4-2

October 13, 2025

1. Use Theorem 4.2.1 to determine which of the following are subspaces of R^3. b)

All vectors of the form $(a, 1, 1)$.

c)

All vectors of the form (a, b, c), where $b = a + c$.

3. Use Theorem 4.2.1 to determine which of the following are subspaces of P^3. b)

All polynomials $a_0 + a_1x + a_2x^2 + a_3x^3$ for which $a_0 + a_1 + a_2 + a_3 = 0$.

c)

All polynomials of the form $a_0 + a_1x + a_2x^2 + a_3x^3$ in which a_0, a_1, a_2, and a_3 are rational numbers.

4. Which of the following are subspaces of $F(-\infty, \infty)$? b)

All functions f in $F(-\infty, \infty)$ for which $f(0) = 1$.

c)

All functions f in $F(-\infty, \infty)$ for which $f(-x) = f(x)$.

5. Which of the following are subspaces of R^∞? a)

All sequences v in R^∞ of the form.

$$v = (v, 0, v, 0, v, 0, \ldots).$$

b)

All sequences $\mathbf{v}$ in R^∞ of the form

$$\mathbf{v} = (v, 1, v, 1, v, 1, \ldots)$$

10. In each part express the vector as a linear combination of $\mathbf{p}_1 = 2 + x + 4x^2$,$\mathbf{p}_2 = 1 - x + 3x^2$, and $\mathbf{p}_3 = 3 + 2x + 5x^2$. a)

$$-9 - 7x - 15x^2$$

11. In each part, determine whether the vectors span R^3 a)

$$\mathbf{v}_1 = (2, 2, 2),\ \mathbf{v}_2 = (0, 0, 3),\ \mathbf{v}_3 = (0, 1, 1)$$

12. Suppose that $\mathbf{v}_1 = (2, 1, 0, 3)$, $\mathbf{v}_2 = (3, -1, 5, 2)$, and $\mathbf{v}_3 = (-1, 0, 2, 1)$. Which of the following vectors are in span $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? a)

$$(2, 3, -7, 3)$$

c)

$$(1, 1, 1, 1)$$

14. Let $\mathbf{f} = \cos^2 x$ and $\mathbf{g} = \sin^2 x$. Which of the following lie in the space spanned by f and g? a)

$$\cos 2x$$

b)

$$(0,\ 0,\ 0,\ 0)$$