Physical Pendulum – A Challenging Problem!

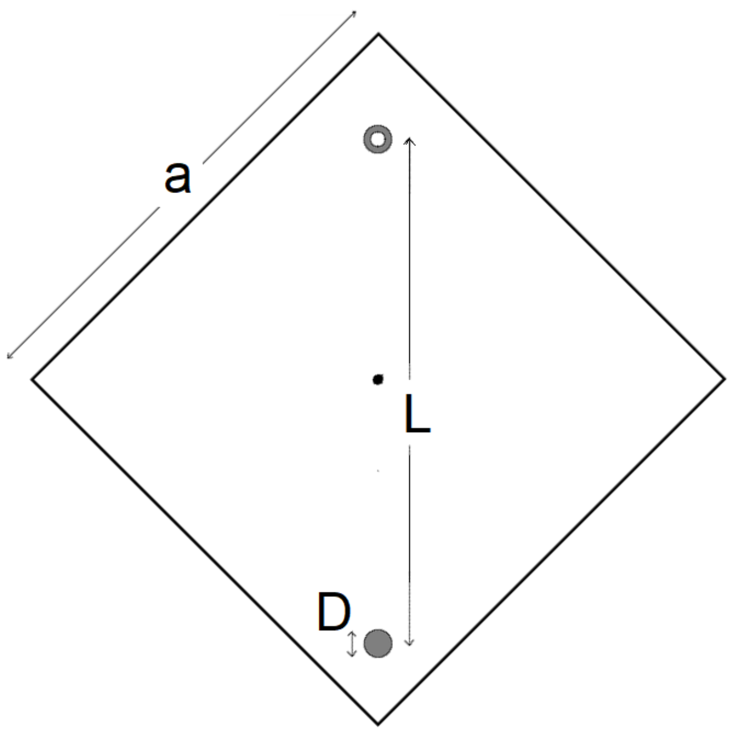
During the construction of the pendulums I went through a bit of trial and error… and some of the pendulums ended up with an extra hole. Now we can ask…

How much does the extra hole affect the moment of inertia of the pendulum?

In other words:

By what percentage is the moment of inertia of a pendulum reduced due to the extra hole?

To answer this question, we can first define a pendulum using the diagram below:



The pendulum has side length ***a***, with a hole of diameter ***D*** that is distance ***L*** from the pivot.

Then we can establish what we are trying to find:

$$\%\,difference = \frac{moment\ of\ intertia\ for\ \text{"}hole\text{"}}{moment\ of\ inertia\ for\ full\ square}$$

If we can construct expressions for this numerator and denominator – in terms of the given ***a*** ***L*** and ***D*** – then we can put these expressions in the fraction, simplify, and end up with an expression for the percent difference.

We already know, from what we used in Lab 2, the expression for the moment of inertia for the full square. This was our derived expression for:

$$I_{pivot} = I_{cm} + md^2 = \frac{1}{6}ma^2 + m(a/2)^2 = \frac{5}{12}ma^2$$

And the moment of inertia of the hole? We have to consider the hole is a disk of radius D/2 and that it is rotating around the pivot. So the moment of inertia of the hole will be:

$$I_{hole} = I_{cm} + md^2 = \frac{1}{2}m(D/2)^2 + mL^2 = \frac{1}{8}mD^2 + mL^2$$

Seems pretty simple, but there's a small twist: ***what is "m" in these expressions?***

In the expression for the full square, "*m*" is the mass of the full square. And in the expression for the "hole", that "*m*" is the missing mass, of the disk that would fit in that hole.

To handle this, consider that the square and the mass missing from the hole have the same density, which we can we define as ***mass per area***. So now:

- Write the expression for the mass of square as density times area, using σ for the density.
- Write the expression for the missing mass of the hole, as the area times the same density.
- Now use the expressions here and above to write the expression for *% difference*

You should be able to simplify this expression so it is only in terms of the three given values: ***a D L***.

Finally, measure the three values – all in cm – for one of the pendulums, and calculate this percentage.

From your calculation, ***is it reasonable to ignore the "missing" mass and the effect it has on the moment of inertia of the pendulum?***