Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. A particle moves horizontally to the right. For $n \in \mathbf{Z}^+$, the distance the particle travels in the $(n+1)$st second is equal to 2 times the distance it travels during the nth second minus the distance it travelled during the $n-1$th. If $x_n, n \geq 0$, denotes the position of the particle at the start of the $(n+1)$st second, find a recurrence relation for x_n, where $x_0 = 2$, $x_1 = 4$ and $x_2 = 7$. Solve it on the computer. Plot with command DiscretePlot the first 10 values of x_n.

2. Plot the unstable 2-periodic orbit in the logistic map with $a = 4$. Use for example $g(g(x))$ to find it. Illustrate with a figure that is is unstable.

3. About a 1D CA that models traffic flow. Black cells are cars, white cells empty space. The cars move to the right if there is an empty space. Investigate the 1D CA with rule number 184. What is going on? Try some different random seeds (then you use command RandomInteger). It can be seen as a very simple traffic model. Black is a car, white is an empty space. What will the cars do? Illustrate with a figure.

4. Investigate the rule B36/S125. B denotes birth and S survival. Game of Life is B3/S23. What is the rule number for B36/S125? Try random seeds and seeds that are Still Life, Oscillators and Gliders in Game of Life (see Wikipedia article about Game of Life). Check also pattern composed of 2 times 2 blocks.**OP**