

October 28, 2025

- 1 1.** Use the method of Example 3 to show that the following set of vectors forms a basis for R^2 .

$$[[(2, 1), (3, 0)]]$$

- 2 3.** Show that the following polynomials form a basis for P_2 .

$$[x^2 + 1, \quad x^2 - 1, \quad 2x - 1]$$

- 3 5.** Show that the following matrices form a basis for M_{22} .

$$\begin{bmatrix} \begin{bmatrix} 3 & 6 \\ 3 & -6 \end{bmatrix}, \\ \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \\ \begin{bmatrix} 0 & -8 \\ -12 & -4 \end{bmatrix}, \\ \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} \end{bmatrix}$$

- 4 7. a)** In each part, show that the set of vectors is not a basis for R^3 .

$$[[(2, -3, 1), (4, 1, 1), (0, -7, 1)]]$$

- 5 9.** Show that the following matrices do not form a basis for M_{22} .

$$\begin{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \end{bmatrix},$$

$$\begin{bmatrix} 2 & -2 \\ 3 & 2 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$$

6 11. a) Find the coordinate vector of w relative to the basis $S = [u_1, u_2]$ for R^2 .

$$[u_1 = (2, -4), u_2 = (3, 8); \quad w = (1, 1)]$$

7 13. a) Find the coordinate vector of v relative to the basis $S = [v_1, v_2, v_4]$ for R^3 .

$$[v = (2, -1, 3); \quad v_1 = (1, 0, 0), v_2 = (2, 2, 0)]$$

8 15. In Exercises 15- 16, first show that the set $S = \{A_1, A_2, A_3, A_4\}$ is a basis for M_{22} , then express A as a linear combination of the vectors in S , and then find the coordinate vector of A relative to S .

$$\begin{aligned} &[\quad A_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \\ &\quad , \quad A_2 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \\ &\quad , \quad A_3 = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \\ &\quad , \quad A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \\ &\quad ; \quad A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \\ &] \end{aligned}$$

- 9 17. In Exercises 17- 18, first show that the set $S = \{p_1, p_2, p_3\}$ is a basis for P_2 , then express p as a linear combination of the vectors in S , and then find the coordinate vector of p relative to S .

$$[p_1 = 1 + x + x^2, \quad p_2 = x + x^2, \quad p_3 = x^2;] \quad [p = 7 - x + 2x^2]$$