Lab 7: *Latent Heat of Fusion for Water*

The objective of this lab exercise is to measure a value for the latent heat of fusion for water.

Ice at an initial temperature of T_i (which is below zero degrees Celsius) is tossed into an insulated container that holds warm water. The ice and water soon come to “thermal equilibrium”, i.e. the same final temperature. This is a common, simple calorimetry problem... but it’s also something that we can do in lab. With a bit of clever algebra and a few careful measurements, we can use this scenario to measure the latent heat of fusion for water.

First, the algebra:

The ice warms to $0^{O}C$, then melts, then warms (as water) to the final temperature

The water simply cools to the final temperature.

Note that the ice goes through a temperature change (as solid ice), then a phase change, then a second temperature change (as liquid water.) So “Q” for the ice can be written as:

$$Q_i = m_i c_i \Delta T_1 + m_i L_f + m_i c_w \Delta T_2$$

$$Q_i = m_i c_i (0 - T_i) + m_i L_f + m_i c_w (T_f - 0)$$

or $$\mathbf{Q_i = - m_i c_i T_i + m_i L + m_i c_w T}$$

where I have dropped the “f” subscript from “L”, used the fact that ΔT_1 is simply zero minus the initial temperature of the ice, and ΔT_2 is the final temperature of the system (for which I’ve just used an unsubscripted “T”) minus zero.

The warm water simply goes through a temperature change, so “Q” for the water can be written:

$$\mathbf{Q_w = m_w c_w (T - T_w)}$$

We can add these expressions and set the sum equal to zero:

$$Q_i + Q_w = 0$$

$$\mathbf{- m_i c_i T_i + m_i L + m_i c_w T + m_w c_w T - m_w c_w T_w = 0}$$

This is pretty ugly... there are a lot of little things in this equation, and they are scattered all over. We need some serious algebra magic to turn this into something useful. Fortunately for us, algebra is full of magic.

For our procedure, you will toss a small amount of ice into the warm water and measure the corresponding final temperature. You will then do it again: toss some more ice in, measure the new final temperature. Due to the nature of calorimetry, i.e. the principle of conservation of energy, we can claim that any particular temperature measurement is due to the accumulated amount of ice introduced to the water. The net effect is that you will be able to collect a series of correlated data: the accumulated mass of ice introduced to the water at every step along the way, and the corresponding temperature of the system.

With this in mind, our equation becomes simpler to look at: everything in the equation is a constant except for the mass of the ice and the final temperature:

$$-m_i c_i T_i + m_i L + m_i c_w T + m_w c_w T - m_w c_w T_w = 0$$

Okay... it's still pretty ugly. We need to manipulate this into a form that allows us a simple comparison between our two variables. That is, we need an equation from which we can create a straight-line graph.

To accomplish this, we can first isolate T on the left side of the equation:

$$T(m_w + m_i) c_w = m_w c_w T_w + m_i c_i T_i - m_i L$$

This helps a little bit... but now we have m_i in three places. We need to simplify somehow. And this is where we use a little trick: instead of measuring the mass of the ice directly, <u>we can measure the *total* *mass of the ice & water system*</u>*.* This is a practical consideration: with the container on the scale, the scale will read the mass of the water. When you add ice to the water, the container will read the mass of the ice and water together.

It also provides some much needed help with the algebra, as we can define:

$$M = m_i + m_w$$

and then

$$T M c_w = m_w c_w T_w + (M - m_w) c_i T_i - (M - m_w) L$$

This might not look like much of an improvement... but it truly is. Because now we can invoke the kind of magic that only algebra can provide. First we rearrange the right side of the equation with a bit of factoring:

$$T M c_w = m_w (c_w T_w - c_i T_i + L) + M (c_i T_i - L)$$

We can then divide both sides of the equation by $M c_w$ to leave just $T = \ldots$ for our equation. Note that when we do this to both terms on the right side, we will have a lot of constants, but M will appear in only one term. The result looks like this:

$$T = \frac{m_w}{c_w}\left(c_w T_w - c_i T_i + L\right) \cdot M^{-1} - \left(\frac{L - c_i T_i}{c_w}\right)$$

Okay, so it's *still* pretty ugly. But at least now it's functional. Note that the equation is essentially:

T = (bunch of constants) M^{-1} - (bunch of constants)

This means that we can measure a series of values for M (the total mass of water and ice combined) and each corresponding T (the final temperature of the system), we can graph T vs M^{-1} and the resulting slope and intercept of the best-fit line should be described by those "bunches" of constants.

Note that the slope of the best-fit line should equal:

$$slope = \frac{m_w}{c_w}\left(c_w T_w - c_i T_i + L\right)$$

If we rearrange this to solve for L, we get:

$$L = c_w \cdot \left(\frac{slope}{m_w} - T_w\right) + c_i T_i$$

To calculate the measured value of L, you will need:

c_w = 1.00 cal/g-^{O}C c_i = 0.49 cal/g-^{O}C

T_i : we will measure this as a group

T_w : you will measure this before adding ice to the warm water

m_w : you will measure this before adding ice to the warm water

Procedure

1. Set up the dry styrofoam container on the scale, with the lid in place, measure its mass.
2. Set up the temperature probe using the black box and Data Studio. Choose the "Digits" display.
3. After your warm water is poured, record the mass of the cup & lid & water together.
4. Measure the temperature of the warm water and record this as T_w.
5. Add one spoonful of ice, approximately 5 grams, to the warm water.
6. With the lid in place (but *not* the temperature probe!), record the total mass.
7. Replace the temperature probe; record the temperature after it has stabilized.
8. Repeat steps 5, 6 and 7 until a total of approximately 50 grams of ice has been added to the water.
9. Use your measured data to create a graph of T vs M^{-1}.

You can now use the value of the slope of your best-fit line, along with the initial data you recorded, to calculate the measured value of L.