

Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. Let $\Sigma = \{0, 1, 2, 3\}$. For $n \geq 1$ let a_n count the number of strings in Σ^n containing an odd number of 1's. Find and solve a RR for a_n . For example 02231 is an allowed sequence of length 5. Hint: All strings of length n are either allowed (a_n) or forbidden (b_n). $a_n + b_n = ?$
2. Consider instead of the logistic map the map $g(x) = a \sin(\pi x)$. Starting value is still between 0 and 1 and a lies also between zero and one. Use pocket calculator and the graph of $g(x)$. When will the left fix-point $x = 0$ loose stability? Same question for the right fix-point. Illustrate with a few iterations on your calculator.
3. Start with one black cell and construct a staircase going down to the right. That is black cells down, right, down, right, down..... . All other cells are white. What is the rule number?
4. Run the outer totalistic rule 90016 starting with a wall, that is a square of black cells. Take for example a 5 times 5 square which means 16 black cells in the wall. Do 4 iterations. What happens? Express 90016 in base 2 and describe in words the rule, when will birth and survival happen?