Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. Determine the number of n-digit quartenary $\{0, 1, 2, 3\}$ sequences, a_n, in which there is never a 3 anywhere to the right of a 0. 20213 is thus a forbidden string of length 5. What is the initial condition? Formulate the RR and solve on the computer. How many such strings of length 50 are there? Plot with command DiscretePlot the first 10 values of a_n. Plot also the logarithm. Hint: Split all strings into those with and without zeros.

2. Try to find a stable 2-cycle in the map with $g(x) = a \sin \pi x, 0 \leq x \leq 1, 0 < a \leq 1$. The program will protest somewhat when you use the command Solve but you can trust the output. **OP**

3. A Garden of Eden state has no predecessor. Show that a single black cell has no predecessor for rule 110 with finite number of black cells. Find a prevoius state for the configuration ..□□■■■□□.... for rule 110. Run it 1000 times forward in time using a string of 1000 cells. Note it evolves only to the left.

4. Consider the circular growth CA, rule 746, with the given seed ■■■□■. Where are the changes from one iteration to another taking place? Produce plots that support your claim! Do it for 400-405 iterations. Hint: In Mathematica one can add and subtract tables like numbers. A table with only ones could be useful. If the table contains 0,1, and 2 then 0 is white, 1 is grey and 2 is black.