Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. Find a RR for the number of ways, a_n, to climb n stairs if the allowed steps are 1 or 3 staircases. What are the initial conditions for this RR? Solve the problem with RSolve. In how many ways can one climb 50 stairs? Plot with command DiscretePlot the first few values of a_n and the logarithm. Small imaginary numbers you get rid of by using the command Re (real part).N[a] gives the numerical value of a.

2. Plot a stable 3-periodic orbit in the logistic map. Give the 3 x values for the orbit. Find the a value for which it goes through a period doubling and loose stability.

3. Consider rule 110 (1D CA) and start with one black cell. Run it up to 1000 iterations so let your string be 1000 cells. Note it evolves to the left only. The moving structures you observe are called spaceships and your task is to look at one of them. Try first the periodic sequence 00010011011111. It consists of 14 cells and repeat it 40 times so the string length should now be a multiple of 14. It is called the background. Finally put 1001111 surrounded by the periodic sequence above. This string has length 7 so start with $14n + 7 + 14n$ cells where n is an integer. What happens? Illustrate with a figure. Useful commands here are: ConstantArray, Flatten, Insert. If you use a For-loop, use AppendTo command.

4. Consider the following 2D CA: Birth in center cell (white->black) if either 3 or 5 of its 8 neigbours were black (alive) on the step before. Otherwise same color as before. What is the rule number? Run it for 400 times for a seed of 7 black cells in a row in a 400 times 400 grid. Is there some activity in the interior? You can investigate this by plotting the difference between two consecutive iterations. CellularAutomaton produces lists and they can be added and subtracted as we do with numbers. 0 gives white, 1 gives grey and 2 gives black color. No activity means 0 to 0 or 1 to 1 could be shown by grey color. What is going on at the boundary?**OP**