

Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. Find a RR for the number of bit strings of length n that contain two consecutive zeros. More zeros are allowed. Somewhere in the string there must be a 00 sequence. What is the initial condition? Solve the problem. How many such strings of length 10 are there? (use the RR) Hint: split the total number of strings of length n into strings with and without 00.

2. Consider the map $x_{n+1} = 2x_n \pmod{1}$ for the starting value $x_0 = 5/7$. Iterate a few times. Convert the values you get to base 2 using for example Wolfram Alpha. How can the iterations be described? Finally find all periodic orbits of length 3.

3. Consider a 1D cellular automata starting with one black cell. Produce a black triangle (90,45,45 degrees) with two corners in the middle and the third to the right. So top row is one black B, next row BB, third BBB and so on. What is the rule number?

4. Consider the CA called Majority Action. The majority of the 9 cells (8 neighbors + middle cell) decides the state of the middle cell in next generation. So 5-9 black cells gives black in the middle in next generation and 0-4 black cells gives white. What is the rule number? Find 2 *almost equal states* which give the same image. Two configurations are said to be almost equal if they differ only in finite number of cells. Try to find a Garden-of-Eden state, that is a state that not be reached. It can only be the initial configuration.