


Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. You borrow 2500 dollars, at 12 percentage compounded monthly, to buy a computer. If the loan is to be paid back over two years, what is your monthly payment? Note you pay back same amount every month. Plot with command `DiscretePlot` the values of the debt a_n over the 2 years.

2. Consider the map $x_{n+1} = 2 x_n \pmod{1}$ for the starting value $x_0 = 11/24$. You have to modify the program with the new function $g(x) = 2x \pmod{1}$. Use the `If` command when you define $g(x)$. Take away the second part with bifurcation diagram since there is no parameter for this map. Convert the numerical (decimal) x_n -values to base base 2. The command `BaseForm` will help you to do this. Try other starting points between zero and one. Check how the decimal expansion in base 2 changes from iteration to iteration. How can the iterations be described in base 2? Can you find some short periodic orbits?

3. Consider rule 110 (1D CA) and start with one black cell. Run it up to 1000 iterations so let your string be 1000 cells. Note it evolves to the left only. The moving structures you observe are called spaceships and your task is to look at one of them. Try first the periodic sequence 00010011011111. It consists of 14 cells and repeat it 40 times so the string length should now be a multiple of 14. It is called the background. Finally put 0001110111 surrounded by the periodic sequence above. This string has length 10 so start with $14n + 10 + 14n$ cells where n is an integer. What happens? Illustrate with a figure. Useful commands here are: `ConstantArray`, `Flatten`, `Insert`. If you use a `For`-loop, use `AppendTo` command.

4. Consider rule 746 (2D CA) and start with the seed . Plot the difference between 301 and 300 iterations, 302 and 301 iterations, 303 and 302 iterations, 304 and 303 iterations and finally 305 and 304 iterations. Describe what you see. What kind of action can you see in the interior? `CellularAutomaton` produce lists and they can be added and subtracted as with numbers. With 0,1 and 2 in the lists you will get 3 colors, white, grey and black, respectively. Take a large enough grid so you don't hit the boundary. **OP**