

Introducere în criptografie

Marios Choudary

What is cryptography ?

- Greek:
 - crypto: hide, make secret (RO: a ascunde)
 - Γραφω [Grafo]: to write (RO: a scrie)
- In older cryptographic systems:
 - “the *art* of secret writing”
- In modern cryptography:
 - “the *science* of secret writing”

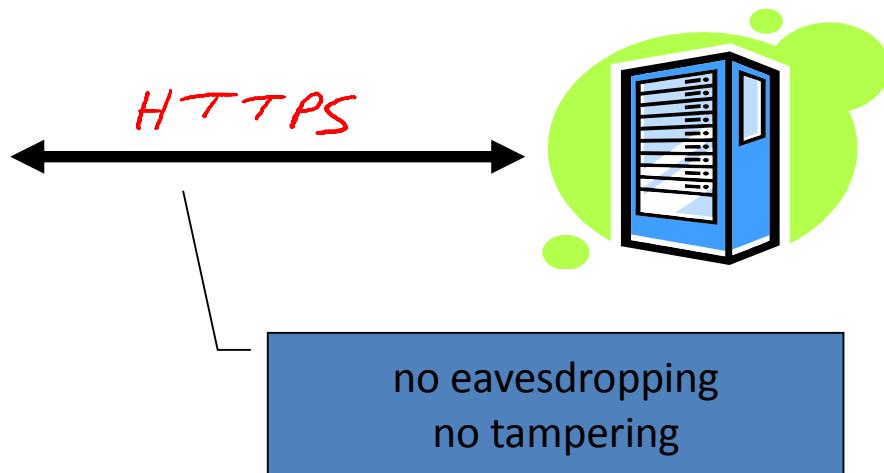
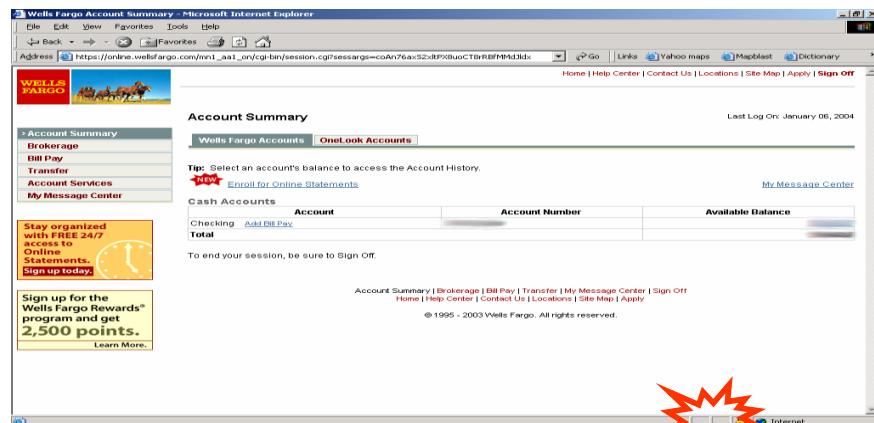
What is cryptography ?

- Cryptography, cryptology, cryptanalysis
 - Some consider cryptology = cryptography + cryptanalysis
 - Cryptanalysis : art/science of analysing (often breaking) the security of cryptographic systems
 - Modern cryptographic algorithms often have a strong cryptanalysis performed

Exemple

APLICATII ALE CRYPTOGRAFIEI

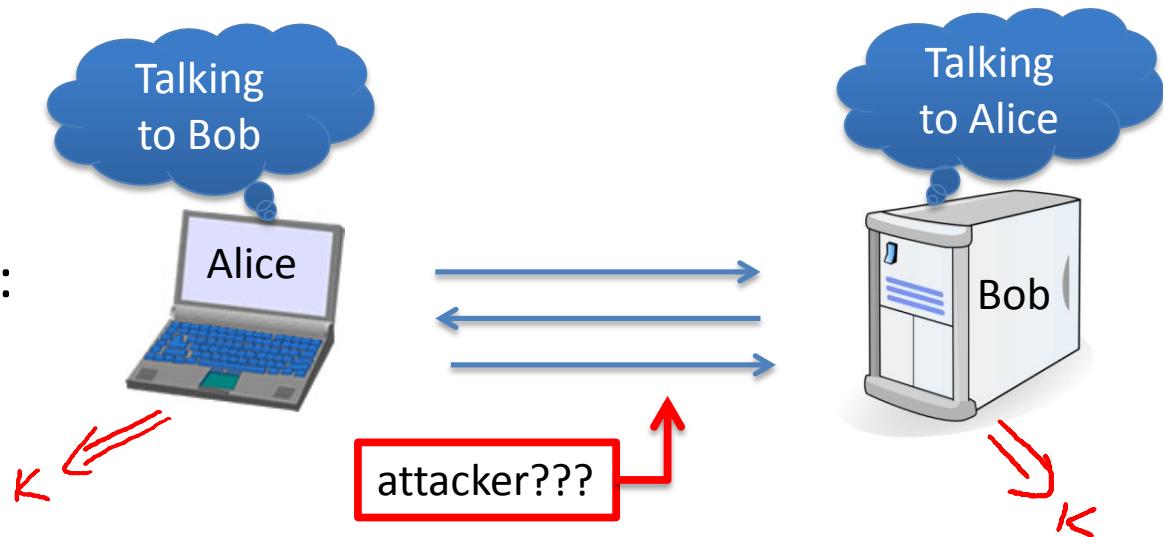
Secure communication



Dan Boneh

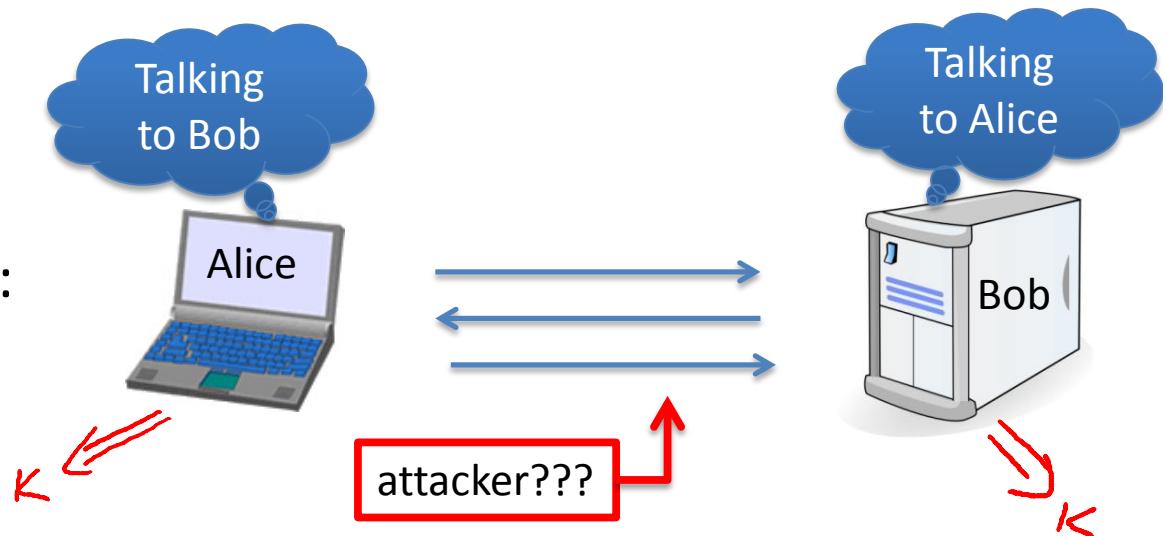
Crypto core

Secret key establishment:

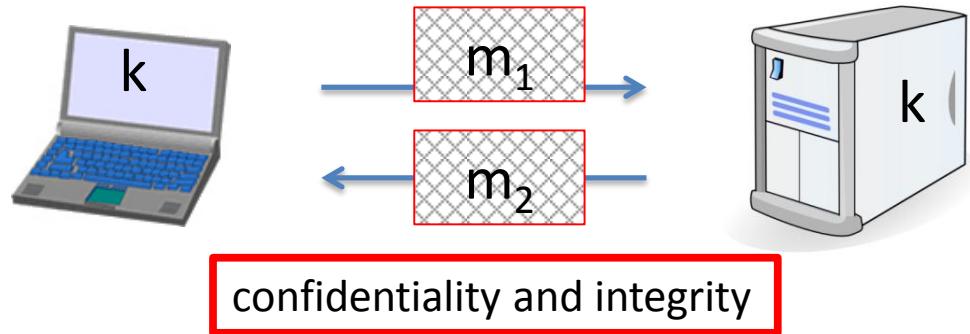


Crypto core

Secret key establishment:

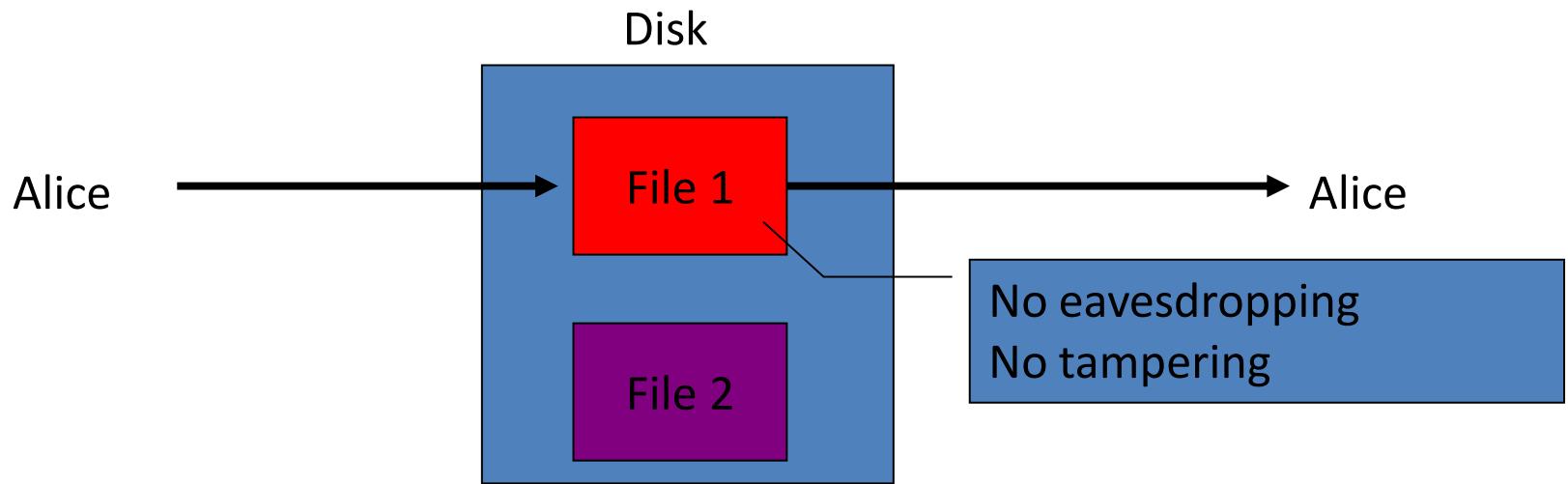


Secure communication:



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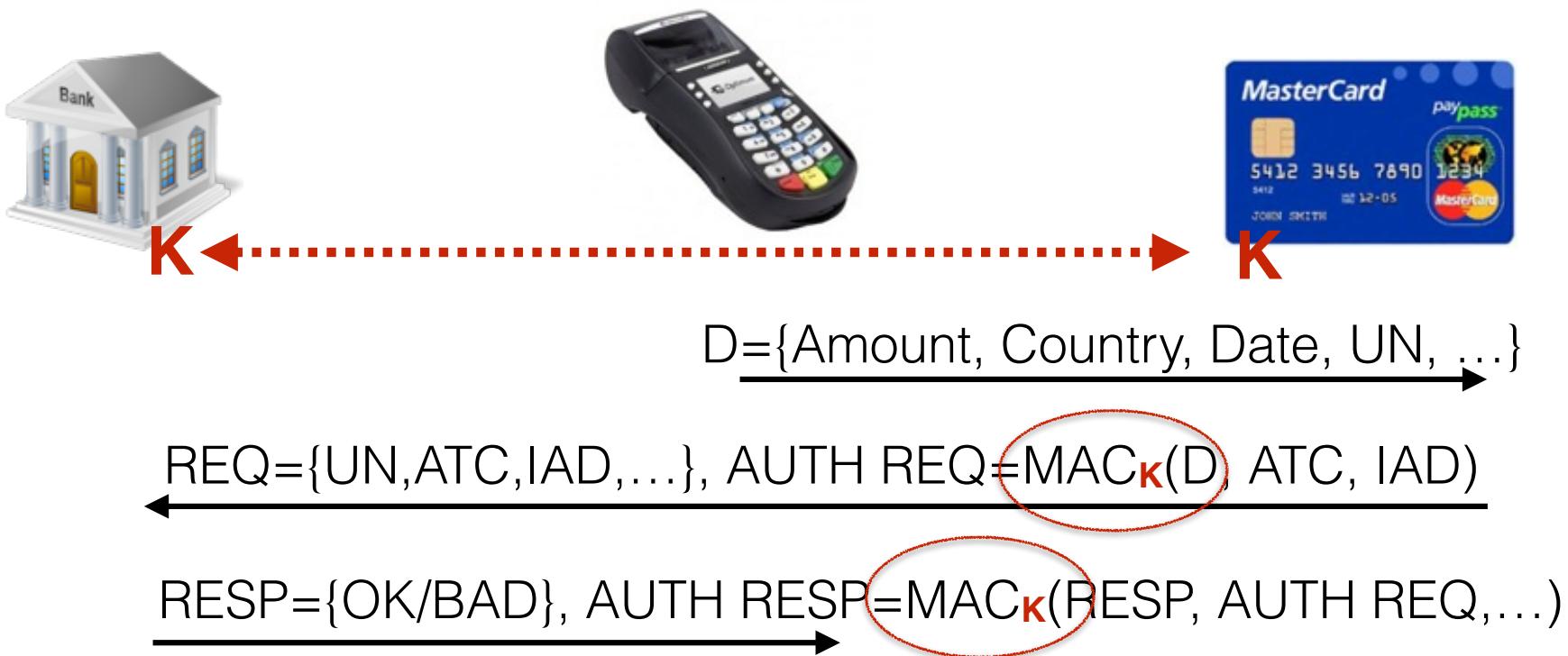
Protected files on disk



Analogous to secure communication:

Alice today sends a message to Alice tomorrow

EMV online authorisation



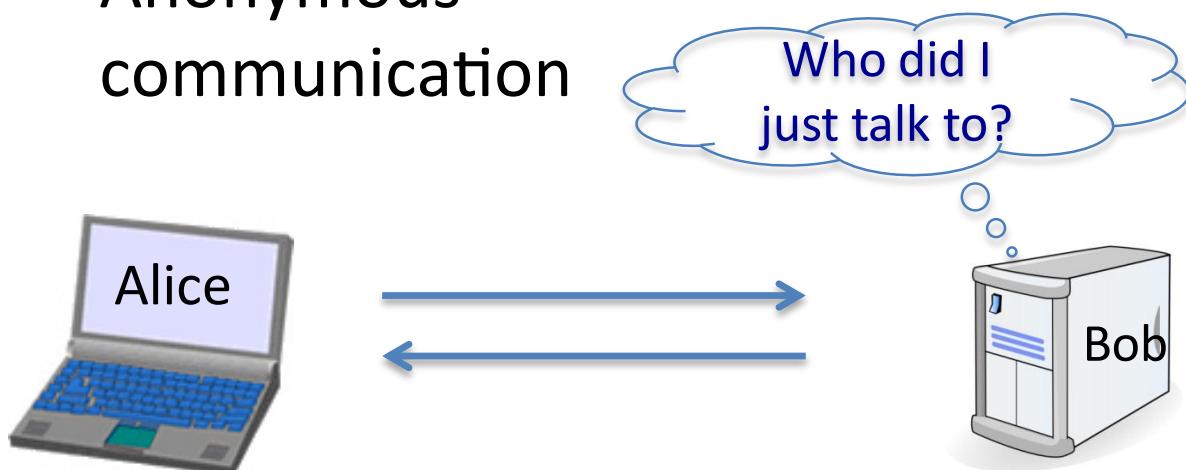
Fancier applications

- Digital signatures



Fancier applications

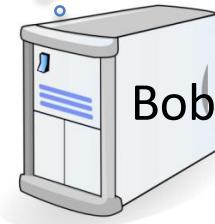
- Digital signatures
- Anonymous communication



Fancier applications

- Digital signatures
- Anonymous communication

Who did I just talk to?

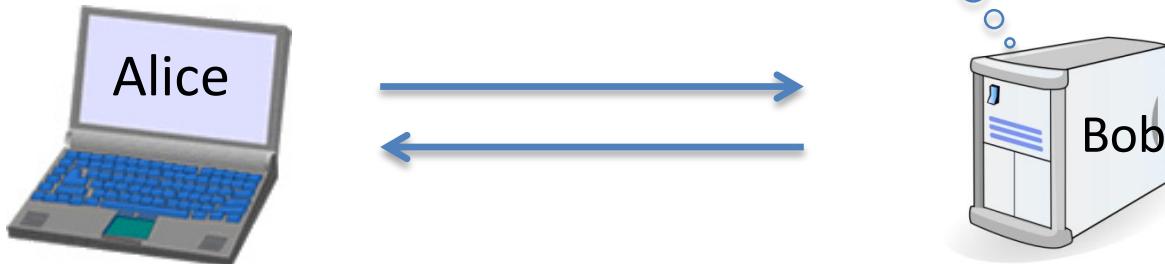


- Digital cash (bitcoin)

Fancier applications

- Digital signatures
- Anonymous communication

Who did I just talk to?



- Digital cash (bitcoin)
- E-voting, etc.

Things to remember

Cryptography is:

- A tremendous tool
- The basis for many security mechanisms

Cryptography is not:

- The solution to all security problems
- Reliable unless implemented and used properly
- Something you should try to invent yourself
 - many many examples of broken ad-hoc designs

Vedeti "The code breakers" de David Kahn

EXEMPLE ISTORICE

Atbash: criptografie din istoria lui Israel

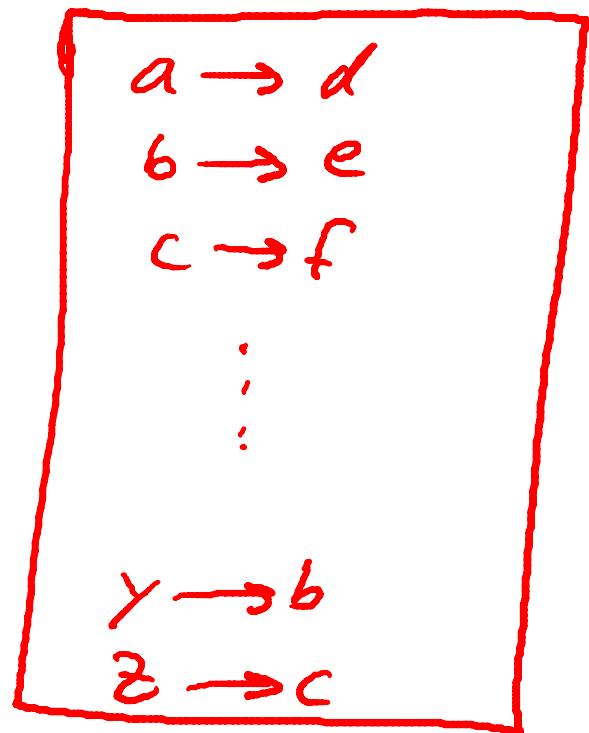
- Ieremia 25, 17-26: “Și am luat cupa din mâna Domnului și am dat să bea tuturor neamurilor la care m-a trimis Domnul: [...] iar regele **Şişacului** va bea după ei.”
- Şişac (ebr: Sheshakh) = Babilon cu litere schimbatе; prima cu ultima, a doua cu penultima, etc.

More popular: Caesar cipher

shift by 3 :
(or K)

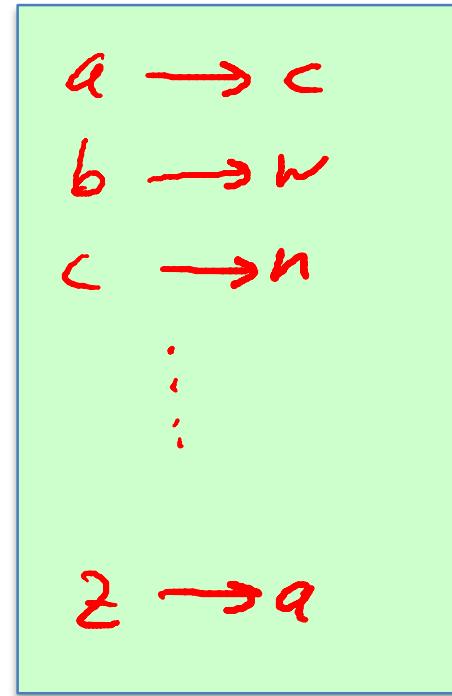
$$\text{Enc}(k, c) = c + k \bmod 26$$

$$\text{Dec}(k, c) = c - k \bmod 26$$



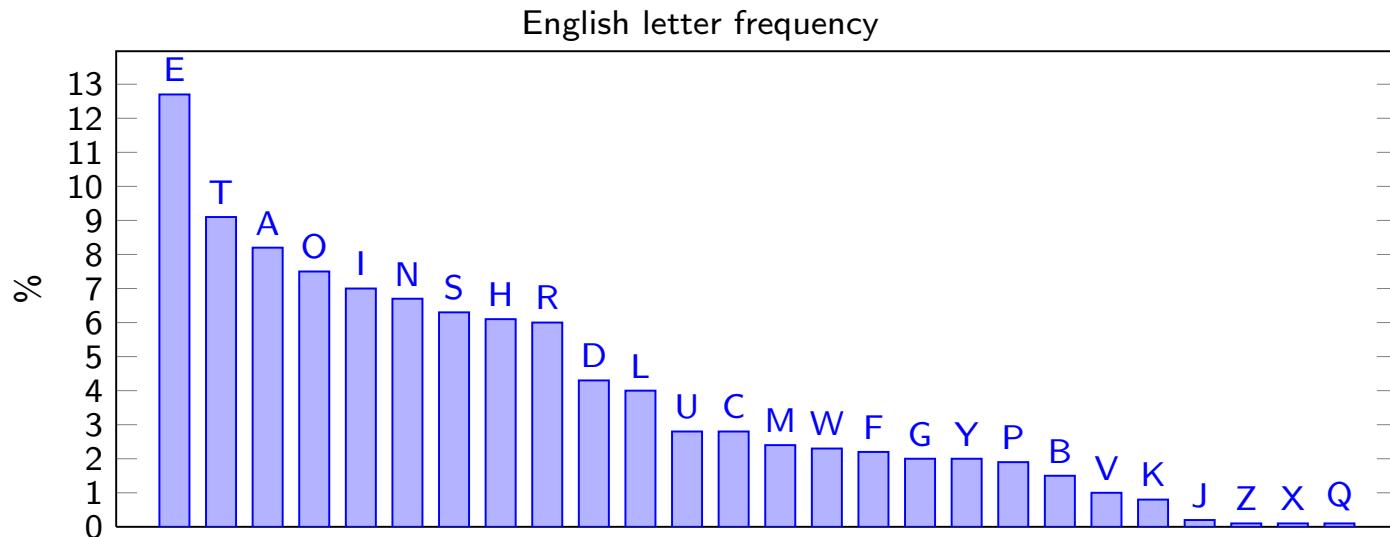
Substitution cipher

$k :=$



Key size: $26! \approx 2^{88}$

Statistical properties of plain text



The most common letters in English:

E, T, A, O, I, N, S, H, R, D, L, U, C, M, W, F, G, Y, P, B, V, K, J, ...

The most common digrams in English:

TH, HE, IN, ER, AN, RE, ED, ON, ES, ST, EN, AT, TO, ...

The most common trigrams in English:

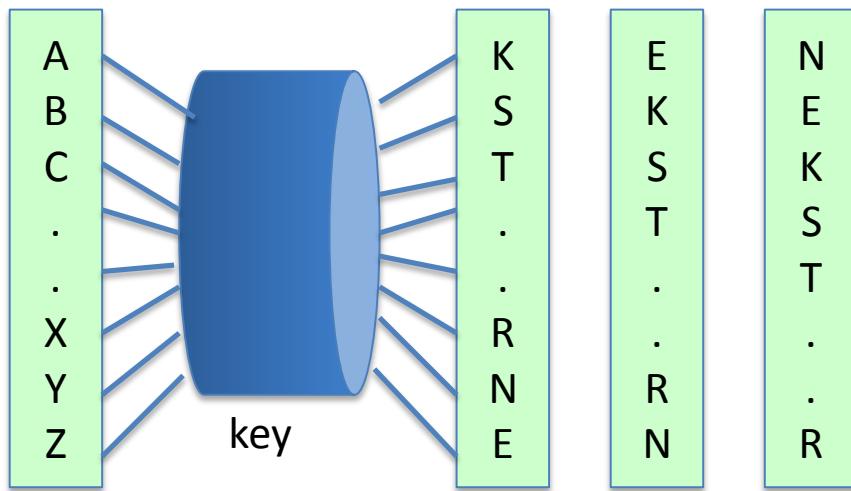
THE, ING, AND, HER, ERE, ENT, THA, NTH, WAS, ETH, ...

English text is highly redundant: very roughly 1 bit/letter entropy.

Monoalphabetic substitution ciphers allow simple ciphertext-only attacks based on digram or trigram statistics (for messages of at least few hundred characters).

Rotor machines

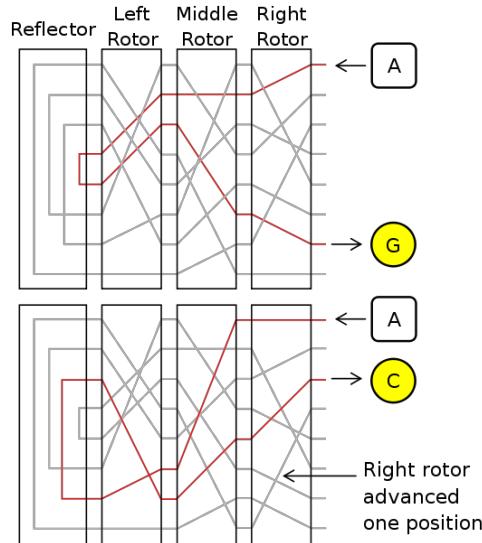
Early example: the Hebern machine (single rotor)



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Rotor machines

Most famous: the Enigma (3-5 rotors)



$$\# \text{ keys} = 26^4 = 2^{18} \quad (\text{actually } 2^{36} \text{ due to plugboard})$$

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Bletchley Park

- Britain's codebreaking centre during WW II
- Alan Turing's and other mathematicians break first versions of Enigma
- 'Bomb': computer specialised in breaking such encryptions
- 'Colossus': first semi-programmable electronic computer
- Great help against Germany and Japan - very important factor in determining the end of the war



<https://www.bletchleypark.org.uk/>

MODERN CRYPTOGRAPHY

What is a secure cipher ?

Possible security requirements:

attempt #1: **attacker cannot recover secret key**

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What is a secure cipher ?

Possible security requirements:

attempt #1: **attacker cannot recover secret key**

$$E(K, m) = m \quad \text{would be secure}$$

attempt #2: **attacker cannot recover all of plaintext**

$$E(K, m_0 || m_1) = m_0 || K \oplus m_1 \quad \text{would be secure}$$

What is a secure cipher ?

Possible security requirements:

attempt #1: **attacker cannot recover secret key**

$$E(K, m) = h \quad \text{would be secure}$$

attempt #2: **attacker cannot recover all of plaintext**

$$E(K, m_0 // m_1) = m_0 // K \oplus m_1 \quad \text{would be secure}$$

Shannon's idea:

CT should reveal no “info” about PT

Approach to modern cryptography

- Precisely specify threat model
- Propose a construction
- Prove that breaking construction under threat mode will solve an underlying hard problem

Symmetric vs asymmetric cryptography

- Symmetric cryptography
 - All parties have the same private key
 - E.g. stream ciphers (Salsa20), block ciphers (AES)
 - Used for bulk encryption
- Asymmetric cryptography
 - Uses private AND public keys
 - Users can publish their public keys without affecting security of private keys
 - Used for key exchange and authentication

STREAM CIPHERS

One time pad (Vernam, 1917)

$$C := E(K, m) = K \oplus m$$

$$D(K, c) = K \oplus c$$

msg: 0 1 1 0 1 1 1

key: 1 0 1 1 0 1 0

CT:



Lemma: OTP has perfect secrecy.

The bad news ...

Thm: perfect secrecy $\Rightarrow |K| \geq |M|$

i.e. perfect secrecy \Rightarrow key-len \geq msg-len

\Rightarrow hard to use in practice !!

Stream ciphers: making OTP practical

idea: replace “random” key by “pseudorandom” key

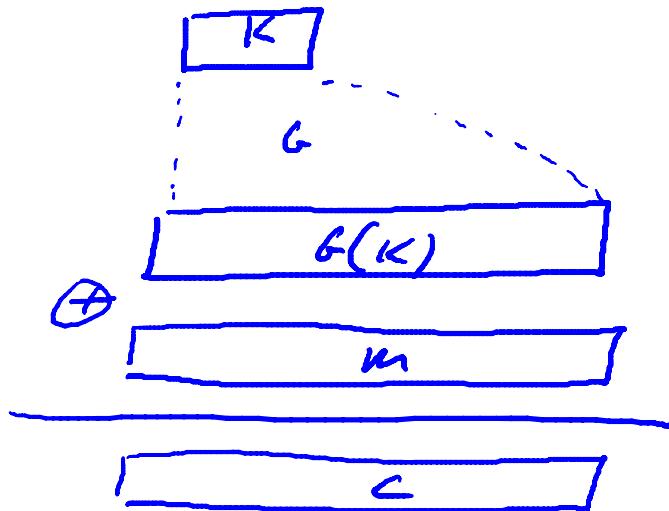
PRG is a function $g: \underbrace{\{0,1\}^s}_{\text{seed space}} \rightarrow \{0,1\}^n$ $n \gg s$

(eff. computable by a deterministic algorithm)

Stream ciphers: making OTP practical

$$c := E(k, m) = m \oplus g(k)$$

$$D(k, c) = c \oplus g(k)$$



Weak PRGs (do not use for crypto)

Lin. Cong. generator with parameters a, b, p :

```
r[i] ← a · r[i-1] + b mod p  
output bits of r[i]  
i++
```

seed = $r[0]$

glibc random():

```
r[i] ← ( r[i-3] + r[i-31] ) % 232  
output r[i] >> 1
```

never use random()
for crypto !!
(e.g. Kerberos V4)

Never use one-time pad (or generated stream) more than once

$$C_1 \leftarrow m_1 \oplus \text{PRG}(k)$$

$$C_2 \leftarrow m_2 \oplus \text{PRG}(k)$$

Eavesdropper does:

$$C_1 \oplus C_2 \rightarrow$$



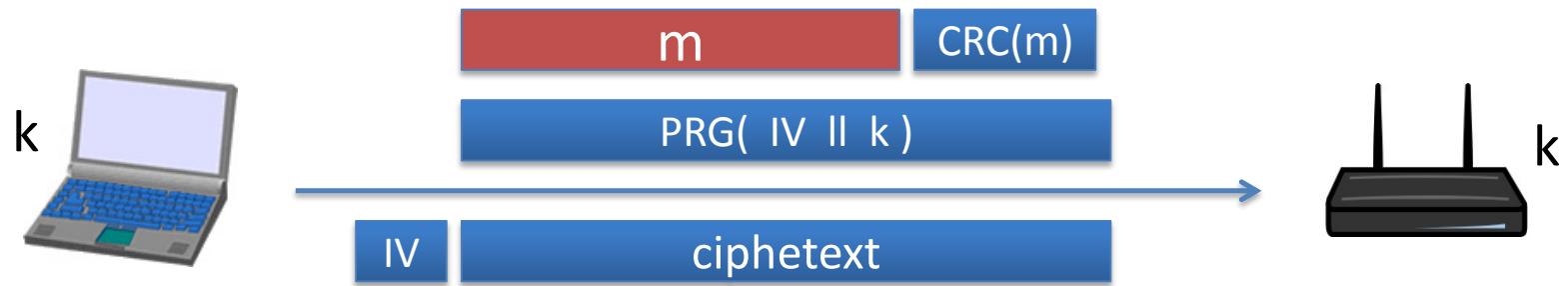
Enough redundancy in English and ASCII encoding that:

$$m_1 \oplus m_2 \rightarrow m_1, m_2$$

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Real world examples

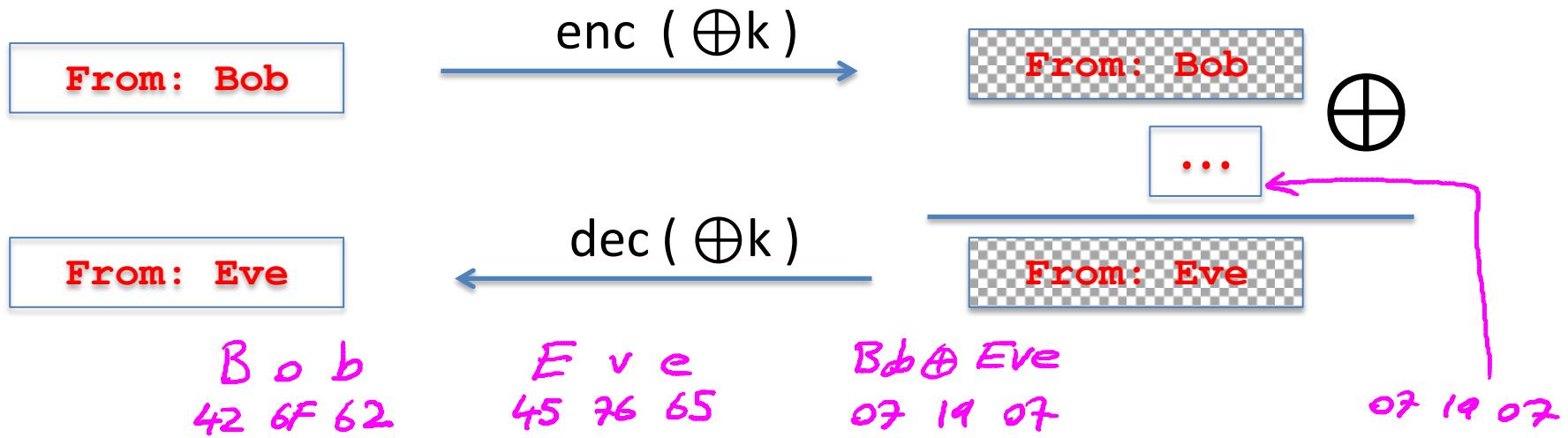
802.11b WEP:



Length of IV: 24 bits

- Repeated IV after $2^{24} \approx 16M$ frames
- On some 802.11 cards: IV resets to 0 after power cycle

Problem 2: OTP is malleable (no integrity)



Modifications to ciphertext are undetected and have predictable impact on plaintext

Modern stream ciphers: eStream

$$\text{PRG: } \underbrace{\{0,1\}^s}_{\text{seed}} \times R \rightarrow \{0,1\}^n$$

Nonce: a non-repeating value for a given key.

$$E(k, m ; r) = m \oplus \text{PRG}(k ; r)$$

The pair (k,r) is never used more than once.

Performance:

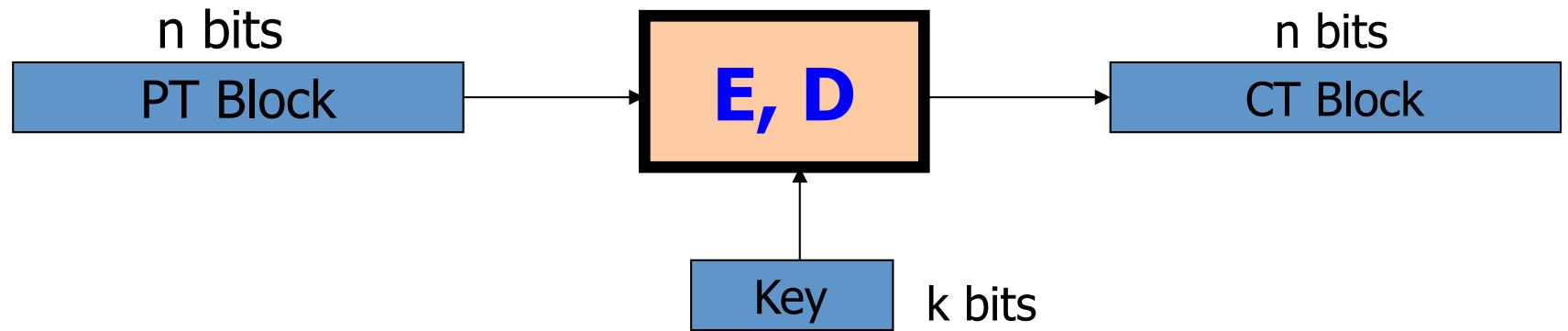
Crypto++ 5.6.0 [Wei Dai]

AMD Opteron, 2.2 GHz (Linux)

<u>PRG</u>	<u>Speed (MB/sec)</u>
RC4	126
eStream	Salsa20/12
	Sosemanuk

BLOCK CIPHERS

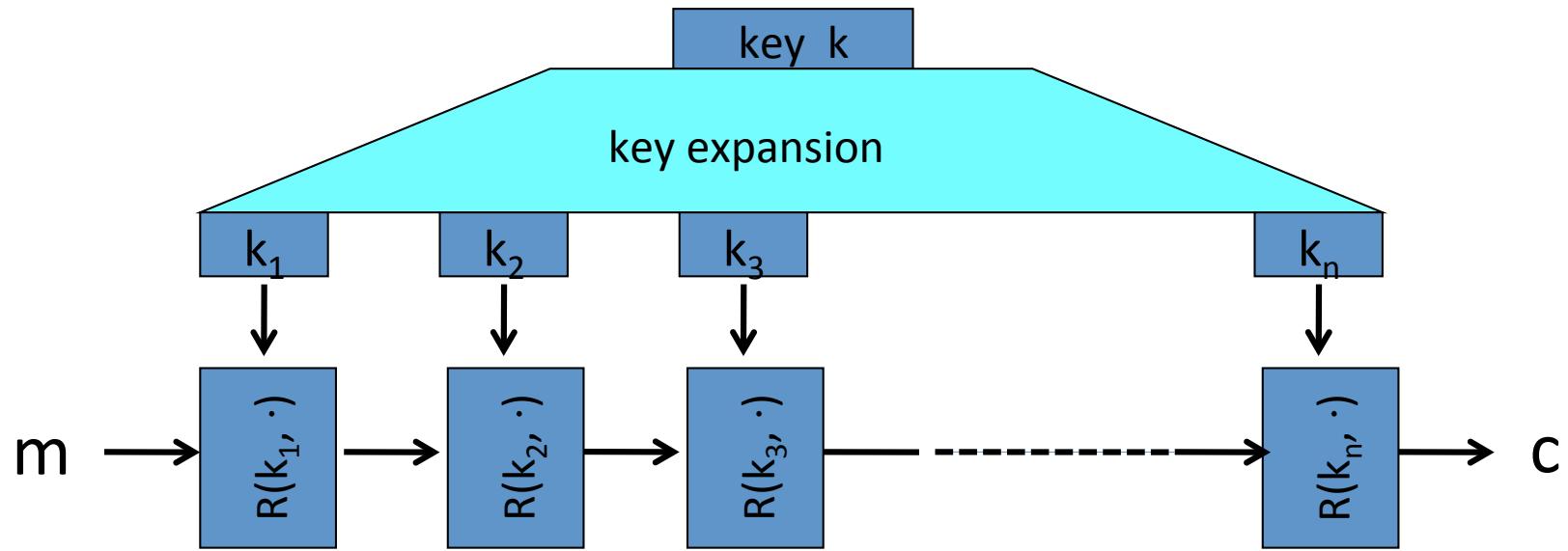
Block ciphers: crypto work horse



Canonical examples:

1. 3DES: n= 64 bits, k = 168 bits
2. AES: n=128 bits, k = 128, 192, 256 bits

Block Ciphers Built by Iteration



$R(k, m)$ is called a round function

for 3DES ($n=48$), for AES-128 ($n=10$)

DES history

- Early 1970s: Horst Feistel designs Lucifer at IBM
key-len = 128 bits ; block-len = 128 bits
- 1973: NBS asks for block cipher proposals.
IBM submits variant of Lucifer.
- 1976: NBS adopts DES as a federal standard
key-len = 56 bits ; block-len = 64 bits
- 1997: DES broken by exhaustive search
- 2000: NIST adopts Rijndael as AES to replace DES

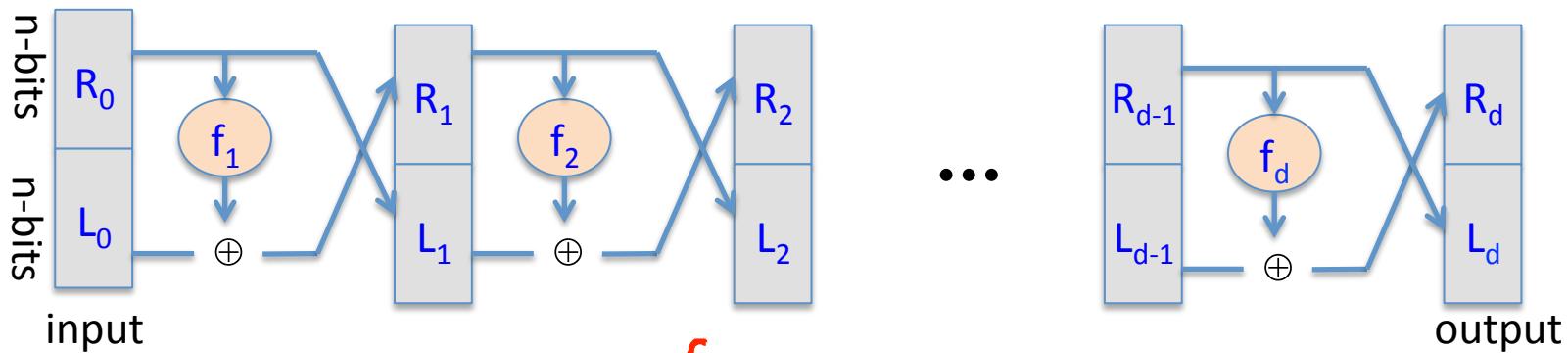
Widely deployed in banking (ACH) and commerce

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DES: core idea – Feistel Network

Given functions $f_1, \dots, f_d: \{0,1\}^n \rightarrow \{0,1\}^n$

Goal: build invertible function $F: \{0,1\}^{2n} \rightarrow \{0,1\}^{2n}$



In symbols:

$$\begin{cases} R_i = f_i(R_{i-1}) \oplus L_{i-1}, \\ L_i = R_{i-1}, \end{cases}$$

The S-boxes

$$S_i: \{0,1\}^6 \rightarrow \{0,1\}^4$$

S₅		Middle 4 bits of input																	
		0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111		
Outer bits	00	0010	1100	0100	0001	0111	1010	1011	0110	1000	0101	0011	1111	1101	0000	1110	1001		
	01	1110	1011	0010	1100	0100	0111	1101	0001	0101	0000	1111	1010	0011	1001	1000	0110		
	10	0100	0010	0001	1011	1010	1101	0111	1000	1111	1001	1100	0101	0110	0011	0000	1110		
	11	1011	1000	1100	0111	0001	1110	0010	1101	0110	1111	0000	1001	1010	0100	0101	0011		

DES challenge

msg = "The unknown messages is: XXXX ... "

CT = $c_1 \quad c_2 \quad c_3 \quad c_4$

Goal: find $k \in \{0,1\}^{56}$ s.t. $\text{DES}(k, m_i) = c_i$ for $i=1,2,3$

1997: Internet search -- **3 months**

1998: EFF machine (deep crack) -- **3 days** (250K \$)

1999: combined search -- **22 hours**

2006: COPACOBANA (120 FPGAs) -- **7 days** (10K \$)

⇒ 56-bit ciphers should not be used !! (128-bit key ⇒ 2^{72} days)

Strengthening DES against ex. search

Method 1: Triple-DES

- Let $E : K \times M \rightarrow M$ be a block cipher
- Define $3E: K^3 \times M \rightarrow M$ as

$$3E((k_1, k_2, k_3), m) = E(k_1, D(k_2, E(k_3, m)))$$
$$k_1 = k_2 = k_3 \Rightarrow \text{single DES}$$

For 3DES: key-size = $3 \times 56 = 168$ bits. 3×slower than DES.

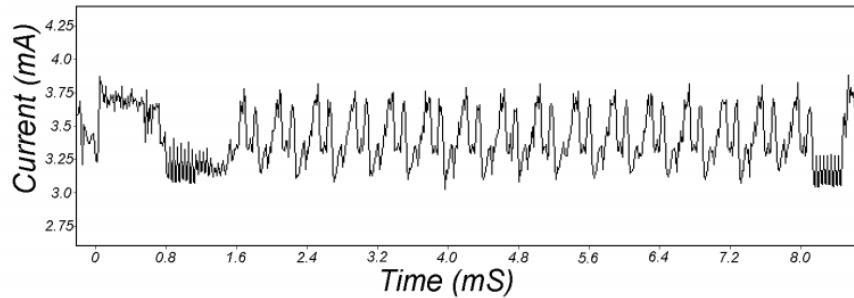
(simple attack in time $\approx 2^{118}$)

Attacks on the implementation

1. Side channel attacks:

We'll talk in detail in ~2 lectures

- Measure **time** to do enc/dec, measure **power** for enc/dec



[Kocher, Jaffe, Jun, 1998]

2. Fault attacks:

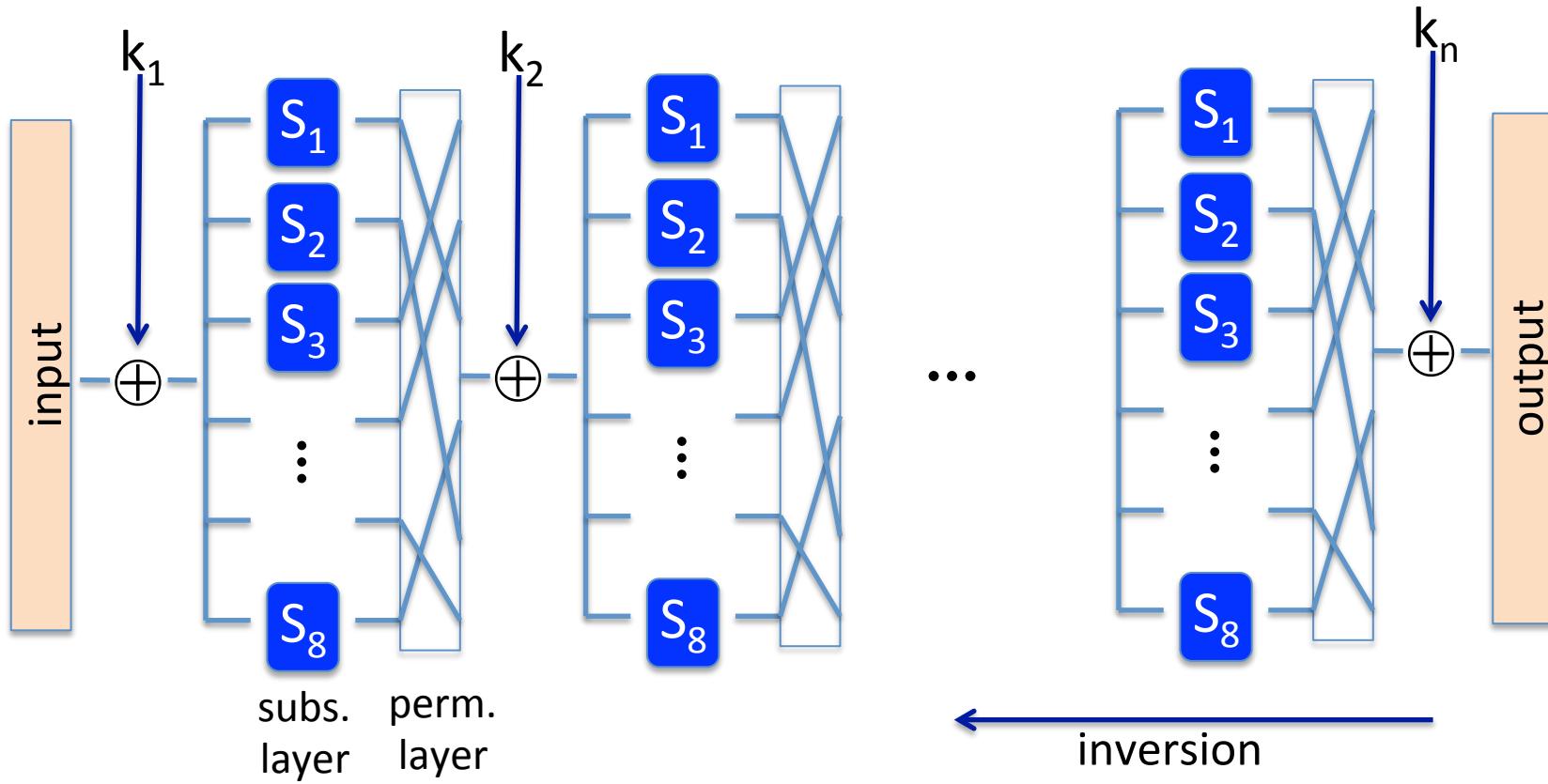
- Computing errors in the last round expose the secret key k

AES history

- 1997: NIST publishes request for proposal
- 1998: 15 submissions. Five claimed attacks.
- 1999: NIST chooses 5 finalists (Rijndael, Serpent, Twofish, RC6, MARS)
- 2000: NIST chooses Rijndael as AES (designed in Belgium)

Key sizes: 128, 192, 256 bits. Block size: 128 bits

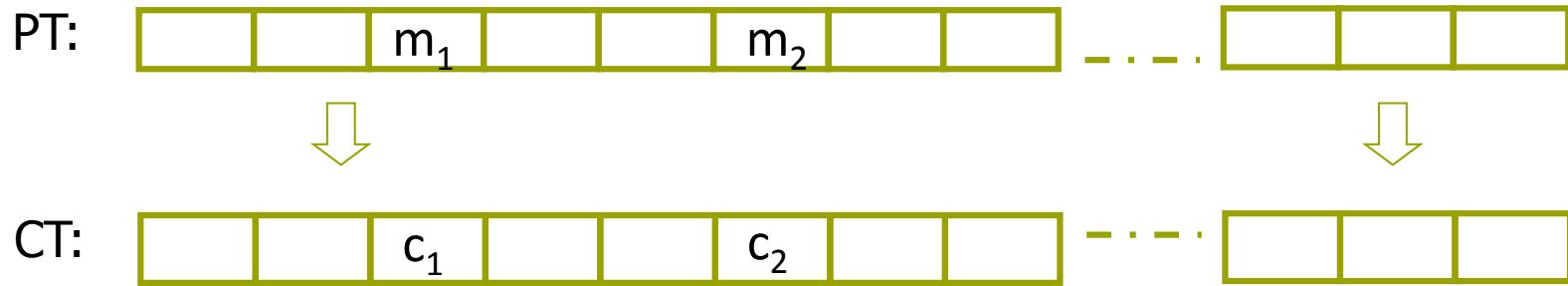
AES is a Subs-Perm network (not Feistel)



MODES OF OPERATION

ECB: do NOT use

Electronic Code Book (ECB):



Problem:

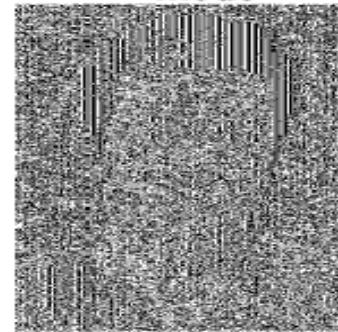
- if $m_1 = m_2$ then $c_1 = c_2$

In pictures

An example plaintext



Encrypted with AES in ECB mode



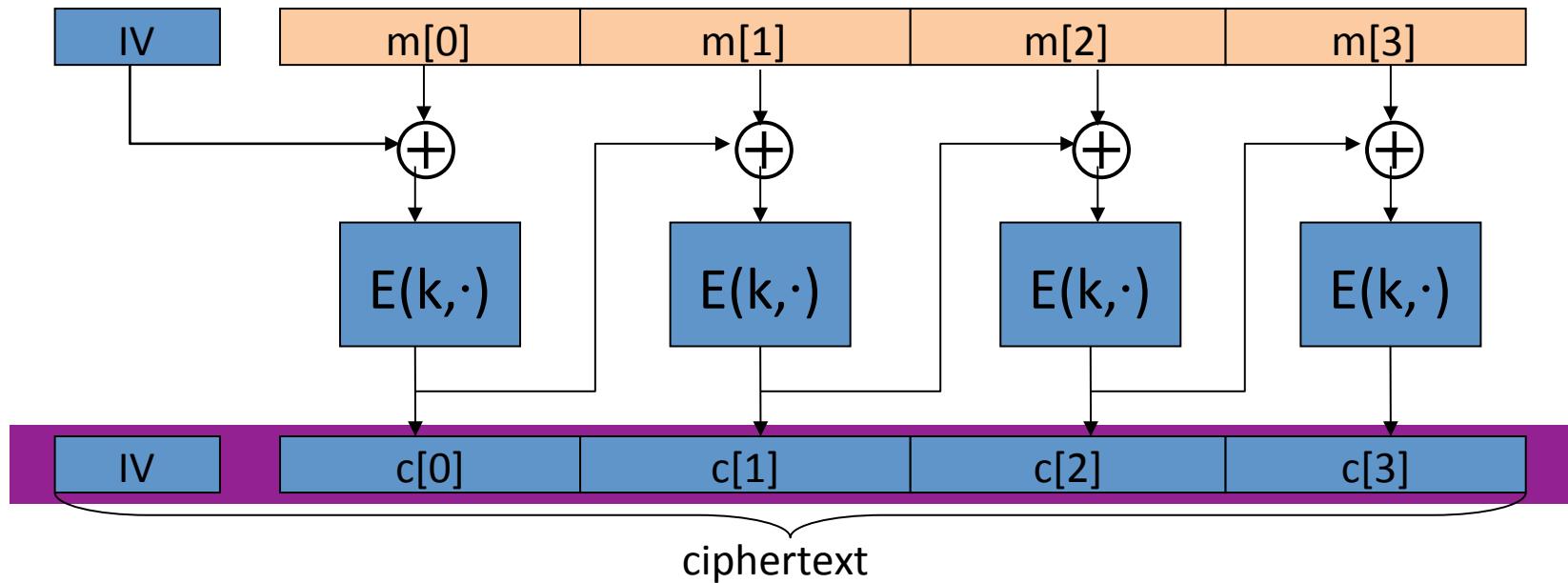
(courtesy B. Preneel)

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Good construction 1: CBC

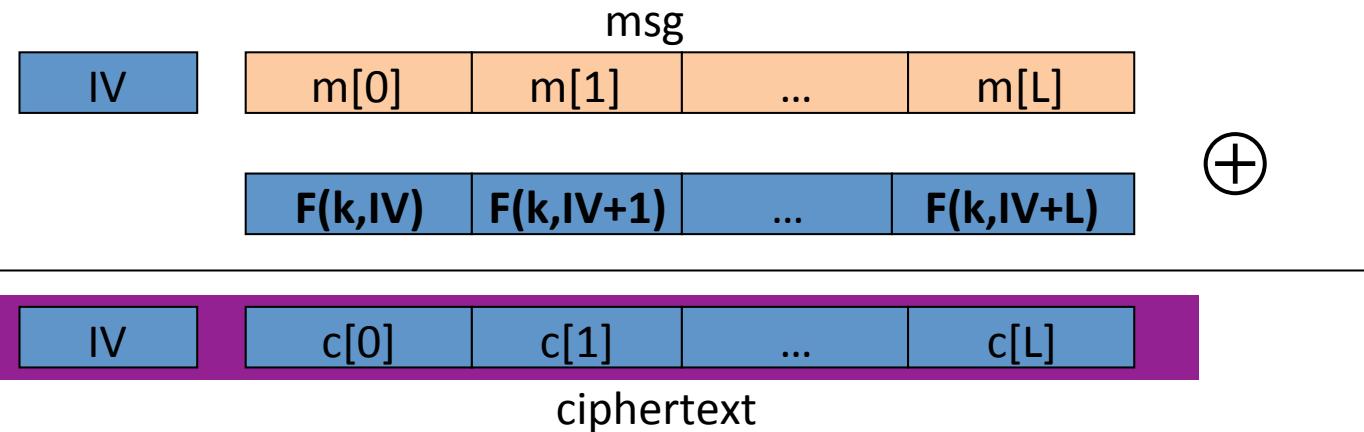
$E: \mathcal{D} \times \{0,1\}^n \rightarrow \{0,1\}^n$

$IV \in \{0,1\}^n$



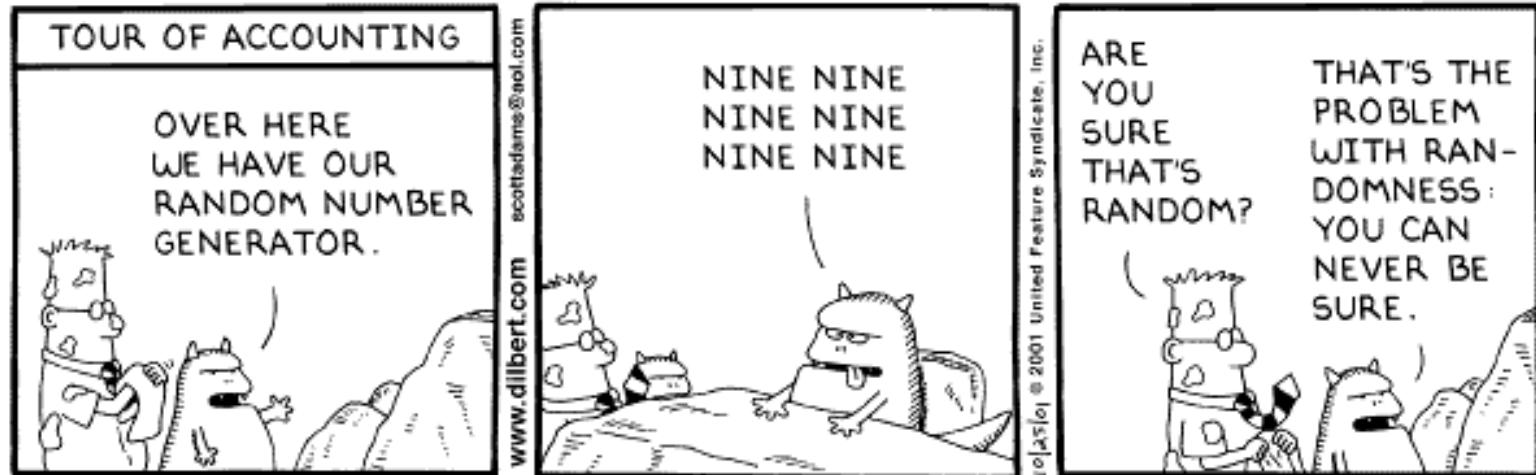
Good construction 2: CTR

$E(k,m)$: choose a random $\text{IV} \in \{0,1\}^n$ and do:



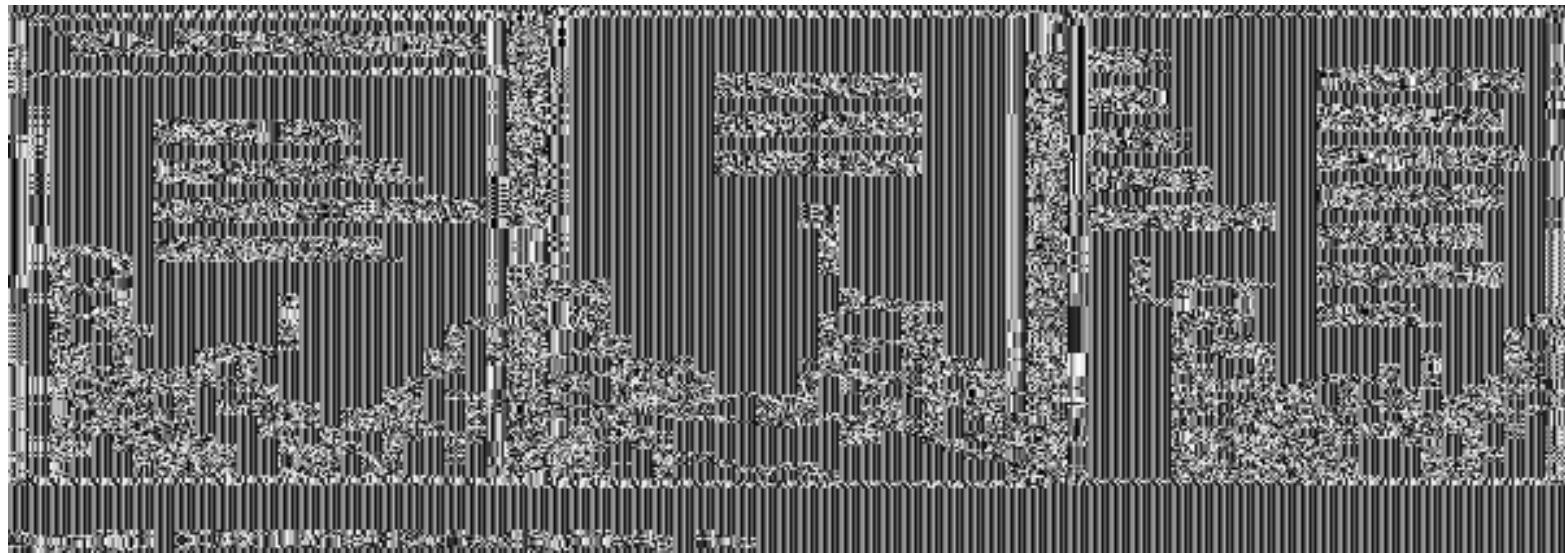
note: parallelizable (unlike CBC)

Plain-text bitmap:

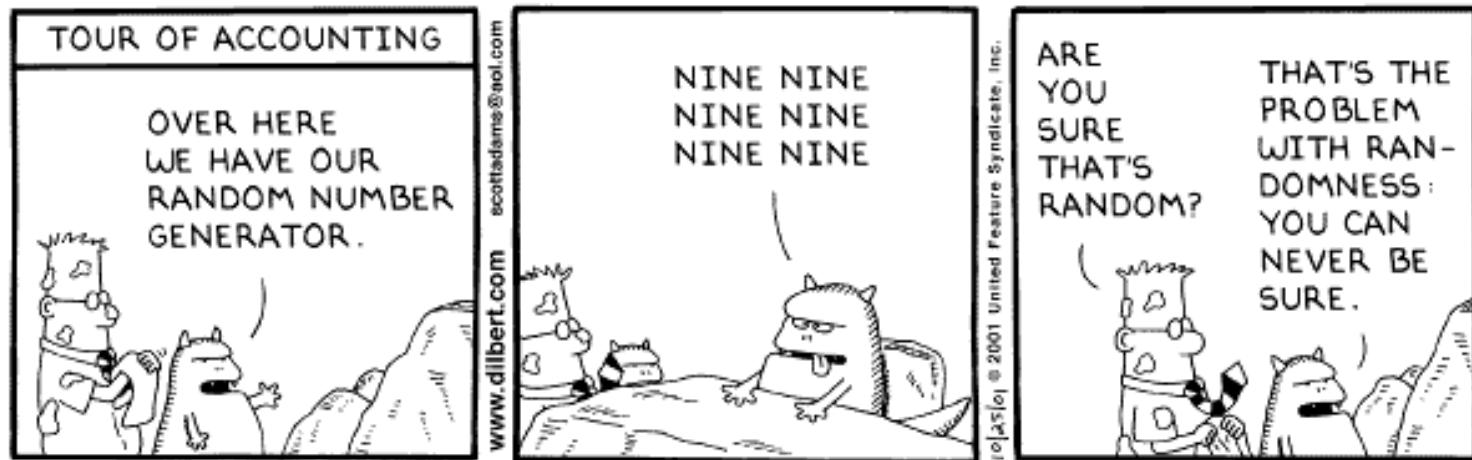


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DES-ECB encrypted:

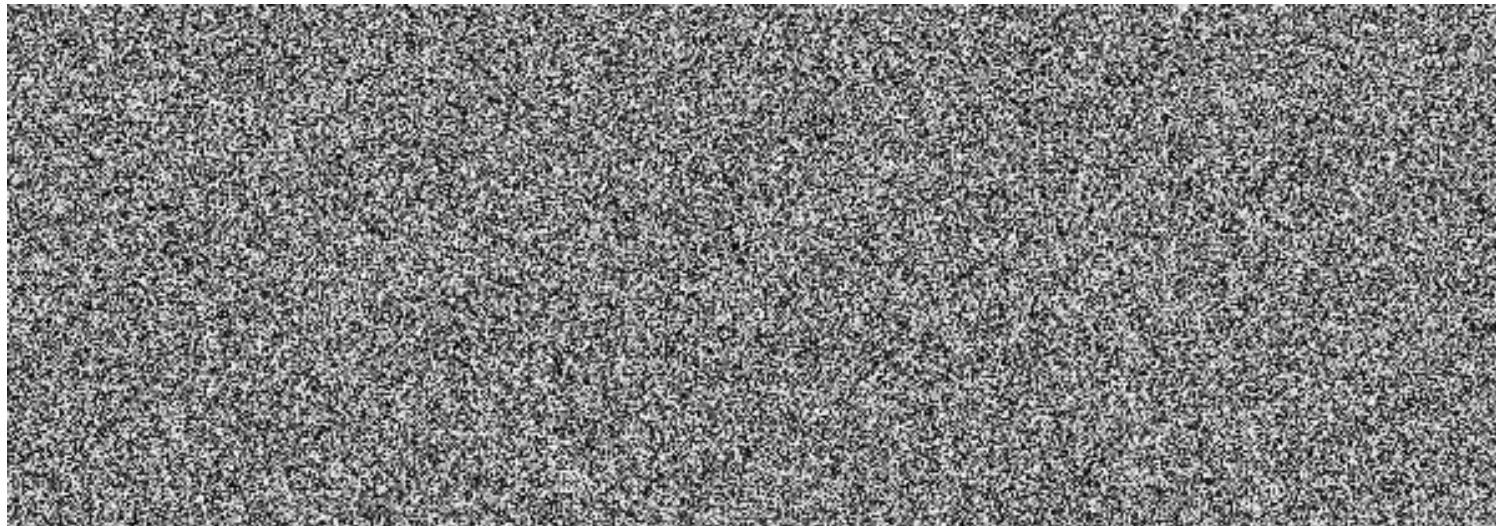


Plain-text bitmap:



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DES-CBC encrypted:



Resurse utile

- Cursul de la Stanford (Dan Boneh):
<https://www.coursera.org/learn/crypto/>
- Jonathan Katz, Yehuda Lindell:
Introduction to Modern Cryptography
CRC Press — 2008 (1st Ed.), 2015 (2nd Ed.)

Resurse utile

- Conferinte
 - IEEE Security & Privacy
 - Usenix Security
 - Crypto, Eurocrypt, Asiacrypt
 - ACM CCS, ESORICS
 - CHES, CARDIS
- <https://eprint.iacr.org/>
- <https://scholar.google.co.uk/>