Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. A particle moves horizontally to the right. For $n \in \mathbf{Z}^+$, the distance the particle travels in the $(n+1)$st second is equal to 2 times the distance it travels during the nth second minus the distance it travelled during the $n-1$th. If $x_n, n \geq 0$, denotes the position of the particle at the start of the $(n+1)$st second, find a recurrence relation for x_n, where $x_0 = 2$, $x_1 = 4$ and $x_2 = 7$. Solve it on the computer. Plot with command DiscretePlot the first 10 values of x_n.

2. Plot the unstable 3-periodic orbit in the logistic map with $a = 4$. Use for example $g(g(g(x)))$ to find it. Illustrate with a figure that is is unstable.

3. About a 1D CA that models traffic flow. Black cells are cars, white cells empty space. The cars move to the right if there is an empty space. Investigate the 1D CA with rule number 184. What is going on? Try some different random seeds (then you use command RandomInteger). It can be seen as a very simple traffic model. Black is a car, white is an empty space. What will the cars do? Illustrate with a figure. **OP**

4. Considering the following rule: Birth if *exactly* three of its neighbors is alive, otherwise it remains unchanged. What is the rule number? This is Life without Death. Start with some small seeds and try to understand what is going on. Try also some random seeds in a box in the middle of your grid. Use a large grid so you do not hit the wall.