Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. You take a loan of S dollars that is to be paid back in T periods of time. If r is the interest rate per period of the loan, what constant payment P do you have to make at the end of each period? This is a boundary value problem. What are the boundary values? Solve it on the computer. Say $r = 0.05$ and $S = 10000$. Try different P and see for which T you have paid back. Plot with command DiscretePlot the values of your debt after period n, a_n, where $a_0 = S$.

2. Plot in the logistic map for $a = 4$ a periodic orbit of length 4. Is it stable? You can start with the rational number in base 2 $\beta = 0.10111011\ldots$. What rational number is this? Do then one iteration in the logistic map for $a = 4$ starting with $x_0 = \sin^2\ 2\pi\beta$. Since β is a real number between zero and one so is also x_0. Move then the decimal point in the base 2 expression for β one step to the right and take away an eventual integer part. Convert this new β in base 2 to base 10 and calculate $\sin^2\ 2\pi\beta$ and compare with the iteration. Now you can find the orbit! Are there other period 4 orbits?**OP**

3. Run and plot the *totalistic* rule 2007 1500 times starting with a random seed which is 800 cells long. 3 colors (0-white, 1- grey and 2-black) and the region is with the 2 nearest neighbors. You have to modify the program a little bit. Read about CellularAutomaton, see details. Totalistic means the rule only depends on the sum of the values in the 3 cells. The sum lies between 0 and 6=2+2+2. Since there are three colors you have to work in base 3. Write 2007 in base 3 using the command BaseForm and try to figure out the rule.

4. Run Game of Life on a rather large grid and with a random seed. Try to find other Still Life, Oscilators and Gliders than these you find in the Wikipedia article about Game of Life.