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Exotic spin-dependent interactions through unparticle exchange

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Abstract: The potential discovery of unparticles could have far-reaching implications for particle physics and cosmology. For over a decade, high-energy physicists have extensively studied the effects of unparticles. In this study, we derive six types of nonrelativistic potentials between fermions induced by unparticle exchange in coordinate space. We consider all possible combinations of scalar, pseudo-scalar, vector, and axial-vector couplings to explore the full range of possibilities. Previous studies have only examined scalar-scalar (SS), pseudoscalar-pseudoscalar (PP), vector-vector (VV), and axial-axial-vector (AA) type interactions, which are all parity even. We propose SP and VA interactions to extend our understanding of unparticle physics, noting that parity conservation is not always guaranteed in modern physics. We explore the possibilities of detecting unparticles through the long-range interactions they may mediate with ordinary matter. Dedicated experiments using precision measurement methods can be employed to search for such interactions. We discuss the properties of these potentials and estimate constraints on their coupling constants based on existing experimental data. Our findings indicate that for some particular values of the scaling dimension $d_{\mathcal{U}}$, the coupling between scalar or vector unparticles and fermions is constrained by several orders of magnitude more tightly than the previous limits. The underlying reason for this improvement is analyzed. Limits are also set on the newly proposed SP and VA interactions for continuous $d_{\mathcal{U}}$ values, allowing the exploration of the $d_{\mathcal{U}}$ dependence of the constraints. It turns out that the bounds exhibit an exponential decay trend with the increasing $d_{\mathcal{U}}$.

Keywords: New Light Particles, Specific BSM Phenomenology, Axions and ALPs

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Contents

1 Introduction

The concept of symmetry has had a profound impact on the development of modern physics. In 2007, Georgi [1] proposed the existence of a hidden conformal symmetry sector beyond the Standard Model (SM) of particle physics, termed unparticle. Since then, the effects of unparticles have been explored in many subjects, including collider physics [2–6], neutrino physics [7, 8], cosmology and astrophysics [9, 10], quantum electrodynamics [11, 12], dark energy [13–16], etc. The recent nonrelativistic extension of unparticle physics has led to a new concept of unnucleus [17] and provided an interpretation of neutral charm mesons near threshold [18].

One of the most intriguing phenomenologies of unparticle physics is the possibility of exotic long-range interactions mediated by unparticles [19]. To date, only electromagnetic and gravitational interactions have been observed as long-range interactions. However, considerable efforts [20–26] have been devoted to searching for extra long-range interactions, which could be mediated by axions [27, 28], paraphotons [29, 30], Z' bosons [31], and graviphotons [32]. The discovery of new long-range interactions would have a tremendous impact on our understanding of nature, such as the strong CP problem [33] and dark matter [34].

Liao and Liu [19] first proposed the idea of unparticle-mediated long-range interactions, using experimental data on the long-range spin-dependent interaction of electrons to constrain the couplings of unparticles to electrons. It was also noted that unparticles could mediate spin-independent long-range interactions, which might modify the inverse-square law (ISL) of gravity [35–37], contribute to the ground-state energy of the hydrogen atom [38], and influence the measurement of spacetime curvature near Earth [39]. The possibility of long-range interaction between neutrinos from unparticle exchange was also explored in solar

JHEP01(2024)083

neutrino experiments [40]. Only long-range interactions originating from the same type of vertex, i.e., scalar-scalar (SS), pseudoscalar-pseudoscalar (PP), vector-vector (VV), and axial-axial-vector (AA), were considered in these works. Different types of couplings between unparticles and SM particles, i.e., scalar-pseudoscalar (SP) and vector-axial-vector (VA) interactions, could also exist. The interactions of SS, PP, VV, and AA types conserve all of the discrete symmetries, including charge conjugation (C), parity (P), and time reversal (T). In contrast, the SP-type interaction violates both P and T, while the VA-type interaction violates P and C. It is important to note that the conservation of discrete symmetries such as P and CP cannot be assumed in modern physics. Taking such interactions into account when exploring extra long-range interactions or constraining them with existing experimental data could deepen our understanding of unparticle physics and provide new insights into new physics searches.

In this paper, we derive the interactions between fermions from unparticle exchange, considering all possible combinations of scalar, vector, pseudoscalar, and axial-vector couplings. In section 2, we present six types of potentials explicitly in coordinate space under the nonrelativistic limit. In section 3, we discuss the properties of the derived potentials and estimate constraints on their coupling constants based on existing experimental data. In section 4, we summarize this work. The details for deriving the unparticle-mediated interactions and their bounds are given in the appendix.

2 Long-range spin-dependent interactions via unparticle exchange

Following the scenario described in ref. [1], at some high energy scale, the SM fields can interact with the $\mathcal{BZ}$ (Banks-Zaks [41]) field having a nontrivial infrared fixed point through the exchange of messenger particles with a large mass scale $M_{\mathcal{U}}$. Below this mass scale, nonrenormalizable couplings are suppressed by inverse powers of $M_{\mathcal{U}}$ and take the generic form of $\frac{1}{M_{\mathcal{U}}^k}\mathcal{O}_{\rm SM}\mathcal{O}_{\mathcal{BZ}}$, where $\mathcal{O}_{\rm SM}$ and $\mathcal{O}_{\mathcal{BZ}}$ denote operators built out of SM and $\mathcal{BZ}$ fields, respectively. The renormalizable couplings of the $\mathcal{BZ}$ fields then induce dimensional transmutation, and the scale-invariant unparticle fields emerge at another energy scale $\Lambda_{\mathcal{U}}$. Below $\Lambda_{\mathcal{U}}$, $\mathcal{BZ}$ operators match onto unparticle operators $\mathcal{O}_{\mathcal{U}}$, and the unparticle interaction with SM particles at low energy has the form

$$\mathcal{L}_{\rm int} \propto \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}+d_{\rm SM}-4}}\mathcal{O}_{\rm SM}\mathcal{O}_{\mathcal{U}}, \tag{2.1}$$

where $d_{\mathcal{U}}$ and d_{SM} are the scaling dimensions of the unparticle operator and the SM particle operator, respectively.

Now, we consider the interactions between unparticles with fermions, specifically, electrons and nucleons. At low energies, nucleons can be described by a heavy fermion field, and thus, the leading interactions in the effective field theory [19] can be generalized as

$$\mathcal{L}_{\rm int} = \sum_{i=e,N}(C_S\bar{\psi}_i\psi_i + C_P\bar{\psi}_i i\gamma_5\psi_i)\Phi_{\mathcal{U}} + (C_V\bar{\psi}_i\gamma_\mu\psi_i + C_A\bar{\psi}_i\gamma_\mu\gamma_5\psi_i)X^\mu_{\mathcal{U}}. \tag{2.2}$$

Here ψ_e (ψ_N) is the electron (nucleon) field, and $\Phi_{\mathcal{U}}$ and $X^\mu_{\mathcal{U}}$ stand for the fields of scalar and vector unparticles, respectively, with $C_{S,P,V,A}$ being the corresponding coupling constants.

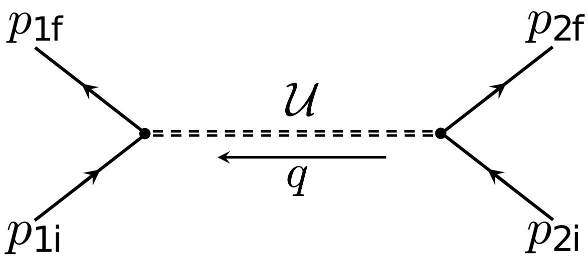

Figure 1. Feynman diagram of scattering between fermions from unparticle exchange. For scalar, pseudoscalar, vector, and axial-vector interaction vertex, 1, $i\gamma_5$, γ_μ, and $\gamma_\mu\gamma_5$ should be attached, respectively. q represents the transferred four-momentum. In the nonrelativistic limit, q is replaced by the three-momentum $\vec{q}$.

They can be parameterized by $C_i = c_i \Lambda_i^{1-d_{\mathcal{U}}}$, where c_i and Λ_i are unknown dimensionless numbers and energy scales, respectively. A simple choice is to put $\Lambda_i = 1\,\text{TeV}$ and constrain c_i because they can be easily converted into each other [19].

The scattering procedure in the nonrelativistic limit can be described by a tree-level Feynman diagram, shown in figure 1. From the Lagrangian (2.2), six types of interaction, i.e., SS, SP, PP, VV, VA, and AA, can be constructed at the tree level. In the laboratory frame, we consider the elastic scattering with $q_0 = 0$ and use notations

$$\begin{aligned} \vec{q} &= \vec{p}_{1f} - \vec{p}_{1i} = \vec{p}_{2i} - \vec{p}_{2f}, \\ \vec{p}_1 &= \frac{\vec{p}_{1f} + \vec{p}_{1i}}{2}, \\ \vec{p}_2 &= \frac{\vec{p}_{2f} + \vec{p}_{2i}}{2}, \end{aligned} \tag{2.3}$$

where $\vec{q}, \vec{p}_1, \vec{p}_2$ are three-momentum components of the corresponding four-momentum. The scalar unparticle propagator takes the form [1, 42]

$$D(q^2) = i\frac{A_{d_{\mathcal{U}}}}{2}\frac{1}{\sin(d_{\mathcal{U}}\pi)}(-q^2 - i\epsilon)^{d_{\mathcal{U}}-2}, \tag{2.4}$$

where $A_{d_{\mathcal{U}}} = \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}}} \frac{\Gamma(d_{\mathcal{U}}+1/2)}{\Gamma(d_{\mathcal{U}}-1)\Gamma(2d_{\mathcal{U}})}$ is the normalization factor. The vector unparticle propagator $D^{\mu\nu}$ can be obtained by further attaching the spin structure $-g^{\mu\nu} + (1-\xi)q^\mu q^\nu/q^2$. The six types of amplitude are given by

$$\begin{aligned} \mathcal{M}_{SS} &= iC_S^2 \bar{u}(p_{2f})u(p_{2i})D(q^2)\bar{u}(p_{1f})u(p_{1i}), \\ \mathcal{M}_{SP} &= iC_S(iC_P)\bar{u}(p_{2f})u(p_{2i})D(q^2)\bar{u}(p_{1f})\gamma_5 u(p_{1i}), \\ \mathcal{M}_{PP} &= i(iC_P)^2\bar{u}(p_{2f})\gamma_5 u(p_{2i})D(q^2)\bar{u}(p_{1f})\gamma_5 u(p_{1i}), \\ \mathcal{M}_{VV} &= iC_V^2\bar{u}(p_{2f})\gamma_\mu u(p_{2i})D^{\mu\nu}(q^2)\bar{u}(p_{1f})\gamma_\nu u(p_{1i}), \\ \mathcal{M}_{VA} &= iC_V C_A\bar{u}(p_{2f})\gamma_\mu u(p_{2i})D^{\mu\nu}(q^2)\bar{u}(p_{1f})\gamma_\nu\gamma_5 u(p_{1i}), \\ \mathcal{M}_{AA} &= iC_A^2\bar{u}(p_{2f})\gamma_\mu\gamma_5 u(p_{2i})D^{\mu\nu}(q^2)\bar{u}(p_{1f})\gamma_\nu\gamma_5 u(p_{1i}), \end{aligned} \tag{2.5}$$

where $u(p)$ is the Dirac spinor. The amplitude in the nonrelativistic limit reads

$$\mathcal{A} = \prod_i \frac{1}{\sqrt{2E_i}} \prod_f \frac{1}{\sqrt{2E_f}} \mathcal{M}, \tag{2.6}$$

where i (f) denotes the index of initial (final) particles, and the coefficients come from the normalization condition. The potential in coordinate space can be calculated through the Fourier transformation,

$$V(\vec{r}) = -\int \frac{d\vec{q}^3}{(2\pi)^3}\mathcal{A}(\vec{q},\vec{p})e^{i\vec{q}\cdot\vec{r}}. \tag{2.7}$$

The six types of coordinate-space potential keeping terms up to $\mathcal{O}(m^{-3})$ are as follows.

$$\begin{aligned} V_{SS}(r) = &-C_S^2\frac{A_{d_\mathcal{U}}}{4\pi^2}r^{1-2d_\mathcal{U}}\left[\Gamma(2d_\mathcal{U}-2)\left(1-\frac{\vec{p}_1^2}{2m_1^2}-\frac{\vec{p}_2^2}{2m_2^2}\right)\right.\\ &\left.+\frac{\Gamma(2d_\mathcal{U})}{(2d_\mathcal{U}-2)}\left(\frac{\vec{\sigma}_1\cdot(\hat{r}\times\vec{p}_1)}{4m_1^2r}-\frac{\vec{\sigma}_2\cdot(\hat{r}\times\vec{p}_2)}{4m_2^2r}\right)\right], \end{aligned} \tag{2.8}$$

$$V_{SP}(r) = -C_SC_P\frac{A_{d_\mathcal{U}}}{4\pi^2}\frac{\Gamma(2d_\mathcal{U})}{2(2d_\mathcal{U}-2)m_1r^{2d_\mathcal{U}}}\vec{\sigma}_1\cdot\hat{r}, \tag{2.9}$$

$$V_{PP}(r) = C_P^2\frac{A_{d_\mathcal{U}}}{4\pi^2}\Gamma(2d_\mathcal{U})\frac{(1+2d_\mathcal{U})(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r})-\vec{\sigma}_1\cdot\vec{\sigma}_2}{4(2d_\mathcal{U}-2)m_1m_2r^{2d_\mathcal{U}+1}}, \tag{2.10}$$

$$\begin{aligned} V_{VV}(r) = &\,C_V^2\frac{A_{d_\mathcal{U}}}{4\pi^2}r^{1-2d_\mathcal{U}}\left[\left(1-\frac{\vec{p}_1\cdot\vec{p}_2}{m_1m_2}\right)\Gamma(2d_\mathcal{U}-2)+\Gamma(2d_\mathcal{U})\left(\frac{1}{8m_1^2r^2}+\frac{1}{8m_2^2r^2}\right)\right.\\ &+\frac{\Gamma(2d_\mathcal{U})}{2d_\mathcal{U}-2}\vec{\sigma}_1\cdot\hat{r}\times\left(\frac{\vec{p}_2}{2m_1m_2r}-\frac{\vec{p}_1}{4m_1^2r}\right)\\ &+\frac{\Gamma(2d_\mathcal{U})}{2d_\mathcal{U}-2}\vec{\sigma}_2\cdot\hat{r}\times\left(\frac{\vec{p}_2}{4m_2^2r}-\frac{\vec{p}_1}{2m_1m_2r}\right)\\ &\left.-\Gamma(2d_\mathcal{U})\frac{(2d_\mathcal{U}+1)(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r})+(1-2d_\mathcal{U})(\vec{\sigma}_1\cdot\vec{\sigma}_2)}{4(2d_\mathcal{U}-2)m_1m_2r^2}\right], \end{aligned} \tag{2.11}$$

$$V_{VA}(r) = C_VC_A\frac{A_{d_\mathcal{U}}}{4\pi^2}r^{1-2d_\mathcal{U}}\left[\Gamma(2d_\mathcal{U}-2)\left(\frac{\vec{\sigma}_1\cdot\vec{p}_1}{m_1}-\frac{\vec{\sigma}_1\cdot\vec{p}_2}{m_2}\right)-\frac{\Gamma(2d_\mathcal{U})}{2d_\mathcal{U}-2}\frac{(\vec{\sigma}_2\times\hat{r})\cdot\vec{\sigma}_1}{2m_2r}\right], \tag{2.12}$$

$$\begin{aligned} V_{AA}(r) = &\,C_A^2\frac{A_{d_\mathcal{U}}}{4\pi^2}r^{1-2d_\mathcal{U}}\left\{\Gamma(2d_\mathcal{U}-2)\left(\frac{\vec{p}_1^2}{2m_1^2}+\frac{\vec{p}_2^2}{2m_2^2}-1\right)\vec{\sigma}_1\cdot\vec{\sigma}_2\right.\\ &+\Gamma(2d_\mathcal{U}-2)\left[\frac{(\vec{\sigma}_1\cdot\vec{p}_1)(\vec{\sigma}_2\cdot\vec{p}_2)}{m_1m_2}-\frac{(\vec{\sigma}_1\cdot\vec{p}_1)(\vec{\sigma}_2\cdot\vec{p}_1)}{2m_1^2}-\frac{(\vec{\sigma}_1\cdot\vec{p}_2)(\vec{\sigma}_2\cdot\vec{p}_2)}{2m_2^2}\right]\\ &-\Gamma(2d_\mathcal{U})\left(\frac{1}{8m_1^2}+\frac{1}{8m_2^2}\right)\frac{(1+2d_\mathcal{U})(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r})-\vec{\sigma}_1\cdot\vec{\sigma}_2}{(2d_\mathcal{U}-2)r^2}\\ &\left.+\frac{\Gamma(2d_\mathcal{U})}{2d_\mathcal{U}-2}\left[\frac{\vec{\sigma}_2\cdot(\vec{p}_1\times\hat{r})}{4m_1^2r}-\frac{\vec{\sigma}_1\cdot(\vec{p}_2\times\hat{r})}{4m_2^2r}\right]\right\}, \end{aligned} \tag{2.13}$$

where the gauge of $\xi = 1$ is adopted [19, 43]. The main derivation of these potentials, with details for the SS type (2.8), is given in appendix A. $\vec{\sigma}_i$, m_i, and $\vec{p}_i$ are the spin, mass, and momentum of the fermion i, respectively. $\hat{r}$ is a unit vector pointing from particle 1 to particle 2, and r is the distance between them. Potentials (2.8), (2.10), (2.11), and (2.13) generated from two same vertexes are invariant under the permutation of particle index.

These potentials can be justified by comparing with previous works [19, 38]. For instance, the dipole-dipole terms in the PP, VV, and AA potentials make up the U^{--}_{spin} term in eq. (5) of ref. [19]. The monopole-monopole terms in the SS and VV potentials are consistent with the U^{--}_{non} term in eq. (5) of ref. [19] and eq. (7) of ref. [38], respectively. When dealing with identical particles, potentials (2.9) and (2.12) should have terms that exchange particle index 1 and 2 added to them. This leads to the following expressions:

$$V_{SP}(r) = -C_S C_P \frac{A_{d_\mathcal{U}}}{4\pi^2} \frac{\Gamma(2d_\mathcal{U})}{2(2d_\mathcal{U}-2)mr^{2d_\mathcal{U}}} (\vec{\sigma}_1 - \vec{\sigma}_2)\cdot\hat{r}, \tag{2.14}$$

$$V_{VA}(r) = C_V C_A \frac{A_{d_\mathcal{U}}}{4\pi^2} r^{1-2d_\mathcal{U}} \left[\Gamma(2d_\mathcal{U}-2)\frac{1}{m}(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot(\vec{p}_1 - \vec{p}_2) - \frac{\Gamma(2d_\mathcal{U})}{2d_\mathcal{U}-2}\frac{(\vec{\sigma}_2\times\hat{r})\cdot\vec{\sigma}_1}{mr}\right]. \tag{2.15}$$

These potentials do not vanish when the wave function is antisymmetric.

The potentials we derived contain terms up to $\mathcal{O}(m^{-3})$ because the mass largely suppresses higher-order effects and is, therefore, irrelevant for observations in low-energy experiments. $\vec{r}$ and $\vec{p}$ should be treated as vectors/operators for macroscopic/microscopic systems, e.g., the term $\vec{\sigma}_1\cdot(\hat{r}\times\vec{p}_1)$ should be replaced by $(\vec{\sigma}_1\times\hat{r})\cdot\vec{p}_1+\vec{p}_1\cdot(\vec{\sigma}_1\times\hat{r})$ for the interaction of SS type at the atomic scale [44]. One can put $\vec{p}_1 = -\vec{p}_2$ to obtain the potentials in the center-of-mass frame. It is important to note that these potentials strongly depend on the continuous scaling dimension $d_\mathcal{U}$. This is due to the exponential factor of r, which dominates the behavior of the potentials at small distances. The significant dependence on $d_\mathcal{U}$ has been highlighted by studying the unparticle contribution to the ground-state energy of the hydrogen atom [38]. In the following discussion, to make better comparisons with previous works, we take a range of $d_\mathcal{U}\in[1,2]$ [2, 19, 36]. In principle, according to the scale invariance and conformal invariance, the lower bound of $d_\mathcal{U}\geq 1$ is imposed by the unitarity condition [3, 45]. Our adopted range was originally introduced by Georgi in one of the seminal works on unparticle physics [2] and commonly used in relevant investigations on unparticle phenomenology later [19, 35–39].

3 Constraints derived from existing experiments

Potentials mediated by unparticles are particularly special as they follow a non-Yukawa $1/r^\alpha$ relationship, where α is a positive real number greater than one and not necessarily an integer, which is a unique characteristic of these potentials. Some other types of non-Yukawa spin-dependent interactions were recently studied in ref. [22]. This could be viewed as a possible origin of the deviation from the gravitational ISL. Previously, such kind of deviation was modeled by including a Yukawa type or $1/r^N (N = 2, 3, \cdots)$ type potential, mediated by a finite-mass particle or N massless particles, into the Newtonian potential [46]. If the violation of ISL is considered due to unparticle effects, constraints on unparticle could be obtained. For instance, treating the nucleon as baryon number current, ref. [36] estimated the limits on the long-range interactions induced by vector unparticle exchange. Similarly, the bounds on such interactions have been obtained from geodetic and frame-dragging measurements by considering the vector unparticle coupling to the electronic (leptonic) and the nucleonic (baryonic) currents [39]. Here, assuming the deviation of the ISL originates from the unparicle-mediated VV interaction between nucleons (N-N), or electrons (e-e), or nucleon and electron (N-e), the data in ref. [46] is used to constrain the VV interaction at $d_{\mathcal{U}} = 1.5$ and 2.0. The details to derive the constraint are given in appendix B.1. Figure 2 shows the corresponding results compared with the constraints derived in ref. [47]. It can be deduced that our derived bounds on $|c_V|$ are about nine orders of magnitude more stringent than that from ref. [47] at $d_{\mathcal{U}} = 1.5$. An intuitive reason for this difference is the different interacting particle pairs, i.e., N-N, e-e, or N-e in this work and quark-electron (q-e) in ref. [47]. In refs. [2, 47], unparticle-mediated interactions are assumed to be flavor-blind. In other studies such as [48, 49], constraints are derived by distinguishing the interacting particle species involved. Hence, one cannot fully rule out the inherent distinction in coupling constants between different particles. In fact, in the field of exploring exotic interactions mediated by axions or axion-like particles (ALPs), the particles involved in the interactions are, in general, specified [50].

On the other hand, the improvement shown in figure 2 could be attributed to the methodology in deriving the bounds. In ref. [47], the bound for the VV interaction is derived by attributing the deviation of the measured scattering amplitude for the llqq (l represents lepton and q represents quark) process from the SM prediction to unparticle effects. Here, we attribute the violation of the ISL to the long-range forces through unparticle exchange. The scattering amplitude is proportional to $1/(\vec{q}^2)^{2-d_{\mathcal{U}}}$ in both cases, where $\vec{q}$ is the momentum transfer. Due to the large $|\vec{q}|$ at high energy in the accelerator, the detection probability can be sequentially lowered. As the process notably favors low $|\vec{q}|$ values, in contrast, the unparticle effects can be more pronounced at a low energy regime. Besides, the estimation of the scattering amplitude in accelerator experiments is performed at the single-particle level. However, when dealing with a macroscopic force mediated by some new particle between two bulk masses — which can comprise a substantially larger number of particles (e.g., $\gtrsim 10^{23}$) — a coherent light boson field can be produced. Thus, the effects of new particle(s) can be substantially enhanced and more detectable. Moody and Wilczek first highlighted this point [20], based upon which the most stringent limits on several kinds of spin-dependent interactions at astronomical distances have been established by utilizing the huge nucleon numbers in the Sun ($\sim 10^{57}$) and Moon ($\sim 10^{49}$) as sources [51]. Another analogous example is the cavity haloscope experiment [52], detecting the conversion of a galactic axion halo that acts as a coherent classical field to the photon. It leads to a limit on the axion-photon coupling about 12 orders of magnitude more stringent than that derived from the constraint on $e^+e^- \to a\gamma$ (a represents the axion) production in the inclusive 2γ search on LEPII (OPAL 2γ) [53].

By comparing eqs. (2.8) and (2.11), one finds that at the leading order, the SS interaction is spin-independent and takes the same form as the VV interaction. Consequently, the constraints on the VV interaction, derived from the results of the torsion balance experiment, are also applicable to the SS interaction. In figure 3, we compare the bounds on the SS interaction with the results from ref. [49], where the SS interaction between neutrino and electron (ν-e) was limited by analyzing the data from neutrino-electron elastic scattering. The significant discrepancy between our findings and those from [49] at $d_{\mathcal{U}} = 1.5$ could potentially be attributed to three factors: 1) the neutrino-electron elastic scattering is a high-energy process with q on the order of MeV, 2) the interacting particle pairs under examination are not the same, and 3) the neutrino-electron elastic scattering is a microscopic process occurring at the individual particle level.

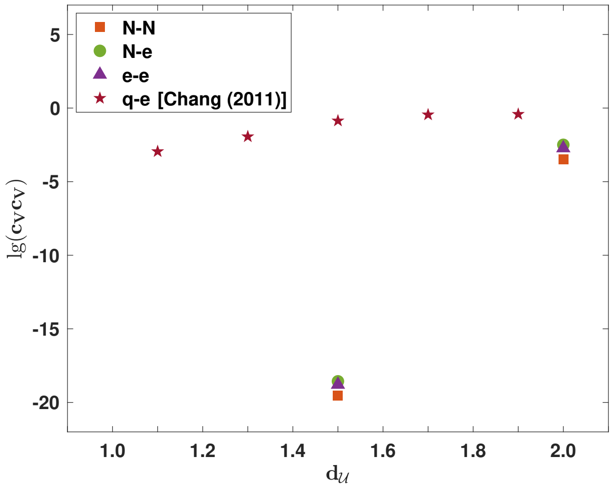

Figure 2. Constraints on the VV interaction between nucleons (N-N), nucleon and electron (N-e), and electrons (e-e) derived from the results of ref. [46], in comparison with those on the VV interaction between quark and electron (q-e) in ref. [47].

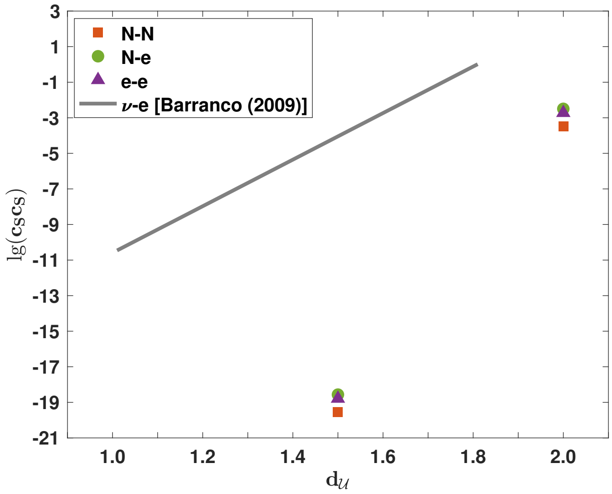

Figure 3. Constraints on the SS interaction between nucleons (N-N), nucleon and electron (N-e), and electrons (e-e) derived from the results of ref. [46], in comparison with those on the SS interaction between neutrino and electron (ν-e) in ref. [49].

We now explore the limits of unparticle-mediated PP and AA interactions. We note that several experiments have been performed to search for the exotic spin-dependent PP [54, 55] and AA [51, 55, 56] interactions mediated by axions or ALPs by employing state-of-the-art atomic magnetometers. These interactions comprise a common factor of e^{-mr}/r, where m is the mass of the mediator. When m is sufficiently small, the factor can be simplified to $1/r$ at the laboratory distance scale. Then, the simplified interactions share the same form as the unparticle-mediated ones at $d_{\mathcal{U}} = 1.0$. As a result, we can determine the bounds on PP and AA interactions at $d_{\mathcal{U}} = 1.0$ by utilizing the established limits on axions [51, 54–56]. The details for deriving these limits are also given in appendix B.1.

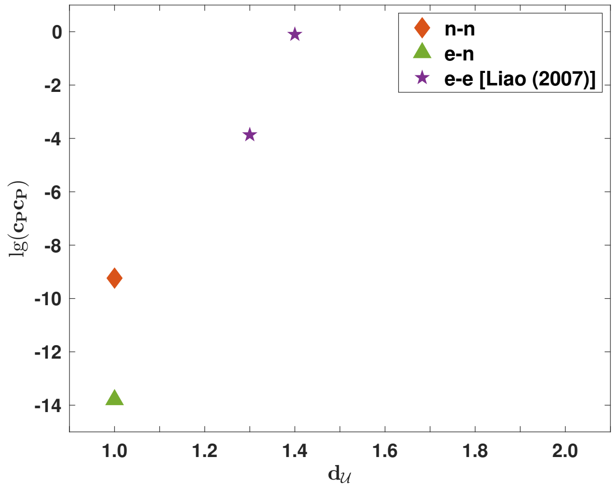

Figure 4. Constraints on the n-n and e-n PP interactions derived from the results of refs. [54, 55], in comparison with those on the e-e interaction in ref. [19].

In figures 4 and 5, the obtained limits on PP and AA interactions are exhibited, respectively, and compared with the results in ref. [19], where the limits on interactions between electrons (e-e) have been set by applying the previously determined bounds on the anomalous spin-spin interaction. While the results of ref. [19] only concern e-e interactions, our results involve various particle pairs, i.e., neutron-neutron (n-n) and electron-neutron (e-n). For the PP interaction, we cannot make a fair comparison with ref. [19], as not only the interacting particle pairs are different, but also the bounds are set at different $d_{\mathcal{U}}$ values.

Besides the limits on the e-n and e-e AA interactions at $d_{\mathcal{U}} = 1.0$, we further derive the bounds with continuous $d_{\mathcal{U}}$ values for the interactions between nucleon and neutron (N-n) or electron (N-e) by using the results of ref. [51] and the raw data of a previous experiment [24]. The details for deriving the bounds with continuous $d_{\mathcal{U}}$ are provided in appendix C. The results for the N-n and N-e AA interactions are also shown in figure 5 by solid and dashed lines, respectively. From figure 5, it is found that for the e-e interaction, the bound at $d_{\mathcal{U}} = 1.0$ derived in this work is comparable to that at $d_{\mathcal{U}} = 1.5$ in ref. [19]. Although the bounds on the N-n and N-e AA interactions derived in this work are less stringent than those on the e-e AA interaction in ref. [19], they enable us to scrutinize the $d_{\mathcal{U}}$ dependence of the constraints, which will be discussed at the end of this section.

The amount of CP violation permitted by the SM has been recognized to be inadequate for explaining the observed asymmetry between matter and antimatter in the Universe. Therefore, searching for novel sources of CP and T violations has become a crucial frontier in modern physics research [57]. Previously, the effects of CP violation from unparticle physics were only explored in collider physics [58, 59]. In this work, the unparticle-mediated SP and VA interactions, V_{SP} and V_{VA}, can offer an additional approach to probe the effects of discrete symmetry violation in precision measurement experiments.

The SP potential, originating from couplings of different vertex types, violates P and T symmetries. A particularly relevant manifestation of the P, T violation effect is the nonzero electric dipole moments (EDMs) of atoms, which could be induced by V_{SP} mixing opposite-parity eigenstates. Based on the precise measurement of the EDM of ^{199}Hg

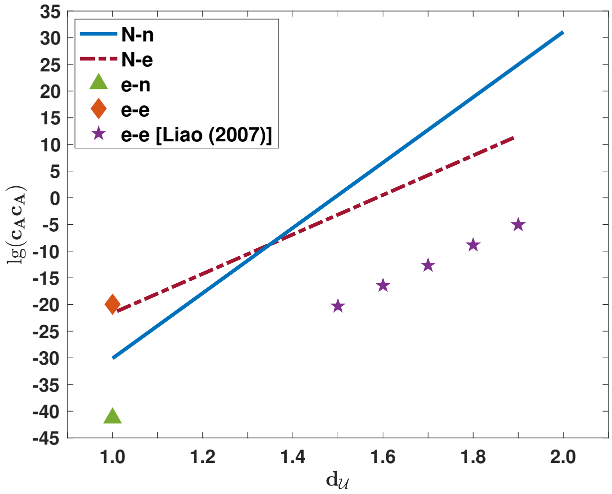

Figure 5. Constraints on the e-e, e-n, N-n, and N-e AA interactions derived from the results of refs. [24, 51, 55, 56], in comparison with those on the e-e interaction in ref. [19].

atom [60], the constraint on coupling constants of the SP interaction between electron and nucleon (e-N), $c_S^e c_P^N$, at $d_\mathcal{U} = 1.0$ can be evaluated. Due to the spin-dependence of the SP interaction, the results of ref. [61], where the exotic spin-dependent interaction is explored, can also be employed to set constraints on the coupling constant of the N-e interaction, $c_S^N c_P^e$. Furthermore, the bounds for the N-n SP interactions with continuous $d_\mathcal{U}$ values can be derived from the results of ref. [51]. Figure 6 shows the constraints on $c_S^e c_P^N$ and $c_S^N c_P^e$ at $d_\mathcal{U} = 1.0$ and those on $c_S^N c_P^n$ as a function of $d_\mathcal{U}$. Since the unparticle-mediated SP interaction is for the first time proposed in this work, no previous limit was available for our comparison. The limits set here will provide valuable guidance for future experimental studies on not only the unparticle phenomenology but also the violation of P and T symmetries.

Finally, we utilize the results of ref. [51] and the raw data of the magnetometer-based experiment [24] to constrain the VA interactions of N-n ($c_V^N c_A^n$) and N-e ($c_V^N c_A^e$) types. The corresponding results are shown in figure 7. A clear linear relationship between the limits and $d_\mathcal{U}$ can be found from not only figure 7 but also the bounds on N-e AA interaction in figure 5 and those on the N-n AA and SP interactions in figures 5 and 6. Note that the results are displayed in logarithmic coordinates, indicating that the bounds exhibit an exponential decay trend as $d_\mathcal{U}$ increases.

4 Conclusion

In summary, six types of coordinate-space potential between fermions from unparticle exchange are derived under the nonrelativistic limit, which could be probed in precision measurement experiments. Previous studies have focused on parity-even interactions, i.e., SS, PP, VV, and AA. We propose new interactions of the SP and VA types, which do not conserve parity. It is important to note that parity conservation is not always satisfied in modern physics. The properties of these potentials are discussed, and constraints on their coupling constants are estimated by utilizing existing experimental data.

By examining the unparticle effects in violating the gravitational ISL, we constrain the coupling constants of the SS and VV type interactions and compare our results with

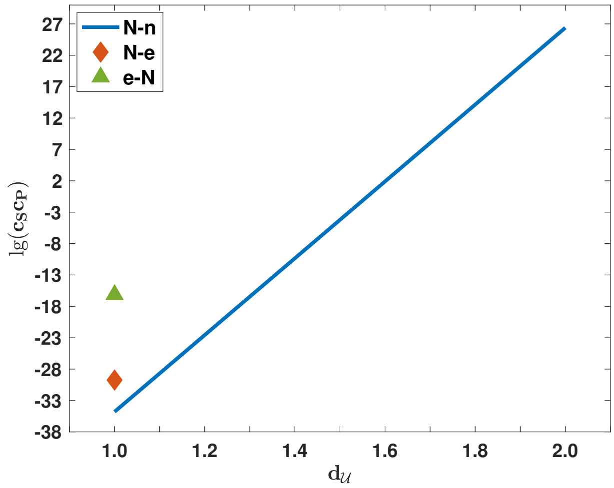

Figure 6. Constraints on the N-e and e-N SP interactions ($c_S^N c_P^e$ and $c_S^e c_P^N$) derived respectively from the results of refs. [60, 61] and those on the N-n SP interaction ($c_S^N c_P^n$) derived from the results of ref. [51].

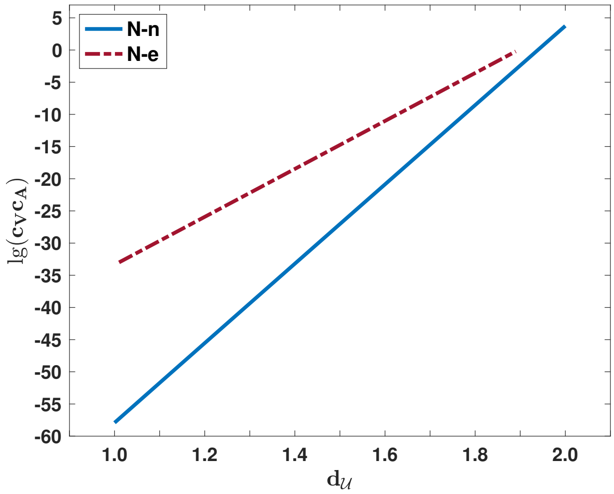

Figure 7. Constraints on the N-n and N-e VA interactions ($c_V^N c_A^n$ and $c_V^N c_A^e$) derived from the results of refs. [24, 51].

the bounds derived in previous works. The significant discrepancy between our results and the previous ones is attributed to two aspects: 1) the different interacting particles under consideration and 2) the advantage of the precision measurement in detecting the macroscopic forces mediated by unparticles or other new particles beyond the SM. By utilizing the previously established limits on exotic spin-dependent interactions mediated by axions or ALPs, we also obtain limits on the PP, AA, SP, and VA interactions. A bound on the SP interaction is also derived by using the measured upper limit for the EDM of the ^{199}Hg atom. The limits on the newly proposed SP and VA interactions through unparticle exchange will provide a valuable reference for future experimental studies on not only the unparticle phenomenology but also the effects of discrete symmetry violation. The dependence of the

JHEP01(2024)083

limits on the scaling dimension $d_{\mathcal{U}}$ is explored for SP, VA, and AA interactions. It turns out that the bounds exhibit an exponential decay trend with the increasing $d_{\mathcal{U}}$.

Acknowledgments

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JHEP01(2024)083

A Derivation of potentials in coordinate space

First, some useful identities are listed as follows.

$$\begin{aligned}(\vec{\sigma}\cdot\vec{A})(\vec{\sigma}\cdot\vec{B}) &= \vec{A}\cdot\vec{B}+i\vec{\sigma}\cdot(\vec{A}\times\vec{B})\\ (\vec{\sigma}_1\times\vec{A})\cdot(\vec{\sigma}_2\times\vec{B}) &= (\vec{A}\cdot\vec{B})(\vec{\sigma}_1\cdot\vec{\sigma}_2)-(\vec{\sigma}_2\cdot\vec{A})(\vec{\sigma}_1\cdot\vec{B})\\ (\vec{\sigma}\cdot\vec{A})\sigma_i(\vec{\sigma}\cdot\vec{B}) &= (\vec{\sigma}\cdot\vec{A})\vec{B}_i+i(\vec{B}\times\vec{A})_i-(\vec{A}\cdot\vec{B})\vec{\sigma}_i+(\vec{\sigma}\cdot\vec{B})\vec{A}_i \end{aligned} \tag{A.1}$$

$$\partial_i\partial_j r^{1-2d} = (1-2d)(-1-2d)r^{-3-2d}x_ix_j+(1-2d)r^{-1-2d}\delta_{ij} \tag{A.2}$$

$$\nabla r^{1-2d} = (1-2d)r^{-2d}\hat{r} \tag{A.3}$$

$$\int\frac{d\vec{q}^3}{(2\pi)^3}\frac{1}{(\vec{q}^2)^k}e^{i\vec{q}\cdot\vec{r}} = \frac{1}{2\pi^2}r^{2k-3}\sin(k\pi)\Gamma(2-2k) \tag{A.4}$$

$$\int\frac{d\vec{q}^3}{(2\pi)^3}\frac{\vec{q}}{(\vec{q}^2)^k}e^{i\vec{q}\cdot\vec{r}} = (-i\nabla)\frac{1}{2\pi^2}r^{2k-3}\sin(k\pi)\Gamma(2-2k) \tag{A.5}$$

In the subsequent sections, we will adopt the natural units $\hbar=c=1$, metric tensor $(+,-,-,-)$, and γ_μ matrices in the Dirac representation. The Dirac spinor (2.5) is defined as

$$u(p) = N\begin{pmatrix}\chi_s\\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_s\end{pmatrix}. \tag{A.6}$$

Here, χ_s represents the spin wave function where $s=0$ or 1. $N=\sqrt{E+m}$ is the normalization factor, and E, $\vec{p}$, and m represent the particle's energy, momentum, and rest mass, respectively. Taking an example from the scalar-type spinor product, we obtain

$$\begin{aligned}\bar{u}(p_{1f})u(p_{1i}) &= u(p_{1f})^\dagger\begin{pmatrix}1&0\\0&-1\end{pmatrix}u(p_{1i})\\ &= N_1^2\left(1-\frac{(\vec{\sigma}_1\cdot\vec{p}_{1f})(\vec{\sigma}_1\cdot\vec{p}_{1i})}{(E_1+m_1)^2}\right)\\ &= N_1^2\left(1-\frac{\vec{p}_{1f}\cdot\vec{p}_{1i}+i\vec{\sigma}_1\cdot(\vec{p}_{1f}\times\vec{p}_{1i})}{(E_1+m_1)^2}\right)\\ &= N_1^2\left(1-\frac{\vec{p}_{1i}^2+\vec{p}_{1i}\cdot\vec{q}+i\vec{\sigma}_1\cdot(\vec{q}\times\vec{p}_{1i})}{(E_1+m_1)^2}\right)\\ &= N_1^2\left(1-\frac{\vec{p}_1^2-\vec{q}^2/4+i\vec{\sigma}_1\cdot(\vec{q}\times\vec{p}_1)}{(E_1+m_1)^2}\right). \end{aligned} \tag{A.7}$$

In this equation, $\vec{p}_{1i}$ ($\vec{p}_{1f}$) is the three-momentum of the initial (final) state of particle 1. For an elastic scattering process, we have $E_{1i} = E_{1f} = E_1$. As defined in eq. (2.3), $\vec{p}_1 = (\vec{p}_{1i} + \vec{p}_{1f})/2$ and $\vec{q} = \vec{p}_{1f} - \vec{p}_{1i}$ represent the average and transferred three-momenta respectively. Other spinor products can be computed similarly as outlined below.

$$\bar{u}(p_{1f})\gamma_5 u(p_{1i}) = N_1^2 \frac{\vec{\sigma}_1\cdot(\vec{p}_{1i}-\vec{p}_{1f})}{E_1+m_1} = -N_1^2 \frac{\vec{\sigma}_1\cdot\vec{q}}{E_1+m_1} \tag{A.8}$$

$$\bar{u}(p_{1f})\gamma_0 u(p_{1i}) = N_1^2 \left(1+\frac{\vec{p}_1^2-\vec{q}^2/4+i\vec{\sigma}_1\cdot(\vec{q}\times\vec{p}_1)}{(E_1+m_1)^2}\right) \tag{A.9}$$

$$\bar{u}(p_{1f})\gamma_i u(p_{1i}) = N_1^2 \left(\frac{2(\vec{p}_1)_i}{E_1+m_1}+\frac{i(\vec{\sigma}_1\times\vec{q})_i}{E_1+m_1}\right) \tag{A.10}$$

$$\bar{u}(p_{1f})\gamma_0\gamma_5 u(p_{1i}) = N_1^2 \frac{2\vec{\sigma}_1\cdot\vec{p}_1}{E_1+m_1} \tag{A.11}$$

$$\bar{u}(p_{1f})\gamma_i\gamma_5 u(p_{1i}) = N_1^2 \left((\vec{\sigma}_1)_i+\frac{2(\vec{\sigma}_1\cdot\vec{p}_1)(\vec{p}_1)_i-(\vec{\sigma}_1\cdot\vec{q})(\vec{q}_1)_i/2+i(\vec{p}_1\times\vec{q})_i-(\vec{p}_1^2-\vec{q}^2/4)(\vec{\sigma}_1)_i}{(E_1+m_1)^2}\right) \tag{A.12}$$

The nonrelativistic scattering amplitude for the interaction of the SS-type can be written as:

$$\begin{aligned}\mathcal{A}_{SS} &= \frac{1}{4E_1E_2}\mathcal{M}_{SS}\\ &= \frac{N_1^2N_2^2}{4E_1E_2} iC_S^2 \frac{A_{d_\mathcal{U}}}{2\sin(\pi d_\mathcal{U})}\frac{i}{(-q^2-i\epsilon)^{2-d_\mathcal{U}}}\left(1-\frac{\vec{p}_1^2-\vec{q}^2/4+i\vec{\sigma}_1\cdot(\vec{q}\times\vec{p}_1)}{(E_1+m_1)^2}\right)\left(1-\frac{\vec{p}_2^2-\vec{q}^2/4-i\vec{\sigma}_2\cdot(\vec{q}\times\vec{p}_2)}{(E_2+m_2)^2}\right).\end{aligned} \tag{A.13}$$

By expanding the energy term as $E = \sqrt{m^2+\vec{p}^2} = m + \frac{\vec{p}^2}{2m} + \mathcal{O}(m^{-2})$, we derive:

$$\mathcal{A}_{SS} = -\frac{A_{d_\mathcal{U}}}{2\sin(\pi d_\mathcal{U})}C_S^2\frac{1}{(\vec{q}^2)^{2-d_\mathcal{U}}}\left(1-\frac{2\vec{p}_1^2+i\vec{\sigma}_1\cdot(\vec{q}\times\vec{p}_1)}{4m_1^2}-\frac{2\vec{p}_2^2-i\vec{\sigma}_2\cdot(\vec{q}\times\vec{p}_2)}{4m_2^2}\right). \tag{A.14}$$

The nonrelativistic amplitudes for other types of interactions are provided below.

$$\mathcal{A}_{SP} = i\frac{A_{d_\mathcal{U}}}{2\sin(\pi d_\mathcal{U})}C_S C_P\frac{1}{(\vec{q}^2)^{2-d_\mathcal{U}}}\frac{\vec{\sigma}_1\cdot\vec{q}}{2m_1}, \tag{A.15}$$

$$\mathcal{A}_{PP} = -\frac{A_{d_\mathcal{U}}}{2\sin(\pi d_\mathcal{U})}C_P^2\frac{1}{(\vec{q}^2)^{2-d_\mathcal{U}}}\frac{(\vec{\sigma}_1\cdot\vec{q})(\vec{\sigma}_2\cdot\vec{q})}{4m_1m_2}, \tag{A.16}$$

$$\begin{aligned}\mathcal{A}_{VV} = \frac{A_{d_\mathcal{U}}}{2\sin(\pi d_\mathcal{U})}C_V^2\frac{1}{(\vec{q}^2)^{2-d_\mathcal{U}}}\Bigg(&1+\frac{-\vec{q}^2/2+i\vec{\sigma}_1\cdot(\vec{q}\times\vec{p}_1)}{4m_1^2}+\frac{-\vec{q}^2/2-i\vec{\sigma}_2\cdot(\vec{q}\times\vec{p}_2)}{4m_2^2}\\ &-\frac{4\vec{p}_1\cdot\vec{p}_2+(\vec{\sigma}_1\times\vec{q})(\vec{\sigma}_2\times\vec{q})+i(2\vec{p}_2\cdot(\vec{\sigma}_1\times\vec{q})-2\vec{p}_1\cdot(\vec{\sigma}_2\times\vec{q}))}{4m_1m_2}\Bigg),\end{aligned} \tag{A.17}$$

$$\mathcal{A}_{VA} = \frac{A_{d_\mathcal{U}}}{2\sin(\pi d_\mathcal{U})}C_V C_A\frac{1}{(\vec{q}^2)^{2-d_\mathcal{U}}}\left(\frac{\vec{\sigma}_1\cdot\vec{p}_1}{m_1}-\frac{\vec{\sigma}_1\cdot\vec{p}_2}{m_2}+\frac{i(\vec{\sigma}_2\times\vec{q})\cdot\vec{\sigma}_1}{2m_2}\right), \tag{A.18}$$

$$\mathcal{A}_{AA} = \frac{A_{d_\mathcal{U}}}{2\sin(\pi d_\mathcal{U})} C_V^2 \frac{1}{(\vec{q}^2)^{2-d_\mathcal{U}}} \{-\vec{\sigma}_1 \cdot \vec{\sigma}_2 + \frac{(\vec{\sigma}_1 \cdot \vec{p}_1)(\vec{\sigma}_2 \cdot \vec{p}_2)}{m_1 m_2}$$
$$- \frac{2(\vec{\sigma}_1 \cdot \vec{p}_1)(\vec{\sigma}_2 \cdot \vec{p}_1) - (\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q})/2 + i\vec{\sigma}_2 \cdot (\vec{p}_1 \times \vec{q}) - 2\vec{p}_1^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2}{4m_1^2}$$
$$- \frac{2(\vec{\sigma}_1 \cdot \vec{p}_2)(\vec{\sigma}_2 \cdot \vec{p}_2) - (\vec{\sigma}_2 \cdot \vec{q})(\vec{\sigma}_1 \cdot \vec{q})/2 - i\vec{\sigma}_1 \cdot (\vec{p}_2 \times \vec{q}) - 2\vec{p}_2^2 \vec{\sigma}_1 \cdot \vec{\sigma}_2}{4m_2^2}\}. \tag{A.19}$$

By applying the Fourier transformation (2.7) to the amplitude (A.14), we arrive at:

$$\begin{aligned}
V_{SS}(\vec{r}) &= -\int \frac{d^3\vec{q}}{(2\pi)^3} \mathcal{A}_{SS}(\vec{q},\vec{p}) e^{i\vec{q}\cdot\vec{r}} \\
&= \frac{A_{d_\mathcal{U}}}{2\sin(\pi d_\mathcal{U})(2\pi)^3} C_S^2 \left\{ \int \frac{1}{(\vec{q}^2)^{2-d_\mathcal{U}}} \left(1 - \frac{\vec{p}_1^2}{2m_1^2} - \frac{\vec{p}_2^2}{2m_2^2}\right) e^{i\vec{q}\cdot\vec{r}} d^3\vec{q} \right. \\
&\quad \left. - \frac{i\vec{\sigma}_1\cdot(\vec{q}\times\vec{p}_1)}{4m_1^2} e^{i\vec{q}\cdot\vec{r}} d^3\vec{q} + \frac{i\vec{\sigma}_2\cdot(\vec{q}\times\vec{p}_2)}{4m_2^2} e^{i\vec{q}\cdot\vec{r}} d^3\vec{q} \right\} \\
&= -\frac{A_{d_\mathcal{U}}}{4\pi^2} C_S^2 \Gamma(2d_\mathcal{U}-2) r^{1-2d_\mathcal{U}} \left\{ \left(1 - \frac{\vec{p}_1^2}{2m_1^2} - \frac{\vec{p}_2^2}{2m_2^2}\right) - \frac{\vec{\sigma}_1\cdot(\nabla\times\vec{p}_1)}{4m_1^2} + \frac{\vec{\sigma}_2\cdot(\nabla\times\vec{p}_2)}{4m_2^2} \right\} \\
&= -\frac{A_{d_\mathcal{U}}}{4\pi^2} C_S^2 \Gamma(2d_\mathcal{U}-2) r^{1-2d_\mathcal{U}} \left\{ \left(1 - \frac{\vec{p}_1^2}{2m_1^2} - \frac{\vec{p}_2^2}{2m_2^2}\right) - (1-2d_\mathcal{U}) \left(\frac{\vec{\sigma}_1\cdot(\hat{r}\times\vec{p}_1)}{4m_1^2 r} - \frac{\vec{\sigma}_2\cdot(\hat{r}\times\vec{p}_2)}{4m_2^2 r} \right) \right\} \\
&= -\frac{A_{d_\mathcal{U}}}{4\pi^2} C_S^2 r^{1-2d_\mathcal{U}} \left\{ \Gamma(2d_\mathcal{U}-2) \left(1 - \frac{\vec{p}_1^2}{2m_1^2} - \frac{\vec{p}_2^2}{2m_2^2}\right) + \frac{\Gamma(2d_\mathcal{U})}{(2d_\mathcal{U}-2)} \left(\frac{\vec{\sigma}_1\cdot(\hat{r}\times\vec{p}_1)}{4m_1^2 r} - \frac{\vec{\sigma}_2\cdot(\hat{r}\times\vec{p}_2)}{4m_2^2 r} \right) \right\}.
\end{aligned} \tag{A.20}$$

This was done using eqs. (A.3)–(A.5). The result is exactly the potential (2.8) with an explicit inclusion of the common factor and the coupling strength. By following a similar process, one can derive the coordinate-space potentials (2.9)–(2.13).

B Deriving constraints at specific $d_\mathcal{U}$ values

B.1 Constraints from the violation of the ISL

In the torsion-balance experiment, the power-law potential,

$$V_{ab}^k(r) = G\frac{M_1 M_2}{r}\beta_k \left(\frac{1\ \text{mm}}{r}\right)^{k-1}, \tag{B.1}$$

was considered as a possible form of the breakdown of the ISL, and some constraints on it were obtained [46]. The total power-law interaction between the molybdenum pendulum and the tantalum attractors is given by

$$U_{ab}^k(r) = \int\int \rho_1 \rho_2 V_{ab}^k(r) dV_1 dV_2, \tag{B.2}$$

where $\rho_1 = 0.01028\ \text{g/mm}^3$ and $\rho_2 = 0.01669\ \text{g/mm}^3$ are the respective mass densities. If the VV interaction is also considered to lead to the violation of the ISL, the corresponding potential reads

$$U_{VV_0}(r) = \int\int \rho_{N1}\rho_{N2} V_{VV_0}(r) dV_1 dV_2, \tag{B.3}$$

where $\rho_{N_1} = 6.45 \times 10^{19}$ mm^{-3} and $\rho_{N_2} = 5.55 \times 10^{19}$ mm^{-3} are the nucleon number densities of molybdenum and tantalum, respectively. Here, $V_{VV_0}(r)$ is the first term of eq. (2.11), which can be expressed as

$$V_{VV_0}(r) = c_V^2 \Lambda^{2-2d_{\mathcal{U}}} \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}} 4\pi^2} \frac{\Gamma(d_{\mathcal{U}}+1/2)\Gamma(2d_{\mathcal{U}}-2)}{\Gamma(d_{\mathcal{U}}-1)\Gamma(2d_{\mathcal{U}})} (\hbar c)^{2d_{\mathcal{U}}-1} (1\text{ mm})^{1-2d_{\mathcal{U}}} \left(\frac{1\text{ mm}}{r}\right)^{2d_{\mathcal{U}}-1}, \tag{B.4}$$

in the international system (SI) of units. By comparing eqs. (B.2) and (B.3), we can obtain the constraint

$$c_V = [G\rho_1\rho_2\beta_k/(\Lambda^{2-2d_{\mathcal{U}}} \frac{16\pi^{5/2}}{(2\pi)^{2d_{\mathcal{U}}} 4\pi^2} \frac{\Gamma(d_{\mathcal{U}}+1/2)\Gamma(2d_{\mathcal{U}}-2)}{\Gamma(d_{\mathcal{U}}-1)\Gamma(2d_{\mathcal{U}})} (\hbar c)^{2d_{\mathcal{U}}-1} (1\text{ mm})^{2-2d_{\mathcal{U}}} \rho_{N_1}\rho_{N_2}]^{1/2}, \tag{B.5}$$

when $k = 2d_{\mathcal{U}} - 1$. If we consider the interaction involving electrons, the nucleon number density should be replaced with the electron number density by multiplying ρ_N with Z/A, where Z (A) is the proton (mass) number of the isotope. For the molybdenum pendulum and tantalum attractors considered here, $Z/A = 0.438$ and 0.403, respectively. The obtained bounds for the VV interaction also apply to the SS interaction, as they share the same form at the leading-order term.

B.2 Constraints derived from the upper limit for the EDM of ^{199}Hg

The most stringent constraint on the EDM of the ^{199}Hg atom [60] is used to constrain the axion mediated P, T violating interaction [62]. The interaction is given by:

$$V(r) = -\frac{g_e^s g_N^p}{8\pi m_N} \vec{\sigma} \cdot \vec{\nabla} \left(\frac{e^{-m_a r}}{r}\right) \gamma^0. \tag{B.6}$$

When considering the limit of a small axion mass, the interaction (B.6) simplifies to:

$$V(r) \overset{m_a \to 0}{\longrightarrow} \frac{g_e^s g_N^p}{8\pi m_N} \vec{\sigma} \cdot \hat{r} \frac{1}{r^2} \gamma^0, \tag{B.7}$$

where m_N and $\vec{\sigma}$ represent the mass and spin of the nucleon, respectively, and the Dirac matrix γ^0 is applied to the electron. The SP potential (2.9) at $d_{\mathcal{U}} = 1.0$ reads:

$$V_{SP}(r) \overset{d_{\mathcal{U}} \to 1}{\longrightarrow} -\frac{c_S^e c_P^N}{8\pi m_N} \vec{\sigma} \cdot \hat{r} \frac{1}{r^2} \gamma^0. \tag{B.8}$$

This potential is similar to that of the potential (B.7), with an appended γ^0 for the relativistic electron. By comparing eqs. (B.7) and (B.8), one can derive the constraint on $|c_S^e c_P^N|$.

B.3 Constraints derived from existing limits on exotic spin-dependent interactions

The exotic spin-dependent pseudoscalar-pseudoscalar (PP) [54, 55], axial-axial (AA) [51, 55, 56], and scalar-pseudoscalar (SP) [60, 61] interactions mediated by axions or ALPs can be used to establish limits on the corresponding interactions mediated by unparticles. For

JHEP01(2024)083

example, the axion-mediated AA interaction, when the mediator mass m is very small, thereby making λ very large [51], is given by:

$$U_{AA}(r) = \frac{g_A^N g_A^n}{16\pi m}\left(\frac{1}{\lambda r}+\frac{1}{r^2}\right)e^{-r/\lambda}\vec{\sigma}\cdot(\vec{v}\times\hat{r}) \xrightarrow{\lambda\to\infty} \frac{g_A^N g_A^n}{16\pi m}\frac{1}{r^2}\vec{\sigma}\cdot(\vec{v}\times\hat{r}). \tag{B.9}$$

This shares the same form as the unparticle-mediated AA interaction at $d_{\mathcal{U}} = 1.0$:

$$V_{AA}(r) = -c_A^N c_A^n \Lambda^{2-2d_{\mathcal{U}}}\frac{A_{d_{\mathcal{U}}}}{4\pi^2}\frac{\Gamma(2d_{\mathcal{U}})}{2d_{\mathcal{U}}-2}\frac{\vec{\sigma}\cdot(\vec{v}\times\hat{r})}{4mr}r^{1-2d_{\mathcal{U}}} \xrightarrow{d_{\mathcal{U}}\to 1} -\frac{c_A^N c_A^n}{16\pi m}\frac{\vec{\sigma}\cdot(\vec{v}\times\hat{r})}{r^2}. \tag{B.10}$$

The difference between these two equations lies in the coefficients. Consequently, one can deduce the limits on $c_A^N c_A^n$ at $d_{\mathcal{U}} = 1.0$ using the existing limit on $g_A^N g_A^n$ when $\lambda \to \infty$. Similarly, constraints on the PP and SP interactions can be obtained.

C Establishing constraints at continuous $d_{\mathcal{U}}$ values

The exotic spin-dependent vector-axial (VA) and axial-axial (AA) interactions mediated by axions or ALPs function as an effective magnetic field on the spin. In ref. [24], an effective magnetic field that is periodically varied and induced by two rotating BGO cylinders at a fixed frequency has been detected by four magnetometers. The upper limit on this effective magnetic field has been used to derive the corresponding constraint on these new interactions. The effective magnetic field generated due to unparticle exchange between the nucleons of BGO and the spin-polarized electrons of the magnetometer can be expressed as:

$$\begin{aligned}
\vec{\mathrm{B}}_{VA}(\vec{r}) &= \frac{2(\hbar c)^{2d_{\mathcal{U}}-2}}{\gamma_e}c_V^N c_A^e \Lambda^{2-2d_{\mathcal{U}}}\frac{A_{d_{\mathcal{U}}}}{4\pi^2}\Gamma(2d_{\mathcal{U}}-2)\int\frac{1}{|\vec{r}-\vec{r'}|^{2d_{\mathcal{U}}-1}}\vec{v}(\vec{r'})d\vec{r'}^3, \\
\vec{\mathrm{B}}_{AA}(\vec{r}) &= \frac{2(\hbar c)^{2d_{\mathcal{U}}-1}}{c^2\gamma_e m_N}c_A^N c_A^e \Lambda^{2-2d_{\mathcal{U}}}\frac{A_{d_{\mathcal{U}}}}{4\pi^2}\frac{\Gamma(2d_{\mathcal{U}})}{2d_{\mathcal{U}}-2}\int\frac{1}{|\vec{r}-\vec{r'}|^{2d_{\mathcal{U}}}}\vec{v}(\vec{r'})\times\frac{\vec{r}-\vec{r'}}{|\vec{r}-\vec{r'}|}d\vec{r'}^3,
\end{aligned} \tag{C.1}$$

where $\vec{r'}$ ($\vec{v}(\vec{r'})$) denotes the position (velocity) of the volume element of the BGO cylinder, $\vec{r}$ is the position of the magnetometer, m_N is the mass of the nucleon, and γ_e is the gyromagnetic ratio of the electron. The integral can be computed through Monte Carlo integration [63] for various values of $d_{\mathcal{U}}$. Due to the rotation, the effective magnetic field in eq. (C.1) can be decomposed into a Fourier series:

$$\begin{aligned}
B_{VAy}(t) &= c_V^N c_A^e \sum_{n=0}^{\infty} a_n \cos(2\pi n f_0 t), \\
B_{AAx}(t) &= c_A^N c_A^e \sum_{n=0}^{\infty} b_n \cos(2\pi n f_0 t),
\end{aligned} \tag{C.2}$$

where $f_0 = 20$ Hz is twice the rotation frequency, a result of the symmetrical placement of the BGO cylinders, and the coefficients a_n and b_n can be calculated through numerical integration. In this scenario, the y component of $\vec{B}_{VA}$ and the x component of $\vec{B}_{AA}$ are the maximum ones sensed by the magnetometers. The signal acquired by the magnetometers can also be expressed as:

$$\begin{aligned}
S(t) &= \sum_{n=0}^{\infty} c_n \cos(2\pi n f_0 t) + N(t) \\
&= \sum_{n=0}^{\infty}(c_n + \delta c_n)\cos(2\pi n f_0 t) + \delta s_n \sin(2\pi n f_0 t),
\end{aligned} \tag{C.3}$$

where $N(t)$ is the background noise, c_n is the Fourier component of the actual pseudo-magnetic field, and δc_n and δs_n are the Fourier components of $N(t)$. By combining eq. (C.2) and eq. (C.3), we can estimate the value of the coupling constant as follows:

$$
\begin{aligned}
c_V^N c_A^e &= \frac{\sum_{n=1}^{4}(c_n+\delta c_n)a_n}{\sum_{n=1}^{4} a_n^2},\\
c_A^N c_A^e &= \frac{\sum_{n=1}^{4}(c_n+\delta c_n)b_n}{\sum_{n=1}^{4} b_n^2},
\end{aligned} \tag{C.4}
$$

where $n=0$ is not included to avoid the $1/f$ noise, and components with $n>4$ are negligible due to their very small weight [63]. After performing M experiments, the average and variance of the coupling constants can be calculated as:

$$
\begin{aligned}
\langle c_V^N c_A^e\rangle \pm \sigma_{\langle c_V^N c_A^e\rangle} &= \frac{\sum_{n=1}^{4}\langle c_n\rangle a_n}{\sum_{n=1}^{4} a_n^2} \pm \frac{1}{\sqrt{\sum_{n=1}^{4} a_n^2}}\sqrt{\frac{S(nf_0)}{2MT}},\\
\langle c_A^N c_A^e\rangle \pm \sigma_{\langle c_A^N c_A^e\rangle} &= \frac{\sum_{n=1}^{4}\langle c_n\rangle b_n}{\sum_{n=1}^{4} b_n^2} \pm \frac{1}{\sqrt{\sum_{n=1}^{4} b_n^2}}\sqrt{\frac{S(nf_0)}{2MT}},
\end{aligned} \tag{C.5}
$$

where T represents the measurement time of a single experiment, and $S(nf_0)$ is the noise power spectrum density at frequency nf_0. To minimize the variance of the estimated coupling constants, it is critical to reduce $S(nf_0)$ as much as possible and to extend the measurement period. The constraints on VA and AA interactions at continuous $d_{\mathcal{U}}$ values are derived in this manner using raw data from the experiment detailed in refs. [24, 63].

If the exotic spin-dependent interactions exist, the Sun, acting as a giant unpolarized mass source, can generate an effective magnetic field that influences polarized spins on Earth. This field is hypothesized to be measurable using a ^{3}He-^{129}Xe comagnetometer in a laboratory setting [51]. The rotation of the Earth causes a sidereal variation in the component of this field that is perpendicular to the Earth's axis of rotation. In the laboratory frame, this sidereal varied component takes on the following form:

$$
\begin{aligned}
B_{SP\perp} &= \frac{2(\hbar c)^{2d_{\mathcal{U}}}N_\odot}{\hbar\gamma_n} c_S^N c_P^n \Lambda^{2-2d_{\mathcal{U}}}\frac{A_{d_{\mathcal{U}}}}{4\pi^2}\frac{\Gamma(2d_{\mathcal{U}})}{2(2d_{\mathcal{U}}-2)m_n c^2}\frac{1}{R^{2d_{\mathcal{U}}}}[\sin(\omega_\oplus t)\hat{x}+\cos(\omega_\oplus t)\hat{y}],\\
B_{VA\perp} &= \frac{2(\hbar c)^{2d_{\mathcal{U}}-1}N_\odot}{\hbar\gamma_n c} c_V^N c_A^n \Lambda^{2-2d_{\mathcal{U}}}\frac{A_{d_{\mathcal{U}}}}{4\pi^2}\Gamma(2d_{\mathcal{U}}-2)\frac{1}{R^{2d_{\mathcal{U}}-1}}v_\oplus\cos\eta[-\cos(\omega_\oplus t)\hat{x}+\sin(\omega_\oplus t)\hat{y}],\\
B_{AA\perp} &= \frac{2(\hbar c)^{2d_{\mathcal{U}}}N_\odot}{\hbar\gamma_n c} c_A^N c_A^n \Lambda^{2-2d_{\mathcal{U}}}\frac{A_{d_{\mathcal{U}}}}{4\pi^2}\frac{\Gamma(2d_{\mathcal{U}})}{4(2d_{\mathcal{U}}-2)m_n c^2}\frac{1}{R^{2d_{\mathcal{U}}}}v_\oplus\sin\eta[\cos(\omega_\oplus t)\hat{x}-\sin(\omega_\oplus t)\hat{y}],
\end{aligned} \tag{C.6}
$$

where γ_n is the gyromagnetic ratio of the neutron, m_n is the mass of the neutron, $N_\odot$ is the number of nucleons in the Sun, R is the distance between the Sun and Earth, η is the inclination of the Earth's equatorial plane, $v_\oplus$ is the orbital velocity of the Earth, and $\omega_\oplus \approx 10^{-5}$ s^{-1} is the Earth's self-rotation frequency. The upper limit estimate on the amplitude of the sidereal variation could be obtained by fitting the comagnetometer experimental data with eq. (C.6). The result at a 95% confidence level is given by [51]:

$$
|B_\perp| < 0.023 \text{ fT}. \tag{C.7}
$$

For various values of $d_{\mathcal{U}}$, by substituting the known values $N_\odot \approx 1.2\times10^{57}$, $R = 1.5\times10^{11}$ m, $\eta = 23.4^\circ$, and $v_\oplus \approx 3.0\times10^4$ m/s into eq. (C.6), we can derive the constraints on $c_S^N c_P^n$, $c_V^N c_A^n$, and $c_A^N c_A^n$.

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