# 4-5

October 29, 2025

In Exercises 1- 6, find a basis for the solution space of the homogeneous linear system, and find the dimension of that space.

**2)**

$$
\begin{aligned}
3x_1 + x_2 + x_3 + x_4 &= 0 \\
5x_1 - x_2 + x_3 - x_4 &= 0
\end{aligned}
$$

**5)**

$$
\begin{aligned}
x_1 - 3x_2 + x_3 &= 0 \\
2x_1 - 6x_2 + 2x_3 &= 0 \\
3x_1 - 9x_2 + 3x_3 &= 0
\end{aligned}
$$

In each part, find a basis for the given subspace of $R^3$, and state its dimension.

**7 a) The plane $3x - 2y + 5z = 0$.**

**c) The line $x = 2t$, $y = -t$, $z = 4t$.**

**d) All vectors of the form** $(a, b, c)$**, where** $b = a + c$**.**

In each part, find a basis for the given subspace of $R^4$, and state its dimension.

**8 a) All vectors of the form** $(a, b, c, 0)$**.**

**c) All vectors of the form $(a, b, c, d)$, where $a = b = c = d$.**

**10) Find the dimension of the subspace of $P_3$ consisting of all polynomials $a_0 + a_1x + a_2x^2 + a_3x^3$ for which $a_0 = 0$.**

Find a standard basis vector for $R^3$ that can be added to the set $\{v_1, v_2\}$ to produce a basis for $R^3$.

**12 b)**

$$v_1 = (1, -1, 0), \quad v_2 = (3, 1, -2)$$

**13) Find standard basis vectors for $R^4$ that can be added to the set $\{v_1, v_2\}$ to produce $R^4$.**

$$v_1 = (1, -4, 2, -3), \quad v_2 = (-3, 8, -4, 6)$$

**15) The vectors $v_1 = (1, -2, 3)$ and $v_2 = (0, 5, -3)$ are linearly independent. Enlarge $\{v_1, v_2\}$ to a basis for $R^3$.**

**17) Find a basis for the subspace of $R^3$ that is spanned by the vectors**

$$v_1 = (1, 0, 0), \quad v_2 = (1, 0, 1), \quad v_3 = (2, 0, 1), \quad v_4 = (0, 0, -1)$$