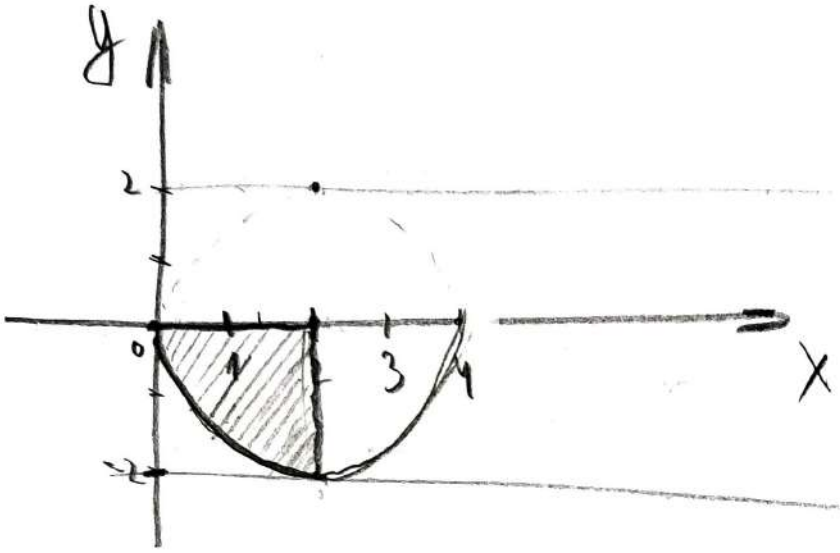
Д31 | ВАР 12 | 26.09.21 | 11:55 — 13:03

N1.

$$I = \int_0^2 dx \int_{-\sqrt{4x-x^2}}^{0} f(x,y)\,dy$$



$y = -\sqrt{4x-x^2}$ — часть окр. зад. ур-ем.

$y^2 = 4x - x^2$

$x^2 - 4x + y^2 = 0$

$x^2 - 4x + 4 + y^2 = 4$

$(x-2)^2 + y^2 = 4$

$\frac{\pi R^2}{4} = \frac{\pi \cdot 4}{4} = \pi$

$$I = \int_{-2}^{0} dy \int_{-\sqrt{4-y^2}+2}^{2} f(x,y)\,dx$$

Проверка

$(x-2)^2 + y^2 = 4$

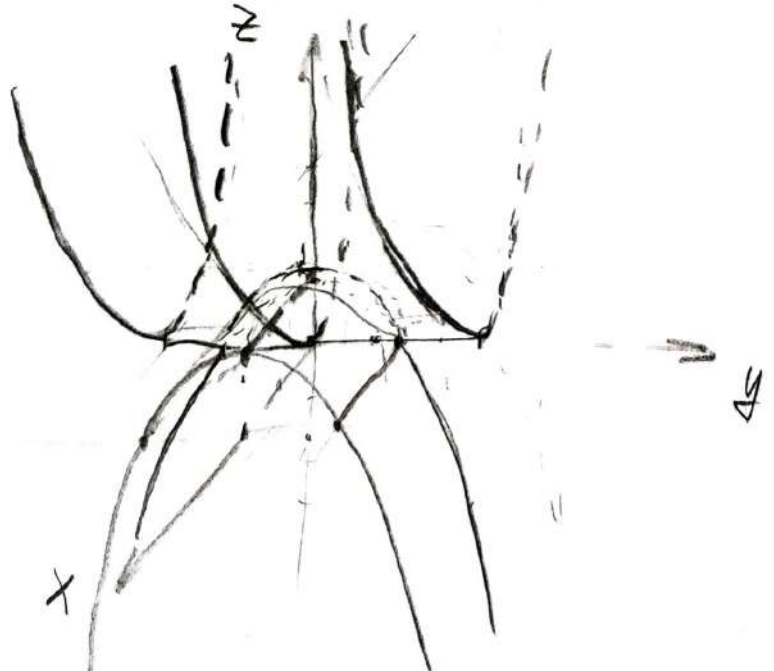
$(x-2)^2 = 4 - y^2$

$x - 2 = \pm\sqrt{4-y^2}$

$x = \pm\sqrt{4-y^2} + 2$

$x = -\sqrt{4-y^2} + 2$

N2 $z = x^2$, $z = 1 - y^2$



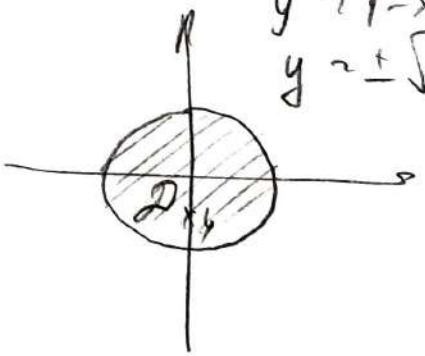
Тело лежит над окружностью

$x^2 = 1 - y^2$

$x^2 + y^2 = 1$

$y^2 = 1 - x^2$

$y = \pm\sqrt{1-x^2}$



$$V = \iint_{D_{xy}} (z_1 - z_2)\,dx\,dy = \iint_{D_{xy}} (1 - y^2 - x^2)\,dx\,dy =$$

$$= \int_{-1}^{1} dx \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} (1 - y^2 - x^2)\,dy = \int_{-1}^{1} \left((1-x^2)y - \frac{y^3}{3} \right) \Big|_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}$$

$$V = \iint_{D_{xy}} (z_2 - z_1)\,dx\,dy = \iint_{D_{xy}} (1 - y^2 - x^2)\,dx\,dy =$$

$$= \iint_{D_{\rho\varphi}} (1 - \rho^2)\,\rho\,d\rho\,d\varphi = \int_0^{2\pi} d\varphi \int_0^1 (\rho - \rho^3)\,d\rho =$$

$$= \int_0^{2\pi} \left(\frac{\rho^2}{2} - \frac{\rho^4}{4} \right) \Big|_{\rho=0}^{\rho=1} d\varphi = \int_0^{2\pi} \left(\frac{1}{2} - \frac{1}{4} - 0 + 0 \right) d\varphi =$$

$$= \int_0^{2\pi} \frac{1}{4}\,d\varphi = \frac{1}{4}\varphi \Big|_{\varphi=0}^{\varphi=2\pi} = \boxed{\frac{\pi}{2}}$$

№ ③ способ. Замена!!!

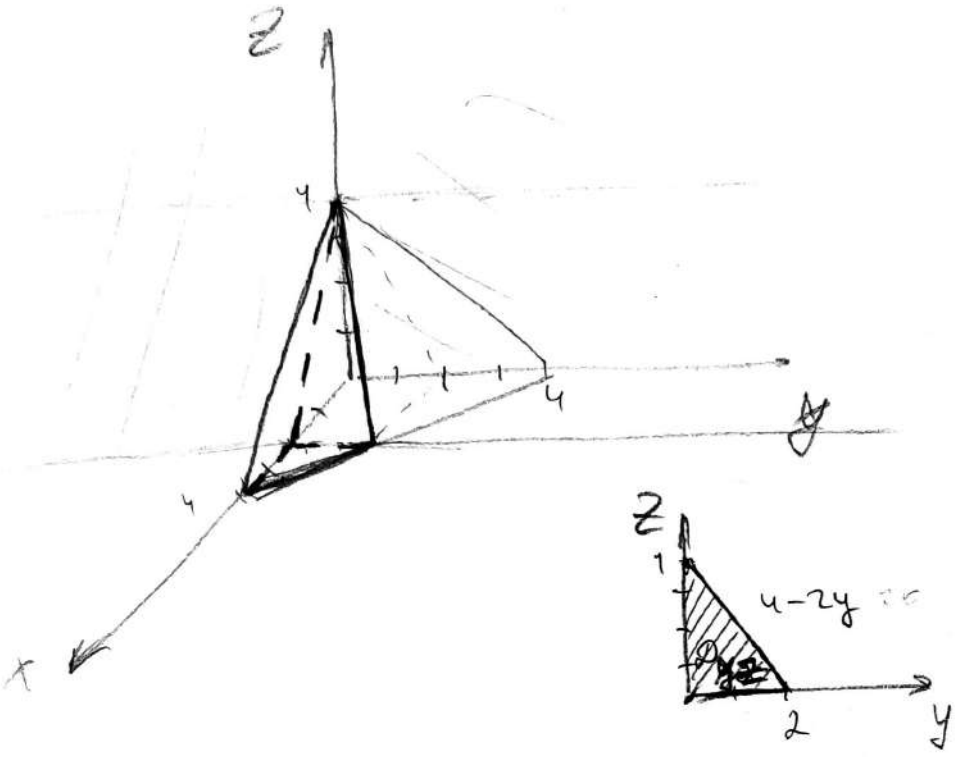
$$2\int_0^2 (y-2)^2\,d(y-2) = 2\,\frac{(y-2)^3}{3}\Big|_0^2 = \left\{ \begin{array}{l} t = y-2 \\ y = 0 \Rightarrow t = -2 \\ y = 2 \Rightarrow t = 0 \end{array} \right.$$

$$= 2\left(0 - \frac{8}{3}\right) = -\frac{16}{3}$$

$$= 2\int_{-2}^{0} t^2\,dt = 2\,\frac{t^3}{3}\Big|_{t=-2}^{t=0} = 2\,\frac{8}{3} = \boxed{\frac{16}{3}}$$

№3

$y=0,\ z=0,\ x+y+z=4,\ 2x+z=4$

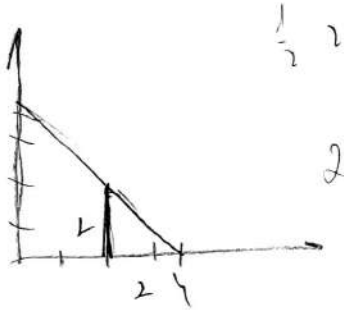


$x=0 \quad z=4$

$x=1 \quad z=2$

$z=0 \quad 2x=4$

$x=2$



$\frac{1}{2} \cdot 2 \cdot 2 = 2$

$2 \cdot 4 = 8$

$\frac{8}{3}$

$x+y+z=4$

$x = 4-y-z$

$2x+z=4$

$2x = 4-z$

$x = \frac{4-z}{2}$

$4-y-z = \frac{4-z}{2}$

$4-y-z-2+\frac{z}{2}=0$

$2-y-\frac{z}{2}=0$

$4-2y-z=0$

$z = 4-2y$

$$V = \iint_{D_{yz}} (x_2(y,z) - x_1(y,z))\,dy\,dz =$$

$$= \iint_{D_{yz}} \left(4-y-z-\frac{4-z}{2}\right) dy\,dz =$$

$$= \int_0^2 dy \int_0^{4-2y} \left(4-y-z-2+\frac{z}{2}\right) dz =$$

$$= \int_0^2 dy \int_0^{4-2y} \left(2-y-\frac{z}{2}\right) dz =$$

$$= \frac{1}{2} \int_0^2 dy \int_0^{4-2y} (4-2y-z)\,dz =$$

$$= \frac{1}{2} \int_0^2 \left((4-2y)z - \frac{z^2}{2}\right)\Big|_{z=0}^{z=4-2y} dy \;\textcircled{=}$$

№3 прод.

$$= \frac{1}{2}\int_0^2 \left((4-2y)(4-2y) - \frac{(4-2y)^2}{2}\right) dy =$$

$$= \frac{1}{2}\int_0^2 \left(\frac{(4-2y)^2}{2}\right) dy = \cancel{\text{...}}$$

$$= \frac{1}{2}\int_0^2 \frac{4(2-y)^2}{2} dy = \frac{1}{2}\int_0^2 2(2-y)^2 dy =$$

$$= \int_0^2 (2-y)^2 dy = -\int_0^2 (2-y)^2 d(2-y) =$$

$$= \begin{cases} t = 2-y \\ y=0 \Rightarrow t=2 \\ y=2 \Rightarrow t=0 \end{cases} = -\int_2^0 t^2 dt =$$

$$= -\frac{t^3}{3}\Big|_{t=2}^{t=0} = \cancel{\text{...}} -\left(0 + \frac{8}{3}\right) = \frac{8}{3}$$

№4

$$= \int_0^2 (y-2)^2 dy = \int_0^2 (y-2)^2 d(y-2) =$$

$$= \begin{cases} t = y-2 \\ y=0 \Rightarrow t=-2 \\ y=2 \Rightarrow t=0 \end{cases} = \int_{-2}^0 t^2 dt = \frac{t^3}{3}\Big|_{-2}^0 =$$

$$= \frac{8}{3}$$