4-3

November 9, 2025

1 Explain why the following form linearly dependent sets of vectors. (Solve this problem by inspection.) b)

$$\mathbf{u}_1 = (3, -1), \quad \mathbf{u}_2 = (4, 5), \quad \mathbf{u}_3 = (-4, 7) \quad \text{in } R^2$$

c)

$$\mathbf{p}_1 = 3 - 2x + x^2 \text{ and } \mathbf{p}_2 = 6 - 4x + 2x^2 \text{ in } P_2$$

d)

$$A = \begin{bmatrix} -3 & 4 \\ 2 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 3 & -4 \\ -2 & 0 \end{bmatrix} \text{ in } M_{22}$$

2. In each part, determine whether the vectors are linearly independent or are linearly dependent in R^3. a)

(-3, 0, 4), (5, -1, 2), (1, 1, 3)

3. In each part, determine whether the vectors are linearly independent or are linearly dependent in R^4. a)

(3,8,7,-3), (1,5,3,-1), (2,-1,2,6), (4,2,6,4)

4. In each part, determine whether the vectors are linearly independent or are linearly dependent in P_2. b)

$$1 + 3x + 3x^2,\ x + 4x^2,\ 5 + 6x + 3x^2,\ 7 + 2x - x^2$$

5. In each part, determine whether the matrices are linearly independent or dependent. a)

$$\begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} \text{ in } M_{22}$$

7. In each part, determine whether the three vectors lie in a plane in R^3. a)

$$\mathbf{v}_1 = (2, -2, 0),\ \mathbf{v}_2 = (6, 1, 4),\ \mathbf{v}_3 = (2, 0, -4)$$

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(a) Show that the three vectors $v_1 = (0, 3, 1, -1)$, $v_2 = (6, 0, 5, 1)$, and $v_3 = (4, -7, 1, 3)$ form a linearly dependent set in R^4. (b) Express each vector in part (a) as a linear combination of the other two.

15. Are the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ in part (a) of the accompanying figure linearly independent? What about those in part (b)? Explain.

(a) Yes, all vector lie on different planes. No two vectors lie on the same plane.
(b) No. All vectors lie on a 2D plane, therefore they cannot be linearly independent.

16. By using appropriate identities, where required, determinewhich of the following sets of vectors in $F(-\infty, \infty)$ are linearly dependent. a)

$$6, 3\sin^2 x, 2\cos^2 x$$

c)

$$1,\ \sin x,\ \sin 2x$$

e)

$$(3-x)^2,\ x^2-6x,\ 5$$

19. (Calculus required) Use the Wronskian to show that the following sets of vectors are linearly independent. a)

$$1, x, e^x$$

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(Calculus required) Use the Wronskian to show that the functions $f_1(x) = \sin x$, $f_2(x) = \cos x$, and $f_3(x) = x \cos x$ are linearly independent vectors in $C^\infty(-\infty, \infty)$.