

# On The Capacity of Opportunistic Cooperative Networks under Adaptive Transmission

Vo Nguyen Quoc Bao\*, Hyung Yun Kong\*, Asaduzzaman\*, Jin-Hee Lee\* and Ji-Hwan Park\*

\*School of Electrical Engineering

University of Ulsan, San 29 of MuGeo Dong, Nam-Gu, Ulsan, Korea 680-749

Email: {baovnq,hkong,asad78,tajimania,hazarders79}@mail.ulsan.ac.kr

**Abstract**—This paper offers a study on Rayleigh channel capacity of opportunistic cooperative networks in conjunction with adaptive transmission where the source adapts its rate and/or power level according to the changing channel conditions while the best relay simply amplifies and forwards the signals. To this end, the upper and lower bound of the end-to-end effective signal-to-noise ratio (SNR) for an arbitrary number of cooperative relays equipped amplify-and-forward protocol is derived and then used to derive the closed-form expressions of the system channel capacity under three different adaptive policies: optimal simultaneous power and rate adaptation (OPRA), constant power with optimal rate adaptation (OPA) and channel inversion with fixed rate (TIFR). Our results are verified through comparison with Monte Carlo simulations where we also illustrate that, among them, for an arbitrary number of relays, TIFR gives the worst channel capacity; OPRA gives the best channel capacity and ORA has a channel capacity quality in between the others.

## I. INTRODUCTION

Adaptive transmission is a prominent and powerful technique by which wireless communication systems can take advantage of the time-varying nature of wireless channels to enhance spectral efficiency without having to sacrifice error rate performance. The key idea is that the transmitter is real-time balancing of the link budget through adaptive variation of the transmitted power level, symbol transmission rate, constellation size, coding rate/scheme, or any combination of these parameters [1], [2].

Recently, this concept has gained new actuality in the context of cooperative wireless communication networks [3], [4], [5]. In particular, in [3], the use of constant-power, rate-adaptive  $M$ -QAM transmission with an amplify-and-forward (AF) cooperative system is proposed and investigated in terms of outage probability, achievable spectral efficiency, and error rate indicating that the system with adaptive discrete rate  $M$ -QAM approaches the Shannon capacity within 5 dB for both independent and identically distributed (i.i.d.) and independent but not identically distributed (i.n.d.) Rayleigh fading channels. In [4], Hwang et. al. proposes an incremental opportunistic relaying with adaptive modulation in which variable-rate transmission is adopted in conjunction with opportunistic incremental relaying. Also in this paper, the authors show that this scheme provides a certain improvement of the spectral efficiency and outage probability while satisfying the required bit error rate (BER) performance. In [5], Tyler et. al. propose the use of repetition-based amplify-and-forward relaying systems with adaptive transmission techniques in which only

the source adapts its rate and/or power level according to the equivalent end-to-end SNR, while the relays simply amplify and forward the signals. However, this cooperation strategy suffers from one disadvantage – a loss in spectral efficiency as multiple orthogonal channel allocations (equals number of relays plus one) are required to transmit one data packet.

Motivated by all of the above, in this paper, we provide a capacity analysis of Rayleigh fading channel for AF opportunistic cooperative networks under adaptive transmission by applying the seminal theory developed in [1], [2]. In particular, the capacity of the opportunistic cooperative network under three different adaptive transmission policies, namely optimal simultaneous power and rate adaptation, constant power with optimal rate adaptation and channel inversion with fixed rate is studied. Because of the relatively complicated statistics of AF opportunistic cooperative relaying, the probability density function (PDF) of the upper and lower bounds of the end-to-end SNR are derived and then used to evaluate the system capacity.

The remainder of this paper is organized as follows. In Section II, the system model under investigation and the upper and lower bounds of the effective end-to-end SNR expressed in a tractable form are provided. In Section III, the optimum and sub-optimum adaptation policies for AF opportunistic cooperative relaying networks are studied in terms of Rayleigh fading capacity. Finally, numerical results are given in Section IV and conclusions are drawn in Section V.

## II. SYSTEM MODEL

Consider a distributed wireless cooperative network in which the source node  $S$  communicates with the destination node  $D$  with the help of  $N$  relay nodes  $R_1, \dots, R_k, \dots, R_N$  employing amplify-and-forward protocol as illustrated in [5, Fig. 1]. Each node is equipped with single antenna and operates in half-duplex mode. All transmissions are assumed orthogonal either in time or in frequency.

Let us denote the channel gains corresponding to the links of the  $S \rightarrow D$ ,  $S \rightarrow R_k$  and  $R_k \rightarrow D$  as  $h_{SD}$ ,  $h_{SR_k}$  and  $h_{R_kD}$ , respectively. Due to Rayleigh fading, the channel powers, denoted by  $\alpha_0 = |h_{SD}|^2$ ,  $\alpha_{1,k} = |h_{SR_k}|^2$  and  $\alpha_{2,k} = |h_{R_kD}|^2$  are exponential random variables whose means are  $\lambda_0$ ,  $\lambda_{1,k}$  and  $\lambda_{2,k}$ , respectively.

To facilitate the explanation, we assume that the cooperative transmission under consideration takes places into two time

slots. In the first time slot, the source broadcasts its symbol  $s$  with an average transmitted power  $\mathcal{P}_s$  to all the relays and the destination. The source adopts the relay selection scheme where the "best" relay is chosen a priori according to the distributed timer fashion [6], [7] with minimum signaling overhead. Specifically, the relay with shortest timer expires first and thus becomes the transmitter (also denoted as the "best" relay) in the second time slot while other relays discard the received signals after receiving this node transmission. Since AF relaying is employed, the best relay, indexed by  $k^*$ , simply amplifies the received signals with a fixed gain  $\mathcal{G}$  and forwards the resultant signal to D. With the assumption that  $R_{k^*}$  transmits the same amount of power as S and the additive white Gaussian noise at all terminals is zero mean and variance of  $\mathcal{N}_0$ , the system model can be described mathematically by the following set of equations as follows:

$$y_{SD} = \sqrt{\mathcal{P}_s} h_{SD} s + n_{SD} \quad (1a)$$

$$y_{SR_{k^*}} = \sqrt{\mathcal{P}_s} h_{SR_{k^*}} s + n_{SR_{k^*}} \quad (1b)$$

$$y_{R_{k^*}D} = \sqrt{\mathcal{P}_s} h_{R_{k^*}D} (\mathcal{G}_i y_{SR_{k^*}}) + n_{R_{k^*}D} \quad (1c)$$

where  $\mathcal{G} = \sqrt{1/[\mathcal{P}_s |h_{SR_{k^*}}|^2 + \mathcal{N}_0]}$ .  $n_{SD}$  and  $n_{SR_{k^*}}$  as well as  $n_{R_{k^*}D}$  are the additive noise. Let us define the effective instantaneous SNRs for  $S \rightarrow D$ ,  $S \rightarrow R_k$  and  $R_k \rightarrow D$  links as  $\gamma_0 = \mathcal{P}_s |h_{SD}|^2$ ,  $\gamma_{1,k} = \mathcal{P}_s |h_{SR_k}|^2$  and  $\gamma_{2,k} = \mathcal{P}_s |h_{R_kD}|^2$ , respectively. Thus, the instantaneous dual-hop SNR of the "best" relay at the destination is  $\gamma_{k^*} = \arg \max_{1 \leq k \leq N} \gamma_k$  where  $\gamma_k$  with  $k = 1, \dots, N$  can be shown to be

$$\gamma_k = \frac{\gamma_{1,k} \gamma_{2,k}}{\gamma_{1,k} + \gamma_{2,k} + 1} \xrightarrow{\mathcal{P}_s \rightarrow \infty} \frac{\gamma_{1,k} \gamma_{2,k}}{\gamma_{1,k} + \gamma_{2,k}} \quad (2)$$

Furthermore, for tractable analysis, (2) can be upper- and lower-bounded as follows [8], [9]:

$$\frac{1}{2} \min(\gamma_{1,k}; \gamma_{2,k}) \triangleq \gamma_k^L \leq \gamma_k < \gamma_k^U \triangleq \min(\gamma_{1,k}; \gamma_{2,k}) \quad (3)$$

Since  $\gamma_{1,k}$  and  $\gamma_{2,k}$  are exponentially distributed random variables with hazard rates  $\mu_{1,k} = 1/\bar{\gamma}_{1,k} = 1/(\mathcal{P}_s \lambda_{1,k})$  and  $\mu_{2,k} = 1/\bar{\gamma}_{2,k} = 1/(\mathcal{P}_s \lambda_{2,k})$ , respectively. Making use the fact that the minimum of two independent exponential random variables is again an exponential random variable with a hazard rate equals to the sum of the two hazard rates [10], i.e.,  $\mu_k = \mu_{1,k} + \mu_{2,k} = \frac{\bar{\gamma}_{1,k} + \bar{\gamma}_{2,k}}{\bar{\gamma}_{1,k} \bar{\gamma}_{2,k}}$ . For brevity, by introducing  $\bar{\gamma}_k = \frac{\mathcal{K}}{\mu_k} = \mathcal{K} \frac{\bar{\gamma}_{1,k} \bar{\gamma}_{2,k}}{\bar{\gamma}_{1,k} + \bar{\gamma}_{2,k}}$  and from (3), we have

$$f_{\gamma_k}(\gamma) = \frac{1}{\bar{\gamma}_k} e^{-\gamma/\bar{\gamma}_k}; F_{\gamma_k}(\gamma) = \int_0^\gamma f_{\gamma_k}(\gamma) d\gamma = 1 - e^{-\gamma/\bar{\gamma}_k} \quad (4)$$

where  $\mathcal{K}$  equals 1 or 1/2 associated with the cases of upper bound and lower bound in (3), respectively. Under the assumption that all links are subject to independent fading, order statistics gives the cumulative distribution function (CDF) of  $\gamma_{k^*}$  as:

$$F_{\gamma_{k^*}}(\gamma) = \Pr(\gamma_1 < \gamma, \dots, \gamma_N < \gamma) = \prod_{k=1}^N F_{\gamma_k}(\gamma) \quad (5)$$

Hence, the joint PDF of  $\gamma_{k^*}$  is given by differentiating (5) with respect to  $\gamma$ .

$$f_{\gamma_{k^*}}(\gamma) = \frac{\partial F_{\gamma_{k^*}}(\gamma)}{\partial \gamma} = \sum_{k=1}^N \left[ f_{\gamma_k}(\gamma) \prod_{j=1, j \neq k}^N F_{\gamma_j}(\gamma) \right] \quad (6)$$

Substituting (4) into (6) and after some manipulation yields [11]

$$f_{\gamma_{k^*}}(\gamma) = \sum_{k=1}^N (-1)^{k-1} \sum_{\substack{n_1, \dots, n_k=1 \\ n_1 < \dots < n_k}}^N \frac{1}{\chi_k} e^{-\frac{\gamma}{\chi_k}} \quad (7)$$

where  $\chi_k = \left( \sum_{l=1}^k \bar{\gamma}_{n_l}^{-1} \right)^{-1}$ . Since maximal ratio combining technique is employed at the destination, the combined instantaneous SNR at the output of the maximal ratio combiner is given by

$$\gamma_\Sigma = \gamma_0 + \gamma_{k^*} \quad (8)$$

Assuming the independence of  $\gamma_0$  and  $\gamma_{k^*}$ , the moment generating function (MGF) of  $\gamma_\Sigma$  can be written as

$$M_{\gamma_\Sigma}(s) = M_{\gamma_0}(s) M_{\gamma_{k^*}}(s) \quad (9)$$

where  $M_{\gamma_0}(s)$  and  $M_{\gamma_{k^*}}(s)$  are MGF functions of  $\gamma_0$  and  $\gamma_{k^*}$ , respectively. Using the definition of the MGF as  $M_\gamma(s) = E\{e^{s\gamma}\}$  [11] where  $E(\cdot)$  is the statistical average operator, we have

$$M_{\gamma_0}(s) = (1 - s\bar{\gamma}_0)^{-1} \quad (10a)$$

$$M_{\gamma_{k^*}}(s) = \sum_{k=1}^N (-1)^{k-1} \sum_{\substack{n_1, \dots, n_k=1 \\ n_1 < \dots < n_k}}^N (1 - s\chi_k)^{-1} \quad (10b)$$

where  $\bar{\gamma}_0 = \mathcal{P}_s \lambda_0$ . Using the partial fraction expansion of the MGF, (9) can be shown that

$$\begin{aligned} M_{\gamma_\Sigma}(s) &= (1 - s\bar{\gamma}_0)^{-1} \sum_{k=1}^N (-1)^{k-1} \sum_{\substack{n_1, \dots, n_k=1 \\ n_1 < \dots < n_k}}^N (1 - s\chi_k)^{-1} \\ &= \sum_{k=1}^N (-1)^{k-1} \sum_{\substack{n_1, \dots, n_k=1 \\ n_1 < \dots < n_k}}^N m(\bar{\gamma}_0, \chi_k) \end{aligned} \quad (11)$$

where  $m(\bar{\gamma}_0, \chi_k)$  is defined as

$$m(\bar{\gamma}_0, \chi_k) = \begin{cases} \frac{\bar{\gamma}_0}{\bar{\gamma}_0 - \chi_k} (1 - s\bar{\gamma}_0)^{-1} + \frac{\chi_k}{\chi_k - \bar{\gamma}_0} (1 - s\chi_k)^{-1}, & \chi_k \neq \bar{\gamma}_0 \\ (1 - s\bar{\gamma}_0)^{-2}, & \chi_k = \bar{\gamma}_0 \end{cases}$$

Finally, the joint PDF of  $\gamma_\Sigma$  is determined by the inverse Laplace transforms of  $M_{\gamma_\Sigma}(s)$  as follows:

$$f_{\gamma_\Sigma}(\gamma) = \sum_{k=1}^N (-1)^{k-1} \sum_{\substack{n_1, \dots, n_k=1 \\ n_1 < \dots < n_k}}^N p(\bar{\gamma}_0, \chi_k) \quad (12)$$

with

$$p(\bar{\gamma}_0, \chi_k) = \begin{cases} \left( \frac{\bar{\gamma}_0}{\bar{\gamma}_0 - \chi_k} \right) \frac{1}{\bar{\gamma}_0} e^{-\frac{\gamma}{\bar{\gamma}_0}} + \left( \frac{\chi_k}{\chi_k - \bar{\gamma}_0} \right) \frac{1}{\chi_k} e^{-\frac{\gamma}{\chi_k}}, & \chi_k \neq \bar{\gamma}_0 \\ \frac{\gamma}{\bar{\gamma}_0^2} e^{-\frac{\gamma}{\bar{\gamma}_0}}, & \chi_k = \bar{\gamma}_0 \end{cases}$$

### III. CAPACITY ANALYSIS

With the PDF of  $\gamma_\Sigma$ , expressed under a tractable form, in hand and using the same approach discussed in [2], we can now evaluate the system capacity under different adaptive transmission policies over Rayleigh fading channels as follows.

#### A. Optimal Simultaneous Power and Rate Adaptation

Since the equivalent end-to-end channel state information (CSI) is tracked perfectly by the destination and then sent back to the source via an error-free delayless feedback link, it therefore allows the source to adapt both its power and rate to the actual channel condition. More specifically, the source allocates high power levels and rates for good channel conditions ( $\gamma_\Sigma$  large), and lower power levels and rates for unfavorable channel conditions ( $\gamma_\Sigma$  small). The achievable channel capacity of opportunistic cooperative networks over Rayleigh fading channels is given by [2, eq. 7]

$$C_{opra} = \frac{B}{2} \int_{\gamma_c}^{+\infty} \log_2 \left( \frac{\gamma}{\gamma_c} \right) f_{\gamma_\Sigma}(\gamma) d\gamma \quad (13)$$

where  $B$  is the channel bandwidth in Hz and  $\gamma_c$  is the optimal cutoff SNR threshold below which data transmission over the network is halted. The ratio 1/2 in (13) is included to reflect that the source-to-destination information transmission via relays will occupy two time slots. The optimal cutoff threshold is determined by using the total average power constraint [2, eq. 8].

$$\int_{\gamma_c}^{+\infty} \left( \frac{1}{\gamma_c} - \frac{1}{\gamma} \right) f_{\gamma_\Sigma}(\gamma) d\gamma = 1 \quad (14)$$

To appreciate  $\gamma_c$  in (13), we insert (12) into (14) and then numerically solve the resulting equation (after some manipulations) given by

$$\sum_{k=1}^N (-1)^{k-1} \sum_{\substack{n_1, \dots, n_k=1 \\ n_1 < \dots < n_k}} \Omega(\bar{\gamma}_0, \chi_k) = 1 \quad (15)$$

with

$$\Omega(\bar{\gamma}_0, \chi_k) = \begin{cases} \frac{\frac{\bar{\gamma}_0}{\gamma_c} e^{-\frac{\bar{\gamma}_0}{\gamma_c}} - E_1\left(\frac{\bar{\gamma}_0}{\gamma_c}\right)}{\bar{\gamma}_0 - \chi_k} + \frac{\frac{\chi_k}{\gamma_c} e^{-\frac{\chi_k}{\gamma_c}} - E_1\left(\frac{\chi_k}{\gamma_c}\right)}{\chi_k - \bar{\gamma}_0}, & \chi_k \neq \bar{\gamma}_0 \\ \left(1 + \frac{\gamma_c}{\bar{\gamma}_0} - \frac{1}{\gamma_c}\right) e^{-\frac{\gamma_c}{\bar{\gamma}_0}}, & \chi_k = \bar{\gamma}_0 \end{cases}$$

where  $E_n(x) = \int_1^{+\infty} t^{-n} e^{-xt} dt$ ,  $x > 0$  is the exponential integral of order [13, p. 228, eq. (5.1.4)]. Substituting (12) into (13), we obtain the channel capacity for OPRA policy in terms of the integral  $\mathcal{J}_n(\mu) = \int_1^{+\infty} t^{n-1} \ln t e^{-\mu t} dt$  [2, eq. (70)] as

$$C_{opra} = \frac{B}{2 \ln 2} \sum_{k=1}^N (-1)^{k-1} \sum_{\substack{n_1, \dots, n_k=1 \\ n_1 < \dots < n_k}} c_{opra}(\bar{\gamma}_0, \chi_k) \quad (16)$$

with

$$c_{opra}(\bar{\gamma}_0, \chi_k) = \begin{cases} \frac{\gamma_c}{\bar{\gamma}_0 - \chi_k} \mathcal{J}_1\left(\frac{\gamma_c}{\bar{\gamma}_0}\right) + \frac{\gamma_c}{\chi_k - \bar{\gamma}_0} \mathcal{J}_1\left(\frac{\gamma_c}{\chi_k}\right), & \chi_k \neq \bar{\gamma}_0 \\ \left(\frac{\gamma_c}{\bar{\gamma}_0}\right)^2 \mathcal{J}_2\left(\frac{\gamma_c}{\chi_k}\right), & \chi_k = \bar{\gamma}_0 \end{cases}$$

According to the operation mode of optimal simultaneous power and rate adaptation, the system will stop transmitting when  $\gamma_\Sigma < \gamma_c$ . Mathematically speaking, the system suffers an outage probability, equal to the probability of the end-to-end SNR falls below the cutoff SNR, given by

$$\begin{aligned} P_o &= \Pr(\gamma_\Sigma < \gamma_c) = \int_0^{\gamma_c} f_{\gamma_\Sigma}(\gamma) d\gamma \\ &= \sum_{k=1}^N (-1)^{k-1} \sum_{\substack{n_1, \dots, n_k=1 \\ n_1 < \dots < n_k}} p_o(\bar{\gamma}_0, \chi_k) \end{aligned} \quad (17)$$

with

$$p_o(\bar{\gamma}_0, \chi_k) = \begin{cases} \left( \frac{\bar{\gamma}_0}{\bar{\gamma}_0 - \chi_k} \right) \left( 1 - e^{-\frac{\bar{\gamma}_0}{\gamma_c}} \right) + \left( \frac{\chi_k}{\chi_k - \bar{\gamma}_0} \right) \left( 1 - e^{-\frac{\chi_k}{\gamma_c}} \right), & \chi_k \neq \bar{\gamma}_0 \\ 1 - \left( 1 + \frac{\gamma_c}{\bar{\gamma}_0} \right) e^{-\frac{\gamma_c}{\bar{\gamma}_0}}, & \chi_k = \bar{\gamma}_0 \end{cases}$$

#### B. Optimal Rate Adaptation with Constant Transmit Power

Under the condition of optimal rate adaptation with constant transmit power, the capacity of Rayleigh fading opportunistic cooperative systems with an arbitrary number of relays is given [2, eq. (29)] as

$$\begin{aligned} C_{ora} &= \frac{B}{2} \int_0^{+\infty} \log_2(1 + \gamma) f_{\gamma_\Sigma}(\gamma) d\gamma \\ &= \frac{B}{2 \ln 2} \sum_{k=1}^N (-1)^{k-1} \sum_{\substack{n_1, \dots, n_k=1 \\ n_1 < \dots < n_k}} c_{ora}(\bar{\gamma}_0, \chi_k) \end{aligned} \quad (18)$$

with

$$c_{ora}(\bar{\gamma}_0, \chi_k) = \begin{cases} \frac{\mathcal{I}_1(1/\bar{\gamma}_0)}{\bar{\gamma}_0 - \chi_k} + \frac{\mathcal{I}_1(1/\chi_k)}{\chi_k - \bar{\gamma}_0}, & \chi_k \neq \bar{\gamma}_0 \\ \frac{\mathcal{I}_2(1/\bar{\gamma}_0)}{\bar{\gamma}_0^2}, & \chi_k = \bar{\gamma}_0 \end{cases}$$

where  $\mathcal{I}_n(\mu) = \int_0^{+\infty} t^{n-1} \ln(1+t) e^{-\mu t} dt$ ,  $\mu > 0$  [2, eq. (78)].

#### C. Channel Inversion with Fixed Rate

In order to maintain a constant received SNR at the destination, the source adapts its transmit power based on the actual channel fading, i.e., it inverts the channel fading. As previously discussed in [1], [2], the power adaptation is commonly used and the related channel capacity is given by [2, eq. (46)]

$$C_{tifr} = \frac{B}{2} \log_2 \left[ 1 + \left( \int_0^{+\infty} \frac{f_{\gamma_\Sigma}(\gamma)}{\gamma} d\gamma \right)^{-1} \right] \quad (19)$$

This technique has the advantage of maintaining a fixed data rate over the channel regardless of channel conditions, however, results in a large capacity loss compared with other adaptation techniques because a large amount of transmitted

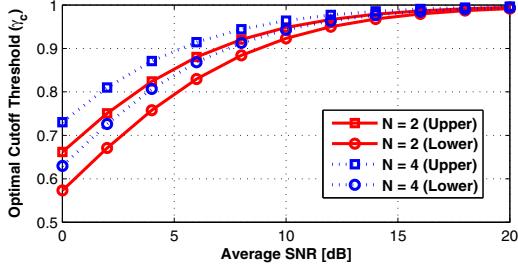


Fig. 1. Optimal cutoff threshold versus average SNR for OPRA.

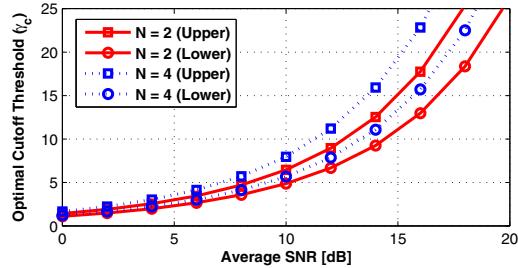


Fig. 2. Optimal cutoff threshold versus average SNR for TIFR.

power is used to compensate for the deep channel fades. In order to achieve a better channel capacity, a modified version of this policy in which the transmission is suspended in unfavorable channel conditions ( $\gamma_{\Sigma} < \gamma_c$ ) is proposed leading to [2, eq. (47)]

$$C_{tifr} = \frac{B}{2} \log_2 \left[ 1 + \left( \int_{\gamma_c}^{+\infty} \frac{f_{\gamma_{\Sigma}}(\gamma)}{\gamma} d\gamma \right)^{-1} \right] (1 - P_o) \\ = \frac{B}{2} \log_2 \left[ 1 + \left( \sum_{k=1}^N (-1)^{k-1} \sum_{\substack{n_1, \dots, n_k=1 \\ n_1 < \dots < n_k}} c_{tifr}(\bar{\gamma}_0, \chi_k) \right)^{-1} \right] (1 - P_o) \quad (20)$$

where  $P_o$  has the same form of (17) and  $c_{tifr}$  is given by

$$c_{tifr}(\bar{\gamma}_0, \chi_k) = \begin{cases} \frac{E_1(\gamma_c/\bar{\gamma}_0)}{\bar{\gamma}_0 - \chi_k} + \frac{E_1(\gamma_c/\chi_k)}{\chi_k - \bar{\gamma}_0}, & \chi_k \neq \bar{\gamma}_0 \\ \frac{1}{\bar{\gamma}_0} e^{-\frac{\gamma_c}{\bar{\gamma}_0}}, & \chi_k = \bar{\gamma}_0 \end{cases}$$

The cutoff threshold  $\gamma_c$  given in (20) can be selected to satisfy the desired outage probability or alternatively to maximize  $C_{tifr}$ . However, due to space limitations, we only consider the latter case in this paper.

#### IV. NUMERICAL RESULTS AND DISCUSSION

In this section, we validate our analysis by comparing with simulation. Numerical results are presented for  $N = 2$  and  $4$  with  $\lambda_0 = 1$ ,  $\{\lambda_{1,k}\}_{k=1}^N = 2$ , and  $\{\lambda_{2,k}\}_{k=1}^N = 3$ .

In Fig. 1 and 2, we investigate the optimal cutoff threshold for OPRA and TIFR, defined as the value that maximizes the channel capacity, as a function of average SNR in dB, respectively. It can be observed from Fig. 1 that the optimal cutoff threshold for OPRA increases as the number of relays increases. Furthermore, the curves also reveal that for a fixed

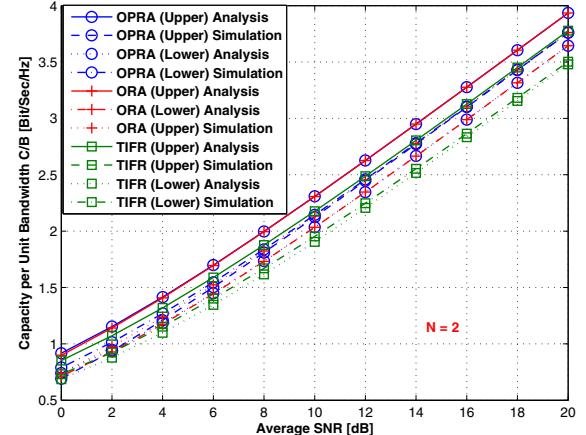


Fig. 3. Channel capacity per unit bandwidth for opportunistic cooperative networks under different adaptation policies.

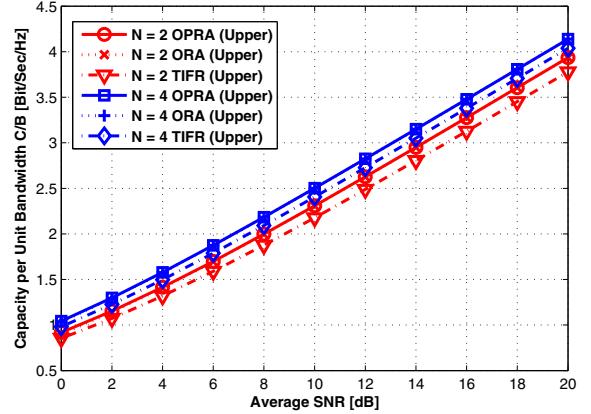


Fig. 4. Channel capacity per unit bandwidth for opportunistic cooperative networks under different adaptation policies.

number of relays, the increment of average SNR makes the optimal cutoff threshold increase, however, all curves are bounded between [0 1]. In Fig. 2, we compare the optimal threshold value for TIFR versus the average SNR for two case of network topologies,  $N = 2$  and  $4$ . Contrary to the behavior of the optimal cutoff threshold of OPRA, which always varies within [0 1], the optimal cutoff threshold for TIFR is linearly related to average SNR as well as to number of relays. However, note that in both figures, for a fixed number of relays and average SNR, there is a distinct gap between the upper and lower curve of optimal cutoff threshold for both OPRA and TIFR. It is worth noting that the behavior of these gaps is completely different at high and low SNR regime. Specifically, with OPRA, the gap tends to diminish as average SNR goes from low to high regimes; however, with TIFR, the contrary result is obtained.

In Fig. 3, the closed-form channel capacity given in (16), (18), and (20) are presented and compared for the case of  $N = 2$ . In general, from Fig. 3 and other numerical experi-

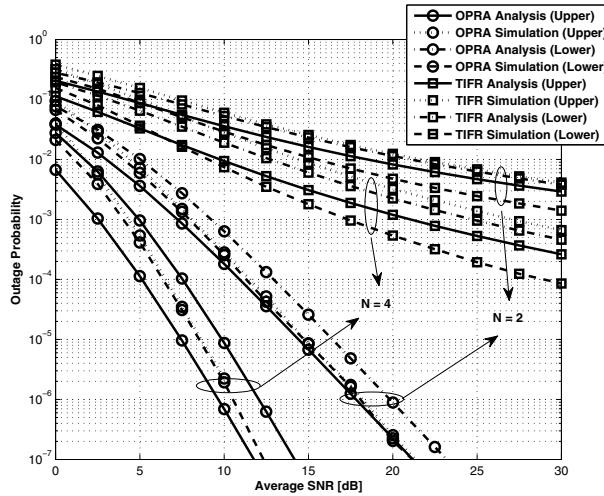


Fig. 5. Outage probability of the OPRA and TIFR for opportunistic cooperative networks.

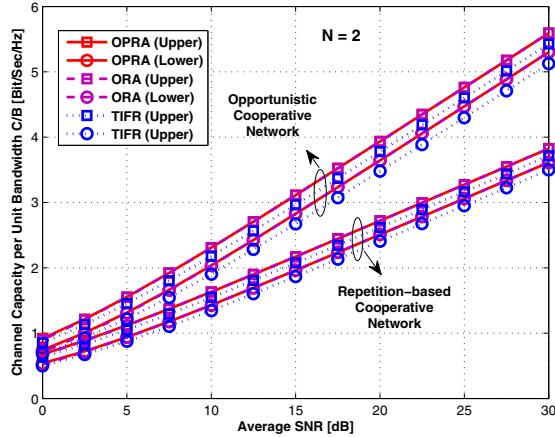


Fig. 6. Channel capacity per unit bandwidth for opportunistic cooperative networks and repetition-base cooperative networks under different adaptation policies.

ments not included here, we notice that, as expected, OPRA slightly outperforms ORA, which, in turn, outperforms TIFR. This observation can be explained by using the standpoint of the tradeoff between capacity and complexity. In addition, it can clearly seen that the simulated and analytical results are in good agreement.

In Fig. 4, we investigate the effects of number of relays on the capacity per unit bandwidth under different adaptation policies. It can be seen that increasing number of relays makes the normalized capacity slightly increase. It is due to the fact that only two time slots is always required for the communication regardless of number relays involved in the cooperative transmission. The corresponding outage probability for the optimum adaptation and truncated channel inversion policies is plotted in Fig. 5. As we can see, the benefit of the maximization of the capacity for TIFR systems takes places

at the cost of increasing probability of outage.

In Fig. 6, we perform numerical investigation and a channel capacity comparison of two schemes, namely repetition-base cooperative relaying and opportunistic cooperative relaying, showing that the channel capacity of repetition-based cooperative network is worse than that of the opportunistic networks. This observation is reasonable due to the fact that in repetition-based cooperative networks, multiple time slots are required to realize orthogonal channelization which normally equals to number of relays. Meanwhile, for opportunistic cooperative relaying networks, the number of necessary time slots for a complete data communication is always two and irrespective of number of relays.

## V. CONCLUSION

The paper has investigated and compared opportunistic cooperative networks under three different adaptive transmission policies: OPRA, ORA and TIFR. The analysis is applicable for general cases, including independent identically distributed and independent but not identically distributed Rayleigh fading channels. The numerical results confirm that among them, OPRA provides the best capacity and TIFR suffers the largest capacity penalty relative to the other policies.

## REFERENCES

- [1] A. J. Goldsmith, and P. P. Varaiya, "Capacity of fading channels with channel side information," *IEEE Trans. Infor. Theory*, , vol. 43, no. 6, pp. 1986-1992, Jun. 1997.
- [2] M. S. Alouini, and A. J. Goldsmith, "Capacity of Rayleigh fading channels under different adaptive transmission and diversity-combining techniques," *IEEE Trans. Vehicular Tech.*, vol. 48, no. 4, pp. 1165-1181, Apr. 1999.
- [3] T. Nechiporenko, K. T. Phan, C. Tellambura et al., "Performance Analysis of Adaptive M-QAM for Rayleigh Fading Cooperative Systems." in Proceeding *IEEE ICC '08*, 2008, pp. 3393-3399.
- [4] K.-S. Hwang, Y.-C. Ko, and M.-S. Alouini, "Performance analysis of incremental opportunistic relaying over identically and non-identically distributed cooperative paths," *IEEE Trans. Wireless Commun.*, vol. 8, no. 4, pp. 1953-1961, Apr. 2009.
- [5] T. Nechiporenko, K. T. Phan, C. Tellambura et al., "On the capacity of Rayleigh fading cooperative systems under adaptive transmission," *IEEE Trans. on Wireless Commun.*, vol. 8, no. 4, pp. 1626-1631, Apr. 2009.
- [6] A. Bletsas, A. Khisti, D. P. Reed et al., "A Simple Cooperative Diversity Method Based on Network Path Selection," *IEEE Journal on Select Areas in Commun.*, vol. 24, no. 3, pp. 659-672, Mar. 2006.
- [7] A. Bletsas, H. Shin, and M. Z. Win, "Cooperative Communications with Outage-Optimal Opportunistic Relaying," *IEEE Trans. on Wireless Commun.*, vol. 6, no. 9, pp. 3450-3460, Sep. 2007.
- [8] P. A. Anghel, and M. Kaveh, "Exact Symbol Error Probability of a Cooperative Network in a Rayleigh-Fading Environment," *IEEE Trans. on Wireless Commun.*, vol. 3, no. 5, pp. 1416-1421, Sep. 2004.
- [9] S. Ikki, and M. H. Ahmed, "Performance Analysis of Cooperative Diversity Wireless Networks over Nakagami-m Fading Channel," *IEEE Commun. Lett.*, vol. 11, no. 4, pp. 334-336, Apr. 2007.
- [10] A. Papoulis, and S. U. Pillai, *Probability, random variables, and stochastic processes*, 4th ed., Boston: McGraw-Hill, 2002.
- [11] V. N. Q. Bao, H. Y. Kong, and S. W. Hong, "Performance Analysis of M-PAM and M-QAM with Selection Combining in Independent but Non-Identically Distributed Rayleigh Fading Paths." in Proceeding *IEEE VTC Fall '08*, pp. 1-5.
- [12] M. K. Simon, and M.-S. Alouini, *Digital communication over fading channels*, 2nd ed., Hoboken, N.J.: John Wiley & Sons, 2005.
- [13] M. Abramowitz, I. A. Stegun, and Knovel (Firm), "Handbook of mathematical functions with formulas, graphs, and mathematical tables," 10th printing, with corrections. ed Washington, D.C.: U.S. Dept. of Commerce : U.S. G.P.O., 1972, pp. xiv, 1046 p.