

Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. Investigate the following system of RRs

$$a_{n+1} = -2a_n - 4b_n, \quad b_{n+1} = 4a_n + 6b_n \quad (1)$$

when initial value is $a_0 = 1$ and $b_0 = 0$. Iterate a couple of times and try to understand what is going on. You can view this as a point starting at $(1,0)$ jumping around in the plane. Try to predict the future for the particle. For which starting values (a_0, b_0) is $(a_1, b_1) = k(a_0, b_0)$? (k is a real number)

2. Plot a stable 3-periodic orbit in the logistic map. Give the three x values for the orbit. Use the bifurcation diagram on page 5 in my notes or see logistic map on Wikipedia. Use a pocket calculator.

3. Consider rule 30 and start with a single black cell. The state before this one: Does it exist? How is it then looking like? Infinite long strings are allowed. Consider finally infinite strings where every fourth cell is black. $\dots \blacksquare \square \square \square \blacksquare \dots$. Describe the future for this seed.

4. Consider the following 2D CA: The center cell becomes black if either 3 or 5 of its 8 neighbors were black on the step before, and should otherwise stay the same color as it was before. What is the rule number? Start with 3 black cells in a row. Run it 3 times.