6-2

November 17, 2025

In Exercises 1-2, find the cosine of the angle between the vectors with respect to the Euclidean inner product.

1 c)

$\mathbf{u} = (1, 0, 1, 0), \mathbf{v} = (-3, -3, -3, -3)$

In Exercises 3-4, find the cosine of the angle between the vectors with respect to the standard inner product on P_2 .

3

$\mathbf{p} = -1 + 5x + 2x^2$ $\mathbf{q} = 2 + 4x - 9x^2$

In Exercises 5-6, find the cosine of the angle between A and B with respect to the standard inner product on M_{22} .

5

$A = \begin{bmatrix} 2 & 6 \\ 1 & -3 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 \\ 1 & 0 \end{bmatrix}$

In Exercises 7-8, determine whether the vectors are orthogonal with respect to the Euclidean inner product.

7 a)

$\mathbf{u} = (-1, 3, 2), \mathbf{v} = (4, 2, -1)$

In Exercises 9-10, show that the vectors are orthogonal with respect to the standard inner product on P_2 .

9

$\mathbf{p} = -1 - x + 2x^2, \mathbf{q} = 2x + x^2$

In Exercises 11-12, show that the matrices are orthogonal with respect to the standard inner product on M_{22} .

11

$U = \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}, V = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}$

In Exercises 21-24, confirm that the Cauchy-Schwarz inequality holds for the given vectors using the stated inner product.

21

$\mathbf{u} = (1, 0, 3), \mathbf{v} = (2, 1, -1)$ using the weighted Euclidean inner product $\langle \mathbf{u}, \mathbf{v} \rangle = 2u_1v_1 + 3u_2v_2 + u_3v_3$ in R^3

23

$\mathbf{p} = -1 + 2x + x^2$ and $\mathbf{q} = 2 - 4x^2$ using the standard inner product on P_2

Answers

1. (a) $-\frac{1}{\sqrt{2}}$

   (b) 0

   (c) -1

   3. 0

   5. $\frac{19}{10\sqrt{7}}$

   7. (a) Orthogonal

   (b) Not orthogonal

   (c) Orthogonal

   13. Orthogonal if $k = \frac{4}{3}$

   15. The weights must be positive numbers such that $w_1 = 4w_2$.

   17. No

   25. No