# Chapter 5

October 29, 2025

## 5.1 Eigen Vectors. Eigen Vectors

**Definition 5.1.**

$$A\vec{x} = \lambda\vec{x}$$

*$\lambda$ can be $\pm$, 0, or complex. This is known as the eigen value.*
*The eigen vector, $\vec{x}$, where $\vec{x} \neq 0$*

### 5.1.1 Examples

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$$\det(\lambda I - A) = 0$$

This is the characteristic equation; yes the same one from diff eq!

$$a = \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{aligned} &\begin{bmatrix} 5 \\ 1 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \end{bmatrix} \\ =& \begin{bmatrix} 4 \\ 4 \end{bmatrix} = 4\begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{aligned}$$

$$\lambda = 4 \qquad \vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

**5 d)**

$$\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$$

$$\lambda = ?$$

$$\begin{aligned} &\det(\lambda I - A) = 0 \\ &\det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}\right) = 0 \\ &\det\begin{bmatrix} \lambda - 1 & 2 \\ 0\lambda - 1 & \end{bmatrix} = 0 \\ &(\lambda - 1)^2 = 0 \\ &\lambda = 1, 1 \end{aligned}$$

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$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & 0 & 0 \\ -2 & 0 & 4 \end{bmatrix}$$

$$\begin{vmatrix} \lambda-1 & 0 & 2 \\ 0 & \lambda & 0 \\ 2 & 0 & \lambda-4 \end{vmatrix} =$$

$$\begin{aligned} &= (\lambda-1)(\lambda)(\lambda-4) - 4\lambda \\ &= \lambda(\lambda^2 - 5\lambda + \cancel{4} - \cancel{4}) \\ &= \lambda^2(\lambda-5) \end{aligned}$$

$$\lambda = 0, \ , 0 \ , 5$$

$\lambda = 0$

$$\begin{bmatrix} -1 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-x_1 + 2x_3 = 0 \qquad x_1 = 2x_3 \qquad x_3 = t$$

$$x_2 = s \qquad x_1 = 2t$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2t \\ s \\ t \end{bmatrix} = t \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + s \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\lambda = 5$

$$\begin{bmatrix} 4 & 0 & 2 \\ 0 & 5 & 0 \\ 2 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$: \begin{bmatrix} \frac{-1}{2} \\ 0 \\ 1 \end{bmatrix}$$

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$$\begin{bmatrix} -2 & 2 & 3 \\ -2 & 3 & 2 \\ -4 & 2 & 5 \end{bmatrix}$$

$$\begin{aligned} \det(\lambda I - A) &= \det\left(\begin{bmatrix} \lambda+2 & -2 & -3 \\ 2 & \lambda-3 & -2 \\ 4 & -2 & \lambda-5 \end{bmatrix}\right) \\ &= (\lambda+2)(\lambda-3)(\lambda-5) + 16 + 12 - (-12(\lambda+2) - 4(\lambda-3) + 4(\lambda-5)) \end{aligned}$$