Experiment 1: Full Model Set and R Specification

Model	Formula	Rstan Code

m0	$Y_i \sim Beta(p_i, \theta)$
	$logit(p_i) = \alpha$
	$\alpha \sim Normal(0, 1)$
	$\theta \sim HalfCauchy(1)$
		`# Intercept Model`
		`m0 <- map2stan(`
		`  alist(`
		`    y ~ dbeta2(p,theta),`
		`    logit(p) <- a,`
		`    a ~ dnorm(0, 1),`
		`    theta ~ dcauchy(0,1)`
		`  ),`
		`  data = dlist,`
		`  constraints=list(theta="lower=0"),`
		`  start=list(theta = 1),`
		`  sample=TRUE , warmup=1000 , iter=1e4 ,`
		`  cores=2 , chains=1 )`

m1	$Y_i \sim Beta(p_i, \theta)$
	$logit(p_i) = \alpha_{\text{FISH}[i]}$
	$\alpha_{\text{FISH}[i]} \sim Normal(0, 1)$
	$\theta \sim HalfCauchy(0, 1)$
		`# Fixed effect model: fish`
		`m1 <- map2stan(`
		`  alist(`
		`    y ~ dbeta2(p,theta),`
		`    logit(p) <- a_fish[fish_id],`
		`    a_fish[fish_id] ~ dnorm(0,1),`
		`    theta ~ dcauchy(0,1)`
		`  ),`
		`  data=dlist,`
		`  constraints=list(theta="lower=0"),`
		`  start=list(theta=1), warmup=1000 , iter=1e4, cores=2)`

m2

$$Y_i \sim Beta(p_i, \theta)$$
$$logit(p_i) = \alpha_{\text{FISH}[i]}$$
$$\alpha_{\text{FISH}[i]} \sim Normal(\alpha, \sigma_{\text{FISH}})$$
$$\alpha \sim Normal(0, 1)$$
$$\sigma_{\text{FISH}} \sim HalfCauchy(0, 1)$$
$$\theta \sim HalfCauchy(0, 1)$$

```
# Varying intercepts: fish
m2 <- map2stan(
  alist(
    y ~ dbeta2(p,theta),
    logit(p) <- a_fish[fish_id],
    a_fish[fish_id] ~ dnorm(a, sigma_fish),
    a ~ dnorm(0, 1),
    sigma_fish ~ dcauchy(0,1),
    theta ~ dcauchy(0,1)
  ),
  data=dlist,
  constraints=list(theta="lower=0"),
  start=list(theta=1), warmup=1000 , iter=1e4 , cores=2 )
```

m3

$$Y_i \sim Beta(p_i, \theta)$$
$$logit(p_i) = \alpha + \alpha_{\text{FISH}[i]} + \beta_{\text{TREATMENT}_{[}i]}$$
$$\alpha_{\text{FISH}[i]} \sim Normal(0, \sigma_{\text{FISH}})$$
$$\alpha \sim Normal(0, 1)$$
$$\sigma_{\text{FISH}} \sim HalfCauchy(0, 1)$$
$$\beta_{\text{TREATMENT}} \sim Normal(0, 1)$$
$$\theta \sim HalfCauchy(0, 1)$$

```
# varying intercepts for fish by (fixed) treatment effect
m3 <- map2stan(
  alist(
    y ~ dbeta2( p, theta ),
    logit(p) <- a + a_fish[fish_id] + b_treatment*treatment ,
    a_fish[fish_id] ~ dnorm(0, sigma_fish),
    a ~ dnorm(0, 1),
    b_treatment ~ dnorm(0, 1),
    theta ~ dcauchy(0,1),
    sigma_fish ~ dcauchy(0,1)
  ),
  data = dlist,
  constraints=list(theta="lower=0"),
  start=list(theta=1), warmup=1000 , iter=1e4 , cores=2 )
```

m4

$$Y_i \sim Beta(p_i, \theta)$$
$$logit(p_i) = \alpha + \alpha_{\text{FISH}[i]} + \beta_{\text{Above}} + \beta_{\text{Below}} + \beta_{\text{Light}} + \beta_{\text{Dark}}$$
$$\alpha_{\text{FISH}[i]} \sim Normal(0, \sigma_{\text{FISH}})$$
$$\beta_{\text{Above, Below, Light, Dark}} \sim Normal(0, 1)$$
$$\sigma_{\text{FISH}} \sim HalfCauchy(0, 1)$$
$$\theta \sim HalfCauchy(0, 1)$$

```
# Varying Intercepts: fish and treatment
m4 <- map2stan(
  alist(
    y ~ dbeta2(p,theta),
    logit(p) <- a_fish[fish_id] + b_above*above + b_below*below
    + b_light*light + b_dark*dark,
    a_fish[fish_id] ~ dnorm(0, sigma_fish),
    c(b_above, b_below, b_light, b_dark) ~ dnorm(0, 1),
    sigma_fish ~ dcauchy(0,1),
    theta ~ dcauchy(0,1)
  ),
  data = dlist,
  constraints=list(theta="lower=0"),
  start=list(theta = 1),
  sample=TRUE , warmup=1000 , iter=1e4 ,
  cores=2, chains = 1 )
```

m4.5

$$Y_i \sim Beta(p_i, \theta)$$
$$logit(p_i) = \alpha + \alpha_{\text{FISH}[i]} + \alpha_{\text{TREATMENT}[i]}$$
$$\alpha_{\text{FISH}[i]} \sim Normal(0, \sigma_{\text{FISH}})$$
$$\alpha_{\text{TREATMENT}[i]} \sim Normal(0, \sigma_{\text{TREATMENT}})$$
$$\alpha \sim Normal(0, 1)$$
$$\sigma_{\text{FISH}} \sim HalfCauchy(0, 1)$$
$$\sigma_{\text{TREATMENT}} \sim HalfCauchy(0, 1)$$
$$\theta \sim HalfCauchy(0, 1)$$

```
# Varying Intercepts: fish and treatment (treatment as a mean)
m4.5 <- map2stan(
  alist(
    #likelihood
    y ~ dbeta2( p, theta ),
    # linear model
    logit(p) <- a + a_fish[fish_id] + a_treat[treatment],
    # adaptive priors
    a_fish[fish_id] ~ dnorm(0,sigma_fish),
    a_treat[treatment] ~ dnorm(0, sigma_treat),
    # fixed priors
    a ~ dnorm(0, 1),
    theta ~ dcauchy(0,1),
    sigma_fish ~ dcauchy(0,1),
    sigma_treat ~ dcauchy(0,1)
  ),
  data = dlist,
  constraints=list(theta="lower=0"),
  start=list(theta=1), warmup=1000 , iter=1e4 , cores=2 )
```

m5

$$Y_i \sim Beta(p_i, \theta)$$
$$logit(p_i) = \alpha + \alpha_{\text{FISH}[i]} + \beta_{\text{VELOCITY}_i}$$
$$\alpha_{\text{FISH}[i]} \sim Normal(0, \sigma_{\text{FISH}})$$
$$\alpha \sim Normal(0, 1)$$
$$\beta_{\text{VELOCITY}} \sim Normal(0, 1)$$
$$\sigma_{\text{FISH}} \sim HalfCauchy(0, 1)$$
$$\theta \sim HalfCauchy(0, 1)$$

```
# Varying Intercepts by fish with fixed velocity effect
m5 <- map2stan(
  alist(
    y ~ dbeta2( p, theta ),
    logit(p) <- a + a_fish[fish_id] + b_velocity*vel,
    a_fish[fish_id] ~ dnorm(0, sigma_fish),
    a ~ dnorm(0, 1),
    b_velocity ~ dnorm(0,1),
    theta ~ dcauchy(0,1),
    sigma_fish ~ dcauchy(0,1)
  ),
  data = dlist,
  constraints=list(theta="lower=0"),
  start=list(theta=1), warmup=1000 , iter=1e4 , cores=2 )
```

m6

$$Y_i \sim Beta(p_i, \theta)$$
$$logit(p_i) = \alpha + \alpha_{\text{FISH}[i]} + \alpha_{\text{VELOCITY}_{[i]}}$$
$$\alpha_{\text{FISH}[i]} \sim Normal(0, \sigma_{\text{FISH}})$$
$$\alpha_{\text{VELOCITY}[i]} \sim Normal(0, \sigma_{\text{VELOCITY}})$$
$$\alpha \sim Normal(0, 1)$$
$$\sigma_{\text{FISH}} \sim HalfCauchy(0, 1)$$
$$\sigma_{\text{VELOCITY}} \sim HalfCauchy(0, 1)$$
$$\theta \sim HalfCauchy(0, 1)$$

```
# Varying Intercepts: fish and velocity
m6 <- map2stan(
  alist(
    #likelihood
    y ~ dbeta2( p, theta ),
    # linear model
    logit(p) <- a + a_fish[fish_id] + a_velocity[vel],
    # adaptive priors
    a_fish[fish_id] ~ dnorm(0,sigma_fish),
    a_velocity[vel] ~ dnorm(0, sigma_vel),
    # fixed priors
    a ~ dnorm(0, 1),
    theta ~ dcauchy(0,1),
    sigma_fish ~ dcauchy(0,1),
    sigma_vel ~ dcauchy(0,1)
  ),
  data = dlist,
  constraints=list(theta="lower=0"),
  start=list(theta=1), warmup=1000 , iter=1e4 , cores=2 )
```

m7

$$Y_i \sim Beta(p_i, \theta)$$
$$logit(p_i) = \alpha + \alpha_{\text{FISH}[i]} + \beta_{\text{VELOCITY}_{[i]}} + \beta_{\text{FISHXVELOCITY}}$$
$$\alpha_{\text{FISH}[i]} \sim Normal(0, \sigma_{\text{FISH}})$$
$$\alpha \sim Normal(0,1)$$
$$\beta_{\text{VELOCITY}} \sim Normal(0,1)$$
$$\beta_{\text{FISHXVELOCITY}} \sim Normal(0,1)$$
$$\sigma_{\text{FISH}} \sim HalfCauchy(0,1)$$
$$\theta \sim HalfCauchy(0,1)$$

```
# varying intercepts (fish) +
# fixed effect interaction (fish X velocity)
m7 <- map2stan(
  alist(
    y ~ dbeta2( p, theta ),
    logit(p) <- a + a_fish[fish_id] +  b_velocity*vel +
    bfXv*fish_id*vel,
    a_fish[fish_id] ~ dnorm(0, sigma_fish),
    a ~ dnorm(0, 1),
    b_velocity ~ dnorm(0, 1),
    bfXv ~ dnorm(0,1),
    theta ~ dcauchy(0,1),
    sigma_fish ~ dcauchy(0,1)
  ),
  data=dlist,
  constraints=list(theta="lower=0"),
  start=list(theta=1) , warmup=1000 , iter=1e4 , cores=2)
```

m1NC

$$Y_i \sim Beta(p_i, \theta)$$
$$logit(p_i) = \alpha + \alpha_{\text{FISH}[i]} + \beta_{\text{FISH}[i]} Velocity_i$$
$$\alpha \sim Normal(0,10)$$
$$\begin{bmatrix} \alpha_{\text{FISH}} \\ \beta_{\text{FISH}} \end{bmatrix} \sim MVNormal\left(\begin{bmatrix} \alpha \\ \beta \end{bmatrix}, S \right)$$
$$\mathbf{S} = \begin{pmatrix} \sigma_\alpha & 0 \\ 0 & \sigma_\beta \end{pmatrix} R \begin{pmatrix} \sigma_\alpha & 0 \\ 0 & \sigma_\beta \end{pmatrix}$$
$$\alpha \sim Normal(0,1)$$
$$\beta \sim Normal(0,1)$$
$$\sigma_{\text{FISH}} \sim Exp(1)$$
$$\theta \sim HalfCauchy(0,1)$$
$$\rho_{FISH} \sim LKJcorr(2)$$

```
# Mixed effects model with varying intercepts on fish,
# varying slopes on fish/velocity
m1NC <- map2stan(
  alist(
    #likelihood
    y ~ dbeta2( p, theta ),
    # linear model
    logit(p) <- a + a_fish[fish_id] + (b_velocity + b_fish)*vel,
    # adaptive non-centered priors
    c(a_fish, b_fish)[fish_id] ~ dmvnormNC(sigma_fish, Rho_fish),
    # fixed priors
    c(a, b_velocity) ~ dnorm(0,1),
    theta ~ dcauchy(0,1),
    sigma_fish ~ dexp(1),
    Rho_fish ~ dlkjcorr(2)
  ),
  # data
  data=dlist,
  constraints=list(theta="lower=0"),
  start=list(theta=1) , warmup=1000 , iter=1e4 ,cores=2 )
```

m2NC

$$Y_i \sim Beta(p_i, \theta)$$

$$logit(p_i) = \beta_{1\text{FISH}}[i]Above_i + \beta_{2\text{FISH}_{[}}i]Below_i + \beta_{3\text{FISH}_{[}}i]Light_i + \beta_{4\text{FISH}_{[}}i]Dark_i$$

$$\begin{bmatrix} \beta_{1\text{FISH}} \\ \beta_{2\text{FISH}} \\ \beta_{3\text{FISH}} \\ \beta_{4\text{FISH}} \end{bmatrix} \sim MVNormal\left(\begin{bmatrix} \alpha \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix}, S \right)$$

$$\beta_1 \sim Normal(0, 1)$$

$$\beta_2 \sim Normal(0, 1)$$

$$\beta_3 \sim Normal(0, 1)$$

$$\beta_4 \sim Normal(0, 1)$$

$$\sigma_{\text{FISH}} \sim Exp(1)$$

$$\theta \sim HalfCauchy(0, 1)$$

$$\rho_{FISH} \sim LKJcorr(2)$$

```
m2NC <- map2stan(
  alist(
    #likelihood
    y ~ dbeta2( p, theta ),

    # linear model
    logit(p) <- (b_above + ba_fish[fish_id])*above +
                (b_below + bb_fish[fish_id])*below +
                (b_light + bl_fish[fish_id])*light +
                (b_dark + bd_fish[fish_id])*dark,

    # adaptive NON-CENTERED priors
    c(ba_fish, bb_fish, bl_fish, bd_fish)[fish_id] ~
       dmvnormNC(sigma_fish, Rho_fish),
    # fixed priors
    c(b_above, b_below, b_light, b_dark) ~ dnorm(0,1),
    theta ~ dcauchy(0,1),
    sigma_fish ~ dexp(1),
    Rho_fish ~ dlkjcorr(2)
  ),
  # data
  data=dlist,
  constraints=list(theta="lower=0"),
  start=list(theta=1), warmup=1000 , iter=1e4 , cores=2 )
```