2-5

September 15, 2025

"In Problems 1–40 find the general solution of the given differential equation. State an interval on which the general solution is defined."

1

$$\frac{dy}{dx} = 5y$$

5

$$\frac{dy}{dx} + 12y = 4$$

9

$$x^2y' + xy = 1$$

13

$$xdy = (x \sin x - y)dx$$

17

$$\cos x \frac{dy}{dx} + y \sin x = 1$$

21

$$x^2 y' + x(x+2)y = e^x$$

25

$$ydx + (xy + 2x - ye^x)dy = 0$$

29

$$ydx - 4(x + y^6)dy = 0$$

33

$$ydx + (x + 2xy^2 - 2y)dy = 0$$

37

$$(x + 2)^2 \frac{dy}{dx} = 5 - 8y - 4xy$$

"In Problems 41–54 solve the given differential equation subject to the indicated initial condition. "

41

$$\frac{dy}{dx} + 5y = 20, \quad y(0) = 2$$

45

$$y' + (\tan x)y = \cos^2 x, \quad y(0) = -1$$

49

$$(x+1)\frac{dy}{dx} + y = \ln x, \quad y(1) = 10$$

Answers:

1

$$y = ce^{5x}, \quad -\infty < x < \infty$$

5

$$y = \frac{1}{4}e^{3x} + ce^{-x}, \quad -\infty < x < \infty$$

9

$$y = x^{-1}\ln x + cx^{-1}, \quad 0 < x < \infty$$

13

$$y = -\cos x + \frac{\sin x}{x} + \frac{c}{x}, \quad 0 < x < \infty$$

17

$$y = \sin x + c\cos x, \quad -\pi/2 < x < \pi/2$$

21

$$y = \frac{1}{2x^2}e^{x} + \frac{c}{x^2}e^{-x}, \quad 0 < x < \infty$$

25

$$x = \frac{1}{2}e^{y} - \frac{1}{2y}e^{y} + \frac{1}{4y^2}e^{y} + \frac{c}{y^2}e^{-y}, \quad 0 < x < \infty$$

29

$$x = 2y^6 + cy^4, \quad 0 < y < \infty$$

33

$$x = \frac{1}{y} + \frac{c}{y}e^{-y^2}, \quad 0 < y < \infty$$

37

$$y = \frac{5}{3}(x+2)^{-1} + c(x+2)^{-4}, \quad -2 < x < \infty$$

41

$$y = 4 - 2e^{-5x}, \quad -\infty < x < \infty$$

45

$$y = \sin x \cos x - \cos x, \quad -\pi/2 < x < \pi/2$$

49

$$(x+1)y = x \ln x - x + 21, \quad 0 < x < \infty$$