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Math 024
03D
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4.6/16, 21, 23; 5.3/ 4, 8, 11, 16

4.6/16. $X_p = A\cos\omega_f t + B\sin\omega_f t$

$= C\cos(\omega_f t - \delta)$

$C = \sqrt{A^2+B^2}$

$\tan\delta \frac{B}{A}$

$F_0 = \cos\omega_f t$

Both the steady-state solution and forcing function have the same frequency and the answer is True

21. $x'' + 2x' + 2x = 2\cos t$

$r^2 + 2r + 2 = 0$

$r_{1,2} = \frac{-2 \pm \sqrt{4-8}}{2} = -1 \pm i$

$X_1(t) = e^{-t}(C_1\cos t) + (C_2 \sin t)$

$X_2 = A\cos t + B\sin t$

$x'_2 = -A\sin t + B\cos t$

$x''_2 = -A\cos t - B\sin t$

$-A\cos t - B\sin t + 2(-A\sin t + B\cos t) + 2(A\cos t + B\sin t) = 2\cos t$

$(A + 2B)\cos t + (B - 2A)\sin t = 2\cos t$

$A + 2B = 2 \qquad B - 2A = 0$

$A = \frac{2}{5} \qquad B = \frac{4}{5}$

$X_2 = \frac{2}{5}\cos t + \frac{4}{5}\sin t$

$C\cos(\omega t - \delta)$

$C = \sqrt{A^2+B^2}$

$= \sqrt{(\frac{2}{5})^2 + (\frac{4}{5})^2}$

$= \frac{2}{\sqrt{5}}$

$\delta = \frac{B}{A}$

$= \frac{\frac{4}{5}}{\frac{2}{5}}$

$= 1.1$

$\omega = 1$

$X = \frac{2}{\sqrt{5}}\cos(t - 1.1)$

$m\ddot{x}+kx=F_0\cos w_f t$

23 $X_1=C_1\cos w_0+C_2\sin w_0 t$

$w_0=\sqrt{\frac{k}{m}}$

$\ddot{x}+12x=F_0\cos w_f t$ 12 lb/ft and mass of 1 slug

$w_f=w_0$

$w_f=\sqrt{12}=2\sqrt{3}$

5.3/ 4. $A=\begin{bmatrix}3 & 4\\ -5 & -5\end{bmatrix}$

$|A-\lambda I|=0$

$\begin{vmatrix}3-\lambda & 4\\ -5 & -5-\lambda\end{vmatrix}=0$

$(3-\lambda)(-5-\lambda)+20=0$

$\lambda^2+5\lambda-3\lambda-15+20=0$

$(\lambda+1)^2+4=0$

$\lambda+1=\pm 2i$

$\lambda_{1,2}=-1\pm 2i$

$(A-\lambda I)v=0$

$\lambda=-1+2i$

$\begin{bmatrix}3-(-1+2i) & 4\\ -5 & -5-(-1+2i)\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

$\begin{bmatrix}4-2i & 4\\ -5 & -4-2i\end{bmatrix}\begin{bmatrix}v_1\\ v_2\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$

RREF. $\begin{bmatrix}4-2i & 4\\ -5 & -4-2i\end{bmatrix}$ is $\begin{bmatrix}1 & \frac{4}{5}+\frac{2}{5}i\\ 0 & 0\end{bmatrix}$

$v_1+\left(\frac{4}{5}+\frac{2}{5}i\right)v_2=0.$

v_2 is free variable

eigenvalue

$V_2 = t$

$V_1 = (-\frac{4}{5} - \frac{2}{5}i)t$

$v = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

$= \begin{bmatrix} (-\frac{4}{5} - \frac{2}{5}i)t \\ t \end{bmatrix}$

$= t \begin{bmatrix} -\frac{4}{5} - \frac{2}{5}i \\ 1 \end{bmatrix}$

$\lambda = -1 + 2i$

$\begin{bmatrix} -\frac{4}{5} - \frac{2}{5}i \\ 1 \end{bmatrix}$

$\lambda_2 = -1 - 2i$ is the conjugate of eigenvector corresponding to the eigenvalue $\lambda_1 = -1 + 2i$

$\lambda_2 = -1 - 2i$ is $\begin{bmatrix} -\frac{4}{5} + \frac{2}{5}i \\ 1 \end{bmatrix}$. corresponding to the eigenvalue

cannot sketch in real space, eigenvector are complex number.

8. $A = \begin{bmatrix} 12 & -6 \\ 15 & -7 \end{bmatrix}$

$|A - \lambda I| = 0$

$\left| \begin{bmatrix} 12 & -6 \\ 15 & -7 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$

$\begin{vmatrix} 12-\lambda & -6 \\ 15 & -7-\lambda \end{vmatrix} = 0$

$\lambda^2 - 5\lambda + 6 = 0$

$(\lambda - 2)(\lambda - 3) = 0$

$\lambda_{1,2} = 2, 3$

$(A - \lambda I)\vec{v} = 0$

$\lambda_1 = 2 \quad \vec{v}_1 = \begin{bmatrix} x \\ y \end{bmatrix}$

$(A - 2I)\vec{v}_1 = \vec{0}$

$\left(\begin{bmatrix} 12 & -6 \\ 15 & -7 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right) = 0$

$\begin{bmatrix} 10 & -6 \\ 15 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

RREF: $\left[\begin{array}{cc|c} 1 & -3/5 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$x = \frac{3}{5} y$

$y = 1 \quad x = \frac{3}{5} \qquad \lambda_2 = 2.$

$v_1 = \begin{bmatrix} 1 \\ \frac{3}{5} \end{bmatrix}$

$\lambda_2 = 3 \quad \vec{v}_2 = \begin{bmatrix} x \\ y \end{bmatrix}$

$(A - 3I)\vec{v}_2 = 0$

$\left(\begin{bmatrix} 12 & -6 \\ 15 & -7 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) = 0$

$\begin{bmatrix} 3 & -6 \\ 15 & -10 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\left[\begin{array}{cc|c} 9 & -6 & 0 \\ 15 & -10 & 0 \end{array}\right]$

RREF

$\left[\begin{array}{cc|c} 1 & -2/3 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$x = \frac{2}{3}y$

$y = 1 \quad x = \frac{2}{3}$

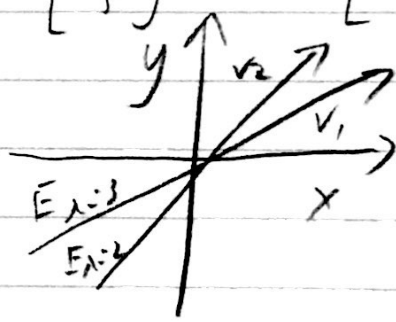
$\lambda_1 = 3$

$\vec{v}_2 = \begin{bmatrix} 1 \\ \frac{2}{3} \end{bmatrix}$

$E_{\lambda_1 = 2} = \text{Span}\left\{\begin{bmatrix} 1 \\ \frac{3}{5} \end{bmatrix}\right\}$

$E_{\lambda_2 = 3} = \text{Span}\left\{\begin{bmatrix} 1 \\ \frac{2}{3} \end{bmatrix}\right\}$

$\vec{v}_1 = \begin{bmatrix} 1 \\ \frac{3}{5} \end{bmatrix} \quad \vec{v}_2 = \begin{bmatrix} 1 \\ \frac{2}{3} \end{bmatrix}$



11. $A = \begin{bmatrix} 3 & 5 \\ -1 & -1 \end{bmatrix}$

$|A - \lambda I| = 0$

$\left| \begin{bmatrix} 3 & 5 \\ -1 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$

$\begin{vmatrix} 3-\lambda & 5 \\ -1 & -1-\lambda \end{vmatrix} = 0$

$\lambda^2 - 2\lambda + 2 = 0$

$a = 1,\ b = -2,\ c = 2$

$\lambda_2 = 1 - i$

$\begin{bmatrix} 2+i & 5 \\ -1 & -2+i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 2-i \\ -1 & -2+i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$R_1 = \frac{1}{2+i} k_1$

$\begin{bmatrix} 1 & 2-i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$R_2 = R_2 + R_1$

$x + (2-i)y = 0$

$x = -(2-i)y$

$y = 1 \quad x = -2+i$

eigenvalue $\lambda_2 = 1-i$

$V_2 = \begin{bmatrix} -2+i \\ 1 \end{bmatrix}$

$\lambda = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{2 \pm \sqrt{4-8}}{2}$

$= 1 \pm i$

$\lambda_{1,2} = 1 \pm i$

$(A - \lambda I)v = 0$

$\lambda_1 = 1+i$

$\begin{bmatrix} 2+i & 5 \\ -1 & -2-i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} 1 & 2+i \\ -1 & -2-i \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $R_1 = \frac{1}{2-i} R_1$

$\begin{bmatrix} 1 & 2+i \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $R_2 = R_2 + R_1$

$x + (2+i)y = 0$

$x = -(2+i)y$

$y = 1 \quad x = -2-i$

eigenvalue $\lambda_1 = 1+i$ is $v_1 = \begin{bmatrix} -2-i \\ 1 \end{bmatrix}$

Cannot sketch the Eigen space.

eignvalues are all complex numbers

16. $A = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}$

$|A - \lambda I| = 0$

$\left| \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$

$\begin{vmatrix} 1-\lambda & 1 \\ -1 & -1-\lambda \end{vmatrix} = 0$

$\lambda^2 = 0$

$\lambda_{1,2} = 0$

$(A - \lambda I)\vec{v} = 0$

$\lambda_1 \quad \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\left[\begin{array}{cc|c} 1 & 1 & 0 \\ -1 & -1 & 0 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$

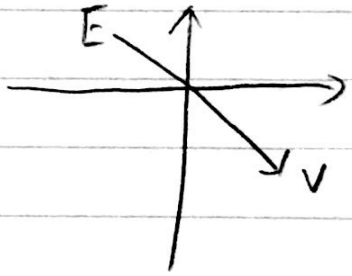
$x + y = 0$

$x = -y$

y is free vrible and only eigenvector

$\vec{v} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. eignvector is real.

Sheer Eigen span



5.3 / 21, 22, 25, 28, 30, 31, 76.

21. $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix}$ $\quad |A - \lambda I| = 0.$

$$\left| \begin{bmatrix} 1 & 2 & 2 \\ 2 & 0 & 3 \\ 2 & 3 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & 2 \\ 2 & -\lambda & 3 \\ 2 & 3 & -\lambda \end{vmatrix} = 0 \qquad (5-\lambda)(\lambda+1)(\lambda+3) = 0$$

$$\lambda_{1,2,3} = 5, -1, -3$$

$(A - \lambda I)\vec{v} = 0.$

$$\lambda_1, \begin{bmatrix} -4 & 2 & 2 \\ 2 & -5 & 3 \\ 2 & 3 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

RREF

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \qquad \begin{array}{l} x = z \\ y = z \end{array}$$

$$v_1 = C \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\lambda_2 \begin{bmatrix} 4 & 2 & 2 \\ 2 & 3 & 3 \\ 2 & 3 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

RREF. $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x = 0$

$y = -z$

$$v_2 = C \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\lambda_3 \begin{bmatrix} 2 & 2 & 2 \\ 2 & 1 & 3 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

RREF $\left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$x + 2z = 0 \quad \cancel{y. z}$

$y - z = 0 \qquad \vec{v}_3 = C \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

$x = -2z$

$E_1 = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ one-dimensional

$E_2 = \text{Span}\left\{ \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix} \right\}$ a single

$E_3 = \text{Span}\left\{ \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \right\}$ vector

22. $A = \begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $|A - \lambda I| = 0$

$$\left| \begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} -\lambda & 1 & -1 \\ 0 & -1-\lambda & 1 \\ 0 & 0 & -\lambda \end{vmatrix} = 0.$$

$-\lambda((-\lambda-1)(-\lambda) - 0) = 0$

$-\lambda(-\lambda-1)(-\lambda) = 0$

$\lambda_{123} = 0, 0, -1$

$(A - \lambda_1 I)\vec{v} = 0$

$\lambda_1 = 0.$ $\begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\left[\begin{array}{ccc|c} 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $R_2 = R_2 + R_1$

$y - z = 0$

$z = k$ $y = k$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ k \\ k \end{bmatrix} = k \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

$\lambda_2 = 0.$ $\begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

$\left[\begin{array}{ccc|c} 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right]$ $R_2 = R_2 + R_1$

$y - z = 0$

$z = k$ $y = k.$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t \\ k \\ k \end{bmatrix} = k \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\lambda_3 = -1$

$(A - (-1)I)u = 0$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad R_1 = R_1 + R_2$$

$x + y = 0 \quad z = 0$

$y = k \quad x = -k$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -k \\ k \\ 0 \end{bmatrix} = k\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$E_1 = \text{Span}\left\{\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right\}$$

$$E_2 = \text{Span}\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right\}$$

$$E_3 = \text{Span}\left\{\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}\right\}$$

25. $$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 3 & 0 \\ -4 & 13 & -1 \end{bmatrix}$$

$|A - \lambda I| = 0$

$-\lambda^3 + \lambda^2 + \lambda + 2 = 0$

$$\begin{bmatrix} -1 & 0 & 1 \\ -1 & 3 & 0 \\ -4 & 13 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = 0$$

$$\begin{vmatrix} -1-\lambda & 0 & 1 \\ -1 & 3-\lambda & 0 \\ -4 & 13 & -1-\lambda \end{vmatrix} = 0$$

$\lambda_1 = 2$

$\boxed{V_1 = [1, 1, 3]}$

$$\Rightarrow \begin{bmatrix} -3 & 0 & 1 \\ -1 & 1 & 0 \\ -4 & 13 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1/3 & 0 \\ 0 & 1 & -1/3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

RREF.

$3x = z$

$3y = z$

$\lambda_2 = -\frac{1}{2} - \frac{1}{2} i\sqrt{3}$

$$\begin{bmatrix} -1-\frac{1}{2}+\frac{1}{2}i\sqrt{3} & 0 & 1 \\ -1 & 3-\frac{1}{2}+\frac{1}{2}i\sqrt{3} & 0 \\ -4 & 13 & -1-\frac{1}{2}+\frac{1}{2}i\sqrt{3} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{2}-\frac{1}{2}i\sqrt{3} & 0 \\ 0 & 1 & -0.1923-0.1999i & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \text{RREF}$$

$x = (-\frac{1}{2}-\frac{1}{2}i\sqrt{3})z$

$y = (-0.1923-0.1999i)z$

$$\left[\begin{array}{ccc|c} -\frac{1}{2}+\frac{1}{2}i\sqrt{3} & 0 & -\frac{1}{2}-\frac{1}{2}i\sqrt{3} & 0 \\ -1 & \frac{7}{2}+\frac{1}{2}i\sqrt{3} & 0 & 0 \\ -4 & 13 & -\frac{1}{2}+\frac{1}{2}i\sqrt{3} & 0 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 4 & 13 & -\frac{1}{2}+\frac{1}{2}i\sqrt{3} & 0 \end{array}\right]$$

$$\vec{V}_2 = \begin{bmatrix} \frac{7}{2}-\frac{1}{2}i\sqrt{3} \\ 1 \\ \frac{5}{2}-\frac{3}{2}i\sqrt{3} \end{bmatrix}$$

$\lambda_3 = -\frac{1}{2}+\frac{1}{2}i\sqrt{3}$

$$\begin{bmatrix} -\frac{1}{2}-\frac{1}{2}i\sqrt{3} & 0 & 1 \\ -1 & \frac{7}{2}-\frac{1}{2}i\sqrt{3} & 0 \\ -4 & 13 & -\frac{1}{2}-\frac{1}{2}i\sqrt{3} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -0.5+0.866i & 0 \\ 0 & 1 & -0.1923+0.19999i & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \text{RREF}$$

$x = (0.5-0.866i)z$

$y = (0.1923-0.19999i)z$

E_1 is one dimensional in R^3

$\lambda_2\ \lambda_3$ no real eigenspaces

$$V_3 = \begin{bmatrix} \frac{7}{2}+\frac{1}{2}i\sqrt{3} \\ 1 \\ \frac{5}{2}-\frac{1}{2}i\sqrt{3} \end{bmatrix}$$

28 $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ $|A - \lambda I| = 0$ $\left| \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$

$\begin{vmatrix} 1-\lambda & 1 & 1 \\ 0 & 1-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$ $(1-\lambda)^3 = 0$

$\lambda_{123} = 1, 1, 1$

$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ RREF $y = 0$ $z = 0$

$v_{1,2,3} = C \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$E_1 = \text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

$E_2 = \text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

$E_3 = \text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

30. $A = \begin{bmatrix} 0 & 0 & 2 \\ -1 & 1 & 2 \\ -1 & 0 & 3 \end{bmatrix}$ $|A - \lambda I| = 0$ $\left| \begin{bmatrix} 0 & 0 & 2 \\ -1 & 1 & 2 \\ -1 & 0 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} \right| = 0$

$\begin{vmatrix} -\lambda & 0 & 2 \\ -1 & 1-\lambda & 2 \\ -1 & 0 & 3-\lambda \end{vmatrix} = 0.$ $(1-\lambda)^2(\lambda-2) = 0$ $\lambda_{123} = 1, 1, 2$

$(A - \lambda_1 I)\vec{v} = 0.$

λ_{12} $\begin{bmatrix} -1 & 0 & 2 \\ -1 & 0 & 2 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\left[\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ RREF $x = 2z$ $\vec{V}_{12} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

$\lambda_3,$ $\begin{bmatrix} -2 & 0 & 2 \\ -1 & -1 & 2 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\left[\begin{array}{ccc|c} -2 & 0 & 2 & 0 \\ -1 & -1 & 2 & 0 \\ -1 & 0 & 1 & 0 \end{array}\right] \xrightarrow{\text{RREF}} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$ $x = z$ $y = z$

$V_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $E_{1,2} = \text{span}\left\{ \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$

$E_3 = \text{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

31. $A=\begin{bmatrix}2&1&8&-1\\0&4&0&0\\0&0&6&0\\0&0&0&4\end{bmatrix}$ $|A-\lambda I|=0$ $\boxed{\begin{bmatrix}2&1&8&-1\\0&4&0&0\\0&0&6&0\\0&0&0&4\end{bmatrix}-\begin{bmatrix}\lambda&0&0&0\\0&\lambda&0&0\\0&0&\lambda&0\\0&0&0&\lambda\end{bmatrix}}$ $\parallel$ 0

$$\begin{bmatrix}2-\lambda&1&8&-1\\0&4-\lambda&0&0\\0&0&6-\lambda&0\\0&0&0&4-\lambda\end{bmatrix}=0.$$

$(\lambda-2)(\lambda-4)^2(\lambda-6)=0$

$\lambda_1=2$ $\lambda_{2,3}=4$ $\lambda_4=6$

$(A-\lambda_1 I)\vec{v}=0.$

λ_1, $\begin{bmatrix}0&1&8&-1\\0&2&0&0\\0&0&4&0\\0&0&0&2\end{bmatrix}\begin{bmatrix}x\\y\\z\\w\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$

$\left[\begin{array}{cccc|c}0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&0\end{array}\right]$ $y,z,w=0$ $\vec{v}_1=\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$

$\lambda_{2,3}$ $\begin{bmatrix}-2&1&8&-1\\0&0&0&0\\0&0&2&0\\0&0&0&0\end{bmatrix}\begin{bmatrix}x\\y\\z\\w\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$

$\left[\begin{array}{cccc|c}1&-0.5&0&0.5&0\\0&0&1&0&0\\0&0&0&0&0\\0&0&0&0&0\end{array}\right]$ RREF $2x-y+v=0$

$z=0$

$\vec{v}_{23}$ $\begin{bmatrix}1\\2\\0\\0\end{bmatrix},\begin{bmatrix}-1\\0\\0\\2\end{bmatrix}$

$$\lambda_4 \begin{bmatrix} -4 & 1 & 8 & -1 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc|c} -4 & 1 & 8 & -1 & 0 \\ 0 & -2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 & 0 \end{array}\right] \xrightarrow{RREF} \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$x = 2z$ $\quad y, w = 0$ $\quad z$ is free variable

$\vec{v}_4 = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ $\quad E_1 = \text{span}\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}\right\}$

$E_{2,3} = \text{span}\left\{\begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 2 \end{bmatrix}\right\}$

$E_4 = \text{span}\left\{\begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \end{bmatrix}\right\}$

E_1 and E_4 are one-dimension they contain one vector each $E_{2,3}$ is two dimension it contain 2 vector

Characteristic equation contains complex roots and solution to DE

76 $y'' + \lambda y = 0.$ $\quad y(t) = C_1 \cos\sqrt{\lambda} t + C_2 \sin\sqrt{\lambda} t$

$r^2 + \lambda = 0$ $\quad y(\pi) = 0.$ boundary condition to solve

$r_{1,2} = i\sqrt{\lambda}$ $\quad C_1 \cos(\sqrt{\lambda}\pi) + C_2 \sin(\sqrt{\lambda}\pi) = 0$

$\alpha, \beta = 0, \sqrt{\lambda}$ $\quad y(0) = 0$

$C_1 \cos(\sqrt{\lambda} \cdot 0) + C_2 \sin(\sqrt{\lambda} \cdot 0) = 0.$

$C_1 = 0$

final solution $y(t) = C_2 \sin\sqrt{\lambda}\, t$

$\sqrt{\lambda} = \frac{2n+1}{2}$

$\lambda = \left(\frac{2n+1}{2}\right)^2$

n is an integer