Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. For $n \geq 1$ let a_n count the number of binary strings of length n, where there are no consecuitive 1's. For example 011 is a fobidden string of length 3. Find and solve the RR for a_n. Solve with RSolve. Plot with command DiscretePlot the first 10 values of a_n. Plot also the logarithm of a_n.

2. Plot in the logistic map for $a = 4$ a periodic orbit of length 3. Is it stable? You can start with the rational number in base 2 $\beta = 0.101101\ldots$. What rational number is this? Do then one iteration in the logistic map for $a = 4$ starting with $x_0 = \sin^2\ 2\pi\beta$. Since β is a real number between zero and one so is also x_0. Move then the decimal point in the base 2 expression for β one step to the right and take away an eventual integer part. Convert this new β in base 2 to base 10 and calculate $\sin^2\ 2\pi\beta$ and compare with the iteration. Now you can find the orbit! Are there other period 3 orbits?

3. Starting with one black cell in a 1D CA and let it produce a straight line (black cells) downwards to the left. Find a rule which does the same thing but to the right. Can there be a rule that produce both? If you take a larger region?

4. Investigate the rule B368/S245. B denotes birth and S survival. Game of Life is B3/S23. What is the rule number for B368/S245? Try random seeds and seeds that are Still Life, Oscillators and Gliders in Game of Life (see Wikipedia article about Game of Life). **OP**