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M S RAMAIAH INSTITUTE OF TECHNOLOGY

(AUTONOMOUS INSTITUTE, AFFILIATED TO VTU)

BANGALORE – 560 054

SEMESTER END EXAMINATIONS – JANUARY 2015

Course & Branch	:	**B.E. – INFORMATION SCIENCE & ENGG.**	**Semester**	:	**III**
Subject	:	**Discrete Mathematical Structures**	**Max. Marks**	:	**100**
Subject Code	:	**IS314**	**Duration**	:	**3 Hrs**

Instructions to the Candidates:

- Answer one full question from each unit.

UNIT – I

1. a) Determine the power sets of the following sets and find the cardinality of each power set. (06)
   i) Φ ii) $\{\Phi, \{\Phi\}\}$ iii) $\{a, b, \{\Phi\}\}$
   b) i) Let A= {0,1}. Show that the following expressions are regular expressions over A. (06)
   i) 00*(0v1)*1 ii) (01)*(01v1*)
   ii) Let S={0,1}.Give the regular expression corresponding to the regular set given
   a) {00,010,0110,011110,.....} b){0,001,000,00001,00000,0000001.....}
   c) Establish the validity of the following premises. (08)
   " A student in this class has not read the book", and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first has not read the book".

2. a) Provide steps and reasons to establish the following logical equivalences. (08)
   i) $\neg[\neg[(p \vee q) \wedge r] \vee \neg q] \Leftrightarrow q \wedge r$
   ii) $p \vee q \vee (\neg p \wedge \neg q \wedge r) \Leftrightarrow p \vee q \vee r$
   b) Let A, B and C be sets .Show that (06)
   i) $(A-C) \cap (C-B) = \Phi$
   ii) $(B-A) \cup (C-A) = (B \cup C) - A$
   c) Using principle of Mathematical Induction, prove that for all $n \in N$: $2^{3n}-1$ is divisible by 7. (06)

UNIT -II

3. a) i) How many strings of length 4 can be formed using the letters ABCDE if repetitions are not allowed? (06)
   ii) How many strings begin with the letter B?
   iii) How many do not begin with the letter B?
   b) Find the explicit formula for the sequence defined by the recurrence relation $b_n = 2b_{n-1}+1$ with initial condition $b_1 = 7$ (07)
   c) Let A=Z , and let (07)
   R={(a,b) $\in$AX A | a and b yield the same remainder when divided by 2 }.
   Show that congruence mod 2 or $a \equiv b \pmod 2$ is an equivalence relation.

4. a) i) Show that ${}^{n+1}C_r = {}^{n}C_{r-1} + {}^{n}C_r$. (06)
   ii) Show that if any 30 people are selected, the none may choose a subset of five so that all five were born on the same day of the week.

b) Let A={1,2,3,4} and R={(2,1),(2,3),(3,2),(3,3),(2,2),(4,2)}. Compute i)M_R^2 ii)$M_{\bar{R}}$ iii) M_R^{-1} iv)Symmetric closure v) Reflexive closure (06)

c) Let A= {1,2,3,4} and let R and S be the relations on A described by R={(1,4),(3,2),(4,3)}; S={(1,1),(1,2)(2,2),(3,3),(4,2),(4,4)}. Use the Warshall's algorithm to compute the transitive closure of R ∪ S and also find the relative set of $(R \cup S)^{\infty}$ (08)

UNIT III

5. a) Let f and g are functions from R→R given below, verify that $(g \circ f)^{-1}=f^{-1} \circ g^{-1}$be defined by (i)f(x)=2x, g(x)=(3x-2) ii) f(x)=(x+1)/2, g(x)=(x-1)/2 (06)

b) i)Decode the message ATEHAOMOMNTI , which was encoded using the permutation (06)
(3,7,1,12)°(2,5,8)°(4,10,6,11,9)
ii) Encode the message MAKE ME AN OFFER using the keyword columnar transposition method. Use the keyword as JONES

c) Draw the Hasse Diagram for the following poset A={2,4,6,8,12,18,24,36,72} with the partial order of divisibility. Determine whether the Hasse diagram represents a lattice or not. Justify your answers. (08)

6. a) Prove the following theorem (06)
Let L be a bounded distributive lattice. If a complement exists, it is unique.

b) Use the hashing function h, which takes the first three digits of the account number as one number and the last four digits as another number, adds them, and then applies the mod -59 function.Assume there are 7500 customer records to be stored using this hashing function. (06)
   i) How many Linked lists will be required for the storageof these records.
   ii) If an approximatelyeven distribution is achieved, roughly how many records will be stored by each linked list
   iii) Determine to which list the given customer account number that is 3759273 should be attached.

c) Draw the Hasse diagram for the partial order "divides" on the set S={2,3,4,6,8,12,18,24}. (08)
   i) Find the upper bound and lower bound for the subset S1={3,4}
   ii) Does S have an upper bound? A lower bound? Justify your answer.
   iii) Does S have any minimal elements ? Any maximal elements? Justify your answer

INIT-IV

7. a) i) Represent the following graphs using an adjacency matrix and incidence matrix (08)

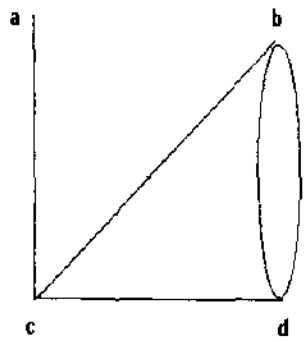


fig 1

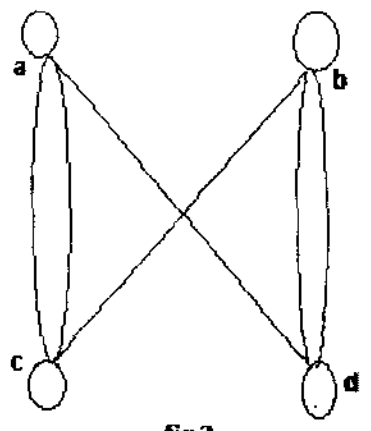
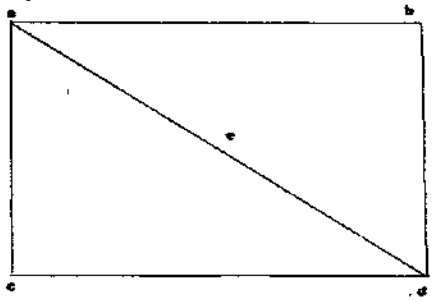


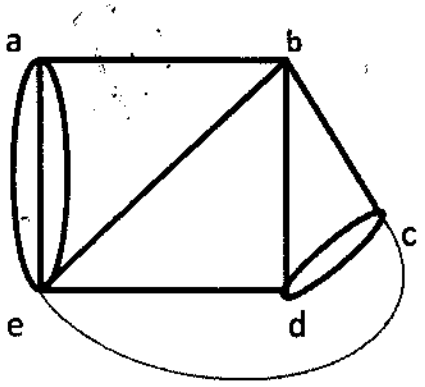
fig 2

b) i) Define the following with an example.(i) Strongly connected Graph ii) Bipartite Graph (12)
ii)Define Euler circuit. Which of the following graphs has an Euler circuit or an Euler path but no Euler circuit or neither? Give reasons for your choice.

a)

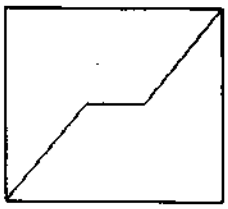


b)



8. a) i)State the handshaking theorem. Prove that in every graph, the number of vertices of odd degree is even. (10)
ii)Show that isomorphism of simple graph is an equivalence relation.

b) i) Discuss any two applications of Bipartite Graph.ii) Define isomorphism of Graphs. Verify that the two graphs shown below are isomorphic. (10)



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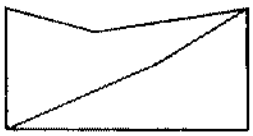

H

UNIT - V

9. a) Prove the following : (06)
I)Let G be a group. Each element a in G has only one inverse in G.
ii)Let G be a Group and let a,b and c be elements of G.Then ab=ac implies that b=c.

b) Consider the binary operation * defined on the set A= {a, b, c, d} by the following table. (08)

*	a	b	c	d
a	a	c	b	d
b	d	a	b	c
c	c	d	a	a
d	d	b	a	c

Compute and Justify your answer.
a) c* d and d*c b) b*d and d*b
c) a*(b*c) and (a*b)*c d) s * commutative? Associative?

c) Let (06)

$$H = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Be a parity check matrix. Determine the (3,6) group code $e_H: B^3 \rightarrow B^6$.

10. a) Define semigroup. Let S={a,b,c} and T={x,y,z} .Verify the following tables are isomorphic. (06)

*	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

*	x	y	z
x	z	x	y
y	x	y	z
z	y	z	x

b) Let m=2,n=5 and (07)

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Determine the Group code $e_H: B^2 - B^5$

c) What is the remainder when 3^{850} is divided by 17? (07)

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