0.14	522.52	**1692.32**	0.16	482.76	**1692.32**
S1_10i6s	20	6	5	1253.32	>3600	1173.48	0.16	6.05	**1173.48**	0.16	8.67	**1173.48**
S2_2i6s	20	6	6	1645.80	>3600	1633.10	0.14	170.59	**1633.10**	0.22	323.69	**1633.10**
S2_4i6s	19	6	6	1505.06	>3600	1505.07	0.14	1081.47	**1505.07**	0.19	599.68	**1505.07**
S2_6i6s	20	6	10	2842.08	>3600	2431.33	0.14	23.82	**2431.33**	0.28	24.09	**2431.33**
S2_8i6s	16	6	9	2549.98	2436.63	**2158.35**	0.06	15.28	**2158.35**	0.08	7.75	**2158.35**
S2_10i6s	16	6	6	1606.65	>3600	1585.46	0.09	129.96	**1585.46**	0.08	461.97	**1585.46**
Average					3483.66		0.13	282.29		0.19	309.62	
S1_4i2s	20	2	6	1582.22	>3600	1582.21	0.13	159.53	**1582.21**	0.11	224.86	**1582.21**
S1_4i4s	20	4	6	1504.10	>3600	1460.09	0.13	105.33	**1460.09**	0.13	190.03	**1460.09**
S1_4i6s	20	6	5	1397.28	>3600	1397.27	0.14	64.66	**1397.27**	0.22	30.42	**1397.27**
S1_4i8s	20	8	6	1376.98	>3600	1397.27	0.16	103.10	**1397.27**	0.17	195.61	**1397.27**
S1_4i10s	20	10	5	1080.16	>3600	1397.27	0.20	186.49	**1396.02**	0.14	378.93	**1396.02**
S2_4i2s	18	2	5	1466.90	>3600	1059.35	0.06	221.25	**1059.35**	0.06	220.76	**1059.35**
S2_4i4s	19	4	6	1454.96	>3600	1446.08	0.13	584.39	**1446.08**	0.12	549.34	**1446.08**
S2_4i6s	20	6	6	1454.96	>3600	1434.14	0.13	418.61	**1434.14**	0.17	1636.43	**1434.14**
S2_4i8s	20	8	6	1454.96	>3600	1470.73	0.14	3063.52	**1434.14**	0.31	1536.05	**1434.14**
S2_4i10s	20	10	6	1454.93	>3600	1434.13	0.17	1498.95	**1434.13**	0.44	3129.49	**1434.13**
Average					3600		0.14	640.58		0.19	809.19	

In these tables, *UB* denotes either the proven optimal value (in bold) or the best known solution, whereas *Gap* is the percentage gap between the best feasible solution and the value of the best remaining node in the branch-and-bound tree.

In Tables 1 and 2 we display: the best solution values that were reported in Erdoğan and Miller-Hooks (2012) for at most 100,000 s of computation, the CPU time, and the best solution values that we are able to obtain by directly solving formulation F_{KK} and the computational results of Formulations F_{RLT1} and F_{RLT2}.

As it appears evident from the tables, on the basis of our results, only 5 out of the 40 instances (12.5%) are optimally solved using F_{KK}. However, more often than F_{EMH}, the feasible solution returned by the solver at the end of the computations turns out to be optimal. The linearized compact formulation F_{RLT1} is able to solve to optimality all, but one, of the test instances (97.5%) with remaining percentage gaps of 15.26% for the instance 20c3sC5. Adding the valid inequality (95)

Table 3
Comparison of solution approaches on the test instances of Scenarios 1 and 2.

	Branch-and-cut (Koç and Karaoglan, 2016)				F_{RLT3}				F_{RLT4}				F_{RLT5}			
Instance	T_{LP}	T_{opt}	*Gap*	*UB*	T_{LP}	T_{opt}	*Gap*	*UB*	T_{LP}	T_{opt}	*Gap*	*UB*	T_{LP}	T_{opt}	*Gap*	*UB*
20c3sU1	0.0	172.3	0.0	**1797.49**	0.08	252.83	0.00	**1797.49**	0.08	315.10	0.00	**1797.49**	0.08	322.16	0.00	**1797.49**
20c3sU2	0.0	>3600	4.7	1574.78	0.08	188.97	0.00	**1574.78**	0.12	121.47	0.00	**1574.78**	0.11	138.69	0.00	**1574.78**
20c3sU3	0.0	1789.0	0.0	**1704.48**	0.08	88.27	0.00	**1704.48**	0.09	48.97	0.00	**1704.48**	0.08	44.56	0.00	**1704.48**
20c3sU4	0.0	>3600	9.5	1482.00	0.08	540.00	0.00	**1482.00**	0.09	399.82	0.00	**1482.00**	0.06	331.53	0.00	**1482.00**
20c3sU5	0.1	2165.5	0.0	**1689.37**	0.06	195.10	0.00	**1689.37**	0.11	142.16	0.00	**1689.37**	0.09	308.47	0.00	**1689.37**
20c3sU6	0.0	>3600	2.7	1618.65	0.08	215.34	0.00	**1618.65**	0.08	324.81	0.00	**1618.65**	0.06	313.00	0.00	**1618.65**
20c3sU7	0.0	>3600	9.8	1713.66	0.08	2419.88	0.00	**1713.66**	0.09	>3600	2.49	1713.66	0.08	>3600	3.69	1713.66
20c3sU8	0.0	1601.3	0.0	**1706.50**	0.05	121.66	0.00	**1706.50**	0.08	60.55	0.00	**1706.50**	0.06	99.60	0.00	**1706.50**
20c3sU9	0.0	>3600	8.3	1708.82	0.06	>3600	6.14	1708.82	0.08	2417.56	0.00	**1708.82**	0.06	>3600	4.98	1708.82
20c3sU10	0.0	2.3	0.00	**1181.31**	0.06	6.30	0.00	**1181.31**	0.08	2.26	0.00	**1181.31**	0.08	4.10	0.00	**1181.31**
Average	0.0	2373.0	3.5		0.07	762.83	0.61		0.09	743.95	0.25		0.08	876.21	0.87	
20c3sC1	0.0	>3600	7.6	1173.57	0.09	57.82	0.00	**1173.57**	0.09	12.36	0.00	**1173.57**	0.08	43.18	0.00	**1173.57**
20c3sC2	0.0	1164.5	0.0	**1539.97**	0.06	36.54	0.00	**1539.97**	0.07	23.40	0.00	**1539.97**	0.06	91.64	0.00	**1539.97**
20c3sC3	0.0	25.4	0.0	**880.20**	0.03	2.70	0.00	**880.20**	0.02	1.01	0.00	**880.20**	0.03	2.43	0.00	**880.20**
20c3sC4	0.0	>3600	11.0	1059.35	0.05	676.27	0.00	**1059.35**	0.06	253.87	0.00	**1059.35**	0.06	341.63	0.00	**1059.35**
20c3sC5	0.0	2246.4	0.0	**2156.01**	0.05	>3600	10.71	2156.01	0.08	1626.20	0.00	**2156.01**	0.08	>3600	9.01	2156.01
20c3sC6	0.0	61.6	0.0	**2758.17**	0.05	12.65	0.00	**2758.17**	0.04	15.54	0.00	**2758.17**	0.03	16.23	0.00	**2758.17**
20c3sC7	0.0	0.1	0.0	**1393.99**	0.02	0.25	0.00	**1393.99**	0.01	0.30	0.00	**1393.99**	0.02	0.55	0.00	**1393.99**
20c3sC8	0.0	53.7	0.0	**3139.72**	0.03	6.27	0.00	**3139.72**	0.03	6.94	0.00	**3139.72**	0.03	6.24	0.00	**3139.72**
20c3sC9	0.0	113.9	0.0	**1799.94**	0.06	49.39	0.00	**1799.94**	0.07	31.92	0.00	**1799.94**	0.06	60.22	0.00	**1799.94**
20c3sC10	0.0	2067.5	0.0	**2583.42**	0.02	16.86	0.00	**2583.42**	0.04	16.88	0.00	**2583.42**	0.03	12.81	0.00	**2583.42**
Average	0.0	1293.3	1.9		0.05	445.88	1.07		0.05	198.84	0.00		0.05	417.49	0.90	

Table 4
Comparison of solution approaches on the test instances of Scenarios 3 and 4.

	Branch-and-cut (Koç and Karaoglan, 2016)				F_{RLT3}				F_{RLT4}				F_{RLT5}			
Instance	T_{LP}	T_{opt}	*Gap*	*UB*	T_{LP}	T_{opt}	*Gap*	*UB*	T_{LP}	T_{opt}	*Gap*	*UB*	T_{LP}	T_{opt}	*Gap*	*UB*
S1_2i6s	0.0	1626.6	0.0	**1578.12**	0.08	781.03	0.00	**1578.12**	0.17	792.83	0.00	**1578.12**	0.13	811.68	0.00	**1578.12**
S1_4i6s	0.0	>3600	4.3	1397.27	0.09	47.83	0.00	**1397.27**	0.10	36.26	0.00	**1397.27**	0.08	36.97	0.00	**1397.27**
S1_6i6s	0.1	523.5	0.0	**1560.49**	0.06	173.61	0.00	**1560.49**	0.09	124.90	0.00	**1560.49**	0.08	83.65	0.00	**1560.49**
S1_8i6s	0.0	817.8	0.0	**1692.32**	0.08	829.57	0.00	**1692.32**	0.11	474.44	0.00	**1692.32**	0.09	397.37	0.00	**1692.32**
S1_10i6s	0.0	1.9	0.0	**1173.48**	0.11	8.10	0.00	**1173.48**	0.13	10.17	0.00	**1173.48**	0.09	5.51	0.00	**1173.48**
S2_2i6s	0.0	66.0	0.0	**1633.10**	0.08	232.19	0.00	**1633.10**	0.10	312.58	0.00	**1633.10**	0.06	139.00	0.00	**1633.10**
S2_4i6s	0.0	>3600	9.5	1505.07	0.06	738.43	0.00	**1505.07**	0.10	687.76	0.00	**1505.07**	0.08	499.63	0.00	**1505.07**
S2_6i6s	0.0	1801.0	0.0	**2431.33**	0.05	21.51	0.00	**2431.33**	0.08	47.49	0.00	**2431.33**	0.08	16.79	0.00	**2431.33**
S2_8i6s	0.0	3.2	0.0	**2158.35**	0.02	22.42	0.00	**2158.35**	0.04	5.40	0.00	**2158.35**	0.03	6.41	0.00	**2158.35**
S2_10i6s	0.0	1.3	0.0	**1585.46**	0.06	123.92	0.00	**1585.46**	0.06	112.25	0.0	**1585.46**	0.05	126.49	0.00	**1585.46**
Average	0.0	1204.1	1.4		0.07	297.86	0.00		0.10	260.41	0.0		0.08	212.35	0.00	
S1_4i2s	0.0	>3600	6.2	1582.21	0.06	291.44	0.00	**1582.21**	0.08	87.77	0.00	**1582.21**	0.09	230.56	0.00	**1582.21**
S1_4i4s	0.1	>3600	3.8	1460.09	0.08	286.41	0.00	**1460.09**	0.08	122.90	0.00	**1460.09**	0.06	194.67	0.00	**1460.09**
S1_4i6s	0.1	>3600	4.3	1397.27	0.08	48.13	0.00	**1397.27**	0.10	19.53	0.00	**1397.27**	0.08	62.54	0.00	**1397.27**
S1_4i8s	0.1	2133.6	0.0	**1397.27**	0.08	187.86	0.00	**1397.27**	0.15	22.59	0.00	**1397.27**	0.11	70.17	0.00	**1397.27**
S1_4i10s	0.1	>3600	5.0	1396.02	0.09	210.25	0.00	**1396.02**	0.12	159.67	0.00	**1396.02**	0.11	137.91	0.00	**1396.02**
S2_4i2s	0.0	>3600	0.0	1059.35	0.06	522.81	0.00	**1059.35**	0.06	270.67	0.00	**1059.35**	0.06	314.67	0.00	**1059.35**
S2_4i4s	0.0	>3600	6.2	1446.08	0.08	807.66	0.00	**1446.08**	0.09	315.87	0.00	**1446.08**	0.08	552.52	0.00	**1446.08**
S2_4i6s	0.0	>3600	6.8	1434.14	0.09	1186.79	0.00	**1434.14**	0.09	570.24	0.00	**1434.14**	0.11	1771.52	0.00	**1434.14**
S2_4i8s	0.1	>3600	6.6	1434.14	0.08	3252.34	0.00	**1434.14**	0.15	1898.60	0.00	**1434.14**	0.11	1305.18	0.00	**1434.14**
S2_4i10s	0.1	>3600	6.7	1434.13	0.09	>3600	3.31	1434.13	0.13	1757.26	0.00	**1434.13**	0.16	>3600	3.59	1434.13
Average	0.0	3453.4	5.4		0.08	1039.37	0.33		0.11	513.78	0.00		0.10	823.97	0.36	

improves on the average the computational performances only for the instances of Scenario 2 and the instance 20c3sC5 can be solved to optimality. However, two instances are unsolved with remaining percentage gaps of 1.89% for the instance 20c3sU7 and of 5.95% for the instance 20c3sU9.

Ultimately, using either formulation F_{RLT1} (that is the basic formulation) or F_{RLT2} (that is the basic formulation with the addition of the valid inequality (95)) all the instances can be solved to optimality.

As already observed in Koç and Karaoglan (2016), we found that the instances in which the customers are uniformly distributed are more difficult to solve than the clustered customers instances and with some exceptions the CPU time increases if the number of charging stations increases.

5.2. Analysis of the impact of the reduction tests

In Tables 3 and 4, we compare the branch-and-cut method results proposed in Koç and Karaoglan (2016) with the results that we are able to obtain solving directly formulations F_{RLT3} and F_{RLT4} (that include preprocessing reduction tests described in Section 4.1). Moreover, we also present the results of formulation F_{RLT5} which is similar to F_{RLT4} except for the relaxation of the constraint on the maximum number of available vehicles (17). We highlight that the results in Koç and Karaoglan (2016) were obtained on an Intel Xeon PC with 8 GB and 3.16 GHz.

From Tables 3 and 4, we can make the following observations:

- F_{RLT3}, F_{RLT4} and F_{RLT5} consistently outperforms the branch-and-cut method. The latter solves only 22 out of 40 instances (55%) within the time limit, whereas the formers provide optimal solutions in 92,5%, 97.5% and 90% of the instance, respectively.
- Comparing the computational results of F_{RLT1} with F_{RLT3}, we observe that the reduction procedure does not reduce the average computation time.
- Comparing the computational results of F_{RLT2} with F_{RLT4}, on the contrary, we observe that the reduction procedure reduces the average computation time in all the scenarios except for Scenario 1.
- Considering all the CPU times, the most efficient formulation is F_{RLT4}.
- The CPU times of F_{RLT5} are, on the average, worse than those of F_{RLT4} for all cases, but the instances of Scenario 3.

As a final experiment, we run F_{RLT5} after dropping the valid inequality (95). In this case, we observed that instances 20c3sU7 and S2_4i10s can be solved to optimality in 1713.49 and 1433.99 s, respectively. Therefore, as a final result, we found that only two instances, namely 20c3sC5 and 20c3sU9, remained unsolved after reaching the 1 h-time limit considering an unbounded fleet of vehicles.

6. Conclusion

We addressed a green vehicle routing problem with time duration limits and energy consumption constraints. We proposed a valid nonlinear formulation that was subsequently linearized using the Reformulation-Linearization Technique. Also, we proposed a preprocessing reduction procedure that turns out to enhance the problem solvability. We presented the results of a computational study that was carried out on a benchmark set of medium-sized instances. These results provide strong empirical evidence that the proposed formulation can be directly solved using a commercial MIP solver while requiring reasonable CPU times. Furthermore, we found that the proposed exact approach consistently outperforms a state-of-the-art branch-and-cut algorithm and it constitutes an appealing, simple, and practical alternative for optimally solving medium-sized green vehicle routing problems.

For future research, we recommend to investigate a variant of the GVRP where the full recharge restriction is relaxed and partial recharging is allowed. This variant is particularly relevant to electric vehicles, where the partial recharging option may positively impact the routing decisions (Keskin and Çatay, 2016). Furthermore, we recommend to embed the proposed exact approach into an optimization-based improvement procedure, in the spirit of the matheuristic that was recently proposed by Leggieri and Haouari (2017), with the aim of providing near-optimal solutions for large-scale green vehicle routing problems. This is a topic of our ongoing research.

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