# СЕМИНАР №1

$f(x) = \begin{cases} 1, \text{ если } aba \in x, \in \{a,b\}^* \text{ ровно 1 раз} \\ 0, \text{ иначе} \end{cases}$

$q_0 ⊛ \to q_0 ⊛, R$

$q_0 a \to q_1 a, R$

$q_0 b \to q_0 b, R$

$q_1 a \to q_1 a, R$

$q_1 b \to q_2 b, R$ // ab

$q_2 a \to q_3 a, R$ // aba - 1е вхождение

$q_2 b \to q_0 b, R$ // abb

$q_3 a \to q_3 a, R$ // abaa

$q_3 b \to q_4 b, R$ // $aba^n b$, $n \geq 1$

$q_4 a \to q_5 a, R$ // $aba^n ba$

$q_4 b \to q_6 b, R$ // $aba^n bb$..

$q_6 a \to q_3 a, R$ // $aba^n b^m a$, $n \geq 1, m > 1$ $\langle q_6 b \to q_6 b, R$

$q_5 \alpha \to q_5 \alpha, R$ // $\alpha \in \{a, b\}$

$q_5 \sqcup \to q_7 \sqcup, L$

$q_7 \alpha \to q_7 \sqcup, L$

$q_7 ⊛ \to q_7 ⊛, R$

$q_7 \sqcup \to q_f 0, L$

$q_i \sqcup \to q_7 \sqcup, L$ // $i = 0, 1, 2$

$q_j \sqcup \to q_8 \sqcup, L$ // $j = 3, 4, 6$

$q_8 \alpha \to q_8 \sqcup, L$

$q_8 ⊛ \to q_8 ⊛, R$ , $q_8 \sqcup \to q_f 1, L$

стр 2 | $(q_0, \lambda, \circledast\square) \vdash (q_0, \circledast, \square) \vdash (q_2, \lambda, \circledast\square) \vdash$

$\vdash (q_2, \circledast, \square) \vdash (q_1, \lambda, \circledast\square)$

Ребр.

$q_3 \xrightarrow{a} q_3 \xrightarrow{b} q_4 \;\cancel{\xrightarrow{b} q_3}\; \xrightarrow{b} q_4 \xrightarrow{a} q_5$ если не ввести $q_6$

$(q_0, \lambda, \circledast ababbba\square) \vdash (q_0, \circledast, ababbba\square) \vdash$

$\vdash (q_1, \circledast a, babbba\square) \vdash (q_2, \circledast ab, abbba\square) \vdash$

$\vdash (q_3, \circledast aba, bbba\square) \vdash (q_4, \circledast abab, bba\square) \vdash$

$\vdash (q_6, \circledast ababb, ba\square) \vdash (q_6, \circledast ababbb, a\square) \vdash$

$\vdash (q_3, \circledast ababbba, \square) \vdash (q_8, \circledast ababbb, a\square) \vdash \ldots$

$(q_0, \lambda, \circledast bababaa\square) \vdash \ldots \vdash$

$(q_7, \circledast bababa, a\square)$

$\uparrow$

при движении назад,
пробел остается !!!

$T: (q_0, \lambda, \circledast x \square) \vdash^* (q_f, \lambda, \circledast \# x \square)$, где $x \in V^*, \# \notin V$

$\boxed{\text{стр 3}}$

$q_0 \circledast \to q_0 \circledast, R$

$q_0 \square \to q_f \#, L$

$q_0 \alpha \to q_\alpha \#, R \quad // \alpha \in V$

$q_\alpha \beta \to q_\beta \alpha, R \quad // \alpha, \beta \in V$

$q_\alpha \square \to q_R \alpha, L \quad // \alpha \in V, R \notin V$

$q_R \alpha \to q_R \alpha, L$

$q_R \# \to q_f \#, L$

$(q_0, \lambda, \circledast abbc\square) \vdash (q_0, \circledast, abbc\square) \vdash (q_a, \circledast\#, bbc\square) \vdash$

$\vdash (q_b, \circledast\# a, bc\square) \vdash (q_b, \circledast\# ab, c\square) \vdash$

$\vdash (q_c, \circledast\# abb, \square) \vdash (q_R, \circledast\# ab, bc\square) \vdash^2$

$\vdash^2 (q_R, \circledast, \# abbc\square) \vdash$ бесконечная вост пробелов

$\vdash (q_f, \lambda, \circledast\# abbc\square$

Реализация сдвига на k позиций $\square$

$(q_0, \lambda, \circledast x \square) \vdash^* (q_f, \lambda, \circledast \underbrace{\# \ldots \#}_{k} x \square)$

k как внешний параметр.
состояние не только буквы, но и номерами итерации

cap 4

$T: (q_0, \lambda, \circledast \underbrace{\# \ldots \#}_{k \geq 1} x \square) \vdash^* (q_f, \lambda, \circledast x \square)$, $x \in V^*$, $\# \in V$

$q_0 \circledast \to q_0 \circledast, R$

$q_0 \alpha \to q_0 \alpha, R, // \alpha \in V \cup \{\#\}$

$q_0 \square \to q_1 \square, L$

$q_1 \# \to q_1 \square, L$

$q_1 \circledast \to q_f \circledast, S$

$q_1 \alpha \to q_\alpha \square, L // \alpha \in V$

$q_\alpha \beta \to q_\beta \alpha, L // \alpha, \beta \in V$

$q_\alpha \# \to q_2 \alpha, L // \alpha \in V$

$q_2 \circledast \to q_f \circledast, S$

$q_2 \# \to q_0 \#, R$

$(q_0, \lambda, \circledast \# \# abc \square) \vdash (q_0, \circledast, \# \# abc \square) \vdash$

$\vdash^5 (q_0, \circledast \# \# abc, \square) \vdash (q_1, \circledast \# \# ab, c \square) \vdash$

$\vdash (q_c, \circledast \# \# a, b \square \square) \vdash (q_b, \circledast \# \#, ac \square \square) \vdash$

$\vdash (q_a, \circledast \#, \# bc \square \square) \vdash (q_2, \circledast, \# abc \square \square) \vdash$

$\vdash (q_0, \circledast \#, abc \square) \vdash^3 (q_0, \circledast \# abc, \square) \vdash$

$\vdash (q_1, \circledast \# ab, c \square) \vdash (q_c, \circledast \# a, b \square) \vdash$

$\vdash (q_b, \circledast \#, ac \square) \vdash (q_a, \circledast, \# bc \square) \vdash$

$\vdash (q_2, \circledast \lambda, \circledast abc \square) \vdash (q_f, \lambda, \circledast abc \square)$

$T: (q_0, \lambda, \circledast x \square) \vdash^* (q_f, \lambda, \circledast x x \square)$

$\# \notin V, \ \overline{V} \cap V = \varnothing \quad V = \{a_1, \dots, a_n\}$

$\overline{V} = \{\bar{a}_1, \dots, \bar{a}_n\}$

алфавит букв двойников

$q_0 \circledast \to q_0 \circledast, R$

$q_0 \square \to q_f \square, L$

$q_0 \alpha \to q_\alpha \#, R \quad // \alpha \in V$

$q_\alpha \beta \to q_\alpha \beta, R \quad // \alpha, \beta \in V$

$q_\alpha \bar{\beta} \to q_\alpha \bar{\beta}, R$

$q_\alpha \square \to q_\alpha^R \bar{\alpha}, L$

$q_\alpha^R \bar{\beta} \to q_\alpha^R \bar{\beta}, L$

$q_\alpha^R \beta \to q_\alpha^R \beta, L$

$q_\alpha^R \# \to q_0 \alpha, R$

$q_0 \bar{\alpha} \to q_1 \alpha, R$

$q_1 \bar{\alpha} \to q_1 \alpha, R$

$q_1 \square \to q_2 \square, L$

$q_2 \alpha \to q_2 \alpha, L$

$q_2 \circledast \to q_f \circledast, S$

$(q_0, \lambda, \circledast abc\square) \vdash (q_0, \circledast, abc\square) \vdash (q_a, \circledast\#, bc\square) \vdash^2$

$\vdash^2 (q_a, \circledast\#bc, \square) \vdash (q_a^R, \circledast\#bc\bar{a}\square) \vdash^2$

$\vdash (q_a^R, \circledast, \#bc\bar{a}\square) \vdash (q_0, \circledast a, bc\bar{a}\square) \vdash$

сор 6

$\vdash (q_0, \circledast a\#, c\bar{a}\square) \vdash^2 (q_b, \circledast, a\#c\bar{a}, \square) \vdash$

$\vdash (q_b^R, \circledast a\#c\bar{a}\bar{b}\square) \vdash^2 (q_b^R, \circledast a, \#c\bar{a}\bar{b}\square) \vdash$

$\vdash (q_0, \circledast ab, c\bar{a}\bar{b}\square) \vdash^3 (q_c, \circledast ab\#, \bar{a}\bar{b}\square) \vdash^2$

$\vdash (q_c, \circledast ab\#\bar{a}\bar{b}, \square) \vdash (q_c^R, \circledast ab\#\bar{a}, \bar{b}\bar{c}\square) \vdash$

$\vdash^2 (q_c^R, \circledast ab, \#\bar{a}\bar{b}\bar{c}\square) \vdash (\underline{\underline{q_b}}, \circledast abc, \underline{\underline{\bar{a}}}\bar{b}\bar{c}\square) \vdash$

$\vdash \ldots$

Сдвиг на k позиций

$(q_0, \lambda, \circledast x\square) \vdash^* (q_0, \lambda, \circledast\underbrace{\#\ldots\#}_{k} x\square)$

$\checkmark\ q_0\circledast \to q_0\circledast, R$

$q_i\square \to q_{i+1}\#, R \quad // i=\overline{0,k-2}$

$q_{k-1}\square \to q_f\#, S$

$q_i\alpha \to q_i^\alpha\#, R \quad // i=\overline{0,k}$

$q_i^\alpha\beta \to q_i^\beta\alpha, R \quad // i=\overline{0,k},\ \alpha,\beta\in V$

$q_i^\alpha\square \to q_i^L\alpha, L \quad // i=\overline{0,k-1}$

$q_i^L\# \to q_{i+1}\#, R \quad // i=\overline{0,k-1}$

$q_k^\alpha\square \to q_f\alpha, S$

$k=1$

$(q_0, \lambda, \circledast\square) \vdash (q_0, \circledast, \square) \vdash (q_1, \circledast\#, \square)$

$q_0 \circledast \to q_0 \circledast, R$

$q_0 \square \to q_1 \#, R$

$q_i \square \to q_{i+1} \#, R \quad // i = \overline{1, k-1}$

$q_k \square \to q_f \square, S \ (L)$

$q_0 \alpha \to q_1^\alpha \#, R \quad // \alpha \in V$

$q_i^\alpha \beta \to q_i^\beta \alpha, R \quad // i = \overline{1, k} \quad // \beta \in V$

$q_i^\alpha \square \to q_i^L \alpha, L \quad // i = \overline{1, k}$

$q_i^L \alpha \to q_i^L \alpha, L \quad // i = \overline{1, k}$

$q_i^L \# \to q_{i+1} \#, R \quad // i = \overline{1, k-1}$

$q_i \alpha \to q_i^\alpha \#, R \quad // i = \overline{1, k-1}$

$q_k^\alpha \# \to q_f \#, S$

$k=1$

$(q_0, \lambda, \circledast \square) \vdash (q_0, \circledast, \square) \vdash (q_1, \circledast \#, \square)$

$k=2$

$(q_0, \lambda, \circledast \square) \vdash (q_0, \circledast, \square) \vdash (q_1, \circledast \#, \square) \vdash$

$\vdash (q_2, \circledast \#\#, \square) \vdash (q_f, \circledast \#\#, \square)$

$k=2$

$(q_0, \lambda, \circledast abc\square) \vdash (q_0, \circledast, abc\square) \vdash (q_1^a, \circledast \#, bc\square) \vdash$

$\vdash (q_1^b, \circledast \# a, c\square) \vdash (q_1^c, \circledast \# ab, \square) \vdash$

$\vdash (q_1^L, \circledast \# a, bc\square) \vdash (q_1^L, \circledast \#, abc\square) \vdash$

$\vdash (q_1^L, \circledast, \# abc\square) \vdash (q_2, \circledast \#, abc\square) \vdash$

$\vdash (q_2^a, \circledast \#\#, bc\square) \vdash (q_2^b, \circledast \#\# a, c\square) \vdash$

$\vdash (q_2^c, \circledast \#\# ab, \square) \vdash (q_2^L, \circledast \#\# a, bc\square) \vdash$

$\vdash (q_2^L, \circledast \#\#, abc\square) \vdash (q_2^L, \circledast \#, \# abc\square)$

# СЕМИНАР №2

$$f(x) = \begin{cases} \lambda, & \text{если } x \neq \lambda \text{ и } u \sqsubseteq x \text{ ровно 2 раза; } x \in V^*, \\ & u = u(1)u(2)\ldots u(k) \in V^+ \\ x, & \text{если } x \neq \lambda \text{ и усл не вып} \\ @, & \text{если } x = \lambda; \quad @ \in V \end{cases}$$

$$\begin{cases} \#\#u \to \cdot u & (1) \\ \#\#\xi \to \xi\#\# & (2) \\ \#\# \to \$ & (3) \\ \xi\$ \to \$ \ (4) \quad \$ \to \cdot & (5) \\ \#u \to u(1)\#\#u(2)\ldots u(k) & (6) \\ \#\xi \to \xi\# \ // \ \xi \in V & (7) \\ u \to u(1)\#u(2)\ldots u(k) \ // \ \# \notin V & (8) \\ \xi \to \cdot\xi & (10) \\ \to \cdot @ & (11) \end{cases}$$

$u = abab \qquad x = cab \vdash_{(10)} \cdot cab$

$a \to \cdot a$

$x = bbabababa \vdash_{(9)} bba\#bababa \vdash_{(7)} bbab\#ababa \vdash_{(6)}$

$\vdash_{(6)} bbabab a\#\#baba \vdash^{y}_{(2)} bbababababa\#\# \vdash_{(3)}$

$\vdash_{(4)} bbababababa\$ \vdash^{9}_{(4)} \$ \vdash^{9}_{(4)} \$ \vdash_{(5)} \cdot\lambda$

$$f(x) = \begin{cases} 1\#x, & \text{если } u \sqsubseteq x \text{ ровно 2 раза}; \ x \in V^*, u \in V^+ \text{ (фикс)} \\ 0\#x, & \text{иначе.} \end{cases}$$

$$\left\{ \begin{array}{ll}
\& u \to 0u & (3) \\
\& \xi \to \xi \& & (4) \\
\& \to 1 & (5) \\
\$ u \to u(1) \& u(2) \ldots u(k) \quad // \ \& \notin V & (6) \\
\$ \xi \to \xi \$ \quad // \ \xi \in V & (7) \\
\$ \to 0 & (?) \\
u \to u(1) \$ u(2) \ldots u(k) \quad // \ \$ \notin V & (9) \\
\to \cdot 0\# & (10)
\end{array} \right.$$

$$\xi \alpha \to \alpha \xi \quad // \ \alpha \in \{0, 1\} \quad (1)$$

$$\alpha \to \cdot \alpha \# \quad (2)$$

$u = aba$

$x = ababaaba \vdash_{(9)} a\$babaaba \vdash_{(7)}$

$\vdash_{(7)} ab\$abaaba \vdash_{(6)} aba\&baaba \vdash^2_{(4)}$

$\vdash^2_{(4)} ababa\&aba \vdash_{(3)} ababa0aba \vdash_{(1)}$

$\vdash_{(1)}$

$f(x) = \begin{cases} \lambda, \text{ если } u \sqsubseteq x \neq \lambda, u \in V^+, x \in V^+ \ (u \text{ - фикс}) \\ x, \text{ если } u \not\sqsubseteq x \neq \lambda \\ \#, \text{ если } x = \lambda \end{cases}$

$\overline{V} \cap V = \varnothing$

$\# \notin V \cup \overline{V}$

$u = u(1)u(2)\dots u(k)$ $k \geq 1$

$q_0 \circledast \to q_0 \circledast, R$

$q_0 \square \to q_f \#, L$ // $x = \lambda$

$q_0 \alpha \to q_0' \alpha, R$ // $\alpha \neq u(1)$

$q_0 u(1) \to q_1 \overline{u(1)}, R$

$q_0' u(1) \to q_1 \overline{u(1)}, R$

$q_i u(i+1) \to q_{i+1} \overline{u(i+1)}, R$ // $i = \overline{1, k-1}$

$q_k \alpha \to q_k \alpha, R$ // $\alpha \in \overline{V}$

$q_k \square \to q_k \square, L$

$q_R \alpha \to q_R \square, L$ // $\alpha \in V \cup \overline{V}$

$q_R \circledast \to q_f \circledast, S$

$q_i \beta \to \overline{q} \beta, L$ // $\beta \neq u(i+1), \beta \neq \square, i < k$

$\overline{q} \alpha \to \overline{q} \alpha, L$ // $\alpha \in \overline{V}$

$\overline{q} \overline{u(1)} \to q_0' u(1), R$

$u = ababaa$ $(k = 6)$

$x = abaabaaab$

$q_0' \square \to r \square, L$

$q_i \square \to r \square, L$ // $i = \overline{1, k-1}$

$r \alpha \to r \alpha, L$

$r \overline{\alpha} \to r \alpha, L$

$r \circledast \to q_f \circledast, S$

$(q_0, \lambda, \otimes ab\square) \vdash (q_0, \otimes, ab\square) \vdash (q_1, \otimes \bar{a}, b\square) \vdash$

$\vdash (q_2, \otimes \bar{a} b, \square) \vdash (r, \otimes \bar{a}, b\square) \vdash (r, \otimes, \bar{a} b\square) \vdash$

$\vdash (r, \lambda, \otimes ab\square) \vdash (q_f, \lambda, \otimes ab\square)$

## Семинар №3

18.03.21

$\cancel{\xi \to \# \xi \quad // \xi \in V}$

$bab \vdash b\#ab$

$u = aba, \quad v = bab$

$x = bbabab \vdash \# bbabab \vdash b\# \underbrace{bab}ab \vdash$

$\vdash b\beta babab \vdash bb \underbrace{\beta ab a}_{\text{б4}} b \; \#\sigma \vdash$

$\vdash bb\nabla b \vdash bb\nabla \vdash^2 \nabla \vdash \cdot$

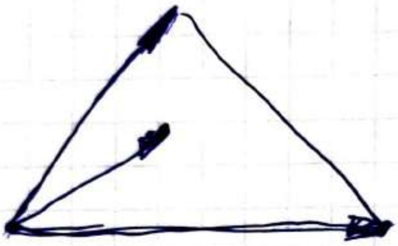
$xx^R$ (Double)

$$\begin{cases} \alpha \xi \to \xi \beta \xi \alpha, \; \xi \in V & (1) \\ \beta \xi \eta \to \eta \beta \xi, \; \eta, \xi \in V & (2) \\ (\beta \to) \; \beta \xi \alpha \to \xi \alpha & (3) \\ \alpha \to \cdot & (4) \\ \to \alpha & (5) \end{cases}$$

$abca \vdash a\beta a \beta b \beta c \beta a \alpha \vdash_{(3)}$

$\vdash_{(3)} abca \beta a \beta b \beta c a \alpha \vdash_{(2)} abca\beta a\beta b a \beta c \alpha \vdash_{(2)}$

$\vdash_{(2)} abca\beta aa\beta b\beta c\alpha \vdash_{(2)} abcaa\beta a\beta b\beta c\alpha \vdash_{(3)}$



$\vdash_{(3)} abcaa\beta a\beta bc\alpha \vdash_{(2)} abcaa\beta ac\beta b\alpha \vdash_{(2)}$

$\vdash_{(1)} abcaac\beta a\beta b\alpha \vdash_{(3)} \vdash_{(3)}$

$\vdash_{(3)} abcaac\beta ab\alpha \vdash_{(2)}$

$\vdash_{(1)} abcaacb\beta a\alpha \vdash_{(3)} abcaacba\alpha \vdash_{(4)}$

$\vdash_{(4)} \cdot abcaacba$

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1) $m > n$

$01^m\$01^n \vdash_{(1)} 01^{m-1}\$01^{n-1} \vdash_{(1)} 01^{m-n}\$0 \vdash_{(3)}$

$\vdash_{(3)} \cdot 01^{m-n}$

2) $m \le n$

$01^m\$01^n \vdash_{(1)} 0\$01^{n-m} \models_{(2)} 0\$0 \vdash_{(3)} \cdot 0$

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No.

Date

1) $m \geq n$

$$01^m \$ 01^n \models_{(1)} 01^{m-n} \$ 0 \vdash \begin{cases} 0\$0 \vdash_4 \cdot 1 & (m=n) \\ 01^{m-n} \$ 0 \vdash_3 0\$0 \vdash_4 \cdot 1 & (m>n) \end{cases}$$

2) $m < n$

$$01^m \$ 01^n \models_{(1)} 0\$01^{n-m} \vdash_{(2)} 0\$a1^{n-m-1} \models_{(5)}$$

$$\models_{(5)} 0\$a \vdash_{(0)} \cdot 0$$

Строгое неравенство.

$1\$01 \to \$0$
$0\$01 \to 0\$a$ // $a \notin \{0,1\}$
$01\$a \to \cdot 1$
$1\$0 \to \$a$
$0\$0 \to \cdot 0$
$a1 \to a$
$\$a \to \cdot$

не сраб.

Если это так?
$m-n-1 \geq 0$

1) $m > n$

$$01^m \$ 01^n \models_{(1)}^{n} 01^{m-n} \$ 0 \vdash_{(4)} 01^{m-n-1} \$ 0 \models_{(4)}^{m-n-2}$$

$$\models_{(4)}^{m-n-2}$$

No.

Date

$$
\begin{aligned}
0 &\to && (1) \qquad \alpha, \beta \notin V_0 \\
1\$1 &\to \$ && (2) \\
1\$ &\to \alpha && (3) \\
1\alpha &\to \alpha && (4) \\
\alpha &\to \cdot 1 && (5) \\
\$1 &\to \beta && (6) \\
\beta 1 &\to \beta && (7) \\
\beta &\to \cdot 0 && (8) \\
\$ &\to \cdot 0 && (9)
\end{aligned}
$$

$01^m\$01^n \models^2_{(1)} 1^m\$1^n \vdash_{(2)} 1^{m-n}\$ \vdash_{(3)}$

$\underline{m>n}$ $\vdash_{(3)} 1^{m-n-1}\alpha \cancel{\$} \models_{(4)} \alpha \vdash \cdot 1$

$\underline{m=n}$: $01^m\$01^n \models^2 1^m\$1^m \models^m_{(1)} \$ \vdash \cdot 0$

$\underline{m<n}$: $01^m\$01^n \models^2 1^m\$1^n \models_{(2)} \$1^{n-m} \vdash_{(6)} \beta 1^{n-m-1} \models_{(7)} \beta \vdash_{(8)} \cdot 0$

Campus

СЕМИНАР N5. 15.04.21

Че.

Чт. 17:25

ЗН

Ср. 17:25

Пт. 15:40

№1

$f = (0011\ 0101\ 1101\ 1010)$ 0110 0111

| $x_1, x_2$ \ $x_3, x_4$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | | | 1 | 1 |
| 01 | | 1 | 1 | |
| 11 | 1 | | | 1 |
| 10 | 1 | 1 | 1 | |

$001x\ (\bar{x}_1\bar{x}_2x_3)$ Я

$0x11\ (\bar{x}_1x_3x_4)$ избыточная

$01x1\ (\bar{x}_1x_2x_4)$ Я

$11x0\ (x_1x_2\bar{x}_4)$ Я

$1x00\ (x_1\bar{x}_3\bar{x}_4)$ $K_1$

$100x\ (x_1\bar{x}_2\bar{x}_3)$ $K_2$

$10x1\ (x_1\bar{x}_2x_4)$ $K_3$

$x011\ (\bar{x}_2x_3x_4)$ $K_4$

Сокращенная ДНФ

$$f = \bar{x}_1\bar{x}_2x_3 \vee \cancel{\bar{x}_1x_3x_4} \vee \bar{x}_1x_2x_4 \vee x_1x_2\bar{x}_4 \vee$$
$$\vee \bar{x}_2x_3x_4 \vee x_1\bar{x}_2x_4 \vee x_1\bar{x}_2\bar{x}_3 \vee x_1\bar{x}_3\bar{x}_4$$

ДНФ Квайна. (Сокр − избыт)

<u>Функция Патрика.</u>

$(K_1 \vee K_2)(K_2 \vee K_3)(K_3 \vee K_4) = (K_1K_2 \vee K_1K_3 \vee \cancel{K_2} \vee K_2K_3)$
$(K_3 \vee K_4) = (K_1K_3 \vee K_2)(K_3 \vee K_4) =$

$= K_1K_3 \vee K_1\cancel{K_3}K_4 \vee K_2K_3 \vee K_2K_4 =$

$= K_1K_3 \vee K_2K_3 \vee K_2K_4$

$$f = \underbrace{\bar{x}_1 \bar{x}_2 x_3 \vee \bar{x}_1 x_2 x_4 \vee x_1 x_2 \bar{x}_4}_{\text{ядро}} \vee \left\{ \begin{array}{l} x_1 \bar{x}_3 \bar{x}_4 \vee x_1 \bar{x}_2 \bar{x}_4 \\ x_1 \bar{x}_2 \bar{x}_3 \vee x_1 \bar{x}_2 x_4 \\ x_1 \bar{x}_2 \bar{x}_3 \vee \bar{x}_2 x_3 x_4 \end{array} \right.$$

Все явл. минимальными

$m = 5 \quad L = 15$

№2 $f = \{0, 2, 3, 4, 7, 11, 12, 15\}$ 6.21

| $x_1 x_2 \backslash x_3 x_4$ | 00 | 01 | 11 | 10 |
|---|---|---|---|---|
| 00 | 1 | | 1 | 1 |
| 01 | 1 | | 1 | |
| 11 | 1 | | 1 | |
| 10 | | | 1 | |

001x $\bar{x}_1 \bar{x}_2 x_3$ $K_1$

00x0 $\bar{x}_1 \bar{x}_2 \bar{x}_4$ $K_2$

xx11 $x_3 x_4$ Я

x100 $x_2 \bar{x}_3 \bar{x}_4$ Я

0x00 $\bar{x}_1 \bar{x}_3 \bar{x}_4$ $K_3$

Ф-ия Патрика:

$$(K_2 \vee K_3)(K_1 \vee K_2) = \cancel{K_1 K_2} \vee K_2 \vee K_1 K_3 \vee \cancel{K_3 K_2} =$$

$$= K_2 \vee K_1 K_3 \qquad m = 3$$

Тупиковая ДНФ $\qquad L = 8$

$$f = x_3 x_4 \vee x_2 \bar{x}_3 \bar{x}_4 \vee \left\{ \begin{array}{l} \bar{x}_1 \bar{x}_2 \bar{x}_4 \text{ — мин} \\ \bar{x}_1 \bar{x}_2 x_3 \vee \bar{x}_1 \bar{x}_3 \bar{x}_4 \end{array} \right.$$

СЕМИНАР №6

29.04.21

$f = x_1 x_2 \oplus x_1 x_3 \oplus x_2 x_3$

| | $T_0$ | $T_1$ | $S$ | $M$ | $L$ |
|---|---|---|---|---|---|
| $f$ | + | + | + | + | — |
| $g$ | — | — | — | — | — |

$g = (1000\ 0010)$

$a_0 = g(0,0,0) = 1$

$0 = g(1,0,0) = a_1 \oplus a_0 = a_1 \oplus 1 \Rightarrow a_1 = 1$

$0 = g(0,1,0) = a_2 \oplus a_0 = a_2 \oplus 1 \Rightarrow a_2 = 1$

$0 = g(0,0,1) = a_3 \oplus a_0 = a_3 \oplus 1 \Rightarrow a_3 = 1$

$1 = g(1,1,0) = a_{12} \oplus a_1 \oplus a_2 \oplus a_0 = a_{12} \oplus 1 = 0;\ a_{12} = 0$

$0 = g(1,0,1) = a_{13} \oplus a_1 \oplus a_3 \oplus a_0 = a_{13} \oplus 1 = 0;\ a_{13} = 1$

$0 = g(0,1,1) = a_{23} \oplus a_2 \oplus a_3 \oplus a_0 = a_{23} \oplus 1 \Rightarrow a_{23} = 1$

$0 = g(1,1,1) = a_{123} \oplus a_{12} \oplus a_{13} \oplus a_{23} \oplus a_1 \oplus a_2 \oplus a_3 =$
$= a_{123} = 0$

$g = x_1 x_3 \oplus x_2 x_3 \oplus x_1 \oplus x_2 \oplus x_3 \oplus 1$

$f(x_1, x_2, 0) = x_1 x_2$

$x_1 \vee x_2 = \overline{f(\bar{x}_1, \bar{x}_2, 0)}$

$g(x, x, x) = \bar{x}$

$g(\bar{x}, x, x) \equiv 0$

$\bar{0} = 1 = g(0,0,0) =$
$= g(g(g(x,x,x), x, x), x, x)$

$g(0, x_2, x_3) = \underbrace{x_2}_{x}\underbrace{x_3}_{y} \oplus x_2 \oplus x_3 \oplus 1$

$a = b = 1 = c$

$\chi(x, y) = xy \oplus x \oplus y \oplus 1$ $\qquad ab \oplus c = 0$

$xy = \chi(x \oplus b, y \oplus a) \oplus ab \oplus c$

$xy = g(0, \bar{x}, \bar{y})$; $\quad x \vee y = \overline{g(0, x, y)}$

$x \oplus y = x\bar{y} \vee \bar{x}y$

$g(x_1, x_2, 0) = x_1 \oplus x_2 \oplus 1 = \overline{x_1 \oplus x_2}$

$x_1 \oplus x_2 = \overline{g(x_1, x_2, 0)}$

---

$f \notin T_0 \;\&\; f \not\equiv 1 \Rightarrow f \notin M,$

$\qquad \& \equiv 1 \Rightarrow f \notin S$

$f \notin T_1 \;\&\; f \neq 0 \Rightarrow f \notin M$

$\qquad \& \; \cancel{f} \equiv 0 \Rightarrow f \notin S$

~~ПК~~ $T_1$

| | $T_0$ | $T_1$ | $S$ | $M$ | $L$ |
|---|---|---|---|---|---|
| $f_1$ | + | + | + | + | − |
| $f_2 \equiv 0$ | + | − | − | + | − |
| $f_3 \equiv 1$ | − | + | − | + | + |
| $f_4 = x_1 \oplus x_2 \oplus x_3$ | + | + | + | − | + |

$f_4(\bar{x}_1, \bar{x}_2, \bar{x}_3) = x_1 \oplus x_2 \oplus x_3 \oplus 1 = \overline{f_4(x_1, x_2, x_3)}$

000
001
010
011
100
1

~~на~~ 100 1
110 0
не монотонно.

$\{f_2, f_3, f_4\} \subseteq L$

$\{f_1, f_3, f_4\} \subseteq T_1$

$\{f_1, f_2, f_4\} \subseteq T_0 \qquad \{f_1, f_2, f_3\} \subseteq M$

СЕМИНАР №7.

13.05.21

$$A \& (B \& C) \equiv (A \& B) \& C$$

$$\neg(A \to \underbrace{\neg\neg(B \to \neg C)}_{B \& C}) \equiv \neg(\neg(A \to \neg B) \to \neg C)$$

$$\boxed{A \to \neg\neg(B \to \neg C) \equiv \neg(A \to \neg B) \to \neg C}$$

$\Lambda \vdash \Pi$

1. $A \to \neg\neg(B \to \neg C)$ – гип
2. $\neg(A \to \neg B)$ – гип
3. $\neg A \to (A \to \neg B)$ – сл 5 при $B := \neg B$
4. $\neg(A \to \neg B) \to \neg\neg A$ – R7, (3)
5. $\neg\neg A$ – MP, (2) и (4)
6. $A$ – R3, (5)
7. $\neg\neg(B \to \neg C)$ – MP, (1) и (6)
8. $B \to \neg C$ – R3, (7)
9. $\neg B \to (A \to \neg B)$ – сх(1) при $A := \neg B$, $B := A$
10. $\neg(A \to \neg B) \to \neg\neg B$ – R7, (9)
11. $\neg\neg B$ – MP, (2), (10)
12. $B$ – R3, (5)
13. $\neg C$ – MP, (8) и (12)

---

$A \to \neg\neg(B \to \neg C), \neg(A \to \neg B) \vdash \neg C \xrightarrow{\text{по т. 8}}$

$\Rightarrow A \to \neg\neg(B \to \neg C) \vdash \neg(A \to \neg B) \to \neg C$

$\Pi \vdash A$

1. $\neg(A \to \neg B) \to \neg C$ - гип
2. $A$ - гип
3. $B$ - гип
4. $\neg\neg B$ - R4, (3)
5. $\neg(A \to \neg B)$ - R8, (4) $B := \neg B$
6. $\neg C$ - MP, (1) и (5)

$$\frac{A, \neg B}{\neg(A \to B)}$$

---

$\neg(A \to \neg B) \to \neg C, A, B \vdash \neg C$.

по т. дедукции:

$\neg(A \to \neg B) \to \neg C, A \vdash B \to \neg C \vdash \neg\neg(B \to \neg C)$, R4

по т. дедукции:

$\neg(A \to \neg B) \to \neg C \vdash A \to \neg\neg(B \to \neg C)$

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$\vdash (p \to (\neg q \vee (r \& s))) \& p \& \neg s \to \neg q$

1. $(p \to (\neg q \vee (r \& s))) \& p \& \neg s$ - гип
2. $(p \to (\neg q \vee (r \& s)))$ - св. ва конъюн (1)
3. $p$
4. $\neg s$
5. $\neg q \vee (r \& s) = \neg\neg q \to (r \& s)$ - MP (3) (2)

6. $\neg(r \& s) \to \neg\neg\neg q$ - R7, (5)

7. $r \& s \to s$ - в-ва конъюнкции

8. $\neg s \to \neg(r \& s)$ - R7, (7)

9. $\neg(r \& s)$ - MP, (4) и (8)

10. $\neg\neg\neg q$ - MP и (6) и (9)

11. $\neg q$ - R3, к (10)

---

$(\neg y \to (\neg x \vee \neg(y \to z))) \vdash (\neg x \vee y)$

$\neg x \vee y = \neg\neg x \to y$

ЛТД

$\neg\neg x \to \neg(y \to z)$

1. $\neg y \to (\neg x \vee \neg(y \to z))$ - гип

2. $\neg\neg x$ - гип

3. $\neg y \to \neg(y \to z)$ - R2 к (1) и (2)

4. $(y \to z) \to y$ - R6 к (3)

5. $\neg y \to (y \to z)$ - св-в 5

6. $\neg(y \to z) \to \neg\neg y$ - R7 (5)

7. $\neg y \to \neg\neg y$ - R1 (3) и (6)

8. $\neg\neg y \to y$ - св-в (8)

9. $\neg y \to y = y \vee y$ R1 (7) (8)

10. $y \vee y \to y$ - теорема

11. $y$ - MP 9 и 10

---

$\neg y \to (\neg x \vee \neg(y \to z)), \neg\neg x \vdash y$

$\neg y \to (\neg x \vee \neg(y \to z)) \vdash \neg\neg x \to y$

~~$x \to \neg z x$~~

1 $\neg x \vee y = \neg\neg x \to y$ – гип

2 $\neg y$ – гип

3. $\neg\neg x$ – гип.

~~4. $y$ – МР (1) (3)~~

~~5. $z$ – R5 (2) и (4)~~

~~6.~~

4. $\neg y \to (y \to \neg z)$ – сед 8

5. ~~...~~ $y$ – МР (1,3)

6. $y \to \neg z$ – МР (2) и (4)

7. $\neg z$ – МР, (5) и (6)

8. $y \,\&\, \neg z$ – в. 1 кон. (7) (5)

9. $y \,\&\, \neg z \to \neg(y \to z)$ – теорема.

10. $\neg(y \to z)$ – МР (7) и (9)

---

$\neg(A \to B) = A \,\&\, \neg B$

$\neg(A \to B) \vdash A \,\&\, \neg B \Leftrightarrow \neg(A \,\&\, \neg B) \vdash \neg\neg(A \to B)$

$\neg\neg(A \to \neg\neg B)$

$A \,\&\, \neg B \vdash \neg(A \to B)$, $\neg\neg(A \to B) \vdash \neg(A \,\&\, \neg B)$

ЛЮБИТ$(x, M) \to \neg$ЛЮБИТ$(M, x)$

$$((\neg(\neg x \to y) \to (x \& (y \vee z))) \equiv (x \vee y)$$