Lab 4: Archimedes' Principle & the Density of Metals

Archimedes lived in Syracusa on the island of Sicily around the year 250 BC. He made important contributions to the study of mathematics, engineering and physics... long before real science and math even existed. (Algebra would only appear in Europe 1500 years later!) Archimedes is best known for his "principle" that describes the effect of the "buoyant force" acting on an object that is partially or completely submerged in a fluid:

> ***An object submerged in a fluid experiences and upward "buoyant force" from the fluid that is equal to the weight of the fluid displaced.***

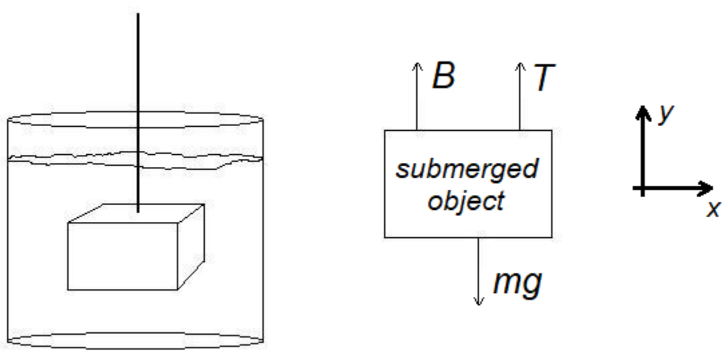
We can write this algebraically, using a "B" as the symbol for the buoyant force:

B = weight of the fluid displaced = "mg" of the fluid displaced

$$\textit{buoyant force:} \qquad B = \rho_{fluid}\, V_{displ}\, g$$

The force the fluid exerts on the object depends on the *density of the fluid* and the *volume of the fluid displaced*, because these two define the *mass of the fluid displaced*.

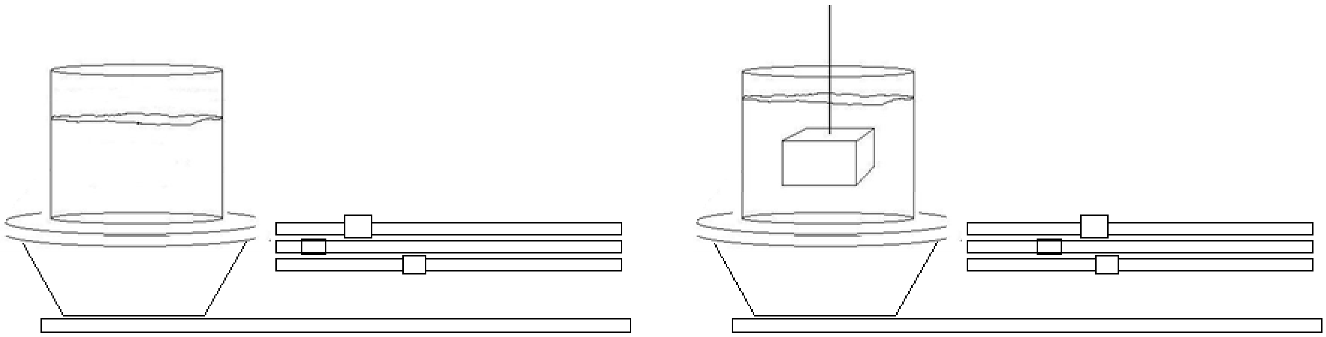
We will use the buoyant force today to determine the density of brass. Consider an object that hangs from a string and is completely submerged in a fluid (e.g. submerged in water.) We can draw a free body diagram for the object; it would look like this:



The water exerts an upward force on the object, and by Newton's Third Law…

the object must exert an equal downward force on the water.

Consider the two situations illustrated here:



If we place a container of water on a scale, the scale should read the weight of the container and the water. If we then place an object in the water, the object will exert a downward force on the water... and the scale reading will increase. **The increase in the reading on the scale must represent the downward force exerted by the object**, which is equal to (and in the opposite direction of) the buoyant force acting on the object.

For today's lab, we will submerge metal weights and measure the buoyant force acting on the weights as the increase in the reading on the scale. We will use the collected data to determine the density of two metals: aluminum (in Part 1) and brass (in Part 2).

The procedure for measuring this data is simple:

- Place a plastic container of water on the scale and record its mass.
- Ensure the **brass weights are dry;** hang them from the string.
- Lower the weights into the cup so they are completely submerged but _do not touch the sides or bottom of the container_.
- Record the reading on the scale.

The reading on the scale is not the buoyant force directly. The scale responds to *force*, but it displays its reading as *mass*. In other words, it displays the mass equivalent of the force that it measures. If we use "m_s" for the "mass reading of the scale", then **the force the scale reads is $m_s g$**.

We can now use this to derive an expression that will allow us to determine the density of brass from our data:

$$B = \rho_w V_{displ}\, g$$

where "ρ_w" is the density of water and V_{displ} is the volume of the water displaced by the brass. Since the brass is completely submerged, the volume displaced is the same as the volume of the brass. The volume can be replaced by using the relationship between the density, volume and mass of the brass:

$$m_s\, g = \rho_w \left(\frac{m_m}{\rho_m}\right) g \qquad \text{or} \qquad m_s = \left(\frac{\rho_w}{\rho_m}\right) m_m$$

Your measured data will be m_m and m_s, the actual mass of the metal and the corresponding reading on the scale when the metal is submerged in water. The expression above includes only these two measurements and two constants, the densities of water and the metal. So a graph of m_s vs m_m and find the best fit line.

Your graph should form a perfectly straight line with an intercept of nearly zero.

The slope of your graph should be equal to the ratio of the density of water to the density of brass. So you can calculate the density of brass using the density of water, which is 1.00 g/cc, and your slope. That is:

$$slope = \frac{\rho_w}{\rho_m} \qquad \text{so} \qquad \rho_m = \frac{\rho_w}{slope}$$

Part 1: Density of Aluminum

The aluminum is only available in 10-gram pieces. Use multiples of these pieces to measure the value of m_s for m_m = 10, 20, … , 70 grams.

Create a graph of m_s vs m_m and find the best fit line. Use the slope of this line to calculate the density of aluminum. Compare your measured value to the expected value of 2.70 g/cc.

Part 2: Density of Brass

The procedure for Part 2 is identical to that of Part 1, but the brass weights are available in 20-gram, 50-gram, and 100-gram pieces.

Use multiples of these pieces to measure the value of m_s for m_m = 50, 100, … , 350 grams.

Create a graph of m_s vs m_m and find the best fit line. Use the slope of this line to calculate the density of brass.

We do not have an expected value for the density of brass!

Brass is an alloy of copper and zinc, which have densities of

copper: $\boldsymbol{\rho_c}$ **= 8.96 g/cc** **zinc:** $\boldsymbol{\rho_z}$ **= 7.14 g/cc**

So while we do not have an “expected value” for the density of brass, we do know that its density must be greater than the density of zinc and less than the density of copper. However, we can do more.
When measuring an alloy, we are often interested in the percentage of each component metal used to create the alloy. In this case, we would like to know what percentage copper is our brass made of. Both

metals (copper and zinc), contribute to the *mass of the brass*, but they also contribute to the *volume of the brass*. But since the copper is more dense than the brass, it will contribute a greater share of the mass than its share of the volume. In other words, the "percent copper by volume" is a different value than the "percent copper by mass."

To illustrate this idea: ***derive an expression (on scratch paper) for the percent copper by volume, and use this expression to calculate the percentage for your brass.*** The expression should include only three values: the densities of copper, zinc and brass.

To accomplish this:

- Draw a picture of a piece of copper, a piece of zinc and a piece of brass.
- Label each with the information you need: volume, mass and density.
- Write an equation for each that connects mass, density and volume.
- The idea is that the piece of copper and piece of zinc are combined to make the piece of brass; so write an equation that connects the volumes of the three pieces, and an equation that connects the masses of the three pieces.

You now have all the equations that you need. But what are you solving for? Remember that a "percentage" is just a ratio. You first must find the "percent copper by volume"; what is this ratio? It is the ratio of the volume of copper to the volume of the brass. You can create this ratio in your equations, and then solve for it, by:

- Divide your "volume equation" by the volume of the brass.
- Define the volume of copper divided by the volume of brass as "x".
- Define the volume of zinc divided by the volume of brass as "y".
- Replace the masses in your "mass equation" with density times volume; divide this equation by volume of brass.
- You should now have two equations with two unknowns, x and y. Solve for "x".

The expression should look like this:

$$x = \frac{\rho_{??} - \rho_{??}}{\rho_{??} - \rho_{??}}$$

where your derivation will reveal which densities to use in this fraction.