

7-1

November 18, 2025

In each part of Exercises 1-4, determine whether the matrix is orthogonal, and if so find it inverse.

**1 a)**

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

**3 a)**

$$\begin{bmatrix} 0 & 1 & \frac{1}{\sqrt{2}} \\ 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

**13 a)**

Let a rectangular  $x'y'$  -coordinate system be obtained by rotating a rectangular  $xy$  -coordinate system counterclockwise through the angle  $\theta = \pi/3$  .

Find the  $x'y'$  -coordinates of the point whose  $xy$  -coordinates are  $(-2, 6)$  .

**b)**

Find the  $xy$  -coordinates of the point whose  $x'y'$  -coordinates are  $(5, 2)$  .

**15 a)**

Let a rectangular  $x'y'z'$ -coordinate system be obtained by rotating a rectangular  $xyz$ -coordinate system counterclockwise about the  $z$ -axis (looking down the  $z$ -axis) through the angle  $\theta = \pi/4$ .

Find the  $x'y'z'$ -coordinates of the point whose  $xyz$ -coordinates are  $(-1, 2, 5)$

**16 a)**

Repeat Exercise 15 for a rotation of  $\theta = 3\pi/4$  counterclockwise about the  $x$ -axis (looking along the positive  $x$ -axis toward the origin).

**17 a)**

Repeat Exercise 15 for a rotation of  $\theta = \pi/3$  counterclockwise about the  $y$ -axis (looking along the positive  $y$ -axis toward the origin).

**Answers**

$$1. \text{ (a) Orthogonal; } A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\text{ (b) Orthogonal; } A^{-1} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

3. (a) Not orthogonal

$$\text{ (b) Orthogonal; } A^{-1} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$13. \text{ (a) } \begin{bmatrix} -1 + 3\sqrt{3} \\ 3 + \sqrt{3} \end{bmatrix}$$

$$\text{ (b) } \begin{bmatrix} \frac{5}{2} - \sqrt{3} \\ 1 + \frac{5}{2}\sqrt{3} \end{bmatrix}$$

$$15. \text{ (a) } \begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{3}{\sqrt{2}} \\ 5 \end{bmatrix}$$

$$\text{ (b) } \begin{bmatrix} -\frac{5}{\sqrt{2}} \\ \frac{\sqrt{2}}{\sqrt{2}} \\ -3 \end{bmatrix}$$

$$17. \text{ (a) } \begin{bmatrix} -\frac{1}{2} - \frac{5\sqrt{3}}{2} \\ 2 \\ -\frac{\sqrt{3}}{2} + \frac{5}{2} \end{bmatrix}$$

$$\text{ (b) } \begin{bmatrix} \frac{1}{2} - \frac{3\sqrt{3}}{2} \\ 6 \\ -\frac{\sqrt{3}}{2} - \frac{3}{2} \end{bmatrix}$$