Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. You borrow 2500 dollars, at 12 percentage compounded monthly, to buy a computer. If the loan is to be paid back over two years, what is your monthly payment? Note you pay back same amount every month. Plot with command DiscretePlot the values of the debt a_n over the 2 years.

2. Consider the map $x_{n+1} = 2\ x_n \pmod 1$ for the starting value $x_0 = 5/7$. You have to modify the program with the new function $g(x) = 2x \pmod 1$. Use the If command when you define $g(x)$. Take away the second part with bifurcation diagram since there is no parameter for this map. Convert the numerical (decimal) x_n-values to base base 2. The command BaseForm will help you to do this. Try other starting points between zero and one. Check how the decimal expansion in base 2 changes from iteration to iteration. How can the iterations be described in base 2? Can you find some short periodic orbits?

3. Consider rule 110 (1D CA) and start with one black cell. Run it up to 1000 iterations so let your string be 1000 cells. Note it evolves to the left only. The moving structures you observe are called spaceships and your task is to look at one of them. Try first the periodic sequence 00010011011111. It consists of 14 cells and repeat it 40 times so the string length should now be a multiple of 14. It is called the background. Finally put 0001110111 surrounded by the periodic sequence above. This string has length 10 so start with $14n + 10 + 14n$ cells where n is an integer. What happens? Illustrate with a figure. Useful commands here are: ConstantArray, Flatten, Insert. If you use a For-loop, use AppendTo command.

4. Investigate the rule B4678/S35678. B denotes birth and S survival. Game of Life is B3/S23. What is the rule number for B4678/S35678? Try random seeds and seeds that are Still Life, Oscillators and Gliders in Game of Life (see Wikipedia article about Game of Life).**OP**