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„Техника интегрирования"

ИУ7-23Б

МАСЛОВА

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ДМИТРИЕВНА

КР №1

БИЛЕТ № 9

(1) $\int \frac{e^x}{1+e^{2x}}\,dx = \int \frac{de^x}{1+e^{2x}} = \frac{1}{1}\,\mathrm{arctg}\,\frac{e^x}{1} + C =$

$= \mathrm{arctg}\, e^x + C$

(2.) $\int x^2 \sqrt[5]{1+x^3}\,dx = \int \sqrt[5]{1+x^3}\, d\frac{x^3}{3} = \frac{1}{3}\int (1+x^3)^{\frac{1}{5}}\, d(x^3+1) =$

$= \frac{1}{3}\cdot\frac{(x^3+1)^{\frac{6}{5}}}{6/5} + C = \frac{5}{18}(x^3+1)^{\frac{6}{5}} + C$

(3.) $\int e^{\sqrt{2x}}\,dx = \left| \begin{array}{l} t=\sqrt{2x} \\ t^2 = 2x \\ x = \frac{t^2}{2} \\ dx = d\frac{t^2}{2} = t\,dt \end{array} \right| = \int t\cdot e^t\,dt = \int t\,de^t =$

$= t\cdot e^t - \int e^t\,dt = t\cdot e^t - e^t + C = e^t(t-1) + C = e^{\sqrt{2x}}(\sqrt{2x}-1) + C$

(4) $\int \frac{2x-8}{\sqrt{1-x-x^2}}\,dx = \int \frac{2x-8}{\sqrt{-(x^2+x-1)}}\,dx = \int \frac{2x-8}{\sqrt{-(x^2+2\cdot\frac{1}{2}x+\frac{1}{4}-\frac{5}{4})}}\,dx =$

$= 2\int \frac{x-4}{\sqrt{\frac{5}{4}-(x+\frac{1}{2})^2}}\,dx = 2\int \frac{x+\frac{1}{2}-\frac{9}{2}}{\sqrt{\frac{5}{4}-(x+\frac{1}{2})^2}}\,d(x+\tfrac{1}{2}) = \left| t = x+\tfrac{1}{2} \right| =$

$= 2\int \frac{t-\frac{9}{2}}{\sqrt{\frac{5}{4}-t^2}}\,dt = 2\left(\int \frac{t\,dt}{\sqrt{\frac{5}{4}-t^2}} - \frac{9}{2}\int \frac{1}{\sqrt{\frac{5}{4}-t^2}}\,dt \right) =$

$= 2\left(-\frac{1}{2}\int \frac{d(\frac{5}{4}-t^2)}{\sqrt{\frac{5}{4}-t^2}} - \frac{9}{2}\arcsin\frac{t}{\sqrt{\frac{5}{4}}} \right) = -2\sqrt{\frac{5}{4}-t^2} - 9\arcsin\frac{2t}{\sqrt{5}} + C =$

$= -2\sqrt{1-x-x^2} - 9\arcsin\frac{2x+1}{\sqrt{5}} + C$

(5.) $\int \mathrm{tg}^{\frac{3}{2}}x\,\sec^4 x\,dx = \int \mathrm{tg}^{\frac{3}{2}}x \cdot \frac{1}{\cos^2 x}\cdot\frac{1}{\cos^2 x}\,dx =$

$= \int \mathrm{tg}^{\frac{3}{2}}x\,(1+\mathrm{tg}^2 x)\,d\,\mathrm{tg}\,x = |t = \mathrm{tg}\,x| = \int t^{\frac{3}{2}}(1+t^2)\,dt =$

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(5.) (продолжение)

$$= \int \left(t^{\frac{3}{2}} + t^{\frac{7}{2}}\right) dt = \frac{t^{\frac{5}{2}}}{5/2} + \frac{t^{\frac{9}{2}}}{9/2} + C =$$

$$= \frac{2}{5} t^{\frac{5}{2}} + \frac{2}{9} t^{\frac{9}{2}} + C = \frac{2}{5} tg^{\frac{5}{2}} x + \frac{2}{9} tg^{\frac{9}{2}} x + C$$

(6.)
$$\int \frac{dx}{1+3\cos x} = \left| \begin{array}{l} t = tg\frac{x}{2} \\ x = 2\operatorname{arctg} t \\ dx = \frac{2\,dt}{1+t^2} \\ \cos x = \frac{1-t^2}{1+t^2} \end{array} \right| = \int \frac{2\,dt}{(1+t^2)\left(1+3\,\frac{1-t^2}{1+t^2}\right)} =$$

$$= 2\int \frac{dt}{1+t^2+3(1-t^2)} = 2\int \frac{dt}{1+t^2+3-3t^2} = 2\int \frac{dt}{-2t^2+4} = 2\int \frac{dt}{2(2-t^2)} =$$

$$= \int \frac{dt}{2-t^2} = -\int \frac{dt}{t^2-2} = -\frac{1}{2\sqrt{2}} \ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| + C =$$

$$= -\frac{1}{2\sqrt{2}} \ln\left|\frac{tg\frac{x}{2} - \sqrt{2}}{tg\frac{x}{2} + \sqrt{2}}\right| + C$$

(7.)
$$\int \frac{2x^2-3x-3}{(x-1)(x^2-2x+5)}\,dx = \int \left(\frac{3x-2}{x^2-2x+5} - \frac{1}{x-1}\right) dx \;(=)$$

$$\frac{2x^2-3x-3}{(x-1)(x^2-2x+5)} = \frac{A}{(x-1)} + \frac{Bx+C}{(x^2-2x+5)} = \frac{Ax^2-2Ax+5A+Bx^2+Cx-Bx-C}{(x-1)(x^2-2x+5)}$$

$$\begin{array}{c|l} x^2 & A+B = 2 \\ x & -2A+C-B = -3 \\ 1 & 5A-C = -3 \end{array} \Rightarrow \begin{cases} A+B = 2 \\ -2A-B+C = -3 \\ 5A - C = -3 \end{cases} ; \quad \begin{cases} B = 2-A \\ C = 5A-3 \\ -2A-2+A+5A+3 = -3 \;* \end{cases}$$

$$*\;\; 4A+4=0 \qquad A+1=0 \qquad A=-1 \qquad \begin{cases} A = -1 \\ B = 3 \\ C = -2 \end{cases}$$

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(7) (продолжение)

$$= 3\int \frac{x-\frac{2}{3}}{x^2-2x+1+4}\,dx - \int \frac{1}{x-1}\,d(x-1) =$$

$$= 3\int \frac{(x-1)+\frac{1}{3}}{(x-1)^2+4}\,d(x-1) - \ln|x-1| =$$

$$= 3\left(\int \frac{x-1}{(x-1)^2+4}\,d(x-1) + \frac{1}{3}\int \frac{1}{(x-1)^2+4}\,d(x-1)\right) - \ln|x-1| =$$

$$= 3\left(\frac{1}{2}\ln\left((x-1)^2+4\right) + \frac{1}{3}\cdot\frac{1}{2}\,\mathrm{arctg}\,\frac{x-1}{2}\right) - \ln|x-1| + C =$$

$$= \frac{3}{2}\ln(x^2-2x+5) + \frac{1}{2}\,\mathrm{arctg}\,\frac{x-1}{2} - \ln|x-1| + C$$

(8) $$\int \frac{x^5}{\sqrt{x^2-1}}\,dx = \int \frac{x^4}{\sqrt{x^2-1}}\,d\frac{x^2}{2} = \frac{1}{2}\int \frac{x^4}{\sqrt{x^2-1}}\,d(x^2-1) =$$

$$= \left|\begin{array}{l} t=\sqrt{x^2-1} \\ t^2 = x^2-1 \\ x^2 = t^2+1 \\ x^4 = (t^2+1)^2 \end{array}\right| = \frac{1}{2}\int \frac{(t^2+1)^2}{t}\,dt^2 = \frac{1}{2}\int \frac{(t^4+2t^2+1)\cdot 2t}{t}\,dt =$$

$$= \int (t^4+2t^2+1)\,dt = \frac{t^5}{5} + 2\frac{t^3}{3} + t + C =$$

$$= \frac{\sqrt{(x^2-1)^5}}{5} + 2\frac{\sqrt{(x^2-1)^3}}{3} + \sqrt{x^2-1} + C = \frac{1}{15}\sqrt{x^2-1}\,\left(3(x^2-1)^2 + \right.$$

$$\left. + 10(x^2-1)+15\right) + C = \frac{1}{15}\sqrt{x^2-1}\,(3x^4+4x^2+8) + C$$