4-1

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- a) $u + v = (2, 6)$, $ku = (0, 12)$
- b) V is a linear transformation where all vectors of the space result in a vector that is also contained in that space.
- c) Axioms 1-5
- d) $k(u + v) = ku + kv$

$$u = (a, b); \qquad v = (c, d)$$

$$\begin{aligned} \text{Axiom 7 states: } k(u + v) &= k(a, b) + v(c + d) \\ k((a, b) + (c, d)) &= (0, kb) + (0, kd) \\ k(a + c, b + d) &= \\ (0, k(b + d)) &= \\ (0, kb + kd)) &= (0, kb + kd) \end{aligned}$$

Axiom 8:

$$\begin{aligned} (k + m)u &= ku + mu \\ (k + m)(a, b) &= k(a, b) + m(a, b) \\ (0, (k + m)b) &= (0, kb) + (0, mb) \\ (0, kb + mb) &= (0, kb + mb) \end{aligned}$$

Axiom 9:

$$\begin{aligned} k(mu) &= (km)(u) \\ k(m(a, b)) &= (km)(a, b) \\ k(0, mb) &= (0, kmb) \\ (0, kmb) &= (0, kmb) \end{aligned}$$

- e)

$$\begin{aligned} 1u &= u \\ 1(a, b) &= (a, b) \\ (0, 0) &\neq (a, b) \end{aligned}$$

5 The set of all pairs of real number of the form (x, y), where $x \geq 0$, with the standard operations on R^2 is not a vector space because it violates axiom 5 which states $-u$ exist in the vector space.

$$(-1, y) \notin \{\boldsymbol{R}^2, (x, y) \mid x >= 0\}$$