1-1

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State wether the given differential equations are linear or nonlinear. Give the order of each.

1

$$(1-x)y'' - 4xy' + 5y = \cos x$$

5

$$x^3 y^{(4)} - x^2 y'' + 4xy' - 3y = 0$$

9

$$(\sin x)y''' - (\cos x)y' = 2$$

Verify that the indicated function is a solution for the given differential equations. Where appropriate c_1 and c_2 denote constants.

13

$$\frac{dy}{dx} - 2y = e^{3x}; \quad y = e^{3x} + 10e^{2x}$$

17

$$y' + y = \sin x; \quad y = \frac{1}{2}\sin x - \frac{1}{2}\cos x + 10e^{-x}$$

21

$$y = 2xy' + y(y')^2; \quad y^2 = c_1(x + \frac{1}{4}c_1)$$

25

$$\frac{dX}{dt} = (2 - X)(1 - X); \quad \ln \frac{2 - X}{1 - X} = t$$

29

$$y'' - 6y' + 13y = 0; \quad y = e^{3x} \cos 2x$$

33

$$y'' + (y')^2 = 0; \quad y = \ln|x + c_1| + c_2$$

37

$$x^2 y'' - 3xy' + 4y = 0; \quad y = x^2 + x^2 \ln x; \quad x > 0$$

In Problems 41 and 42 verify that the indicated piecewise-defined function is a solution of the given differential equation.

41

$$xy' - 2y = 0; \quad y = \begin{cases} -x^2 & x < 0 \\ x & x \geq 0 \end{cases}$$