

Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. For $n \geq 1$ let a_n count the number of binary strings of length n , where there are no consecutive 1's. For example 011 is a forbidden string of length 3. Find and solve the RR for a_n . Solve with RSolve. Plot with command DiscretePlot the first 10 values of a_n . Plot also the logarithm of a_n . **OP**

2. Try to estimate a value of the Feigenbaum constant from your observations of the a values of the first 3 period doublings (fix point \rightarrow 2-cycle \rightarrow 4-cycle \rightarrow 8-cycle). Compare with exact value.

3. Starting with one black cell in a 1D CA and let it produce a straight line (black cells) downwards to the left. Find a rule which does the same thing but to the right. Can there be a rule that produce both? If you take a larger region?

4. Consider the following 2D CA: Birth in center cell (white \rightarrow black) if either 3 or 5 of its 8 neighbours were black (alive) on the step before. Otherwise same color as before. What is the rule number? Run it for 400 times for a seed of 7 black cells in a row in a 400 times 400 grid. Is there some activity in the interior? You can investigate this by plotting the difference between two consecutive iterations. CellularAutomaton produces lists and they can be added and subtracted as we do with numbers. 0 gives white, 1 gives grey and 2 gives black color. No activity means 0 to 0 or 1 to 1 could be shown by grey color.