3-2

November 9, 2025

In Exercises 1-2, find the norm of $\mathbf{v}$, and a unit vector that is oppositely directed to $\mathbf{v}$.

1 b)

$\mathbf{v} = (1, 0, 2, 1, 3)$

In Exercises 3-4, evaluate the given expression with $\mathbf{u} = (2, -2, 3)$, $\mathbf{v} = (1, -3, 4)$, and $\mathbf{w} = (3, 6, -4)$.

3 d)

$\|3\mathbf{u} - 5\mathbf{v} + \mathbf{w}\|$

In Exercises 5-6, evaluate the given expression with $\mathbf{u} = (-2, -1, 4, 5)$, $\mathbf{v} = (3, 1, -5, 7)$, and $\mathbf{w} = (-6, 2, 1, 1)$.

5 b)

$\|3\mathbf{u}\| - 5\|\mathbf{v}\| + \|\mathbf{w}\|$

7)

Let $\mathbf{v} = (-2, 3, 0, 6)$. Find all scalars k such that $\|k\mathbf{v}\| = 5$.

In Exercises 9-10, find $\mathbf{u} \cdot \mathbf{v}$, $\mathbf{u} \cdot \mathbf{u}$, and $\mathbf{v} \cdot \mathbf{v}$.

9 a)

$\mathbf{u} = (3, 1, 4)$, $\mathbf{v} = (2, 2, -4)$

b)

$\mathbf{u} = (1, 1, 4, 6)$ $\mathbf{v} = (2, -2, 3, -2)$

In Exercises 11-12, find the Euclidean distance between $\mathbf{u}$ and $\mathbf{v}$ and the cosine of the angle between those vectors. State whether that angle is acute, obtuse, or $90°$.

11 a)

$\mathbf{u} = (3, 3, 3)$, $\mathbf{v} = (1, 0, 4)$

b)

$\mathbf{u} = (0, -2, -1, 1)$, $\mathbf{v} = (-3, 2, 4, 4)$

In Exercises 17-18, verify that the Cauchy-Schwarz inequality holds.

17 a)

$\mathbf{u} = (-3, 1, 0), \mathbf{v} = (2, -1, 3)$

b)

$\mathbf{u} = (0, 2, 2, 1)$ $\mathbf{v} = (1, 1, 1, 1)$

Answers

1. (a) $\|\mathbf{v}\| = 2\sqrt{3}$; $\frac{1}{\|\mathbf{v}\|}\mathbf{v} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$; $-\frac{1}{\|\mathbf{v}\|}\mathbf{v} = \left(-\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$

(b) $\|\mathbf{v}\| = \sqrt{15}$; $\frac{1}{\|\mathbf{v}\|}\mathbf{v} = \left(\frac{1}{\sqrt{15}}, 0, \frac{2}{\sqrt{15}}, \frac{1}{\sqrt{15}}, \frac{3}{\sqrt{15}}\right)$; $\frac{1}{\|\mathbf{v}\|}\mathbf{v} = \left(-\frac{1}{\sqrt{15}}, 0, -\frac{2}{\sqrt{15}}, -\frac{1}{\sqrt{15}}, -\frac{3}{\sqrt{15}}\right)$

3. (a) $\sqrt{83}$ (b) $\sqrt{17} + \sqrt{26}$ (c) $2\sqrt{3}$ (d) $\sqrt{466}$

5. (a) $\sqrt{2570}$ (b) $3\sqrt{46} - 10\sqrt{21} + \sqrt{42}$ (c) $2\sqrt{966}$

7. $k = \frac{5}{7}$ or $k = -\frac{5}{7}$

9. (a) $\mathbf{u} \cdot \mathbf{v} = -8$; $\mathbf{u} \cdot \mathbf{u} = 26$; $\mathbf{v} \cdot \mathbf{v} = 24$ (b) $\mathbf{u} \cdot \mathbf{v} = 0$; $\mathbf{u} \cdot \mathbf{u} = 54$; $\mathbf{v} \cdot \mathbf{v} = 21$

11. (a) $d(\mathbf{u}, \mathbf{v}) = \sqrt{14}$; $\cos\theta = \frac{5}{\sqrt{51}}$; the angle is acute 13. $\frac{45\sqrt{3}}{2}$

(b) $d(\mathbf{u}, \mathbf{v}) = \sqrt{59}$; $\cos\theta = \frac{-4}{\sqrt{6}\sqrt{45}}$; the angle is obtuse

15. (a) Does not make sense; $\mathbf{v} \cdot \mathbf{w}$ is a scalar, whereas the dot product is only defined for vectors

(b) Makes sense (c) Does not make sense; $\mathbf{u} \cdot \mathbf{v}$ is a scalar, whereas the norm is only defined for vectors (d) Makes sense

25. $71°, 61°, 36°$