Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. Determine the number of n-digit quartenary $\{0, 1, 2, 3\}$ sequences, a_n, in which there is never a 3 anywhere to the right of a 0. 20213 is thus a forbidden string of length 5. What is the initial condition? Formulate the RR and solve on the computer. How many such strings of length 50 are there? Plot with command DiscretePlot the first 10 values of a_n. Plot also the logarithm. Hint: Split all strings into those with and without zeros.

2. Try to find a stable 2-cycle in the map with $g(x) = a \sin \pi x, 0 \leq x \leq 1, 0 < a \leq 1$. The program will protest somewhat when you use the command Solve but you can trust the output. **OP**

3. A Garden of Eden state has no predecessor. Show that a single black cell has no predecessor for rule 110 with finite number of black cells. If you allow for infinite strings? Find a prevoius state for the configuration ..□□■■■□□.... for rule 110. Run it 900 times forward in time using a string of 1000 cells. Note it evolves only to the left so start to the right.

4. Investigate the rule B2/S. B denotes birth and S survival. Game of Life is B3/S23. What is the rule number for B2/S? Note no survival here! Try random seeds and seeds that are Still Life, Oscillators and Gliders in Game of Life (see Wikipedia article about Game of Life).