

## Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. In how many ways,  $a_n$ , can a sum of 1s and 2s sum up to  $n$ ? For  $n = 3$  there are 3 ways.  $3=1+1+1=2+1=1+2$ . Find a RR for  $a_n$  and solve it on the computer. What is  $a_{20}$ ? Plot with command `DiscretePlot` the first 10 values of  $a_n$  and its logarithm.

2. Find and plot an unstable 2-cycle in the logistic map for  $a = 4$ . Iterate a couple of times to illustrate the instability. You can locate the 2-cycle by the fix points of  $g(g(x))$ . **OP**

3. Consider the CA called Majority Action. The majority of the 3 cells (left,middle and right) decides the state of the middle cell in next generation. So 2-3 black cells gives black and 0-1 black cells gives white. What is the rule number? Run it for a random seed a couple of times. What happens? Extend to 2 neighbors on each side. Then you have to modify the program somewhat. Read about the command `CellularAutomata`. What is the rule number for majority action this time? Run it also for random seeds. Any difference compared to 2 neighbors?

4. Investigate the rule B35678/S5678. B denotes birth and S survival. Game of Life is B3/S23. What is the rule number for B35678/S5678? Try random seeds and seeds that are Still Life, Oscillators and Gliders in Game of Life (see Wikipedia article about Game of Life). Can you see diamonds? **OP**