2.3

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In Problems 1–10 determine whether the given function is homogeneous. If so, state the degree of homogeneity.

1

$$x^3 + 2xy^2 - y^4/x$$

3

$$\frac{x^3y - x^2y^2}{(x + 8y)^2}$$

5

$$\cos \frac{x^2}{x+y}$$

7

$$\ln x^2 - 2\ln y$$

9

$$(x^{-1} + y^{-1})^2$$

In Problems 11–30 solve the given differential equation by using an appropriate substitution.

13

$$x \, dx + (y - 2x) \, dy = 0$$

17

$$\frac{dy}{dx} = \frac{y - x}{y + x}$$

21

$$2x^2y \ dx = (3x^3 + y^3)dy$$

25

$$y \frac{dx}{dy} = x + 4ye^{-2x/y}$$

29

$$(x^2 + xy - y^2)dx + xydy = 0$$

In Problems 31–44 solve the given differential equation subject to the indicated initial condition.

33

$$2x^2\frac{dy}{dx} = 3xy + y^2, \quad y(1) = -2$$

37

$$(y2 + 3xy)dx = (4x2 + xy)dy, \quad y(1) = 1$$

41

$$y^2 dx + (x^2 + xy + y^2)dy = 0, \quad y(0) = 1$$

Answers:

1 Yes, degree 3

3 Yes, degree 2

5 No.

7 Yes, degree 0

9 Yes, degree -2

13

$$(x - y) \ln |x - y| = y + c(x - y)$$

17

$$\ln(x^2 + y^2) + 2 \tan^{-1}(y/x) = c$$

21

$$y^9 = c(x^3 + y^3)^2$$

25

$$e^{2x/y} = 8 \ln |y| + c$$

29

$$y + x = cx^2 e^{y/x}$$

33

$$y^2 = 4x(x + y)^2$$

37

$$4x \ln |y/x| + x \ln x + y - x = 0$$

41

$$(1 + \cos x)(1 + e^y) = 4$$