

4-1

October 13, 2025

1)

Let V be the set of all ordered pairs of real numbers, and consider the following addition and scalar multiplication operations on $u = (u_1, u_2)$ and $v = (v_1, v_2)$:

$$u + v = (u_1 + v_1, u_2 + v_2), \quad ku = (0, ku_2)$$

- (a) Compute $u + v$ and ku for $u = (-1, 2)$, $v = (3, 4)$, and $k = 3$.
- (b) In words, explain why V is closed under addition and scalar multiplication.
- (c) Since addition on V is the standard addition operation on R^2 , certain vector space axioms hold for V because they are known to hold for R^2 . Which axioms are they?
- (d) Show that Axioms 7, 8, and 9 hold.
- (e) Show that Axiom 10 fails and hence that V is not a vector space under the given operations.

5) In Exercises 3–12, determine whether each set equipped with the given operations is a vector space. For those that are not vector spaces identify the vector space axioms that fail.

The set of all pairs of real numbers of the form (x, y) , where $x \geq 0$, with the standard operations on R^2 .

7)

The set of all triples of real numbers with the standard vector addition but with scalar multiplication defined by

$$k(x, y, z) = (k_2x, k_2y, k_2z)$$

9)

The set of all 2×2 matrices of the form

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

11)

The set of all pairs of real numbers of the form $(1, x)$ with the operations

$$(1, y) + (1, y') = (1, y + y') \quad \text{and} \quad k(1, y) = (1, ky)$$