Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. Determine the number of n-digit quartenary $\{0, 1, 2, 3\}$ sequences in which there is never a 3 anywhere to the right of a 0. 20213 is thus a forbidden string of length 5. What is the initial condition? Solve the RR. How many such strings of length 50 are there? Hint: Split all strings into allowed and forbidden ones.

2. Find the fix points for the tent map. Then $g(x) = a\mathrm{Min}(x, 1-x), 0 \leq x \leq 1, 0 \leq a \leq 2$. Plot the location of the right fix point as function of a when $a > 1$. Is this fix point stable or unstable? To investigate this you can use the plot and make a cobweb diagram or iterate on a pocket calculator a few times.

3. Run the totalistic rule 1599 4 times starting with one grey (1) cell. Use 3 colors and only in the region with the nearest neighbors. Totalistic means the rule only depends on the sum of the cells. Since there are three colors you have to work in base 3. Write 1599 in base 3 (use for example Wolfram alpha) and try to figure out the rule.

4. Modify the 2D cellular automata in such a way that only neighbors to the left and right, up and down, influence the next state of the middle cell (the cells on the diagonal have no influence). Start with one single black cell. Birth (W to B) if 1 or 4 of the neighbors are black, otherwise stay the same color. What is the rule number? Run it 4 steps.