5-1

November 12, 2025

In Exercises 1-4, confirm by multiplication that $\mathbf{x}$ is an eigenvector of A , and find the corresponding eigenvalue.

1)

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}; \mathbf{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

3)

$$A = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}; \mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

In each part of Exercises 5-6, find the characteristic equation, the eigenvalues, and bases for the eigenspaces of the matrix.

5 a)

$$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

b)

$$\begin{bmatrix} -2 & -7 \\ 1 & 2 \end{bmatrix}$$

In Exercises 7-12, find the characteristic equation, the eigenvalues, and bases for the eigenspaces of the matrix.

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$$\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$$

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$$\begin{bmatrix} 6 & 3 & -8 \\ 0 & -2 & 0 \\ 1 & 0 & -3 \end{bmatrix}$$

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Prove: If λ is an eigenvalue of an invertible matrix A and $\mathbf{x}$ is a corresponding eigenvector, then $1/\lambda$ is an eigenvalue of A^{-1} and $\mathbf{x}$ is a corresponding eigenvector.

Answers

1. eigenvalue: -1

3. eigenvalue: 5

5. (a) Characteristic equation: $(\lambda - 5)(\lambda + 1) = 0$; eigenvalue: 5, basis for eigenspace: $\{(1, 1)\}$; eigenvalue: -1 , basis for eigenspace: $\{(-2, 1)\}$

(b) Characteristic equation: $\lambda^2 + 3 = 0$; no real eigenvalues

(c) Characteristic equation: $(\lambda - 1)^2 = 0$ eigenvalue: 1, basis for eigenspace: $\{(1, 0), (0, 1)\}$

(d) Characteristic equation: $(\lambda - 1)^2 = 0$ eigenvalue: $\lambda = 1$ basis for eigenspace: $\{(1, 0)\}$

7. Characteristic equation: $(\lambda - 1)(\lambda - 2)(\lambda - 3) = 0$ eigenvalue: 1, basis for eigenspace: $\{(0, 1, 0)\}$ eigenvalue: 2, basis for eigenspace: $\{(-1, 2, 2)\}$ eigenvalue: 3, basis for eigenspace: $\{(-1, 1, 1)\}$

9. Characteristic equation: $(\lambda + 2)^2(\lambda - 5) = 0$ eigenvalue: -2 basis for eigenspace: $\{(1, 0, 1)\}$ eigenvalue: 5, basis for eigenspace: $\{(8, 0, 1)\}$