Lab 5: *Conductivity of PVC*

The objective of this lab exercise is to measure the *thermal conductivity* of PVC.

To measure the thermal conductivity of PVC, we will use a section of PVC pipe that we will fill with warm water. The pipe will be surrounded by ice water, so heat will flow through the wall of the pipe from the warm water to the ice water.

If the cylinder has inner diameter “a”, outer diameter “b” and length “L”, the thermal resistance of the cylinder is given by:

$$R = \frac{\ln\left(\frac{b}{a}\right)}{2\pi k L}$$

We can then write that the rate of heat flow through the wall of the cylinder is:

$$H = \frac{\Delta T}{R}$$

where ΔT is the difference between the temperature of the warm water and the temperature of the ice water. We assume the temperature of the ice water is 0 ^{O}C, so the value of ΔT is the same as the temperature of the warm water in ^{O}C. So for our system, we can replace ΔT with simply the temperature of the warm water.

As heat flows through the wall of the cylinder, the temperature of the warm water decreases because the warm water loses heat. We can express the rate of heat loss using the definition of “H”. That is:

$$H = \frac{Q}{t}$$

Or if we consider the continuous heat loss, in small “bits”, we can write H as a rate of heat loss:

$$H = \frac{dQ}{dt}$$

We can now relate the amount of heat lost by the warm water to the temperature change of the water, using what we know about calorimetry. For the warm water:

$$Q = mc\Delta T$$

Or for a tiny bit of heat loss, a corresponding tiny change in temperature:

$$dQ = mc\ dT$$

We can now use this expression in our heat flow expression:

$$H = \frac{mc\ dT}{dt}$$

And we can use this in place of H in our conduction equation. We have to be careful though. The water is losing heat, so dQ and dT are negative for the water. But in our conduction equation, H is assumed to be positive because ΔT is always defined as the hot temperature minus the cold temperature. To reconcile these, we simply insert a minus sign:

$$\frac{mc\ dT}{dt} = -\frac{T}{R}$$

Or:

$$\frac{dT}{dt} = -\frac{T}{mc\ R}$$

And:

$$\frac{dT}{dt} = -\frac{T}{mc\ [ln(b/a)/2\pi kL]}$$

We can simplify this by recognizing that everything in the denominator on the right side is a constant. All of these constants combined can be replaced by one letter; I will choose "α".

So: $$\frac{dT}{dt} = -\alpha\ T$$ where $$\alpha = \frac{2\pi\ kL}{mc\ ln(b/a)}$$

This is a simple differential equation. It tells us that the rate at which the temperature is decreasing is directly proportional to the temperature itself. That is, the hotter the temperature, the faster the water will cool off.

This is arguably the most important, as well as the simplest, differential equation in science. The behavior of many systems, both physical and biological, is described by this equation: the rate of change of the quantity of the system is directly proportional to the quantity itself. For example, population growth: in general, the number of babies born (i.e. increase in the population) in a given year is directly related to the size of the existing population.

If we divide both sides by T, multiply both sides by "dt" and then integrate both sides (i.e. add the "bits" of dT on one side, add the "bits" of dt on the other side), we get

$$\int \frac{dT}{T} = -\alpha \int dt \quad \text{or} \quad lnT - lnT_0 = -\alpha\, t$$

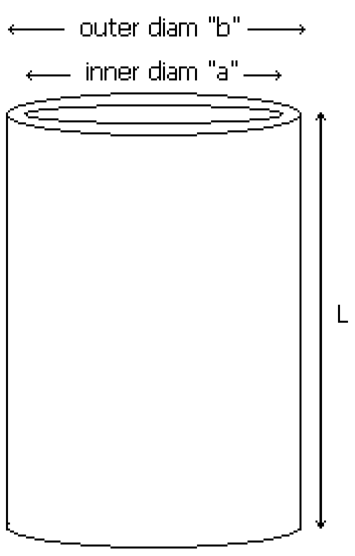
We can rearrange this equation to read:

$$\text{or} \quad lnT = -\alpha\, t + lnT_0$$

Note that in this equation, α and T_O are constants. Only T and t are changing. If we measure the temperature of the water as it decreases with time, we can create a table of t and lnT, then graph lnT vs t and the slope of the resulting plot should be equal to $-\alpha$. We can then use the value of α to calculate the conductivity of PVC.

Procedure

1. Use the caliper to carefully measure the outer diameter "b" and the inner diameter "a" in centimeters. Record the results in the first data table. Use a ruler to carefully measure L (inside the cylinder), in centimeters; record the result in meters.



2. Pack the outside of the PVC pipe with ice, then fill the ice with tap water to a level of approximately half a centimeter below the top of the PVC pipe.

3. Connect the temperature probe to the black box and set up the Data Studio software. Choose *Table* from the display and change the *Sampling Rate* to **15 seconds**. This will allow the software to automatically record the temperature in the table every 15 seconds.

4. I will pour warm water into your PVC cylinder and will measure the value of "m", the mass of your water. Record this value in the first data table in kilograms.

5. Cover the warm water and insert the temperature probe. Allow the system to sit for approximately 20 seconds to give the probe time to reach the same temperature as the water. Then click *Start*.

6. After 10 minutes (i.e. 600 seconds) the computer will have collected 40 measurements in the table. Click *Stop*. Transfer the values of temperature to your second data table in Excel.

7. Finish your data table by having Excel calculate lnT for each temperature value.

8. Create a graph of lnT vs t and find the best-fit line.

9. The slope of the best-fit line is your value of $-\alpha$. Use the slope of the line, without the negative sign, along with your values of a b L m and c, to calculate the conductivity of PVC.

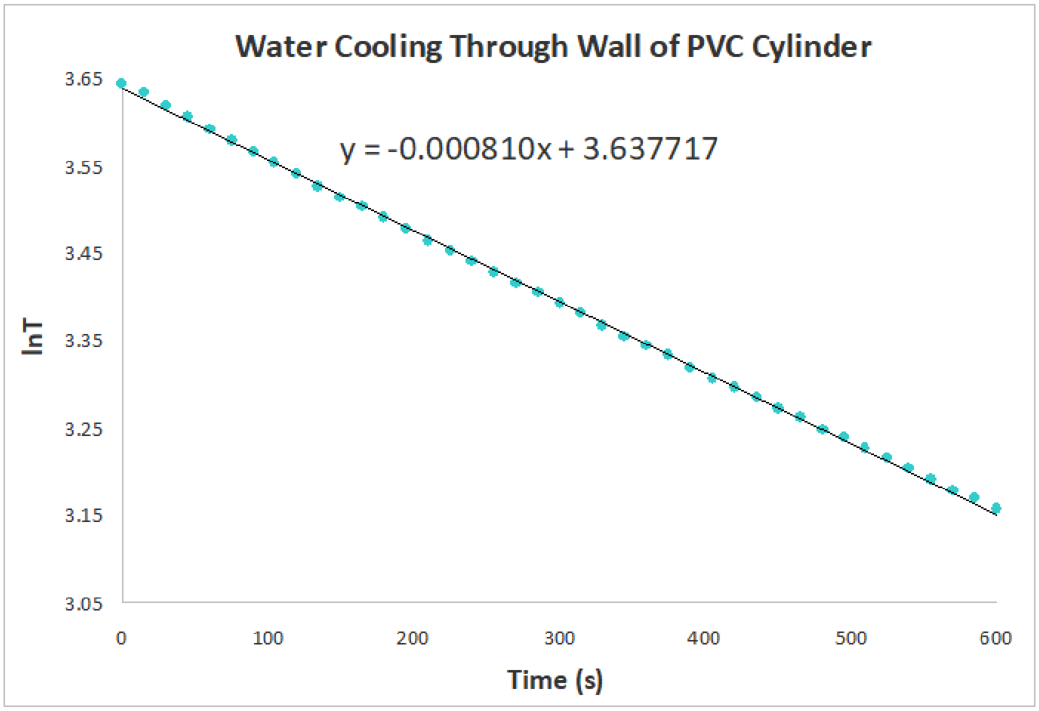
Samples of the data tables, graph and calculations are below:

Conductivity of PVC

a	4.03	cm
b	4.87	cm
L	0.2243	m
m_w	0.270	kg
c_w	4190	J/kg-K

Water Cooling Through Wall of PVC Cylinder

t (s)	T (C^o)	lnT
0	38.2	3.64
15	37.8	3.63
30	37.3	3.62
45	36.8	3.61
60	36.3	3.59
⋮	⋮	⋮
570	24.0	3.18
585	23.8	3.17
600	23.5	3.16



Calculations

Calculate the value of k for PVC:

$$k = \frac{\alpha\, m\, c\, \ln(b/a)}{2\pi L} = \frac{(0.000810\ s^{-1})\ (0.270\ kg)\ (4190\ J/kg.K)\ \ln(4.87\ cm/4.03\ cm)}{2\pi\ (0.2243\ m)} = 0.123 \quad W/m\text{-}K$$