Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. You take a loan of S dollars that is to be paid back in T periods of time. If r is the interest rate per period of the loan, what constant payment P do you have to make at the end of each period? This is a boundary value problem. What are the boundary values? Solve it on the computer. Say $r = 0.05$ and $S = 10000$. Try different P and see for which T you have paid back. Plot with command DiscretePlot the values of your debt after period n, a_n, where $a_0 = S$.

2. Run the logistic map for $a = 4$ for 30 iterations for any start value between 0 and 1. Do you think the orbit will cover the whole interval smoothly if the iterations where continued? Take then $1/3$ and express it in base 2 using BaseForm. Call this number β. Do then one iteration in the logistic map for $a = 4$ starting with $x_0 = \sin^2\ 2\pi\beta$. Since $\beta = 1/3$ is a real number between zero and one so is also x_0. Move then the decimal point in the base 2 expression for β one step to the right and take away an eventual integer part. Convert this new β in base 2 to base 10 and calculate $\sin^2\ 2\pi\beta$ and compare with the iteration. How can you now describe the iterations for $a = 4$? Test your idea for another rational number like $2/5$.

3. Run and plot the *totalistic* rule 2007 1500 times starting with a random seed which is 800 cells long. 3 colors (0-white, 1- grey and 2-black) and the region is with the 2 nearest neighbors. You have to modify the program a little bit. Read about CellularAutomaton, see details. Totalistic means the rule only depends on the sum of the values in the 3 cells. The sum lies between 0 and 6=2+2+2. Since there are three colors you have to work in base 3. Write 2007 in base 3 using the command BaseForm and try to figure out the rule.

4. Investigate the rule B36/S23. B denotes birth and S survival. Game of Life is B3/S23. What is the rule number for B36/S23? Try random seeds and seeds that are Still Life, Oscillators and Gliders in Game of Life (see Wikipedia article about Game of Life). Can you find gliders also in this case?**OP**.