Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA) XXIV

1. A particle moves horizontally to the right. For $n \in \mathbf{Z}^+$, the distance the particle travels in the $(n+1)$st second is equal to 4/5 the distance it travels during the nth second. If $x_n, n \geq 0$, denotes the position of the particle at the start of the $(n+1)$st second, find a recurrence relation for x_n, where $x_0 = 1$ and $x_1 = 10$. Solve it on the computer. Plot with command Plot the first 10 values of x_n.

2. When (for which a value) becomes the right fix point unstable in the sine map? For this map $g(x) = a \sin \pi x, 0 \leq x \leq 1, 0 \leq a \leq 1$. So you have to change function in the program. Plot the iterations for a somewhat larger a, starting close to the unstable fix point, and show that it approaches the stable 2-cycle.

3. Run the totalistic rule 588 400 times starting with one grey (1) cell and also with a black cell (2). Your string can be 800 cells. Use 3 colors (0-white, 1-black, 2-black) and consider only the region with the nearest neighbors. You have to modify the program a little bit. Read about CellularAutomaton, see details. Totalistic means the rule only depends on the sum of the cells, even the one in the middle. Since there are three colors you have to work in base 3. Write 588 in base 3, use BaseForm, and try to figure out how the rule works. To do this run the CA just a few iterations so you can clearly see the individual cells.

4. Run Game of Life on a 500 times 500 grid with a randomly generated seed. Each cell is white/black with probability 1/2. Run it 1, 50, 100, 500, 1000 times. Repeat it a couple of times for different seeds. What do you observe? Make an illustrative plot. Use command RandomInteger for the seed. **OP**