

4-4

October 28, 2025

1. Use the method of Example 3 to show that the following set of vectors forms a basis for R^2 .

$$[(2, 1), (3, 0)]$$

3. Show that the following polynomials form a basis for P_2 .

$$x^2 + 1, \quad x^2 - 1, \quad 2x - 1$$

5. Show that the following matrices form a basis for M_{22} .

$$\begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix}, \quad \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$$

7. a) In each part, show that the set of vectors is not a basis for R^3 .

$$[(2, -3, 1), (4, 1, 1), (0, -7, 1)]$$

9. Show that the following matrices do not form a basis for M_{22} .

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \quad \begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}, \quad \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}, \quad \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$$

11. a) Find the coordinate vector of \mathbf{w} relative to the basis $S = [\mathbf{u}_1, \mathbf{u}_2]$ for R^2 .

$$\mathbf{u}_1 = (2, -4), \quad \mathbf{u}_2 = (3, 8); \quad \mathbf{w} = (1, 1)$$

13. a) Find the coordinate vector of \mathbf{v} relative to the basis $S = [\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4]$ for R^3 .

$$\mathbf{v} = (2, -1, 3); \quad \mathbf{v}_1 = (1, 0, 0), \quad \mathbf{v}_2 = (2, 2, 0)$$

15. In Exercises 15- 16, first show that the set $S = \{A_1, A_2, A_3, A_4\}$ is a basis for M_{22} , then express A as a linear combination of the vectors in S , and then find the coordinate vector of A relative to S .

$$A_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}; \quad A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$$

17. In Exercises 17- 18, first show that the set $S = \{p_1, p_2, p_3\}$ is a basis for P_2 , then express p as a linear combination of the vectors in S , and then find the coordinate vector of p relative to S .

$$p_1 = 1 + x + x^2, \quad p_2 = x + x^2, \quad p_3 = x^2;$$

$$p = 7 - x + 2x^2$$