

Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA) I

1. Find a RR for the number of *infinite* regions that are created when n lines are drawn in the plane. Find the solution to the RR. The lines are not parallel and no three lines cross each other. What are the initial conditions for this RR? How many such regions are there for 100 lines?

2. Do one iteration in the logistic map for $a = 4$ starting with $x_0 = \sin^2 2\pi\beta$. Here β is a real number between zero and one so also x_0 lies in this interval. Use trigonometric formulas to get an expression for x_1 . What happens to the angle in one iteration? And in next, and in next...? If $\beta = 0.10110100001.....$ in base 2, what happens when you iterate? Remember that a multiple of 2π in the angle does not change the value. If due to measurement error you are uncertain about fifth decimal in β what can you say about x_5 ?

3. Consider the CA called Majority Action. The majority of the 3 cells (left,middle and right) decides the state of the middle cell in next generation. So 2-3 black cells gives black and 0-1 black cells gives white. What is the rule number? Find 2 *almost equal states* which give the same image. Two configurations are said to be almost equal if they differ only in finite number of cells.

4. Find an infinite pattern (initial state) such that after one iteration of Game of Life the whole plane (all cells) are black. What happens after one more iteration?