Бmuc 0

№1 $\int_0^1 dx \int_0^{x^2} f(x,y)\,dy$ $\qquad \frac{1}{3} \qquad \frac{1}{3}$

Б3

$y = x^2$ 17

$\cancel{\rho}\sin\varphi = \rho^{\cancel{2}}\cos^2\varphi$ 18

$\sin\varphi = \rho\cos^2\varphi$ 20

$\rho = \frac{\sin\varphi}{\cos^2\varphi}$ 22

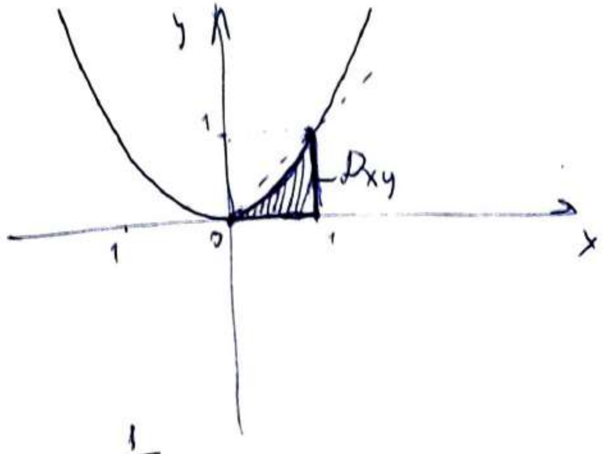
$D = \{(x,y) : 0 \le x \le 1;\ 0 \le y \le x^2\}$

$y = x^2 \Rightarrow x = \pm\sqrt{y}$

$x = 1$

$\rho\cos\varphi = 1$

$\rho = \frac{1}{\cos\varphi}$



$I = \int_0^1 dy \int_{\sqrt{y}}^1 f(x,y)\,dx =$

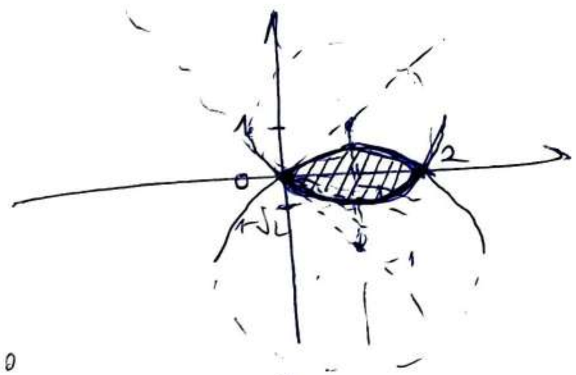
$= \int_0^{\frac{\pi}{4}} d\varphi \int_{\frac{\sin\varphi}{\cos^2\varphi}}^{\frac{1}{\cos\varphi}} f(\rho\cos\varphi, \rho\sin\varphi)\,\rho\,d\rho$

$\frac{\frac{1}{\sqrt{2}}}{\frac{1}{2}} = \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\frac{\frac{\sqrt{2}}{2}}{\frac{2}{4}}$ $4\sqrt{2}$ $\qquad \frac{2}{\sqrt{2}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

$\frac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$

~~1~~, ~~2~~, ~~3~~, 4, ~~6~~, 7, 9, 12, 14, 16, 17, 18, 20, 22



Б1)

$$I = \int_0^2 dx \int_{1-\sqrt{2-(x-1)^2}}^{-1+\sqrt{2-(x-1)^2}} f(x,y)\,dy$$

$(\rho\cos\varphi-1)^2+(\rho\sin\varphi-1)^2=2$

$\rho^2\cos^2\varphi-2\rho\cos\varphi+\cancel{1}+\rho^2\sin^2\varphi-2\rho\sin\varphi+\cancel{1}=\cancel{2}$

$\rho^2-2\rho(\cos\varphi+\sin\varphi)=0$

$\rho(\rho-2(\cos\varphi+\sin\varphi))=0$

$x=1\pm\sqrt{2-(y-1)^2}$

$y=1-\sqrt{2-(x-1)^2}$

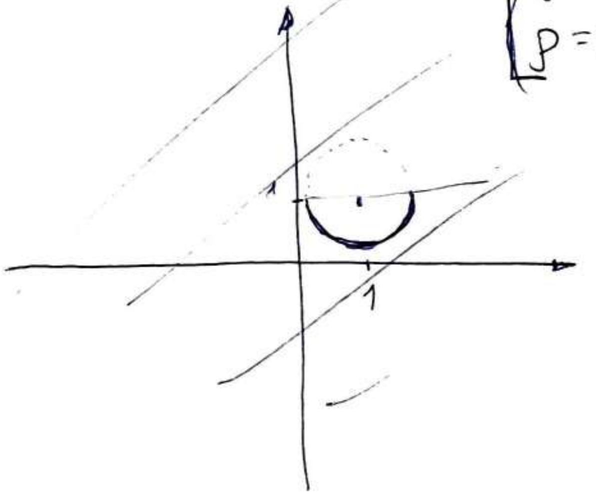
$y-1=-\sqrt{2-(x-1)^2} \iff \begin{cases}(y-1)^2=2-(x-1)^2\\ y-1\le 0\end{cases}$

$\begin{cases}(x-1)^2+(y-1)^2=2\\ y\le 1\end{cases}$ $y=1\pm\sqrt{2}$

$\left[\begin{array}{l}\rho=0\\ \rho=2(\cos\varphi+\sin\varphi)\end{array}\right.$

$y=-1+\sqrt{2-(x-1)}$

$y+1=\sqrt{2-(x-1)^2} \Rightarrow \begin{cases}(y+1)^2=2-(x-1)^2\\ y+1\ge 0\end{cases}$



$\begin{cases}(x-1)^2+(y+1)^2=2\\ y\ge -1\end{cases}$ $y+1=\pm\sqrt{2}$, $y=-1+\sqrt{2}$

$$I=\int_{1-\sqrt{2}}^{1+\sqrt{2}} dy \int_{1-\sqrt{2-(y-1)^2}}^{1+\sqrt{2-(y-1)^2}} f(x,y)\,dx \neq$$

$$\neq \int_0^{-1+\sqrt{2}} dy \int_{1-\sqrt{2-(y+1)^2}}^{1+\sqrt{2-(y+1)^2}} f(x,y)\,dx$$

$y=0$

$(x-1)^2+1=2$

$(x-1)^2=1$

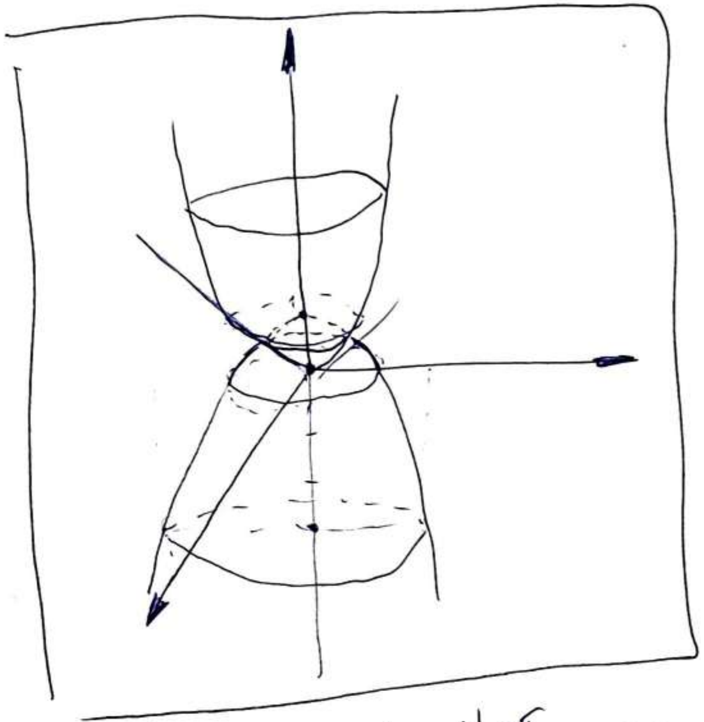
$x-1=\pm 1$

$x_{1,2}=0,2$

$$I=\int_{-\frac{\pi}{4}}^{0} d\varphi \int_0^{2(\cos\varphi+\sin\varphi)} f(\rho\cos,\rho\sin)\,\rho\,d\rho+\int_0^{\frac{\pi}{4}} d\varphi\int_0^{2(\cos\varphi-\sin\varphi)} f(\rho c,\rho s)\,\rho\,d\rho$$

№2

$Z = x^2 + y^2 \qquad Z = -x^2 - y^2 + 1$



$z = 0 \qquad x^2 + y^2 = 1.$

$5 - 1 = -x^2 - y^2 \qquad x^2 + y^2 = 1 - Z$

$4 = -x^2 - y^2$

$1 - Z \geq 0$

$Z = 1 \qquad x^2 + y^2 = 0 \qquad -Z \geq -1$

$Z \leq 1$

$+3$

$z = -3$

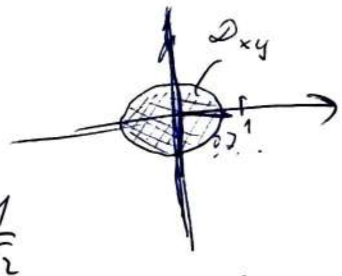
$x^2 + y^2 = 4$

Проекция линии пересечения:

$x^2 + y^2 = -x^2 - y^2 + 1$

$2x^2 + 2y^2 = 1$

$\boxed{x^2 + y^2 = \frac{1}{2}} \qquad R = \frac{1}{\sqrt{2}}$



$$\int_0^{2\pi} d\varphi \int_0^{\frac{1}{\sqrt{2}}} (-2\rho^2 + 1)\rho \, d\rho \, d\varphi =$$

$$= \int_0^{2\pi} \left(-2\frac{\rho^4}{4} + \frac{\rho^2}{2}\right)\Big|_0^{\frac{1}{\sqrt{2}}} d\varphi =$$

$$\circleddash \int_0^{2\pi} \left(-\frac{\rho^4}{2} + \frac{\rho^2}{2}\right)\Big|_0^{\frac{1}{\sqrt{2}}} d\varphi \circleddash$$

$$\circleddash \int_0^{2\pi} \left(-\frac{1}{8} + \frac{1}{4}\right) d\varphi = \frac{1}{8}\varphi \Big|_0^{2\pi} = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$V = \iint_{D_{xy}} [Z_2(x,y) - Z_1(x,y)]\, dx\, dy = \iint_{D_{xy}} (-x^2 - y^2 + 1 - x^2 - y^2)\, dx\, dy =$$

$$= \iint_{D_{xy}} (-2x^2 - 2y^2 + 1)\, dx\, dy = \iint_{D_{\rho\varphi}} \left(-2(\rho^2\cos^2\varphi + \rho^2\sin^2\varphi) + 1\right)\rho\, d\rho\, d\varphi = \iint_{D_{\rho\varphi}}$$

$(x-1)^2+(y-1)^2=2$

$(\rho\cos\varphi-1)^2+(\rho\sin\varphi-1)^2=2$

$\rho^2\cos^2-2\rho\cos\varphi+\cancel{1}+\rho^2\sin^2\varphi-2\rho\sin\varphi+\cancel{1}=\cancel{2}$

$\rho^2-2\rho\cos\varphi=0$

$\rho=2(\cos\varphi+\sin\varphi)$

$(x-1)^2+(y+1)^2=2$

$\rho^2\cos^2\varphi-2\rho\cos\varphi+\cancel{1}\ \rho^2\sin^2\varphi+2\rho\sin\varphi+\cancel{1}=\cancel{2}$

$\rho^2+2\rho(\sin\varphi-\cos\varphi)=0$

$\rho^2-2\rho(\cos\varphi-\sin\varphi)=0$

$\rho=2(\cos\varphi-\sin\varphi)=0$

№2 $x^2+y^2=z$ $\rho^2=$

$(x-1)^2+y^2=1$

$z=0$

$\rho^2-2\rho+\cancel{1}\ldots=\cancel{1}$

$\rho^2-2\rho=0$

$\rho=0$

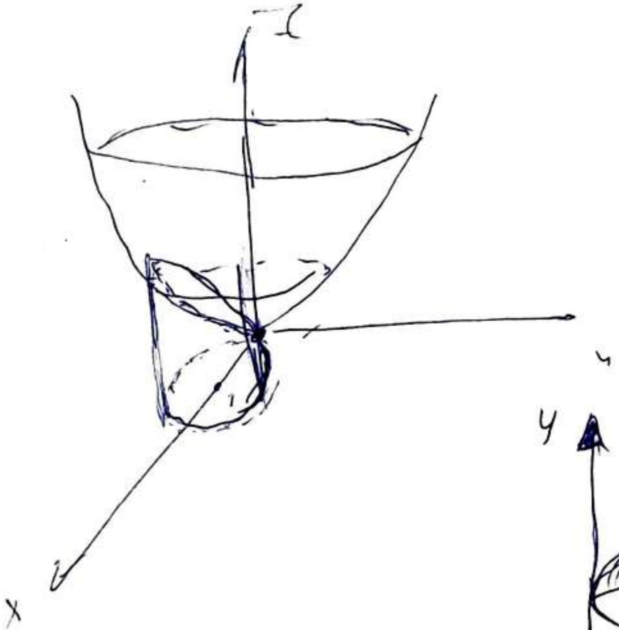
$\rho=2$

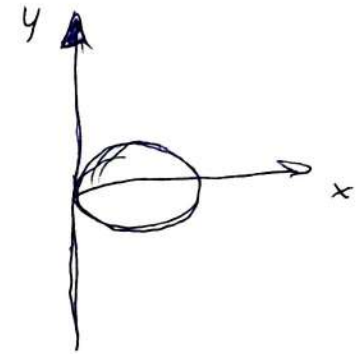
$(x-1)^2+y^2=1$

$\rho^2\cos^2\varphi-2\rho\cos\varphi+\cancel{1}+\rho^2\sin^2\varphi=1$

$\rho^2-2\rho\cos\varphi=0$

$\rho=2\cos\varphi$





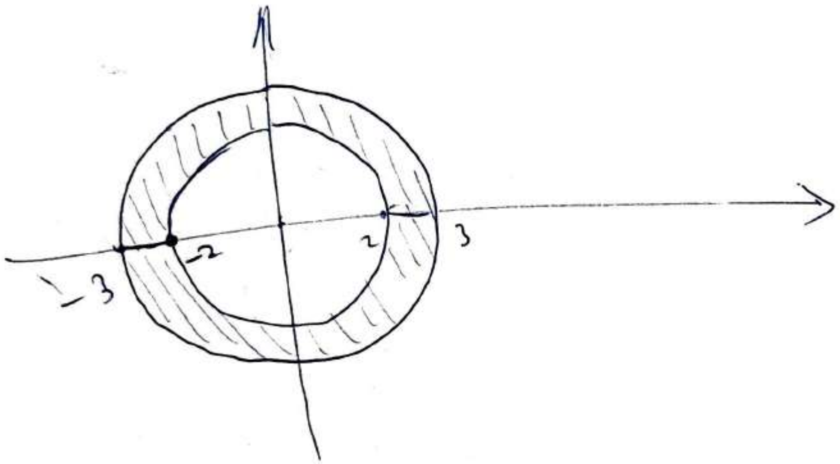
$\iint\limits_{D_{xy}} x^2+y^2\,dxdy=$

$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}d\varphi\int_0^{2\cos\varphi}\rho^3\,d\rho=\int\frac{16\cos^4\varphi}{4}\,d\varphi=$

$=\int 4\cos^4\varphi\,d\varphi=\ldots=\frac{3\pi}{2}$

долго

$\boxed{62}$ $\iint\limits_D f(x,y)\,dx\,dy$ $x^2+y^2=4$ $x^2+y^2=9$



$$I = \int_{-3}^{3} dx \int_{+\sqrt{4-x^2}}^{+\sqrt{9-x^2}} f\,dy + \int_{-3}^{3} dx \int_{-\sqrt{9-x^2}}^{-\sqrt{4-x^2}}$$

$$I = \int_{-3}^{3} dy \int f\,dx + \int dy \int f\,dx$$

$$I = \int_0^{2\pi} d\varphi \int_2^3 f(\rho\cos\varphi, \rho\sin\varphi)\,\rho\,d\rho$$

$\rho^2 = 4$ $\rho = 3$

$\rho = 2$

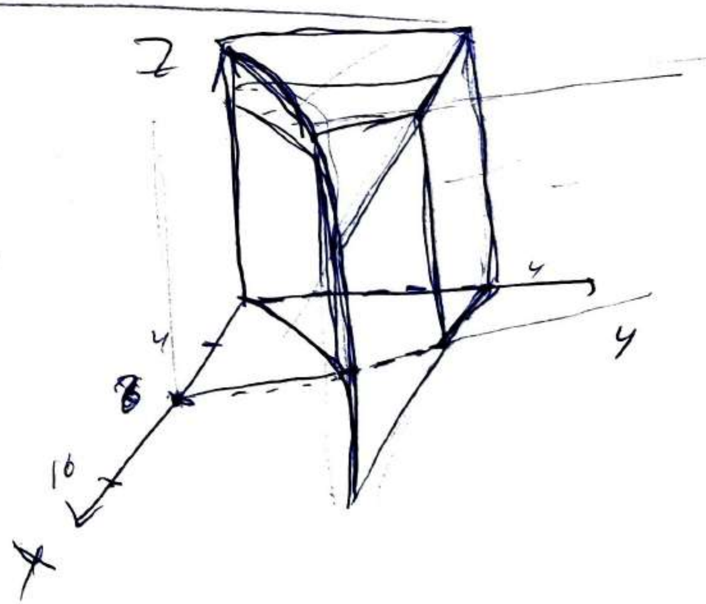
$y = 4$ $x = 0$ $y = \sqrt{x}$ $\cancel{z} + y = 8$

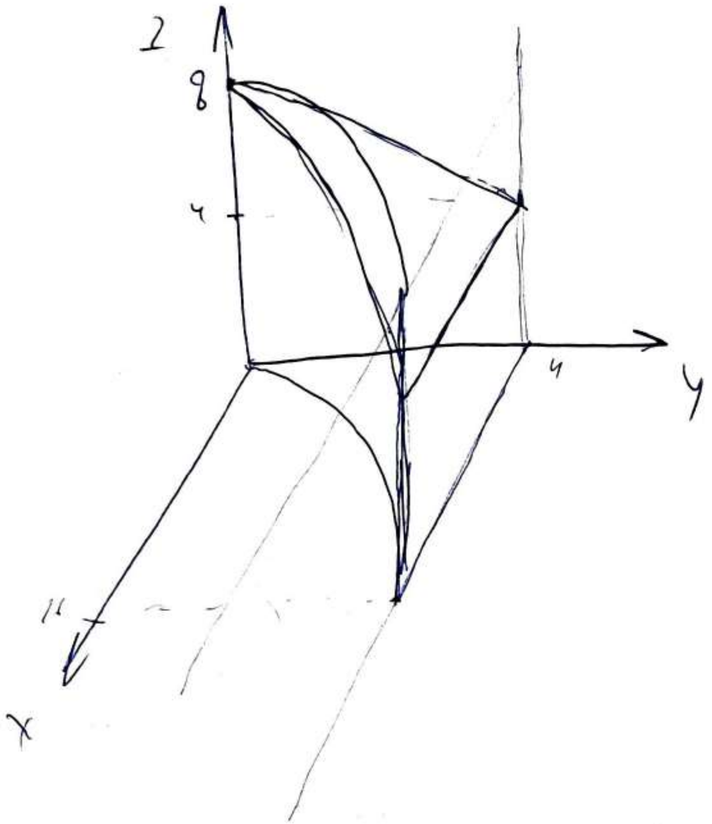
$y = 16$ $y = 8 - x$

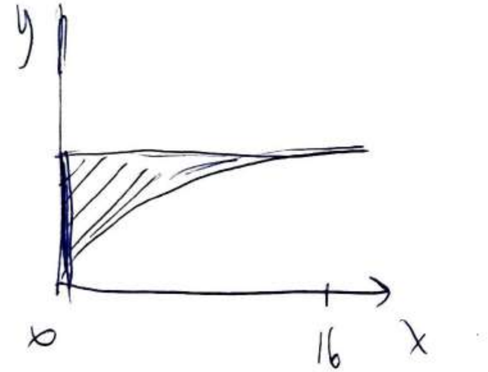
~~$0 = 8 - x$~~

$y = 8 - x$

$x = 4$







$$\iint_{D_{xy}} 8-y \; dx\,dy = \int_0^{16} dx \int_{\sqrt{x}}^{4} (8-y)\,dy$$

$$= \int_0^{16} \left(8y - \frac{y^2}{2}\right)\Big|_{\sqrt{x}}^{4} dx =$$

$$= \int_0^{16} \left(32 - 8 - 8\sqrt{x} + \frac{x}{2}\right) dx =$$

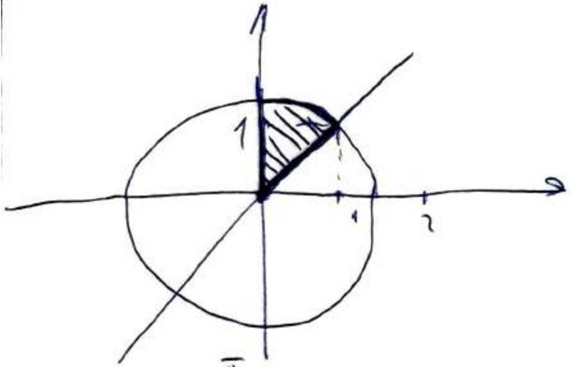
$$= \int_0^{16} \left(24 - 8\sqrt{x} + \frac{x}{2}\right) dx =$$

$$= \left(24x - 8\,\frac{2x^{\frac{3}{2}}}{3} + \frac{x^2}{4}\right)\Big|_0^{16} =$$

$$= \ldots = \frac{320}{3}$$

$\boxed{5)}$ $\int\limits_0^1 dx \int\limits_x^{\sqrt{2-x^2}} f(x,y)\,dy$

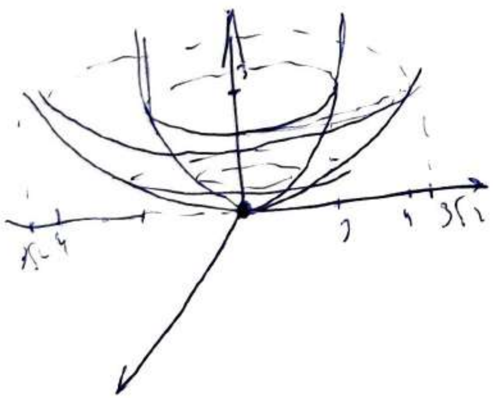
$y=\sqrt{2-x^2} \Rightarrow \begin{cases} y^2 = 2-x^2 \\ y\geq 0\end{cases} \quad \begin{cases} y^2+x^2=2\\ y\geq 0\end{cases}$

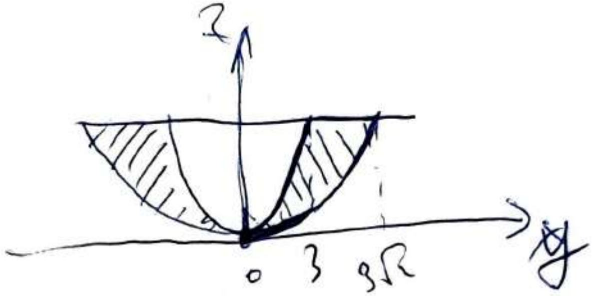


$I = \int\limits_0^1 dy \int\limits_0^y f(x,y)\,dx + \int\limits_1^{\sqrt2} dy \int\limits_0^{\sqrt{2-y^2}} f(x,y)\,dx$

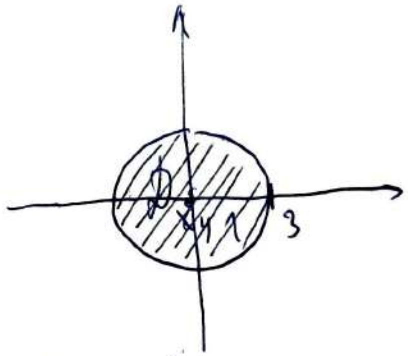
$I = \int\limits_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\varphi \int\limits_0^1 f(\rho\cos, \rho\sin)\,\rho\, d\rho$

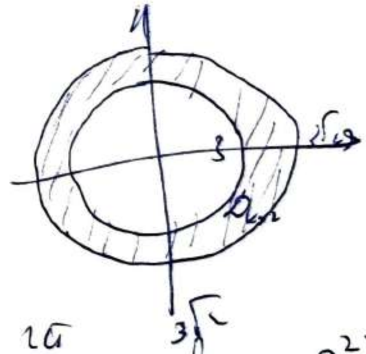
N2 $x^2+y^2=3z$ ($z\geq 0$) $\quad x^2+y^2=6z$ ($z\geq 0$), $z=3$





$V = \iint\limits_{D_{xy1}} \left(\frac{x^2+y^2}{3} - \frac{x^2+y^2}{6}\right) dx\,dy + \iint\limits_{D_{xy2}} \left(3 - \frac{x^2+y^2}{6}\right) dx\,dy$





$V = \int\limits_0^{2\pi} d\varphi \int\limits_0^3 \left(\frac{\rho^2}{3} - \frac{\rho^2}{6}\right)\rho\, d\rho + \int\limits_0^{2\pi} d\varphi \int\limits_3^{3\sqrt2} \left(3 - \frac{\rho^2}{6}\right)\rho\, d\rho$

$= \frac{27\pi}{4} + \frac{27\pi}{4} = \frac{27\pi}{2}$

$$y = \sqrt{1+y^2}$$

$$x^2 = 1+y^2$$

$$\cancel{x^2+y^2=1} \qquad x^2-y^2=1$$

$$y = \sqrt{3}$$

$$\rho \sin\varphi = \sqrt{3}$$

$$\rho = \frac{\sqrt{3}}{\sin\varphi}$$

$$x^2 - 3 = 1$$

$$x^2 = 4$$

$$x = 2$$

$$x^2-y^2=1$$

$$y^2 = x^2-1$$

$$y = \pm\sqrt{x^2-1}$$

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Tuur Stuyck
July 2018

$$x^2-y^2=1$$

$$\rho^2\cos^2\varphi - \rho^2\sin^2\varphi = 1$$

$$\rho^2(\cos^2\varphi - \sin^2\varphi) = 1$$

$$\rho^2\cos 2\varphi = 1$$

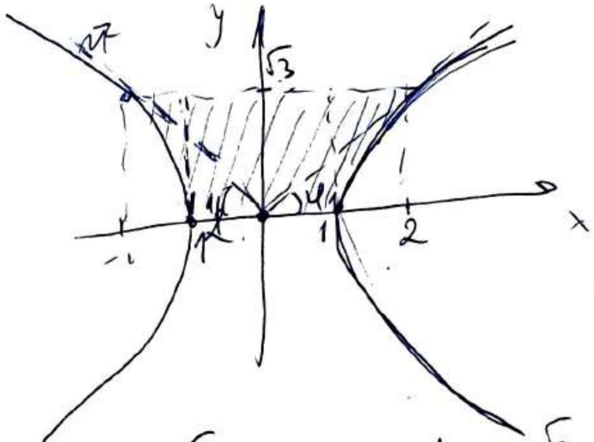
$$\cancel{\rho^2 = \frac{1-\cos 2\varphi}{2\sin^2\varphi}}$$

$$\cancel{\rho = \sqrt{2\sin^2\varphi}}$$

$$\rho^2 = \frac{1}{\cos 2\varphi}$$

$\boxed{64}$

$$\int_0^{\sqrt{3}} dy \int_{-\sqrt{1+y^2}}^{\sqrt{1+y^2}} f(x,y)\,dx =$$



$$\cancel{\varphi = \operatorname{arctg}\sqrt{3}}$$

$$\varphi_1 = \operatorname{arctg}\frac{\sqrt{3}}{2} =$$

$$\varphi_2 = \pi - \operatorname{arctg}\frac{\sqrt{3}}{1}$$

$$I = \int_{-2}^{-1} dx \int_{\sqrt{x^2-1}}^{\sqrt{3}} f\,dy + \int_{-1}^{1} dx \int_0^{\sqrt{3}} f\,dy + \int_1^2 dx \int_{\sqrt{x^2-1}}^{\sqrt{3}} f\,dy$$

$$I = \int_0^{\operatorname{arctg}\frac{\sqrt{3}}{2}} d\varphi \int^{\frac{1}{\sqrt{\cos 2\varphi}}} f(\rho\cos, \rho\sin)\rho\,d\rho +$$

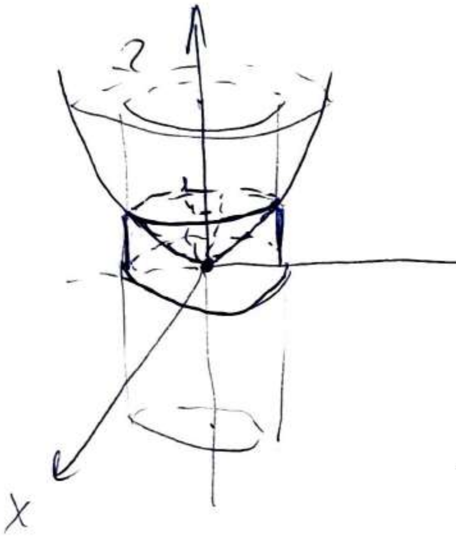
$$+ \int_{\operatorname{arctg}\frac{\sqrt{3}}{2}}^{\pi - \operatorname{arctg}\frac{\sqrt{3}}{2}} d\varphi \int_0^{\frac{\sqrt{3}}{\sin\varphi}} f(\ldots)\rho\,d\rho +$$

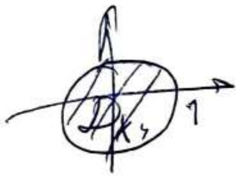
$$+ \int_{\pi - \operatorname{arctg}\frac{\sqrt{3}}{2}}^{\pi} d\varphi \int_0^{\frac{1}{\sqrt{\cos 2\varphi}}} f(\ldots)\rho\,d\rho$$

$\boxed{б4}$

$x^2+y^2=1$; $x^2+y^2=4z$, $z=0$

$4z=1$
$z=\frac{1}{4}$





$$V=\iint\limits_{D_{xy}}\frac{x^2+y^2}{4}\,dx\,dy=\int_0^{2\pi}d\varphi\int_0^1\frac{\rho^3}{4}\,d\rho=$$

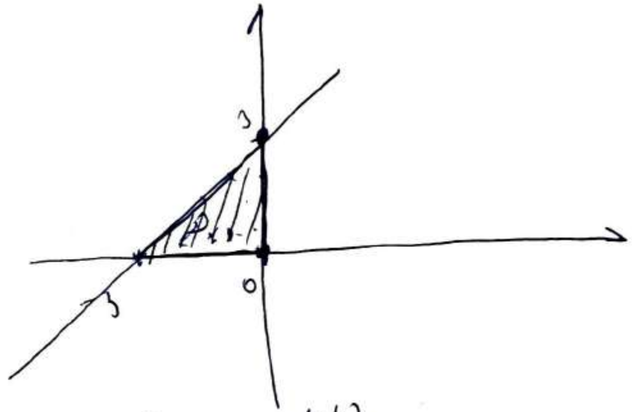
$$=\frac{\pi}{8}$$

$\boxed{5б}$

$$\int_0^3 dy\int_{y-3}^{0} f\,dx$$

$x=y-3$

$y=x+3$



$$I=\int_{-3}^{0}dx\int_0^{x+3}f\,dy$$

ПСК:

$y=x+3$

$\rho\sin\varphi=\rho\cos\varphi+3$

$\rho(\sin\varphi-\cos\varphi)=3$

$$\boxed{\rho=\frac{3}{\sin\varphi-\cos\varphi}}$$

$$I=\int_{\frac{\pi}{2}}^{\pi}d\varphi\int_0 f\,d\rho$$

№2 Б6

$x^2+y^2-z^2=9$; $z=4$; $z=0$

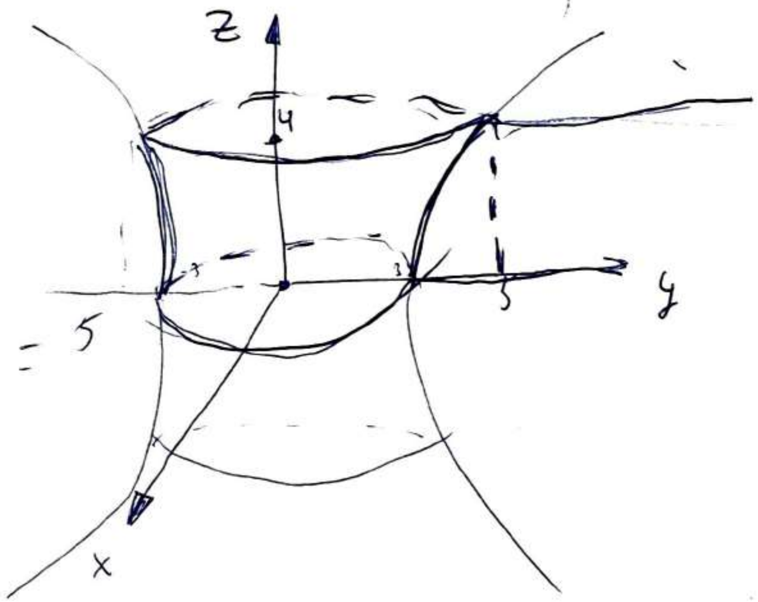
$x=0$

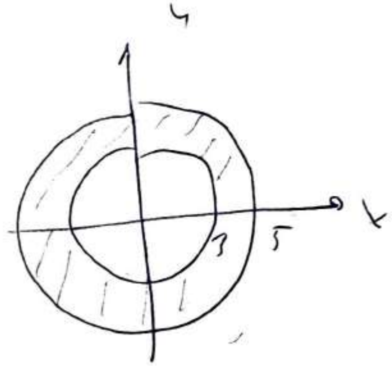
$y^2-z^2=9$

$x^2+y^2=25$

$z^2=x^2+y^2-9$

$z=\sqrt{x^2+y^2-9}$





$f(x,y)=\sqrt{x^2+y^2-9}$

$f(\rho,\varphi)=\sqrt{\rho^2-9}$

$$V_{\text{цилиндр}}=\int_0^{2\pi}d\varphi\int_0^5 4\rho\,d\rho-\int_0^{2\pi}d\varphi\int_3^5\rho\sqrt{\rho^2-9}\,d\rho=100\pi-\frac{128}{3}\pi=57\frac{1}{3}$$

$\boxed{67}$

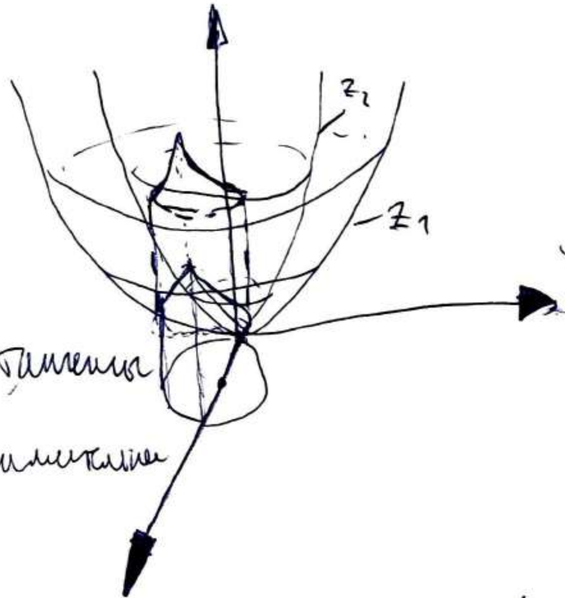
$z_1 = x^2+y^2$ $\quad$ $z_2 = 2(x^2+y^2)$ $\quad$ $(x-1)^2+y^2 = 1$

$\rho^2\cos^2\varphi - 2\rho\cos\varphi + \cancel{1} + \rho^2\sin^2\varphi$

$\cancel{z(x,y)}$ $\quad f(x,y) = x^2+y^2$ $\quad \rho^2 - 2\rho\cos\varphi = 0$

$\rho = 2\cos\varphi$

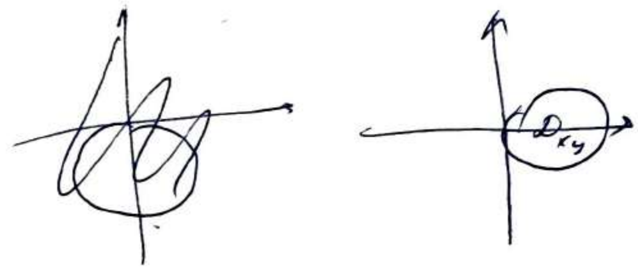
~~угол~~



• угол через [illegible]

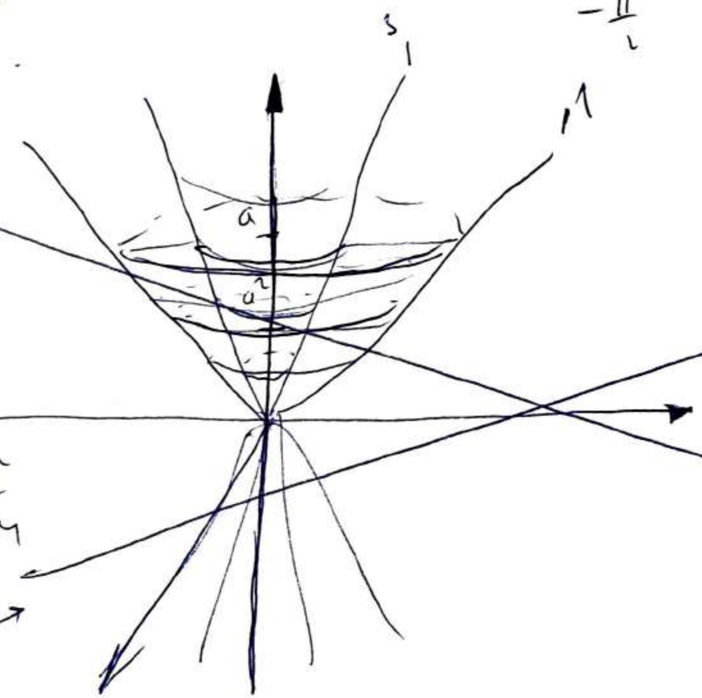
• рисунок [illegible]

D_{xy}



$$V = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\varphi \int_0^{2\cos\varphi} \rho^3\, d\rho = \frac{3\pi}{2}$$

$\boxed{5\,п}$



$x^2+y^2 = ax$

$x^2 - ax + \frac{a^2}{4} + y^2 = \frac{a^2}{4}$

$(x-\frac{a}{2})^2 + y^2 = \frac{a^2}{4}$

$$\cancel{\int_0^{2\pi} d\varphi \int_{\frac{a}{2}}^{a} 2\rho^2\, d\rho =}$$

$z = \sqrt{3(x^2+y^2)}$

$z = \sqrt{x^2+y^2}$

$$\int_0^{2\pi} d\varphi \int_{\frac{a}{2}}^{a} 2\rho^2(\sqrt{3}-1)\, d\rho$$

$z = \sqrt{3\rho^2} - \sqrt{\rho^2} = \rho\sqrt{3} - \rho = \rho(\sqrt{3}-1)$

$y = -1 + \sqrt{2-(x-1)^2}$

$(y + 1 = \sqrt{2-(x+1)^2}$

$(y+1)^2 = 2-(X-1)^2 \quad (y+1 \geqslant 0)$

$(y+1^2 + (x-1)^2 = 2 \quad (y \geqslant -1$