6-1

November 17, 2025

1 c)

Let R^2 have the weighted Euclidean inner product

$$\langle \mathbf{u}, \mathbf{v} \rangle = 2u_1v_1 + 3u_2v_2$$

and let $\mathbf{u} = (1,1), \mathbf{v} = (3,2), \mathbf{w} = (0,-1)$ and $k = 3$. Compute the stated quantities.

$\langle \mathbf{u} + \mathbf{v}, \mathbf{w} \rangle$

f)

$\|\mathbf{u} - k\mathbf{v}\|$

In Exercises 7-8, use the inner product on R^2 generated by the matrix A to find $\langle \mathbf{u}, \mathbf{v} \rangle$ for the vectors $\mathbf{u} = (0, -3)$ and $\mathbf{v} = (6, 2)$

7

$$A = \begin{bmatrix} 4 & 1 \\ 2 & -3 \end{bmatrix}$$

In Exercises 9-10, compute the standard inner product on M_{22} of the given matrices.

9

$$U = \begin{bmatrix} 3 & -2 \\ 4 & 8 \end{bmatrix}, V = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$$

In Exercises 11-12, find the standard inner product on P_2 of the given polynomials.

11

$\mathbf{p} = -2 + x + 3x^2, \mathbf{q} = 4 - 7x^2$

In Exercises 13-14, a weighted Euclidean inner product on R^2 is given for the vectors $\mathbf{u} = (u_1, u_2)$ and $\mathbf{v} = (v_1, v_2)$. Find a matrix that generates it.

13

$\langle \mathbf{u}, \mathbf{v} \rangle = 3u_1v_1 + 5u_2v_2$

In Exercises 17-18, find $\|\mathbf{u}\|$ and $d(\mathbf{u}, \mathbf{v})$ relative to the weighted Euclidean inner product $\langle \mathbf{u}, \mathbf{v} \rangle = 2u_1v_1 + 3u_2v_2$ on R^2

17

$\mathbf{u} = (-3, 2)$a n d$\mathbf{v} = (1, 7)$

In Exercises 19-20, find $\|\mathbf{p}\|$ and $d(\mathbf{p}, \mathbf{q})$ relative to the standard inner product on P_2 .

19

$\mathbf{p} = -2 + x + 3x^2, \mathbf{q} = 4 - 7x^2$

In Exercises 21-22, find $\|U\|$ and $d(U,V)$ relative to the standard inner product on M_{22}

21

$$U = \begin{bmatrix} 3 & -2 \\ 4 & 8 \end{bmatrix}, V = \begin{bmatrix} -1 & 3 \\ 1 & 1 \end{bmatrix}$$

In Exercises 33-34, let $\mathbf{u} = (u_1, u_2, u_3)$ and $\mathbf{v} = (v_1, v_2, v_3)$. Show that the expression does not define an inner product on R^3 , and list all inner product axioms that fail to hold.

33

$$\langle \mathbf{u}, \mathbf{v} \rangle = u_1^2 v_1^2 + u_2^2 v_2^2 + u_3^2 v_3^2$$

37 a)

(Calculus required) Let the vector space P_2 have the inner product

$$\langle \mathbf{p}, \mathbf{q} \rangle = \int_{-1}^{1} p(x)q(x)dx$$

Find the following for $\mathbf{p} = 1$ and $\mathbf{q} = x^2$.
$\langle \mathbf{p}, \mathbf{q} \rangle$

b) $d(\mathbf{p}, \mathbf{q})$

c) $\|\mathbf{p}\|$

d) $\|\mathbf{q}\|$

45

Prove that the following identity holds for vectors in any inner product space.

$$\|\mathbf{u}+\mathbf{v}\|^2+\|\mathbf{u}-\mathbf{v}\|^2=2\|\mathbf{u}\|^2+2\|\mathbf{v}\|^2$$

Answers

1. (a) 12

(b) -18

(c)-9

(d) $\sqrt{30}$

(e) $\sqrt{11}$

(f) $\sqrt{203}$

3. (a) 34

(b) -39

(c)-18

(d) $\sqrt{89}$

(e) $\sqrt{34}$

(f) $\sqrt{610}$

2 0 5. 0 3

7. -24

9. 3

11. -29

13. $\begin{bmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{5} \end{bmatrix}$

15. -50

17. $\|\mathbf{u}\| = \sqrt{30}, d(\mathbf{u}, \mathbf{v}) = \sqrt{107}$

19. $\|\mathbf{p}\| = \sqrt{14}, d(\mathbf{p}, \mathbf{q}) = \sqrt{137}$

21. $\|U\| = \sqrt{93}, d(U, V) = \sqrt{99} = 3\sqrt{11}$

23. $\|\mathbf{p}\| = 6\sqrt{3}, d(\mathbf{p}, \mathbf{q}) = 11\sqrt{2}$

25. $\|\mathbf{u}\| = \sqrt{65}, d(\mathbf{u}, \mathbf{v}) = 12\sqrt{5}$

27. (a) -101

(b) 3

31. $\langle \mathbf{u}, \mathbf{v} \rangle = \frac{1}{9}u_1v_1 + u_2v_2$

33. Axioms 2 and 3 do not hold.

35. $14\langle \mathbf{u}, \mathbf{v} \rangle - 4\|\mathbf{u}\|^2 - 6\|\mathbf{v}\|^2$

37. (a) $\frac{2}{3}$

(b) $\frac{4}{\sqrt{15}}$

(c) $\sqrt{2}$

(d) $\sqrt{\frac{2}{5}}$

39. 0

43. (b) k_1 and k_2 must both be positive.