

2.4

September 15, 2025

”Determine whether the given equation is exact. If it is find the solve.”

1

$$(2x - 1)dx + (3y + 7)dy = 0$$

3

$$(5x + 4y)dx + (4x - 8y^3)dy = 0(5x + 4y)dx + (4x - 8y^3)dy = 0$$

5

$$(2y^2x - 3)dx + (2yx^2 + 4)dy = 0$$

7

$$(x + y)(x - y)dx + x(x - 2y)dy = 0$$

9

$$(y^3 - y^2 \sin x - x)dx + (3xy^2 + 2y \cos x)dy = 0$$

13

$$x \frac{dy}{dx} = 2xe^x - y + 6x^2$$

17

$$\left(x^2y^3 - \frac{1}{1+9x^2}\right) \frac{dx}{dy} + x^3y^2 = 0$$

21

$$(1 - 2x^2 - 2y) \frac{dy}{dx} = 4x^3 + 4xy$$

Solve the differential equation with the initial condition.

25

$$(x + y)^2 dx + (2xy + x^2 - 1)dy = 0, \quad y(1) = 1$$

29

$$(y^2 \cos x - 3x^2 y - 2x)dx + (2y \sin x - x^3 + \ln y)dy = 0, \quad y(0) = e$$

Find the value of k so that the given differential equation is exact.

33

$$(2xy^2 + ye^x)dx + (2x^2y + ke^x - 1)dy = 0$$

”Solve the differential equation by verifying that the indicated function $\mu(xy)$ is an integrating factor.”

37

$$6xydx + (4y + 9x^2)dy = 0, \mu(x, y) = y^2$$

41

$$(2y^2 + 3x)dx + 2xydy = 0, \mu(x, y) = x$$

Answers:

1

$$x^2 - x + \frac{3}{2}y^2 + 7y = c$$

3

$$\frac{5}{2}x^2 + 4xy - 2y^4 = c$$

5

$$x^2y^2 - 3x + 4y = c$$

7

not exact, but is homogeneous

9

$$xy^3 + y^2 \cos x - \frac{1}{2}x^2 = c$$

13

$$xy - 2xe^x + 2e^x - 2x^3 = c$$

17

$$x^3y^3 - \tan^{-1} 3x = c$$

21

$$y - 2x^2y - y^2 - x^4 = c$$

25

$$\frac{1}{3}x^3 + x^2y + xy^2 - y = \frac{4}{3}$$

29

$$y^2 \sin x - x^3 y - x^2 + y \ln y - y = 0$$

33

$$k = 1$$

37

$$\begin{aligned} M(x, y) &= 6xy^3 \\ N(x, y) &= 4y^3 + 9x^2y^2 \\ \frac{\partial M}{\partial y} &= 18xy^2 = \frac{\partial N}{\partial x} \\ \text{Solution is } 3x^2y^3 + y^4 &= c. \end{aligned}$$

41

$$\begin{aligned} M(x, y) &= 2xy^2 + 3x^2 \\ N(x, y) &= 2x^2y \\ \frac{\partial M}{\partial y} &= 4xy = \frac{\partial N}{\partial x} \\ \text{Solution is } x^2y^2 + x^3 &= c. \end{aligned}$$