5-2

November 12, 2025

In Exercises 5-8, find a matrix P that diagonalizes A , and check your work by computing $P^{-1}AP$.

5)

$$A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$$

7)

$$A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

In Exercises 11-14, find the geometric and algebraic multiplicity of each eigenvalue of the matrix A , and determine whether A is diagonalizable. If A is diagonalizable, then find a matrix P that diagonalizes A , and find $P^{-1}AP$.

11)

$$A = \begin{bmatrix} -1 & 4 & -2 \\ -3 & 4 & 0 \\ -3 & 1 & 3 \end{bmatrix}$$

14)

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 1 & 5 & 0 \\ 0 & 1 & 5 \end{bmatrix}$$

In Exercises 17-18, use the method of Example 6 to compute the matrix A^{10} .

17)

$A = \begin{bmatrix} 0 & 3 \\ 2 & -1 \end{bmatrix}$

19)

Let

$$A = \begin{bmatrix} -1 & 7 & -1 \\ 0 & 1 & 0 \\ 0 & 15 & -2 \end{bmatrix} \quad \text{a n d} \quad P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 5 \end{bmatrix}$$

Confirm that P diagonalizes A , and then compute A^{11} .

28 a)

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ Show that A is diagonalizable if $(a-d)^2 + 4bc > 0$.

28 b)

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ Show that A is not diagonalizable if $(a-d)^2 + 4bc < 0$.

Answers

Exercise Set 5.2 (page 313)

5. $P = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$ (answer is not unique)

7. $P = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (answer is not unique)

11. eigenvalues: 1, 2 and 3; each has algebraic multiplicity 1 and geometric multiplicity 1;

A is diagonalizable; $P = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ (answer is not unique); $P^{-1}AP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

17. $\begin{bmatrix} 24,234 & -34,815 \\ -23,210 & 35,839 \end{bmatrix}$

19. $A^{11} = \begin{bmatrix} -1 & 10,237 & -2,047 \\ 0 & 1 & 0 \\ 0 & 10,245 & -2,048 \end{bmatrix}$