8-1

November 19, 2025

In Exercises 1-2, suppose that T is a mapping whose domain is the vector space M_{22} . In each part, determine whether T is a linear transformation, and if so, find its kernel.

1 a)

$T(A) = A^2$

b)

$T(A) = \mathrm{tr}(A)$

In Exercises 3-9, determine whether the mapping T is a linear transformation, and if so, find its kernel.

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$T : M_{22} \to M_{23}$, where B is a fixed 2×3 matrix and $T(A) = AB$

6 a)

$T : M_{22} \to R$, where $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = 3a - 4b + c - d$

7 a)

$T : P_2 \to P_2$, where $T(a_0 + a_1x + a_2x^2) = a_0 + a_1(x+1) + a_2(x+1)^2$

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Consider the basis $S = \{\mathbf{v}_1, \mathbf{v}_2\}$ for R^2 , where $\mathbf{v}_1 = (1,1)$ and $\mathbf{v}_2 = (1,0)$, and let $T : R^2 \to R^2$ be the linear operator for which

$$T(\mathbf{v}_1) = (1,-2)\text{a n d}T(\mathbf{v}_2) = (-4,1)$$

Find a formula for $T(x_1, x_2)$, and use that formula to find $T(5,-3)$

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Consider the basis $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for R^3 , where
$\mathbf{v}_1 = (1,1,1), \mathbf{v}_2 = (1,1,0)$, and $\mathbf{v}_3 = (1,0,0)$ and let $T : R^3 \to R^3$ be the linear operator for which

$$T(\mathbf{v}_1) = (2,-1,4), \quad T(\mathbf{v}_2) = (3,0,1),$$
$$T(\mathbf{v}_3) = (-1,5,1)$$

Find a formula for $T(x_1, x_2, x_3)$, and use that formula to find $T(2,4,-1)$

Answers

1. (a) Nonlinear

5. Linear; kernel consists of all 2×2 matrices whose rows are orthogonal to all columns of B

7. (a) Linear; $\ker(T) = \{0\}$

17. (a) $(1, 0, 1)$

19. $T(x_1, x_2) = (-4x_1 + 5x_2, x_1 - 3x_2)$. $T(5, -3) = (-35, 14)$

21. $T(x_1, x_2, x_3) = (-x_1 + 4x_2 - x_3, 5x_1 - 5x_2 - x_3, x_1 + 3x_3); T(2, 4, -1) = (15, -9, -1)$