# Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. You take a loan of $S$ dollars that is to be paid back in $T$ periods of time. If $r$ is the interest rate per period of the loan, what constant payment $P$ do you have to make at the end of each period. This is a boundary value problem. What are the boundary values? Solve the RR. Find a formula $P = P(T, S, r)$.

2. Instead of the logistic map take the following non-linear dynamical system

$$x_{n+1} = a \sin \pi x_n. \tag{1}$$

We are still on the interval from zero to one but the parameter lies in the interval $0 < a \leq 1$. Find the right fix point for $a = 0.75$. For example by using Wolfram alpha. Is it stable or unstable? Do some iterations (using pocket calculator) starting close to the fix point and see what happens. Can you find the stable 2-cycle for $a = 0.75$?

3. Starting with one black cell in a 1D CA produce a black right-angled triangle when time evolves. That is, 1+2+3+4+5+.... black cells in the rows. How should the rule look like? Which is the rule number?

4. Modify the 2D cellular automata in such a way that only neighbors to the left and right, up and down, influence the next state of the middle cell (the cells on the diagonal have no influence). Start with one single black cell. Birth (W to B) if 1 or 4 of the neighbors are black, otherwise stay the same color. What is the rule number? Run it 4 steps. This is an outer-totalistic CA like Game of Life but now with 0,1,2,3 or 4 neighbors.