4.2 Integer Representation and Algorithms

(31.) decimal of $a = (a_{n-1}a_{n-2} \ldots a_1a_0)_{10}$, thus
$a = 10^{n-1}a_{n-1} + 10^{n-2}a_{n-2} + \ldots + 10a_1 + a_0$

$10 \equiv 1 \pmod 3$ Since being divisible by 3 is the same as being congruent to $0 \pmod 3$ we proved that a positive int is divisible by 3 iff sum of its decimal digits is divisible by 3.

(33.) $a = (a_{n-1}a_{n-2} \ldots a_1a_0)_2$ Thus $a = a_0 + 2a_1 + 2^2a_2 + \ldots + 2^{n-1}a_{n-1}$.

$2^2 \equiv 1 \pmod 3$ $\quad 2^k \equiv 1 \pmod 3$ when k is even
$2^k \equiv 2 \equiv -1 \pmod 3$ when k is odd

Therefore $a \equiv a_0 - a_1 + a_2 - a_3 + \ldots \pm a_{n-1} \pmod 3$

Thus $a \equiv 0 \pmod 3$ if the sum of the binary digits in the even-numbered positions minus the sum of the binary digits in the odd-numbered positions is congruent to 0 modulo 3. Divisible by 3 is same as being congruent to $0 \pmod 3$, proof is complete

(37.) Assume n bits being used so that -2^{n-1} and 2^{n-1} can be represented.

$-5 + 3$
$1010 + 0011$
$1101 = -2$

$-4 + -3$
$1011 + 1100$
$10111 \rightarrow 1000 = -7$

(43.) To obtain two's complement representation of the sum of two integers given in two's complement representation, add them as if they were binary integers, and ignore any carry out of the left-most column. ~~However~~

However, if the left-most digits of the two addends agree and the left-most digit of the answer is different from their common value, then an overflow has occured, and the answer is not valid.

55. procedure compare(a, b: nonegative integers)

$i := n-1$

while $i>0$ and $a_i = b_i$

$i := i-1$

if $a_i > b_i$ then answer := "$a>b$"

else if $a_i < b_i$ then answer := "$a<b$"

else answer := "$a=b$"

return answer