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ISMAT301

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M S RAMAIAH INSTITUTE OF TECHNOLOGY

(AUTONOMOUS INSTITUTE, AFFILIATED TO VTU)

BANGALORE – 560 054

SEMESTER END EXAMINATIONS – JANUARY 2015

Course & Branch	: **B.E: Information Science and Engineering**	**Semester**	: **III**
Subject	: **Engineering Mathematics-III**	**Max. Marks**	: **100**
Subject Code	: **ISMAT301**	**Duration**	: **3 Hrs**

Instructions to the Candidates:

- Answer one full question from each unit.

UNIT – I

1. a) (i) Write Lagrange's Interpolation formula for the set of values $(x_0,y_0),(x_1,y_1)$ and (x_2,y_2). (02)

   (ii) If $y(2) = -4$, $y(4) = 26$, $y(6) = 501$, $y(8) = 1021$, then find $\nabla^2 y_2$. (03)

   b) Find $y'(1.2)$ and $y''(1.8)$ form the following table. (08)

x	1.0	1.2	1.4	1.6	1.8	2.0
f(x)	2.72	3.32	4.06	4.96	6.05	7.39

   c) Use Simpson's 3/8th rule to obtain approximate value of $\int_0^1 e^{x^2}dx$, by considering six equal intervals. (07)

2. a) (i) Define interpolation and extrapolation. (02)

   ii) Construct the divided difference table for the following numerical observations (03)

x	-1	3	5
y	20	16	10

   b) Find the radius of curvature at x = 2 from the following numerical data (08)

x	2	4	5	6
f(x)	10	96	196	350

   c) A survey conducted in a slum locality reveals the following information as classified below: (07)

Income per day(Rs)	Under 10	10 - 20	20 - 30	30 - 40	40 - 50
No. of persons	20	45	115	210	115

   Estimate the probable number of persons in the income group 18 to 23.

UNIT – II

3. a) (i) State Raabe's test for the series of positive terms. (02)

(ii) Find the Fourier coefficient b_n for the function $f(x)=3x$ in $(-\pi,\pi)$. (03)

b) Find the Fourier series of the function $f(x)=\begin{cases}-\pi & in\ -\pi<x<0\\ x & in\ 0<x<\pi\end{cases}$,hence deduce that $\frac{\pi^2}{8}=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}$. (08)

c) For the following values of x and y, find the Fourier series up to first harmonics in (0,24). (07)

x:	0	4	8	12	16	20
y:	9.0	18.2	24.4	27.8	27.5	22

4. a) (i) State p-series test for the positive term series. (02)

(ii) Find the Fourier coefficient a_0 for the function $f(x) = 2x - x^2$ defined in (0, 3). (03)

b) Discuss the nature of the series $\frac{5}{2}\frac{x^3}{3}+\frac{5.7}{2.4}\frac{x^5}{5}+\frac{5.7.9}{2.4.6}\frac{x^7}{7}+\ldots\ldots (x>1)$ (08)

c) Obtain the half-range Fourier sine series for the function (07)

$$f(x)=\begin{cases}\frac{1}{4}-x, \text{ in } 0<x<\frac{1}{2}\\ x-\frac{3}{4}, \text{ in } \frac{1}{2}<x<1\end{cases}.$$

UNIT – III

5. a) (i) Write the Parsevals identities for Fourier transforms. (02)

(ii) Find the inverse Z – transform of $\frac{z}{(z-a)^2}$ (03)

b) Find the complex Fourier transform of $f(x)=e^{-a^2x^2}$ where a is a positive constant. Hence deduce that $e^{\frac{-x^2}{2}}$ is self reciprocal in respect of complex Fourier transform. (08)

c) Solve the difference equations by using Z-transforms $u_{n+2}+4u_{n+1}+3u_n=3^n$ given that $u_0=0$; $u_1=1$. (07)

6. a) (i) Find the Z – transform of ne^{3n}. (02)

(ii) Find $f(x)$, given that $\int_0^{\infty} f(x)\,Cos(\alpha x)\,dx = e^{-\alpha}$. (03)

b) Find the Inverse Z-Transform of $\dfrac{4z^2-2z}{z^3-5z^2+8z-4}$. (08)

c) Employing Parseval's identity to the function $f(x)=\begin{cases}1-x^2 & if \quad |x|<1 \\ 0 & if \quad |x|>1\end{cases}$, (07)

Show that $\int_0^{\infty}\frac{(\sin x - x\cos x)^2}{x^6}dx=\frac{\pi}{15}$

UNIT – IV

7. a) (i) Form a PDE by eliminating arbitrary constants, from $z = a\log\left\{\frac{b(y-1)}{1-x}\right\}$. (02)

(ii) Form a partial differential equation by eliminating the arbitrary functions, given $Log\, z = f(x^2+yz)$. (03)

b) Solve $32u_t = u_{xx}$ subject to the conditions u(0,t) = 0 = u(1,t) and (08)
u(x,0) $=\begin{cases}3x, & 0\le x\le 1/2 \\ 1-x^2, & 1/2<x\le 1\end{cases}$. Compute the values of u for two time levels taking h=1/4 and k=1/2.

c) Solve $4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$ given that $u(0,y)=2e^{5y}$ by the method of separation of variables. (07)

8. a) (i) Write the explicit scheme to solve one-dimensional wave equation. (02)

(ii) Classify the PDE, $x^2u_{xx}+(1-y^2)\,u_{yy}=0, -\infty<x<\infty, -1<y<1$. (03)

b) Find the general solution of the PDE $(x^2-yz)p+(y^2-xz)q=z^2-xy$. (08)

c) Solve the PDE $\nabla^2 u=-10(x^2+y^2+10)$ over the square with sides x = 0 = y, x = 3 = y with u = 0 on the boundary and mesh length =1. Perform three iterations of Gauss-Siedel method. (07)

UNIT – V

9. a) (i) Define Basis and Dimension. (02)

(ii) Prove that the transformation T: $\mathbf{R}^2 \to \mathbf{R}^2$ given by T(x, y) = (x – y, 3x) is linear. (03)

b) Define kernel and range of a linear transformation. Verify Rank-nullity theorem for the transformation matrix $\begin{bmatrix}1 & 2 & 3\\ 0 & -1 & 1\\ 1 & 1 & 4\end{bmatrix}$. (08)

c) Find the coordinate vectors of **u** = $5x^2 + x - 3$ relative to the bases B and B′ of P_2. a) B = { x^2, x, 1 } b) B′ = { $x^2 - x + 5$, $3x^2 - 1$, $2x^2 + 4x - 2$ } (07)

10 a) (i) Define coordinate vector of a vector **u** relative to the given basis B. (02)

(ii) Define Reflection about x-axis and hence find its standard matrix. (03)

b) Show that the transformation T: $\mathbf{P_2} \rightarrow \mathbf{P_1}$ defined as $T(ax^2 + bx + c) = (a+b)x + c$ is linear. Find the image of $3x^2 - x + 2$. Find another element of $\mathbf{P_2}$ that has the same image. (08)

c) State Rank and Nullity theorem and use it to find the dimension of the kernel and range of the inear transformations defined by the matrix $\begin{bmatrix} 1 & 8 & 2 \\ 0 & 1 & -4 \\ 0 & 0 & 0 \end{bmatrix}$. (07)

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