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In[134]:= Clear["`*"]
(* This program solves the logistic map. It is a non-linear problem,
note -x^2 on next line, and RSolve can not be used.
Instead Nest and NestList are used *)
a =.; (* another way to clear parameter a. 0 <
a < 4. The really interesting region is between 3 and 4 *)
g[x_] := a x (1 - x);
(* x_(n+1) = g(x_n), parameter a is in example below 3.1. x_0 =
0.5 = start and we do 50 iterations in this example *)
solg1 = Solve[x == g[x], x] // Simplify
solg2 = Solve[x == g[g[x]], x] // Simplify
(* but first we find fix points of g with solg1. Especially the
second element in the list is interesting. It is the only fix point
which is different from zero. It is created at a =
1 when x = 0 fix point becomes unstable. solg2 gives the period 2 solutions. That is
the fix points of g(g(x)). Here we are interested of the genuine
period 2 solution which are the third and fourth elements in the list
At http://en.wikipedia.org/wiki/Logistic\_map you can read about this
map. There you see that the period 2 solution was created at a = 3
when the fix point to the right became unstable. *)
a = 3.1; start = 0.5; iter = 50;
xfp = x /. solg1[[2]]
x2c1 = x /. solg2[[3]]
x2c2 = x /. solg2[[4]]
(* Last lines give numerical values for fix point and period 2 solution *)
(* The following lines you don't have to think about. It produces an illuminating plot
of the iterations *)
points = NestList[g, start, iter - 1]
lines[x_] := Line[{x, x}, {x, g[x]}, {g[x], g[x]}]
listlines = lines /@ points;
Plot[g[x], {x, 0, 1},
  AspectRatio -> Automatic,
  PlotLabel -> "a=3.1",
  PlotRange -> {{0, 1}, {0, 1}},
  Epilog -> {Line[{0, 0}, {1, 1}], listlines}]
(* Below follows commands that generate the
bifurcation diagram in the interesting region 3 < a < 4. *)
nstart = 500; number = 128;
(* It works like this I
think: after 500 iterations the program takes a look at the following 128
iterations. Union takes away doublets etc. Left
is the stable periodic orbit. In the region from
a = 3.6 to 4 there is a lot of chaos, that is no stable
periodic orbit at all. But in some places
there is a "window". A sign of a short stable periodic orbit *)
afixpoint :=
  Union[
    Map[
      {a, #} &,
      NestList[g, Nest[g, start, nstart], number - 1]
    ]
  ];
amin = 3.; amax = 3.999; step = 0.002;
fixpoints = Flatten[Table[afixpoint, {a, amin, amax, step}], 1];
ListPlot[fixpoints,
  PlotLabel -> "fix points",
  PlotRange -> {{3, 4}, {0, 1}},
  PlotStyle -> PointSize[0.001]]

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Out[136]= $\left\{ \{x \rightarrow 0\}, \left\{ x \rightarrow \frac{-1+a}{a} \right\} \right\}$

Out[137]= $\left\{ \{x \rightarrow 0\}, \left\{ x \rightarrow \frac{-1+a}{a} \right\}, \left\{ x \rightarrow \frac{1+a-\sqrt{-3-2a+a^2}}{2a} \right\}, \left\{ x \rightarrow \frac{1+a+\sqrt{-3-2a+a^2}}{2a} \right\} \right\}$

Out[139]= 0.677419

Out[140]= 0.558014

Out[141]= 0.764567

Out[142]= {0.5, 0.775, 0.540563, 0.7699, 0.549178, 0.767503, 0.553171, 0.766236, 0.555267, 0.765531, 0.556429, 0.765129, 0.557091, 0.764896, 0.557473, 0.76476, 0.557696, 0.76468, 0.557827, 0.764634, 0.557904, 0.764606, 0.557949, 0.76459, 0.557976, 0.76458, 0.557992, 0.764575, 0.558001, 0.764571, 0.558006, 0.764569, 0.558009, 0.764568, 0.558011, 0.764568, 0.558013, 0.764567, 0.558013, 0.764567, 0.558014, 0.764567, 0.558014, 0.764567, 0.558014, 0.764567, 0.558014, 0.764567}

