

FACULTY OF SCIENCE AND ENGINEERING

ASSESSMENT COURSEWORK 2019/20



UNIT CODE: 6G4Z1102	UNIT DESC: Computer Systems Fundamentals	
ASSESSMENT ID: 2CWK25	ASSESSMENT NAME: coursework 25%	WEIGHT FACTOR: 25%
<p>Tasks for this coursework:</p> <ul style="list-style-type: none">• Answer all 7 questions.• The marks awarded for each question are shown in square brackets.• To obtain full marks ALL working must be shown. The report structure and presentation will also be marked.• You must submit your coursework electronically, as pdf file, through Moodle area, assessments.• The hand-out date is 2nd December 2019. This coursework must be completed and submitted by 24th January 2020.• Your mark will be scaled down by a maximum of 10% if sufficient presentation effort has not been made in your submitted coursework. <ul style="list-style-type: none">• Tariq Jarad: office hours, Tuesday 10:00-12:00, Wednesday 10:00-11:00. <u>Room JD E115</u>• Saeed Abuzour office hours, Tuesday 11:00-12:00, Wednesday 10:00- 11:00, Thursday 11:00-12:00. <u>Room JD E115</u>• Frank Bierbrauer: office hours, Monday 15:00-16:00, Thursday 14:00-16:00 <u>Room JD E114a</u>• Jon Borresen: office hours, Monday 10.00-11.00, Tuesday 11.00-12.00, Wednesday 10.00-11.00. <u>Room JD E116</u>		
NAME OF STAFF SETTING ASSIGNMENT: Dr Tariq Jarad & Dr Saeed AbuZour		

1. Given that $U = \{1, 2, 3, \dots, 10\}$ is the universal set, $A = \{x : x \in N, x \text{ is odd number}\}$
 $B = \{x : x \in N, x \text{ is square number}\}$ and $C = \{x : x \in N, 2 \leq x < 5\}$, find the
 following:

- a) $|A|, |A \cap B|, |C|$; ~~$\{5\}$~~ $\{5, 3\}$ $\{5, 13, 3\}$
 b) $P(B)$; $\{ \emptyset, \{1\}, \{1, 4\}, \{1, 9\}, \{4, 9\}, \{1, 4, 9\} \}$ [3]
 c) $C', B \times C, A \cap C, C \setminus B$. $\{1, 5, 7, 9, 13\}$ [8]
 $\{ (1, 4), (2, 3), (1, 2), (1, 3), (2, 4) \}$
 $\{3\}$
 $\{2\}$

[15 marks for this question]

2.

- a) For the universal set $U = \{1, 2, 3, \dots, 36\}$, draw a Venn diagram to represent the
 following sets: [6]

$$F = \{x : x \in N, x \text{ is prime number}, 0 \leq x \leq 15\}; \quad (\text{on extra piece of paper})$$

$$G = \{x : x \in N, x \text{ is multiple of } 3, 1 \leq x \leq 20\};$$

$$H = \{x : x \in N, x \text{ is factor of } 36\}.$$

- b) Use a Venn diagram to illustrate the following: [4]

- (i) $(A \cap B) \cap C'$; $(\text{on extra piece of paper})$
 (ii) $(B \cup C) \setminus A'$.

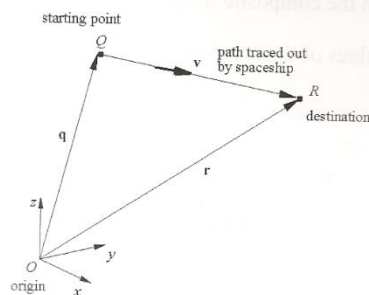
[10 marks for this question]

3. For the three vectors $\mathbf{a} = 2\hat{i} - 3\hat{j} - \hat{k}$, $\mathbf{b} = \hat{i} - 2\hat{j} - \hat{k}$ and $\mathbf{c} = \hat{i} - \hat{j} - \hat{k}$:

- a) calculate $\mathbf{a} + 2\mathbf{c}$; ~~$(-2, 3)$~~ $(4, 5, 3)$ [3]
 b) the unit vector in the direction of \mathbf{c} ; $\frac{\mathbf{c}}{\sqrt{3}}$ [3]
 c) calculate $\mathbf{a} \cdot \mathbf{c}$ and find the angle between the two vectors; 0° [5]
 d) calculate $\mathbf{a} \times \mathbf{b}$. $(1, 1, -1)$ [4]

[15 marks for this question]

4. In a computer game, a spaceship travels on a straight line passing through the points Q , given by the position vector $\mathbf{q} = (4, -2, 10)$ and P , given by the position vector $\mathbf{p} = (6, 3, 5)$. It arrives at its destination, the point R , after t seconds.



- a) Write down the vector equation of the new position R in terms of t ; [6]
 b) Calculate the coordinates of the point R where the spaceship reaches its destination after a time of $t = 4$ seconds. [4]

[10 marks for this question]

5. You are given the following matrices:

$$U = \begin{pmatrix} 1 & 3 & 1 \\ 0 & 2 & -1 \\ 3 & -2 & 5 \end{pmatrix} \quad V = \begin{pmatrix} -1 & 5 & 2 \\ 0 & 3 & 1 \end{pmatrix} \quad W = \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix} \quad Z = \begin{pmatrix} 3 & -2 & 0 \\ -1 & 4 & 2 \end{pmatrix}$$

- a) Write down matrix elements u_{31} , u_{21} , v_{32} , v_{12} , w_{22} and z_{33} if possible. Explain why if not possible. [6]

- b) Calculate the following if possible and explain why if not possible:

- (i) $V + W$; [2]
 (ii) $V - Z$; [3]
 (iii) UV^T ; [6]
 (iv) WZ . [4]

(all on extra piece of paper)

- c) If a point $(3, 1)$ is reflected about the line $y = (\tan \theta)x$ with $\theta = 120^\circ$, find the reflection line and the reflection point. [4]

Reflection point $(3, -4)$ [25 marks for this question]

6. For the three functions $h: R \rightarrow R$, $h(x) = 2x - 1$, $u: R \rightarrow R$, $u(x) = \frac{x}{2} - 1$ and $v: R \rightarrow R$, $v(x) = x^2 + 3$, where R is the set of the real numbers:

- a) write down the composite functions $v \circ h$ and $v \circ u$; [5]
- b) find the values of the composite function $v \circ h$, for $x = -1, 0, 2$. [5]

[10 marks for this question]

7. Given the relation below, defined on $R \rightarrow R$, where R is the set of the real numbers:

$$b(x) = \frac{2(x+1)}{x-1}.$$

- a) explain when b is a function and write down the three values of $b(x)$, for $x = 0, 2, 3$, using the ordered pair representation; [5]
- b) if the relation is a function, state whether it is a total or partial function, and classify it as an injection, surjection, bijection or a combination of these; [4]
- c) find the inverse relation $b^{-1}: R \rightarrow R$ and determine the domain and the range that make the relation a function; [4]
- d) explain why the values $b(1)$ and $b^{-1}(2)$ are not valid for these functions. [2]

[15 marks for this question]

[Total:100]

1)

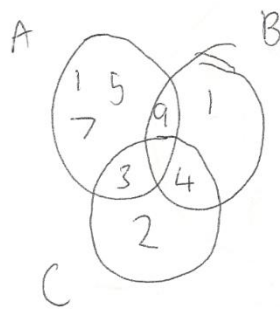
$$A = \{1, 3, 5, 7, 9\}$$

a) $|A| = 5$

$$A \cap B = \{1, 3\}$$

$$|C| = \{3\}$$

$$C = \{2, 3, 4\}$$



b) $P(B) = \{\emptyset, \{1\}, \{1, 4\}, \{1, 9\}, \{4, 9\}, \{4\}, \{9\}, \{1, 4, 9\}\}$

c) $C' = \{1, 5, 7, 9, 13\}$

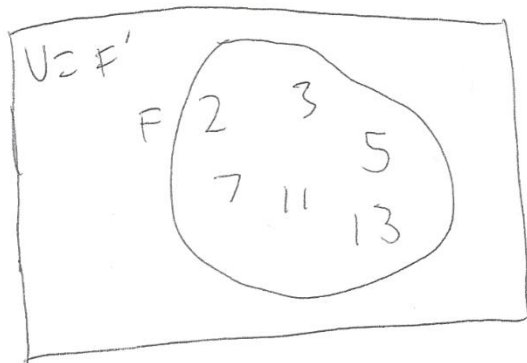
$$B \times C = \{(1, 4), (2, 3), (1, 2), (1, 3), (2, 4)\}$$

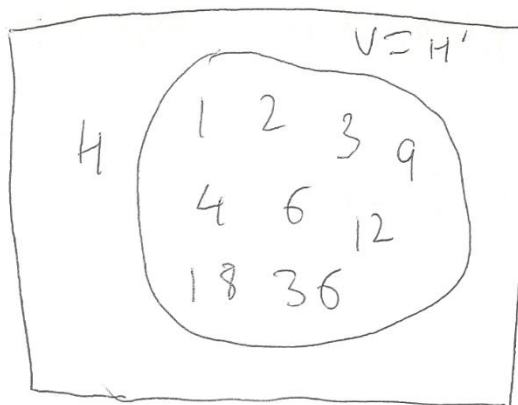
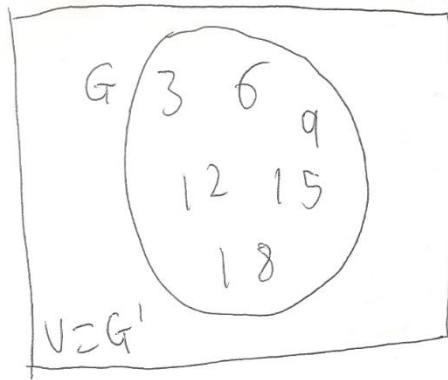
$$A \cap B = \{3\}$$

$$C \setminus B = \{2\}$$

2)

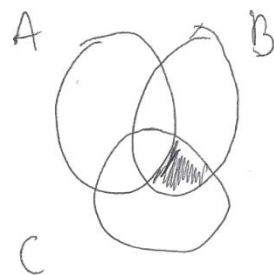
a)



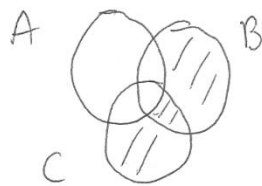


b)

i)



ii)



3)

$$a) \quad a = 2\hat{i} - 3\hat{j} - \hat{k}$$

$$b = \hat{i} - 2\hat{j} - \hat{k}$$

$$c = \hat{i} - \hat{j} - \hat{k}$$

$$a + 2c = (2, 3, 1) + 2(1, 1, 1) = (2, 3, 1) + (2, 2, 2)$$

$$b) \quad \hat{c} = \frac{c}{\|c\|} = \frac{(1, 1, 1)}{\sqrt{1^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} = \frac{c}{\sqrt{3}}$$

$$c) \quad a \cdot c = (2\hat{i} \times \hat{i}), (-3\hat{j} \times -\hat{j}), (-\hat{k} \times -\hat{k})$$

$$= 2 + 3 + 1 = 6$$

$$\|a\| \sqrt{(2)^2, (-3)^2, (1)^2} = 6$$

$$\|c\| \sqrt{(1)^2, (-1)^2, (-1)^2} = 1$$

$$\Theta = \cos^{-1} \left(\frac{6}{6 \times 1} \right) = 0^\circ$$

$$d) \quad a \times b = (2 \times 1) - (3 \times 2),$$

$$a \times b = (2 \times 1), (-3 \times -2), (-1 \times -1) = (2, 6, 1)$$

$$\vec{a} \times \vec{b} = (1, 1, -1)$$

$$\vec{a} = (2, -3, 1)$$

$$\vec{b} = (1, -2, -1)$$

$$\begin{array}{cccccc} & \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ \begin{array}{c} 2 \\ -3 \\ 1 \end{array} & \times & \begin{array}{c} 1 \\ -2 \\ -1 \end{array} & = & \begin{array}{c} 2 \\ 3 \\ -2 \end{array} \end{array}$$

$$a \times b = 2\hat{i} - 6\hat{j} - 2\hat{k}$$

$$a \times b = \hat{i}(-3 \times -1) - \hat{j}(-1 \times 1) - \hat{k}(2 \times -2)$$

$$= \hat{i}(3) - \hat{j}(-1) - \hat{k}(-4)$$

$$= 3\hat{i} + \hat{j} + 4\hat{k}$$

$$= 3\hat{i} + \hat{j} - 1\hat{k} = (3, 1, -1)$$

5)

a) $U_{31} = 3, U_{21} = 0, V_{32} = 1/a, V_{12} = 0, W_{22} = 5,$
 $Z_{33} = 1/a$

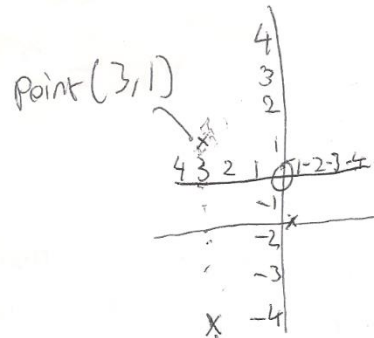
b) $V = \begin{pmatrix} -1 & 5 & 2 \\ 0 & 3 & 1 \end{pmatrix} = 2 \times 3$
 $W = \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix} = 2 \times 2$ = not possible because the number of columns do not match

bii) $V = \begin{pmatrix} -1 & 5 & 2 \\ 0 & 3 & 1 \end{pmatrix} = 2 \times 3$
 $Z = \begin{pmatrix} 3 & -2 & 0 \\ -1 & 4 & 2 \end{pmatrix} = 2 \times 3$ = $\begin{pmatrix} 4 & 7 & 2 \\ 1 & -1 & 1 \end{pmatrix}$

biii) $U = \begin{pmatrix} 1 & 3 & 1 \\ 0 & 2 & -1 \\ 3 & -2 & 5 \end{pmatrix} = 3 \times 3$
 $V^T = \begin{pmatrix} -1 & 5 & 0 \\ 5 & 3 \\ 2 & 1 \end{pmatrix} = 2 \times 3$ = Not possible because the number of columns do not match

biv) $W = \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix} = 2 \times 2$
 $Z = \begin{pmatrix} 3 & -2 & 0 \\ -1 & 4 & 2 \end{pmatrix} = 2 \times 3$ = not possible because the number of columns do not match

c)



$$y = \tan(120^\circ) x \approx -1.74 \dots$$

6)

a)