

Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. A particle moves horizontally to the right. For $n \in \mathbf{Z}^+$, the distance the particle travels in the $(n + 1)$ st second is equal to half the distance it travels during the n th second. If $x_n, n \geq 0$, denotes the position of the particle at the start of the $(n + 1)$ st second, find a recurrence relation for x_n , where $x_0 = 1$ and $x_1 = 7$. Solve it on the computer. Plot with command Plot the first 10 values of x_n .

2. When (for which a value) becomes the right fix point unstable in the sine map? For this map $g(x) = a \sin \pi x, 0 \leq x \leq 1, 0 \leq a \leq 1$. So you have to change function in the program. Plot the iterations for a somewhat larger a , starting close to the unstable fix point, and show that it approaches the stable 2-cycle.

3. One can run asymmetric neighborhoods. Two to the left and one to the right is $r = 3/2$. Read about it in CellularAutomaton. Here we use three colors. Starting with one grey (1) cell in the middle of string with 1000 cells. Run rule 401620738716 100, 200 and 500 times. Write the rule number in base 3 and try to understand the rule. Use very small grid and just 2-3 iterations starting with one black or one grey cell and compare with output from BaseForm, this is the way to understand the rule. **OP**

4. Investigate the rule B34/S34. B denotes birth and S survival. Game of Life is B3/S23. What is the rule number for B34/S34? Do first a few iterations on a small seed so you see you have the right CA. Try random seeds and seeds that are Still Life, Oscillators and Gliders in Game of Life (see Wikipedia article about Game of Life). Can you find gliders also in this case?