Mathematics | Discrete Mathematics
1ma162
Linneaus University | spring 2018

Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. Find a RR for the number of *derangements*, D_n, of n numbers. A derangement is a permutation of 1,2,3,...,n where no element is at its original place. So $D_2 = 1$ since 2,1 is the only derangement of 1,2. Hint: One way to find the RR is to use the inclusion-exclusion principle for some small n. see also internet for good explanations. Solve it with RSolve and plot the first few D_n. Since Mathematica wants to be exact real initial values is a good idea.**OP**

2. Plot in the logistic map for $a = 4$ a periodic orbit of length 4. Is it stable? You can start with the rational number in base 2 $\beta = 0.10111011\ldots$. What rational number is this? Do then one iteration in the logistic map for $a = 4$ starting with $x_0 = \sin^2\ 2\pi\beta$. Since β is a real number between zero and one so is also x_0. Move then the decimal point in the base 2 expression for β one step to the right and take away an eventual integer part. Convert this new β in base 2 to base 10 and calculate $\sin^2\ 2\pi\beta$ and compare with the iteration. Now you can find the orbit! Are there other period 4 orbits?

3. Run rule 90 and the seed is ■ inserted in middle of a background of ■■□ blocks. Iterate 500 times and your seed can be 1000 cells long. That means 333 blocks + 1 for the black cell. What is happening? Start with a smalle string and fewer iterations. Useful commands here for construction of the seed area: ConstantArray, Flatten, Insert. Or if you use a For-loop, use AppendTo command, but ConstantArray is better.

4. Investigate the rule B1357/S1357. B denotes birth and S survival. Game of Life is B3/S23. What is the rule number for B1357/S1357? Try random seeds and seeds that are Still Life, Oscillators and Gliders in Game of Life (see Wikipedia article about Game of Life). Do you see copies of the pattern?