8-2

November 19, 2025

In Exercises 1-2, determine whether the linear transformation is one-to-one by finding its kernel and then applying Theorem 8.2.1.

1 a)

$T : R^2 \to R^2$, where $T(x, y) = (y, x)$

b)

$T : R^2 \to R^3$, where $T(x, y) = (x, y, x + y)$

c)

$T : R^3 \to R^2$, where $T(x, y, z) = (x + y + z, x - y - z)$

3 a)

$$A = \begin{bmatrix} 1 & -2 \\ 2 & -4 \\ -3 & 6 \end{bmatrix}$$

b)

$$A = \begin{bmatrix} 1 & 3 & 1 & 7 \\ 2 & 7 & 2 & 4 \\ -1 & -3 & 0 & 0 \end{bmatrix}$$

In Exercises 11-12, compute $(T_2 \circ T_1)(x, y)$.

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$T_1(x, y) = (2x, 3y), T_2(x, y) = (x - y, x + y)$

In Exercises 13-14, compute $(T_3 \circ T_2 \circ T_1)(x, y)$.

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$T_1(x, y) = (-2y, 3x, x - 2y), T_2(x, y, z) = (y, z, x), T_3(x, y, z) = (x + z, y - z)$

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Suppose that the linear transformations $T_1 \colon P_2 \to P_2$ and $T_2 \colon P_2 \to P_3$ are given by the formulas $T_1(p(x)) = p(x + 1)$ and $T_2(p(x)) = xp(x)$. Find $(T_2 \circ T_1)(a_0 + a_1x + a_2x^2)$.

19 a)

Let $T : P_1 \to R^2$ be the function defined by the formula

$$T(p(x)) = (p(0), p(1))$$

Find $T(1 - 2x)$.

d)

Find $T^{-1}(2, 3)$, and sketch its graph.

Answers

1. (a) $\ker(T) = \{\mathbf{0}\}$; T is one-to-one
   (b) $\ker(T) = \{\mathbf{0}\}$; T is one-to-one
   (c) $\ker(T) = \{\text{span}(0, 1, 1)\}; T$ is not one-to-one
   3. (a) nullity $(A) = 1$; not one-to-one
   (b) nullity $(A) = 1$; not one-to-one
   11. $(T_2 \circ T_1)(x, y) = (2x - 3y, 2x + 3y)$
   13. $(T_3 \circ T_2 \circ T_1)(x, y) = (3x - 2y, x)$
   15. (a) $a + d$
   (b) $(T_2 \circ T_1)(A)$ does not exist because $T_1(A)$ is not a 2×2 matrix
   17. $a_0x + a_1x(x + 1) + a_2x(x + 1)^2$
   19. (a) $(1, -1)$
   (d) $T^{-1}(2, 3) = 2 + x$