

Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. Find a RR for the number of strings, a_n , of length n using the letters A, B, C, D such that no A appears after first B . For example an allowed string of length 4 is $ABDC$ but $BBCA$ is forbidden. What is the initial condition? Solve the problem on the computer. How many such strings of length 30 are there? Plot with command `DiscretePlot` the first 10 values of a_n and the logarithm.

Hint: split the total number of strings of length n into strings with and without B .

2. Plot a stable 2-periodic orbit in the logistic map. Give the two x values for the orbit. Do it both when it is newly born from the right fix point and when it is near to bifurcate to a stable 4-cycle. Finally when it is unstable. Iterate so many times so you see that it is unstable. **OP**

3. Which rule is the mirror image of rule 30? Explain how you got the rule number. Run it and rule 30 twenty times starting with one black cell in the middle of the string. Mirroring with respect to a vertical line through the middle cell.

4. A 2D CA is called *totalistic* if the next state in the middle cell only depends on the *sum* of the number of black cells in the neighbourhood. We include here the middle cell in this neighbourhood and we consider only the four closest neighbours: UP, DOWN, LEFT and RIGHT. If you look in the details about the command `CellularAutomaton` you will see how to run such a 5-neighbourhood totalistic 2D cellular automata. How many rules are there with 2 colors, white and black? Start with only a black cell in the middle and iterate $1, 2, \dots, 10$ times. What kind of behaviour do you observe for the different rules? Make a plot for some interesting cases.