Lab 2: Physical Pendulum

A *pendulum* is an oscillator that rotates: a *spring & mass* system oscillates in linear motion, a pendulum oscillates in rotational motion. This means a pendulum must experience a *restoring torque*, instead of the restoring force of a spring & mass system. A *physical pendulum* gets this restoring torque from gravity… i.e. a *physical pendulum* is any object that is hung from a pivot and rotates back and forth due to the pull of gravity.

For today's lab we will investigate the behavior of a pendulum in the shape of a thin square that oscillates in the plane of the square. We will develop a theoretical expectation of the behavior of this system, and compare our measured results to our expectation.

In the Chapter 13 Notes for the pendulum, we derived an expression for the angular frequency of the motion of the pendulum :

$$\omega^2 = \frac{mgd}{I_{pivot}}$$

where "*m*" is the mass of the pendulum, "*g*" is our old friend, "*d*" is the distance from the center of mass of the pendulum to the axis of rotation, an "I_{pivot}" is the moment of inertia of the pendulum relative to the pivot. If the pendulum is a simple shape (e.g. our squares for this lab), it is useful to replace "I_{pivot}" with an expression that takes advantage of the parallel-axis theorem:

$$\omega^2 = \frac{mgd}{I_{cm} + md^2} \; .$$

where "I_{cm}" is the moment of inertia of the pendulum about its center of mass. For most simple objects, the expression for I_{cm} can be found in a table of moments of inertia.

In the Chapter 10 Notes for moment of inertia, we derived the expression for I_{cm} for a rectangular plate (which rotates in the plane of the plate), where the plate has dimensions "a" and "b":

$$I_{cm} = \frac{1}{12} m(a^2 + b^2)$$

So for a square of side length "a", the corresponding expression would be:

$$I_{cm} = \frac{1}{12} m(a^2 + a^2) = \frac{1}{6} ma^2$$

The value of "*d*" for the squares we will use today was intentionally chosen to be equal to a/2; this will keep our algebra simple. If we use this value of "*d*" along with the above expression for I_{cm} in our original expression above for the angular frequency, we get:

$$\omega^2 = \frac{mg(a/2)}{\frac{1}{6}ma^2 + m(a/2)^2} = \frac{\frac{1}{2}ga}{\frac{1}{6}a^2 + \frac{1}{4}a^2} = \frac{6ga}{2a^2 + 3a^2} = \frac{6g}{5a}$$

Now we can replace ω with $2\pi/T$: $$\left(\frac{2\pi}{T}\right)^2 = \frac{6g}{5a}$$

Or: $$\frac{4\pi^2}{T^2} = \frac{6g}{5a}$$

Or: $$T^2 = \frac{10\pi^2}{3g}a$$

Notice what this last expression tells us: the period of the oscillations depends only on how big the pendulum is (i.e. “a”) and how strong gravity is. The larger the pendulum, the slower the oscillation; and the stronger the pull of gravity, the faster the oscillations.

To test this idea, we can measure the period of square pendulums of several different sizes. By capturing the data of the size of the square, i.e. “a”, and the corresponding period of oscillation, we can make a graph of T^2 vs a and determine how close the slope of this graph matches the expected value In the equation above.

Procedure

1. Format your Excel sheet properly before you begin recording data.
   - Fill in the data table labels and title, enter calculations in the data table where necessary.
   - Create a graph (which will be empty) with axes properly formatted.
   - Complete the *Calculations* section as much as possible.
   - ***Use the example at the end of this outline as a guide to your formatting.***

2. Set up the apparatus, including the photogate system. To do this:
   - ❑ Plug the black box into the computer (USB cable) and the wall (AC adapter.)
   - ❑ Turn on the black box and wait for the driver to download (small icon on lower right screen.)
   - ❑ Open ***Capstone*** software. Verify that it recognizes the black box..
   - ❑ Connect photogate to the black box in input A.
   - ❑ On the picture of the black box on the screen, click input A.
   - ❑ From the menu, select ***Photogate and Pendulum.***
   - ❑ From menu on the right, select ***Table***. Expand the table as necessary.
   - ❑ Test the photogate by clicking ***Record*** in the lower left screen. Click ***Stop*** when done.

3. Choose one of the five pendulums. Record the unique letter and number of the pendulum, e.g. "D3", in your data table. Measure and record, in cm, the length of each of the four sides of the pendulum.

4. Hang the pendulum on the horizontal rod; adjust the photogate so that the screw at the center of the pendulum passes through the photogate.

5. Carefully set the pendulum oscillating with a small amplitude and oriented in the plane of the square (i.e. no wobbling back and forth.) Click ***Start*** to allow the photogate to measure the period of the pendulum.

6. The software will record repetitive measurements of the period in the table on the screen (in *Data Studio.*) After 10 measurements, click ***Stop***. Inspect the measurements for consistency; if any are inconsistent, simply repeat steps 5 and 6.

7. After confirming the measurements are consistent, find the average of the measurements by clicking the "Σ" button at the top of the table. The average (i.e. mean) will appear at the bottom of your list of measurements. Record the average in your data table as the value of the period.

8. Repeat steps 3 through 7 for a pendulum of each of the other four sizes. You only need to measure one pendulum of each size.

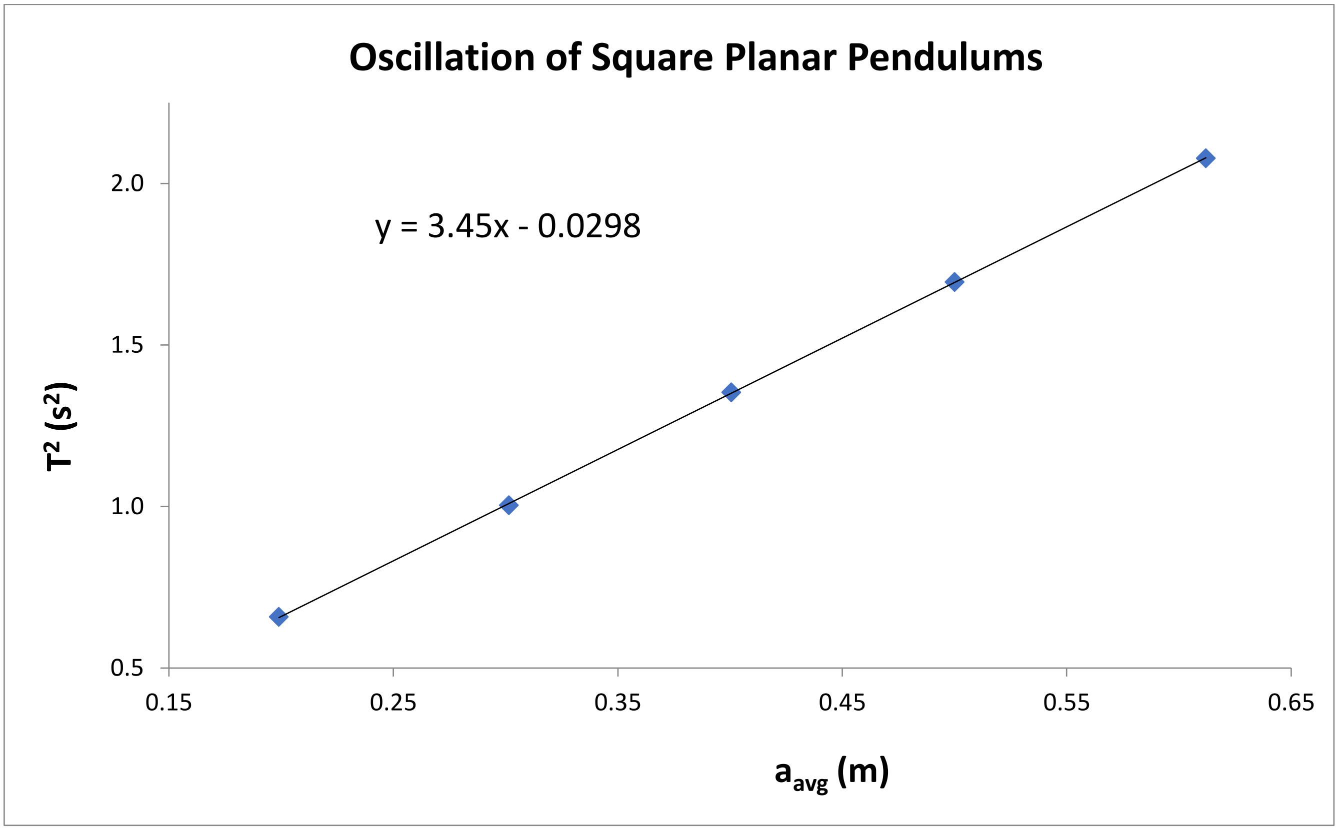
9. Your column of "T^2" should be calculated by Excel. ***Make sure your decimal places are formatted to the correct number of significant figures.***

10. As the data is entered in your table, it should also appear on your graph. If the pendulums behaved as we expected, your data points should form a straight line. Add the "best-fit straight line" to your graph.

11. The equation suggests that the slope of the graph should be equal to $10\pi^2 / 3\,g$. In the ***Calculations*** section of your Excel sheet, calculate the expected value of the slope (including units) and compare this to the measured value of your slope (i.e. calculate the percent difference.)

Oscillation of Square Planar Pendulums

	Side Lengths, all in cm						
Square	a_1	a_2	a_3	a_4	a_{avg} (m)	T (s)	T^2 (s^2)
A1	19.9	20.0	19.9	20.1	0.200	0.8114	0.6584
B3	30.0	30.2	30.1	30.2	0.301	1.0022	1.0044
C2	40.0	40.0	40.0	40.1	0.400	1.1628	1.3521
D2	50.1	50.0	50.2	50.0	0.501	1.3021	1.6955
E1	61.0	61.2	61.1	61.0	0.611	1.4423	2.0802



CALCULATIONS

Measured slope: 3.45 s^2/m

Calculate the expected value of the slope:

$$\textit{slope} = \frac{10\,\pi^2}{3\,g} = \frac{10\,\pi^2}{3\,(9.80\ m/s^2)} = 3.36\quad s^2/m$$

Compare the measured and expected values:

$$\textit{\% difference} = \frac{\textit{measured - expected}}{\textit{expected}} = \frac{3.45\ s^2/m - 3.36\ s^2/m}{3.36\ s^2/m} = 2.8\%$$