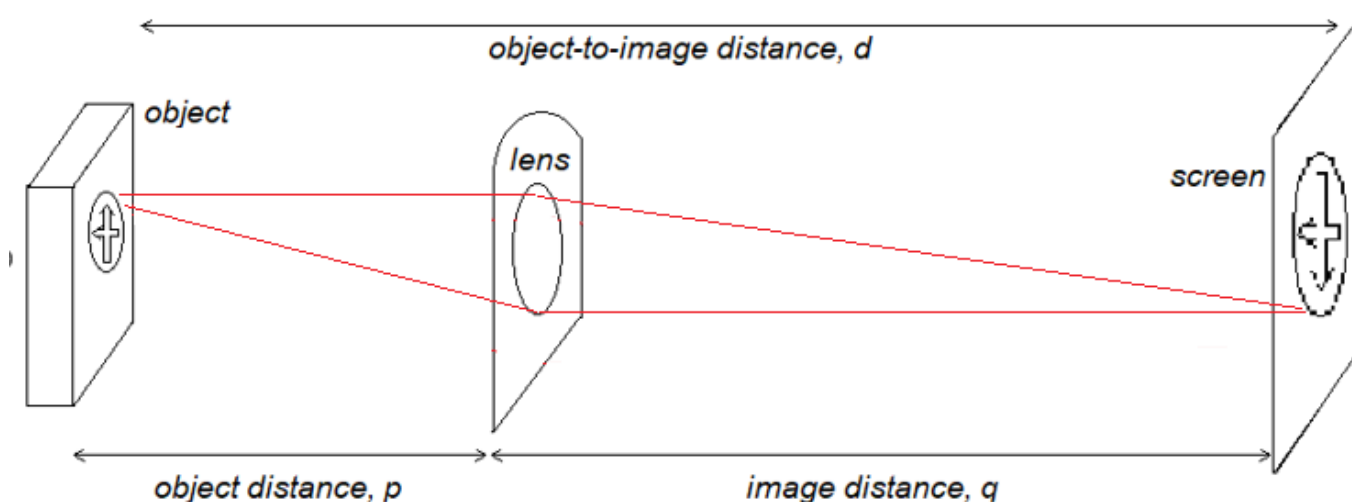


The objective of this lab exercise is to observe and measure real images produced by a converging lens. We will attempt to confirm the relationship between the object distance, image distance and focal length of the lens, and we will use our measured data to calculate the focal length of the lens.

From our Chapter 34 lecture we discover that a converging lens can create an enlarged virtual image, if the object distance is less than the focal length, or a real image, if the object distance is greater than the focal length. In this lab we will consider the latter of these two: we will place an object in front of a converging lens, ensure that the object distance is greater than the focal length, and project the resulting real image onto a screen.

The components of our apparatus will be an *object*, a light source with a slide of an arrow-and-circle design; a lens, in a black plastic lens holder; and a screen, a flat piece of white plastic onto which we can project an image. A diagram of these three components:



The three components are mounted on a black aluminum rack which has a yellow measuring tape along its edge. We will set our object at “zero” on the measuring tape. The position of the lens on the tape will be our object distance, and the position of the screen will be “ $d$ ”, the object-to-image distance.

For each of seven values of the object distance, we will focus the resulting image on the screen and measure the value of “ $d$ ”. The image distance,  $q$ , can then be calculated by using the fact that

$$q = d - p \quad (\text{because } p \text{ and } q \text{ are both positive values.})$$

To determine the seven values of object distance, we will first use the fact that the smallest possible value of  $d$  is equal to  $4f$ . This can be proven with a simple optimization problem. At this distance, the object distance and image distance are each equal to  $2f$ . We will approximate this value of  $2f$  for the

middle of our seven values of object distance; we will then choose a range of three more values that are greater than  $2f$  and three more values that are smaller than  $2f$ .

For each of our seven values of  $p$ , we will focus the image on the screen and record the value of  $d$ . We can then calculate the value of  $q$ , the image distance, for each value of  $p$ . Finally, we can calculate the inverse of each  $p$  and  $q$ .

The equation that allows us to relate the object distance, image distance and focal length is:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \text{or} \quad p^{-1} + q^{-1} = f^{-1}$$

We can rearrange this equation to read:

$$q^{-1} = -p^{-1} + f^{-1}$$

which has the form  $y = (\text{slope})x + (\text{y-int})$

where  $y$  is  $q^{-1}$   $x$  is  $p^{-1}$  the *slope* should be  $-1$  and the *y-intercept* is  $f^{-1}$

This means if we create a graph of  $q^{-1}$  vs  $p^{-1}$ , we expect to find a straight line, with a slope of  $-1$  and an intercept that is the inverse of the focal length.

We will use the y-intercept to calculate the focal length, but a graph with a slope of  $-1$  has a special feature: since that  $-1$  means the line progresses in the  $+x$  direction one unit for every unit it declines (i.e. slope is “rise over run”, then *the x-intercept should also be equal to  $f^{-1}$* . Which means we can calculate the focal length two ways: using the y-intercept for one and the x-intercept for the other.

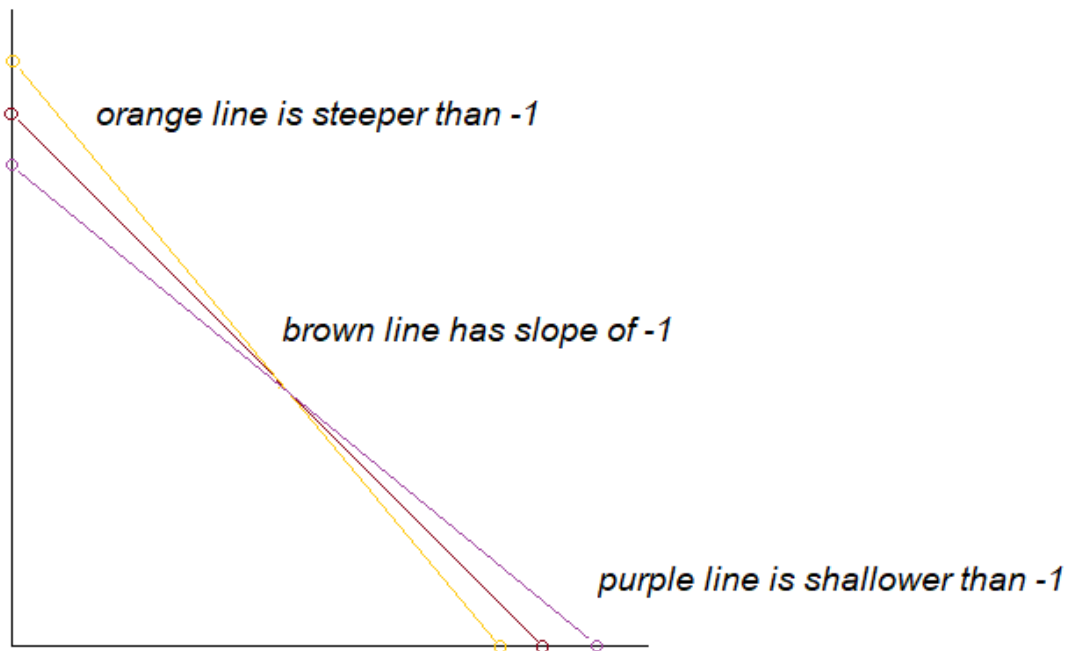
How do we determine the x-intercept? Using our general equation for a straight line, above, by definition the x-intercept is the value of  $x$  when  $y$  is zero. So:

$$y = (\text{slope})x + (\text{y-int})$$

$$0 = (\text{slope})(x - \text{int}) + (\text{y-int})$$

$$\text{So: } x - \text{int} = \frac{-(\text{y-int})}{\text{slope}}$$

Calculating the focal length using both intercepts has a real advantage that will help us to mitigate measurement uncertainties. Consider the simple chart below:



The brown line has a slope of exactly -1, so its y-intercept and x-intercept are equal. The orange line is a bit steeper than -1... and notice the effect on the two intercepts: while the y-intercept is slightly higher than that of the brown line, the x-intercept is slightly lower. Similarly, as the purple line has a slope a bit shallower than -1, its y-intercept is a little lower and x-intercept a little higher than those of the brown line.

If the slope of our data is not exactly -1, we can assume that one of our intercepts will be skewed slightly higher and one will be skewed slightly lower. By calculating the corresponding focal length for each and taking the average of the two results, we can mitigate the effect of a slope that is slightly greater or less than -1 and arrive at a precise value of the focal length of the lens.

## Procedure

For each of two lenses:

- Visually estimate the focal length by adjusting the object distance and image distance until they are approximately equal. The object distance will then be approximately twice the focal length.
- Establish a range of seven viable values of object distance, using a value that is close to  $2f$  as the middle value of the range. The lowest value must be greater than the estimated focal length. Choose the remaining values so they have regular spacing.
- For each of the seven chosen values of the object distance, adjust the location of the screen to find the sharpest focus of the image. Record the object distance,  $p$ , and the position of the screen,  $d$ .

- Finish each data table by calculating the image distance,  $q$ , as well as  $p^{-1}$  and  $q^{-1}$  for each measurement.

*Watch your units! While your measurements will be in cm, it is useful to calculate  $p^{-1}$  and  $q^{-1}$  in inverse meters. This means your calculation will need to convert from cm to meters, and then invert.*

- Create a graph of  $q^{-1}$  vs  $p^{-1}$ .
- Calculate the focal length of the lens twice: first using the y-intercept and then using the x-intercept. Calculate the average of these two results for the best measurement of the focal length. *Again, watch your units. Notice that the y-intercept is in inverse meters, but it will be most useful to express the focal length in cm.*