

## Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. Find a RR for the number regions,  $a_n$ , that are created when  $n$  lines are drawn in the plane. The lines are not parallel and no three lines cross each other. What are the initial conditions for this RR? Note that also infinite regions are counted. How many such regions are there for 100 lines? Maybe you can solve the RR by hand but do it also with RSolve and plot with command Plot the first 10 values of  $a_n$ .

2. Plot a stable 5-periodic orbit in the logistic map. Hint: Look for a tiny window. Give the five  $x$  values for the orbit. Try to find the  $a$  value for which it goes through a period doubling and loose stability.

3. Run the totalistic rule 1815 1500 times starting with one grey (1) cell and also with a random initial state. Your string can be 800 cells. Use 3 colors (0-white, 1-black, 2-black) and consider only the region with the nearest neighbors. You have to modify the program a little bit. Read about CellularAutomaton, see details. Totalistic means the rule only depends on the sum of the cells, even the one in the middle. Since there are three colors you have to work in base 3. Write 1815 in base 3, use BaseForm, and try to figure out how the rule works. To do this run the CA just a few iterations so you can clearly see the individual cells. Has the CA starting with one grey cell a period? **OP**

4. Consider rule 746 and estimate how the (average) radius grow with the number of iterations for the particular seed used in the program. Try other one-dimensional seeds with 3,4 or 5 black cells, will you always see circular growth? Illustrate with figures. Take a sufficiently large grid so the growth will not hit the boundary. Game of life is B3/S23. What is birth an survival conditions for rule 746?