4-7

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In Exercises 3- 4, determine whether **b** is in the column space of A , and if so, express **b** as a linear combination of the column vectors of A

3) a)

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 3 \end{bmatrix}; \quad b = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

b)

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 9 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix}; \quad b = \begin{bmatrix} 5 \\ 1 \\ -1 \end{bmatrix}$$

In Exercises 7–8, find the vector form of the general solution of the linear system $A\mathbf{x} = \mathbf{b}$, and then use that result to find the vector form of the general solution of $A\mathbf{x} = \mathbf{0}$.

7) b)

$$\begin{array}{rrrr} x_1+ & x_2+ & 2x_3 = & 5 \\ x_1+ & + & x_3 = & -2 \\ 2x_1+ & x_2+ & 3x_3 = & 3 \end{array}$$

In Exercises 9- 10, find bases for the null space and row space of A.

9 a)

$$A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$$

10 a)

$$A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$$

In Exercises 11- 12, a matrix in row echelon form is given. By inspection, find a basis for the row space and for the column space of that matrix.

11 b)

$$\begin{bmatrix} 1 & -3 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

12 a)

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 1 & -3 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

13) a) Use the methods of Examples 6 and 7 to find bases for the row space and column space of the matrix

$$A = \begin{bmatrix} 1 & -2 & 5 & 0 & 3 \\ -2 & 5 & -7 & 0 & -6 \\ -1 & 3 & -2 & 1 & -3 \\ -3 & 8 & -9 & 1 & -9 \end{bmatrix}$$

b) Use the method of Example 9 to find a basis for the row space of A that consists entirely of row vectors of A.

In Exericses 16–17, find a subset of the given vectors that forms a basis for the space spanned by those vectors, and then express each vector that is not in the basis as a linear combination of the basis vectors.

17) $v_1 = (1, -1, 5, 2)$, $v_2 = (-2, 3, 1, 0)$, $v_3 = (4, -5, 9, 4)$, $v_4 = (0, 4, 2, -3)$, $v_5 = (-7, 18, 2, -8)$

In each part, let $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & -1 & 4 \end{bmatrix}$. For the given vector $\mathbf{b}$, find the general form of all vectors $\mathbf{x}$ in R^3 for which $T_A(\mathbf{x}) = \mathbf{b}$ if such vectors exist.

21 a) $\mathbf{b} = (0, 0)$

c) $\mathbf{b} = (-1, 1)$

24 a) Find a 3 $\times$ 3 matrix whose null space is a point.

b) Find a 3 $\times$ 3 matrix whose null space is a line.