Aircraft Equations of Motion: Flight Path Computation

Robert Stengel, Aircraft Flight Dynamics, MAE 331, 2016

Learning Objectives

- How is a rotating reference frame described in an inertial reference frame?
- Is the transformation singular?
- Euler Angles vs. quaternions
- What adjustments must be made to expressions for forces and moments in a non-inertial frame?
- How are the 6-DOF equations implemented in a computer?
- Aerodynamic damping effects

Reading:
Flight Dynamics
161-180

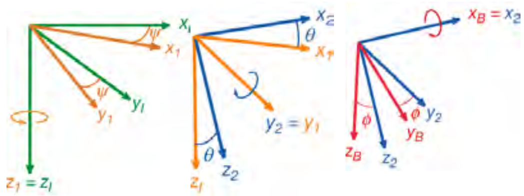
Assignment #5
due: November 11, 2016

- *Takeoff from Princeton Airport, fly over Princeton and Lake Carnegie, and land at Princeton Airport*
- *"HotSeat" cockpit simulation of the Cessna 172*
- *3- and 4-member teams; each member successfully flies the circuit*
- *Individual flight testing reports*

Review Questions

- What are the differences between NTV, NTI, LTV, and LTI ordinary differential equations?
- What good is a rotation matrix?
- What is a "3-2-1" rotation sequence?
- What are the pros and cons of Euler angles for representing rotational attitude(i.e., position)?
- What is a "cross-product-equivalent" matrix?
- How is the "inertia matrix" defined?
- What is the difficulty in transforming from an inertial to a body frame of reference?
- How did increasing engine power affect the stability and Control of World War II airplanes?

Euler Angle Rates



Euler-Angle Rates and Body-Axis Rates

Body-axis angular rate vector (orthogonal)

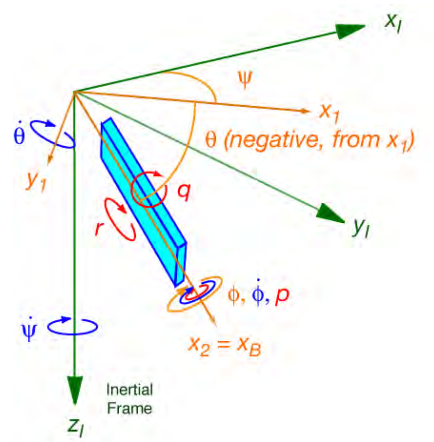
$$\boldsymbol{\omega}_B = \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}_B = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

Euler angles form a non-orthogonal vector

$$\boldsymbol{\Theta} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}$$

Euler-angle rate vector is not orthogonal

$$\dot{\boldsymbol{\Theta}} = \begin{bmatrix} \dot\phi \\ \dot\theta \\ \dot\psi \end{bmatrix} \neq \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}_I$$



Relationship Between Euler-Angle Rates and Body-Axis Rates

- $\dot\psi$ is measured in the Inertial Frame
- $\dot\theta$ is measured in Intermediate Frame #1
- $\dot\phi$ is measured in Intermediate Frame #2
- **... which is**

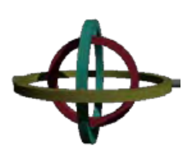
$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \mathbf{I}_3 \begin{bmatrix} \dot\phi \\ 0 \\ 0 \end{bmatrix} + \mathbf{H}_2^B \begin{bmatrix} 0 \\ \dot\theta \\ 0 \end{bmatrix} + \mathbf{H}_2^B \mathbf{H}_1^2 \begin{bmatrix} 0 \\ 0 \\ \dot\psi \end{bmatrix}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} \dot\phi \\ \dot\theta \\ \dot\psi \end{bmatrix} = \mathbf{L}_I^B \dot{\boldsymbol{\Theta}}$$

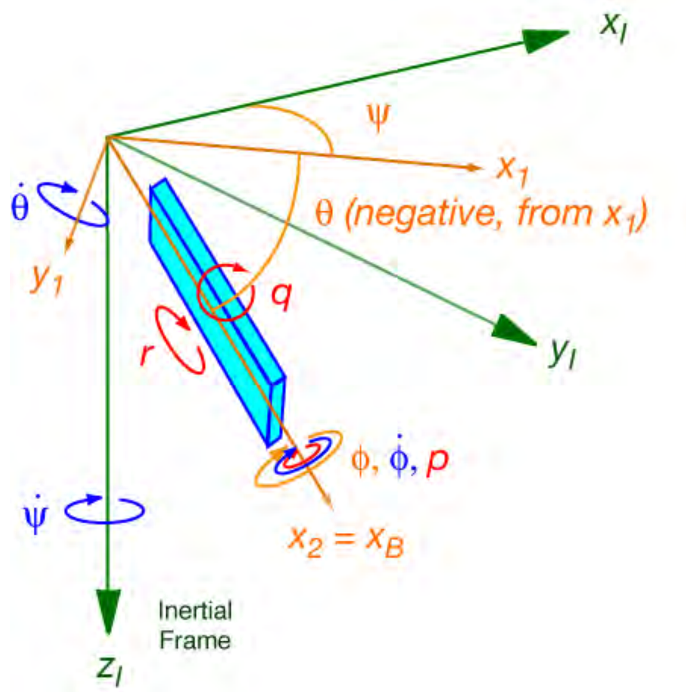
Can the inversion become singular?
What does this mean?

Inverse transformation $[(.)^{-1} \neq (.)^T]$

$$\begin{bmatrix} \dot\phi \\ \dot\theta \\ \dot\psi \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \mathbf{L}_B^I \boldsymbol{\omega}_B$$



Euler-Angle Rates and Body-Axis Rates



Avoiding the Euler Angle Singularity at $\theta = \pm 90°$

- **Alternatives to Euler angles**
  - Direction cosine (rotation) matrix
  - Quaternions

Propagation of direction cosine matrix (9 parameters)

$$\dot{\mathbf{H}}_B^I \mathbf{h}_B = \tilde{\boldsymbol{\omega}}_I \mathbf{H}_B^I \mathbf{h}_B$$

Consequently

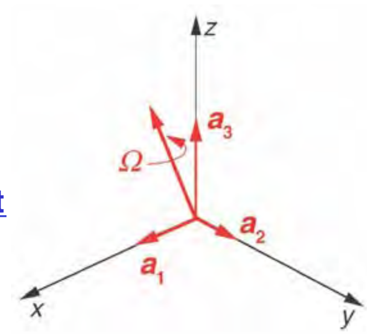
$$\dot{\mathbf{H}}_I^B(t) = -\tilde{\boldsymbol{\omega}}_B(t)\mathbf{H}_I^B(t) = -\begin{bmatrix} 0 & -r(t) & q(t) \\ r(t) & 0 & -p(t) \\ -q(t) & p(t) & 0(t) \end{bmatrix}_B \mathbf{H}_I^B(t)$$

$$\mathbf{H}_I^B(0) = \mathbf{H}_I^B(\phi_0, \theta_0, \psi_0)$$

Avoiding the Euler Angle Singularity at $\theta = \pm 90°$

Propagation of quaternion vector: single rotation from inertial to body frame (4 parameters)

- Rotation from one axis system, I, to another, B, represented by
  - Orientation of axis vector about which the rotation occurs (3 parameters of a unit vector, a_1, a_2, and a_3)
  - Magnitude of the rotation angle, Ω, rad

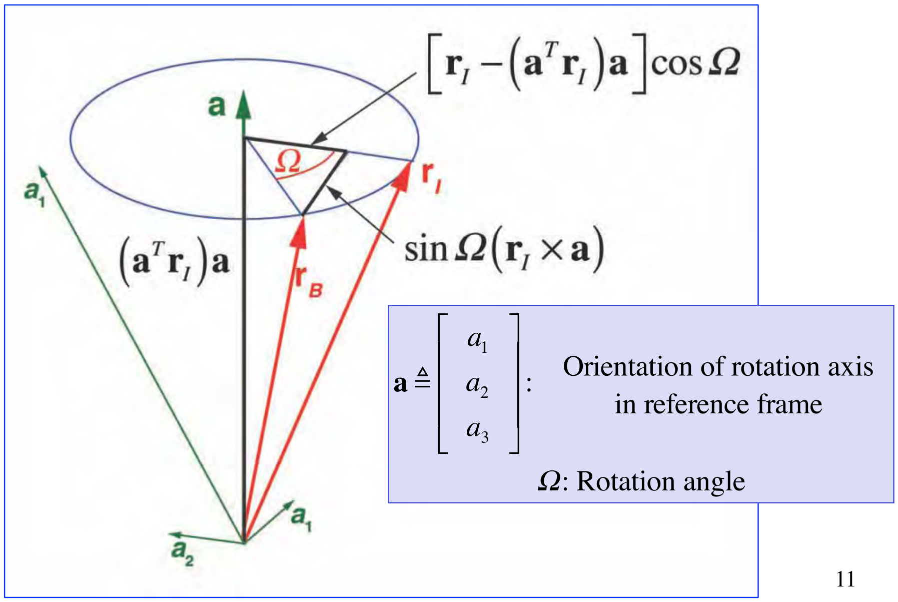


Checklist

- ❑ Are the components of the Euler Angle rate vector orthogonal to each other?
- ❑ Is the inverse of the transformation from Euler Angle rates to body-axis rates the transpose of the matrix?
- ❑ What complication does the inverse transformation introduce?

Euler Rotation of a Vector

Rotation of a vector to an arbitrary new orientation can be expressed as a single rotation about an axis at the vector's base



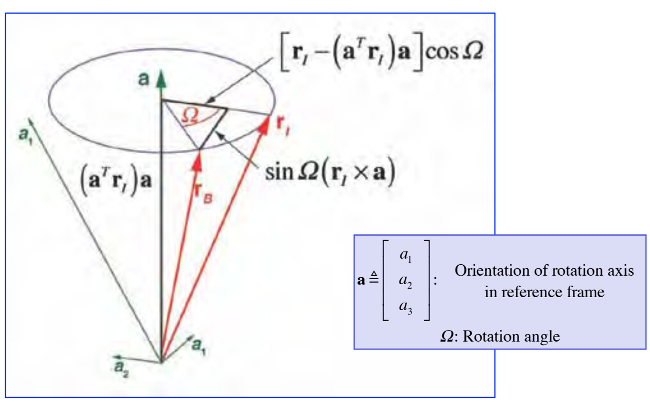
Euler's Rotation Theorem

Vector transformation involves 3 components

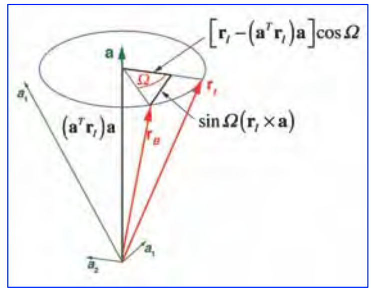
$$\mathbf{r}_B = \mathbf{H}_I^B \mathbf{r}_I$$

$$= \left(\mathbf{a}^T\mathbf{r}_I\right)\mathbf{a} + \left[\mathbf{r}_I - \left(\mathbf{a}^T\mathbf{r}_I\right)\mathbf{a}\right]\cos\Omega + \sin\Omega\left(\mathbf{r}_I \times \mathbf{a}\right)$$

$$= \cos\Omega\,\mathbf{r}_I + \left(1 - \cos\Omega\right)\left(\mathbf{a}^T\mathbf{r}_I\right)\mathbf{a} - \sin\Omega\left(\mathbf{a} \times \mathbf{r}_I\right)$$



Rotation Matrix Derived from Euler's Formula



$$\mathbf{r}_B = \mathbf{H}_I^B \mathbf{r}_I = \cos\Omega\, \mathbf{r}_I + \left(1-\cos\Omega\right)\left(\mathbf{a}^T\mathbf{r}_I\right)\mathbf{a} - \sin\Omega\left(\tilde{\mathbf{a}}\mathbf{r}_I\right)$$

Identity

$$\left(\mathbf{a}^T\mathbf{r}_I\right)\mathbf{a} = \left(\mathbf{a}\mathbf{a}^T\right)\mathbf{r}_I$$

Rotation matrix

$$\mathbf{H}_I^B = \cos\Omega\, \mathbf{I}_3 + \left(1-\cos\Omega\right)\mathbf{a}\mathbf{a}^T - \sin\Omega\, \tilde{\mathbf{a}}$$

Quaternion Derived from Euler Rotation Angle and Orientation

Quaternion vector

4 parameters based on Euler's formula

$$\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \triangleq \begin{bmatrix} \mathbf{q}_3 \\ \hdashline q_4 \end{bmatrix} = \begin{bmatrix} \sin(\Omega/2)\mathbf{a} \\ \hdashline \cos(\Omega/2) \end{bmatrix} = \begin{bmatrix} \sin(\Omega/2)\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \\ \hdashline \cos(\Omega/2) \end{bmatrix} \qquad (4\times 1)$$

4-parameter representation of 3 parameters; hence, a constraint must be satisfied

$$\begin{aligned} &\mathbf{q}^T\mathbf{q} = q_1^{\,2} + q_2^{\,2} + q_3^{\,2} + q_4^{\,2} \\ &= \sin^2\left(\Omega/2\right) + \cos^2\left(\Omega/2\right) = 1 \end{aligned}$$

Rotation Matrix Expressed with Quaternion

From Euler's formula

$$\boxed{\mathbf{H}_I^B = \left[q_4{}^2 - \left(\mathbf{q}_3{}^T\mathbf{q}_3\right)\right]\mathbf{I}_3 + 2\mathbf{q}_3\mathbf{q}_3{}^T - 2q_4\tilde{\mathbf{q}}_3}$$

Rotation matrix from quaternion

$$\mathbf{H}_I^B =$$

$$\begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2\left(q_1q_2 + q_3q_4\right) & 2\left(q_1q_3 - q_2q_4\right) \\ 2\left(q_1q_2 - q_3q_4\right) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2\left(q_2q_3 + q_1q_4\right) \\ 2\left(q_1q_3 + q_2q_4\right) & 2\left(q_2q_3 - q_1q_4\right) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix}$$

Checklist

- ❑ What is an "Euler Rotation"?
- ❑ Why would we use a quaternion vector to express angular attitude instead of an Euler Angle vector?
- ❑ How many components does a quaternion vector have?

Quaternion Expressed from Elements of Rotation Matrix

Initialize q(0) from Direction Cosine Matrix or Euler Angles

$$\mathbf{H}_I^B(0) = \begin{bmatrix} h_{11}\left(=\cos\delta_{11}\right) & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} = \mathbf{H}_I^B\left(\phi_0, \theta_0, \psi_0\right)$$

$$q_4(0) = \frac{1}{2}\sqrt{1 + h_{11}(0) + h_{22}(0) + h_{33}(0)}$$

Assuming that $q_4 \neq 0$

$$\mathbf{q}_3(0) \triangleq \begin{bmatrix} q_1(0) \\ q_2(0) \\ q_3(0) \end{bmatrix} = \frac{1}{4q_4(0)} \begin{bmatrix} \left[h_{23}(0) - h_{32}(0)\right] \\ \left[h_{31}(0) - h_{13}(0)\right] \\ \left[h_{12}(0) - h_{21}(0)\right] \end{bmatrix}$$

Quaternion Vector Kinematics

$$\dot{\mathbf{q}} = \frac{d}{dt}\begin{bmatrix} \mathbf{q}_3 \\ q_4 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} q_4 \boldsymbol{\omega}_B - \tilde{\boldsymbol{\omega}}_B \mathbf{q}_3 \\ \hdashline -\boldsymbol{\omega}_B{}^T \mathbf{q}_3 \end{bmatrix} \qquad (4 \times 1)$$

Differential equation is linear in either q or $\boldsymbol{\omega}_B$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix}_B \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix}$$

Propagate Quaternion Vector Using Body-Axis Angular Rates

$$\begin{bmatrix} p(t) & q(t) & r(t) \end{bmatrix} \triangleq \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}$$

$$\frac{d\mathbf{q}(t)}{dt} = \begin{bmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \\ \dot{q}_3(t) \\ \dot{q}_4(t) \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 0 & r(t) & -q(t) & p(t) \\ -r(t) & 0 & p(t) & q(t) \\ q(t) & -p(t) & 0 & r(t) \\ -p(t) & -q(t) & -r(t) & 0 \end{bmatrix}_B \begin{bmatrix} q_1(t) \\ q_2(t) \\ q_3(t) \\ q_4(t) \end{bmatrix}$$

Digital integration to compute $\mathbf{q}(t_k)$

$$\mathbf{q}_{\text{int}}(t_k) = \mathbf{q}(t_{k-1}) + \int_{t_{k-1}}^{t_k} \frac{d\mathbf{q}(\tau)}{dt} d\tau$$

Euler Angles Derived from Quaternion

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} \operatorname{atan2}\left\{2(q_1 q_4 + q_2 q_3), \left[1 - 2(q_1^2 + q_2^2)\right]\right\} \\ \sin^{-1}\left[2(q_2 q_4 - q_1 q_3)\right] \\ \operatorname{atan2}\left\{2(q_3 q_4 + q_1 q_2), \left[1 - 2(q_2^2 + q_3^2)\right]\right\} \end{bmatrix}$$

- **atan2: generalized arctangent *algorithm,* 2 arguments**
  - **returns angle in proper quadrant**
  - **avoids dividing by zero**
  - **has various definitions, e.g., (MATLAB)**

$$\operatorname{atan2}(y,x) = \begin{cases} \tan^{-1}\left(\frac{y}{x}\right) & \text{if } x > 0 \\ \pi + \tan^{-1}\left(\frac{y}{x}\right), -\pi + \tan^{-1}\left(\frac{y}{x}\right) & \text{if } x < 0 \text{ and } y \geq 0, < 0 \\ \pi/2\ , -\pi/2 & \text{if } x = 0 \text{ and } y > 0, < 0 \\ 0 & \text{if } x = 0 \text{ and } y = 0 \end{cases}$$

Checklist

- ❑ **Can propagation of quaternion become singular?**
- ❑ **Can quaternion replace Euler Angles to express angular orientation?**
- ❑ **Can we define one from the other?**

Rigid-Body Equations of Motion

Point-Mass Dynamics

- **Inertial rate of change of translational position**

$$\dot{\mathbf{r}}_I = \mathbf{v}_I = \mathbf{H}_B^I \mathbf{v}_B$$

$$\mathbf{v}_B = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

- **Body-axis rate of change of translational velocity**
  - **Identical to angular-momentum transformation**

$$\dot{\mathbf{v}}_I = \frac{1}{m}\mathbf{F}_I$$

$$\dot{\mathbf{v}}_B = \mathbf{H}_I^B \dot{\mathbf{v}}_I - \tilde{\boldsymbol{\omega}}_B \mathbf{v}_B = \frac{1}{m}\mathbf{H}_I^B \mathbf{F}_I - \tilde{\boldsymbol{\omega}}_B \mathbf{v}_B$$
$$= \frac{1}{m}\mathbf{F}_B - \tilde{\boldsymbol{\omega}}_B \mathbf{v}_B$$

$$\mathbf{F}_B = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_B = \begin{bmatrix} C_X \bar{q} S \\ C_Y \bar{q} S \\ C_Z \bar{q} S \end{bmatrix}$$

Rigid-Body Equations of Motion (Euler Angles)

- **Translational Position**

$$\mathbf{r}_I = \begin{bmatrix} x \\ y \\ z \end{bmatrix}_I$$

- **Rate of change of Translational Position**

$$\dot{\mathbf{r}}_I(t) = \mathbf{H}_B^I(t)\mathbf{v}_B(t)$$

- **Angular Position**

$$\boldsymbol{\Theta}_I = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}_I$$

- **Rate of change of Angular Position**

$$\dot{\boldsymbol{\Theta}}_I(t) = \mathbf{L}_B^I(t)\boldsymbol{\omega}_B(t)$$

- **Translational Velocity**

$$\mathbf{v}_B = \begin{bmatrix} u \\ v \\ w \end{bmatrix}_B$$

- **Rate of change of Translational Velocity**

$$\dot{\mathbf{v}}_B(t) = \frac{1}{m(t)}\mathbf{F}_B(t) + \mathbf{H}_I^B(t)\mathbf{g}_I - \tilde{\boldsymbol{\omega}}_B(t)\mathbf{v}_B(t)$$

- **Angular Velocity**

$$\boldsymbol{\omega}_B = \begin{bmatrix} p \\ q \\ r \end{bmatrix}_B$$

- **Rate of change of Angular Velocity**

$$\dot{\boldsymbol{\omega}}_B(t) = \mathbb{I}_B^{-1}(t)\left[\mathbf{M}_B(t) - \tilde{\boldsymbol{\omega}}_B(t)\mathbb{I}_B(t)\boldsymbol{\omega}_B(t)\right]$$

Aircraft Characteristics Expressed in Body Frame of Reference

Aerodynamic and thrust force

$$\mathbf{F}_B = \begin{bmatrix} X_{aero} + X_{thrust} \\ Y_{aero} + Y_{thrust} \\ Z_{aero} + Z_{thrust} \end{bmatrix}_B = \begin{bmatrix} C_{X_{aero}} + C_{X_{thrust}} \\ C_{Y_{aero}} + C_{Y_{thrust}} \\ C_{Z_{aero}} + C_{Z_{thrust}} \end{bmatrix}_B \frac{1}{2}\rho V^2 S = \begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix}_B \bar{q} S$$

Aerodynamic and thrust moment

$$\mathbf{M}_B = \begin{bmatrix} L_{aero} + L_{thrust} \\ M_{aero} + M_{thrust} \\ N_{aero} + N_{thrust} \end{bmatrix}_B = \begin{bmatrix} \left(C_{l_{aero}} + C_{l_{thrust}}\right) b \\ \left(C_{m_{aero}} + C_{m_{thrust}}\right) \bar{c} \\ \left(C_{n_{aero}} + C_{n_{thrust}}\right) b \end{bmatrix}_B \frac{1}{2}\rho V^2 S = \begin{bmatrix} C_l b \\ C_m \bar{c} \\ C_n b \end{bmatrix}_B \bar{q} S$$

Inertia matrix

$$\mathbb{I}_B = \begin{bmatrix} \mathbb{I}_{xx} & -\mathbb{I}_{xy} & -\mathbb{I}_{xz} \\ -\mathbb{I}_{xy} & \mathbb{I}_{yy} & -\mathbb{I}_{yz} \\ -\mathbb{I}_{xz} & -\mathbb{I}_{yz} & \mathbb{I}_{zz} \end{bmatrix}_B$$

Reference Lengths

b = wing span

$\bar{c}$ = mean aerodynamic chord

Rigid-Body Equations of Motion: Position

Rate of change of Translational Position

$$\dot{x}_I = (\cos\theta\cos\psi)u + (-\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi)v + (\sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi)w$$
$$\dot{y}_I = (\cos\theta\sin\psi)u + (\cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi)v + (-\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi)w$$
$$\dot{z}_I = (-\sin\theta)u + (\sin\phi\cos\theta)v + (\cos\phi\cos\theta)w$$

Rate of change of Angular Position

$$\dot{\phi} = p + (q\sin\phi + r\cos\phi)\tan\theta$$
$$\dot{\theta} = q\cos\phi - r\sin\phi$$
$$\dot{\psi} = (q\sin\phi + r\cos\phi)\sec\theta$$

Rigid-Body Equations of Motion: Rate

Rate of change of Translational Velocity

$$
\begin{aligned}
\dot{u} &= X/m - g\sin\theta + rv - qw \\
\dot{v} &= Y/m + g\sin\phi\cos\theta - ru + pw \\
\dot{w} &= Z/m + g\cos\phi\cos\theta + qu - pv
\end{aligned}
$$

Rate of change of Angular Velocity

$$
\begin{aligned}
\dot{p} &= \left(I_{zz}L + I_{xz}N - \left\{I_{xz}\left(I_{yy} - I_{xx} - I_{zz}\right)p + \left[I_{xz}^2 + I_{zz}\left(I_{zz} - I_{yy}\right)\right]r\right\}q\right) \Big/ \left(I_{xx}I_{zz} - I_{xz}^2\right) \\
\dot{q} &= \left[M - \left(I_{xx} - I_{zz}\right)pr - I_{xz}\left(p^2 - r^2\right)\right] \Big/ I_{yy} \\
\dot{r} &= \left(I_{xz}L + I_{xx}N - \left\{I_{xz}\left(I_{yy} - I_{xx} - I_{zz}\right)r + \left[I_{xz}^2 + I_{xx}\left(I_{xx} - I_{yy}\right)\right]p\right\}q\right) \Big/ \left(I_{xx}I_{zz} - I_{xz}^2\right)
\end{aligned}
$$

Mirror symmetry, $I_{xz} \neq 0$

Checklist

- ❑ Why is it inconvenient to solve momentum rate equations in an inertial reference frame?
- ❑ Are angular rate and momentum vectors aligned?
- ❑ How are angular rate equations transformed from an inertial to a body frame?
- ❑ After all the fuss about quaternions, why have we gone back to Euler Angles?

FLIGHT - Computer Program to Solve the 6-DOF Equations of Motion

FLIGHT - MATLAB Program

```
%   FLIGHT --  6-DOF Trim, Linear Model, and Flight Path Simulation
%   October 19, 2008
%   ===============================================================
%   Copyright 1993-2008 by ROBERT F. STENGEL.  All rights reserved.

    clear
    global GEAR CONTROL SPOIL u x V parhis

%   This is the SCRIPT FILE.  It contains the Main Program, which:
%       Defines initial conditions
%       Calculates longitudinal trim condition
%       Calculates stability-and-control derivatives
%       Simulates flight path using nonlinear equations of motion

%   Functions used by FLIGHT:
%       AeroModel.m     Aerodynamic coefficients of the aircraft, thrust mode
%                       and geometric and inertial properties
%       Atmos.m         Air density, sound speed
%       ControlSystem.m Control law
%       DCM.m           Direction-cosine matrix
%       EoM.m           Equations of motion for integration
%       LinModel.m      Equations of motion for linear model definition
%       TrimCost.m      Cost function for trim solution
%       WindField.m     Wind velocity components

%   DEFINITION OF THE STATE VECTOR
%       x(1) =      Body-axis x inertial velocity, ub, m/s
%       x(2) =      Body-axis y inertial velocity, vb, m/s
%       x(3) =      Body-axis z inertial velocity, wb, m/s
%       x(4) =      North position of center of mass WRT Earth, xe, m
%       x(5) =      East position of center of mass WRT Earth, ye, m
%       x(6) =      Negative of c.m. altitude WRT Earth, ze = -h, m
%       x(7) =      Body-axis roll rate, pr, rad/s
%       x(8) =      Body-axis pitch rate, qr, rad/s
%       x(9) =      Body-axis yaw rate, rr,rad/s
%       x(10) =     Roll angle of body WRT Earth, phir, rad
%       x(11) =     Pitch angle of body WRT Earth, thetar, rad
%       x(12) =     Yaw angle of body WRT Earth, psir, rad
```

http://www.princeton.edu/~stengel/FlightDynamics.html

FLIGHT - MATLAB Program

```
%    DEFINITION OF THE CONTROL VECTOR
%        u(1) =      Elevator, dEr, rad
%        u(2) =      Aileron, dAr, rad
%        u(3) =      Rudder, dRr, rad
%        u(4) =      Throttle, dT, %
%        u(5) =      Asymmetric Spoiler, dASr, rad
%        u(6) =      Flap, dFr, rad
%        u(7) =      Stabilator, dSr, rad

%    BEGINNING of MAIN PROGRAM
%    =========================

     'FLIGHT'
     date

%    FLIGHT Flags (1 = ON, 0 = OFF)
     TRIM =       1;       % Trim flag (= 1 to calculate trim)
     LINEAR =     1;       % Linear model flag (= 1 to calculate F and G)
     SIMUL =      1;       % Flight path flag (= 1 for nonlinear simulation)
     GEAR =       0;       % Landing gear DOWN (= 1) or UP (= 0)
     SPOIL =      0;       % Symmetric Spoiler DEPLOYED (= 1) or CLOSED (= 0)
     CONTROL =    0;       % Feedback control ON (= 1) or OFF (= 0)
     dF =         0;       % Flap setting, deg
```

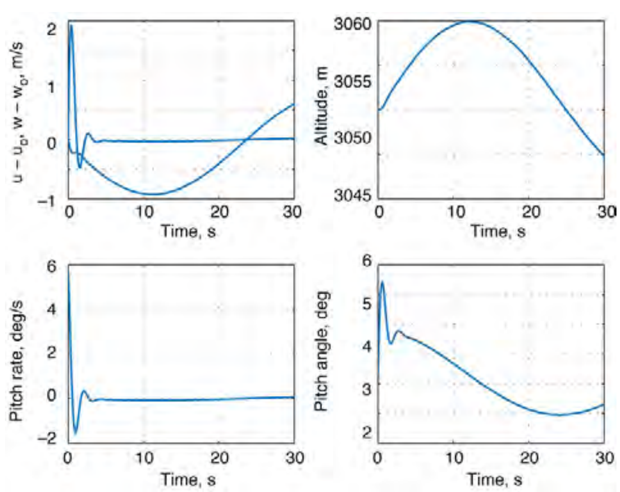
http://www.princeton.edu/~stengel/FlightDynamics.html

FLIGHT, Version 2 *(FLIGHTver2.m)*

- **Provides option for calculating rotations with *quaternions* rather than *Euler angles***
- **Input and output via Euler angles in both cases**
- **Command window output clarified**
- **On-line at *http://www.princeton.edu/~stengel/FDcodeB.html***

Examples from FLIGHT

Longitudinal Transient Response to Initial Pitch Rate

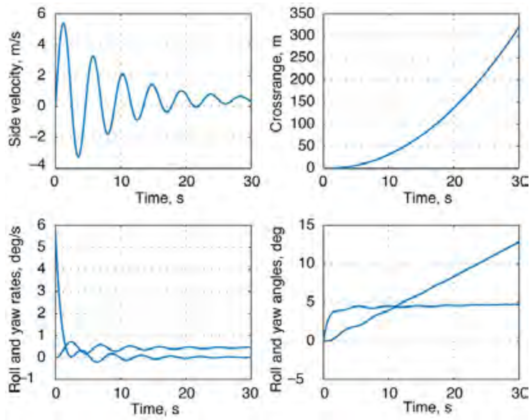


Bizjet, M = 0.3, Altitude = 3,052 m

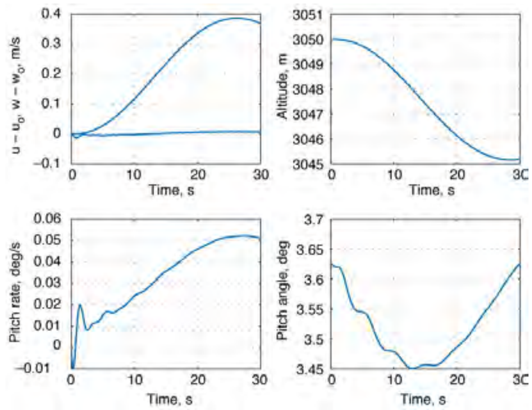
- **For a symmetric aircraft, longitudinal perturbations <u>do not</u> induce lateral-directional motions**

Transient Response to Initial Roll Rate

Lateral-Directional Response



Longitudinal Response

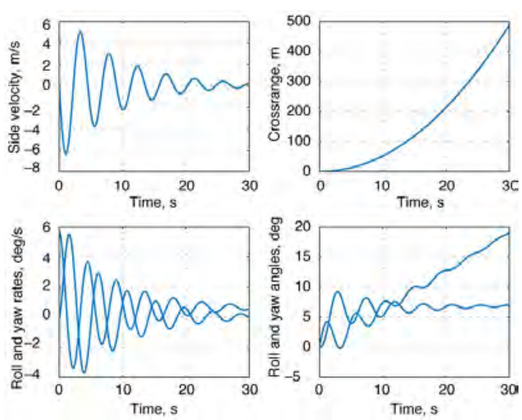


Bizjet, M = 0.3, Altitude = 3,052 m

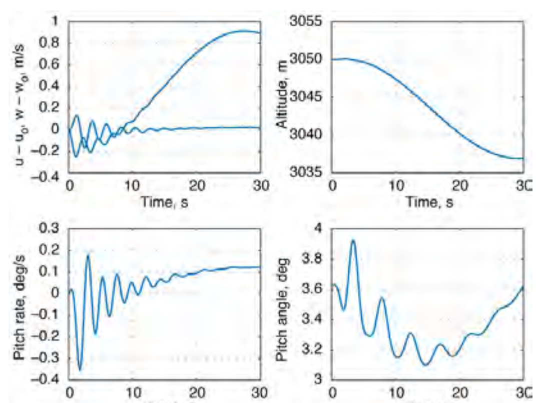
For a symmetric aircraft, lateral-directional perturbations do induce longitudinal motions

Transient Response to Initial Yaw Rate

Lateral-Directional Response



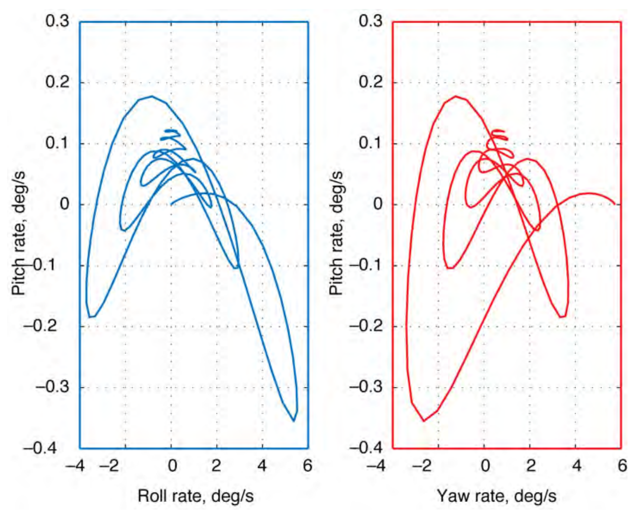
Longitudinal Response



Bizjet, M = 0.3, Altitude = 3,052 m

Crossplot of Transient Response to Initial Yaw Rate

Longitudinal-Lateral-Directional Coupling



Bizjet, M = 0.3, Altitude = 3,052 m

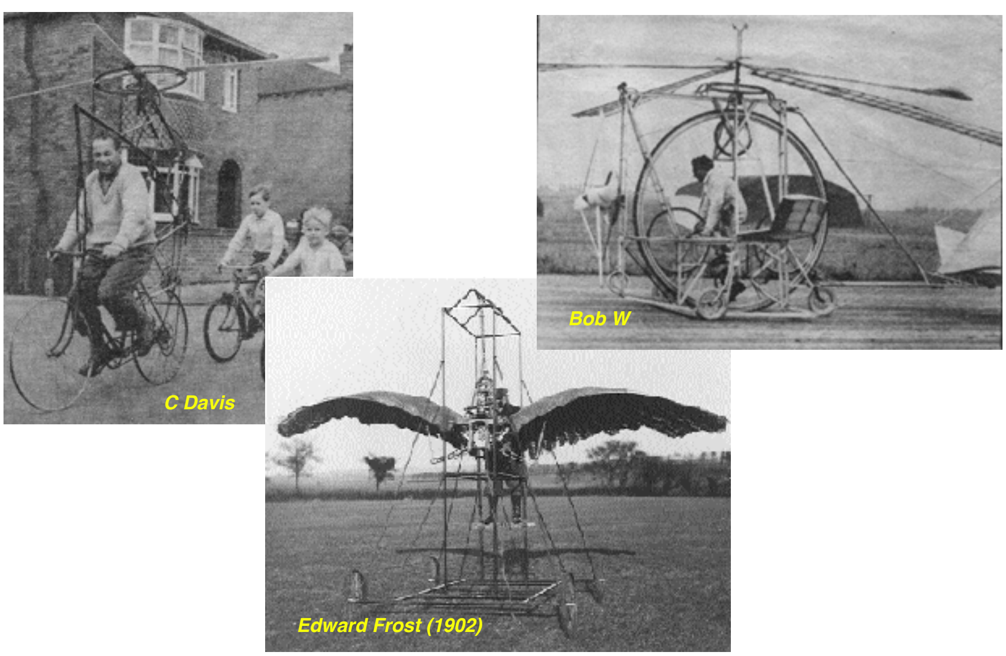
Checklist

- ❑ **Does longitudinal response couple into lateral-directional response?**
- ❑ **Does lateral-directional response couple into longitudinal response?**

Mythical/Historical Factoids

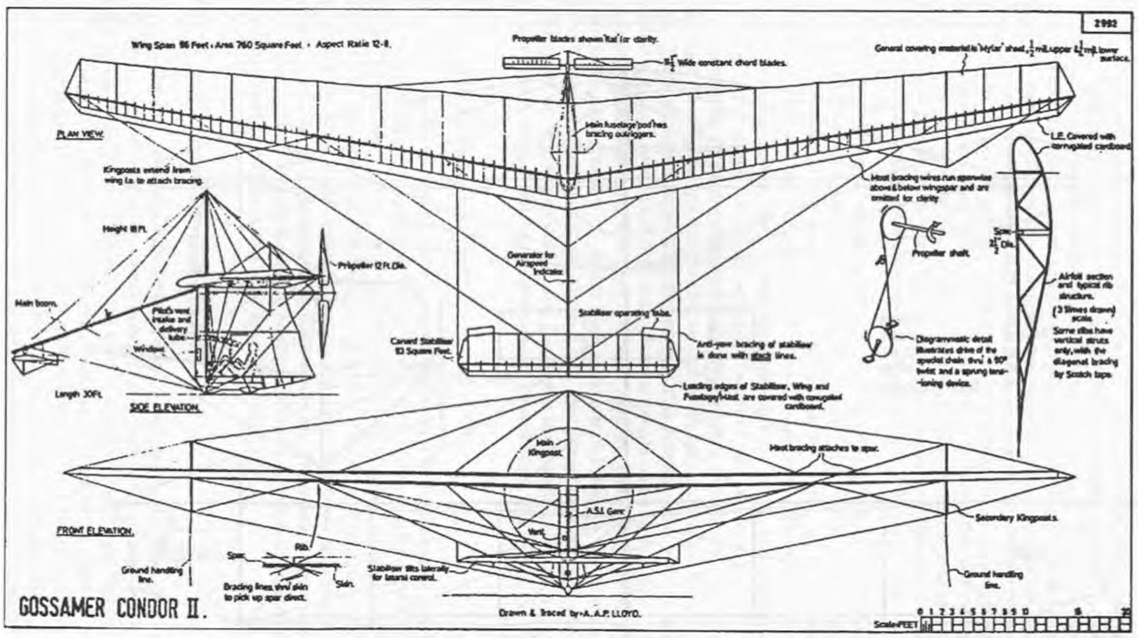
Daedalus and Icarus, father and son, Attempt to escape from Crete *(<630 BC)*

Other human-powered airplanes that didn't work



AeroVironment Gossamer Condor

Winner of the 1st Kremer Prize: *Figure 8* around pylons half-mile apart (1977)



AeroVironment Gossamer Albatross and MIT Monarch

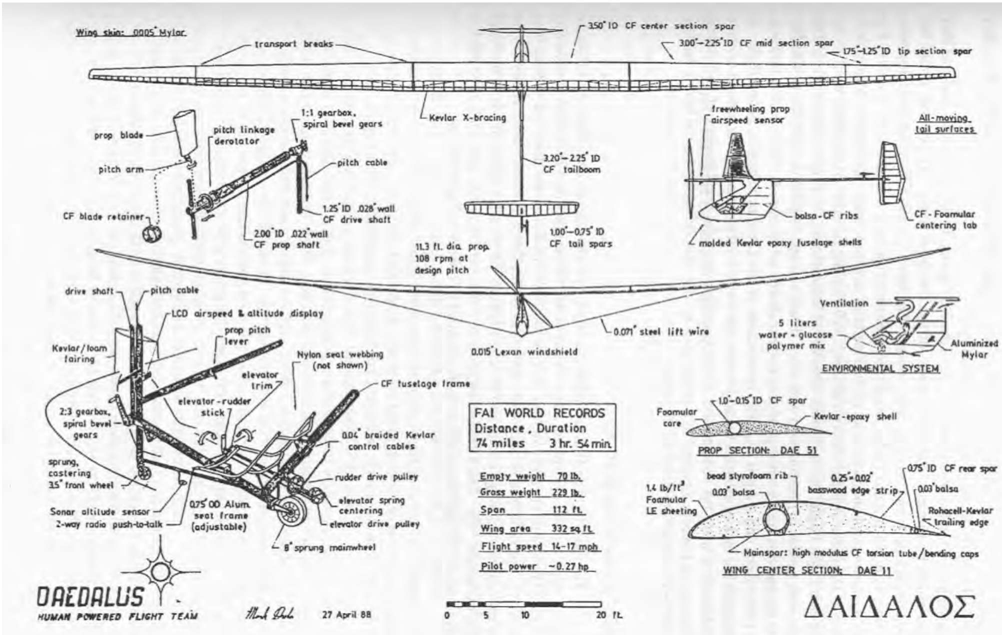
2nd Kremer Prize: crossing the English Channel (1979)

3rd Kremer Prize: 1500 m on closed course in < 3 min (1984)

Gene Larrabee, 'Mr. Propeller' of human-powered flight, dies at 82 (1/11/2003)
“The Albatross' pilot could stay aloft only 10 minutes at first. With (Larrabee’s) propeller, he stayed up for over an hour on his first flight …. There was no way the Albatross could cross the Channel, which took almost three hours, without (Larrabee’s) propeller.” *(D. Wilson)*

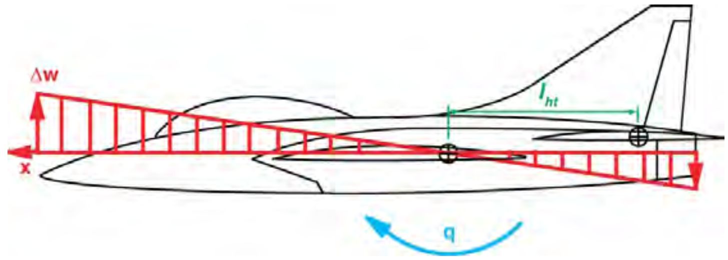
MIT Daedalus

Flew 74 miles across the Aegean Sea, completing Daedalus' s intended flight (1988)



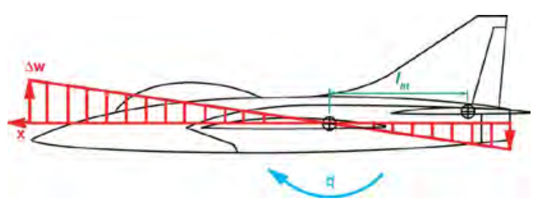
Aerodynamic Damping

Pitching Moment due to Pitch Rate



$$M_B = C_m \bar{q} S\bar{c} \approx \left(C_{m_o} + C_{m_q} q + C_{m_\alpha}\alpha\right)\bar{q} S\bar{c}$$
$$\approx \left(C_{m_o} + \frac{\partial C_m}{\partial q} q + C_{m_\alpha}\alpha\right)\bar{q} S\bar{c}$$

Angle of Attack Distribution Due to Pitch Rate



Aircraft pitching at a constant rate, *q* rad/s, produces a normal velocity distribution along *x*

$$\Delta w = -q\Delta x$$

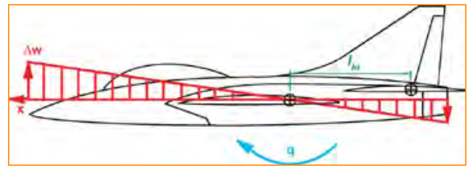
Corresponding angle of attack distribution

$$\Delta\alpha = \frac{\Delta w}{V} = \frac{-q\Delta x}{V}$$

Angle of attack perturbation at tail center of pressure

$$\boxed{\Delta\alpha_{ht} = \frac{q l_{ht}}{V}} \qquad \boxed{l_{ht} = \text{horizontal tail distance from c.m.}}$$

Horizontal Tail Lift Due to Pitch Rate



Incremental tail lift due to pitch rate, referenced to tail area, S_{ht}

$$\Delta L_{ht} = \left(\Delta C_{L_{ht}}\right)_{ht} \frac{1}{2} \rho V^2 S_{ht}$$

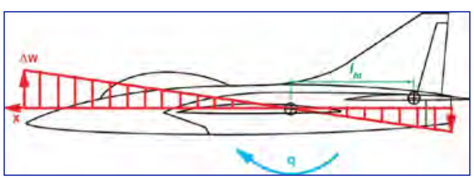
Incremental tail lift coefficient due to pitch rate, referenced to wing area, S

$$\left(\Delta C_{L_{ht}}\right)_{aircraft} = \left(\Delta C_{L_{ht}}\right)_{ht} \left(\frac{S_{ht}}{S}\right) = \left[\left(\frac{\partial C_{L_{ht}}}{\partial \alpha}\right)_{aircraft} \Delta\alpha\right] = \left(\frac{\partial C_{L_{ht}}}{\partial \alpha}\right)_{aircraft} \left(\frac{q l_{ht}}{V}\right)$$

Lift coefficient sensitivity to pitch rate referenced to wing area

$$C_{L_{q_{ht}}} \equiv \frac{\partial\left(\Delta C_{L_{ht}}\right)_{aircraft}}{\partial q} = \left(\frac{\partial C_{L_{ht}}}{\partial \alpha}\right)_{aircraft} \left(\frac{l_{ht}}{V}\right)$$

Moment Coefficient Sensitivity to Pitch Rate of the Horizontal Tail



Differential pitch moment due to pitch rate

$$\frac{\partial \Delta M_{ht}}{\partial q} = C_{m_{q_{ht}}} \frac{1}{2} \rho V^2 S\bar{c} = -C_{L_{q_{ht}}} \left(\frac{l_{ht}}{V}\right) \frac{1}{2} \rho V^2 S\bar{c}$$

$$= -\left[\left(\frac{\partial C_{L_{ht}}}{\partial \alpha}\right)_{aircraft} \left(\frac{l_{ht}}{V}\right)\right] \left(\frac{l_{ht}}{\bar{c}}\right) \frac{1}{2} \rho V^2 S\bar{c}$$

Coefficient derivative with respect to pitch rate

$$C_{m_{q_{ht}}} = -\frac{\partial C_{L_{ht}}}{\partial \alpha} \left(\frac{l_{ht}}{V}\right)\left(\frac{l_{ht}}{\bar{c}}\right) = -\frac{\partial C_{L_{ht}}}{\partial \alpha} \left(\frac{l_{ht}}{\bar{c}}\right)^2 \left(\frac{\bar{c}}{V}\right)$$

Coefficient derivative <u>with respect to normalized pitch rate</u> is insensitive to velocity

$$C_{m_{\hat{q}_{ht}}} = \frac{\partial C_{m_{ht}}}{\partial \hat{q}} = \frac{\partial C_{m_{ht}}}{\partial\left(q\bar{c}/2V\right)} = -2 \frac{\partial C_{L_{ht}}}{\partial \alpha} \left(\frac{l_{ht}}{\bar{c}}\right)^2$$

Pitch-Rate Derivative Definitions

- Pitch-rate derivatives are often expressed in terms of a normalized pitch rate

$$\hat{q} \triangleq \frac{q\bar{c}}{2V}$$

- Then

$$C_{m_{\hat{q}}} = \frac{\partial C_m}{\partial \hat{q}} = \frac{\partial C_m}{\partial\left(q\bar{c}/2V\right)} = \left(\frac{2V}{\bar{c}}\right) C_{m_q}$$

Pitching moment sensitivity to pitch rate

$$C_{m_q} = \frac{\partial C_m}{\partial q} = \left(\frac{\bar{c}}{2V}\right) C_{m_{\hat{q}}}$$

But dynamic equations require $\partial C_m/\partial q$

$$\frac{\partial M}{\partial q} = C_{m_q}\left(\rho V^2/2\right) S\bar{c} = C_{m_{\hat{q}}}\left(\frac{\bar{c}}{2V}\right)\left(\frac{\rho V^2}{2}\right) S\bar{c} = C_{m_{\hat{q}}}\left(\frac{\rho V S \bar{c}^2}{4}\right)$$

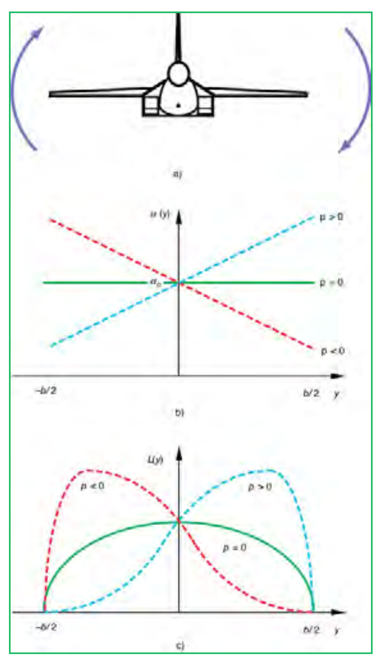
Roll Damping Due to Roll Rate

$$C_{l_p}\left(\frac{\rho V^2}{2}\right) Sb = C_{l_{\hat{p}}}\left(\frac{b}{2V}\right)\left(\frac{\rho V^2}{2}\right) Sb$$

$$= C_{l_{\hat{p}}}\left(\frac{\rho V}{4}\right) Sb^2$$

< 0 for stability

$$\hat{p} = \frac{pb}{2V}$$



- Vertical tail, horizontal tail, and wing are principal contributors

$$C_{l_{\hat{p}}} \approx \left(C_{l_{\hat{p}}}\right)_{Vertical\ Tail} + \left(C_{l_{\hat{p}}}\right)_{Horizontal\ Tail} + \left(C_{l_{\hat{p}}}\right)_{Wing}$$

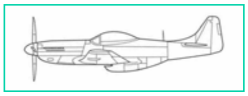
- Wing with taper

$$\left(C_{l_{\hat{p}}}\right)_{Wing} = \frac{\partial\left(\Delta C_l\right)_{Wing}}{\partial \hat{p}} = -\frac{C_{L_\alpha}}{12}\left(\frac{1+3\lambda}{1+\lambda}\right)$$

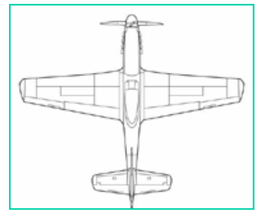
- Thin triangular wing

$$\left(C_{l_{\hat{p}}}\right)_{Wing} = -\frac{\pi AR}{32}$$

NACA-TR-1098, 1952
NACA-TR-1052, 1951



Roll Damping Due to Roll Rate



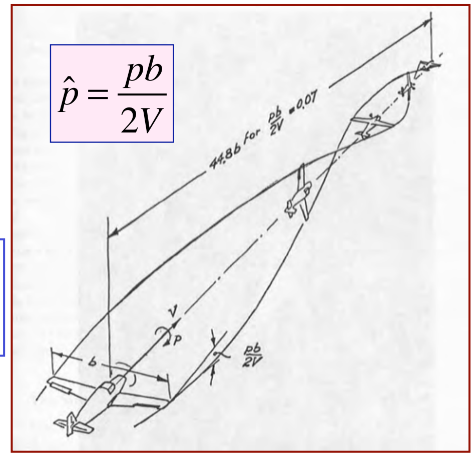
Tapered vertical tail

$$\left(C_{l_{\hat{p}}}\right)_{vt} = \frac{\partial\left(\Delta C_l\right)_{vt}}{\partial \hat{p}} = -\frac{C_{Y_{\beta_{vt}}}}{12}\left(\frac{S_{vt}}{S}\right)\left(\frac{1+3\lambda}{1+\lambda}\right)$$

Tapered horizontal tail

$$\left(C_{l_{\hat{p}}}\right)_{ht} = \frac{\partial\left(\Delta C_l\right)_{ht}}{\partial \hat{p}} = -\frac{C_{L_{\alpha_{ht}}}}{12}\left(\frac{S_{ht}}{S}\right)\left(\frac{1+3\lambda}{1+\lambda}\right)$$

pb/2V **describes helix angle for a steady roll**



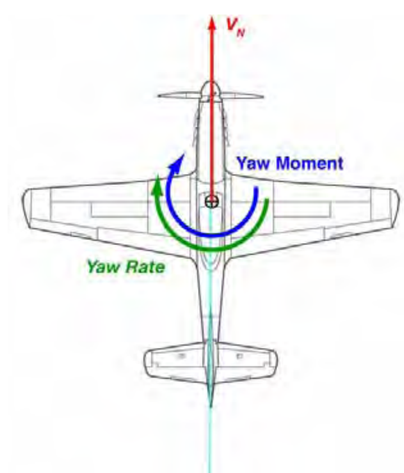
Yaw Damping Due to Yaw Rate

$$C_{n_r}\left(\frac{\rho V^2}{2}\right)Sb = C_{n_{\hat{r}}}\left(\frac{b}{2V}\right)\left(\frac{\rho V^2}{2}\right)Sb$$

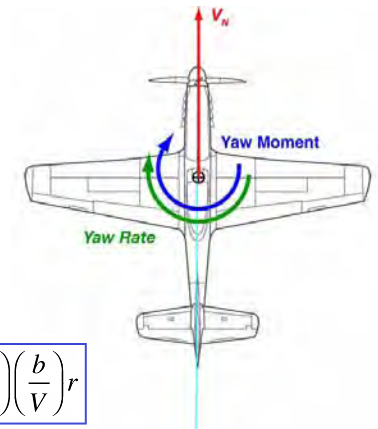
$$= C_{n_{\hat{r}}}\left(\frac{\rho V}{4}\right)Sb^2$$

< 0 for stability

$$\hat{r} = \frac{rb}{2V}$$



Yaw Damping Due to Yaw Rate



$$C_{n_{\hat{r}}} \approx \left(C_{n_{\hat{r}}}\right)_{Vertical\ Tail} + \left(C_{n_{\hat{r}}}\right)_{Wing}$$

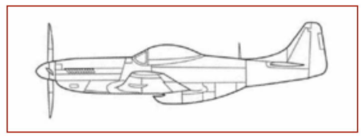
$$\hat{r} = \frac{rb}{2V}$$

Vertical tail contribution

$$\Delta\left(C_n\right)_{Vertical\ Tail} = -\left(C_{n_\beta}\right)_{Vertical\ Tail}\left(\frac{rl_{vt}}{V}\right) = -\left(C_{n_\beta}\right)_{Vertical\ Tail}\left(\frac{l_{vt}}{b}\right)\left(\frac{b}{V}\right)r$$

$$\left(C_{n_{\hat{r}}}\right)_{vt} = \frac{\partial\Delta\left(C_n\right)_{Vertical\ Tail}}{\partial\left(rb/2V\right)} = \frac{\partial\Delta\left(C_n\right)_{Vertical\ Tail}}{\partial\hat{r}} = -2\left(C_{n_\beta}\right)_{Vertical\ Tail}\left(\frac{l_{vt}}{b}\right)$$

Wing contribution



$$\left(C_{n_{\hat{r}}}\right)_{Wing} = k_0 C_L^2 + k_1 C_{D_{Parasite,\ Wing}}$$

k_0 and k_1 are functions of aspect ratio and sweep angle

NACA-TR-1098, 1952
NACA-TR-1052, 1951

Checklist

- ❑ What is the primary source of pitch damping?
- ❑ What is the primary source of yaw damping?
- ❑ What is the primary source of roll damping?
- ❑ What is the difference between

$$\boxed{C_{m_q} \text{ and } C_{m_{\hat{q}}}\ ?}$$

Next Time: Aircraft Control Devices and Systems

Reading:
Flight Dynamics
214-234

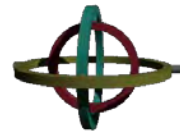
Learning Objectives

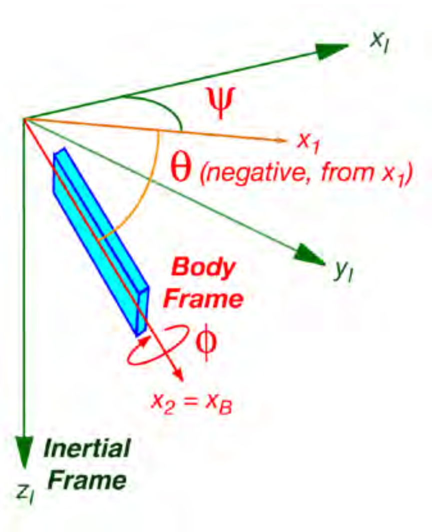
Control surfaces
Control mechanisms
Powered control
Flight control systems
Fly-by-wire control
Nonlinear dynamics and aero/mechanical instability

Supplemental Material

Airplane Angular Attitude (Position)

$$\left(\psi,\theta,\phi\right)$$





- **Euler angles**
  - 3 angles that relate one Cartesian coordinate frame to another
  - defined by sequence of 3 rotations about individual axes
  - intuitive description of angular attitude
  - Euler angle rates have a nonlinear relationship to body-axis angular rate vector
  - Transformation of rates is singular at 2 orientations, ±90°

Airplane Angular Attitude (Position)

$$\mathbf{H}_I^B(\phi,\theta,\psi) = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix}_I^B$$

$$\mathbf{H}_I^B(\phi,\theta,\psi) = \mathbf{H}_2^B(\phi)\mathbf{H}_1^2(\theta)\mathbf{H}_I^1(\psi)$$

$$\left[\mathbf{H}_I^B(\phi,\theta,\psi)\right]^{-1} = \left[\mathbf{H}_I^B(\phi,\theta,\psi)\right]^T = \mathbf{H}_B^I(\psi,\theta,\phi)$$

- **Rotation matrix**
  - orthonormal transformation
  - inverse = transpose
  - linear propagation from one attitude to another, based on body-axis rate vector
  - 9 parameters, 9 equations to solve
  - solution for Euler angles from parameters is intricate

Airplane Angular Attitude (Position)

Rotation Matrix

$$\boxed{\mathbf{H}_I^B(\phi,\theta,\psi) = \mathbf{H}_2^B(\phi)\mathbf{H}_1^2(\theta)\mathbf{H}_I^1(\psi)}$$

$$\mathbf{H}_I^B(\phi,\theta,\psi) =$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{H}_I^B(\phi,\theta,\psi) =$$

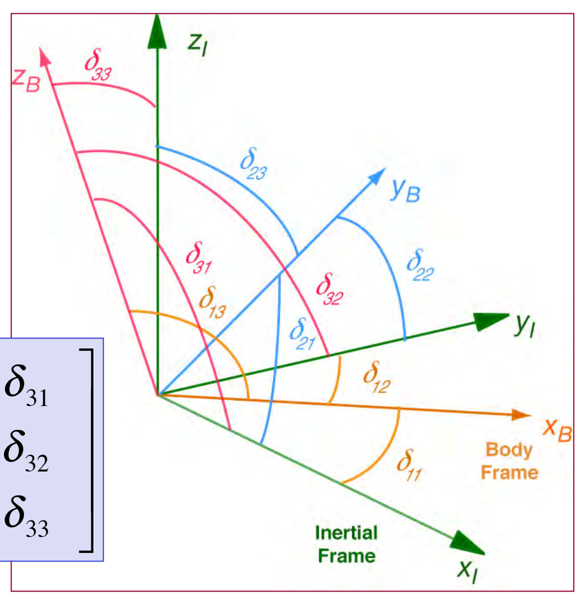
$$\left[\begin{array}{c|c|c} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ \hline -\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & \sin\phi\cos\theta \\ \hline \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi & -\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi & \cos\phi\cos\theta \end{array}\right]$$

$\mathbf{H}_I^B\mathbf{H}_B^I = \mathbf{I}$ for all (ϕ,θ,ψ), i.e., No Singularities

Airplane Angular Attitude (Position)

Rotation Matrix = Direction Cosine Matrix

Angles between each *I* axis and each *B* axis



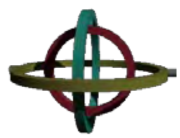
$$\mathbf{H}_I^B = \begin{bmatrix} \cos\delta_{11} & \cos\delta_{21} & \cos\delta_{31} \\ \cos\delta_{12} & \cos\delta_{22} & \cos\delta_{32} \\ \cos\delta_{13} & \cos\delta_{23} & \cos\delta_{33} \end{bmatrix}$$

Euler Angle Dynamics

$$\dot{\Theta} = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \mathbf{L}_B^I \boldsymbol{\omega}_B$$

$\mathbf{L}_B^I$ is not orthonormal

$\mathbf{L}_B^I$ is singular when $\theta = \pm 90°$



Rigid-Body Equations of Motion (Euler Angles)

$$\dot{\mathbf{r}}_I(t) = \mathbf{H}_B^I(t)\mathbf{v}_B(t)$$

$$\dot{\Theta}_I(t) = \mathbf{L}_B^I(t)\boldsymbol{\omega}_B(t)$$

$\mathbf{H}_B^I, \mathbf{H}_I^B$ are functions of $\boldsymbol{\Theta}$

$$\dot{\mathbf{v}}_B(t) = \frac{1}{m(t)}\mathbf{F}_B(t) + \mathbf{H}_I^B(t)\mathbf{g}_I - \tilde{\boldsymbol{\omega}}_B(t)\mathbf{v}_B(t)$$

$$\dot{\boldsymbol{\omega}}_B(t) = \boldsymbol{I}_B^{-1}(t)\left[\mathbf{M}_B(t) - \tilde{\boldsymbol{\omega}}_B(t)\boldsymbol{I}_B(t)\boldsymbol{\omega}_B(t)\right]$$

Rotation Matrix Dynamics

$$\boxed{\dot{\mathbf{H}}_B^I \mathbf{h}_B = \tilde{\boldsymbol{\omega}}_I \mathbf{h}_I = \tilde{\boldsymbol{\omega}}_I \mathbf{H}_B^I \mathbf{h}_B}$$

$$\boxed{\tilde{\boldsymbol{\omega}} = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}}$$

$$\boxed{\dot{\mathbf{H}}_B^I = \tilde{\boldsymbol{\omega}}_I \mathbf{H}_B^I}$$

$$\boxed{\dot{\mathbf{H}}_I^B = -\tilde{\boldsymbol{\omega}}_B \mathbf{H}_I^B}$$

Rigid-Body Equations of Motion (Attitude from 9-Element Rotation Matrix)

$$\boxed{\dot{\mathbf{r}}_I = \mathbf{H}_B^I \mathbf{v}_B}$$

$$\boxed{\dot{\mathbf{H}}_I^B = -\tilde{\boldsymbol{\omega}}_B \mathbf{H}_I^B}$$

$$\boxed{\dot{\mathbf{v}}_B = \frac{1}{m}\mathbf{F}_B + \mathbf{H}_I^B \mathbf{g}_I - \tilde{\boldsymbol{\omega}}_B \mathbf{v}_B}$$

$$\boxed{\dot{\boldsymbol{\omega}}_B = \boldsymbol{I}_B^{-1}\left(\mathbf{M}_B - \tilde{\boldsymbol{\omega}}_B \boldsymbol{I}_B \boldsymbol{\omega}_B\right)}$$

No need for Euler angles to solve the dynamic equations

Successive Rotations Expressed by Products of Quaternions and Rotation Matrices

Rotation from Frame A to Frame C through Intermediate Frame B

$\mathbf{q}_A^B$: Rotation from A to B
$\mathbf{q}_B^C$: Rotation from B to C
$\mathbf{q}_A^C$: Rotation from A to C

Matrix Multiplication Rule

$$\mathbf{H}_A^C\left(\mathbf{q}_A^C\right) = \mathbf{H}_B^C\left(\mathbf{q}_B^C\right)\mathbf{H}_A^B\left(\mathbf{q}_A^B\right)$$

Quaternion Multiplication Rule

$$\mathbf{q}_A^C = \begin{bmatrix} \mathbf{q}_3 \\ q_4 \end{bmatrix}_A^C = \mathbf{q}_B^C \mathbf{q}_A^B \triangleq \begin{bmatrix} (q_4)_B^C \mathbf{q}_{3A}^{\ B} + (q_4)_A^B \mathbf{q}_{3B}^{\ C} - (\tilde{\mathbf{q}}_3)_B^C \mathbf{q}_{3A}^{\ B} \\ (q_4)_B^C (q_4)_A^B - \left(\mathbf{q}_{3B}^{\ C}\right)^T \mathbf{q}_{3A}^{\ B} \end{bmatrix}$$

Rigid-Body Equations of Motion (Attitude from 4-Element Quaternion Vector)

$$\dot{\mathbf{r}}_I = \mathbf{H}_B^I \mathbf{v}_B$$

$$\boxed{\dot{\mathbf{q}} = \mathbf{Q}\mathbf{q}}$$

$\mathbf{H}_B^I, \mathbf{H}_I^B$ are functions of $\mathbf{q}$

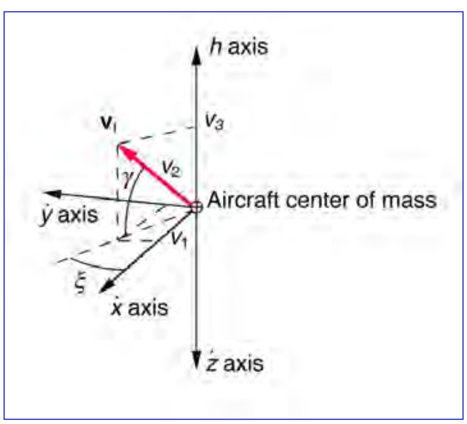
$$\dot{\mathbf{v}}_B = \frac{1}{m}\mathbf{F}_B + \mathbf{H}_I^B \mathbf{g}_I - \tilde{\boldsymbol{\omega}}_B \mathbf{v}_B$$

$$\dot{\boldsymbol{\omega}}_B = \boldsymbol{I}_B^{-1}\left(\mathbf{M}_B - \tilde{\boldsymbol{\omega}}_B \boldsymbol{I}_B \boldsymbol{\omega}_B\right)$$

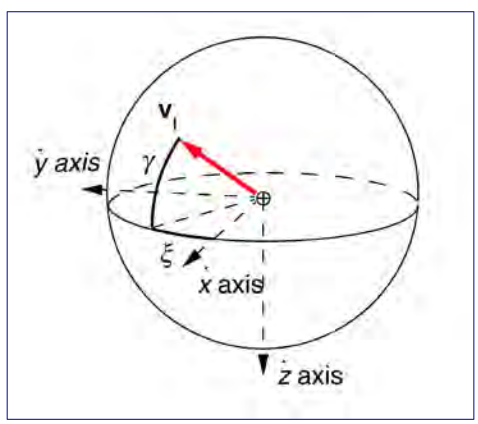
Alternative Reference Frames

Velocity Orientation in an Inertial Frame of Reference

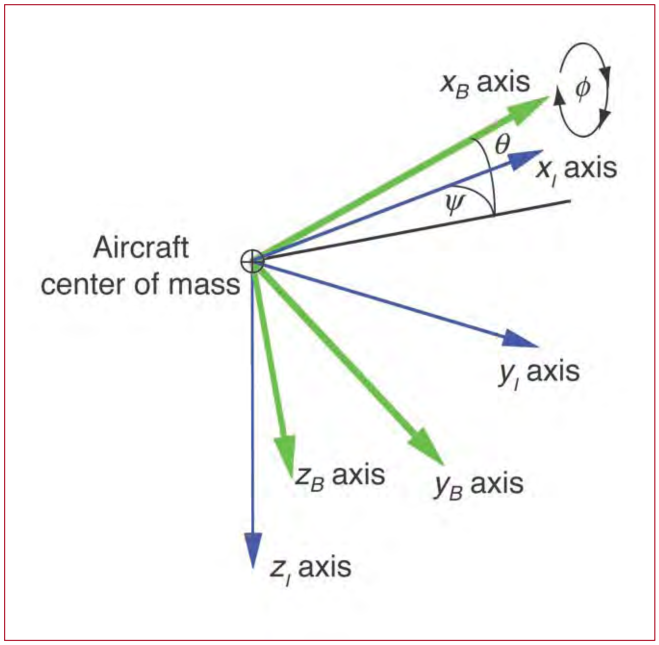
Polar Coordinates



Projected on a Sphere

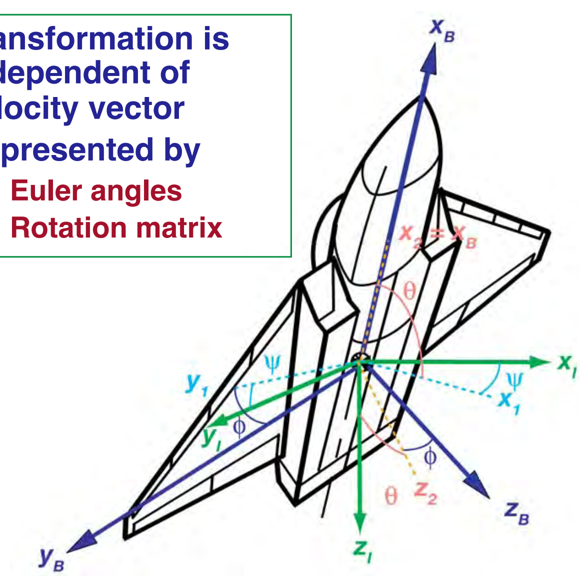


Body Orientation with Respect to an Inertial Frame



Relationship of Inertial Axes to Body Axes

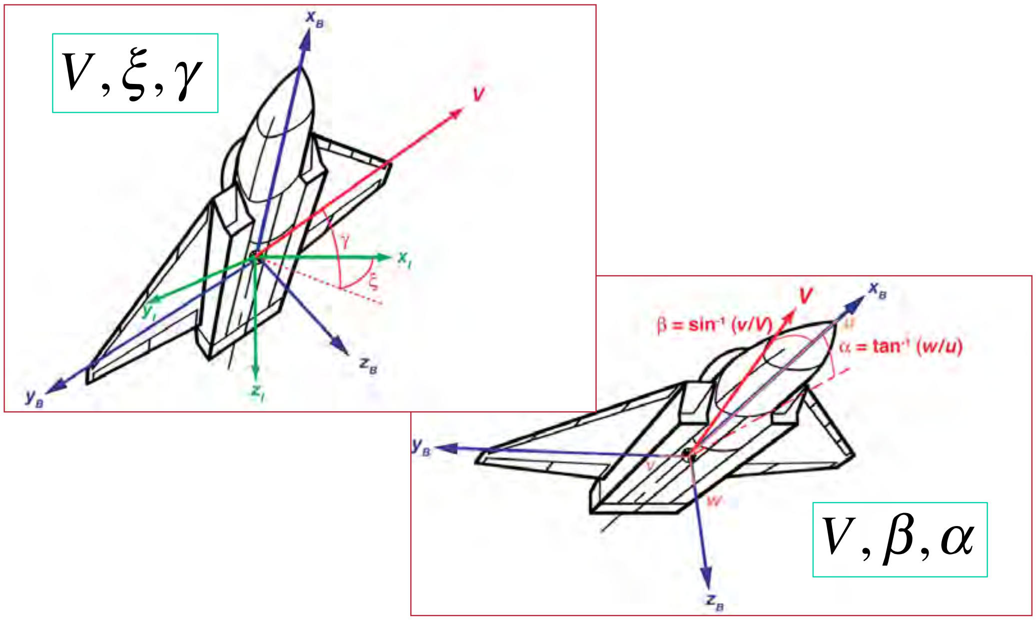
- **Transformation is independent of velocity vector**
- **Represented by**
  - **Euler angles**
  - **Rotation matrix**



$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \mathbf{H}_I^B \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}$$

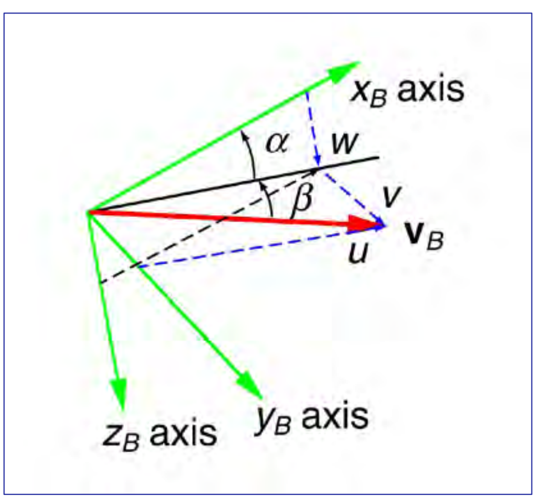
$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \mathbf{H}_B^I \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

Velocity-Vector Components of an Aircraft

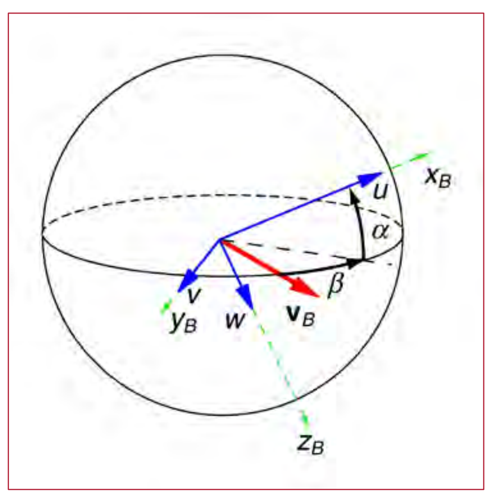


Velocity Orientation with Respect to the Body Frame

Polar Coordinates



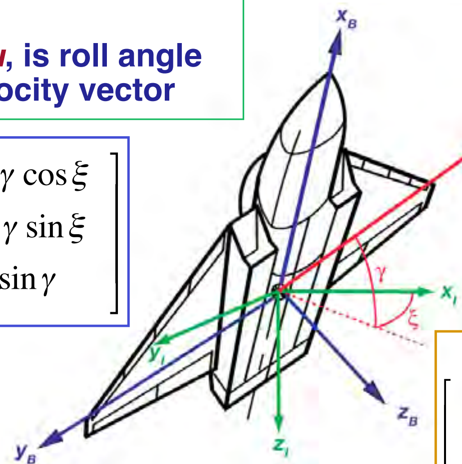
Projected on a Sphere



Relationship of Inertial Axes to Velocity Axes

- No reference to the body frame
- Bank angle, μ, is roll angle about the velocity vector

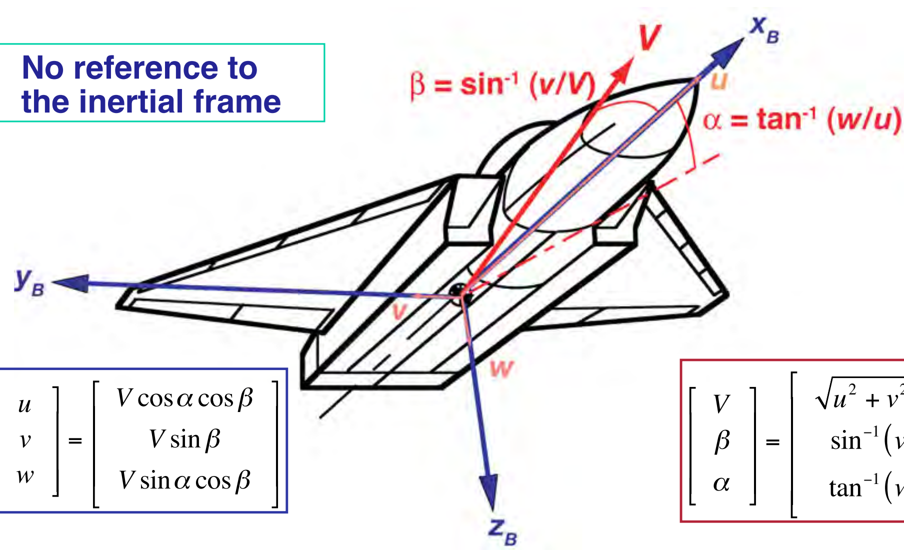
$$\begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix}_I = \begin{bmatrix} V\cos\gamma\cos\xi \\ V\cos\gamma\sin\xi \\ -V\sin\gamma \end{bmatrix}$$



$$\begin{bmatrix} V \\ \xi \\ \gamma \end{bmatrix} = \begin{bmatrix} \sqrt{v_x^2 + v_y^2 + v_z^2} \\ \sin^{-1}\left[v_y / \left(v_x^2 + v_y^2\right)^{1/2}\right] \\ \sin^{-1}\left(-v_z / V\right) \end{bmatrix}$$

Relationship of Body Axes to Wind Axes

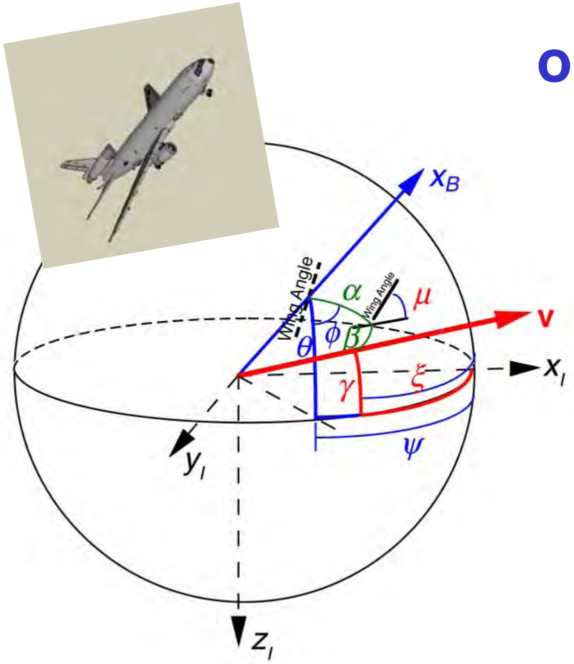
- No reference to the inertial frame



$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} V\cos\alpha\cos\beta \\ V\sin\beta \\ V\sin\alpha\cos\beta \end{bmatrix}$$

$$\begin{bmatrix} V \\ \beta \\ \alpha \end{bmatrix} = \begin{bmatrix} \sqrt{u^2 + v^2 + w^2} \\ \sin^{-1}\left(v / V\right) \\ \tan^{-1}\left(w / u\right) \end{bmatrix}$$

Angles Projected on the Unit Sphere



Origin is airplane's center of mass

α : *angle of attack*
β : *sideslip angle*
γ : *vertical flight path angle*
ξ : *horizontal flight path angle*
ψ : *yaw angle*
θ : *pitch angle*
ϕ : *roll angle (about body x – axis)*
μ : *bank angle (about velocity vector)*

Alternative Frames of Reference

- **Orthonormal transformations connect all reference frames**