Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. Determine the number of n-digit quartenary $\{0, 1, 2, 3\}$ sequences, a_n, with an even number of 1's by finding a RR and solve it with RSolve. What is the initial condition? How many such strings of length 50 are there? Plot with command DiscretePlot the first 10 values of a_n. Hint: You can split the strings into those with even or odd number of ones. For example, some allowed 6-digit strings are 002333, 122201, 301111 and 111111.

2. Plot in the logistic map for $a = 4$ a periodic orbit of length 5. Is it stable? (Iterate many times). You can start with the rational number in base 2 $\beta = 0.1011110111\ldots$. What rational number is this? Do then one iteration in the logistic map for $a = 4$ starting with $x_0 = \sin^2\ 2\pi\beta$. Since β is a real number between zero and one so is also x_0. Move then the decimal point in the base 2 expression for β one step to the right and take away an eventual integer part. Convert this new β in base 2 to base 10 and calculate $\sin^2\ 2\pi\beta$ and compare with the iteration. Now you can find the orbit! Are there other period 5 orbits?

3. In a long random string of zeros and ones you want to calculate how often the substring 010 appears. For example in 00110100011100100 it appears twice. How can you use the simplest CA to spot them? Which rule spots the substrings 010? Go on with a longer sequence like 01010. Then you have to modify the program somewhat.

4. Investigate the rule B3/S012345678. B denotes birth and S survival. Game of Life is B3/S23. What is the rule number for B3/S012345678? Note this is Life without death. Try random seeds and seeds that are Still Life, Oscillators and Gliders in Game of Life (see Wikipedia article about Game of Life). Can you find gliders also in this case?**OP**