2.4

September 9, 2025

Find the general solution of the given differential equation. State an interval on which the general solution is defined.

1

$$(2x - 1)dx + (3y + 7)dy = 0$$

3

$$(5x + 4y)dx + (4x - 8y^3)dy = 0$$

5

$$(2y^2x - 3)dx + (2yx^2 + 4)dy = 0$$

7

$$y' + 3x^2y = x^2$$

9

$$x^2y' + xy = 1$$

13

$$x\,dy = (x\,\sin\,x - y)dx$$

17

$$\cos x \frac{dy}{dx} + y \sin x = 1$$

21

$$x^2y' + x(x+2)y = e^x$$

25

$$y\,dx + (xy + 2x - ye^y)dy = 0$$

29

$$y\,dx - 4(x + y^6)\,dy = 0$$

33

$$y\ dx + (x + 2xy^2 - 2y)\ dy = 0$$

37

$$(x+2)^2\frac{dy}{dx} = 5 - 8y - 4xy$$

41

$$\frac{dy}{dx} + 5y = 20, \quad y(0) = 2$$

Answers:

1

$$x^2 - x + \frac{3}{2}y^2 + 7y = c$$

3

$$\frac{5}{2}x^2 + 4xy - 2y^4 = c$$

5

$$x^2y^2 - 3x + 4y = c$$

7

Not exact, but homogenous.

9

$$xy^3 + y^2 \cos x - \frac{1}{2}x^2 = c$$

13

$$xy - 2xe^x + 2e^x - 2x^3 = c$$

17

$$x^3y^3 - \tan^{-1} 3x = c$$

21

$$y - 2x^2y - y^2 - x^4 = c$$

25

$$\frac{1}{3}x^3 + x^2y + xy^2 - y = \frac{4}{3}$$

29

$$y^2 \sin x - x^3 y - x^2 + y \ln y - y = 0$$

33

$$k = 1$$

37

$$\begin{aligned} M(x,y) &= 6xy^3 \\ N(x,y) &= 6xy^3 \\ N(x,y) &= 4y^3 + 9x^2y^2 \\ \frac{\partial M}{\partial y} &= 18xy^2 = \frac{\partial N}{\partial x} \\ Solution is 3x^2y^3 + y^4 &= c. \end{aligned}$$

41

$$\begin{aligned} M(x,y) &= 2xy^2 + 3x^2 \\ N(x,y) &= 2xy^2 + 3x^2 \\ N(x,y) &= 2x^2y \\ \frac{\partial M}{\partial y} &= 4xy = \frac{\partial N}{\partial x} \\ Solution is x^2y^2 + x^3 &= c. \end{aligned}$$