

Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA) XII

1. You borrow 2500 dollars, at 12 percentage compounded monthly, to buy a computer. If the loan is to be paid back over two years, what is your monthly payment? Note you pay back same amount every month. Plot with the values of the debt a_n over the 2 years.

2. Consider the map $x_{n+1} = 2 x_n \pmod{1}$ for the starting value $x_0 = 11/24$. Iterate it 4 times. At each step convert x_n to base 2. How can the iterations be described in base 2?

3. Consider the following rule for a 1D CA with 3 colors (0-white, 1-grey, 2-black): You take the sum of the of the middle cell and its neighbors and take the rest by division of 3, that is you take the sum mod 3. An example: $\blacksquare\blacksquare\square$ gives sum $2+2+0=4$ which is congruent 1 (mod 3) so the middle cell is grey in next generation. On the other hand $\blacksquare\blacksquare\blacksquare$ gives a white middle cell in next generation since $2 + 2 + 2 = 6 \equiv 0 \pmod{3}$. Iterate this last seed 3 times if it is surrounded by white cells to the left and right. If two different configurations are *almost equal*, that is they differ only in finite number of cells then they are for this particular CA different also in the next generation. Show that by considering the cell to the very right, n , that have different values. What happens in cell $n + 1$ in the next generation? Take for example the two states above, both with white cells to the right.

4. Consider rule 746 (2D CA and 2 colors). Iterate a 2×2 block with black cells. All other cells are white. Find an almost equal configuration (see above) that gives the same image as the block. Compared to the block this configuration differs only at a finite number of points.