Chapter 6-3

1-21 odd

In Problems 1–14 for each differential equation find two linearly independent power series solutions about the ordinary point x=0.

1. $y'' = xy$
2. $y'' - 2xy' + y = 0$
3. $y'' + x^2y' + xy = 0$
4. $(x-1)y'' + y' = 0$
5. $(x^2-1)y'' + 4xy' + 2y = 0$
6. $(x^2+2)y'' + 3xy' - y = 0$
7. $y'' - (x+1)y' - y = 0$

In Problems 15-18 use the power series method to solve the given differential equation subject to the indicated initial conditions.

15. $(x-1)y'' - xy' + y = 0, \quad y(0) = -2, y'(0) = 6$
16. $y'' - 2xy' + 8y = 0, \quad y(0) = 3, y'(0) = 0$

In Problems 19-22 use the procedure illustrated in Example 9 to find two power series solutions of the given differential equation about the ordinary point $x = 0$.

19. $y'' + (sinx)y = 0$
20. $y'' + e^{-x}y = 0$

Answers

1. $y_1(x) = c_0 \left[1 + \frac{1}{3\cdot 2}x^3 + \frac{1}{6\cdot 5\cdot 3\cdot 2}x^6 + \frac{1}{9\cdot 8\cdot 6\cdot 5\cdot 3\cdot 2}x^9 + \dots\right.$

$$y_2(x) = c_1 \left[x + \frac{1}{4\cdot 3}x^4 + \frac{1}{7\cdot 6\cdot 4\cdot 3}x^7 + \dots\right.$$

2. $$y_1(x) = c_0 \left[1 - \frac{1}{27}x^3 - \frac{3}{4!}x^4 - \frac{21}{6!}x^6 - \dots\right]$$
$$y_2(x) = c_1 \left[x + \left(\frac{1}{3!}x^3 + \frac{5}{5!}x^5 + \frac{45}{7!}x^7 + \dots\right)\right.$$

3. $y_1(x) = c_0 \left[1 - \frac{1}{3!}x^3 + \frac{4^2}{6!}x^6 - \frac{7^2\cdot 4^2}{9!}x^9 + \dots\right]$

$$y_2(x) = c_1 \left[x - \frac{2^2}{4!}x^4 + \frac{5^2\cdot 2^2}{7!}x^7 - \frac{8^2\cdot 5^2\cdot 2^2}{10!}x^{10} + \dots\right]$$

4. $y_1(x) = c_0; \quad y_2(x) = c_1 \sum_{n=1}^{\infty} \frac{1}{n}x^n$
5. $y_1(x) = c_0 \sum_{n=0}^{\infty} x^{2n}; \quad y_2(x) = c_1 \sum_{n=0}^{\infty} x^{2n+1}$
6. $y_1(x) = c_0 \left[1 + \frac{x^2}{4} - \frac{7}{4\cdot 4!}x^4 + \frac{23\cdot 7}{8\cdot 6!}x^6 - \dots\right]$

$$y_2(x) = c_1 \left[x - \left(\frac{1}{6}x^3 + \frac{14}{2\cdot 5!}x^5 - \frac{34\cdot 14}{4\cdot 7!}x^7 - \dots\right)\right]$$

7. $y_1(x) = c_0 \left[1 + \frac{\frac{1}{2!}x^2}{x^2} + \frac{\frac{1}{3!}x^3}{x^3} + \frac{\frac{1}{4!}x^4}{x^4} + \dots\right]$

$$y_2(x) = c_1 \left[x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots\right]$$

8. $y(x) = -2\left[1 + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots\right] + 6x$

$$= 8x - 2e^x$$

9. $y(x) = 3 - 12x^2 + 4x^4$
10. $y_1(x) = c_0 \left[1 - \frac{\frac{1}{6}x^3}{x^3} + \frac{1}{120}x^5 + \dots\right]$

$$y_2(x) = c_1 \left[x - \frac{1}{12}x^4 + \frac{1}{180}x^6 + \dots\right]$$

11. $y_1(x) = c_0 \left[1 - \frac{x^2}{6} + \frac{x^3}{6} - \frac{x^4}{40} + \dots\right]$