4-5

November 10, 2025

In Exercises 1-6, find a basis for the solution space of the homogeneous linear system, and find the dimension of that space.

2

$$\begin{aligned}
&3x_1 + x_2 + x_3 + x_4 = 0 \\
&-2x_1 - x_2 + 2x_3 = 0 \\
&-x_1 \quad + x_3 = 0 \\
&5x_1 - x_2 + x_3 - x_4 = 0
\end{aligned}$$

5

$$\begin{aligned}
&x_1 - 3x_2 + x_3 = 0 \\
&2x_1 - 6x_2 + 2x_3 = 0 \\
&3x_1 - 9x_2 + 3x_3 = 0
\end{aligned}$$

7 a) In each part, find a basis for the given subspace of R^3 , and state its dimension.

The plane $3x - 2y + 5z = 0$

c)

The line $x = 2t, y = -t, z = 4t$

d)

All vectors of the form (a, b, c) , where $b = a + c$.

8 a) In each part, find a basis for the given subspace of R^4 , and state its dimension.

All vectors of the form $(a, b, c, 0)$

c)

All vectors of the form (a, b, c, d) , where $a = b = c = d$.

10

Find the dimension of the subspace of P_3 consisting of all polynomials $a_0 + a_1 x + a_2 x^2 + a_3 x^3$ for which $a_0 = 0$.

12 b) Find a standard basis vector for R^3 that can be added to the set $\{\mathbf{v}_1, \mathbf{v}_2\}$ to produce a basis for R^3 .

$\mathbf{v}_1 = (1, -1, 0)$, $\mathbf{v}_2 = (3, 1, -2)$

13) Find standard basis vectors for R^4 that can be added to the set $\{\mathbf{v}_1, \mathbf{v}_2\}$ to produce a basis for R^4 .

$$\mathbf{v}_1 = (1, -4, 2, -3), \quad \mathbf{v}_2 = (-3, 8, -4, 6)$$

15) The vectors $\mathbf{v}_1 = (1, -2, 3)$ and $\mathbf{v}_2 = (0, 5, -3)$ are linearly independent. Enlarge $\{\mathbf{v}_1, \mathbf{v}_2\}$ to a basis for R^3 .

17) Find a basis for the subspace of R^3 that is spanned by the vectors

$$\mathbf{v}_1 = (1, 0, 0), \quad \mathbf{v}_2 = (1, 0, 1), \quad \mathbf{v}_3 = (2, 0, 1), \quad \mathbf{v}_4 = (0, 0, -1)$$

Answers

1. Basis: $\{(1,0,1)\}$; dimension: 1

3. No basis; dimension: 0

5. Basis: $\{(3,1,0),(-1,0,1)\}$; dimension: 2

7. (a) Basis: $\left\{\left(\frac{2}{3},1,0\right),\left(-\frac{5}{3},0,1\right)\right\}$; dimension: 2

(b) Basis: $\{(1,1,0),(0,0,1)\}$; dimension: 2

(c) Basis: $\{(2,-1,4)\}$; dimension: 1

(d) Basis: $S=\{(1,1,0),(0,1,1)\}$; dimension: 2

9. (a) n

(b) $\frac{n(n+1)}{2}$

(c) $\frac{n(n+1)}{2}$

11. (b) Dimension: 2

(c) Basis: $\{-1+x,-1+x^2\}$

13. $\mathbf{e}_2$ and $\mathbf{e}_3$ (the answer is not unique)

15. $\mathbf{v}_1,\mathbf{v}_2$, and $\mathbf{e}_1$ form a basis for R^3 (the answer is not unique)

17. $\{\mathbf{v}_1,\mathbf{v}_2\}$ (the answer is not unique)