6-3

November 17, 2025

In each part, determine whether the set of vectors is orthogonal and whether it is orthonormal with respect to the Euclidean inner product on R^2 .

1 c)

$\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

d)

$(0,0)$, $(0,1)$

In each part, determine whether the set of vectors is orthogonal with respect to the standard inner product on P_2 (see Example 7 of Section 6.1).

3 a)

$p_1(x) = \frac{2}{3} - \frac{2}{3}x + \frac{1}{3}x^2$ $p_2(x) = \frac{2}{3} + \frac{1}{3}x - \frac{2}{3}x^2$,

In Exercises 5-6, show that the column vectors of A form an orthogonal basis for the column space of A with respect to the Euclidean inner product, and then find an orthonormal basis for that column space.

5

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 5 \\ -1 & 2 & 0 \end{bmatrix}$$

7

Verify that the vectors

$$\mathbf{v}_1 = \left(-\frac{3}{5}, \frac{4}{5}, 0\right), \mathbf{v}_2 = \left(\frac{4}{5}, \frac{3}{5}, 0\right), \mathbf{v}_3 = (0, 0, 1)$$

form an orthonormal basis for R^3 with respect to the Euclidean inner product, and then use Theorem 6.3.2(b) to express the vector $\mathbf{u} = (1, -2, 2)$ as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$.

In Exercises 11-14, find the coordinate vector $(\mathbf{u})_S$ for the vector $\mathbf{u}$ and the basis S that were given in the stated exercise.

11

Exercise 7

In Exercises 19-22, let R^3 have the Euclidean inner product.

19

$\mathbf{u}=(4,2,1)$ $\mathbf{v}_1=\left(\frac{1}{3},\frac{2}{3},-\frac{2}{3}\right)$, $\mathbf{v}_2=\left(\frac{2}{3},\frac{1}{3},\frac{2}{3}\right)$

In Exercises 27-28, let R^2 have the Euclidean inner product and use the Gram-Schmidt process to transform the basis $\{\mathbf{u}_1,\mathbf{u}_2\}$ into an orthonormal basis. Draw both sets of basis vectors in the xy -plane.

27

$\mathbf{u}_1=(1,-3)$, $\mathbf{u}_2=(2,2)$ 28. $\mathbf{u}_1=(1,0)$, $\mathbf{u}_2=(3,-5)$

In Exercises 29-30, let R^3 have the Euclidean inner product and use the Gram-Schmidt process to transform the basis $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ into an orthonormal basis.

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$\mathbf{u}_1 = (1, 1, 1)$, $\mathbf{u}_2 = (-1, 1, 0)$, $\mathbf{u}_3 = (1, 2, 1)$

Answers

1. (a) Orthogonal but not orthonormal (b) Orthogonal and orthonormal (c) Not orthogonal and not orthonormal (d) Orthogonal but not orthonormal

3. (a) Orthogonal (b) Not orthogonal

5. An orthonormal basis: $\left\{ \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}} \right), \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right) (0,1,0) \right\}$

7. $\mathbf{u} = -\frac{11}{5}\mathbf{v}_1 - \frac{2}{5}\mathbf{v}_2 + 2\mathbf{v}_3$ 9. $\mathbf{u} = 0\mathbf{v}_1 - \frac{2}{3}\mathbf{v}_2 + \frac{1}{3}\mathbf{v}_3$

11. $\left(-\frac{11}{5}, -\frac{2}{5}, 2\right)$

13. $\left(0, -\frac{2}{3}, \frac{1}{3}\right)$

15. (a) $\left(\frac{63}{25}, \frac{84}{25}\right)$
(b) $\left(-\frac{88}{25}, \frac{66}{25}\right)$

17. (a) $\left(\frac{5}{2}, \frac{5}{2}\right)$
(b) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

19. (a) $\left(\frac{10}{3}, \frac{8}{3}, \frac{4}{3}\right)$
(b) $\left(\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}\right)$

21. (a) $\left(\frac{22}{15}, -\frac{14}{15}, \frac{2}{3}\right)$
(b) $\left(-\frac{7}{15}, \frac{14}{15}, \frac{7}{3}\right)$

23. $\left(\frac{3}{2}, \frac{3}{2}, -1, -1\right)$

25. $\left(\frac{23}{18}, \frac{11}{6}, -\frac{1}{18}, -\frac{17}{18}\right)$

27. $\mathbf{q}_1 = \left(\frac{1}{\sqrt{10}}, -\frac{3}{\sqrt{10}}\right), \mathbf{q}_2 = \left(\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}}\right)$

29. $\left\{ \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right) \right\}$