Mathematica Problems on Recurrence Relations (RR) and Cellular Automata (CA)

1. Find a RR for the number of ways, a_n, to completeley cover a 2 times n checkerboard with 1x2 dominoes. What are the initial conditions for this RR? Solve the problem on the computer. How many such coverings are there on a 2x50 board? Plot with command DiscretePlot the first 20 values of a_n. Plot also logarithm. Hint: Upper right conner can be covered horizontally and vertically. Treat these cases separateley.

2. Plot a stable 4-periodic orbit in the logistic map. Give the four x values for the orbit. Find the a value for which it goes through a period doubling and loose stability.

3. Consider rule 110 (1D CA) and start with one black cell. Run it up to 1000 iterations so let your string be 1000 cells. Note it evolves to the left only. The moving structures you observe are called spaceships and your task is to look at one of them. Try first the periodic sequence 00010011011111. It consists of 14 cells and repeat it 40 times so the string length should now be a multiple of 14. It is called the background. Finally put 0001110111 surrounded by the periodic sequence above. This string has length 10 so start with $14n + 10 + 14n$ cells where n is an integer. What happens? Illustrate with a figure. You can read about it on the Wikipedia article about rule 110. Useful commands here are: ConstantArray, Flatten, Insert. If you use a For-loop, use AppendTo command. Is it possible to get a spaceship moving to the left? Can you get a collision between two spaceships? **OP**.

4. Investigate the rule B34/S34. B denotes birth and S survival. Game of Life is B3/S23. What is the rule number for B34/S34? Try random seeds and seeds that are Still Life, Oscillators and Gliders in Game of Life (see Wikipedia article about Game of Life). Can you find gliders also in this case?