Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA) XI

1. Consider the following system of RRs

$$a_{n+1} = -2a_n - 4b_n, \qquad b_{n+1} = 4a_n + 6b_n \tag{1}$$

when initial value is $a_0 = 1$ and $b_0 = 0$. Plot the first 5 points in the (a, b)-plane. Predict the future. Try to find a mathematical model, on internet or in books, where a system of RRs appears. Describe the model briefly.

2. Find the unstable 2-periodic orbit in the logistic map with $a = 4$. Use $g(g(x))$. Illustrate that is is unstable by calculating, using pocket calculator, an orbit starting close to the 2-cycle.

3. Investigate the 1D CA with rule number 184. What is going on? Try some different random seeds like ...WWWWWBBBWBBWWWBBBB-WWWBBWBBWWWWW.... It can be seen as a very simple traffic model. Black is a car, white is an empty space. What will the cars do? Iterate a seed like the one above 5 times.

4. Considering the following rule: Birth if *exactly* one of its neighbors is alive, otherwise it remains unchanged. What is the rule number? Start with one black cell and iterate 4 times.