Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA) XV

1. Find a RR for the number regions, a_n, that are created when n lines are drawn in the plane. The lines are not parallel and no three lines cross each other. What are the initial conditions for this RR? How many such regions are there for 100 lines? Maybe you can solve the RR by hand but do it also with RSolve and plot with command Plot the first 10 values of a_n.

2. Find an unstable 2-cycle for the sine map $g(x) = a \sin \pi x$. x lies as before between 0 and 1 but the real parameter a is positive but not greater than 1. Use the bifurcation diagram to locate the 2-cycle. The plot should illustrate the instability of the 2-cycle. You have to change function in the program. Solve will protest somewhat when you run but you can trust the output. (You also can take away the Solve-part).

3. Run the *totalistic* rule 1599 4000 times starting with one grey (1) cell in the middle surrounded by white cells. You can for example start with a string of 800 cells. 3 colors (0-white, 1- grey and 2-black) and the region is with the 2 nearest neighbors. You have to modify the program a little bit. Read about CellularAutomaton, see details. Totalistic means the rule only depends on the sum of the values in the 3 cells. The sum lies between 0 and 6=2+2+2. Since there are three colors you have to work in base 3. Write 1599 in base 3 using the command BaseForm and try to figure out the rule. Compare with output from the program. **OP**

4. Consider the following rule: If 5 or more of the neighborhood of nine cells (including the cell itself) are alive, then this cell will also become or remain alive. Otherwise it will die or remain dead. What is the rule number? Run it at least 50 times in a 200 times 200 grid starting with a random distribution of black and white cells. Use command RandomInteger for generating the seed.