8-4

November 19, 2025

3 a)

Let $T : P_2 \to P_2$ be the linear operator defined by

$$T\left(a_0 + a_1 x + a_2 x^2\right) = a_0 + a_1(x-1) + a_2(x-1)^2$$

Find the matrix for T relative to the standard basis $B = \{1, x, x^2\}$ for P_2

b)

Verify that the matrix $[T]_B$ obtained in part (a) satisfies Formula (8) for every vector $\mathbf{x} = a_0 + a_1 x + a_2 x^2$ in P_2

4 a)

Let $T : R^2 \to R^2$ be the linear operator defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 \\ x_1 + x_2 \end{bmatrix}$$

and let $B = \{\mathbf{u}_1, \mathbf{u}_2\}$ be the basis for which

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{a n d} \quad \mathbf{u}_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

Find $[T]_B$

b)

Verify that Formula (8) holds for every vector $\mathbf{x}$ in R^2

5 a)

Let $T : R^2 \to R^3$ be defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + 2x_2 \\ -x_1 \\ 0 \end{bmatrix}$$

Find the matrix $[T]_{B',B}$ relative to the bases $B = \{\mathbf{u}_1, \mathbf{u}_2\}$ and $B' = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ where

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$$
$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$$

b)

Verify that Formula (5) holds for every vector in R^2

7 a)

Let $T : P_2 \to P_2$ be the linear operator defined by

$$T(p(x)) = p(2x+1), \text{ t h a t i s,}$$
$$T\left(c_0 + c_1 x + c_2 x^2\right) = c_0 + c_1(2x+1) + c_2(2x+1)^2$$

Find $[T]_B$ with respect to the basis $B = \{1, x, x^2\}$.

b)

Use the three-step procedure illustrated in Example 2 to compute $T(2-3x+4x^2)$
.

c)

Check the result obtained in part (b) by computing $T(2 - 3x + 4x^2)$ directly.

9 a)

Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$, and let $A = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ be the matrix for $T : R^2 \to R^2$ relative to the basis $B = \{\mathbf{v}_1, \mathbf{v}_2\}$.

Find $[T(\mathbf{v}_1)]_B$ and $[T(\mathbf{v}_2)]_B$

b)

Find $T(\mathbf{v}_1)$ and $T(\mathbf{v}_2)$

c)

Find a formula for $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right)$

d)

Use the formula obtained in (c) to compute $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right)$

Answers

3. (a) $\begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

5. (a) $\begin{bmatrix} 0 & 0 \\ -\frac{1}{2} & 1 \\ \frac{8}{3} & \frac{4}{3} \end{bmatrix}$

7. (a) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & 4 \end{bmatrix}$

(b),(c) $3 + 10x + 16x^2$

9. (a) $[T(\mathbf{v}_1)]_B = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$; $[T(\mathbf{v}_2)]_B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

(b) $T(\mathbf{v}_1) = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$; $T(\mathbf{v}_2) = \begin{bmatrix} -2 \\ 29 \end{bmatrix}$

(c) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} \frac{18}{7} & \frac{1}{7} \\ -\frac{107}{7} & \frac{24}{7} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

(d) $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} \frac{19}{7} \\ -\frac{83}{7} \end{bmatrix}$