Class: CSc 335

Date: Mar 7, 2023

Strong induction

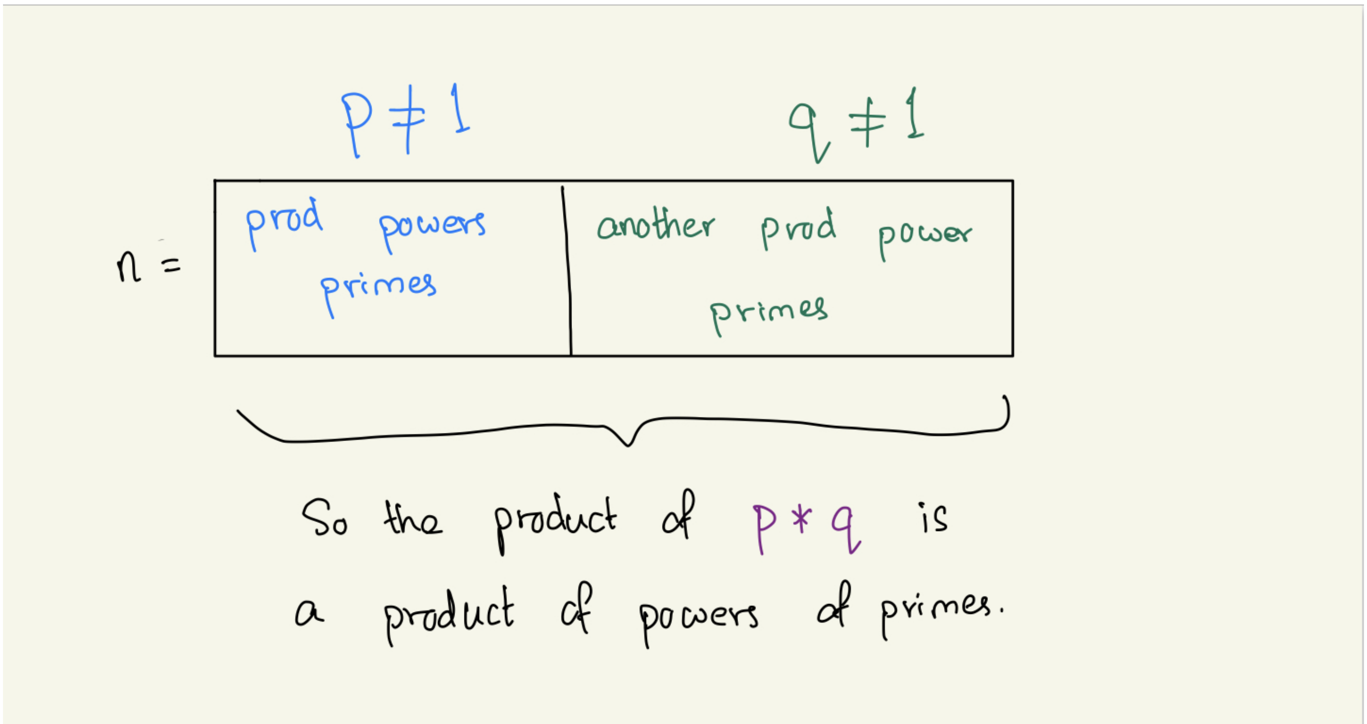
Motivation: We will need strong induction when we begin discussing data strctures.

Prove: every integer $n \geq 2$ can be written as a product of powers of primes (proof by strong induction)

Try inducting on n, with divide & conquer: if $n = p * q$, with neither p nor q equal to 1. Then assume p is a product of powers of primes and also that q is a product of powers of primes.

Now that we have the divide & conquer strategy, here is the full argument

- Consider $n \geq 2$
  - If $n = 2$, there's nothing to show, since $2 = 2'$ is a product of powers of primes.
- Consider $n > 2$
  - Either n is prime or not n i.e. composite
  - If n is prime, we're done.
  - Otherwise, $n = p * q$ for some integers $p, q > 1$ [def. of composite number]
  - Now the argument is the one we just gave.



This is a strong induction because

1. We know neither p nor q is $n-1$
2. We don't know in advance the values of the factors p and q - So we actually need all of the hypotheses.

Tree Recusion

```
;; Fib-sequence

(define (fib n)
  (cond ((zero? n) 0)
        ((one? n) 1)
        (else (+ (fib (- n 1)) (fib (- n 2))))))
```

- a cool technique is `memoization` → `to transform the results of a function into something to remember`
  - example: creating a dictionary to remember the value of `fib 2` or `fib 3` in computing `fib 5`

What about an iterative solution?

A design idea flows from the def of $ fib(n+1) $ as $ fib(n) + fib(n-1) $ when $ n \ngeq 2 $. where

```
fib(n+1) = fib(n) + fib(n-1)
  |           |        |
 next     =  curr  +  prev
```

If we introduce `curr` and `prev` as variables with design roles:

```
curr = (fib n)
prev = (fib (- n 1))
```

with the idea of maintaining these with as count increases from 0 up to n.

i.e.: Take the GI as the logical AND of the design roles:

- GI:

```
next = curr + prev
where next = fib(count)
```