

**M S RAMAIAH INSTITUTE OF TECHNOLOGY**

(AUTONOMOUS INSTITUTE, AFFILIATED TO VTU)

BANGALORE – 560 054

SEMESTER END EXAMINATIONS – January 2013**Course & Branch : B.E.- (Information Science and Engineering)****Semester : III****Subject : Engineering Mathematics-III****Max. Marks : 100****Subject Code : ISMAT301****Duration : 3 Hrs****Instructions to the Candidates:**

- Answer one full question from each unit.

UNIT – I

1. a) From the following table, estimate the number of students who obtained marks between 40 and 45. (06)

Marks	30-40	40 - 50	50 - 60	60-70	70-80
No. of students	31	42	51	35	31

- b) Determine $f(x)$ as the polynomial in x for the following data, using the Lagrange's interpolation formula and hence find $f(3)$. (07)

x	0	1	2	5
y	2	3	12	147

- c) Use Simpson's 1/3 rule and 3/8 rule to evaluate $\int_0^1 \frac{dx}{1+x^2}$ considering seven equidistant ordinates. Hence find the approximate value of π in each case. (07)

2. a) Given $y_0 = -4, y_1 = -2, y_4 = 220, y_5 = 546, y_6 = 1148$ then find y_2 and y_3 corresponding to the equidistant values of x . (06)

- b) Find the equation of the cubic curve for the following data using Newton's divided difference formula and hence find $f(10)$. (07)

x	4	7	9	12
y	-43	83	327	1053

- c) Compute $y'(2)$ and $y'(4.5)$ for the data given below using Newton's backward interpolation formula. (07)

x	-2	-1	0	1	2	3
y	0	0	6	24	60	120

UNIT – II

3. a) Discuss the nature of the series $\frac{x}{1.2} + \frac{x^2}{2.3} + \frac{x^3}{3.4} + \dots$ (06)



- b) Obtain the Fourier series for the function $f(x) = |x|$, over the interval $-\pi < x < \pi$ and hence deduce that $\frac{\pi^2}{8} = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2}$. (07)
- c) Obtain the Fourier series for the function y up to the second harmonic given by the following table (07)
- | | | | | | | |
|------|---|---|----|---|---|---|
| $x:$ | 0 | 1 | 2 | 3 | 4 | 5 |
| $y:$ | 4 | 8 | 15 | 7 | 6 | 2 |
4. a) Find the half range Sine series for the function $f(x) = \begin{cases} \frac{1}{4} - x, & 0 < x < 1/2 \\ x - \frac{3}{4}, & 1/2 < x < 1 \end{cases}$. (06)
- b) Obtain the Fourier series for the function $f(x) = 2x - x^2$ in $0 \leq x \leq 2$ and hence deduce that $\frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$. (07)
- c) Obtain the Fourier series for the function y up to the first harmonic from the following table (07)
- | | | | | | | | | |
|------------|---|-----|----|-----|-----|-----|-----|-----|
| $x^\circ:$ | 0 | 45 | 90 | 135 | 180 | 225 | 270 | 315 |
| $y:$ | 2 | 3/2 | 1 | 1/2 | 0 | 1/2 | 1 | 3/2 |

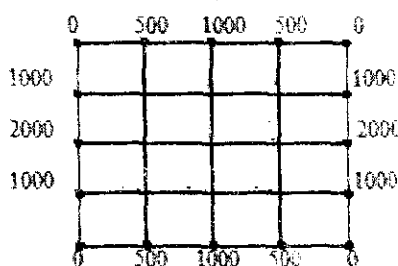
UNIT - III

5. a) Find the Fourier Sine transform of $e^{-|x|}$ and hence evaluate $\int_0^{\infty} \frac{x \sin ax}{1+x^2} dx$, $a > 0$. (06)
- b) Find the complex Fourier transform of $f(x) = e^{-a^2 x^2}$ where 'a' is a positive constant. Hence deduce that $e^{-x^2/2}$ is self reciprocal in respect of complex Fourier transform (07)
- c) Solve the difference equation $y_{n+2} - 4y_n = 0$ with $y_0 = 0$, $y_1 = 2$ by using Z - transforms. (07)
6. a) Find the inverse Z - transforms of the following. (06)
- (i) $z \left(e^z - 1 \right)$ (ii) $\frac{3z^2 + z}{(5z-1)(5z+2)}$
- b) Prove that $Z_T(n^p) = -z \frac{d}{dz} [Z_T(n^{p-1})]$ and hence obtain the Z - transform of n and n^2 . (07)
- c) Employing Parseval's identity to the function $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$, Show that (07)
- $$\int_0^{\infty} \frac{\sin^2 x}{x^2} dx = \frac{\pi}{2}$$



UNIT - IV

7. a) Form a partial differential equation by eliminating the arbitrary functions (06)
from:
i) $\phi(x+y+z, x^2+y^2-z^2)=0$ ii) $z=f(x+iy)+\phi(x-iy)$
- b) Find the general solution of the equation (07)
 $x^2(y-z)p+y^2(z-x)q=z^2(x-y)$
- c) Solve the equation $u_t = u_{xx}$ subject to the conditions $u(x,0)=\sin \pi x$, $0 \leq x \leq 1$; (07)
 $u(0,t)=u(1,t)=0$, using Schmidt method. Carryout computations for two
levels taking $h=1/3$ and $k=1/36$.
8. a) Solve $(y^2+z^2-x^2)p-2xyq+2xz=0$ (06)
- b) Solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, given that $u(x,0)=6e^{-3x}$ by method of separation of (07)
variables.
- c) Solve the Laplace equation $u_{xx} + u_{yy} = 0$ in the square region shown in the (07)
following figure, with the boundary values as indicated in the figure carry
out three iterations.



UNIT - V

9. a) Determine whether the set $\{x^2+3x-1, x+3, 2x^2-x+1\}$ is linearly independent (06)
over the vector space P_2 of polynomials of degree less than or equal to 2.
- b) Prove that the set $\{(1,0,-1), (1,1,1), (1,2,4)\}$ of vectors is a basis for the vector (07)
space \mathbb{R}^3 .
- c) Determine the kernel and range of the transformation defined by the (07)
following matrix.
- $$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 1 & 1 & 4 \end{bmatrix}$$
10. a) Prove that the following transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is Linear. (06)
 $T(x,y) = (2x, x+y)$.
- b) Define coordinate vector. Also determine whether the vector $(-3,3,7)$ is a (07)
linear combination of the vectors $(1,-1,2)$, $(2,1,0)$ and $(-1,2,1)$.
- c) Define dilation and contraction of a linear transformation. Find the matrix A (07)
of a linear operator $T(X)=AX$ from $\mathbb{R}^2 \rightarrow \mathbb{R}^2$ that rotates a vector X through an
angle θ about the origin.
