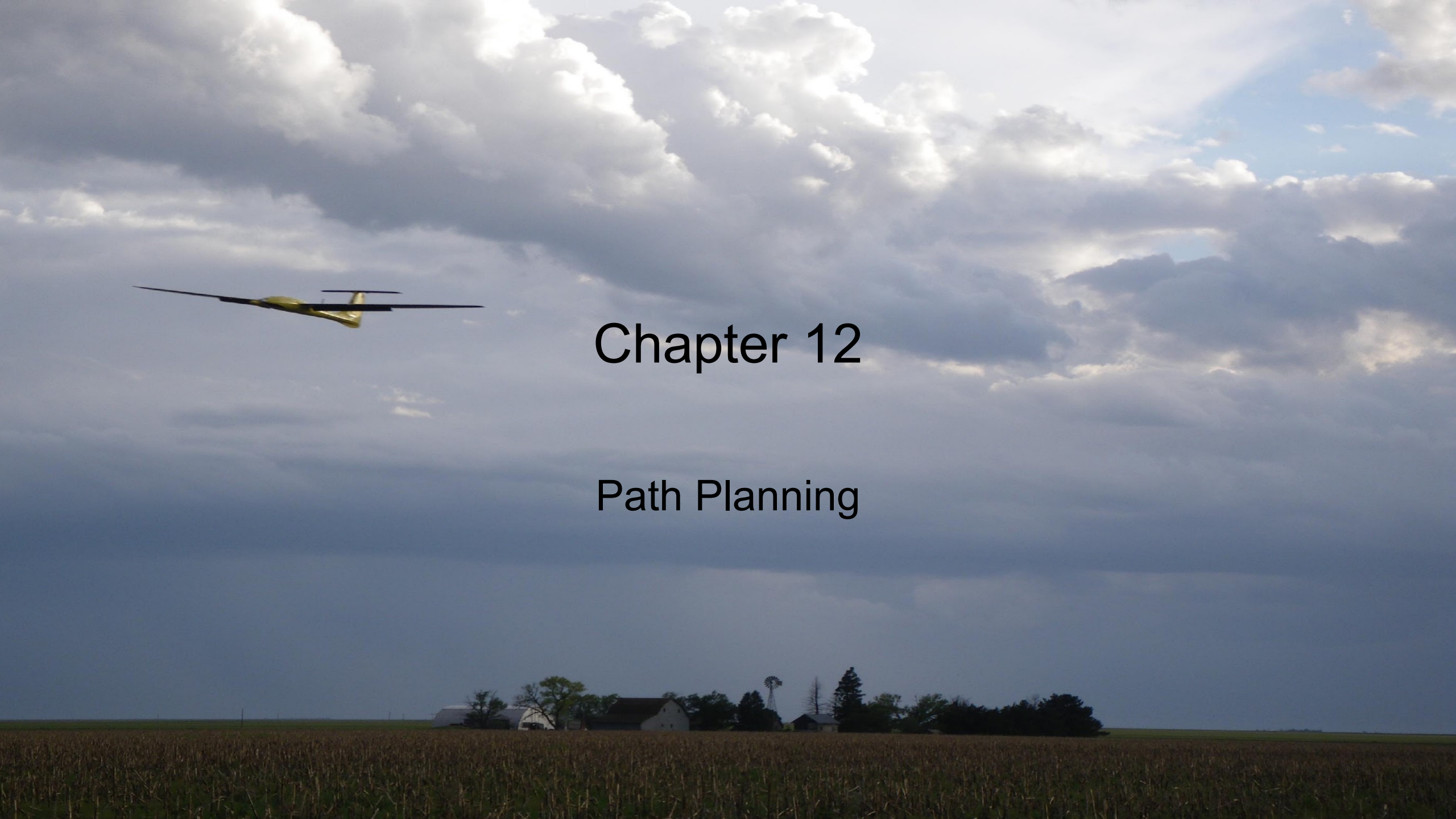
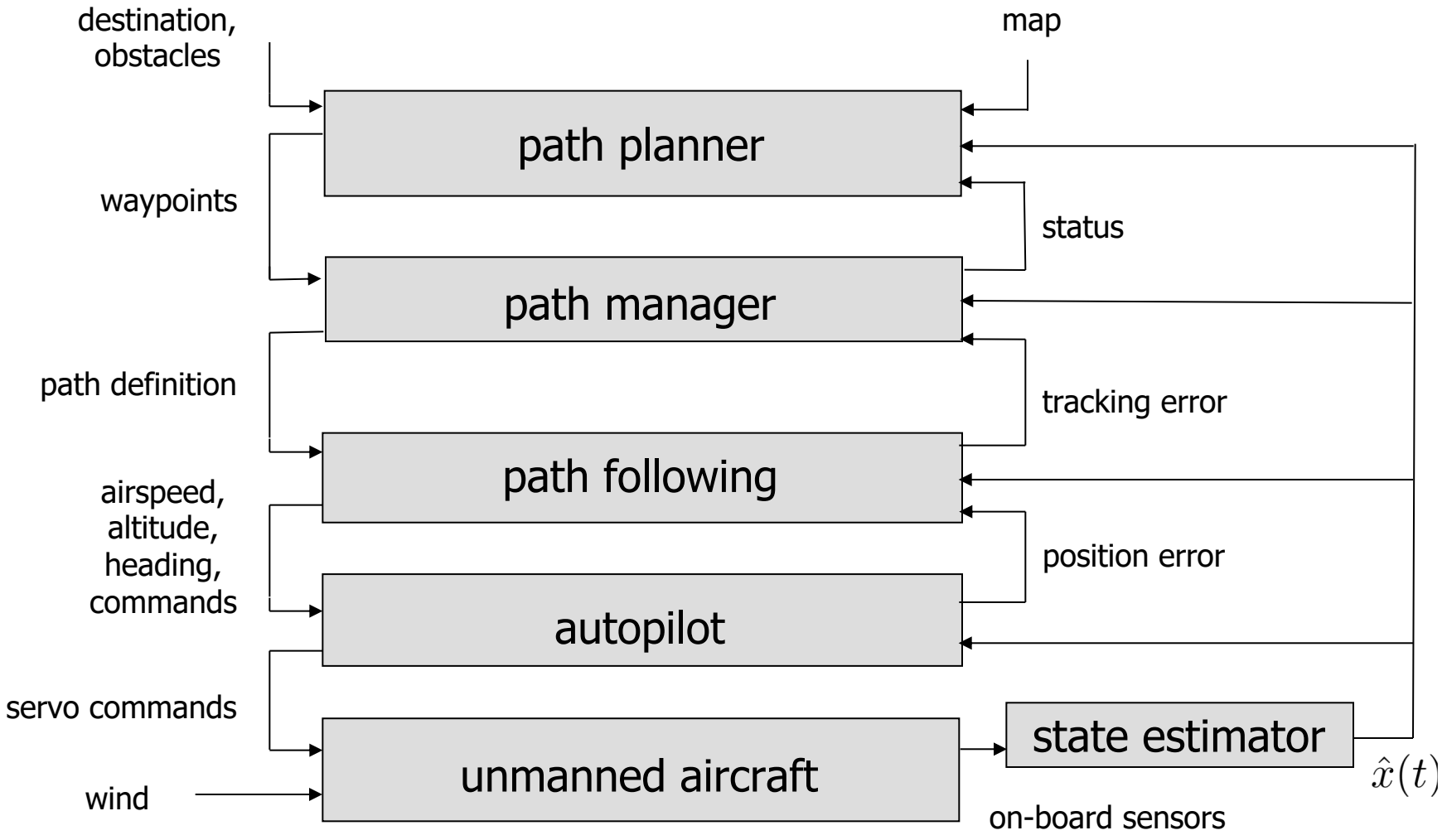


Chapter 12

Path Planning

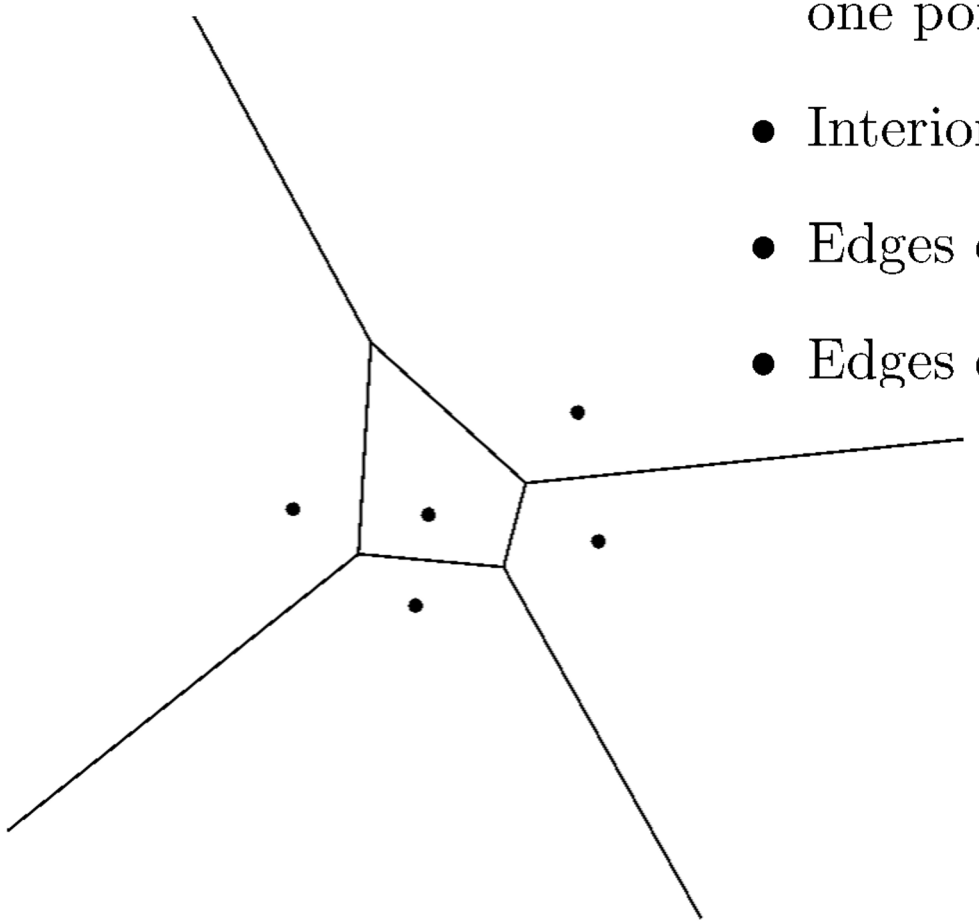
Control Architecture



Path Planning Approaches

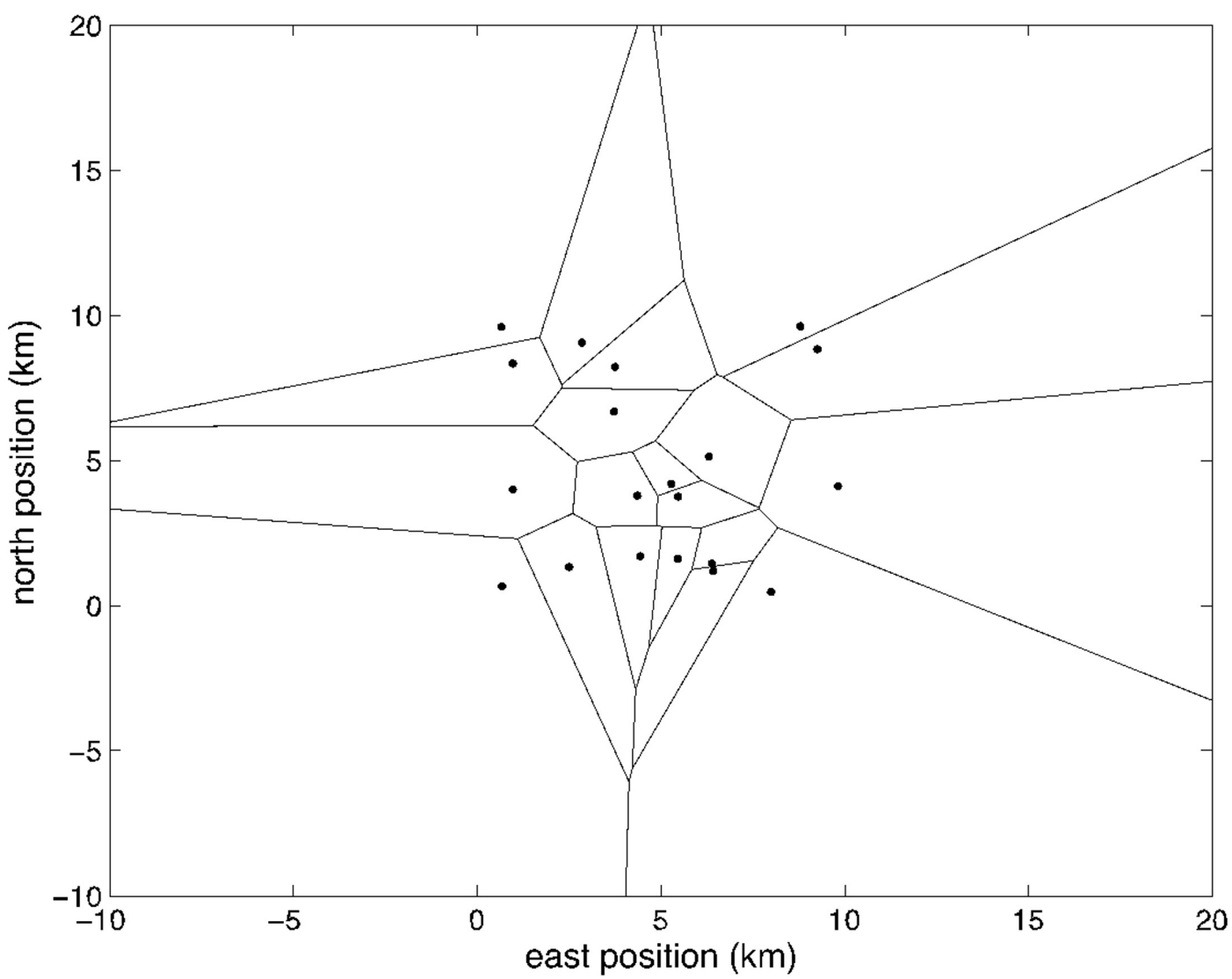
- Deliberative
  - Based on global world knowledge
  - Requires a good map of terrain, obstacles, etc.
  - Can be too computationally intense for dynamic environments
  - Usually executed before the mission
- Reactive
  - Based on what sensors detect on immediate horizon
  - Can respond to dynamic environments
  - Not usually used for entire mission

Voronoi Graphs



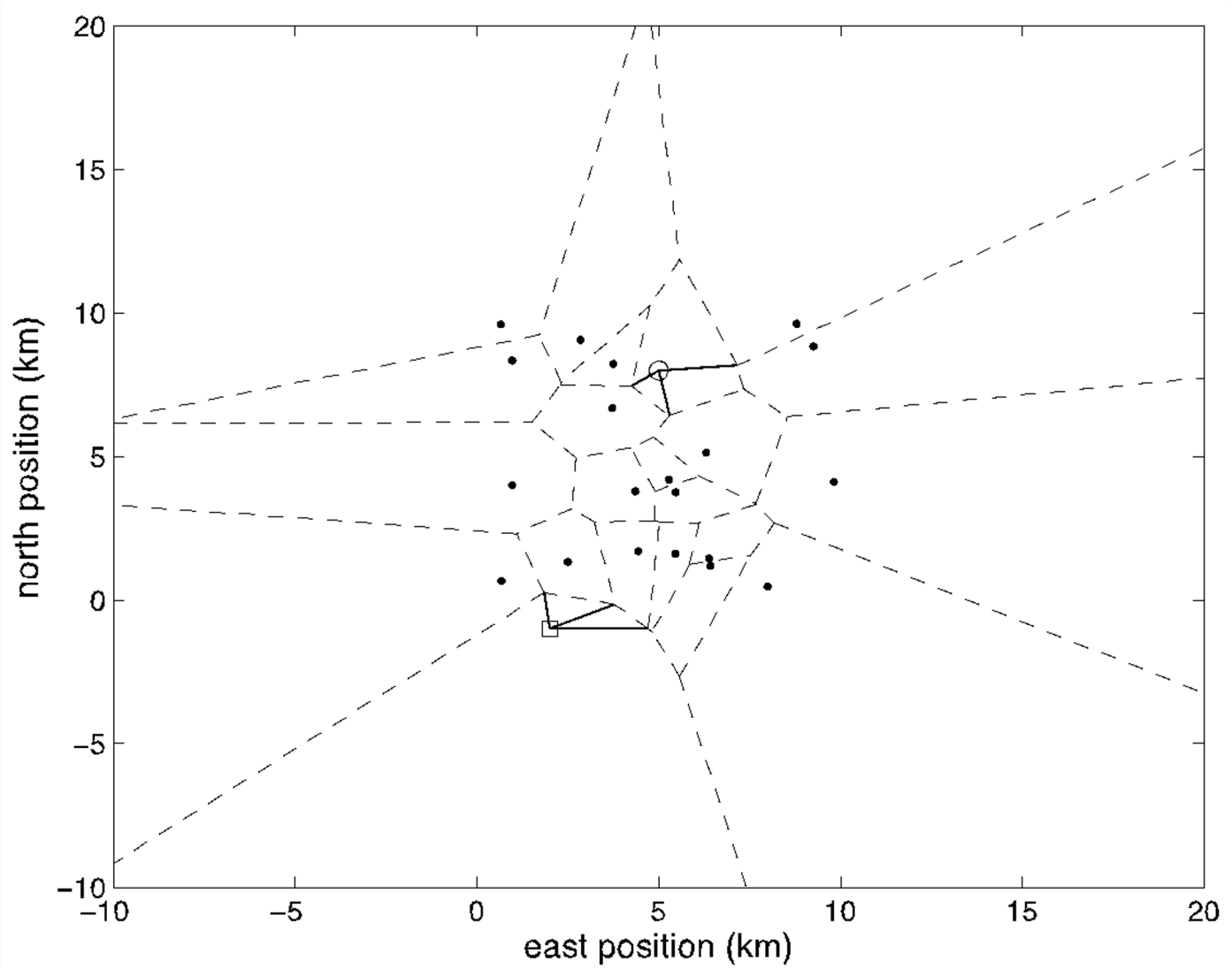
- For finite number $\mathcal{Q}$ point obstacles, Voronoi graph divides search plane into $\mathcal{Q}$ convex cells, each containing one point obstacle
- Interior of cell is closer to its point than any other point in $\mathcal{Q}$
- Edges of graph are perpendicular bisectors between points in $\mathcal{Q}$
- Edges of path maximize distance from point obstacles

Voronoi Graph Example



- Graph generated using voronoi command in Matlab
- 20 point obstacles
- Start and end points of path not shown

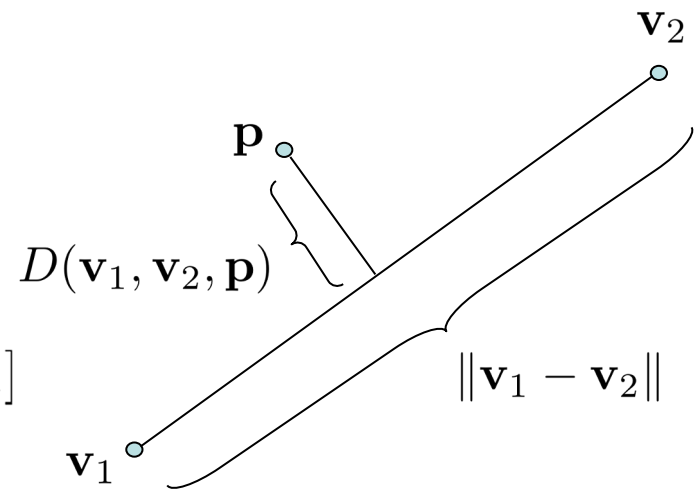
Voronoi Graph Example



- Add *start* and *end* points to graph
- Find 3 closest graph nodes to *start* and *end* points
- Add graph edges to *start* and *end* points
- Search graph to find “best” path
- Must define “best”
  - Shortest?
  - Furthest from obstacles?

Path Cost Calculation

Nodes of graph edge: $\mathbf{v}_1$ and $\mathbf{v}_2$
Point obstacle: $\mathbf{p}$



Length of edge: $\|\mathbf{v}_1 - \mathbf{v}_2\|$

Any point on line segment: $\mathbf{w}(\sigma) = (1-\sigma)\mathbf{v}_1 + \sigma\mathbf{v}_2$ where $\sigma \in [0,1]$

Minimum distance between $\mathbf{p}$ and graph edge

$$
\begin{aligned}
D(\mathbf{v}_1, \mathbf{v}_2, \mathbf{p}) &\stackrel{\triangle}{=} \min_{\sigma \in [0,1]} \|\mathbf{p} - \mathbf{w}(\sigma)\| \\
&= \min_{\sigma \in [0,1]} \sqrt{(\mathbf{p} - \mathbf{w}(\sigma))^\top (\mathbf{p} - \mathbf{w}(\sigma))} \\
&= \min_{\sigma \in [0,1]} \sqrt{\|\mathbf{p} - \mathbf{v}_1\|^2 + 2\sigma(\mathbf{p} - \mathbf{v}_1)^\top (\mathbf{v}_1 - \mathbf{v}_2) + \sigma^2 \|\mathbf{v}_1 - \mathbf{v}_2\|^2}
\end{aligned}
$$

Path Cost Calculation

Value for σ that minimizes D is

$$\sigma^* = \frac{(\mathbf{v}_1 - \mathbf{p})^\top (\mathbf{v}_1 - \mathbf{v}_2)}{\|\mathbf{v}_1 - \mathbf{v}_2\|^2}$$

Location along edge for which D is minimum:

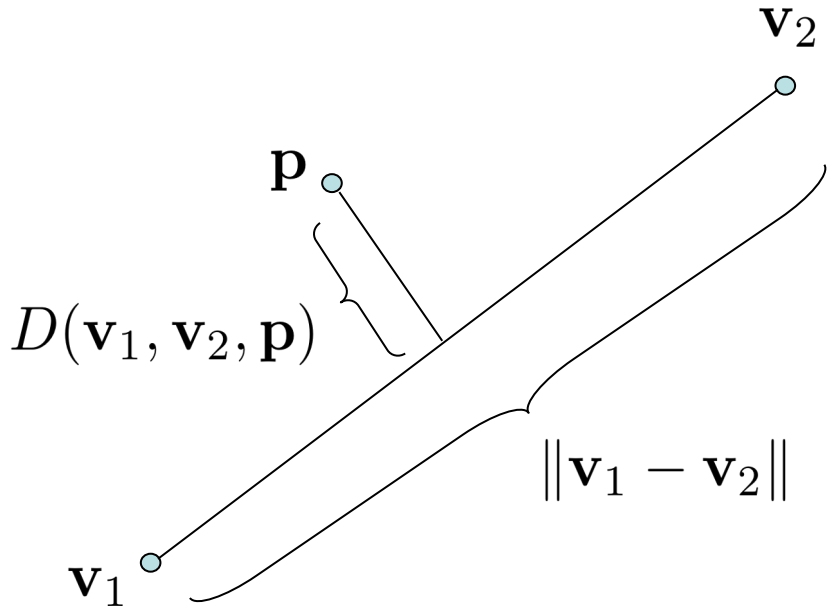
$$\mathbf{w}(\sigma^*) = \sqrt{\|\mathbf{p} - \mathbf{v}_1\|^2 - \frac{\left((\mathbf{v}_1 - \mathbf{p})^\top (\mathbf{v}_1 - \mathbf{v}_2)\right)^2}{\|\mathbf{v}_1 - \mathbf{v}_2\|^2}}$$

Define distance to edge for $\sigma^* < 0$, $\sigma^* > 1$:

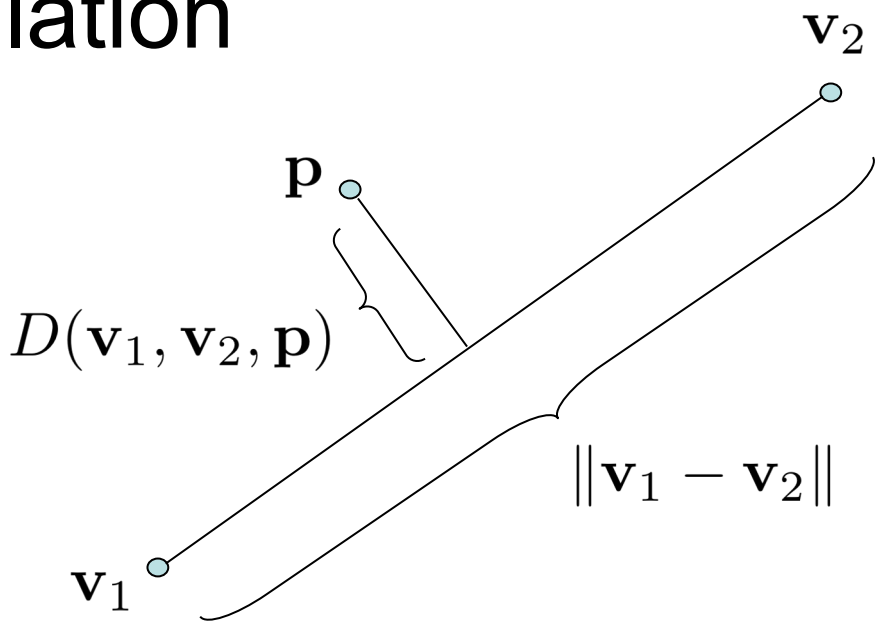
$$D'(\mathbf{v}_1, \mathbf{v}_2, \mathbf{p}) \triangleq \begin{cases} \mathbf{w}(\sigma^*) & \text{if } \sigma^* \in [0, 1] \\ \|\mathbf{p} - \mathbf{v}_1\| & \text{if } \sigma^* < 0 \\ \|\mathbf{p} - \mathbf{v}_2\| & \text{if } \sigma^* > 1 \end{cases}$$

Distance between point set $\mathcal{Q}$ and line segment $\overline{\mathbf{v}_1\mathbf{v}_2}$:

$$D(\mathbf{v}_1, \mathbf{v}_2, \mathcal{Q}) = \min_{\mathbf{p} \in \mathcal{Q}} D'(\mathbf{v}_1, \mathbf{v}_2, \mathbf{p})$$



Path Cost Calculation



Cost for traveling along edge $(\mathbf{v}_1, \mathbf{v}_2)$ is assigned as

$$J(\mathbf{v}_1, \mathbf{v}_2) = \underbrace{k_1 \|\mathbf{v}_1 - \mathbf{v}_2\|}_{\text{length of edge}} + \underbrace{\frac{k_2}{D(\mathbf{v}_1, \mathbf{v}_2, \mathcal{Q})}}_{\text{reciprocal of distance to closest point in } \mathcal{Q}}$$

k_1 and k_2 are positive weights

Choice of k_1 and k_2 allow tradeoff between path length and proximity to threats.

Voronoi Path Planning Algorithm

Algorithm 9 Plan Voronoi Path: $\mathcal{W} = \text{planVoronoi}(\mathcal{Q}, \mathbf{p}_s, \mathbf{p}_e)$

Input: Obstacle points $\mathcal{Q}$, start position $\mathbf{p}_s$, end position $\mathbf{p}_e$
Require: $|\mathcal{Q}| \geq 10$ Randomly add points if necessary.

1: $(V, E) = \text{constructVoronoiGraph}(\mathcal{Q})$
2: $V^+ = V \bigcup \{\mathbf{p}_s\} \bigcup \{\mathbf{p}_e\}$
3: Find $\{\mathbf{v}_{1s}, \mathbf{v}_{2s}, \mathbf{v}_{3s}\}$, the three closest points in V to $\mathbf{p}_s$, and $\{\mathbf{v}_{1e}, \mathbf{v}_{2e}, \mathbf{v}_{3e}\}$, the three closest points in V to $\mathbf{p}_e$
4: $E^+ = E \bigcup_{i=1,2,3} (\mathbf{v}_{is}, \mathbf{p}_s) \bigcup_{i=1,2,3} (\mathbf{v}_{ie}, \mathbf{p}_e)$
5: **for** Each element $(\mathbf{v}_a, \mathbf{v}_b) \in E$ **do**
6: Assign edge cost $\mathbf{J}_{ab} = J(\mathbf{v}_a, \mathbf{v}_b)$ according to equation (12.1).
7: **end for**
8: $\mathcal{W} = \text{DijkstraSearch}(V^+, E^+, \mathbf{J})$
9: **return** $\mathcal{W}$

Dijkstra's algorithm is used to search the graph

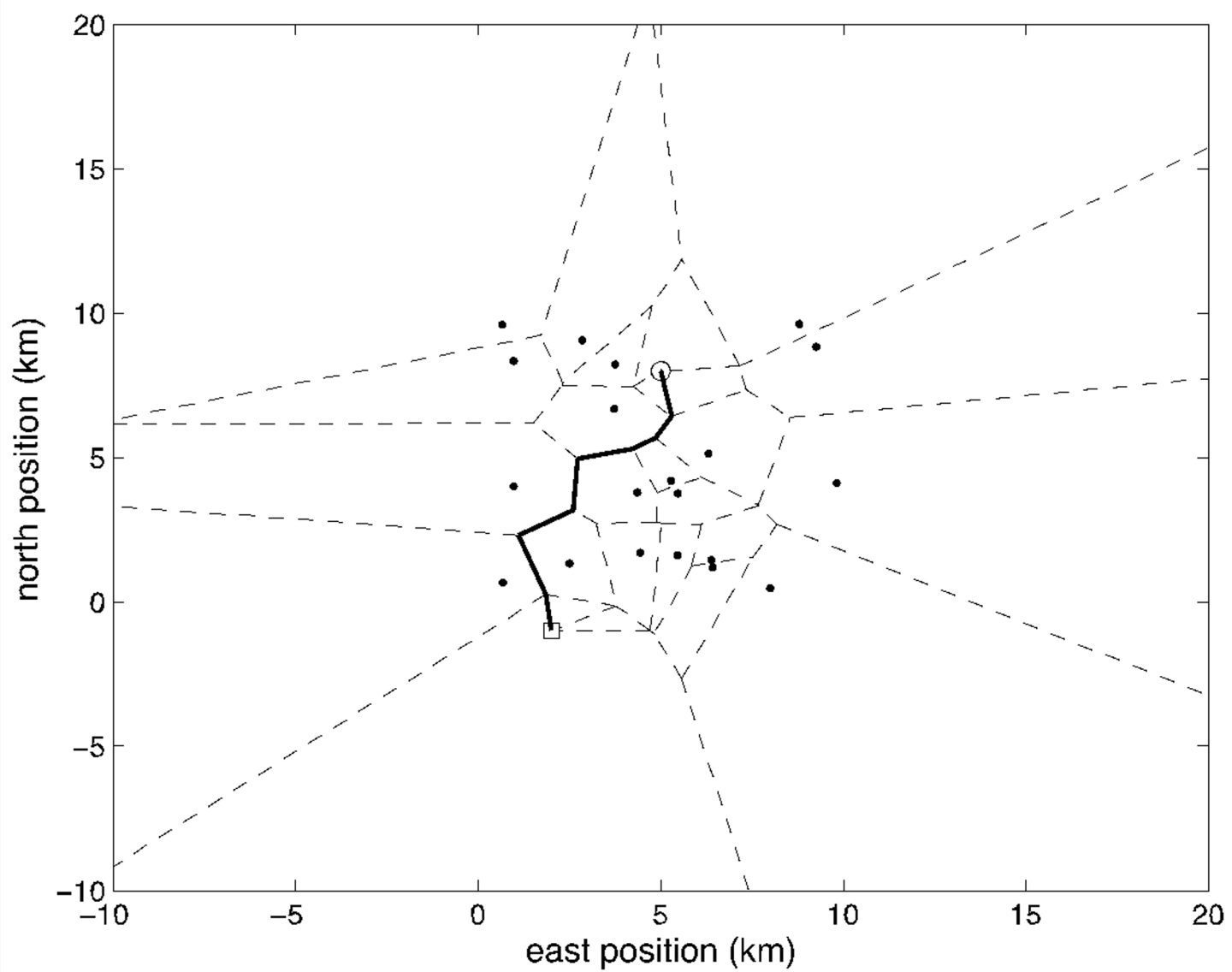
Voronoi graph and Dijkstra's algorithm code are commonly available

Matlab:

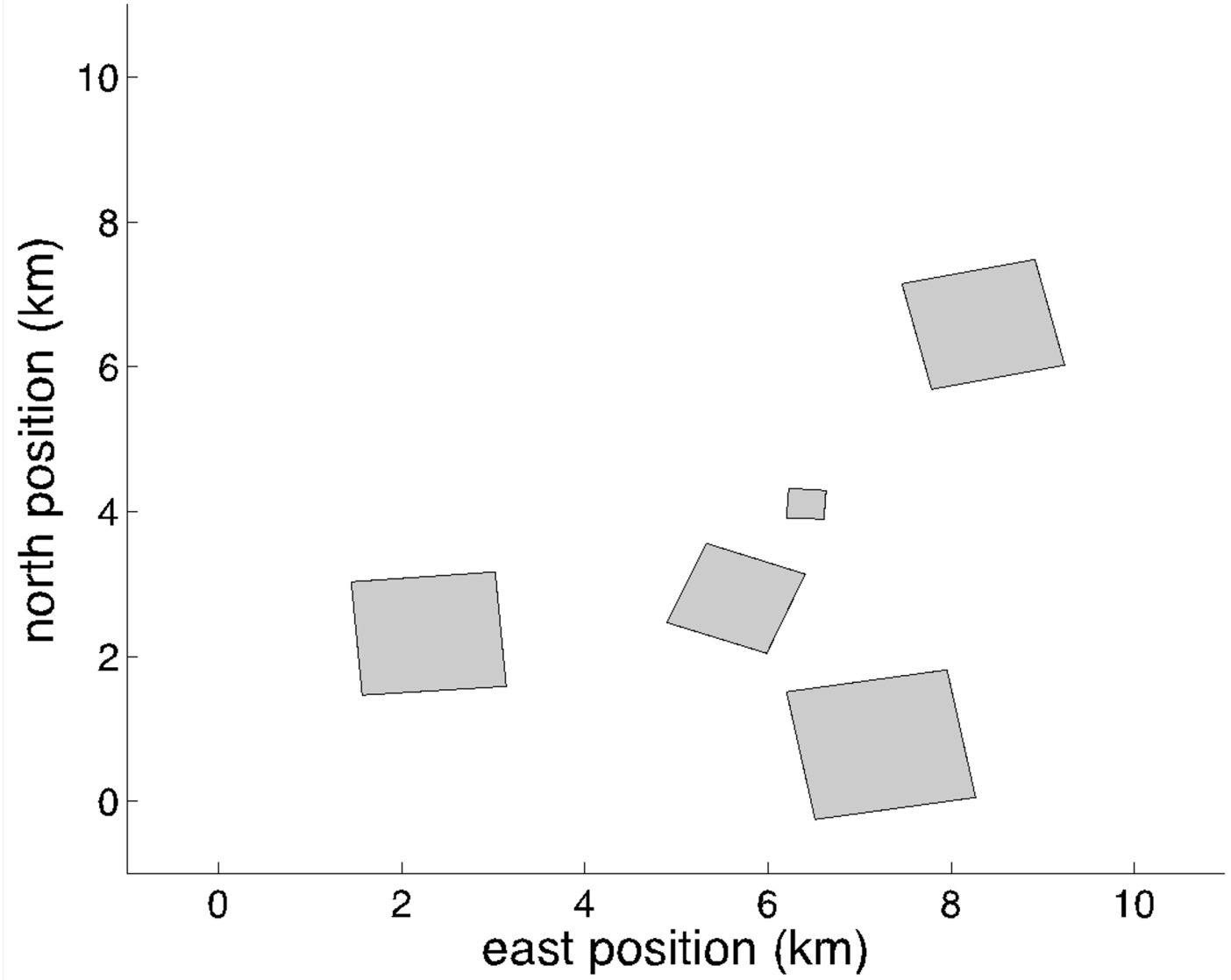
Voronoi -> *voronoi*

Dijkstra -> *shortestpath*

Voronoi Path Planning Result



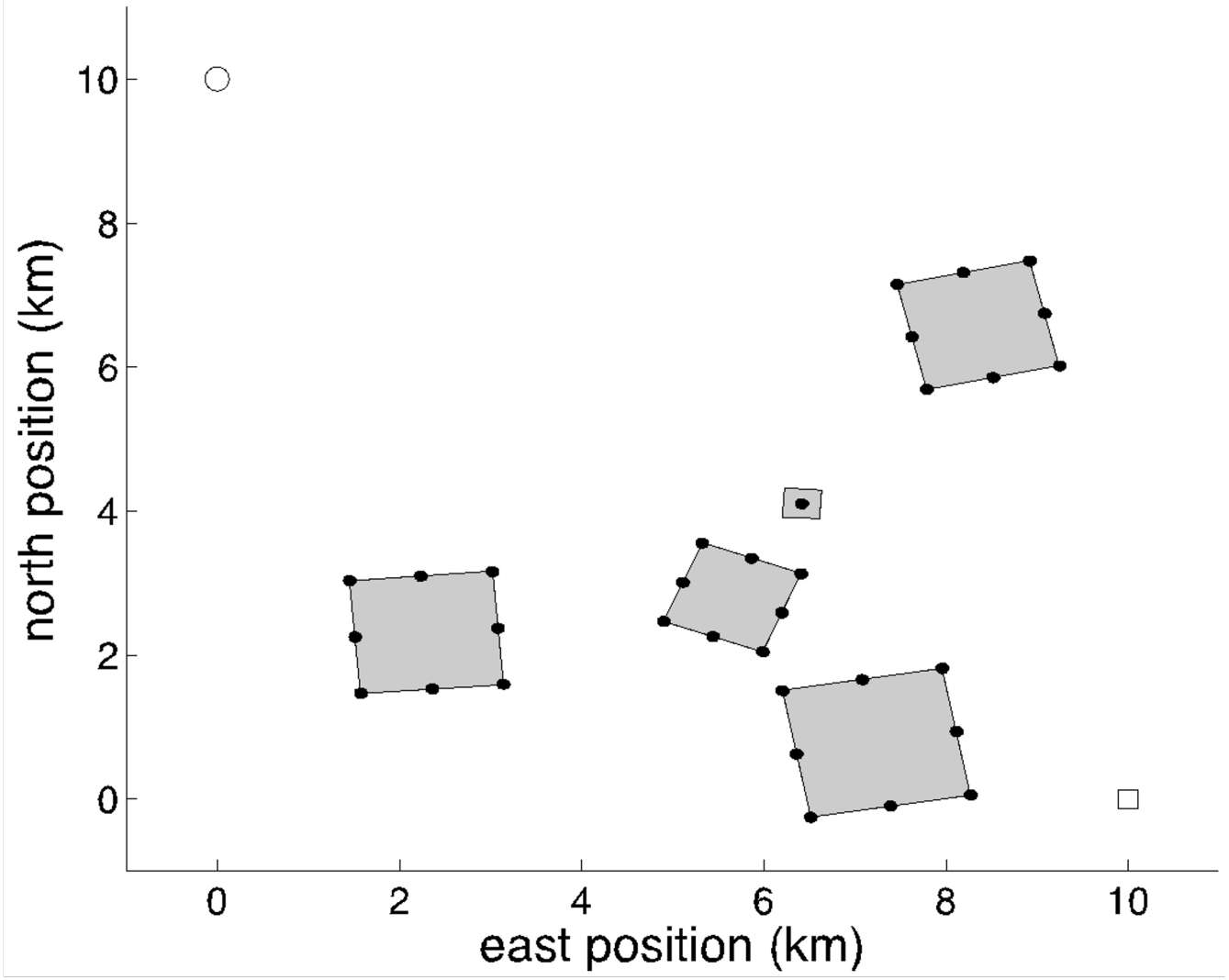
Voronoi Path Planning – Non-point Obstacles



- How do we handle solid obstacles?
- Point obstacles at center of spatial obstacles won't provide safe path options around obstacles

Non-point Obstacles – Step 1

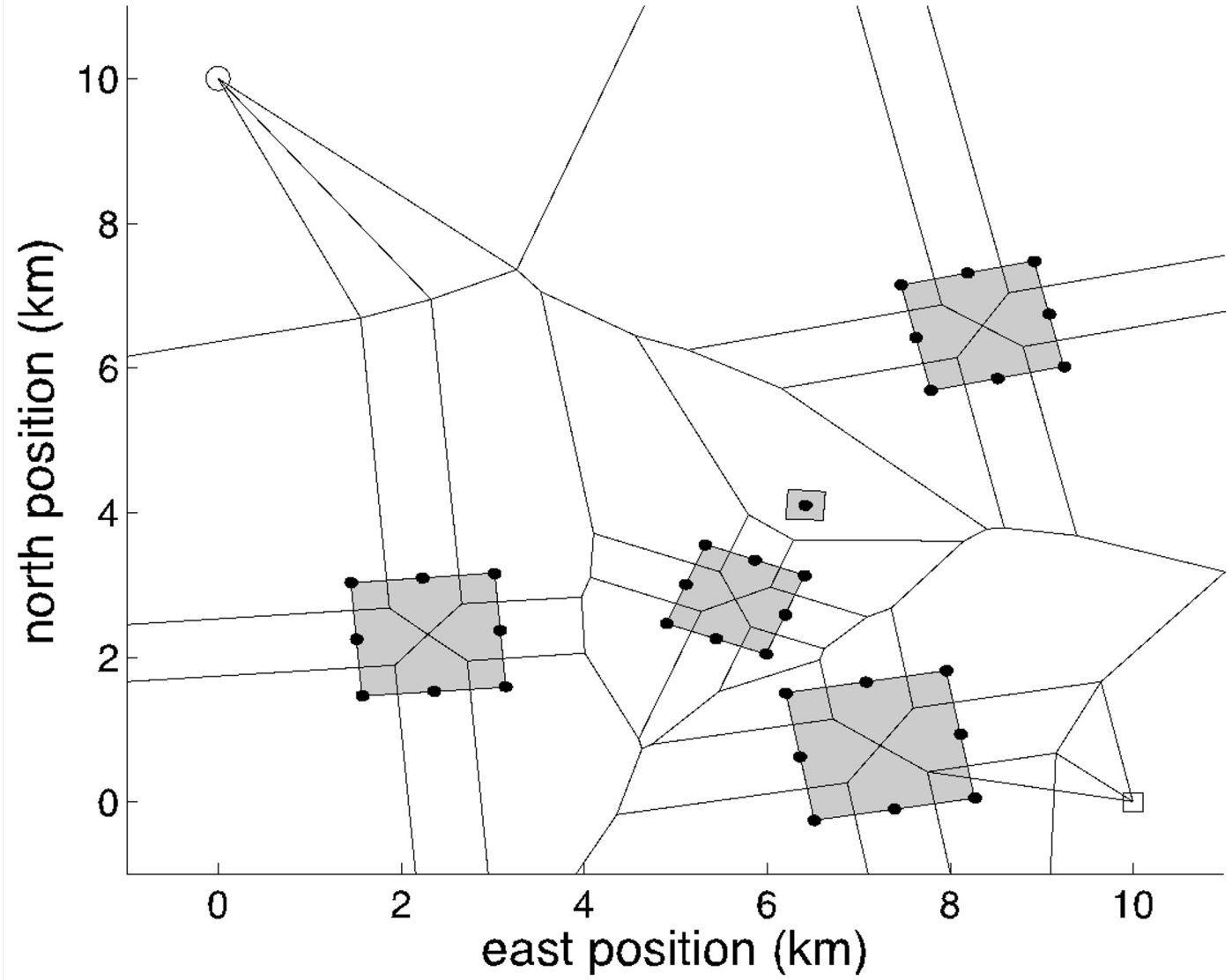
Insert points around perimeter of obstacles



Non-point Obstacles – Step 2

Construct Voronoi graph

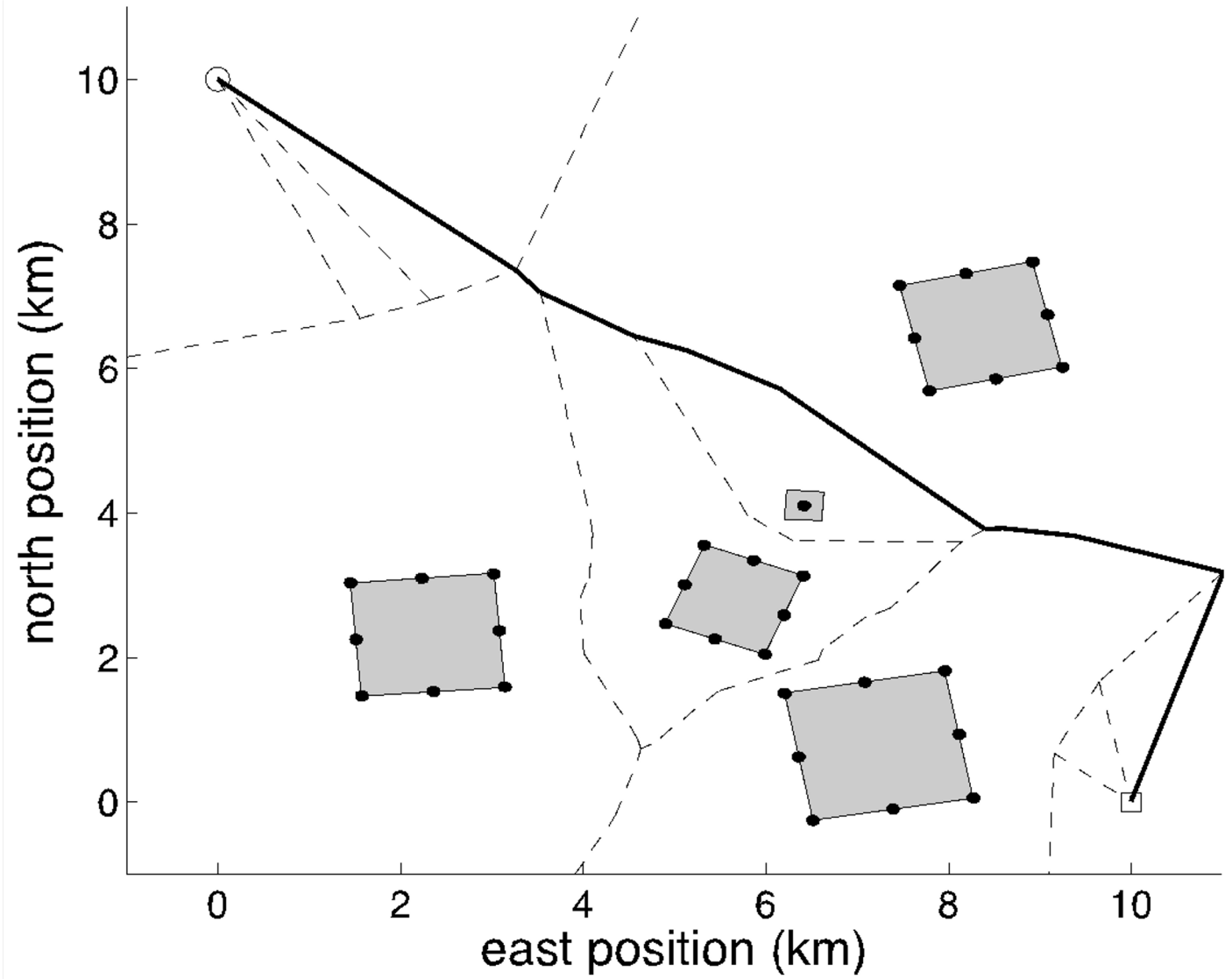
Note infeasible path edges inside obstacles



Non-point Obstacles – Step 3

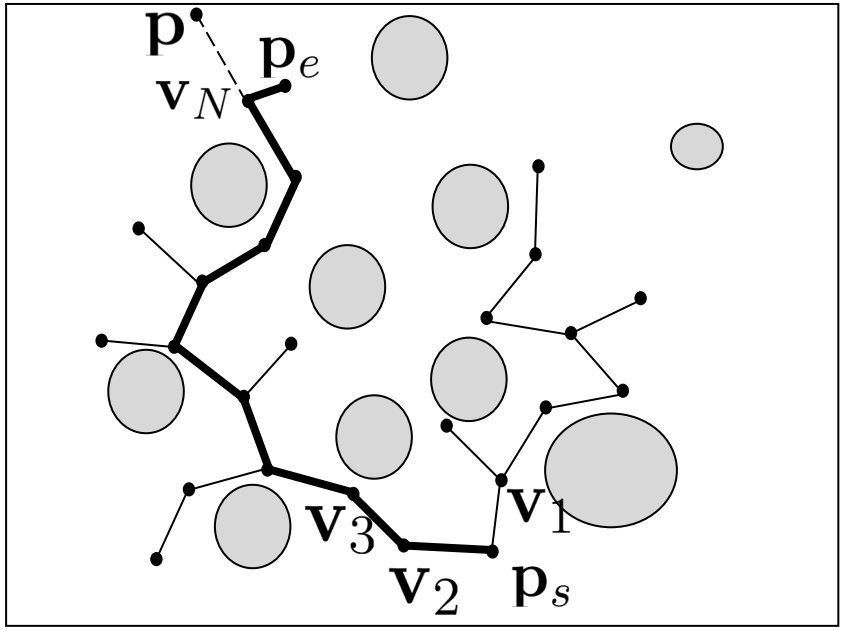
Remove infeasible path edges from graph

Search graph for best path



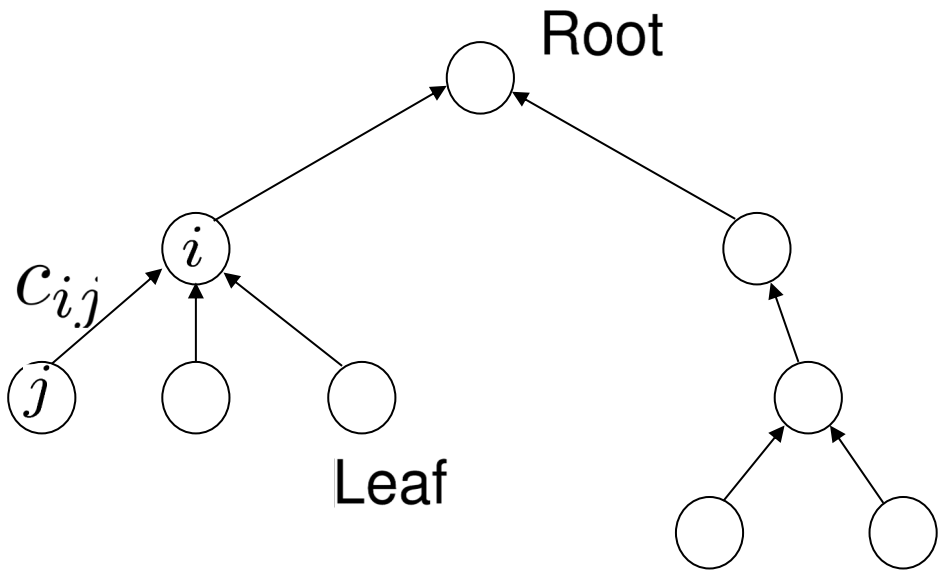
Rapidly Exploring Random Trees (RRT)

- Exploration algorithm that randomly, but uniformly explores search space
- Can accommodate vehicles with complicated, nonlinear dynamics
- Obstacles represented in a terrain map
- Map can be queried to detect possible collisions
- RRTs can be used to generate a single feasible path or a tree with many feasible paths that can be searched to determine the best one
- If algorithm runs long enough, the optimal path through the terrain will be found



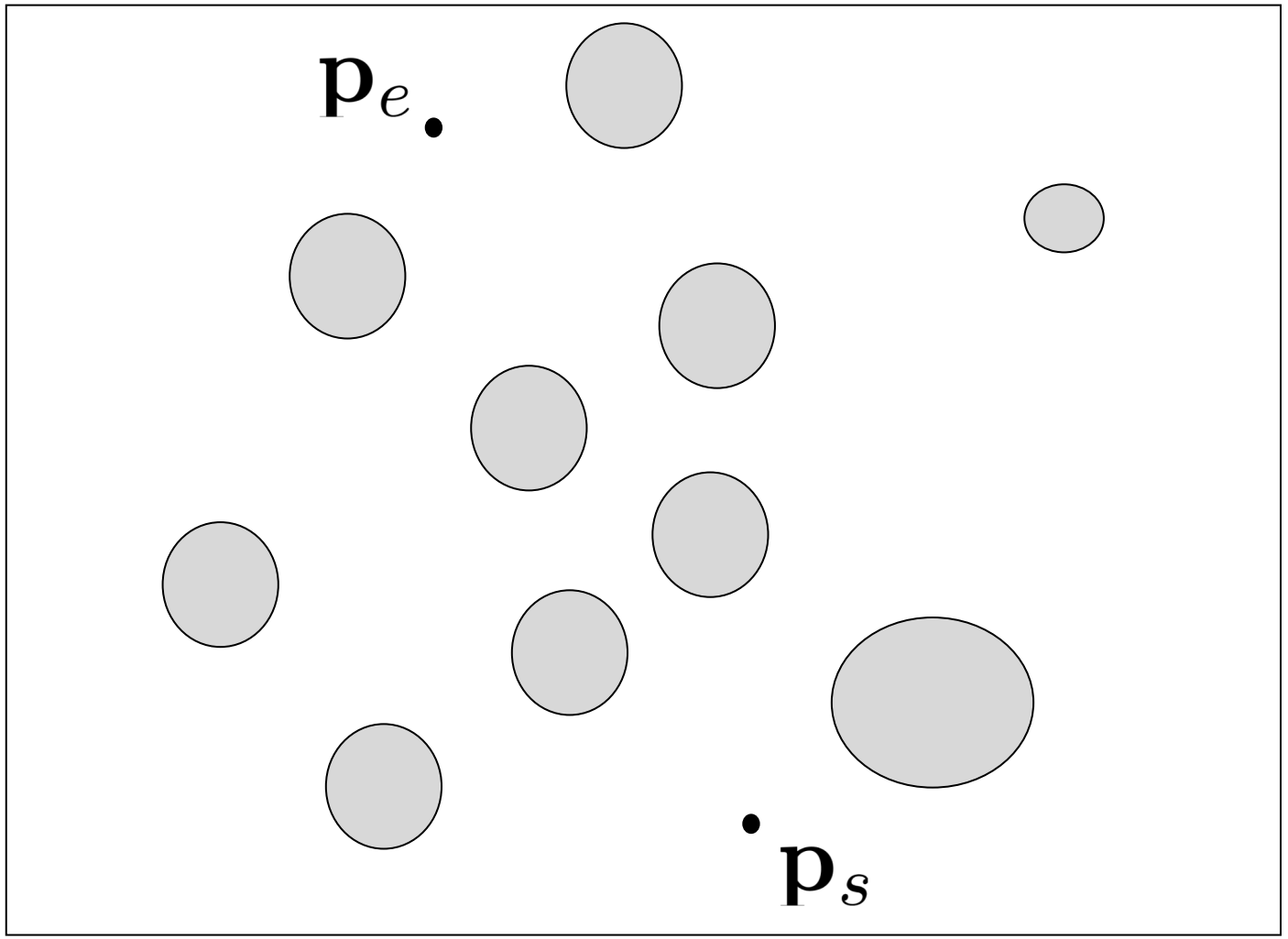
RRT Tree Structure

- RRT algorithm is implemented using tree data structure
- Tree is special case of directed graph
- Edges are directed from child node to parent
- Every node has one parent, except root
- Nodes represent physical states or configurations
- Edges represent feasible paths between states
- Each edge has cost associated traversing feasible path between states



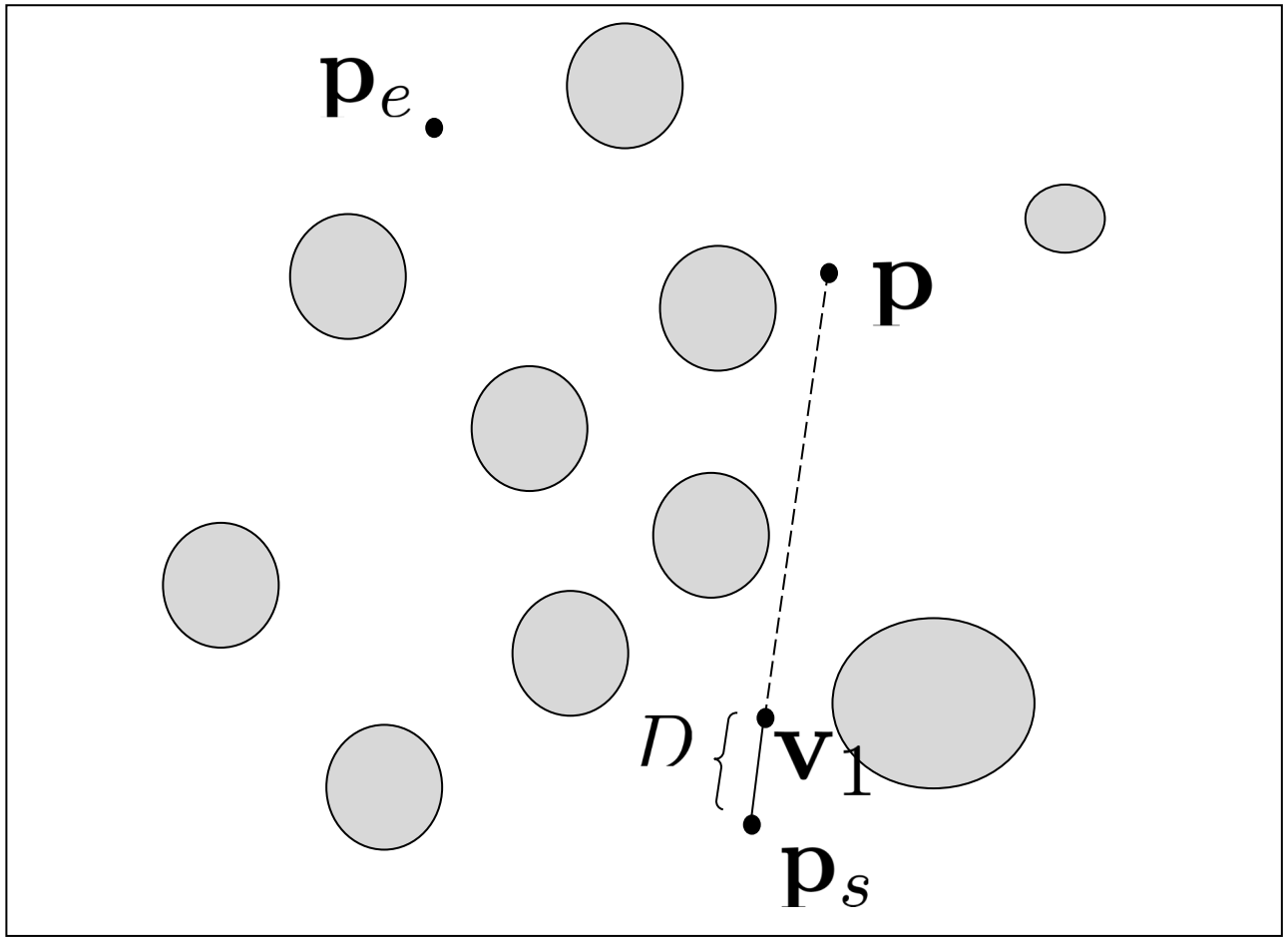
RRT Algorithm

- Algorithm initialized
  - start node
  - end node
  - terrain/obstacle map



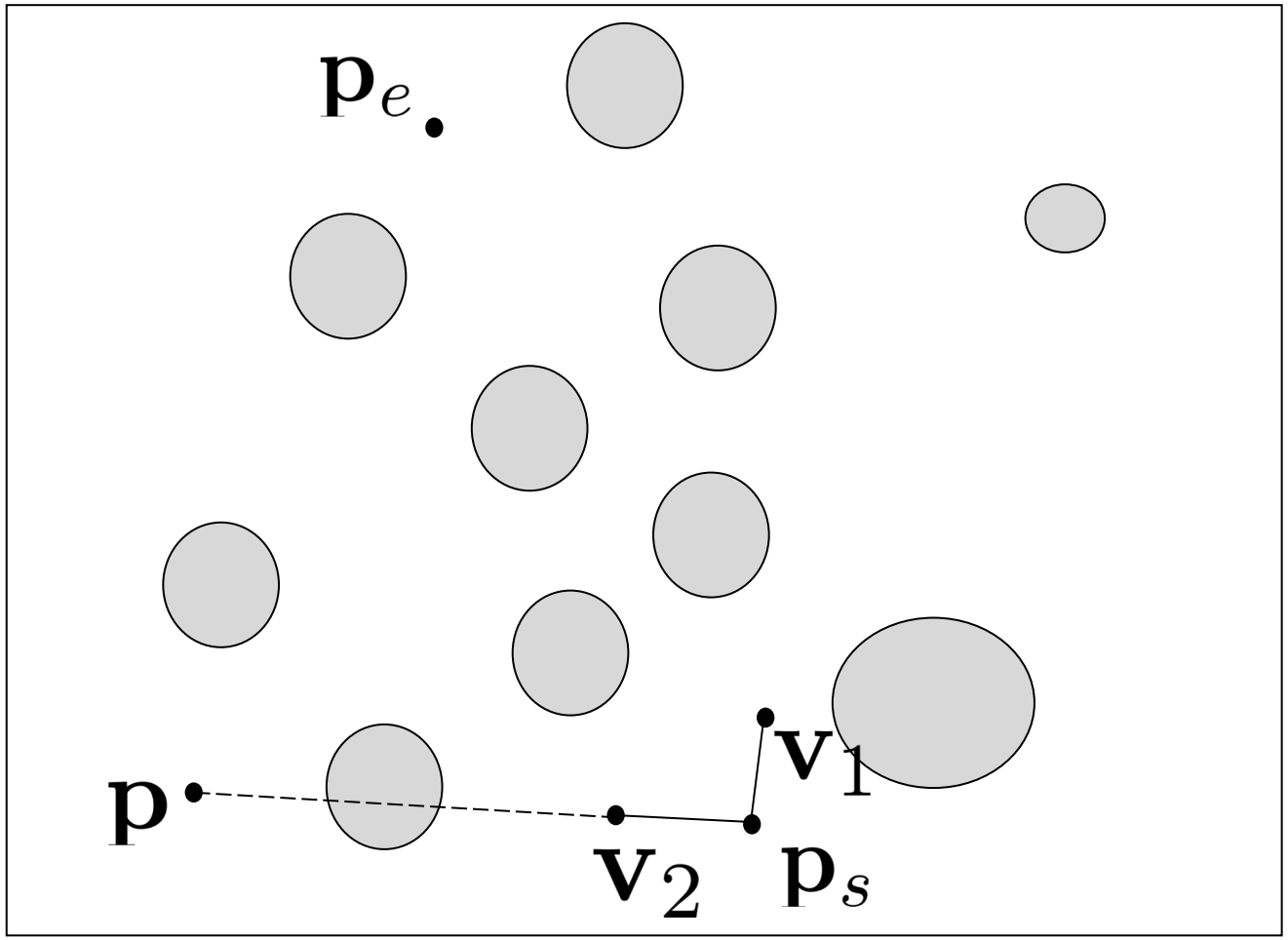
RRT Algorithm

- Randomly select configuration $\mathbf{p}$ in workspace
- Select new configuration $\mathbf{v}_1$ fixed distance D from start point along line connecting $\mathbf{p}_s$ and $\mathbf{p}$
- Check new segment for collisions
- If collision-free, insert $\mathbf{v}_1$ intc tree.



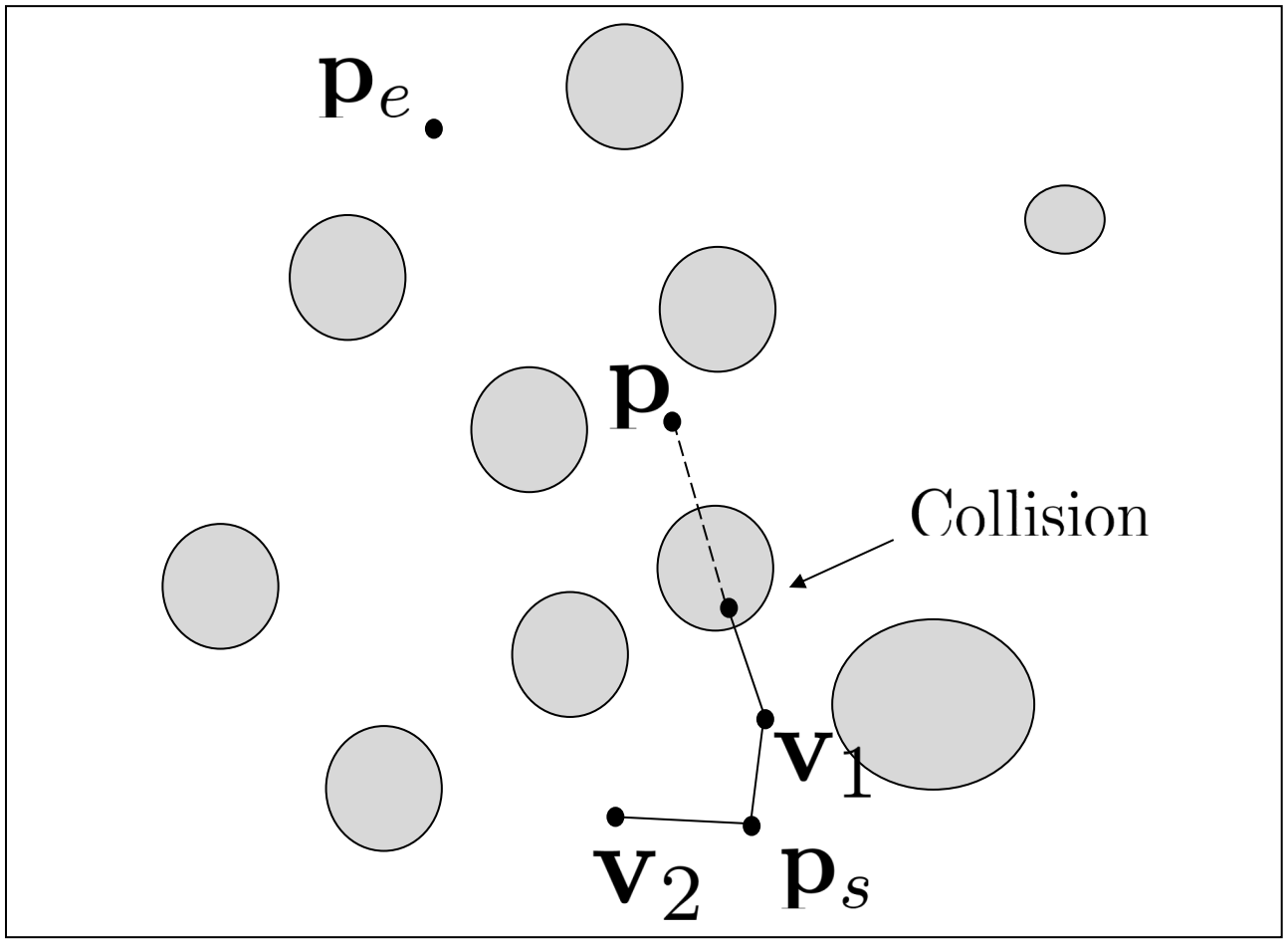
RRT Algorithm

- Generate random configuration $\mathbf{p}$ in workspace
- Search tree to find node closest to $\mathbf{p}$
- From node closest to $\mathbf{p}$, move distance D along line connecting node and $\mathbf{p}$ to establish configuration $\mathbf{v}_2$
- Check new segment for collisions
- If collision-free, insert $\mathbf{v}_2$ intc tree



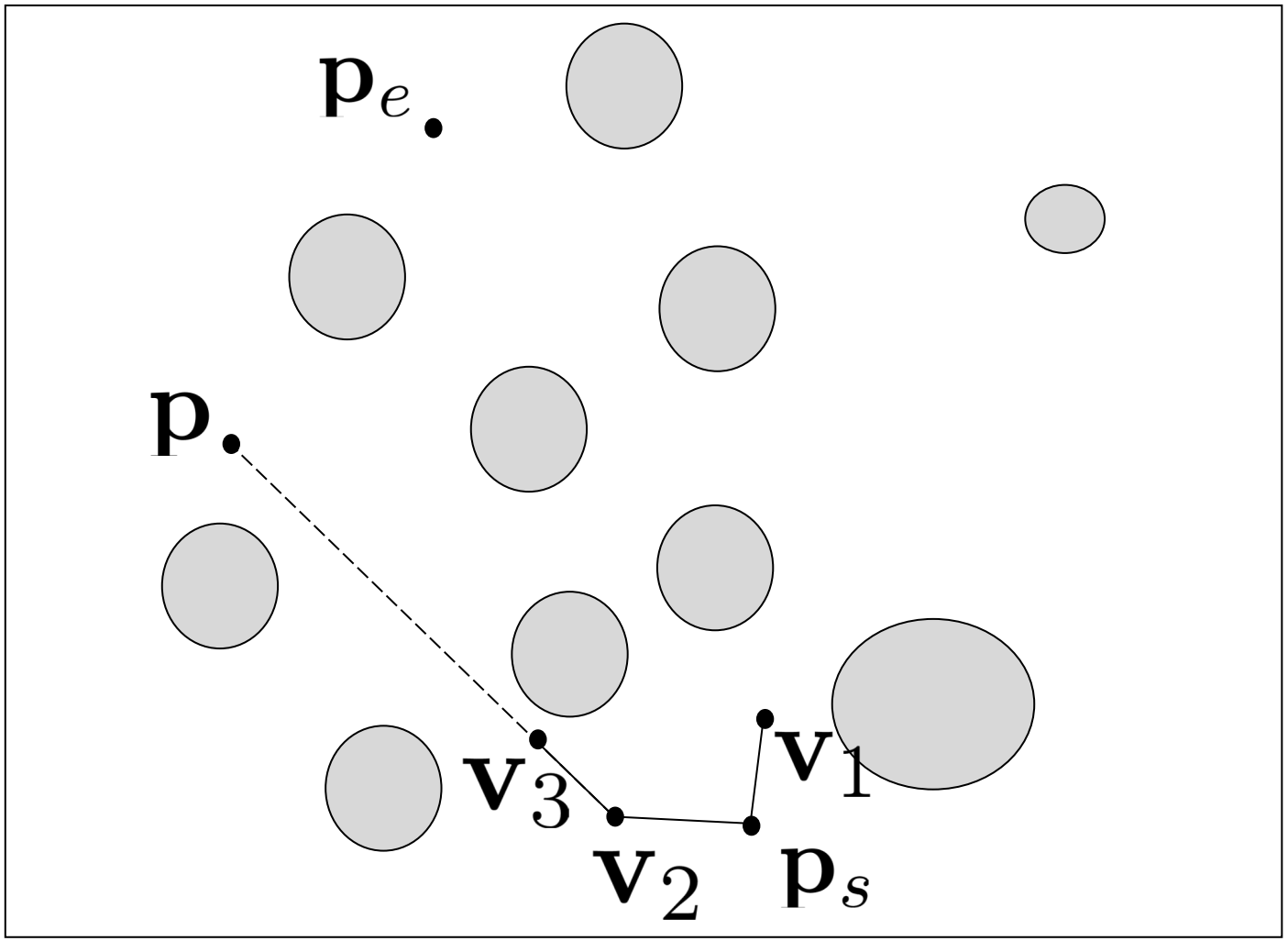
RRT Algorithm

- Generate random configuration $\mathbf{p}$ in workspace
- Search tree to find node closest to $\mathbf{p}$
- From node closest to $\mathbf{p}$, move distance D along line connecting node and $\mathbf{p}$ to establish new node
- Check new segment for collisions
- If collision-free, insert new node into tree



RRT Algorithm

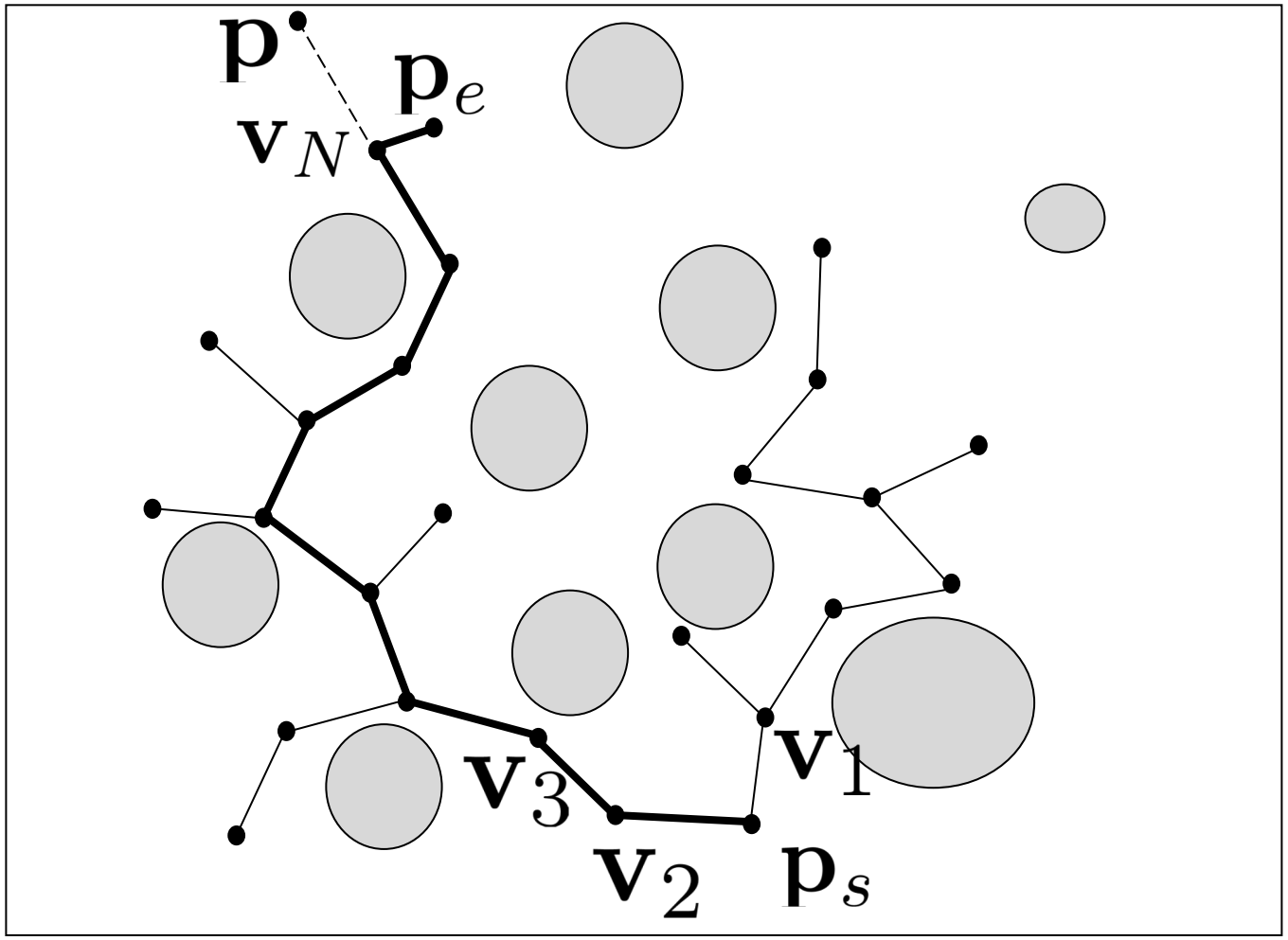
- Continue adding nodes and checking for collisions



RRT Algorithm

- Continue until node is generated that is within distance D of end node

- At this point, terminate algorithm or search for additional feasible paths



RRT Algorithm

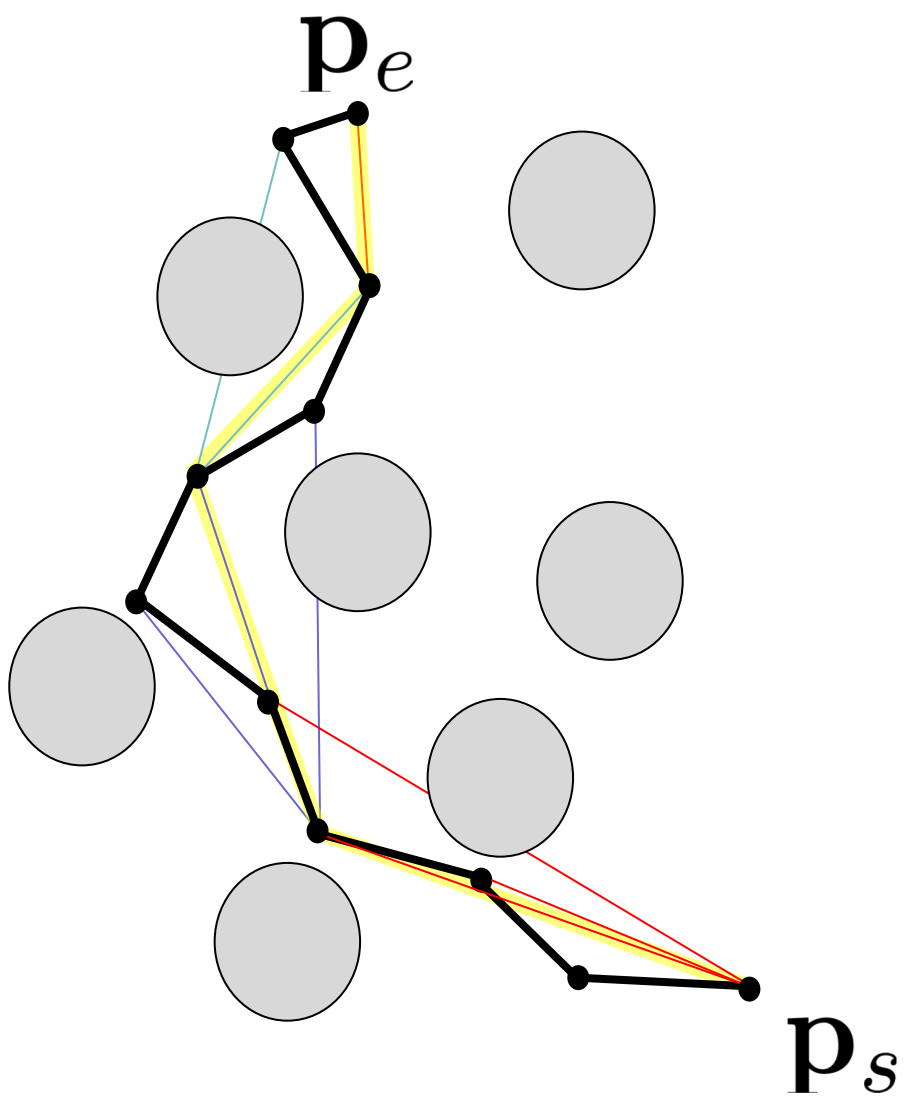
Algorithm 12 Plan RRT Path: $\mathcal{W} = \text{planRRT}(\mathcal{T}, \mathbf{p}_s, \mathbf{p}_e)$

Input: Terrain map $\mathcal{T}$, start configuration $\mathbf{p}_s$, end configuration $\mathbf{p}_e$

Initialize RRT graph $G = (V, E)$ as $V = \{\mathbf{p}_s\}$, $E = \emptyset$.
while The end node $\mathbf{p}_e$ is not connected to G, i.e., $\mathbf{p}_e \notin V$ **do**
    $\mathbf{p} \leftarrow \text{generateRandomConfiguration}(\mathcal{T})$
    $\mathbf{v}^* \leftarrow \text{findClosestConfiguration}(\mathbf{p}, V)$
    $\mathbf{v}^+ \leftarrow \text{planPath}(\mathbf{v}^*, \mathbf{p}, D)$
    if $\text{existFeasiblePath}(\mathcal{T}, \mathbf{v}^*, \mathbf{v}^+)$ **then**
        Update graph $G = (V, E)$ as $V \leftarrow V \bigcup \{\mathbf{v}^+\}$,
      $E \leftarrow E \bigcup \{(\mathbf{v}^*, \mathbf{v}^+)\}$
        Update edge costs as $C[(\mathbf{v}^*, \mathbf{v}^+)] \leftarrow \text{pathLength}(\mathbf{v}^*, \mathbf{v}^+)$
        if $\text{existFeasiblePath}(\mathcal{T}, \mathbf{v}^+, \mathbf{p}_e)$ **then**
            Update graph $G = (V, E)$ as $V \leftarrow V \bigcup \{\mathbf{p}_e\}$,
      $E \leftarrow E \bigcup \{(\mathbf{v}^+, \mathbf{p}_e)\}$
            Update edge costs as $C[(\mathbf{v}^+, \mathbf{p}_e)] \leftarrow \text{pathLength}(\mathbf{v}^+, \mathbf{p}_e)$
        end if
    end if
end while
$\mathcal{W} = \text{findShortestPath}(G, C)$ **return** $\mathcal{W}$

Path Smoothing

- Start with initial point (1)
- Make connections to subsequent points in path (2), (3), (4) …
- When connection collides with obstacle, add previous waypoint to smoothed path
- Continue smoothing from this point to end of path



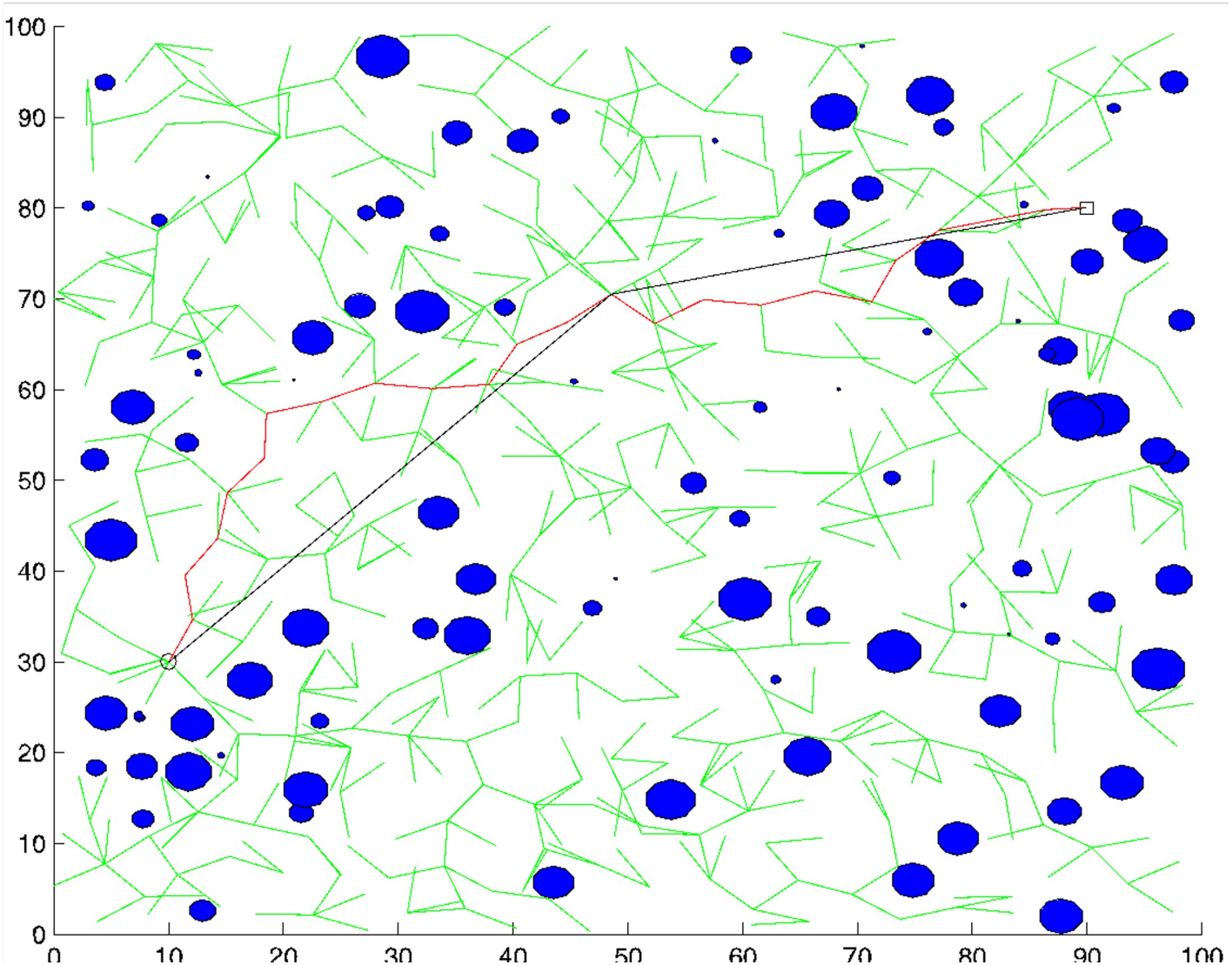
Path Smoothing Algorithm

Algorithm 13 Smooth RRT Path: $(\mathcal{W}_s, C_s) = \text{smoothRRT}(\mathcal{T}, \mathcal{W}, C)$

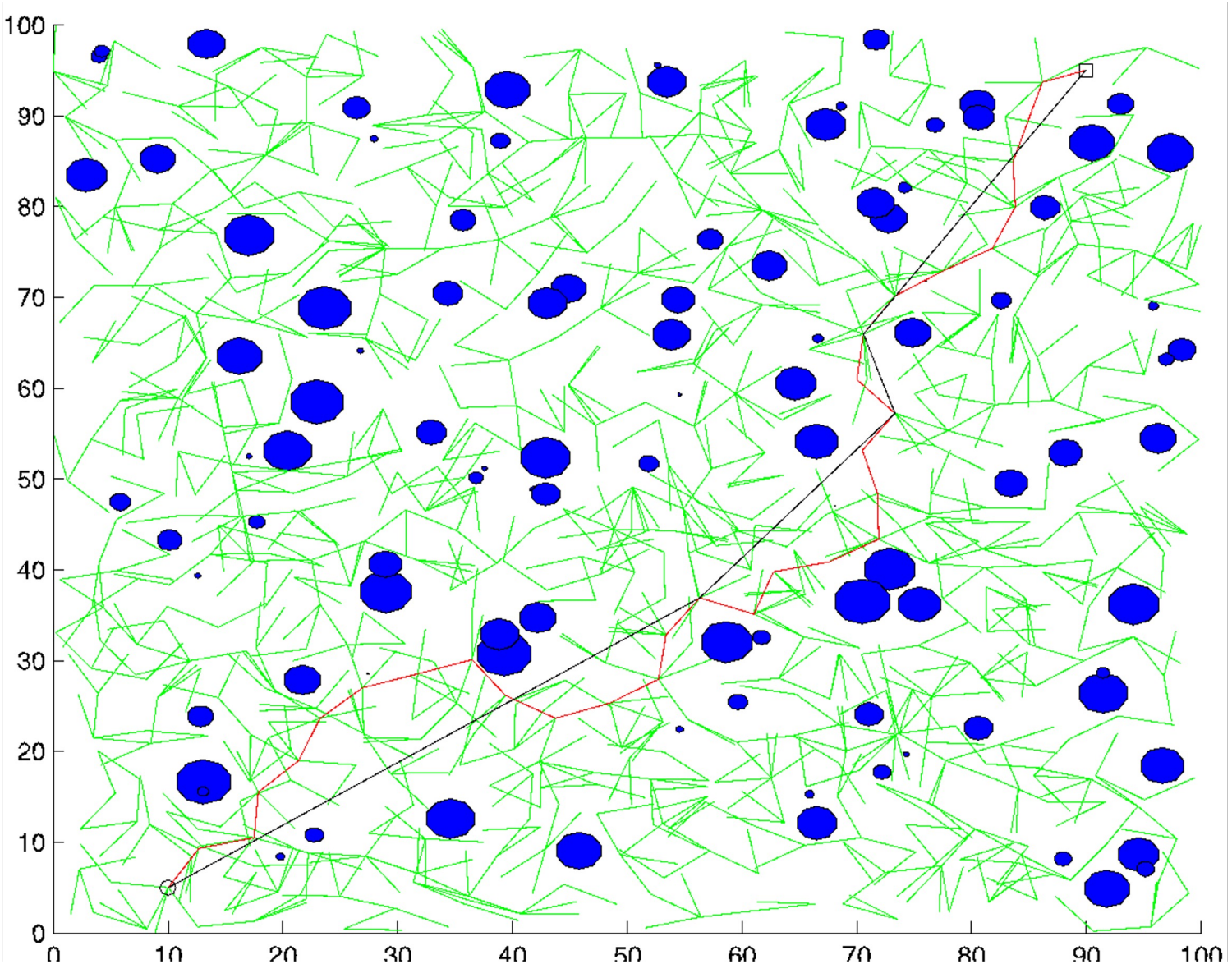
Input: Terrain map $\mathcal{T}$, waypoint path $\mathcal{W} = \{\mathbf{w}_1, \ldots, \mathbf{w}_N\}$, cost matrix C

Initialized smoothed path $\mathcal{W}_s \leftarrow \{\mathbf{w}_1\}$
Initialize pointer to current node in $\mathcal{W}_s$: $i \leftarrow 1$
Initialize pointer to next node in $\mathcal{W}$: $j \leftarrow 2$
while $j < N$ **do**
    $\mathbf{w}_s \leftarrow \text{getNode}(\mathcal{W}_s, i)$
    $\mathbf{w}^+ \leftarrow \text{getNode}(\mathcal{W}, j+1)$
    if $\text{existFeasiblePath}(\mathcal{T}, \mathbf{w}_s, \mathbf{w}^+) = \text{FALSE}$ **then**
        Get last node: $\mathbf{w} \leftarrow \text{getNode}(\mathcal{W}, j)$
        Add deconflicted node to smoothed path: $\mathcal{W}_s \leftarrow \mathcal{W}_s \bigcup \{\mathbf{w}\}$
        Update smoothed cost: $C_s[(\mathbf{w}_s, \mathbf{w})] \leftarrow \text{pathLength}(\mathbf{w}_s, \mathbf{w})$
        $i \leftarrow j$
    end if
    $j \leftarrow j + 1$
end while
Add last node from $\mathcal{W}$: $\mathcal{W}_s \leftarrow \mathcal{W}_s \bigcup \{\mathbf{w}_N\}$
Update smoothed cost: $C_s[(\mathbf{w}_i, \mathbf{w}_N)] \leftarrow \text{pathLength}(\mathbf{w}_i, \mathbf{w}_N)$ **return** $\mathcal{W}_s$

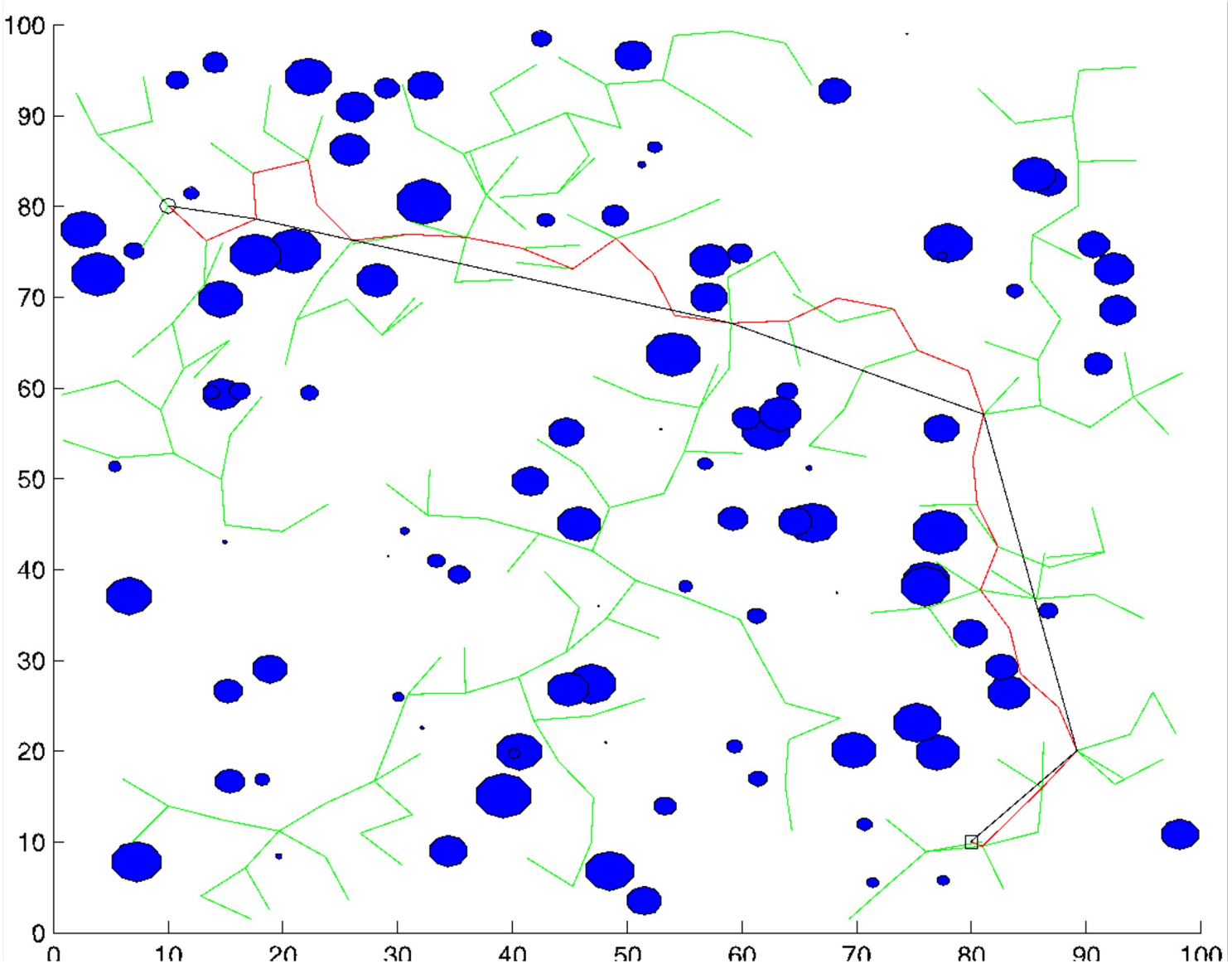
RRT Results



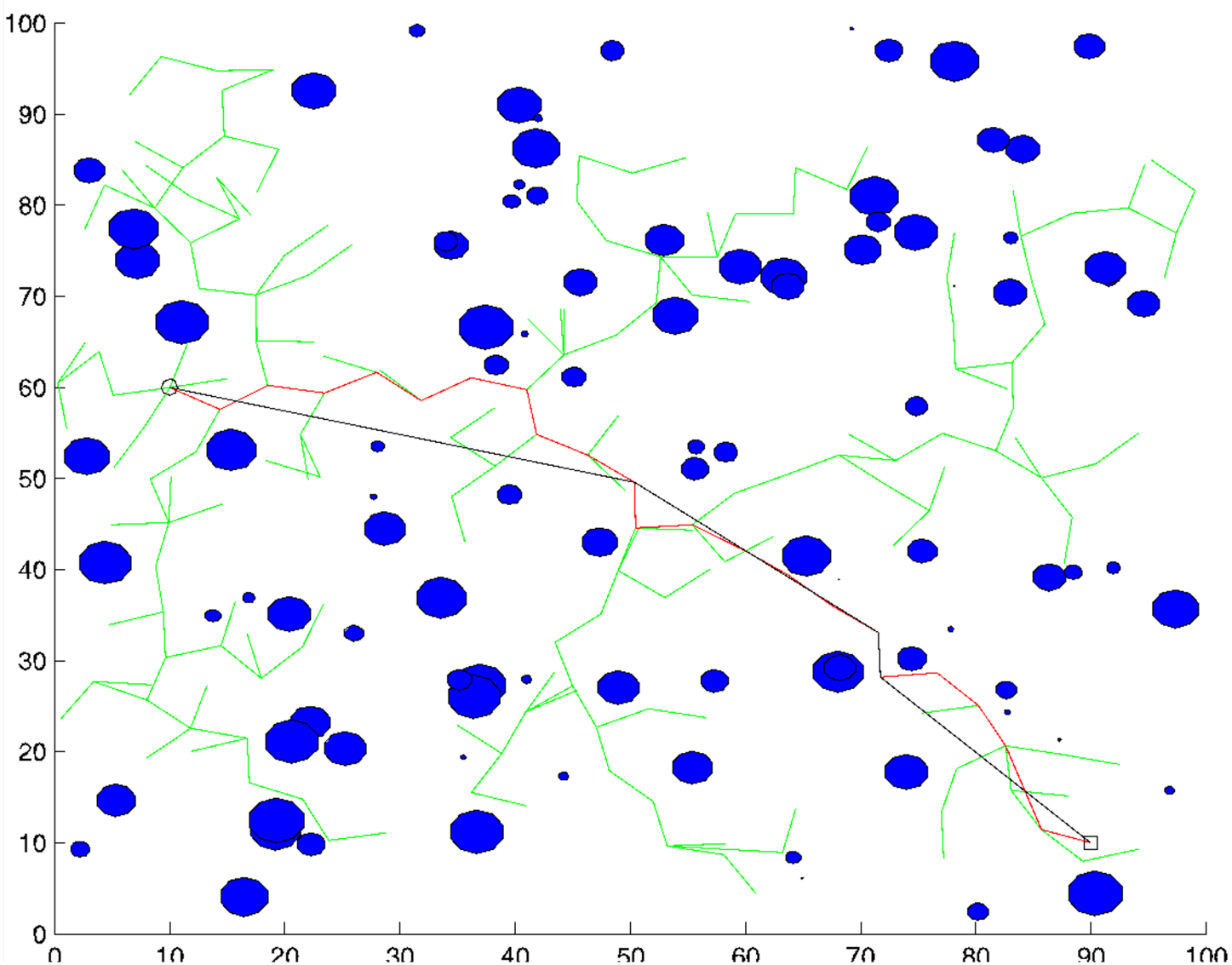
RRT Results



RRT Results

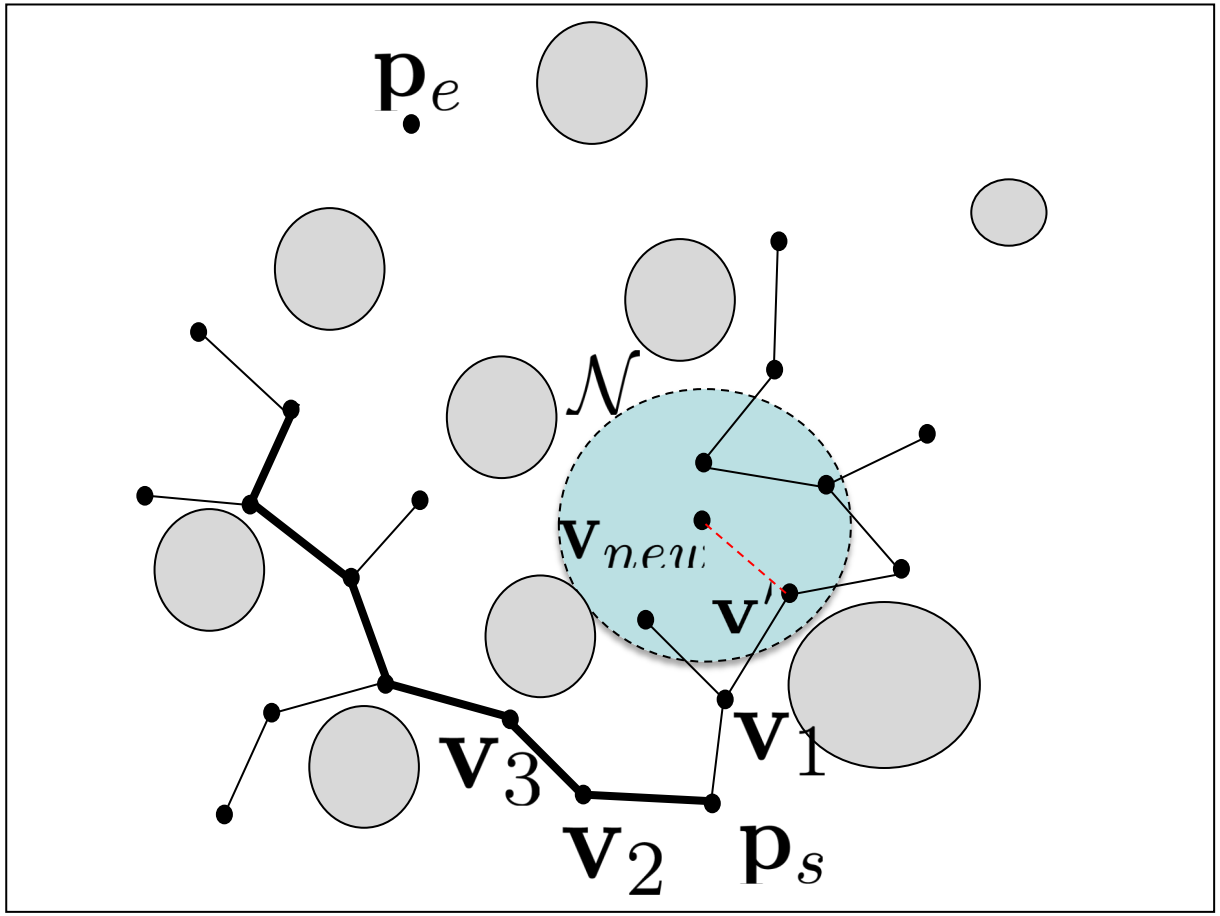


RRT Results



RRT* Algorithm – Extend Step

- Generate new potential node $\mathbf{v}_{new}$ identical to RRT.
- Instead of finding the closest node in the tree, find all nodes within neighborhood $\mathcal{N}$.
- Let $C(\mathbf{v})$ be the path cost from $\mathbf{p}_s$ to $\mathbf{v}$, and let $c(\mathbf{v}_1, \mathbf{v}_2)$ be the path cost from $\mathbf{v}_1$ to $\mathbf{v}_2$.
- Let $\mathbf{v}' = \arg\min_{\mathbf{v}\in\mathcal{N}} (C(\mathbf{v}) + c(\mathbf{v}_{new}, \mathbf{v}))$ (i.e., closest node in $\mathcal{N}$ to $\mathbf{v}_{new}$).
- Add node $V \leftarrow V \bigcup \{\mathbf{v}_{new}\}$.
- Add edge $E \leftarrow E \bigcup \{(\mathbf{v}', \mathbf{v}_{new})\}$.
- Set $C(\mathbf{v}_{new}) = C(\mathbf{v}') + c(\mathbf{v}_{new}, \mathbf{v})$.



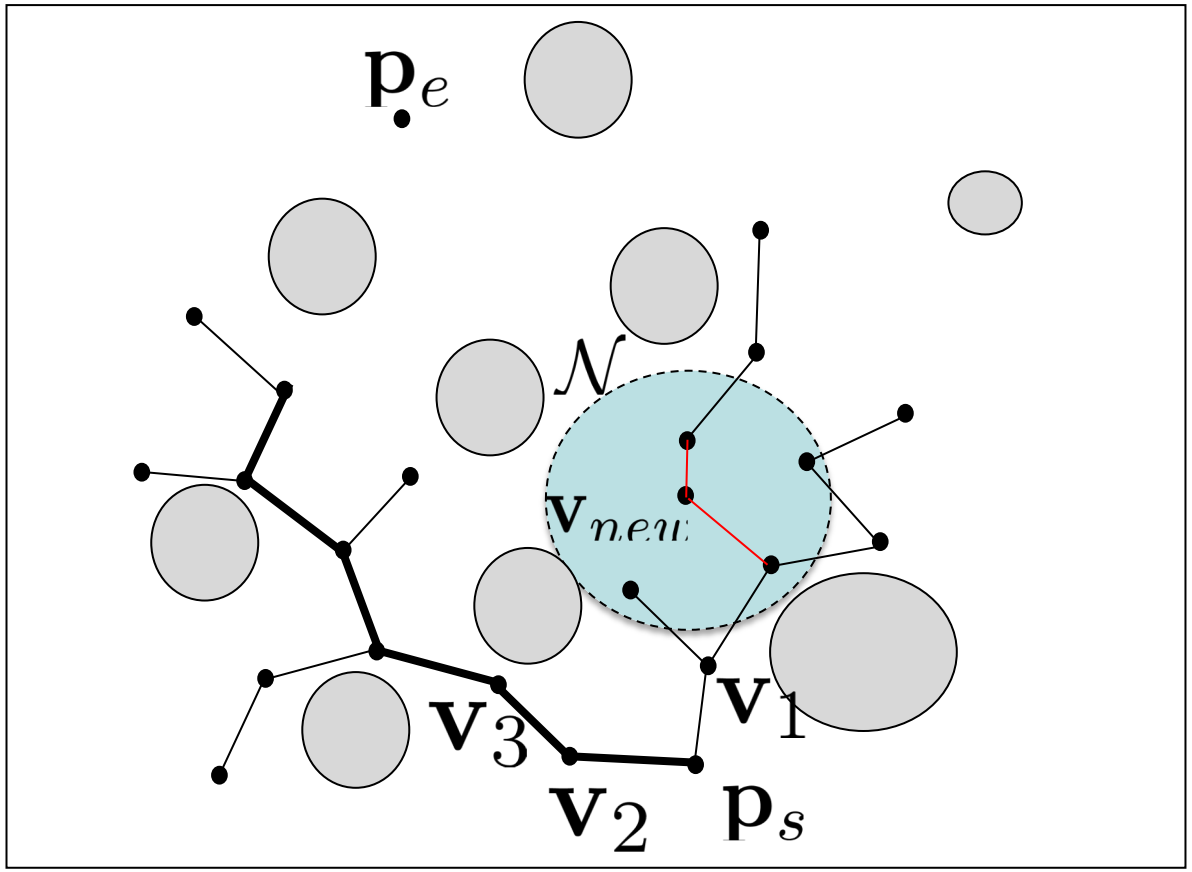
RRT* Algorithm – Re-wire Step

- Check all nodes in $\mathbf{v} \in \mathcal{N}$ to see if

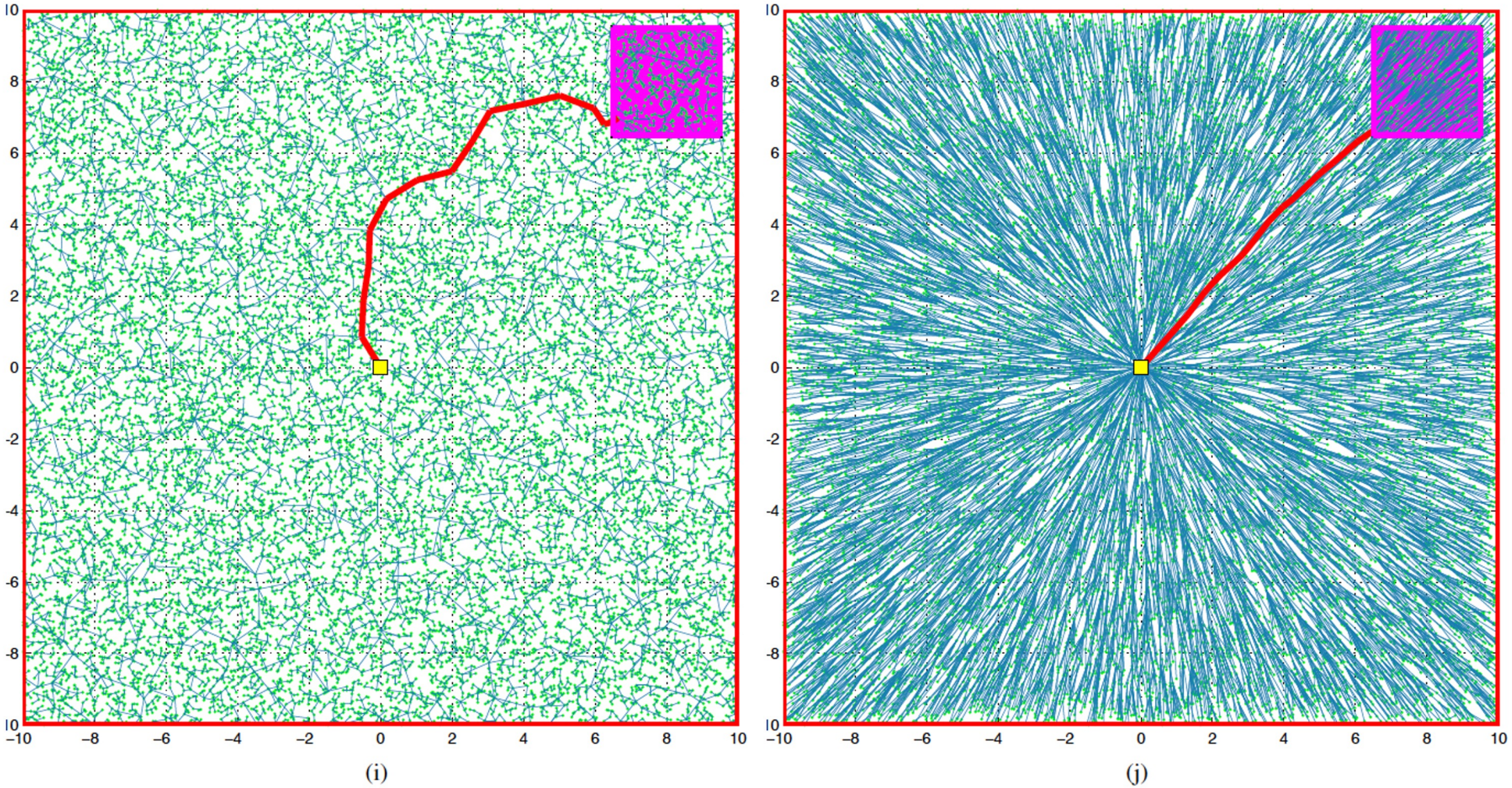
$$C(\mathbf{v}_{new}) + c(\mathbf{v}_{new}, \mathbf{v}) < C(\mathbf{v})$$

  i.e., if re-routing through $\mathbf{v}_{new}$ reduces the path length.

- If so, remove edge between $\mathbf{v}$ and its parent, and add edge between $\mathbf{v}$ and $\mathbf{v}_{new}$.

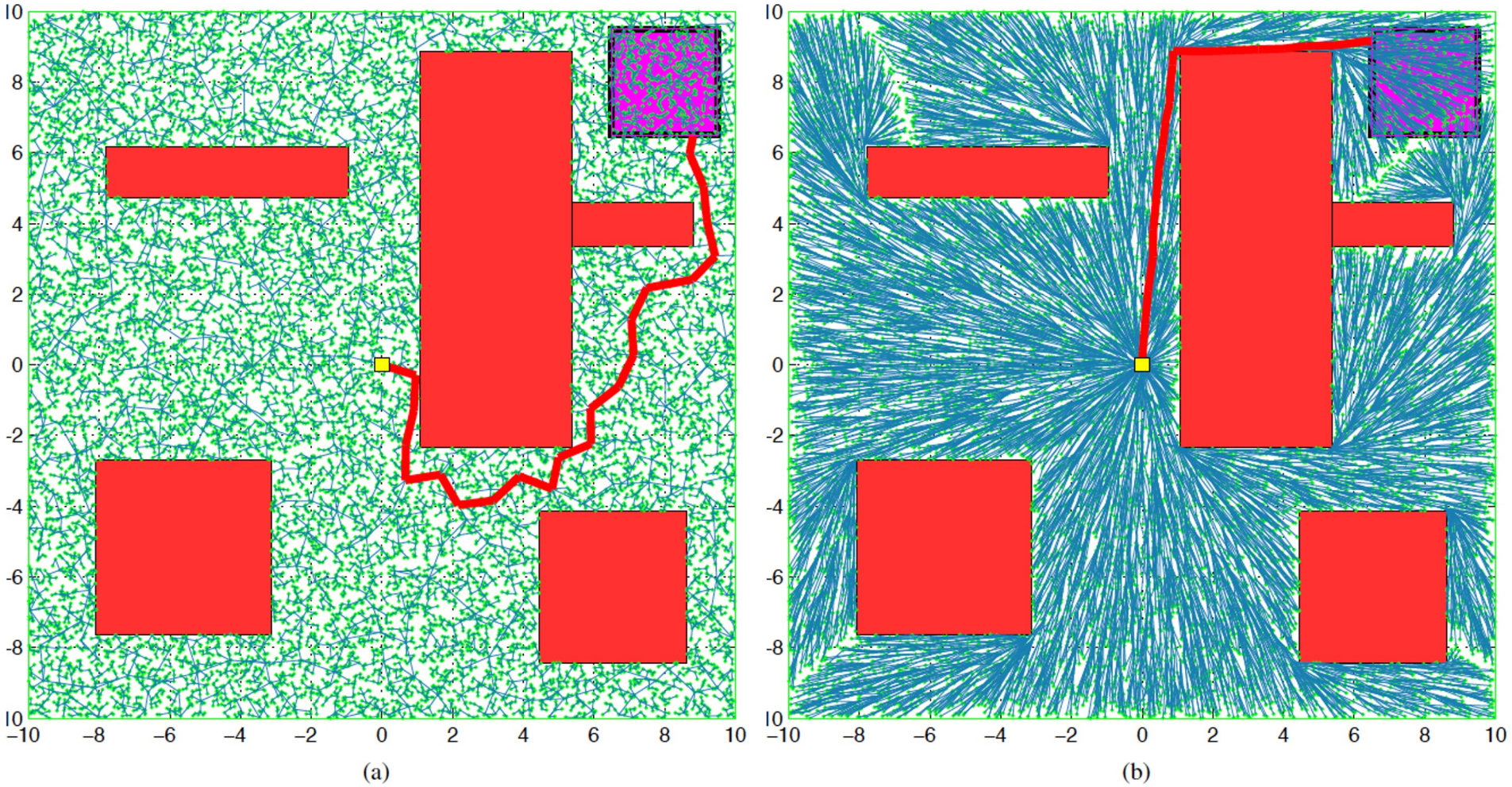


RRT vs. RRT*



From S. Karaman and E. Frazzoli, "Incremental Sampling-based Algorithms for Optimal Motion Planning," International Journal of Robotic Research, 2010.

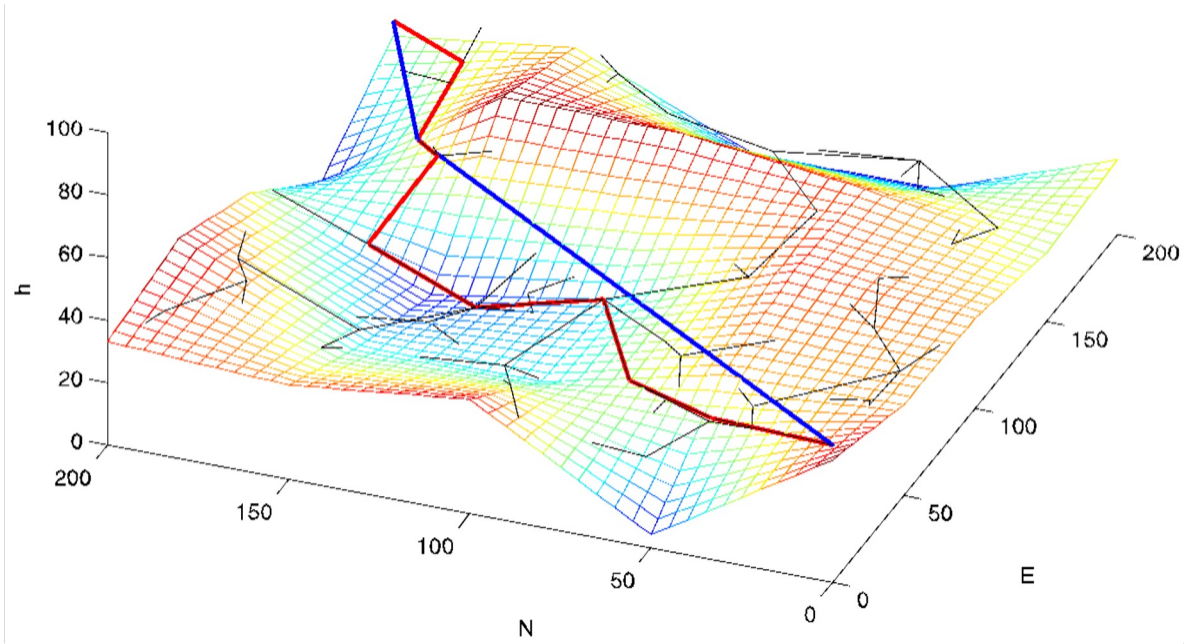
RRT vs. RRT*



From S. Karaman and E. Frazzoli, "Incremental Sampling-based Algorithms for Optimal Motion Planning," International Journal of Robotic Research, 2010.

RRT Path Planning Over 3D Terrain

- Assume terrain map can be queried to determine altitude of terrain at any north-east location
- Must be able to determine altitude for random configuration $\mathbf{p}$ in RRT algorithm
- Must be able to detect collisions with terrain – reject random candidate paths leading to collision
- Options:
  - random altitude within predetermined range
  - random selection of discrete altitudes in desired range
  - set altitude above ground level
- Test candidate paths to ensure flight path angles are feasible – reject if infeasible



RRT Algorithm – 3D Terrain

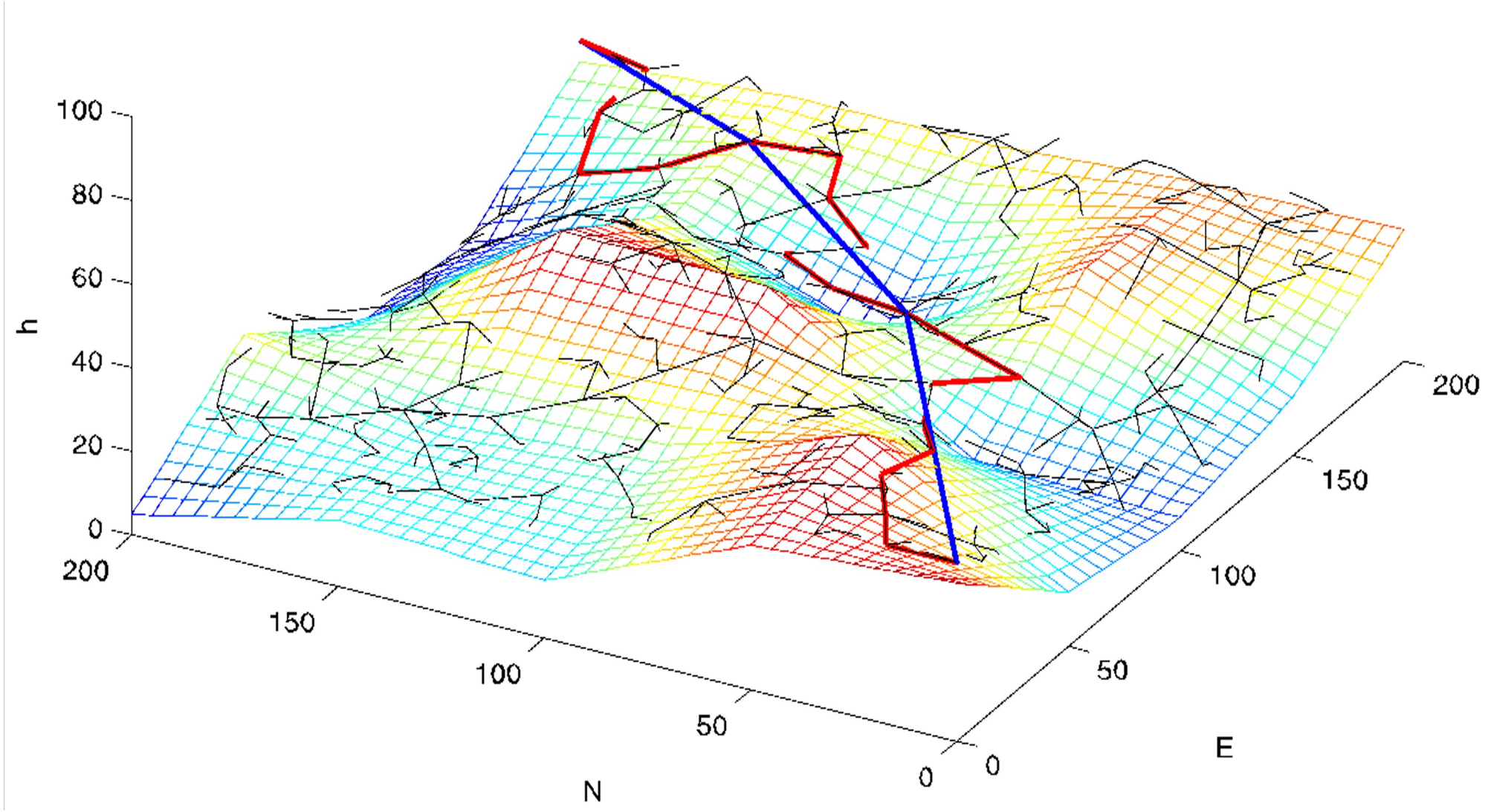
Algorithm 10 Plan RRT Path: $\mathcal{W} = \text{planRRT}(\mathcal{T}, \mathbf{p}_s, \mathbf{p}_e)$

Input: Terrain map $\mathcal{T}$, start configuration $\mathbf{p}_s$, end configuration $\mathbf{p}_e$

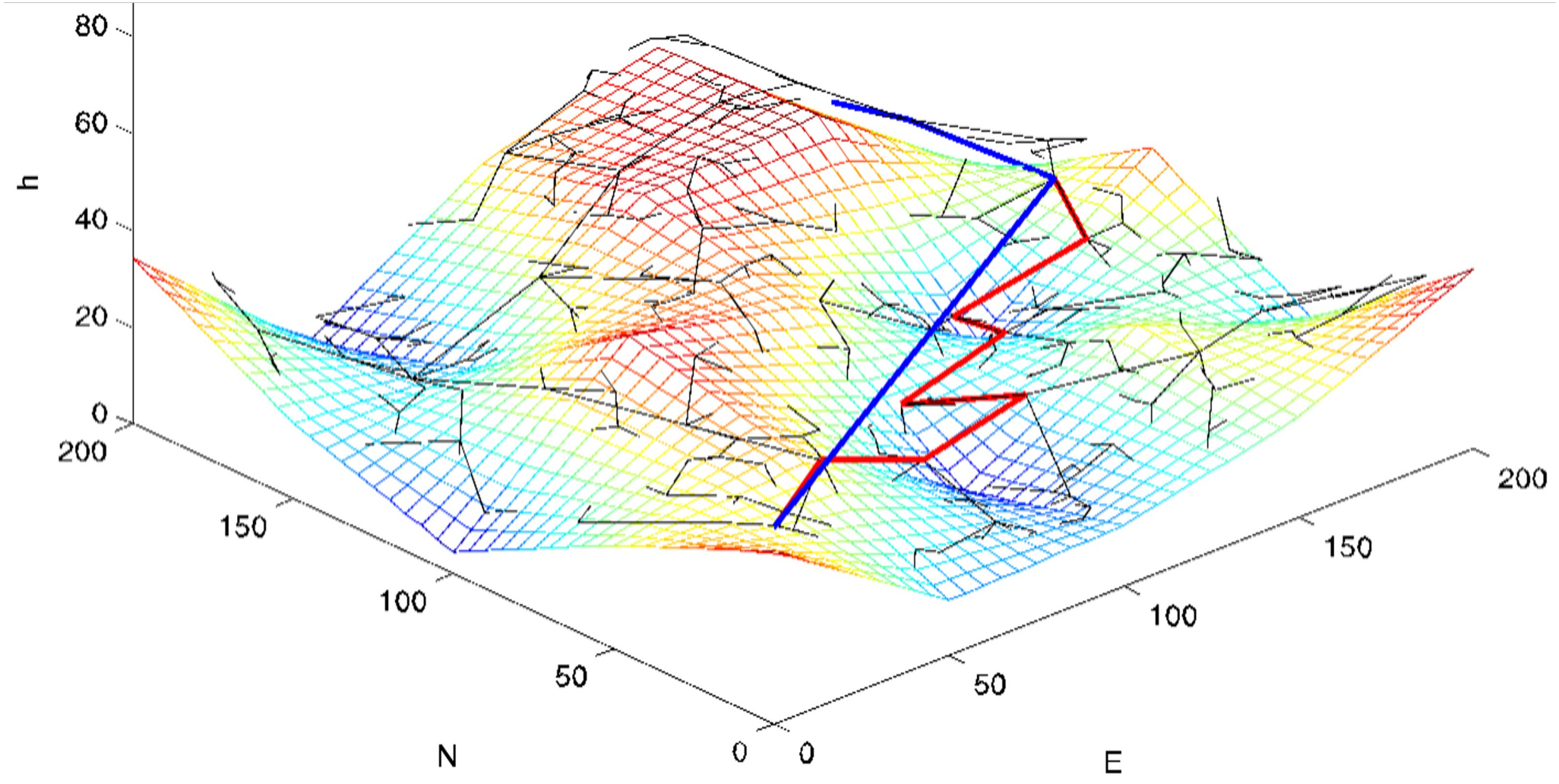
Initialize RRT graph $G = (V, E)$ as $V = \{\mathbf{p}_s\}$, $E = \emptyset$
while The end node $\mathbf{p}_e$ is not connected to G, i.e., $\mathbf{p}_e \notin V$ **do**
  $\mathbf{p} \leftarrow \text{generateRandomConfiguration}(\mathcal{T})$
  $\mathbf{v}^* \leftarrow \text{findClosestConfiguration}(\mathbf{p}, V)$
  $\mathbf{v}^+ \leftarrow \text{planPath}(\mathbf{v}^*, \mathbf{p}, D)$
  if $\text{existFeasiblePath}(\mathcal{T}, \mathbf{v}^*, \mathbf{v}^+)$ **then**
    Update graph $G = (V, E)$ as $V \leftarrow V \bigcup \{\mathbf{v}^+\}$, $E \leftarrow E \bigcup \{(\mathbf{v}^*, \mathbf{v}^+)\}$
    Update edge costs as $C[(\mathbf{v}^*, \mathbf{v}^+)] \leftarrow \text{pathLength}(\mathbf{v}^*, \mathbf{v}^+)$
  end if
  if $\text{existFeasiblePath}(\mathcal{T}, \mathbf{v}^+, \mathbf{p}_e)$ **then**
    Update graph $G = (V, E)$ as $V \leftarrow V \bigcup \{\mathbf{p}_e\}$ $E \leftarrow E \bigcup \{(\mathbf{v}^*, \mathbf{p}_e)\}$
    Update edge costs as $C[(\mathbf{v}^*, \mathbf{p}_e)] \leftarrow \text{pathLength}(\mathbf{v}^*, \mathbf{p}_e)$
  end if
end while
$\mathcal{W} = \text{findShortestPath}(G, C)$.
return $\mathcal{W}$

modify to test for collisions with terrain and flight path angle feasibility

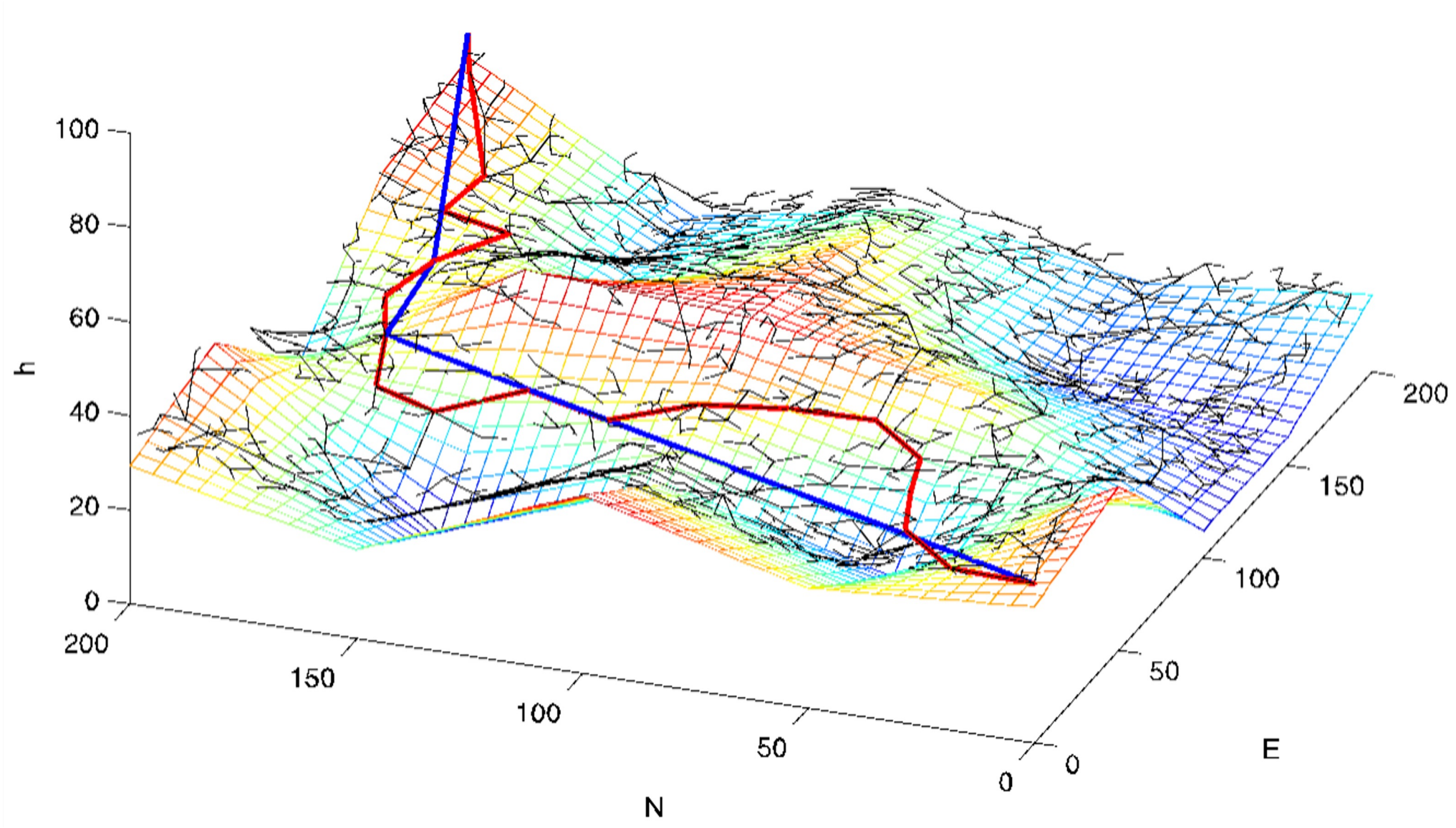
RRT 3D Terrain Results



RRT 3D Terrain Results



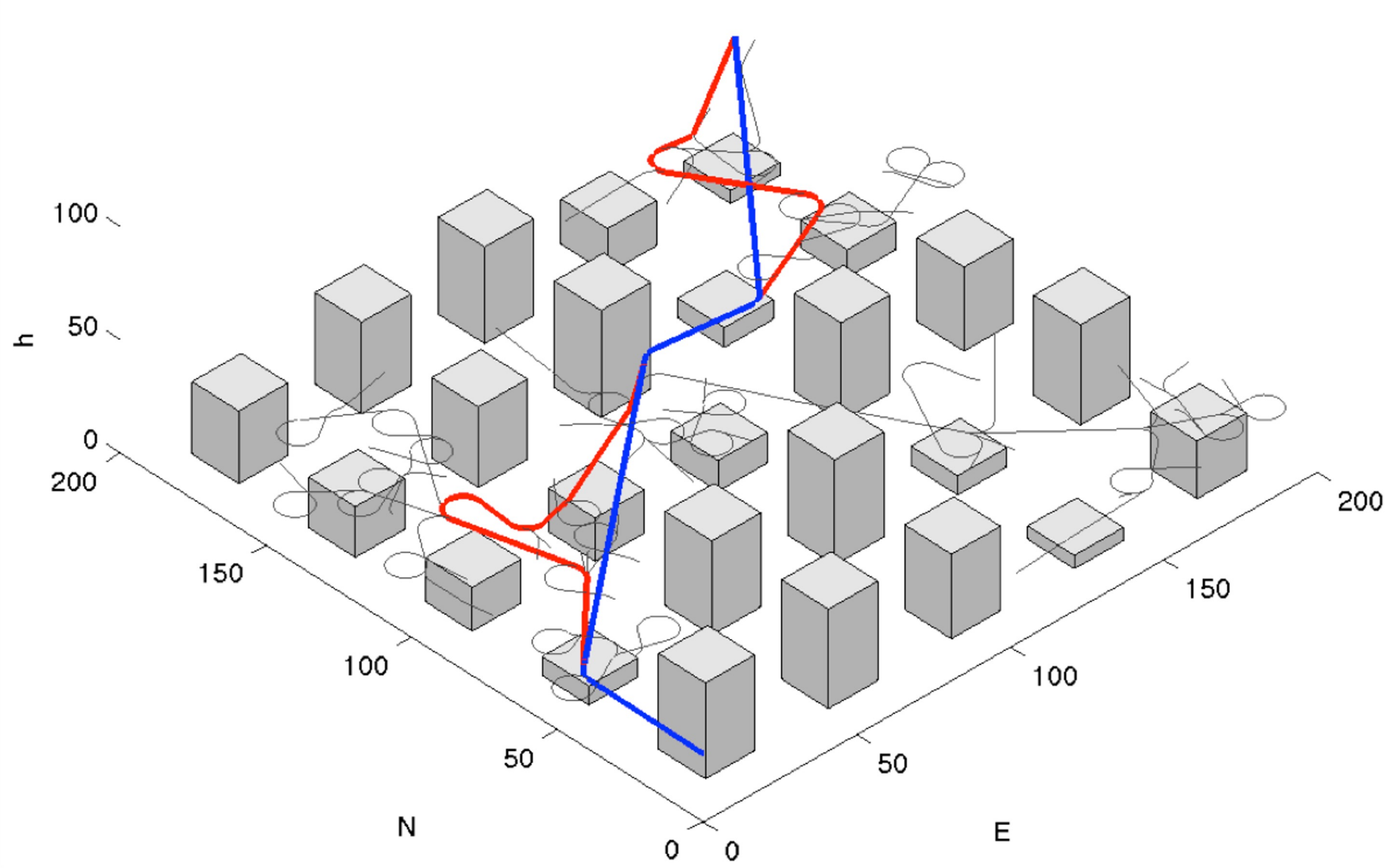
RRT 3D Terrain Results



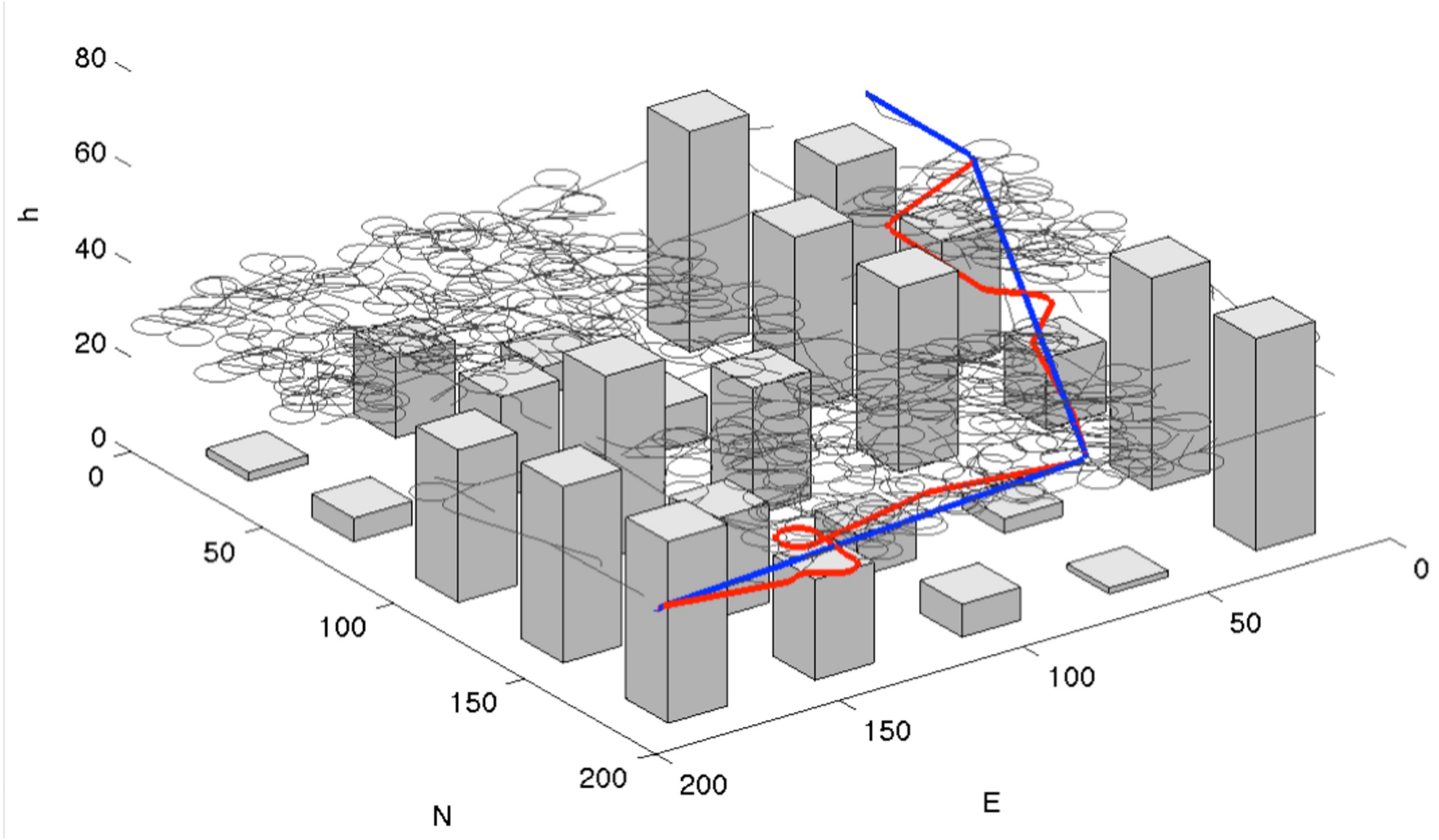
RRT Dubins Approach

- To generate a new random configuration for tree:
  - Generate random N-E position in environment
  - Find closest node in RRT graph to new random point
  - Select a position of distance *L* from the closest RRT node in direction of new point – use this position as N-E coordinates of new configuration
  - Define course angle for new configuration as the angle of the line connecting the RRT graph to the new configuration

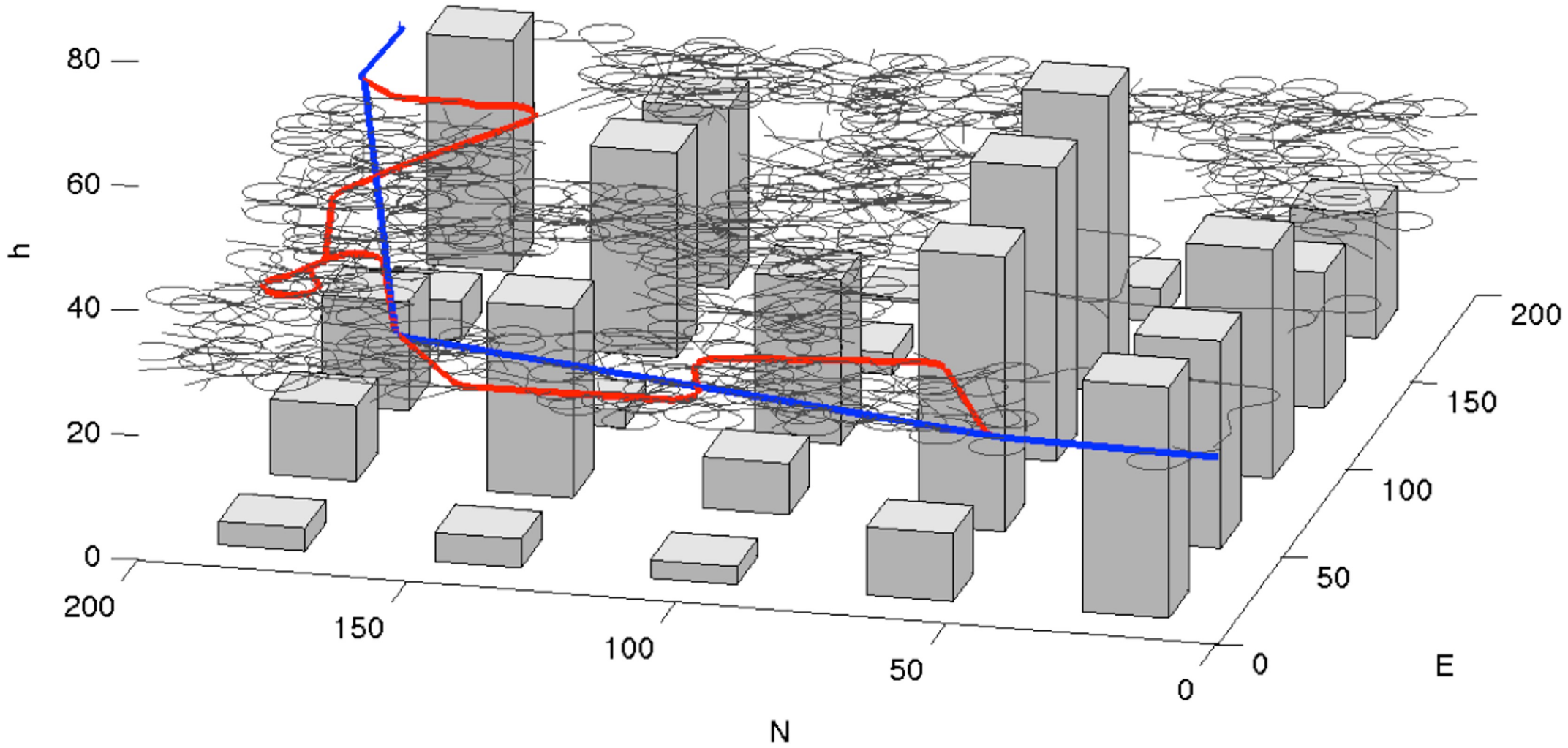
RRT Dubins Results



RRT Dubins Results

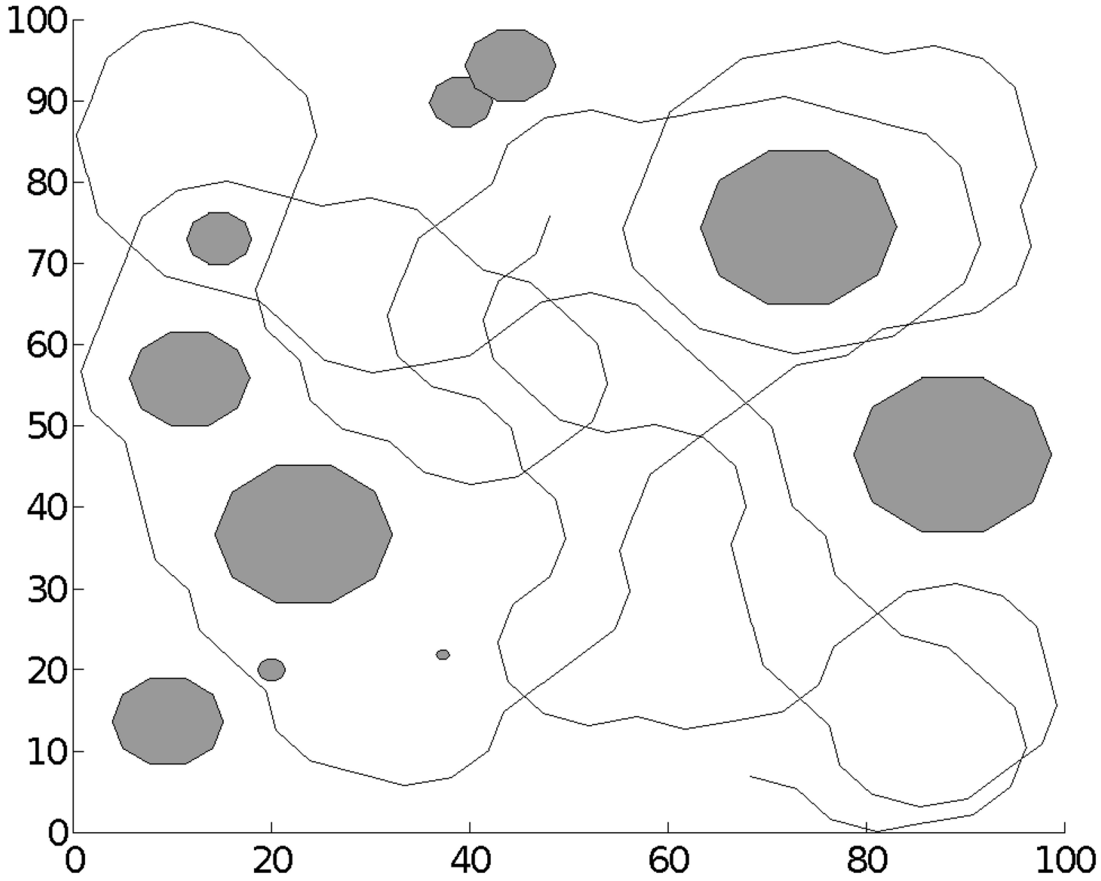


RRT Dubins Results



Coverage Algorithms

- Goal: Survey an area
  - Pass sensor footprint over entire area
- Algorithms often cell based
  - Goal: visit every cell



Coverage Algorithm

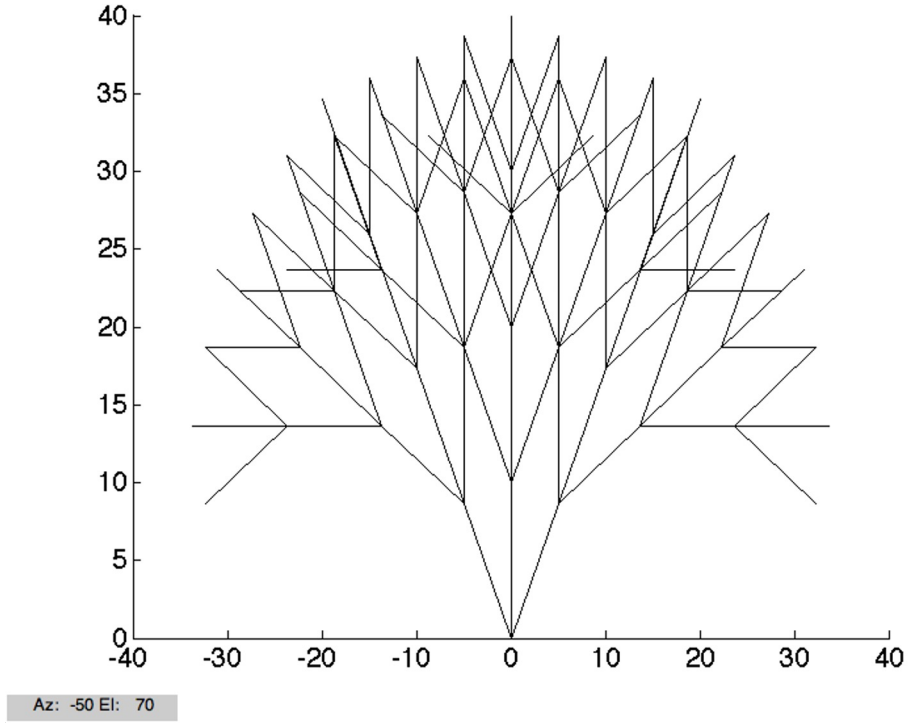
- Two maps in memory
  - terrain map
    - used to detect collisions with environment
  - coverage or return map
    - used to track coverage of terrain

- Return map stores value of returning to particular location
  - Return map initialized so that all locations have same return value
  - As locations are visited, return value of that location is decremented by fixed amount:

$$\Upsilon_i[k] = \Upsilon_i[k-1] - c$$

Coverage Algorithm

- Finite look ahead tree search used to determine where to go
- Tree generated from current MAV configuration
- Tree searched to determine path that maximizes return value
- Two methods for look ahead tree
  - Uniform branching
    - Predetermined depth
    - Uniform branch length
    - Uniform branch separation
  - RRT



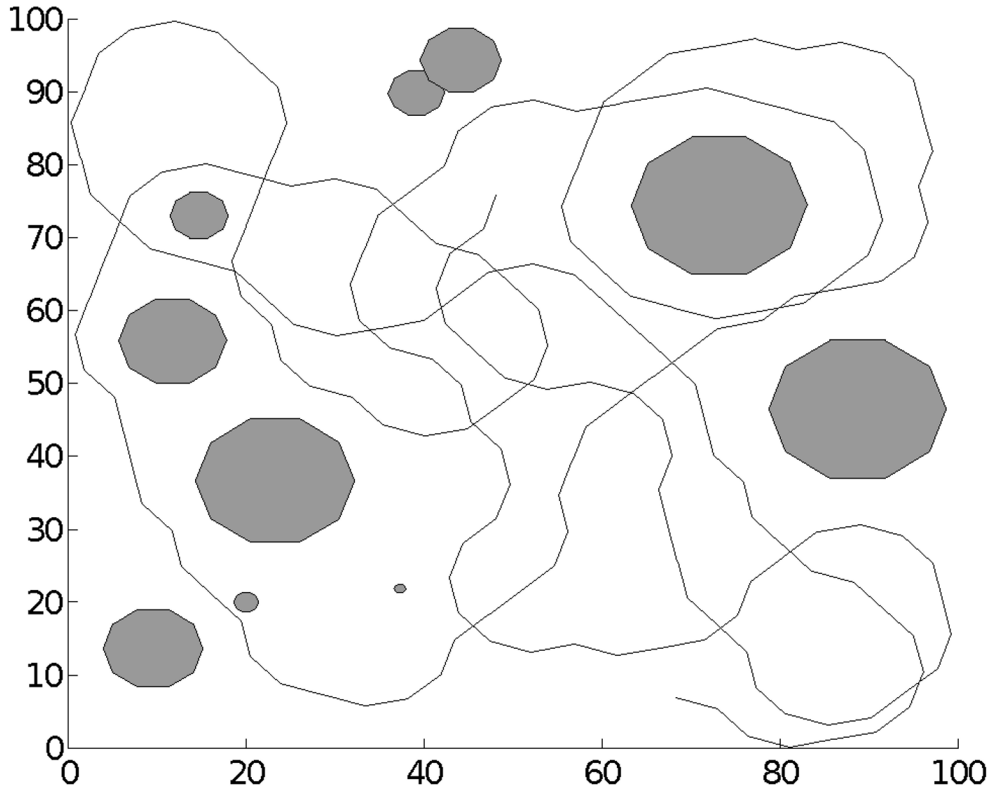
Coverage Algorithm

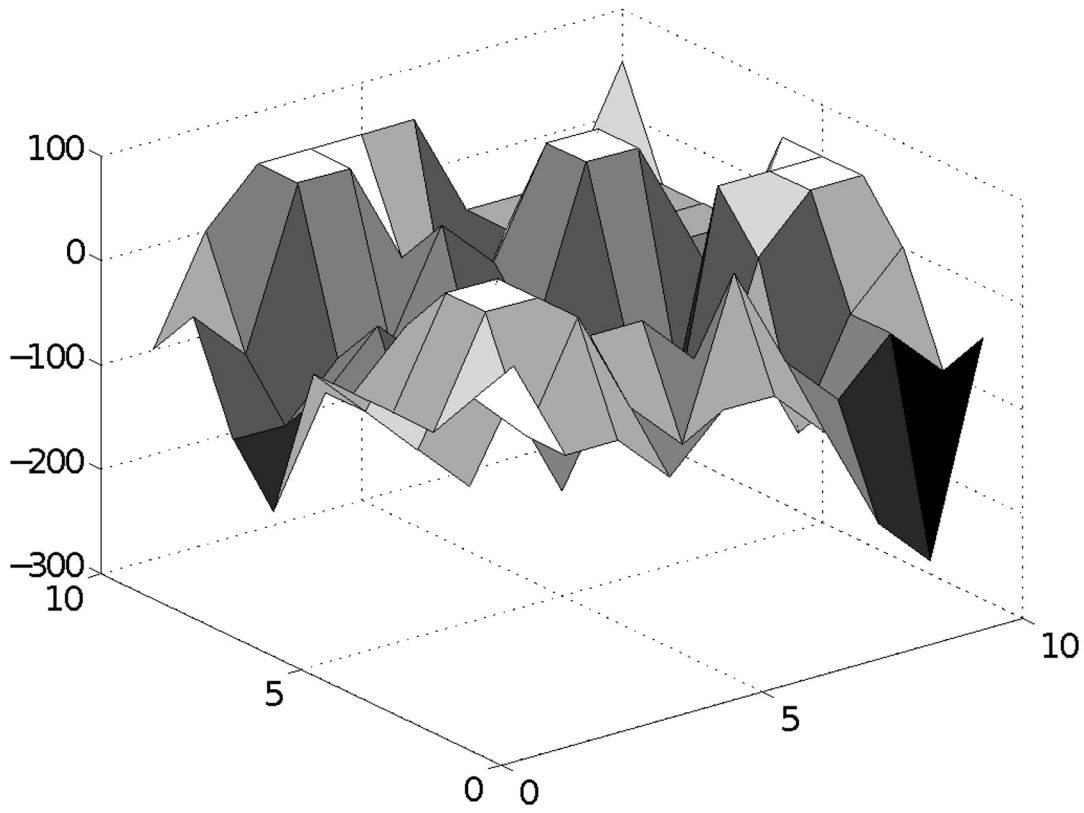
Algorithm 12 Plan Cover Path: planCover($\mathcal{T}$, $\boldsymbol{\Upsilon}$, $\mathbf{p}$)

Input: Terrain map $\mathcal{T}$, return map $\boldsymbol{\Upsilon}$, initial configuration $\mathbf{p}_s$

Initialize look-ahead tree $G = (V, E)$ as $V = \{\mathbf{p}_s\}$, $E = \emptyset$
Initialize return map $\boldsymbol{\Upsilon} = \{\Upsilon_i : i \text{ indexes the terrain}\}$
$\mathbf{p} = \mathbf{p}_s$
for Each planning cycle **do**
    $G =$ generateTree($\mathbf{p}$, $\mathcal{T}$, $\boldsymbol{\Upsilon}$)
    $\mathcal{W} =$ highestReturnPath(G)
    Update $\mathbf{p}$ by moving along the first segment of $\mathcal{W}$
    Reset $G = (V, E)$ as $V = \{\mathbf{p}\}$, $E = \emptyset$
    $\boldsymbol{\Upsilon} =$ updateReturnMap($\boldsymbol{\Upsilon}$, $\mathbf{p}$)
end for

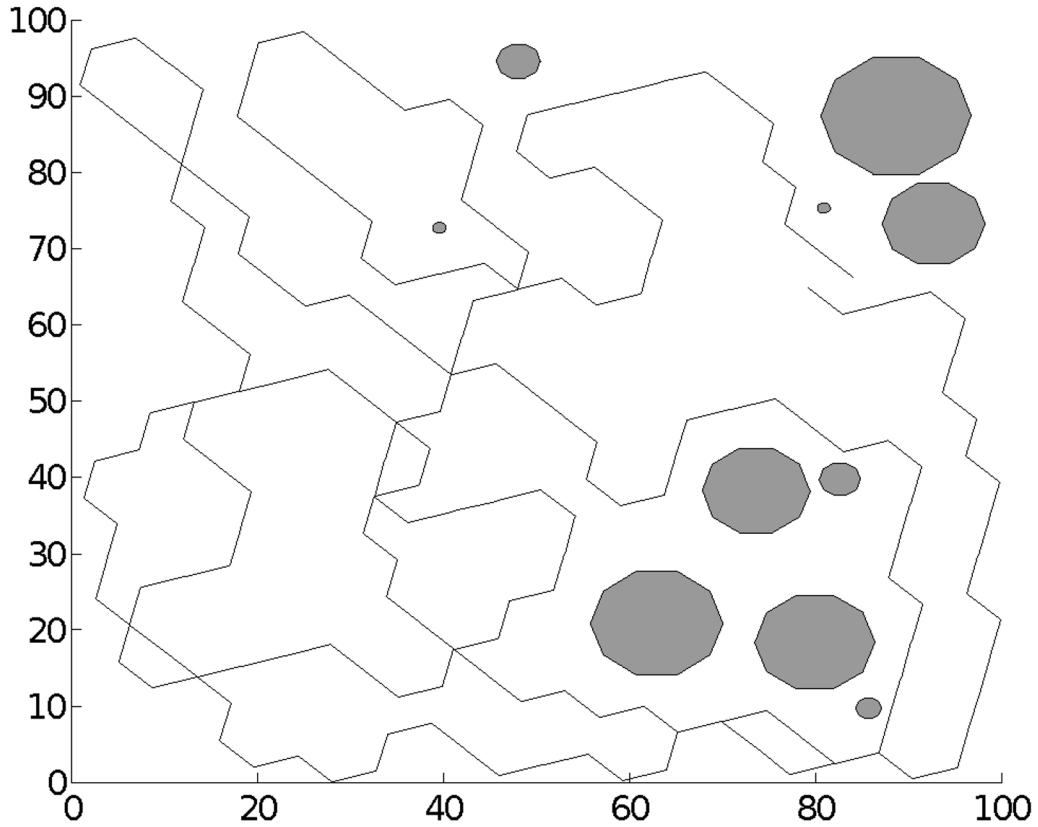
Coverage Planning Results – Uniform Tree

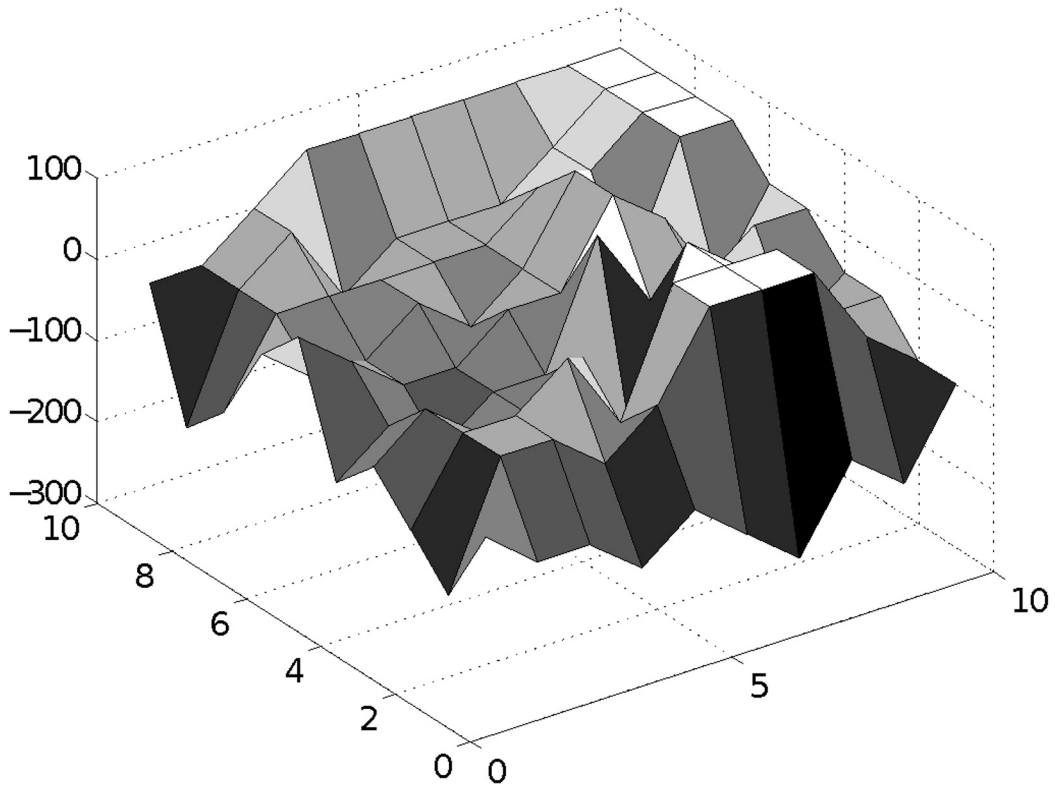




Look ahead length = 5
Heading change = 30 deg
Tree depth = 3
Iterations = 200

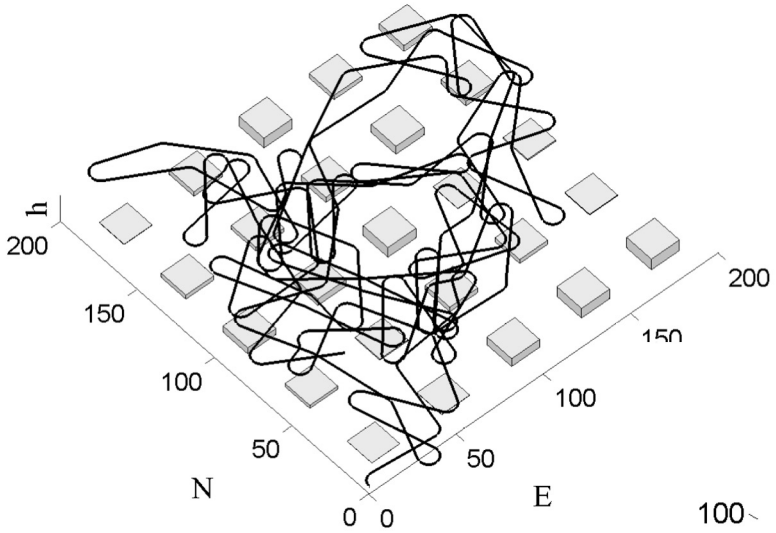
Coverage Planning Results – Uniform Tree

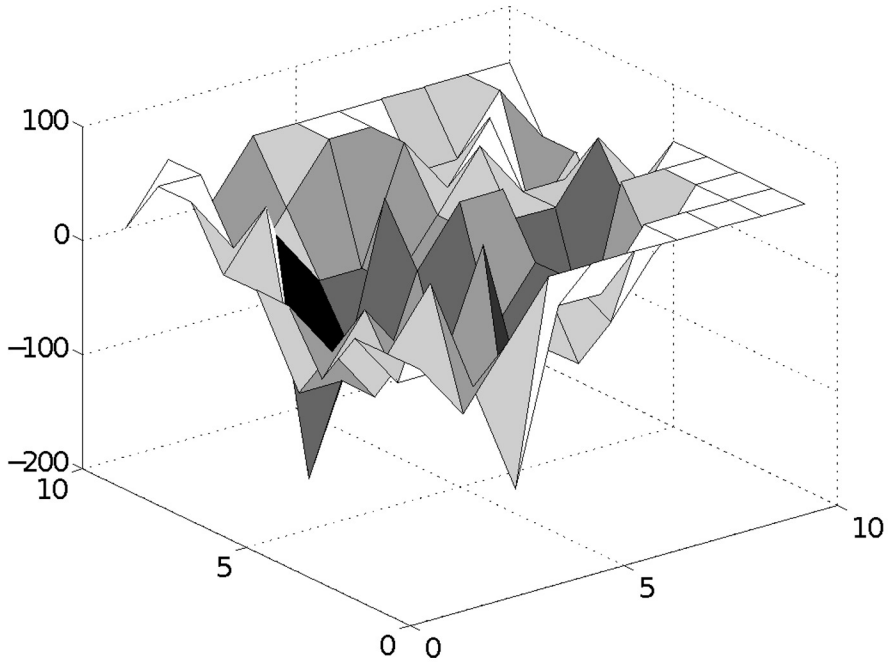


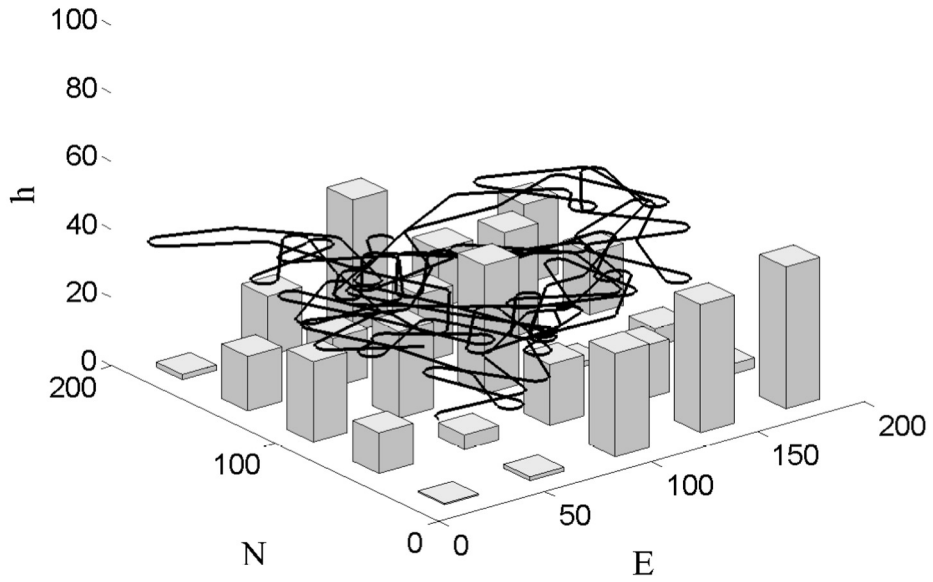


Look ahead length = 5
Heading change = 60 deg
Tree depth = 3
Iterations = 200

Coverage Planning Results – RRT Dubins







Coverage Planning Results – RRT Dubins