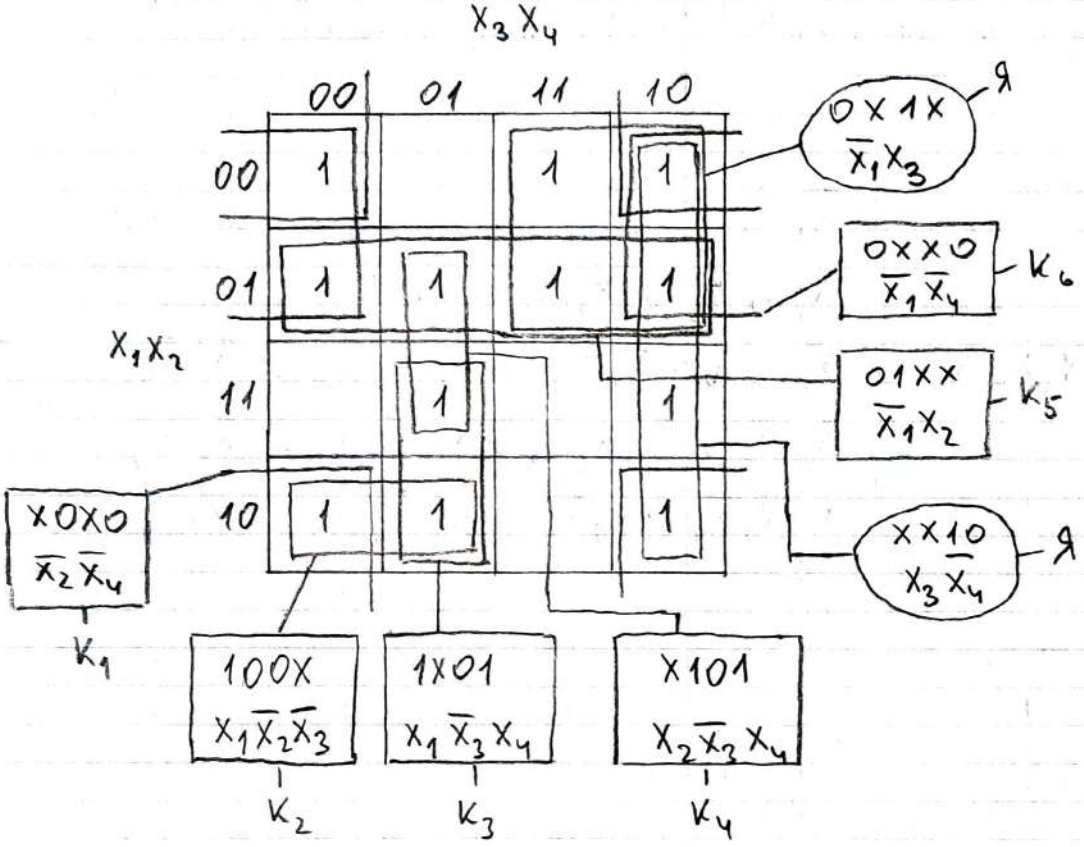
Маслова М.А. ИУ7-43Б Вариант №11

Булевы функции.

Задача 1.

$f = (1, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 1, 1, 0)$



а) Сокращенная ДНФ:

$$f = \bar{x}_1 x_3 \vee x_3 \bar{x}_4 \vee \bar{x}_2 \bar{x}_4 \vee x_1 \bar{x}_2 \bar{x}_3 \vee x_1 \bar{x}_3 x_4 \vee$$
$$\vee x_2 \bar{x}_3 x_4 \vee \bar{x}_1 x_2 \vee \bar{x}_1 \bar{x}_4$$

б) Ядро составляют склейки: $\bar{x}_1 x_3$ и $x_3 \bar{x}_4$

в) Функция Патрика:

$$(K_1 \vee K_2)(K_2 \vee K_3)(K_3 \vee K_4)(K_4 \vee K_5)(K_5 \vee K_6)(K_1 \vee K_6) =$$

$$= (K_1 K_2 \vee K_1 K_3 \vee K_2 \vee K_2 K_3) \wedge$$
$$\wedge (K_3 K_4 \vee K_3 K_5 \vee K_4 \vee K_4 K_5) \wedge$$
$$\wedge (K_5 K_1 \vee K_5 K_6 \vee K_6 K_1 \vee K_6) =$$

$= (K_1 K_3 \vee K_2)(K_3 K_5 \vee K_4)(K_5 K_1 \vee K_6) =$

$= (K_1 K_3 K_5 \vee K_1 K_3 K_4 \vee K_2 K_3 K_5 \vee K_2 K_4)(K_5 K_1 \vee K_6) =$

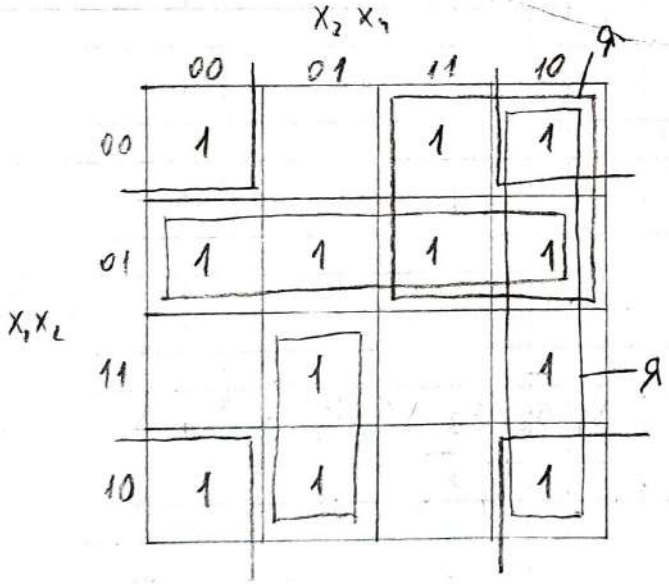
$= K_1 K_3 K_5 \vee K_1 \cancel{K_3 K_5} K_6 \vee K_1 \cancel{K_3 K_4} K_5 \vee K_1 K_3 K_4 K_6 \vee$
$\vee K_1 \cancel{K_2 K_3} K_5 \vee K_2 K_3 K_5 K_6 \vee K_1 K_2 K_4 K_5 \vee K_2 K_4 K_6 =$

$= K_1 K_3 K_5 \vee K_2 K_4 K_6 \vee K_1 K_3 K_4 K_6 \vee K_1 K_2 K_4 K_5 \vee K_2 K_3 K_5 K_6$

Тупиковые ДНФ:

$$f = \bar{x}_1 x_3 \vee x_3 \bar{x}_4 \vee \begin{cases} \bar{x}_2 \bar{x}_4 \vee x_1 \bar{x}_3 x_4 \vee \bar{x}_1 x_2 & \text{— минимальная} \\ x_1 \bar{x}_2 \bar{x}_3 \vee x_2 \bar{x}_3 x_4 \vee \bar{x}_1 \bar{x}_4 \\ \bar{x}_2 \bar{x}_4 \vee x_1 \bar{x}_3 x_4 \vee x_2 \bar{x}_3 x_4 \vee \bar{x}_1 \bar{x}_4 \\ \bar{x}_2 \bar{x}_4 \vee x_1 \bar{x}_2 \bar{x}_3 \vee x_2 \bar{x}_3 x_4 \vee \bar{x}_1 x_2 \\ x_1 \bar{x}_2 \bar{x}_3 \vee x_1 \bar{x}_3 x_4 \vee \bar{x}_1 x_2 \vee \bar{x}_1 \bar{x}_4 \end{cases}$$

2)



Задача 2.

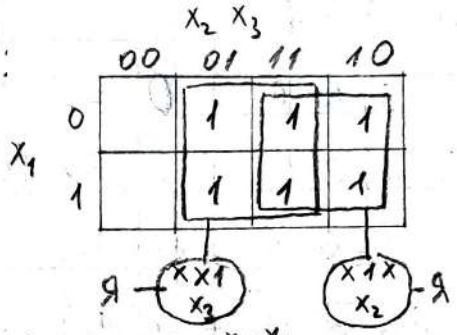
$$f = ((x_2 \to (x_1 \oplus x_3)) \oplus (\overline{x_2} \sim x_3)) \to (\overline{x_2} \mid \overline{x_3})$$

$$w = (1, 1, 0, 1, 0, 1, 1, 1)$$

а)

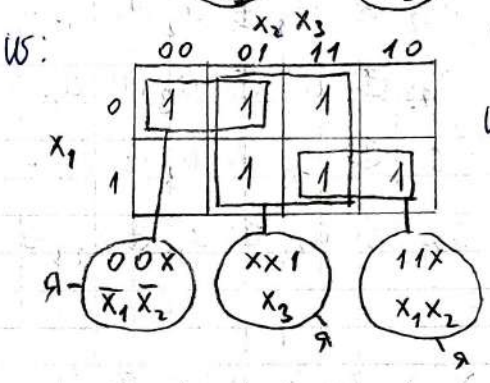
x_1	x_2	x_3	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

б) f:



$$f = x_2 \vee x_3$$

w:



$$w = x_1x_2 \vee \overline{x_1}\,\overline{x_2} \vee x_3$$

в)

	T_0	T_1	S	M	L
f	+	+	−	+	−
w	−	+	−	−	−
g	+	−	−	−	−

Система $\{f, w\}$ неполна, т.к обе ф-ии сохр. 1.

Дополним систему ф-ей g:

$$g = (00010000)$$

Полиномы Жегалкина:

1) f:

$$a_0 = f(0,0,0) = 0$$
$$f(1,0,0) = a_1 \oplus a_0 = 0 \Rightarrow a_1 = 0$$
$$f(0,1,0) = a_2 \oplus a_0 = 1 \Rightarrow a_2 = 1$$
$$f(0,0,1) = a_3 \oplus a_0 = 1 \Rightarrow a_3 = 1$$
$$f(1,1,0) = a_{12} \oplus a_1 \oplus a_2 \oplus a_0 = 1 \Rightarrow a_{12} = 0$$
$$f(1,0,1) = a_{13} \oplus a_1 \oplus a_3 \oplus a_0 = 1 \Rightarrow a_{13} = 0$$
$$f(0,1,1) = a_{23} \oplus a_2 \oplus a_3 \oplus a_0 = 1 \Rightarrow a_{23} = 1$$

$f(1,1,1) = a_{123} \oplus a_{12} \oplus a_{13} \oplus a_{23} \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_0 = 1 \Rightarrow a_{123} = 0$

$f = x_2 x_3 \oplus x_2 \oplus x_3$

2) w:

$a_0 = w(0,0,0) = 1$

$w(1,0,0) = a_1 \oplus a_0 = 0 \Rightarrow a_1 = 1$

$w(0,1,0) = a_2 \oplus a_0 = 0 \Rightarrow a_2 = 1$

$w(0,0,1) = a_3 \oplus a_0 = 1 \Rightarrow a_3 = 0$

$w(1,1,0) = a_{12} \oplus a_1 \oplus a_2 \oplus a_0 = 1 \Rightarrow a_{12} = 0$

$w(1,0,1) = a_{13} \oplus a_1 \oplus a_3 \oplus a_0 = 1 \Rightarrow a_{13} = 1$

$w(0,1,1) = a_{23} \oplus a_2 \oplus a_3 \oplus a_0 = 1 \Rightarrow a_{23} = 1$

$w(1,1,1) = a_{123} \oplus a_{12} \oplus a_{13} \oplus a_{23} \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_0 = 1 \Rightarrow a_{123} = 0$

$w = x_1 x_3 \oplus x_2 x_3 \oplus x_2 \oplus x_1 \oplus 1$

3) g:

$a_0 = g(0,0,0) = 0$

$g(1,0,0) = a_1 \oplus a_0 = 0 \Rightarrow a_1 = 0$

$g(0,1,0) = a_2 \oplus a_0 = 0 \Rightarrow a_2 = 0$

$g(0,0,1) = a_3 \oplus a_0 = 0 \Rightarrow a_3 = 0$

$g(1,1,0) = a_{12} \oplus a_1 \oplus a_2 \oplus a_0 = 0 \Rightarrow a_{12} = 0$

$g(1,0,1) = a_{13} \oplus a_1 \oplus a_3 \oplus a_0 = 0 \Rightarrow a_{13} = 0$

$g(0,1,1) = a_{23} \oplus a_2 \oplus a_3 \oplus a_0 = 1 \Rightarrow a_{23} = 1$

$g(1,1,1) = a_{123} \oplus a_{12} \oplus a_{13} \oplus a_{23} \oplus a_1 \oplus a_2 \oplus a_3 \oplus a_0 = 0 \Rightarrow a_{123} = 1$

$g = x_1 x_2 x_3 \oplus x_2 x_3$

2) $0 = g(x,x,x)$ $\quad x \vee y = f(0,x,y) = f(g(x,x,x), x, y)$

$1 = w(x,x,x)$ $\quad x \wedge y = g(0,x,y) = g(g(x,x,x), x, y)$

$\bar{x} = w(x,0,0) = w(x, g(x,x,x), g(x,x,x))$