Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. Let $\Sigma = \{0, 1, 2\}$. For $n \geq 1$ let a_n count the number of strings in Σ^n , strings with n digits using 0,1, and 2, containing an even number of 1's. Find and solve a RR for a_n. For example 1220 is one of the 3^4 elements in Σ^4 but is not allowed. Plot with command DiscretePlot the first 10 values of a_n. Hint: Split in strings with even and odd number of ones.

2. Plot in the logistic map for $a = 4$ a periodic orbit of length 4. Is it stable? You can start with the rational number in base 2 $\beta = 0.10111011\ldots$. What rational number is this? Do then one iteration in the logistic map for $a = 4$ starting with $x_0 = \sin^2\ 2\pi\beta$. Since β is a real number between zero and one so is also x_0. Move then the decimal point in the base 2 expression for β one step to the right and take away an eventual integer part. Convert this new β in base 2 to base 10 and calculate $\sin^2\ 2\pi\beta$ and compare with the iteration. Now you can find the orbit! Are there other period 4 orbits?

3. Start with one black cell and construct a staircase going down to the right and plot it. Only black cells down, right, down, right, down..... What is the rule number for this 1D CA? If starting cell splits into two cells, one to the left one to the right, in the next generation, what is the rule number then? What happens after second iteration?

4. Modify the 2D cellular automata in such a way that only neighbors to the left and right, up and down, influence the next state of the middle cell (the cells on the diagonal have no influence). Start with one single black cell. Birth (W to B) if 1 or 4 of the neighbors are black, otherwise stay the same color. What is the rule number? Run it 30 steps. Here you have to modify the code somewhat. Read about CellularAutomaton with Help function. It is a 5-neighbor outer totalistic rule. **OP**