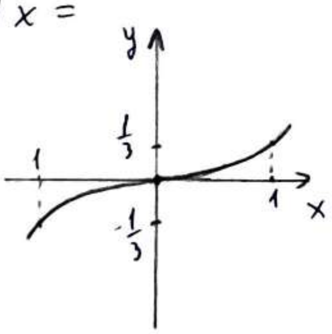
Занятие 14

6.520

$$y = \frac{1}{3}x^3 \qquad x \in [-1, 1]$$

$$S_x = 4\pi \int_0^1 \frac{x^3}{3}\sqrt{1+\left(\frac{3}{3}x^2\right)^2}\,dx = 4\pi\int_0^1 \frac{x^3}{3}\sqrt{1+x^4}\,dx =$$

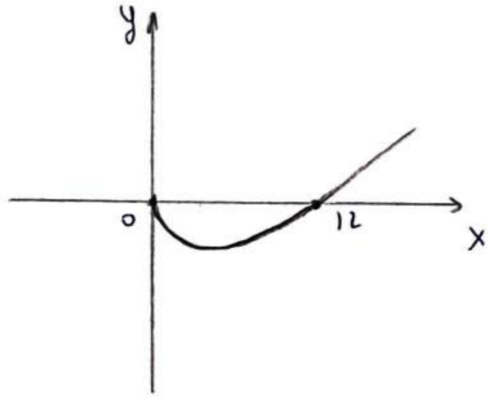


$$= \frac{4\pi}{3}\int_0^1 \sqrt{1+x^4}\,d\,\frac{x^4}{4} = \frac{4\pi}{3\cdot 4}\int_0^1 \sqrt{x^4+1}\,d\,(x^4+1) =$$

$$= \frac{\pi}{3}\cdot\frac{(x^4+1)^{\frac{3}{2}}}{3/2}\Bigg|_0^1 = \frac{2\pi}{9}\sqrt{(x^4+1)^3}\Bigg|_0^1 = \frac{2\pi}{9}\left(2\sqrt{2}-1\right)$$

6.521

$$y = \frac{1}{6}\sqrt{x}\,(x-12)$$



Точки пересечения с Ox:

$$\frac{1}{6}\sqrt{x}\,(x-12) = 0$$

$$\left[\begin{array}{l}\sqrt{x} = 0\\ x-12 = 0\end{array}\right. \qquad \left[\begin{array}{l} x = 0\\ x = 12\end{array}\right.$$

$$y' = \frac{1}{6}\left(x^{\frac{3}{2}} - 12x^{\frac{1}{2}}\right)' = \frac{1}{6}\left(\frac{3}{2}\cdot x^{\frac{1}{2}} - \frac{12}{2}x^{-\frac{1}{2}}\right) = \frac{1}{6}\left(\frac{3\sqrt{x}}{2} - \frac{6}{\sqrt{x}}\right) =$$

$$= \frac{\sqrt{x}}{4} - \frac{1}{\sqrt{x}} = \frac{x-4}{4\sqrt{x}}$$

$$S_x = \left|2\pi\int_0^{12}\frac{1}{6}\sqrt{x}\,(x-12)\sqrt{1+\left(\frac{(x-4)}{4\sqrt{x}}\right)^2}\,dx\right| = \cancel{\left|\frac{\pi}{3}\int_0 \sqrt{x}\,(x-12)\right.}$$

$$= \left|\frac{\pi}{3}\int_0^{12}\sqrt{x}\,(x-12)\sqrt{\frac{16x+x^2-8x+16}{16x}}\,dx\right| = \left|\frac{\pi}{3}\int_0^{12}\cancel{\sqrt{x}}(x-12)\,\frac{(x+4)}{4\cancel{\sqrt{x}}}\,dx\right| =$$

$$= \left|\frac{\pi}{12}\int_0^{12}\left(x^2-8x-48\right)dx\right| = \left|\frac{\pi}{12}\left(\frac{x^3}{3} - 8\,\frac{x^2}{2} - 48x\right)\Bigg|_0^{12}\right| =$$

$$= \left|-\frac{576\pi}{12}\right| = \left|-48\pi\right| = 48\pi$$

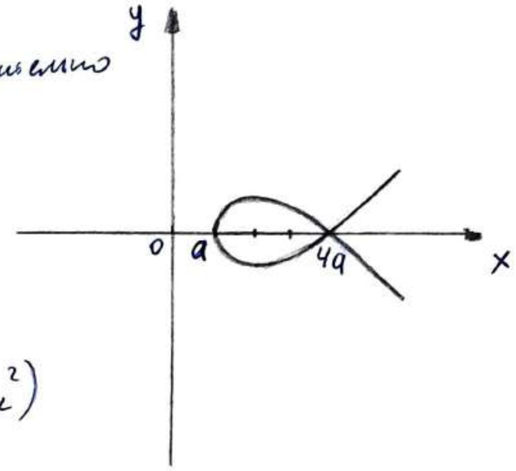
6.526

$$\begin{cases} x = a(t^2+1) \\ y = \frac{at}{3}(3-t^2) \end{cases}$$

$$x' = 2at$$

$$y' = \frac{a}{3}(3t-t^3)' = \frac{a}{3}(3-3t^2)$$

Точка самопересечения находится на оси Ox, петля симметрична относительно оси Ox.



Найдем точку самопересечения:

$$\begin{cases} x(t_1) = x(t_2) \\ y(t_1) = y(t_2) \\ t_1 \neq t_2 \end{cases}, \quad \begin{cases} \cancel{a}(t_1^2+1) = \cancel{a}(t_2^2+1) \\ \frac{\cancel{a}t_1}{\cancel{3}}(3-t_1^2) = \frac{\cancel{a}t_2}{\cancel{3}}(3-t_2^2) \\ t_1 \neq t_2 \end{cases}$$

$$\begin{cases} t_1^2 = t_2^2 \\ t_1(3-t_1^2) = t_2(3-t_2^2) \\ t_1 \neq t_2 \end{cases} \quad \begin{cases} t_1 = -t_2 \\ t_1(3-t_1^2) = t_2(3-t_2^2) \\ t_1 \neq t_2 \end{cases} \Rightarrow$$

$$\Rightarrow -t_2(3-t_2^2) = t_2(3-t_2^2) \ \Big|: t_2 \neq 0 \ (\text{т.к } t_2 \neq t_1)$$

$$-3+t_2^2 = 3-t_2^2$$

$$2t_2^2 = 6$$

$$t_2^2 = 3$$

$$t_{1,2} = \pm\sqrt{3}$$

Точки пересечения с Ox:

$$0 = \frac{at}{3}(3-t^2)$$

$$\left[\begin{array}{l} t = 0 \\ t = \pm\sqrt{3} \end{array}\right.$$

Т.к петля симметрична относительно Ox

$$S_x = 2\pi\int_0^{\sqrt{3}} \frac{at}{3}(3-t^2)\sqrt{(2at)^2 + \frac{a^2}{9}(3-3t^2)^2}\,dt =$$

$$= 2\pi \int_0^{\sqrt{3}} \frac{at}{3} (3-t^2) \sqrt{4a^2t^2 + \frac{a^2}{\cancel{9}}\cancel{9}(1-t^2)^2}\, dt =$$

$$= 2\pi \frac{a^2}{3} \int_0^{\sqrt{3}} t(3-t^2)\sqrt{4t^2+1-2t^2+t^4}\, dt =$$

$$= 2\pi \frac{a^2}{3} \int_0^{\sqrt{3}} t(3-t^2)\sqrt{t^4+2t^2+1}\, dt = \frac{2\pi a^2}{3}\int_0^{\sqrt{3}} t(3-t^2)\sqrt{(t^2+1)^2}\, dt =$$

$$= \frac{2\pi a^2}{3}\int_0^{\sqrt{3}} (3t-t^3)(t^2+1)\, dt = \frac{2\pi a^2}{3}\int_0^{\sqrt{3}} (3t^3+3t-t^5-t^3)\, dt =$$

$$= \frac{2\pi a^2}{3}\cdot\left(\frac{3t^4}{4} + 3\frac{t^2}{2} - \frac{t^6}{6} - \frac{t^4}{4}\right)\Bigg|_0^{\sqrt{3}} = \frac{2\pi a^2}{3}\left(\frac{t^4}{2} + 3\frac{t^2}{2} - \frac{t^6}{6}\right)\Bigg|_0^{\sqrt{3}} =$$

$$= \frac{2\pi a^2}{3}\cdot\frac{9}{2} = \frac{9\pi a^2}{3} = 3\pi a^2.$$

<u>6.529</u>

$r = 2a\sin\varphi$

$$S_x = 2\pi\int_0^{\pi} 2a\sin^2\varphi\sqrt{4a^2\sin^2\varphi + 4a^2\cos^2\varphi}\, d\varphi =$$

$$= 2\pi\int_0^{\pi} 4a^2\sin^2\varphi\, d\varphi = 8\pi a^2\int_0^{\pi}\sin^2\varphi\, d\varphi = 8\pi a^2\int_0^{\pi}\frac{1-\cos 2\varphi}{2}\, d\varphi =$$

$$= 2\pi a^2\int_0^{\pi}(1-\cos 2\varphi)\, d(2\varphi) = 2\pi a^2(2\varphi - \sin 2\varphi)\Big|_0^{\pi} =$$

$$= 4\pi^2 a^2$$