

7-1

November 18, 2025

In each part of Exercises 1-4, determine whether the matrix is orthogonal, and if so find its inverse.

1 a)

$$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

3 a)

$$\begin{bmatrix} 0 & 1 & \frac{1}{\sqrt{2}} \\ 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$$

13 a)

Let a rectangular $x'y'$ -coordinate system be obtained by rotating a rectangular xy -coordinate system counterclockwise through the angle $\theta = \pi/3$.

Find the $x'y'$ -coordinates of the point whose xy -coordinates are $(-2, 6)$.

b)

Find the xy -coordinates of the point whose $x'y'$ -coordinates are $(5, 2)$.

15 a)

Let a rectangular $x'y'z'$ -coordinate system be obtained by rotating a rectangular xyz -coordinate system counterclockwise about the z -axis (looking down the z -axis) through the angle $\theta = \pi/4$.

Find the $x'y'z'$ -coordinates of the point whose xyz -coordinates are $(-1, 2, 5)$

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16 a)

Repeat Exercise 15 for a rotation of $\theta = 3\pi/4$ counterclockwise about the x -axis (looking along the positive x -axis toward the origin).

17 a)

Repeat Exercise 15 for a rotation of $\theta = \pi/3$ counterclockwise about the y -axis (looking along the positive y -axis toward the origin).

Answers

1. (a) Orthogonal; $A^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(b) Orthogonal; $A^{-1} = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$

3. (a) Not orthogonal

(b) Orthogonal; $A^{-1} = \begin{bmatrix} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{bmatrix}$

13. (a) $\begin{bmatrix} -1 + 3\sqrt{3} \\ 3 + \sqrt{3} \end{bmatrix}$

(b) $\begin{bmatrix} \frac{5}{2} - \sqrt{3} \\ 1 + \frac{5}{2}\sqrt{3} \end{bmatrix}$

15. (a) $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{\sqrt{2}}{3} \\ \frac{1}{\sqrt{2}} \\ 5 \end{bmatrix}$

(b) $\begin{bmatrix} -\frac{5}{\sqrt{2}} \\ \frac{7}{\sqrt{2}} \\ -3 \end{bmatrix}$

17. (a) $\begin{bmatrix} -\frac{1}{2} - \frac{5\sqrt{3}}{2} \\ 2 \\ -\frac{\sqrt{3}}{2} + \frac{5}{2} \end{bmatrix}$

(b) $\begin{bmatrix} \frac{1}{2} - \frac{3\sqrt{3}}{2} \\ 6 \\ -\frac{\sqrt{3}}{2} - \frac{3}{2} \end{bmatrix}$