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**IS314**

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# M S RAMAIAH INSTITUTE OF TECHNOLOGY

(AUTONOMOUS INSTITUTE, AFFILIATED TO VTU)

BANGALORE – 560 054

## SEMESTER END EXAMINATIONS –JANUARY 2016

| | | | | | |
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| **Course & Branch** | : | **B.E.- Information Science & Engg.** | **Semester** | : | **III** |
| **Subject** | : | **Discrete Mathematical Structures** | **Max. Marks** | : | **100** |
| **Subject Code** | : | **IS314** | **Duration** | : | **3 Hrs** |

**Instructions to the Candidates:**

- Answer one full question from each unit.

### UNIT – I

1. a) Which of the following set are equal to the null set? C01 (04)

(i) $\{x \mid x \in Z,\ 3x+5=9\}$ (ii) $\{x \mid x \in R,\ x^3=-1\}$

(iii) $\{x \mid x \in Z,\ x^2+4=6\}$ (iv) $\{x \mid x \in R,\ x=x+1\}$

b) For any sets A,B,C,D prove that C01 (08)

i) $(A \cap B) \cup (A \cap B \cap \overline{C} \cap D) \cup (\overline{A} \cap B) = B$

ii) $(A-B)-C = A-(B \cup C) = (A-C)-(B-C)$

c) A survey of a sample of 25 new cars being sold by an auto-dealer was conducted to see which of the three popular options: AC, radio and power windows, were already installed. The survey found: 15 had AC, 12 had radio, 11 had power windows, 5 had AC & power windows, 9 had AC & radio, 4 had radio & power window, and 3 had all three options. Find the number of cars that had: i) only power window ii) atleast one option iii)only one of the options iv) none of the options. C01 (08)

2 a) i) Determine whether the following argument is a valid argument. C01 (12)

I will get grade A in this course or I will not graduate

If I do not graduate, I will join the army

I got grade A

$\therefore$ I will not join the army

ii) Prove that the following statement is true using mathematical induction.

$1+2^1+2^2+\ldots\ldots\ldots\ldots\ldots+2^n=2^{n+1}-1$

b) For the universe of all integers, let C01 (04)

p(x): x>0 r(x): x is a perfect square

q(x): x is even s(x): x is divisible by 3

Write down the following quantified statements in symbolic form:

i) At least one integer is even.

ii) Every integer is either even or odd.

iii) Some even integers are divisible by 3.

iv) If x is even and a perfect square, then x is not divisible by 3.

c) Construct truth table to determine whether the given statement is tautology, contingency or absurdity. C01 (04)

$(p \rightarrow q) \Leftrightarrow (\sim q \rightarrow \sim p)$

## UNIT-II

3. a) Find the explicit formula for the sequence defined by $f_n = f_{n-1} + f_{n-2},\ f_1 = 1,\ f_2 = 1$ C02 (08)

b) State the pigeon hole principle. What is the minimum number of students required in a DMS class to be sure that at least six will receive the same grade, if there are 5 possible grades A, B, C, D and F. C02 (06)

c) State the multiplication principle of counting. Suppose that there are 9 faculty members in the maths department and 11 in ISE department, How many ways are there to select a committee to develop a DMS course if the committee is to consist of 3 faculty members from the maths department and 4 from the ISE department. C02 (06)

4. a) Let A = {1,3,5,7,9} and B={2,4,6,8}; $_aR_b$ iff b<a. C02 (06)
Find (i)set R (ii) Domain (R) (iii) Range(R) (iv) $M_R$ (v) R(9) (vi) R({3,7})

b) Let A = Z and relation R is defined by $R = \{(a,b) \in A \times A \mid a \equiv b (\mathrm{mod}\ 2)\}$. Determine whether the given relation R is an equivalence relation. C02 (06)

c) What is transitive closure of a relation? Compute the transitive closure of the relation given below using Warshall's algorithm. C02 (08)

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

## UNIT-III

5. a) Let A={1,2,3,4,5,6} and P = $\begin{pmatrix} 1\,2\,3\,4\,5\,6 \\ 4\,3\,5\,1\,2\,6 \end{pmatrix}$ be a permutation of A. C03 (10)

i) Write p as a product of disjoint cycles.
ii) Compute $p^{-1}$
iii) Compute $p^2$
iv) Determine whether the given permutation is even or odd
v) Find the smallest positive integer k such that $p^k = I_A$

b) Let f and g be the fuzzy sets whose definition are given below: C03 (06)

$$f(x) = \begin{cases} \frac{1}{9}x + \frac{2}{3} & \text{for } 0 \le x < 3 \\ 1 - \frac{(x-3)^2}{4} & \text{for } 3 \le x < 5 \\ 0 & \text{for } 5 \le x < 7 \\ \frac{(x-7)^2}{4} & \text{for } 7 \le x < 9 \\ -\frac{2}{3}x + 7 & \text{for } 9 \le x \le 10 \end{cases}$$

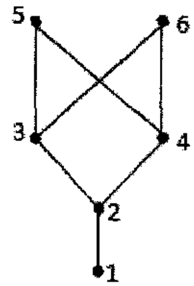
$$g(x) = \begin{cases} 0 & \text{for } 0 \le x < 2 \\ \frac{1}{2}x - 1 & \text{for } 2 \le x < 4 \\ -x+5 & \text{for } 4 \le x < 5 \\ 0 & \text{for } 5 \le x < 7 \\ x - 7 & \text{for } 7 \le x < 8 \\ -\frac{1}{2}x + 5 & \text{for } 8 \le x \le 10 \end{cases}$$

Find the degree of membership of 3.5 in (i) $f \cap g$ (ii) $f \cup g$ (iii) $\overline{f}$

c) Let f: A→B be any function. Then prove that: C03 (04)
(i) $I_B of = f$ (ii) $f^{-1} of = I_A$

6. a) What is partially ordered set? If $(A, \leq)$ and $(B, \leq)$ are posets, then prove that $(A X B, \leq)$ is a poset with the partial order $\leq$ defined by $(a,b) \leq (a',b')$ if $a \leq a'$ in A and $b \leq b'$ in B — C03 (08)

b) For the poset shown in the Hasse diagram, find the following for the set $B=\{3,4,5\}$ : — C03 (08)
(i) All upper bounds (ii) All lower bounds (iii) LUB (iv) GLB

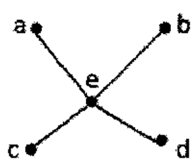



c) Let A be a finite nonempty poset with partial order $\leq$. Then prove that A has at least one maximal element and at least one minimal element. — C03 (04)

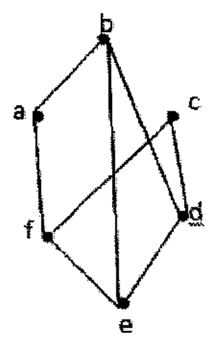
## UNIT-IV

7. a) Explain any two applications of graph. — C04 (06)

b) Determine whether the following graphs are bipartite. — C04 (06)




(i)



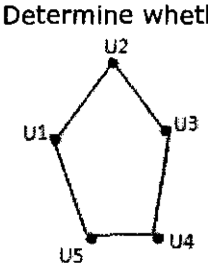

(ii)

c) Define the following with one example for each. — C04 (08)
i) Wheel
ii) Incidence matrix
iii) Hamilton path
iv) Euler circuit

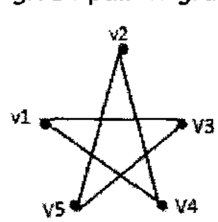
8. a) Define binary operation on a set. Determine whether the * is a valid definition of a binary operation on the set. — C04 (04)
i) On Z, where a*b is $a^b$
ii) on $Z^+$, where a*b is a-b

b) Let T be the set of all even integers. Show that the semigroups (Z, +) and (T, +) are isomorphic. — C04 (08)

c) Determine whether the given pair of graphs G and H is isomorphic. — C04 (08)




G




H

## UNIT-V

| | | | | |
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| 9. | a) | Let G be a group and let a and b element of G. Then prove that: (i) $\left(a^{-1}\right)^{-1} = a$ (ii) $(ab)^{-1} = b^{-1}a^{-1}$ | C05 | (06) |
| | b) | Let G be the set of real numbers not equal to -1 and * be defined by a*b=a+b+ab. Prove that (G,*) is an abelian group. | C05 | (08) |
| | c) | Find the multiplicative inverse of $\overline{25}$ in $Z_{384}$. | C05 | (06) |
| 10. | a) | State Fermat's Little theorem. What is the remainder when $7^{293}$ is divided by 65. | C05 | (06) |
| | b) | Consider the (3, 9) encoding function e: | C05 | (07) |
| | c) | Consider the following parity check matrix H and determine the (3,6) group code $e_H: B^3 \to B^6$. | C05 | (07) |

Question 10 b), continued:

e(000)=000000000    e(100)=010011010
e(001)=011100101    e(101)=111101011
e(010)=010101000    e(110)=001011000
e(011)=110010001    e(111)=110000111

i) Find the minimum distance of e.

ii) How many errors will e detect.

Question 10 c), continued:

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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