Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. Find a RR for the number of ways, a_n, to climb n stairs if the allowed steps are 1 or 3 staircases. What are the initial conditions for this RR? Solve the problem with RSolve. In how many ways can one climb 30 stairs? Give the formula for a_n. Plot with command DiscretePlot the first 10 values of a_n and the logarithm.

2. Consider the logistic map for $a = 2$. Where is the fix point located? Is it stable or unstable? Illustrate with a plot. $a = 2$ is one of the rare cases where there is an exact solution. Try to find it! In fact even RSolve can find it even if it is a non-linear RR, but it will complain a little! It is a polynomial in x_0.**OP**

3. Run the totalistic rule 1815 1500 times starting with one grey (1) cell and also with a random initial state. Your string can be 800 cells. Use 3 colors (0-white, 1-black, 2-black) and consider only the region with the nearest neighbors. You have to modify the program a little bit. Read about CellularAutomaton, see details. Totalistic means the rule only depends on the sum of the cells, even the one in the middle. Since there are three colors you have to work in base 3. Write 1815 in base 3, use BaseForm, and try to figure out the rule. Check with output from program. With one grey cell, is it periodic? Before you try to understand the random seed, consider two grey cells separated by 1,2,5,10,20,50 cells. Iterate the random seed just a few times and see what kind of structures that survives. Can you know understand what happens for the random seed?

4. Run the outer totalistic rule 258752 starting with a random state, that is each cell can be black or white with probablity 1/2. Use RandomInteger to construct the seed. Do 50,100,200 and 500 iterations. You can use a 500 times 500 grid. What happens? Repeat for a new random seed. Express 258752 in base 2 and describe in words the rule, when will birth and survival happen? Becomes the structure static after a while? Is it possible to understand this concentration of white and black cells? **OP**