4-5

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In Exercises 1- 6, find a basis for the solution space of the homogeneous linear system, and find the dimension of that space.

2)

$$3x_1 + x_2 + x_3 + x_4 = 0$$
$$5x_1 - x_2 + x_3 - x_4 = 0$$

5)

$$x_1 - 3x_2 + x_3 = 0$$
$$2x_1 - 6x_2 + 2x_3 = 0$$
$$3x_1 - 9x_2 + 3x_3 = 0$$

In each part, find a basis for the given subspace of R^3, and state its dimension.

7 a) The plane $3x - 2y + 5z = 0$.

c) The line $x = 2t$, $y = -t$, $z = 4t$.

d) All vectors of the form (a, b, c), where $b = a + c$.

In each part, find a basis for the given subspace of R^4, and state its dimension.

8 a) All vectors of the form $(a, b, c, 0)$.

c) All vectors of the form (a, b, c, d), where $a = b = c = d$.

10) Find the dimension of the subspace of P_3 consisting of all polynomials $a_0 + a_1x + a_2x^2 + a_3x^3$ for which $a_0 = 0$.

Find a standard basis vector for R^3 that can be added to the set $\{v_1, v_2\}$ to produce a basis for R^3.

12 b)

$$v_1 = (1, -1, 0), \quad v_2 = (3, 1, -2)$$

13) Find standard basis vectors for R^4 that can be added to the set $\{v_1, v_2\}$ to produce R^4.

$$v_1 = (1, -4, 2, -3), \quad v_2 = (-3, 8, -4, 6)$$

15) The vectors $v_1 = (1, -2, 3)$ and $v_2 = (0, 5, -3)$ are linearly independent. Enlarge $\{v_1, v_2\}$ to a basis for R^3.

17) Find a basis for the subspace of R^3 that is spanned by the vectors

$$v_1 = (1, 0, 0), \quad v_2 = (1, 0, 1), \quad v_3 = (2, 0, 1), \quad v_4 = (0, 0, -1)$$