41

Завфа — уделал 5

Реализовал
памяти (x_1, x_2, x_3) перебор

$N = 10 \cdot 9 \cdot 8$

$N_A \; 8 \cdot 7 \cdot 6$

X	0	1	2
p	$\frac{7}{15}$	$\frac{7}{15}$	$\frac{1}{15}$

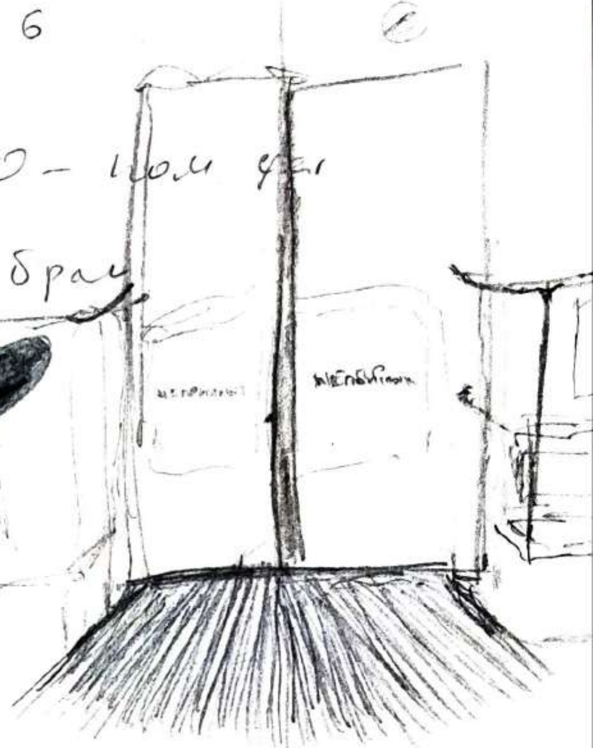
1 . 10 — ном дет

1, 2 — брак

1 Исход $\overset{2\cdot}{\{x_1, x_2, x_3\}}$

сочетания
без
повт из 10 по 3

$$C_{10}^{3} = \frac{10!}{3!\,7!}$$



2. $X = 0$

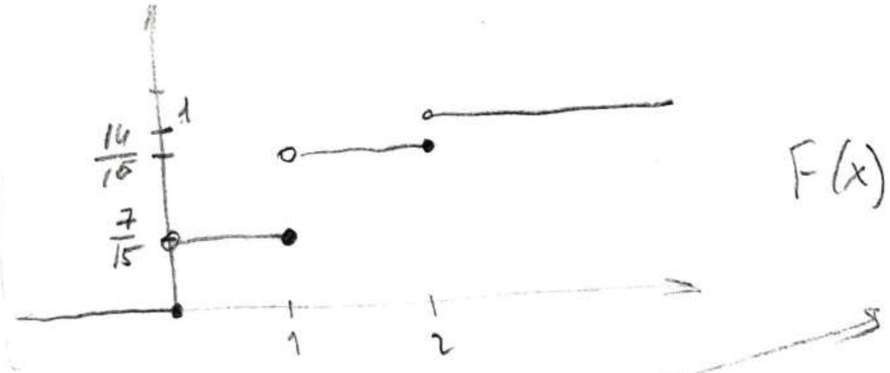
$$N_A = C_8^3 = \frac{8!}{3!\,5!} \qquad P_A = \frac{8!\,\cancel{3!}\,7!}{\cancel{3!}\,5!\,10!} = \frac{7}{15}$$

3. $X = 1$

$$N_B = 2 \cdot C_8^2 = \cancel{2}\,\frac{8!}{\cancel{2}!\,6!} = \qquad P_B = \frac{8!\,3!\,7!}{6!\,10!} = \frac{7}{15}$$

4. $X = 2$

$$N_C = C_8^1 = \frac{8!}{7!} \qquad \frac{8!\,3!\,\cancel{7!}}{\cancel{7!}\,10!} = \frac{1}{15}$$



$$F(x) = \begin{cases} 0, & x \le 0 \\ \frac{7}{15}, & 0 < x \le 1 \\ \frac{14}{15}, & 1 < x \le 2 \\ 1, & x > 2 \end{cases}$$

Ответ

①

4.2

$$P = \{X = k\} = \frac{\lambda^k}{k!} e^{-\lambda}, \quad k = 0, 1, ..$$

$\lambda = 5$

а) $P\{X=2\} = \frac{5^2}{2!} e^{-5} = \frac{25}{2} e^{-5} \approx 0,08$

назали

б) $P\{X \leq 2\} = P\{X=0\} + P\{X=1\} + P\{X=2\} =$

$= \frac{5^0}{0!} e^{-5} + \frac{5^1}{1!} e^{-5} + \frac{5^2}{2!} e^{-5} =$

$= e^{-5}\left(1 + 5 + \frac{25}{2}\right) = 18,5e^{-5} \approx 0,12$

в) $P\{X \geq 2\} = P\{X=2\} + P\{X=3\} + \ldots =$

$= P\{X=2\} + 1 - P\{X \leq 2\} =$

$= \frac{25}{2} e^{-5} + 1 - 18,5 e^{-5} \approx 0,96$

4.3

$$f(x) = \begin{cases} \frac{c}{\sqrt[3]{x}}, & x \in (0,2) \\ 0, & x \notin (0,2) \end{cases}$$

а) Ус-е нормирани:

$$x^{1-\frac{1}{3}} = \frac{x^{\frac{2}{3}}}{\frac{2}{3}}$$

$$1 = \int_{-\infty}^{+\infty} f(x)\,dx = \int_0^2 \frac{c}{\sqrt[3]{x}}\,dx = c\int_0^2 x^{-\frac{1}{3}}\,dx =$$

$$= \frac{3c\sqrt[3]{x^2}}{2}\Big|_0^2 = \frac{3c\sqrt[3]{4}}{2} = 1$$

$$c = \frac{\sqrt[3]{2}}{3}$$

$3c\sqrt[3]{4} = 2$

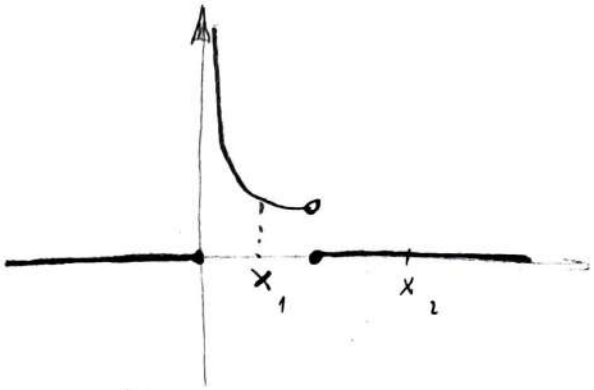
$c\sqrt[3]{4} = \frac{2}{3}$

$$c = \frac{2}{3\sqrt[3]{4}} = \frac{\cancel{2}\sqrt[3]{4}\sqrt[3]{4}}{3\cancel{4}_2} = \frac{\sqrt[3]{16}}{6} = \frac{2\sqrt[3]{2}}{6} = \frac{\sqrt[3]{2}}{3}$$

б)

$F\ f(x)\ ?\ !$

$$F(x) = \int_{-\infty}^{x} f(t)\,dt$$



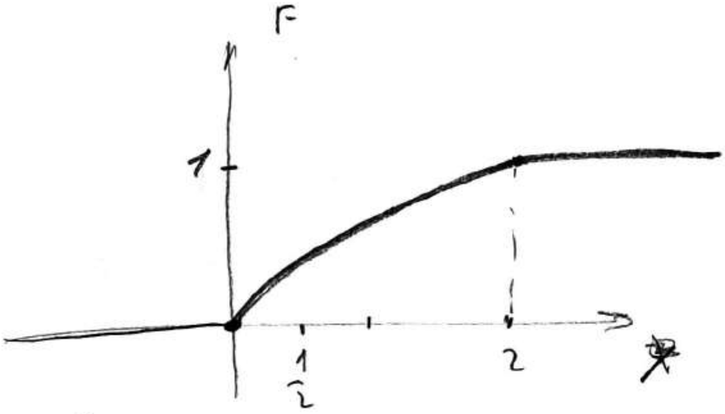
$x \leq 0 \quad F(x) = 0$

$0 < x < 2 \quad F(x) = \int_{-\infty}^{0} 0 + \int_{0}^{x} f(t)\,dt = \int_{0}^{x} \frac{c}{\sqrt[3]{t}}\,dt =$

$= \frac{3c\sqrt[3]{t^2}}{2}\Big|_0^x = \frac{\cancel{3}\frac{\sqrt[3]{2}}{\cancel{3}}}{2}\sqrt[3]{x^2} = \frac{\sqrt[3]{2}}{2}\sqrt[3]{x^2}$

$x \geq 2$ [illegible]

$F(x) = 1$



в) $P\{\frac{1}{2} \leq X \leq \frac{3}{2}\} = F(\frac{3}{2}) - F(\frac{1}{2}) = \frac{\sqrt[3]{36}}{4} - \frac{\sqrt[3]{4}}{4} =$

$= \frac{\sqrt[3]{36} - \sqrt[3]{4}}{4} \approx 0,43$

$\Phi_0^{\,0}(0) - \Phi_0(-\infty)^{-\frac{1}{2}} = \frac{1}{2}$

<u>№44</u>

$m = 2; \quad \sigma^2 = 100$

$P\{X \leq 2\} = P\{-\infty < X \leq 2\} = \Phi\left(\frac{2-2}{10}\right) - \Phi\left(\frac{-\infty-2}{10}\right) =$

$= \Phi(0) - \Phi(-\infty) = \frac{1}{2}$?

4.5

$$X \sim N(0, \sigma^2)$$

$$P\{-0.3 < X < 0.3\} = 0.5$$

$$\Phi_0\left(\frac{0.3-0}{\sigma^2}\right) - \Phi_0\left(\frac{-0.3}{\sigma^2}\right) = 0.5$$

$$\Phi_0\left(\frac{0.3}{\sigma^2}\right) + \Phi_0\left(\frac{0.3}{\sigma^2}\right) = 0.5$$

$$2\Phi_0\left(\frac{0.3}{\sigma^2}\right) = 0.5$$

$$\Phi_0\left(\frac{0.3}{\sigma^2}\right) = \frac{1}{4}$$

$$\frac{0.3}{\sigma^2} = 0.67$$

$$\sigma^2 = \frac{0.3}{0.67} = \frac{30}{67}$$

5.1

а)

X \ Y	10	20	30	40	P_x
0.5	0.05	0.12	0.08	0.04	0.29
2.5	0.09	0.3	0.11	0.21	0.71
P_y	0.14	0.42	0.19	0.25	1

б) $F(2.5, 25) = P(X < 2.5, Y < 25) =$

$= 0.05 + 0.12 = 0.17$

$F(9, 11) = P\{\underset{\substack{\uparrow \\ \text{фоисов}}}{X < 9}, Y < 11\} = P(Y < 11) = P(Y = 10) =$

$= 0.14$

в)

$P\{2 \le X < 9, 10 \le Y \le 30\} =$

$= 0.09 + 0.3 + 0.11 =$

$= 0.5$

г) $F(x, y) = F_X(x) \cdot F_Y(y)$

$p_{ij} = p_{x_i} \cdot p_{y_j}$

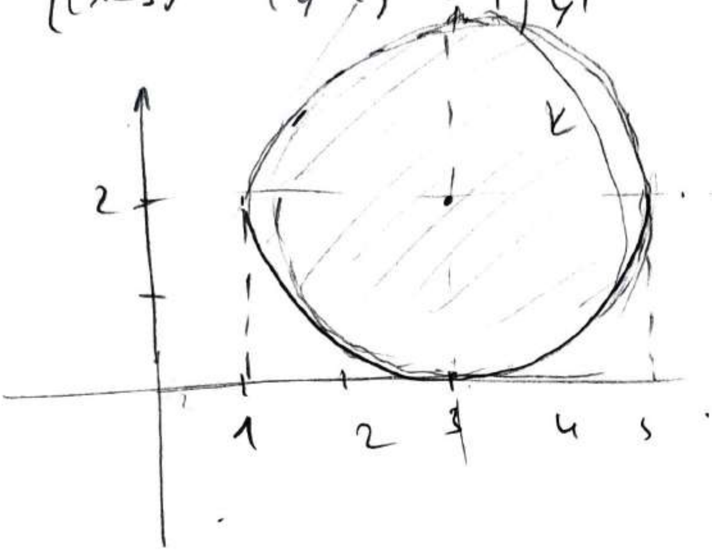
$0.05 \ne 0.29 \cdot 0.14 \Rightarrow$

$\Rightarrow X$ и Y - зависи

5.2

$$f(x,y)=\begin{cases}0, & (x,y)\notin k\\ C\left(2-\sqrt{(x-3)^2+(y-2)^2}\right) & (x,y)\in k\end{cases}$$

$$k=\{(x-3)^2+(y-2)^2\le 4\}$$



$$\Rightarrow (x-3)^2+(y-2)^2=4$$

$$(y-2)^2=4-(x-3)^2$$

$$(y-2)=\pm\sqrt{4-(x-3)^2}$$

$$y=2\pm\sqrt{4-(x-3)^2}$$

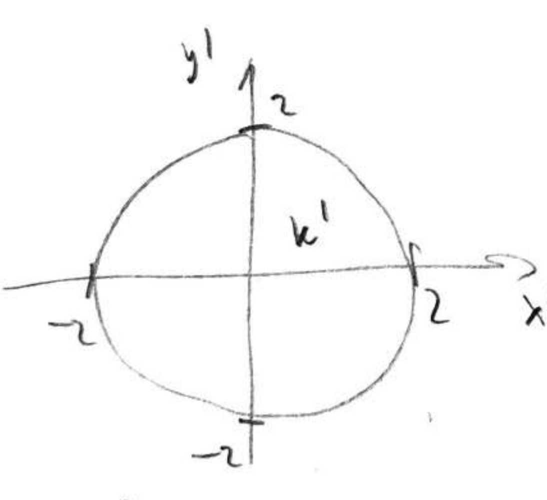
a) Усл. норм.

$$1=\iint\limits_{\mathbb{R}^2} f(x,y)\,dx\,dy=\iint\limits_{k} C\left(2-\sqrt{(x-3)^2+(y-2)^2}\right)dx\,dy=$$

$$=C\int_1^5 dx\int_{2-\sqrt{4-(x-3)^2}}$$

$$=\left\{\begin{array}{ll} x'=x-3; & x=x'+3\\ y'=y-2 & y=y'+2\end{array}\right.$$

$$J=\begin{vmatrix}\frac{\partial(x'+3)}{\partial x'} & \frac{\partial(x'+3)}{\partial y'}\\ \frac{\partial(y'+2)}{\partial x'} & \frac{\partial(y'+2)}{\partial y'}\end{vmatrix}=\begin{vmatrix}1 & 0\\ 0 & 1\end{vmatrix}=1$$



$$=C\iint\limits_{k'}\left(2-\sqrt{x'^2+y'^2}\right)dx'\,dy'=$$

$$=C\int_0^{2\pi}d\varphi\int_0^2\left(2-\sqrt{\rho^2\cos^2\varphi+\rho^2\sin^2\varphi}\right)\rho\,d\rho=C\int_0^{2\pi}d\varphi\int_0^2(2-\rho)\rho\,d\rho\ominus$$

$$\ominus\ C\int_0^{2\pi}\frac{4}{3}\,d\varphi=$$

$$=C\,\frac{4}{3}\varphi\Big|_0^{2\pi}=$$

$$=C\,\frac{8\pi}{3}=1$$

$$\boxed{C=\frac{3}{8\pi}}$$

$\boxed{c = \frac{3}{8\pi}}$

б) $f_X(x) = \int\limits_{-\infty}^{+\infty} f(x,y)\,dy = \begin{cases} 0, & x \notin (3,5) \\ \cancel{\int \frac{3}{8\pi}(-\pi-)}\, dy\ *, & x \in (3,5) \end{cases}$

ВПСК $y = \rho\sin\varphi$

$x = \rho\cos\varphi$

$$\int\limits_{2-\dots}^{2+\dots} \frac{3}{8\pi}\left(2-\sqrt{(x-3)^2+(y-2)^2}\right)dy$$

$* \ \cancel{\text{ПСК}}\ \cancel{\frac{3}{8\pi}} \left\{ \begin{array}{l} x = x'+3 \\ y = y'+2 \end{array} \right\} = \cancel{2+\sqrt{4-(x'^2+y'^2)}^2}$

$$= \int\limits_{2-\sqrt{4-x'^2}}^{2+\sqrt{4-x'^2}} 2 - \sqrt{x'^2+y'^2}\; dy' \;+2) \Big| \; x = \int\limits_{2-\sqrt{4-\rho^2\sin^2\varphi}}^{2+\sqrt{4-\rho^2\cos^2\varphi}} 2-\rho \; d\rho\sin\varphi$$

$$f(x,y) = \begin{cases} 0, & (x,y) \notin K \\ \frac{3}{8\pi}\left(2-\sqrt{(x-3)^2+(y-2)^2}\right), & (x,y) \in K \end{cases}$$

$x = x'+3 \quad y = y'+2$

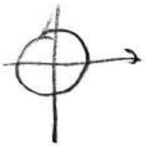
$d\rho\cos\varphi \; d\rho\sin\varphi =$

$= \rho$

$$f(x',y') = \begin{cases} 0, & (x',y') \in K' \\ \frac{3}{8\pi}\cdot\left(2-\sqrt{x'^2+y'^2}\right), & (x,y) \in K \end{cases}$$

В ПСК

$$f(\cancel{x}\rho\cos\varphi, \rho\sin\varphi) = \begin{cases} 0, & (\rho\cos\varphi, \rho\sin\varphi) \notin K \\ \frac{3}{8\pi}\left(2-\rho\right), & (\rho\cos\varphi, \rho\sin\varphi) \in K \end{cases}$$



а) $\cancel{\iint_{\mathbb{R}^2}}$ б) $f_X(x) = \int\limits_{-\infty}^{+\infty} f(x,y)\,dy = \Big\{ \; 0, \; x \notin (3,5)$

$$б)\ f_x(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy = \begin{cases} 0, & x \notin (1,5) \\ \displaystyle\int_{2-\sqrt{4-(x-3)^2}}^{2+\sqrt{4-(x-3)^2}} \frac{3}{8\pi}\left(2-\sqrt{(x-3)^2+(y-2)^2}\right) dy & x \in (1,5) \end{cases} =$$

$$= \left\{\begin{array}{l} \text{т.к } x \text{ от } y = const \\ a = 2-\sqrt{4-(x-3)^2} \\ b = 2+\sqrt{4-(x-3)^2} \\ (x-3)^2 = c \end{array}\right\} = \frac{3}{8\pi}\int_a^b \left(2-\sqrt{c+(y-2)^2}\right) dy =$$

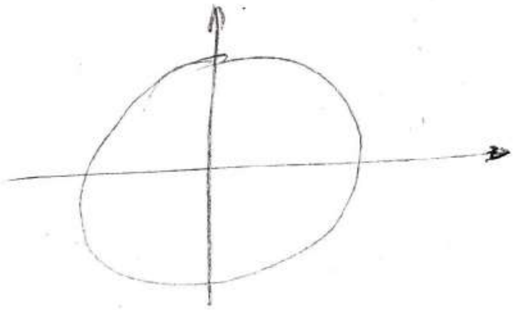
$$= \frac{3}{8\pi}\left(\left(\int_a^b 2\,dy\right) - \int_a^b \sqrt{c+(y-2)^2}\,d(y-2)\right) =$$

$$= \frac{3}{8\pi}\left(2y\Big|_a^b - \frac{1}{2}\left((y-2)\sqrt{c+(y-2)^2} + c\ln\left(\sqrt{c+(y-2)} + y-2\right)\right)\right)\Bigg|_a^b$$

$=$

Мехов!

$f = 2 - \rho$



$f(\rho, \varphi) =$

$(x-3)^2 + (y-2)^2 < 4$

$x'^2 + y'^2 < 4$

$\rho^2\cos^2\varphi + \rho^2\sin^2\varphi < 4$

$\rho^2 < 4$

$\rho < 2$

2) ~~не [illegible]~~
зависимые, т.к
область – не прямоуг.

$$f(\rho,\varphi) = \begin{cases} 0, & \rho > 2, \ \varphi - \text{любое} \\ C(2-\rho), & \rho < 2, \ \varphi - \text{любое} \end{cases}$$

$\pi R^2 = \pi \cdot 4$

$$1 = \iint f(\rho,\varphi)\, d\rho\, d\varphi = \iint_T C(2-\rho)\, d\rho\, d\varphi =$$

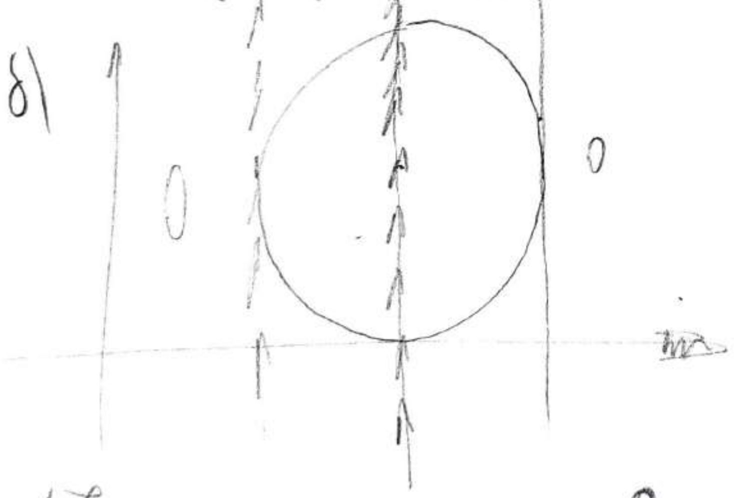
$$= \int_{-\infty}^{+\infty} d\varphi \int_0^2 C(2-\rho)\, d\rho = C \int_{-\infty}^{+\infty} 2\, d\varphi = 4\pi C = 1$$

$$C = \frac{1}{4\pi}$$

~~$\int_0^{+\infty} f(\rho)\, d\rho = C \int_0^2 (2-\rho)\, d\rho = 2C = 1$~~

~~$C = \frac{1}{2}$~~

б)



[redacted]

$$f_X(x) = \int_{-\infty}^{+\infty} f(x,y)\, dy =$$

$$\frac{3}{8\pi}(2-\rho)\overset{\sin\varphi}{d\rho}$$

5.3

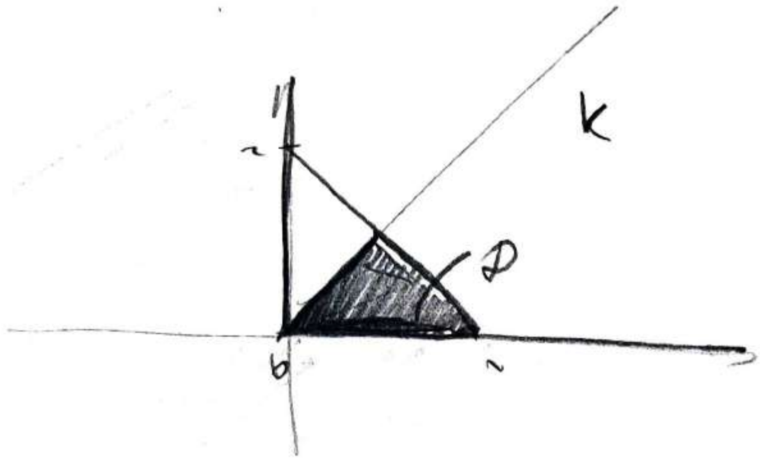
$$f(x,y) = \begin{cases} 0, & x \le 0, \text{или } y \le 0 \\ C e^{-4x-2y}, & x>0, y>0 \end{cases}$$

$\circledast\ (e^{4x}-1)(-e^{2y}) + (e^{4x}-1) =$

$y = 2 - x$

$$\frac{-4+4e^{2y}}{e^{2y+4t_1}}$$

$= (e^{4x}-1)\cdot(1-e^{2y})$



д) если $f_x(x)\cdot f_y(y) = f(x,y)$ незав

$$1 = \iint_{\mathbb{R}^2} f(x,y)\,dx\,dy = \iint_K Ce^{-4x-2y}\,dx\,dy =$$

$$= \int_0^{+\infty} dx \int_0^{+\infty} Ce^{-4x-2y}\,dy = C\int_0^{+\infty}\frac{e^{-4x}}{2}\,dx = \frac{C}{8} = 1 \quad \boxed{C=8}$$

$$б)\ F(x,y) = \int_{-\infty}^{x} dt_1 \int_{-\infty}^{y} f(t_1,t_2)\,dt_2 = \begin{cases} 0, & x \le 0\ y \le 0 \\ \int_0^x dt_1 \int_0^y Ce^{-4t_1-2t_2}\,dt_2 = & x>0, y>0 \end{cases}$$

Нафига? А главное - зачем?

$$= \begin{cases} \int_0^x \left(-4e^{4t_1-2y}+4e^{4t_1}\right)dt_1 \ \circledast \end{cases}$$

$$в)\ \boxed{f_x(x)} = \int_{-\infty}^{+\infty} f(x,y)\,dy = \begin{cases} 0, & x \le 0,\ y \le 0 \\ \int_0^{+\infty} Ce^{-4x-2y}\,dy, \ \boxed{= 4e^{-4x}} & x>0,\ y>0 \end{cases}$$

$$\boxed{f_y(y)} = \int_{-\infty}^{+\infty} f(x,y)\,dx = \begin{cases} 0, & x \le 0,\ y \le 0 \\ \int_0^{+\infty} Ce^{-4x-2y}\,dx \ \boxed{2e^{-2y}} & x>0\ y>0 \end{cases}$$

$$г)\ P\{(X,Y)\in D\} = \iint_D f(x,y)\,dx\,dy = \int_0^1 dy \int_y^{2-y} 8e^{-4x-2y}\,dx =$$

$$= \int_0^1 2e^{-6y} - 2e^{2y-8}\,dy = \frac{1}{3} + \frac{1}{e^8} - \frac{4}{3e^6} \approx 0{,}33$$

⑧

$2e^{-2y}\cdot 4e^{-4x} = \boxed{8e^{-4x-2y}}$

$P\{\underset{\underset{D}{\parallel}}{(x-3)^2 - (y-2)^2 < 1}\} = \iint_D \underset{\uparrow}{f(x,y)}\,dx\,dy =$

везде внутри K =)

$= \int$ перенос $(x \to x', y') =$ используем $C(r-\sqrt{\dots}$

$= \int . \pi c u \int =$ То чаш

5.2

б) ??? не интеграл, а кошмар

5.4

$x \backslash y$	1 0.1	2 0.15	3 0.20	P_x
0.3	0.25	0.15	$\boxed{0.32}$	0,72
0.6	0.10	0.05	0.13	0.28
P_y	0,35	0,2	0.45	1

$P\{X = x_i \mid y = y_j\}$

$\pi_{i1} = \dfrac{p_{i1}}{p_{y1}}$ $\qquad \pi_{i2} = \dfrac{p_{i2}}{p_{y2}}$

а)

	$x \backslash y$	1 0.1	2 0.15	3 0.20
1	0.3	$\frac{0.25}{0.35} = \frac{5}{7}$	$\frac{0.15}{0.2}$	$\frac{0.32}{0.45}$
2	0.6	$\frac{0.1}{0.35} = \frac{2}{7}$	$\frac{0.05}{0.2}$	$\frac{0.13}{0.45}$
		1	1	1

условн. распр-е X при y = 0.1 —||—

б)

$y \backslash x$	0.3	0.6
0.1	$\frac{0.25}{0.72}$	$\frac{0.10}{0.28}$
0,15	$\frac{0.15}{0.72}$	$\frac{0.05}{0.28}$
0.20	$\frac{0.32}{0.72}$	$\frac{0.13}{0.28}$

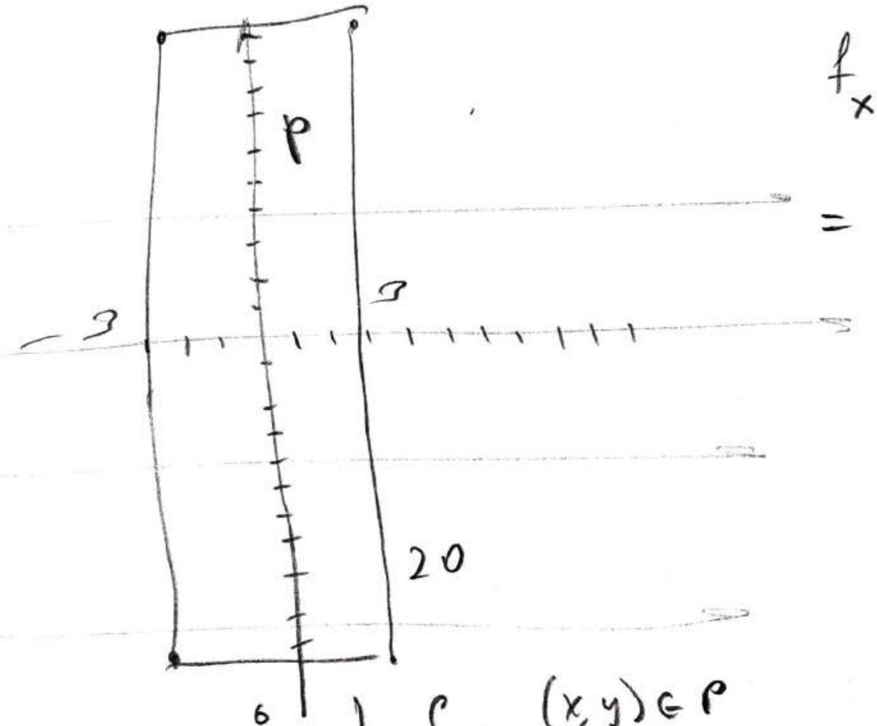
в)

$$\pi_{13} = P\{X=1 \mid Y=0\} = \frac{32}{45} \neq 0.\cancel{7}22$$

$P_{X_1}\{X=0\}$

???

5.5



$$f_X(x \mid Y=y) = \frac{f(x,y)}{f_Y(y)}$$

$$A \ \cancel{A} \ f(x,y) = \begin{cases} C, & (x,y)\in P \\ 0, & \text{иначе} \end{cases}$$

$$\int_{-3}^{3} dx \int_{-10}^{10} c\, dy$$

1) Усл-е нормировки

$$1 = \iint_{\mathbb{R}^2} f(x,y)\,dx\,dy = \iint_P c\,dx\,dy = C\cdot \text{Площ}(P) =$$

$$= C\cdot 120 = 120C$$

$$120C = 1$$

$$C = \frac{1}{120}$$

$$f(x,y) = \begin{cases} \frac{1}{120}, & (x,y)\in P \\ 0, & \text{иначе} \end{cases}$$

2) $$f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx = \begin{cases} 0, & \text{если } y\notin(-10,10) \\ \int_{-3}^{3} \frac{1}{120}\,dx, & y\in(-10,10) \end{cases} =$$

$$= \begin{cases} \frac{1}{20}, \text{ если } y \in (-10, 10) \\ 0, \text{ иначе.} \end{cases}$$

$$f_x(X|Y=y) = \frac{f(x,y)}{f_Y(y)} = |f_Y(y) \neq 0| =$$

$$= \begin{cases} \text{не опр}, & y \notin (-10, 10) \\ 20 f(x,y), & y \in (-10, 10) \end{cases} = \begin{cases} \text{не, опр}, & y \notin (-10, 10) \\ 0, & y \in (-10, 10) \\ & x \notin (-3, 3) \\ \frac{20}{120} = \frac{2}{12} = \frac{1}{6}, & y \in (-10, 10) \\ & x \in (-3, 3) \end{cases}$$

$$f(Y|X=x) = \frac{f(x,y)}{f_x(x)}$$

$$f_x(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy = \Big\{$$

$$f_Y(y|X=x) \neq f_Y(y) \Rightarrow \text{зависимы}$$

5.6 $f(x,y) = \begin{cases} Cy, & (x,y) \in D \\ 0, & (x,y) \notin D \end{cases}$

а) усл-е норм

$$1 = \iint_{R^2} f(x,y)\,dx\,dy =$$

$$= \iint_D Cy\,dx\,dy =$$

$$= \int_0^1 dx \int_{x^2}^1 Cy\,dy = \frac{2C}{5} = 1 \qquad \boxed{C = \frac{5}{2}}$$

б)

$$f_X(x|Y=y)=\frac{f(x,y)}{f_Y(y)}$$

$$f_Y(y)=\int_{-\infty}^{+\infty} f(x,y)\,dx=\left\{\begin{array}{l} 0 \text{ если } y\notin(0,1) \\ \int_0^{\sqrt{y}} \frac{5}{2}y\,dx\ ,\ y\in(0,1)\end{array}\right|=$$

$$=\left\{\begin{array}{l}\frac{5}{2}yx\Big|_1^{\sqrt{y}}\ ,\ y\in(0,1)\\ 0\ ,\text{ иначе}\end{array}\right.=\left\{\begin{array}{l}\frac{5}{2}y\sqrt{y}-\frac{5}{2}y\ ,\text{ иначе}\\ 0,\ y\notin(0,1)\end{array}\right.$$

$$=\left\{\begin{array}{l}\frac{5}{2}y(\sqrt{y}-1),\\ 0\end{array}\right.$$

$\square$

в) $f_Y(y|X=x)=\frac{f(x,y)}{f_X(x)}$

$$f_X(x)=\int_{-\infty}^{+\infty} f(x,y)\,dy=\left\{\begin{array}{l}0,\text{ если } x\notin(0,1)\\ \int_{x^2}^{1}\frac{5}{2}y\,dy\ ,\ x\in(0,1)\end{array}\right.$$

$$=\left\{\begin{array}{l}0,\text{ если } x\notin\\ \frac{5y^2}{4}\Big|_{x^2}^{1}=\frac{5}{4}-\frac{5x^4}{4}=\frac{5}{4}(1-x^4)\end{array}\right.$$

Разработки

$$*\ f_X(x|Y=y)=\frac{f(x,y)}{f_Y(y)}=\left\{\begin{array}{l}\text{не опр.},\ y\notin(0,1)\\ \dfrac{f(x,y)}{\frac{5}{2}y(\sqrt{y}-1)}\ ,\ y\in(0,1)\end{array}\right.=$$

$$=\left\{\begin{array}{l}\text{не опр.},\ y\notin(0,1)\\ y(\sqrt{y}-1)\ ,\ y\in(0,1)\ ,\ x\in(0,\sqrt{y})\\ 0,\ y\in(0,1)\ \ x\notin(0,\sqrt{y})\end{array}\right.$$

$f_X(x|Y=y)\ne f_X(x)$ – завис.