Dirichlet Process Mixture Models (DPMMs)

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Dirichlet Process Mixture Models (DPMMs) is a **flexible clustering method** that can automatically decide the number of clusters based on the data. Unlike traditional methods like **K-means** which require you to specify the number of clusters.

It offers a **probabilistic** and **nonparametric** approach to clustering which allows the model to figure out number of groups on its own based complexity of the data.

Key Concepts in DPMMs

To understand DPMMs it's important to understand two key concepts:

1. Beta Distribution

The **Beta distribution** models probabilities for two possible outcomes such as success or failure. It is defined by two parameters α and β that shape the distribution. The probability density function (PDF) is given by:

$$f(x, \alpha, \beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\mathrm{B}(\alpha\beta)}$$

Where B(α, β) is the beta function.

2. Dirichlet Distribution

The **Dirichlet distribution** is a generalization of the Beta distribution for multiple outcomes. It represents the probabilities of different categories like rolling a dice with unknown probabilities for each side. The PDF of the Dirichlet distribution is:

- p=(p1,p2,..., pK) is a vector representing a probability distribution over K categories. Each pi is a probability and $\sum$Kpi=1.
- α=(α1,α2,...,αK) is a vector of positive shape parameters. This determines the shape of the distribution
- B(α) is a beta function.

How α Affects the Distribution

- Higher α values result in probabilities concentrated around the mean.
- Equal α values produce symmetric distributions.
- Different α values create skewed distributions.

Dirichlet Process (DP)

A **Dirichlet Process** is a stochastic process that generates probability distributions over infinite categories. It enables clustering without specifying the number of clusters in advance. The Dirichlet Process is defined as:

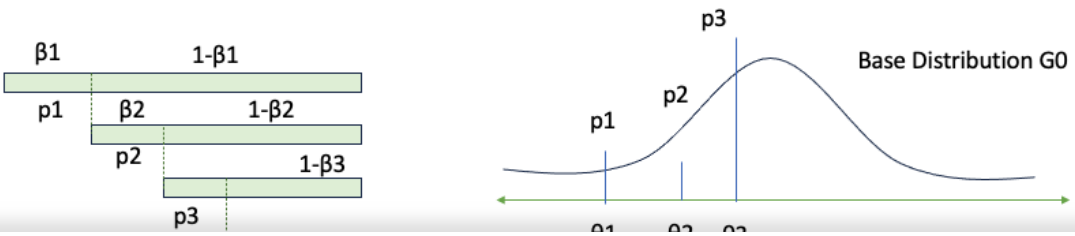
$$DP(\alpha, G0)$$

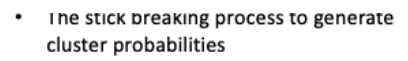
Where:

- **α:** Concentration parameter controlling cluster diversity.
- **G_0:** Base distribution representing the prior belief about cluster parameters.

Stick-Breaking Process

The **stick-breaking process** is a method to generate probabilities from a Dirichlet Process. The concept is shown in the image below:





Stick-Breaking Process

- We take a stick of length unit 1 representing our base probability distribution
- Using marginal distribution property we break it into two. We use beta distribution. Suppose the length obtained is p1
- The conditional probability of the remaining categories is a Dirichlet distribution
- The length of the stick that remains is 1-p1 and using the marginal property again
- Repeat the above steps to obtain enough pi such that the sum is close to 1
- Mathematically this can be expressed as
  - For $k=1, p1=\beta(1,\alpha)$
  - For $k=2, p2=\beta(1,\alpha)*(1-p1)$
  - For $k=3, p3=\beta(1,\alpha)*(1-p1-p2)$

For each categories sample we also sample μ from our base distribution. This becomes our cluster parameters.

How DPMMs Work?

DPMM is an extension of Gaussian Mixture Models where the number of clusters is not fixed. It uses the Dirichlet Process as a prior for the mixture components.

1. **Initialize:** Assign random clusters to data points.
2. **Iteration:**
   - Pick one data point.
   - Fix other cluster assignments.
   - Assign the point to an existing cluster or a new cluster based on probabilities.
3. **Repeat:** Continue until cluster assignments no longer change.

The probability of assigning a point to a new cluster is: $\frac{\alpha}{n-1+\alpha}\mathrm{N}(0,1)$

Where:

- **n_k:** Number of points in cluster k.
- **α:** Concentration parameter.
- **N(μ, σ):** Gaussian distribution.

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Implementing Dirichlet Process Mixture Models using Sklearn

Now let us implement DPMM process in scikit learn and we'll use the **Mall Customers Segmentation Data**. Let's understand this step-by-step:

Step 1: Import Libraries and Load Dataset

In this step we will import all the necessary libraries. This dataset contains customer information, including age, income and spending score. You can download the dataset from here.

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.mixture import BayesianGaussianMixture
from sklearn.decomposition import PCA


data = pd.read_csv('/content/Mall_Customers (1).csv')
print(data.head())
```

Output:

	CustomerID	Genre	Age	Annual Income (k$)	Spending Score (1-100)
0	1	Male	19	15	39
1	2	Male	21	15	81
2	3	Female	20	16	6
3	4	Female	23	16	77
4	5	Female	31	17	40

Dataset

In this step we select features that are likely to influence customer clusters.

```
X = data[['Age', 'Annual Income (k$)', 'Spending Score
(1-100)']].values
```

Step 3: Dimensionality Reduction

We will use PCA algorithm to reduces the data's dimensions to 2 for easy visualization.

```
pca = PCA(n_components=2)
X_pca = pca.fit_transform(X)
```

Step 4: Fit Bayesian Gaussian Mixture Model

The model automatically determines the optimal number of clusters based on the data.

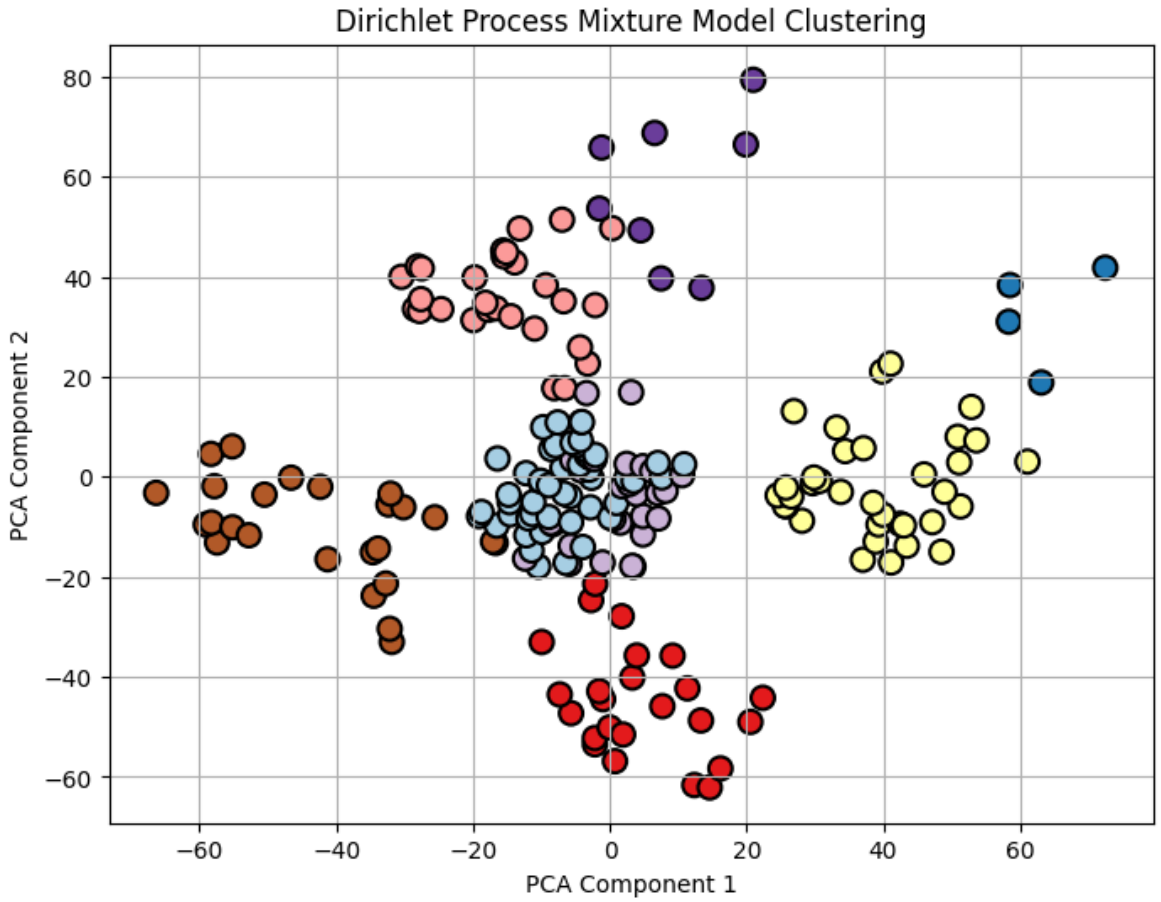
```
dpmm = BayesianGaussianMixture(
    n_components=10,
    covariance_type='full',
    weight_concentration_prior_type='dirichlet_process',
    weight_concentration_prior=1e-2,
    random_state=42
)

dpmm.fit(X)
labels = dpmm.predict(X)
```

Step 5: Visualization

Clusters are visualized with different colors making patterns easier to interpret.

```
plt.figure(figsize=(8, 6))
plt.scatter(X_pca[:, 0], X_pca[:, 1], c=labels,
cmap=plt.cm.Paired, edgecolors='k', s=100, linewidth=1.5)
plt.title('Dirichlet Process Mixture Model Clustering')
plt.xlabel('PCA Component 1')
plt.ylabel('PCA Component 2')
```



DPMM

The clustering of mall customers using DPMM highlights distinct groups where average customers in the center and extreme spenders on the edges. Overlapping clusters suggest some customers share similar behaviors.

Advantages over Traditional Methods

- One of the primary advantage of DPMMs is their ability to automatically determine the number of clusters in the data. Traditional methods often require the pre-specification of the number of clusters like in k-means which can be challenging in real-world applications.
- It operate within a probabilistic framework allowing for the quantification of uncertainty. Traditional methods often provide "hard" assignments of data points to clusters while DPMMs give probabilistic cluster assignments capturing the uncertainty inherent in the data.
- DPMMs find applications in a wide range of fields including natural language processing, computer vision, bioinformatics and finance. Their flexibility makes them applicable to diverse datasets and problem