2.3

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In Exercises 1–4, verify that $det(kA) = k^n det(A)$.

1

$$A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}; k = 2$$

In Exercises 7–14, use determinants to decide whether the given matrix is invertible.

7

$$A = \begin{bmatrix} 2 & 5 & 5 \\ -1 & -1 & 0 \\ 2 & 4 & 3 \end{bmatrix}$$

9

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$$

11

$$A = \begin{bmatrix} 4 & 2 & 8 \\ -2 & 1 & -4 \\ 3 & 1 & 6 \end{bmatrix}$$

In Exercises 15–18, find the values of k for which the matrix A is invertible.

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$$A = \begin{bmatrix} k-3 & -2 \\ -2 & k-2 \end{bmatrix}$$

In Exercises 24–29, solve by Cramer's rule, where it applies.

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$$\begin{aligned} 4x + 5y &= 2 \\ 11x + y + 2z &= 3 \\ x + 5y + 2z &= 3 \\ x + 5y + 2z &= 1 \end{aligned}$$

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$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Assuming that $\det(A) = -7$, find

(a) $\det(3A)$

(b) $\det(A^{-1})$

(c) $\det(2A^{-1})$

(d) $\det((2A)^{-1})$

(e) $\det \begin{bmatrix} a & g & d \\ b & h & e \\ c & i & f \end{bmatrix}$

In each part, find the determinant given that A is a 4×4 matrix for which $det(A) = -2$.

34 b)

$det(A - 1)$

c)

$det(2AT)$

Prove that if A is a square matrix, then $det(A^T A) = det(AA^T)$.

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