4-6

October 29, 2025

Consider the bases $B = \{u_1, u_2\}$ and $B' = \{u'_1, u'_2\}$ for R^2, where

$$u_1 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 4 \\ -1 \end{bmatrix}, \quad u'_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad u'_2 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

1 a)

Find the transition matrix from B' to B.

b)

Find the transition matrix from B to B'.

c)

Compute the coordinate vector $[w]_B$, where

$$w = \begin{bmatrix} 3 \\ -5 \end{bmatrix}$$

and use (12) to compute $[w]'_B$.

d)

Check your work by computing $[w]'_B$ directly.

Consider the bases $B = \{u_1, u_2, u_3\}$ and $B' = \{u_1', u_2', u_3'\}$ for R^3, where

$$u_1 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \quad u_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$u_1' = \begin{bmatrix} 3 \\ 1 \\ -5 \end{bmatrix}, \quad u_2' = \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}, \quad u_3' = \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}$$

3 a)

Find the transition matrix B to B'.

b)

Compute the coordinate vector $[w]_B$, where

$$w = \begin{bmatrix} -5 \\ 8 \\ -5 \end{bmatrix}$$

and use (12) to compute $[w]'_B$.

c)

Check your work by computing $[w]'_B$ directly.

Let S be the standard basis for R^3, and let $B = \{v_1, v_2, v_3\}$ be the basis in which $v_1 = (1, 2, 1)$, $v_2 = (2, 5, 0)$, and $v_3 = (3, 3, 8)$.

9 a)

Find the transition matrix $P_{B \to S}$ by inspection.

b)

Use Formula (14) to find the transition matrix $P_{S \to B}$.

c)

Confirm that $P_{B \to S}$ and $P_{S \to B}$ are inverses of one another.

d)

Let $w = (5, -3, 1)$. Find $[w]_B$ and then use Formula (11) to compute $[w]_S$.

e)

Let $w = (3, -5, 0)$. Find $[w]_S$ and then use Formula (12) to compute $[w]_B$.

Formulas

(11)

$$[v]_B = P_{B' \to B} [v]'_B$$

(12)

$$[v]'_B = P_{B \to B'} [v]'_B$$

(14)

$$[\text{new basis} \mid \text{old basis}] \xrightarrow{\text{row operations}} [I \mid \text{transition from old to new}]$$