

gcd function in sml, C, and asm

Saw Thinkar Nay Htoo

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gcd

The greatest common divisor (gcd) of two positive natural numbers is the largest natural number that exactly divides both numbers. The gcd of 14 and 12 is 2, while the gcd of 14 and 11 is 1. The gcd is given by this specification:

$$\text{gcd} : (\mathbb{N} \times \mathbb{N}) \rightarrow \mathbb{N}$$

$$\text{gcd}(m, n) = \max\{d \in \mathbb{N} \mid m \bmod d = 0 \wedge n \bmod d = 0\}$$

One *algorithm* for calculating the gcd follows Euclid's method. If, for two positive natural numbers m and n , we have that $m > n$, then the gcd of m and n is defined by:

$$\text{euclid} : (\mathbb{N} \times \mathbb{N}) \rightarrow \mathbb{N}$$

$$\text{euclid}(m, n) = \begin{cases} \text{euclid}(n, m \bmod n), & \text{if } n > 0 \\ m, & \text{otherwise} \end{cases}$$

gcd in sml

This can be written in SML like this:

SML

```
fun euclid m n = if n > 0
                  then euclid n (m mod n)
                  else m ;
euclid 558 198; (* expect 18 *)
```

Although a short program, we may not be familiar with the use of recursion because it is not commonly used for C programs due to its inefficiency. But let's implement it that way anyway to follow the math definition more closely.

SML code result

```
> fun euclid m n = if n > 0
                    then euclid n (m mod n)
                    else m ;
euclid 558 198; (* expect 18 *)
# # val euclid = fn: int -> int -> int
> val it = 18: int
```

gcd in c

C source code written to file lab3.c

```
#include <stdio.h>
int euclid(int m, int n)
{
    if( n > 0) return euclid(n, m % n);
    else return m;
}
int main()
{
    printf("GCD output = %i\n",euclid(558,198));
    return 0;
}
```

```
debian@debian:~/labs/lab3$ ./lab3
GCD output = 18
```

The first line of this C code is an include to get access to the library. We need it here to use printf.

```
#include <stdio.h>
```

In ASM, we do not need to include any files in order to link to the C library, so we can skip this.

gcd in c: euclid function

```
int euclid(int m, int n)
{
    if( n > 0) return euclid(n, m % n);
    else return m;
}
```

These lines are the function definition. There are several techniques to be studied to implement it in assembler:

- syntax for function definitions

C:

```
int euclid(){}
```

ASM: function is declared using "euclid:"

```
euclid:
```

- parameter passing

C:

```
int m, int n
```

ASM: variables are placed in stack.

```
push $198
```

```
push $558
```

- decision (if statement)

C:

```
if() return euclid(); else return m; ASM: the procedure will simply  
jump to endif.
```

```
jmp endif
```

```
...
```

```
endif:
```

- conditional (relational expression to compare values)

C:

```
if (n>0) return euclid()
```

```
else reutrn m;
```

ASM: `eax` is compared with zero. If `eax` is equal to zero, it will go to "else". If not it will keep repeating the first function.

```
cmp $0, %eax
jle else
...
else:
...
```

- return statement

```
C:
return
ASM: "ret" is simply used as "return"
ret
```

- calling a function (recursively in this case)

```
C:
euclid(558,198)
ASM: In ASM, function called using "call function-name"
call euclid
```

- calculating modulus

C:

`m % n`

ASM: two numbers are kept in the stack for one operation then placed in ebx and eax registers. edx is set to zero to clear off the previous value. idiv command is used to divide eax by, ebx giving out the remainder of edx. Click [here](#) for detailed explanation for idiv.

```
mov 12(%ebp), %ebx
```

```
mov 8(%ebp), %eax
```

```
mov $0, %edx
```

```
idiv %ebx
```


gcd in asm: main

The main function is simpler, but we need to also learn how to:

- call printf
- end the program

ASM code is put in the “text” section. The entry point is named “_start”. It is a label (indicated by the colon). We make it global so the linker will make it visible to be called externally (by the operating system).

ASM source code written to file lab3.s

```
| .text  
| .globl _start  
| _start:
```

printf needs 2 parameters: a format string and a value. That value must be determined by called our function euclid. Return values are found in the EAX register. The euclid function also requires 2 parameters which must be pushed onto the system stack so they can be retrieved within the function. Parameters are pushed right to left (the C convention). Immediate (literal) values are prefixed with the \$ sign. Register names are prefixed with the % sign.

ASM source code appended to file lab3.s

```
push $198
push $558
call euclid
add $8, %esp #two of them 4 bytes each
.data
fmt: .string "GCD output = %d\n"
.text
push %eax
push $fmt
call printf
add $8, %esp
```

The program is ended by calling software interrupt 0x80. The 1 in EAX means exit command and the 0 in EBX is the convention to mean program had no errors.

ASM source code appended to file lab3.s

```
| mov $1,%eax  
| mov $0,%ebx  
| int $0x80
```

gcd in asm: euclid function

The euclid function uses the same technique of creating a label to indicate start of function. It ends with the ret instruction. The first 2 instructions set up a stack frame base pointer (EBP) to give us access to paramters even if the stack pointer (ESP) moves.

ASM source code appended to file lab3.s

```
| euclid:  
| push %ebp  
|
```

Now we can access the 2 parameters using register EBP. 4 bytes above EBP is the return address, 8 is the 1st parm, and 12 is the second. i.e.

n is on stack at 12(%ebp)

m is on stack at 8(%ebp)

The if statement has to be simulated by branching to labels depending of results of doing the comparison of n to zero.

ASM source code appended to file lab3.s

```
| mov %esp,%ebp  
| mov 12(%ebp),%eax  
| cmp $0,%eax  
| jle else
```

This is the “then” part of the if statement. We need to calculate $m \bmod n$ and call `euclid` again! Integer division is done by putting dividend in EDX:EAX as 64-bit value, and divisor in EBX. The remainder will be in EDX.

ASM source code appended to file `lab3.s`

```
| mov 12(%ebp), %ebx  
| mov 8(%ebp), %eax  
| mov $0, %edx # clear upper 32 bits of the 64-bit dividend edx:eax  
| idiv %ebx # modulus is in edx (quotient is in eax)  
| push %edx  
| push %ebx  
| call euclid  
| add $8,%esp  
| jmp endif  
| else:  
| mov 8(%ebp),%eax  
| endif:
```

These last 2 instructions undo the first 2—they restore the original stack as it was found on entry to the function.

ASM source code appended to file lab3.s

```
mov %ebp,%esp  
pop %ebp  
ret
```

Here is output from running the ASM program:

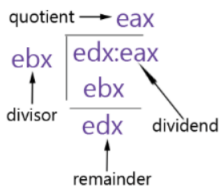
```
debian@debian:~/labs/lab3$ ./labasm  
GCD output = 18
```

This is my own lab explanation.

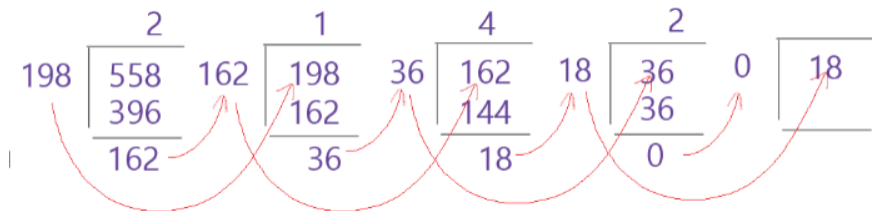
I made this lab report as simple as possible so that anyone without computing knowledge will be able to understand the content of this lab. In this lab, there are two most important things to understand fully: Euclidean Algorithm and Stack-frame.

What is Euclidean Algorithm

According to wikipedia, Euclidean algorithm is an efficient method for computing the greatest common divisor(gcd) of two numbers, the largest number that divides both of them without leaving a remainder. Below is the sample long division components compared with registers. Ok, what are those eax, ebx and edx? They are general purpose registers, where values are stored to do the calculation, and they have specific purposes. We will have to deal more with these registers when we write this algorithm using assembly language.



Lets say there are two numbers: 558 and 198. As we can see below, in the first step, dividend, 558 is divided by divisor, 198, resulting the remainder of 162. Then the remainder, 162 becomes the divisor in the second step, dividing the new dividend, 198 to give the new result of the remainder which is 36. The same procedure is repeated until the divisor becomes zero. So in this case, the GCD of 558 and 198 is 18.



This link is the youtube video explanation for Euclidean Algorithm.

Now if you feel confident enough with your Euclidean Algorithm understanding, try to answer GCD for these numbers: (255,245), (531,234) and (126,186).

Using this online GCD calculator you can check your answer here.

[Click here for the Euclidean Algorithm in mathematical expression.](#)

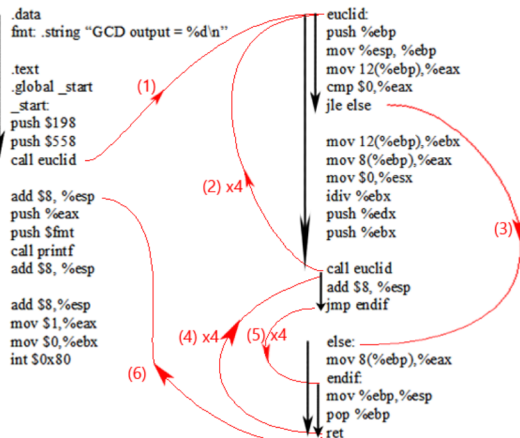
[Click here for the alogrithm written in SML functional programming language.](#)

[Click here for the alogirthm written in C programming language.](#)

Euclidean Algorithm in Assembly Language(ASM)

Now, let's begin with what assembly language is. It is a low-level programming language. This programming language is simple to write codes, if you understand the concept well, but you will have to write a lot just for a short program of high-level programming languages such as C, C++, and so on. Below is the Euclidean Algorithm in ASM.

Euclidean Algorithm written in assembly language



[illegible]

```
(gdb) info r
eax                0x1c          28
ecx                0xb7fffc1c     -1207960548
edx                0xb7fedc90     -1208034160
ebx                0xb7fff000     -1207963648
esp                0xbffff670     0xbffff670
ebp                0x0            0x0
esi                0xbffff67c     -1073744260
edi                0x8048200       134513152
eip                0x8048200       0x8048200 <_start>

(gdb) x/8w $esp
0xbffff670: 1 -1073743937 0 -1073743907
0xbffff680: -1073743889 -1073743871 -1073743855 -1073743844

(gdb) backtrace
#0  _start () at lab3.s:4
```

[illegible]

```
(gdb) info r
eax                0x1c                28
ecx                0xb7fffc1c          -1207960548
edx                0xb7fædc90          -1208034160
ebx                0xb7ffff00          -1207963648
esp                0xbffff66c          0xbffff66c
ebp                0x0                0x0
esi                0xbffff67c          -1073744260
edi                0x8048200          134513152
eip                0x8048205          0x8048205 <_start+5>

(gdb) x/8w $esp
0xbffff66c:    198      1      -1073743937      0
0xbffff67c:    -1073743907  -1073743889  -1073743871  -1073743855

(gdb) backtrace
#0  _start () at lab3.s:5
```

[illegible]

```
(gdb) info r
eax      0x1c      28
ecx      0xb7fffc1c  -1207960548
edx      0xb7fedc90  -1208034160
ebx      0xb7fff000  -1207963648
esp      0xbffff668  0xbffff668
ebp      0x0        0x0
esi      0xbffff67c  -1073744260
edi      0x8048200   134513152
eip      0x804820a   0x804820a <_start+10>

(gdb) x/8w $esp
0xbffff668:  558      198      1      -1073743937
0xbffff678:  0      -1073743907  -1073743889  -1073743871

(gdb) backtrace
#0  _start () at lab3.s:6
```


[illegible]

```
(gdb) info r
eax          0x1c      28
ecx          0xb7fff1c  -1207960548
edx          0xb7fedc90 -1208034160
ebx          0xb7fff000 -1207963648
esp          0xbffff660 0xbffff660
ebp          0xbffff660 0xbffff660
esi          0xbffff67c -1073744260
edi          0x8048200 134513152
eip          0x804822f 0x804822f <euclid+3>
```

(gdb) x/8w \$ebp				
0xbfffffff60:	0	134513167	558	198
0xbfffffff60:	1	-1073743937	0	-1073743907
(gdb) x/8w \$esp				
0xbfffffff60:	0	134513167	558	198
0xbfffffff60:	1	-1073743937	0	-1073743907

[illegible]

```
(gdb) info r
eax             0xc6      198
ecx             0xb7fffc1c   -1207960548
edx             0xb7fedc90   -1208034160
ebx             0xb7fff000   -1207963648
esp             0xbffff660   0xbffff660
ebp             0xbffff660   0xbffff660
esi             0xbffff67c   -1073744260
edi             0x8048200     134513152
eip             0x8048232     0x8048232 <euclid+6>
```

```
(gdb) x/8w $ebp
0xbffff660: 0 134513167 558 198
0xbffff670: 1 -1073743937 0 -1073743907
(gdb) x/8w $esp
0xbffff660: 0 134513167 558 198
0xbffff670: 1 -1073743937 0 -1073743907
```


[illegible]

```
(gdb) info r
eax          0xc6          198
ecx          0xb7ffffc1c      -1207960548
edx          0xb7fedc90      -1208034160
ebx          0xb7ffff000      -1207963648
esp          0xbffff660      0xbffff660
ebp          0xbffff660      0xbffff660
esi          0xbffff67c      -1073744260
edi          0x8048200      134513152
eip          0x8048237      0x8048237 <euclid+11>
```

```
(gdb) x/8w $esp
0xbffff660: 0 134513167 558 198
0xbffff670: 1 -1073743937 0 -1073743907
(gdb) x/8w $ebp
0xbffff660: 0 134513167 558 198
0xbffff670: 1 -1073743937 0 -1073743907
```

```
(gdb) backtrace
#0  euclid () at lab3.s:25
#1  0x0804820f in _start () at lab3.s:6
```

[illegible]

```
(gdb) bt
#0  euclid () at lab3.s:26
#1  0x0804820f in _start () at lab3.s:6
(gdb) x/8w $esp
0xbffff660: 0 134513167 558 198
0xbffff670: 1 -1073743937 0 -1073743907
(gdb) x/8w $ebp
0xbffff660: 0 134513167 558 198
0xbffff670: 1 -1073743937 0 -1073743907
(gdb) info r
eax 0xc6 198
ecx 0xb7fffc1c -1207960548
edx 0xb7fedc90 -1208034160
ebx 0xc6 198
esp 0xbffff660 0xbffff660
ebp 0xbffff660 0xbffff660
esi 0xbffff67c -1073744260
edi 0x8048200 134513152
eip 0x804823a 0x804823a <euclid+14>
```

[illegible]

```
(gdb) bt
#0  euclid () at lab3.s:27
#1  0x0804820f in _start () at lab3.s:6
(gdb) x/8w $esp
0xbffff660: 0 134513167 558 198
0xbffff670: 1 -1073743937 0 -1073743907
(gdb) x/8w $ebp
0xbffff660: 0 134513167 558 198
0xbffff670: 1 -1073743937 0 -1073743907
(gdb) info r
eax 0x22e 558
ecx 0xb7fffc1c -1207960548
edx 0xb7fedc90 -1208034160
ebx 0xc6 198
esp 0xbffff660 0xbffff660
ebp 0xbffff660 0xbffff660
esi 0xbffff67c -1073744260
edi 0x8048200 134513152
eip 0x804823d 0x804823d <euclid+17>
```

[illegible]

```
(gdb) bt
#0  euclid () at lab3.s:28
#1  0x0804820f in _start () at lab3.s:6
(gdb) x/8w $esp
0xbffff660: 0 134513167 558 198
0xbffff670: 1 -1073743937 0 -1073743907
(gdb) x/8w $ebp
0xbffff660: 0 134513167 558 198
0xbffff670: 1 -1073743937 0 -1073743907
(gdb) info r
eax 0x22e 558
ecx 0xb7fffc1c -1207960548
edx 0x0 0
ebx 0xc6 198
esp 0xbffff660 0xbffff660
ebp 0xbffff660 0xbffff660
esi 0xbffff67c -1073744260
edi 0x8048200 134513152
eip 0x8048242 0x8048242 <euclid+22>
```

[illegible]

```
(gdb) bt
#0  euclid () at lab3.s:29
#1  0x0804820f in _start () at lab3.s:6
(gdb) x/8w $esp
0xbffff660:    0      134513167      558      198
0xbffff670:    1     -1073743937      0     -1073743907
(gdb) x/8w $ebp
0xbffff660:    0      134513167      558      198
0xbffff670:    1     -1073743937      0     -1073743907
(gdb) info r
eax          0x2          2
ecx          0xb7fffc1c   -1207960548
edx          0xa2        162
ebx          0xc6        198
esp          0xbffff660   0xbffff660
ebp          0xbffff660   0xbffff660
esi          0xbffff67c   -1073744260
edi          0x8048200     134513152
eip          0x8048244     0x8048244 <euclid+24>
```


[illegible]

```
(gdb) bt
#0  euclid () at lab3.s:22
#1  0x0804824b in euclid () at lab3.s:31
#2  0x0804820f in _start () at lab3.s:6
(gdb) x/8w $esp
0xbffff650:      -1073744288      134513227      198      162
0xbffff660:      0      134513167      558      198
(gdb) x/8w $ebp
0xbffff650:      -1073744288      134513227      198      162
0xbffff660:      0      134513167      558      198
(gdb) info r
eax      0x2      2
ecx      0xb7fffc1c      -1207960548
edx      0xa2      162
ebx      0xc6      198
esp      0xbffff650
ebp      0xbffff650
esi      0xbffff67c      -1073744260
edi      0x8048200      134513152
eip      0x804822f      0x804822f <euclid+3>
```

```
(gdb) bt
#0  euclid () at lab3.s:23
#1  0x0804824b in euclid () at lab3.s:31
#2  0x0804820f in _start () at lab3.s:6
(gdb) x/8w $esp
0xbffff650:      -1073744288      134513227      198      162
0xbffff660:      0      134513167      558      198
(gdb) x/8w $ebp
0xbffff650:      -1073744288      134513227      198      162
0xbffff660:      0      134513167      558      198
(gdb) info r
eax      0xa2      162
ecx      0xb7fffc1c      -1207960548
edx      0xa2      162
ebx      0xc6      198
esp      0xbffff650      0xbffff650
ebp      0xbffff650      0xbffff650
esi      0xbffff67c      -1073744260
edi      0x8048200      134513152
eip      0x8048232      0x8048232 <euclid+6>
```

```
(gdb) bt
#0  euclid () at lab3.s:24
#1  0x0804824b in euclid () at lab3.s:31
#2  0x0804820f in _start () at lab3.s:6
(gdb) x/8w $esp
0xbffff650:      -1073744288      134513227      198      162
0xbffff660:      0      134513167      558      198
(gdb) x/8w $ebp
0xbffff650:      -1073744288      134513227      198      162
0xbffff660:      0      134513167      558      198
(gdb) info r
eax      0xa2      162
ecx      0xb7fffc1c      -1207960548
edx      0xa2      162
ebx      0xc6      198
esp      0xbffff650      0xbffff650
ebp      0xbffff650      0xbffff650
esi      0xbffff67c      -1073744260
edi      0x8048200      134513152
eip      0x8048235      0x8048235 <euclid>9>
```

```
(gdb) bt
#0  euclid () at lab3.s:25
#1  0x0804824b in euclid () at lab3.s:31
#2  0x0804820f in _start () at lab3.s:6
(gdb) x/8w $esp
0xbffff650:      -1073744288      134513227      198      162
0xbffff660:      0      134513167      558      198
(gdb) x/8w $ebp
0xbffff650:      -1073744288      134513227      198      162
0xbffff660:      0      134513167      558      198
(gdb) info r
eax      0xa2      162
ecx      0xb7fffc1c      -1207960548
edx      0xa2      162
ebx      0xc6      198
esp      0xbffff650
ebp      0xbffff650
esi      0xbffff67c      -1073744260
edi      0x8048200      134513152
eip      0x8048237      0x8048237 <euclid+11>
```

```
(gdb) bt
#0  euclid () at lab3.s:26
#1  0x0804824b in euclid () at lab3.s:31
#2  0x0804820f in _start () at lab3.s:6
(gdb) x/8w $esp
0xbffff650:      -1073744288      134513227      198      162
0xbffff660:      0      134513167      558      198
(gdb) x/8w $ebp
0xbffff650:      -1073744288      134513227      198      162
0xbffff660:      0      134513167      558      198
(gdb) info r
eax      0xa2      162
ecx      0xb7fffc1c      -1207960548
edx      0xa2      162
ebx      0xa2      162
esp      0xbffff650      0xbffff650
ebp      0xbffff650      0xbffff650
esi      0xbffff67c      -1073744260
edi      0x8048200      134513152
eip      0x804823a      0x804823a <euclid+14>
```

```
(gdb) bt
#0  euclid () at lab3.s:27
#1  0x0804824b in euclid () at lab3.s:31
#2  0x0804820f in _start () at lab3.s:6
(gdb) x/8w $esp
0xbffff650:      -1073744288      134513227      198      162
0xbffff660:      0      134513167      558      198
(gdb) x/8w $ebp
0xbffff650:      -1073744288      134513227      198      162
0xbffff660:      0      134513167      558      198
(gdb) info r
eax      0xc6      198
ecx      0xb7fffc1c      -1207960548
edx      0xa2      162
ebx      0xa2      162
esp      0xbffff650
ebp      0xbffff650
esi      0xbffff67c      -1073744260
edi      0x08048200      134513152
eip      0x0804823d      0x0804823d <euclid+17>
```

```
(gdb) bt
#0  euclid () at lab3.s:28
#1  0x0804824b in euclid () at lab3.s:31
#2  0x0804820f in _start () at lab3.s:6
(gdb) x/8w $esp
0xbffff650:    -1073744288    134513227    198    162
0xbffff660:    0    134513167    558    198
(gdb) x/8w $ebp
0xbffff650:    -1073744288    134513227    198    162
0xbffff660:    0    134513167    558    198
(gdb) info r
eax            0xc6    198
ecx            0xb7fff1c    -1207960548
edx            0x0    0
ebx            0xa2    162
esp            0xbffff650    0xbffff650
ebp            0xbffff650    0xbffff650
esi            0xbffff67c    -1073744260
edi            0x8048200    134513152
eip            0x8048242    0x8048242 <euclid+22>
```

```
(gdb) bt
#0  euclid () at lab3.s:29
#1  0x0804824b in euclid () at lab3.s:31
#2  0x0804820f in _start () at lab3.s:6
(gdb) x/8w $esp
0xbffff650:      -1073744288      134513227      198      162
0xbffff660:      0      134513167      558      198
(gdb) x/8w $ebp
0xbffff650:      -1073744288      134513227      198      162
0xbffff660:      0      134513167      558      198
(gdb) info r
eax      0x1      1
ecx      0xb7fffc1c      -1207960548
edx      0x24      36
ebx      0xa2      162
esp      0xbffff650      0xbffff650
ebp      0xbffff650      0xbffff650
esi      0xbffff67c      -1073744260
edi      0x8048200      134513152
eip      0x8048244      0x8048244 <euclid+24>
```



```
(gdb) bt
#0  euclid () at lab3.s:30
#1  0x0804824b in euclid () at lab3.s:31
#2  0x0804820f in _start () at lab3.s:6
(gdb) x/8w $esp
0xbffff64c:      36      -1073744288      134513227      198
0xbffff65c:     162      0      134513167      558
(gdb) x/8w $ebp
0xbffff650:     -1073744288      134513227      198      162
0xbffff660:      0      134513167      558      198
(gdb) info r
eax      0x1      1
ecx      0xb7fffc1c      -1207960548
edx      0x24      36
ebx      0xa2      162
esp      0xbffff64c      0xbffff64c
ebp      0xbffff650      0xbffff650
esi      0xbffff67c      -1073744260
edi      0x08048200      134513152
eip      0x08048245      0x08048245 <euclid+25>
```

[illegible]

push %ebx

```
(gdb) bt
#0  euclid () at lab3.s:31
#1  0x0804824b in euclid () at lab3.s:31
#2  0x0804820f in _start () at lab3.s:6
(gdb) x/8w $esp
0xbffff648:      162      36      -1073744288      134513227
0xbffff658:      198      162      0      134513167
(gdb) x/8w $ebp
0xbffff650:      -1073744288      134513227      198      162
0xbffff660:      0      134513167      558      198
(gdb) info r
eax      0x1      1
ecx      0xb7fffc1c      -1207960548
edx      0x24      36
ebx      0xa2      162
esp      0xbffff648      0xbffff648
ebp      0xbffff650      0xbffff650
esi      0xbffff67c      -1073744260
edi      0x8048200      134513152
eip      0x8048246      0x8048246 <euclid+26>
```

Text written to file labcode.sh

```
docsm1 lab3.doc  
as -gstabs -o lab.o lab3.s  
ld -dynamic-linker /lib/ld-linux.so.2 -o labasm lab.o -lc -lX11  
gcc -Wall -o labc lab3.c
```

Text written to file labcode2.sh

```
gcc -Wall -o labc lab.c  
gcc -Wall -o labasm lab.s
```

Text written to file labdoc.sh

```
doctex lab3.doc  
pptexenv /home/debian/texfot.pl pdflatex lab3.tex
```

Bourne Shell

```
chmod 755 labcode2.sh  
chmod 755 labcode.sh  
chmod 755 labdoc.sh
```