

8-5

November 19, 2025

In Exercises 1-2, use a property from Table 1 to show that the matrices A and B are not similar.

1 a)

$$A = \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$$

2 a)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

3

Let $T : R^2 \rightarrow R^2$ be a linear operator, and let B and B' be bases for R^2 for which

$$[T]_B = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad P_{B \rightarrow B'} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

Find the matrix for T relative to the basis B'

5

Let $T : R^2 \rightarrow R^2$ be a linear operator, and let B and B' be bases for R^2 for which

$$[T]_{B'} = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \quad \text{and} \quad P_{B \rightarrow B'} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

Find the matrix for T relative to the basis B

In Exercises 7-14, find the matrix for T relative to the basis B , and use Theorem 8.5.2 to compute the matrix for T relative to the basis B' .

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$T : R^2 \rightarrow R^2$ is defined by

$$T \left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_1 - 2x_2 \\ -x_2 \end{bmatrix}$$

and $B = \{\mathbf{u}_1, \mathbf{u}_2\}$ and $B' = \{\mathbf{v}_1, \mathbf{v}_2\}$, where

$$\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \quad \mathbf{v}_1 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

Answers

1. (a) $\det(A) = -2$ does not equal $\det(B) = -1$

3. $\begin{bmatrix} 6 & -10 \\ 2 & -3 \end{bmatrix}$

5. $\begin{bmatrix} -2 & -2 \\ 6 & 5 \end{bmatrix}$

7. $[T]_B = \begin{bmatrix} 1 & -2 \\ 0 & -1 \end{bmatrix}; [T]_{B'} = \begin{bmatrix} 11 & 20 \\ -6 & -11 \end{bmatrix}$