Lists $\mathbf{F} = (F_1, \ldots, F_n)$ of orthogonality functions are examples of *"orthogonality fans"* (cf. Definition 3.8), and there is a notion for when a chain $\sigma = [y_0 < \cdots < y_r]$ is orthogonal to a fan $\mathbf{F}$, written $\sigma \perp \mathbf{F}$ (cf. Definition 3.9). Using discrete Morse theory (cf. [For98], [Fre09]), we prove:

Theorem **3.14** *(Complementary collapse).* Let $\mathbf{F} = (F_1, \ldots, F_n)$ *be an orthogonality fan on* P *with* $F_1([\hat{0}]) \neq \hat{0}, \hat{1}$. *There is a* G*-equivariant simple homotopy equivalence*

$$|\overline{\mathcal{P}}| \xrightarrow{\simeq} \bigvee_{[y_0 < \cdots < y_r] \perp \mathbf{F}} |\overline{\mathcal{P}}_{(\hat{0}, y_0)}|^{\diamond} \wedge \Sigma |\overline{\mathcal{P}}_{(y_0, y_1)}|^{\diamond} \wedge \cdots \wedge \Sigma |\overline{\mathcal{P}}_{(y_{r-1}, y_r)}|^{\diamond} \wedge |\overline{\mathcal{P}}_{(y_r, \hat{1})}|^{\diamond}.$$