Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA) I

1. Find a RR for the number of bit strings of length n, a_n, that contain two consecutive zeros. More zeros are allowed. Somewhere in the string there must be a 00 sequence. What is the initial condition? Solve the problem on the computer. How many such strings of length 30 are there? Plot with command Plot the first 10 values of a_n. Hint: split the total number of strings of length n into strings with and without 00.

2. Plot a stable 4-periodic orbit in the logistic map. Give the four x values for the orbit. Try to find the a value for which it goes through a period doubling and loose stability. **OP**

3. Consider rule 30 and start with one black cell in the middle. Your string can be 1000 cells long. Run it for 500 iterations. Plot the value in the middle cell, $b(n)$, where n is generation n. $b(0) = 1$ since you start with one black cell. Black is equal to 1 and white is 0 in the list. Does it look random? Equally many black and white cells?

4. Find an infinite pattern (initial state) such that after one iteration of Game of Life the whole plane (all cells) are black. What happens after one more iteration? Run this configuration 1 time. Of course you have to make a cut off but use PlotRange to only show the interesting parts.