

Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA) XIX

1. A particle moves horizontally to the right. For $n \in \mathbf{Z}^+$, the distance the particle travels in the $(n + 1)$ st second is equal to half the distance it travels during the n th second. If $x_n, n \geq 0$, denotes the position of the particle at the start of the $(n + 1)$ st second, find a recurrence relation for x_n , where $x_0 = 1$ and $x_1 = 7$. Solve it on the computer. Plot with command Plot the first 10 values of x_n .

2. When (for which a value) becomes the right fix point unstable in the sine map? For this map $g(x) = a \sin \pi x, 0 \leq x \leq 1, 0 \leq a \leq 1$. So you have to change function in the program. Plot the iterations for a somewhat larger a , starting close to the unstable fix point, and show that it approaches the stable 2-cycle.

3. One can run asymmetric neighborhoods. Two to the left and one to the right is $r = 3/2$. Read about it in CellularAutomaton. Here we use three colors. Starting with one grey (1) cell in the middle of string with 1000 cells. Run rule 401620738716 100, 200 and 500 times. Write the rule number in base 3 and try to understand the rule. Use very small grid and just 2-3 iterations starting with one black or one grey cell and compare with output from BaseForm, this is the way to understand the rule. **OP**

4. Run Game of Life on a 500 times 500 grid with a randomly generated seed. Each cell is white/black with probability $1/2$. Run it 1, 50, 100, 500, 1000 times. Repeat it a couple of times for different seeds. What do you observe? Make an illustrative plot. Use command RandomInteger for the seed.