

# Chapter 6

November 12, 2025

## 6.1

## 6.2

### 6.3 Orthonormal bases.

**Definition 6.1.** *Orthonormal* - This is when two vectors say,  $\vec{u}_1$  and  $\vec{u}_2$  are orthogonal (perpendicular), and both are unit vectors (of length 1). They are called **orthonormal**

To convert two vectors to orthonormal,  $\vec{u}_1$ ;  $\vec{u}_2$ , which we are using as basis for some vector space  $S$ , we first project them to make them orthogonal, then we normalize both vectors, which we can then use as a coordinate system.

Basis  $\rightarrow$  Orthogonal  $\rightarrow$  Orthonormal  $\rightarrow$  Coordinate System.

**From this point forward, orthogonal means orthonormal.**

When we want to perform a dot product we are used to the normal notation, but this is limited to Euclidian geometry. As a result we have some new notation.

Euclidian Notation	General Notation
$\vec{u} \cdot \vec{v}$	$\langle \vec{u}, \vec{v} \rangle$

#### 6.3.1 Gram-Schmit Process

This process is used to form orthogonal vectors.

First we will start with an example in 2D.

#### Example 6.2.

$$u_1 \qquad \qquad \qquad u_2$$

