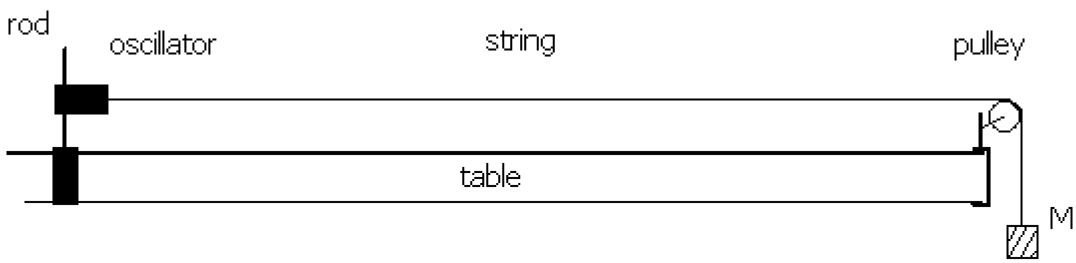
Lab 3: Standing Waves on a String

Today's lab has two parts. In "Part A", we will measure the tension and density of a string to calculate the speed at which waves *should* travel on the string, i.e. the "expected" value of the speed of the waves. In "Part B", we will measure the frequency and, indirectly, the wavelength of the waves that travel on the string. Using these measurements, we will be able to determine the actual speed at which the waves travel on the string, i.e. the "measured" value of the speed of the waves. We should find that our expected and measured values match very well.

Our apparatus consists of a mechanical oscillator clamped to a vertical post; a string, with one end tied to the oscillator and the other end over a pulley; a weight hung from the end of the string.



Part A

To calculate the *expected* value of the speed of the waves, we will need to know the tension in the string and the *linear density* of the string, i.e. its mass per length.

- Choose how much mass, a value between 600 and 800 grams you will use for the tension of your string. The tension will be equal to the weight of the hanging mass, i.e. "Mg"

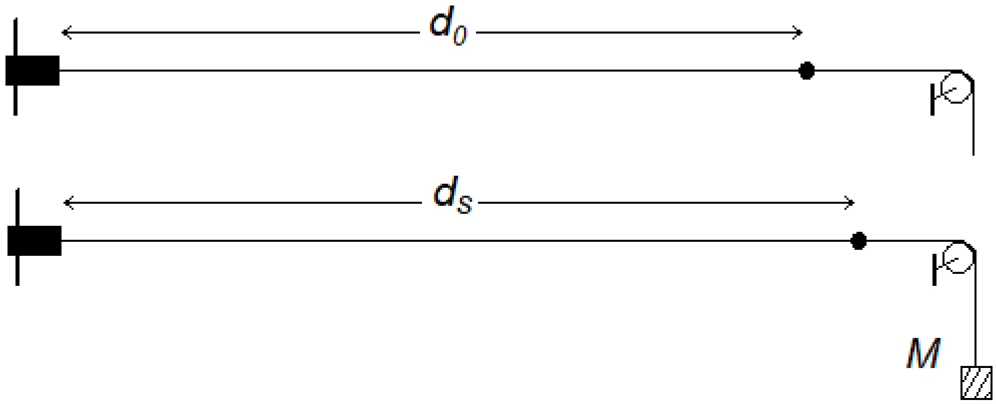
- To determine the linear density:
    - Measure the mass and length of the entire string you are using.
    - Use these measurements to calculate the "unstretched" density, "μ_0"

Bu the "unstretched" linear density is not the density we need for the waves traveling on our string. This is because the string stretches slightly when we hang the mass from it, i.e. when we apply tension. We will need to calculate the "stretched" density of the string for each of our two trials.

To accomplish this, we will consider an arbitrary length of string, d_0 , from the oscillator to a point that is marked on the string. We measure d_0, i.e. the length of this segment of string with no tension applied. We know that this length of string has the density μ_0 so we can write the mass of this length of string as:

$$m = \mu_0 \, d_0$$

We can now add the hanging mass to apply tension to the string. This will stretch the string slightly, which will decrease the density of the string. We need this new "stretched" density for the calculation of the speed of the wave.



We can label the new (i.e. "stretched") distance from the oscillator to the point on the string as "d_s" and we acknowledge that this is the same part of the string which previously had an unstretched length of d_0. This part of the string has the same mass when stretched and unstretched, so we can write:

$$m = \mu_s d_s$$

If we set these two expressions for the mass of that section of string equal to each other, we get a simple equation to relate μ_0 d_0 μ_s and d_s :

$$\mu_s d_s = \mu_0 d_0$$

Or: $$\mu_s = \mu_0 \frac{d_0}{d_s}$$

To calculate your "stretched" density:

- Choose a marked point on your string near the pulley. Measure d_0 and d_s .
- Using μ_0 d_0 and d_s , calculate the value of μ_s.

Recall that for a wave on a string: $v^2 = \frac{tension}{\mu}$ so $v^2 = \frac{Mg}{\mu_s}$

- Use the above expression to calculate v^2, then calculate v, the *expected speed* of the waves on your string. Then calculate v_1.

Watch your units! Everything must be in standard units for your speed to be in m/s.

Part B

For Part B, we will observe the phenomenon of “standing waves”. A standing wave can be formed in any medium when a wave reflects from a boundary that medium shares with another medium. The reflected wave will have properties (speed, frequency, wavelength) identical to that of the incoming wave. These two identical waves can “interfere” and produce a simple, symmetrical pattern. It is this pattern that we refer to as a “standing wave”.

The condition for a standing wave to form on a string is that the reflection must occur when the incoming wave is at a “node”, i.e. when the wave reaches the boundary its position of oscillation is at the center. (*Note: the word “node” refers to a position of zero displacement, i.e. at the center of the oscillatory motion*.) If a cycle of oscillation starts at the center, goes one direction, then back to the center, goes the other way and then finally back to the center again, we can see that the oscillator is at the center every half-cycle. In terms of distance, this means as the wave travels down the string, ***there is a node every half-wavelength.*** Since the standing wave must have a node at the boundary where it reflects and there is a node every half-wavelength along the string:

the length of the string must be an integer number of half-wavelengths

Algebraically, we can write:

$$L = n\,\frac{1}{2}\,L \qquad \text{and} \qquad v = \lambda\, f \qquad \text{so} \qquad L = n\,\frac{1}{2}\,\frac{v}{f}$$

Or:

$$f = \frac{v}{2L}\,n \qquad \text{where} \quad n = 1, 2, 3...$$

From our definition above, “n” is the “number of half-wavelengths”. This is a very easy thing to measure, since the “half-wavelengths” of a standing wave (i.e. the “bumps”) are the defining feature of the wave. If we can measure the frequency and “n” for our standing waves, and we can measure “L”, the length of the string from the oscillator to the top of the pulley, we can calculate “v”, the speed of the waves.

(Note: measuring “n” along with “L” is a proxy for measuring the wavelength, as shown by the expression for “L” above. So by measuring n and L we are indirectly measuring the wavelength. When we combine these with the frequency, we can determine the speed of the wave.)

- Adjust the frequency of the oscillator until a standing wave forms on the string. Record the frequency for the corresponding value of *n* for the standing wave.
- Repeat this process until you have the frequency that corresponds to *n* = 1 through 12.
- Create a graph of *f* vs *n* , find the best fit line and its equation. Format the equation so the slope displays the appropriate number of significant figures.

Note: the sig figs for the slope are determined by the sig figs of your measured data. Since *n* is a *counting number*, i.e. it did not come from a measuring device and it is exact, it is exempt from sig figs. Only the sig figs of your frequency data should be used to determine the allowed sig figs for your slope.

- Using the slope of the best fit line, calculate the *measured* value of the speed of the waves.

From the equation above, we can see that a graph of f vs n should result in a slope that is equal to:

$$slope = \frac{v}{2L}$$

And so we can calculate the speed by: $v = 2L\ (slope)$

- Compare the measured and expected values for the speed of waves on your string, i.e. calculate the percent difference.