

Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. What is the risk for ruin if you play an unfair game with another person, playing about 1 crown each time. You start with 6 crowns and your opponent has 4 crowns. When someone got 10 crowns the other one is ruined. The chance that you win each play is p where p lies between 0.1 and 0.5. Plot the risk for ruin as a function of p . Hint: In the first game you can win and later on be ruined (these probabilities can be multiplied) or you lose and later one become ruined (these probabilities can also be multiplied).

2. Plot in the logistic map for $a = 4$ a periodic orbit of length 3. Is it stable? You can start with the rational number in base 2 $\beta = 0.101101101\dots$. What rational number is this? Do then one iteration in the logistic map for $a = 4$ starting with $x_0 = \sin^2 2\pi\beta$. Since β is a real number between zero and one so is also x_0 . Move then the decimal point in the base 2 expression for β one step to the right and take away an eventual integer part. Convert this new β in base 2 to base 10 and calculate $\sin^2 2\pi\beta$ and compare with the iteration. Now you can find the orbit! Are there other period 3 orbits?

3. Consider rule 170. Run it for a randomly generated seed. This is an example of a reversible CA, that is, each state has a unique predecessor. Try to understand that by considering how the rule works. Explain the rule by words

4. Investigate the rule B4678/S35678. B denotes birth and S survival. Game of Life is B3/S23. What is the rule number for B4678/S35678? Try random seeds and seeds that are Still Life, Oscillators and Gliders in Game of Life (see Wikipedia article about Game of Life). **OP**