Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. Find a RR for the number of bit strings of length n, a_n, that contain three consecutive zeros. More zeros are allowed. Somewhere in the string there must be a 000 sequence. What is the initial condition? Solve the problem on the computer. How many such strings of length 30 are there? Plot with DiscretePlot the first 10 values of a_n. Hint: split the total number of strings of length n into strings with and without 000. **OP**

2. Plot a stable 3-periodic orbit in the logistic map. Give the three x values for the orbit. Try to find the a value for which it goes through a period doubling and loose stability.

3. Consider rule 30 and start with one black cell in the middle. Your string can be 1000 cells long. Run it for 300,400,500 iterations. Plot the value in the middle cell, $b(n)$, where n is generation n. $b(0) = 1$ since you start with one black cell. Black is equal to 1 and white is 0 in the list. Does it look random? Equally many black and white cells?

4. Consider rule 746 and estimate how the (average) radius grow with the number of iterations for the particular seed used in the program. Try other one-dimensional seeds with 3,4 or 5 black cells, will you always see circular growth? Try also a random seed in the interior, for example a five times five box with black and white cells. Illustrate with figures. Take a sufficiently large grid so the growth will not hit the boundary.