Chapter 1

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0.1 Basic Definitions and Terminology

There are different ways of notating derivatives equations.

$$\begin{aligned} y &= f(x) \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= f'(x) \\ y' &= f'(x) \end{aligned}$$

An example of a differential equation is:

$$\begin{aligned} y &= e^{x^3} \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= 3x^2 e^{x^3} \\ \frac{\mathrm{d}y}{\mathrm{d}x} &= 3x^2 y \end{aligned}$$

Def: Differential Equations (DE) $\rightarrow$ An equation continaing the derivative(s) or differential(s) of one or more dependent variables, with respect to (wrt) one or more independent variables.

0.1.1 Types of DEs

Type	Order	Linearity
ODE	Highest Derivative involved in the DE.	?
PDE		

Example of ODE:

$$\begin{aligned} x dy - 4x^2 y dx &= 0 \\ \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} + 6y = 0 \quad &\rightarrow \quad y'' - 2y' + 6y = 0 \end{aligned}$$

0.1.2 General nth order ODE:

$$\begin{aligned} F(x, y, \frac{\mathrm{d}y}{\mathrm{d}x}, \frac{\mathrm{d}^2 y}{\mathrm{d}x^2}, \frac{\mathrm{d}^3 y}{\mathrm{d}x^3}, \ldots, \frac{\mathrm{d}^n y}{\mathrm{d}x^n}) &= 0 \\ F(x, y, y', y'', y^{(3)}, \ldots, y^{(n)}) &= 0 \end{aligned}$$

0.1.3 Linear, nth order, ODE

$$a_n(x)y^{(n)} + a_{n-1}(x)y^{(n-1)} + \cdots + a_1(x)y' + a_0(x)y = g(x)$$

0.1.4 Non-linear, nth order, ODE:

$$yy'' - 2y' = x^2$$

We have y times a y'', which is not allowed for linear ODEs

Ex: Verify that $y = e^{3x}$ is a sol'n to:

$$\begin{aligned} y'' - 2y' - 3y &= 0 \\ y = e^{3x} \to y' = 3e^{3x} \to y'' &= 9e^{3x} \\ 9e^{3x} - 2(3e^{3x}) - 3(e^{3x}) &=^{?} 0 \end{aligned}$$

Ex:

$$(y'')^2 + y^2 + 4 = 0$$

Second order ODE, no solution. $f(x)^2 > 0$. Positive + Positive + Positive = 0 ???

Solutions to DE's are usually of the form $G(x, y, c_1, \ldots, c_n)$. For Example:

$$y'' + y = 0$$
$$y = c_1 cosx + c_2 sinx$$

c_1 and c_2 can be any constants and it will satisfy the original DE. This kind of solution is called a general solution.